Code Design for Broadband Indoor Wireless Communications Using Commutation Signaling

by

Hong Ren

Department of Electrical Engineering

McGill University, Montreal

Quebec, Canada

Oct. 1996

A thesis submitted to the Faculty of Graduate Studies and Research

in partial fulfillment of the requirements for the degree of Master of Engineering

0-612-29626-1

Abstract

Commutation signaling is an anti-multipath modulation technique that employs bandwidth spreading to overcome multipath induced intersymbol interference. In this work, a wireless communication system using differential binary phase shift keyed commutation signaling has been considered for a two-wave Rayleigh fading channel. Commutation signaling code parameters that characterize the system performance (average and worst case) are revealed and codebooks have been constructed based on these parameters. For binary codebooks constructed based on average performance, the maximum probability of error may be very large at certain inter-wave delays. On the other hand, binary and four phase codebooks constructed based on the worst case performance not only ensure an acceptable maximum probability of error, but also give very good average performance. Normally, systems using codebooks that require more bandwidth expansion have better anti-multipath capability. It is also found that with the same bandwidth expansion, the anti-multipath capability is increased by using four phase codebooks instead of binary codebooks. For a performance degradation of about 3dB with respect to ideal two-fold diversity, four phase codebooks of length 4 can be used for inter-wave delays of up to nearly three times the symbol duration, while codebooks of length 8 can cope with delays as large as nearly sixteen times the symbol duration.

Résumé

La signalisation par commutation est une méthode de modulation qui réduit l'interférence due aux chemins multiples dans un canal, grâce à une extension du spectre. Dans ce mémoire, un système de communications sans fil qui utilise la signalisation par commutation avec DBPSK est analysé, sur un canal de transmission Rayleigh à deux chemins. Les paramètres des codes de commutations qui caractérisent la performance du système sont mis à jour (cas moyen et pire cas), et des familles de codes sont construites en se basant sur ces paramètres. Pour les familles de codes binaires construites à partir d'un critère de performance moyenne, la probabilité d'erreur maximale peut-être très élevée pour certaines valeurs du delai inter-ondes. Cependant, les familles de codes binaires et quaternaires (quatre phases) construites pour minimiser la probabilité d'erreur maximale donnent une très bonne performance moyenne. Normalement, plus le code étend le spectre, et plus les effets de chemins multiples sont réduits. Pour une même extension du spectre, il ressort que les codes quaternaires réduisent les effets de chemins multiples dans le canal mieux que les codes binaires. Contre une dégradation de 3dB environ par rapport à une diversité idéale d'ordre 2, des familles de codes quaternaires de longueur 4 peuvent être utilisées pour des délais inter-ondes de presque 3 fois la durée d'un symbole. De même, des familles de codes de longueur 8 peuvent supporter des délais aussi longs que seize fois la durée d'un symbole.

Acknowledgement

I would like to thank my supervisor, Prof. Harry Leib, for giving me the opportunity to pursue my graduate studies under his guidance and introducing me to the field of telecommunications. His understanding and continuous support throughout the course of the research is greatly appreciated.

I want to thank the Canadian Institute for Telecommunication Research (CITR) for the financial support. The computer facilities in the Telecommunication and Signal Processing (TSP) Lab is a great help to this work.

Thanks are extended to all my friends in the communication group, especially Jean-Paul Chaib, who translated the abstract. It has been my pleasure and privilege to be associated with them.

Finally, I am extremely grateful to my wife, Huaxian. Her love and support made the completion of this thesis possible. My special thanks are due to my son, John, who brings me so much joy.

Contents

Abstract i

Résumé ii

Acknowledgement iii

List of Figures vi

List of Tables viii

1 Introduction 1

2 System Analysis 6

2.1 System Model 6

2.2 Performance 12

3 Code Construction 23

3.1 Code Design Rules 23

3.1.1 Average Signal-to-Noise Ratio 24

3.1.2 Worst Case Performance 32

3.2 Code Construction 35

3.2.1 Code Construction Based on Average Performance 35

3.2.2 Code Construction Based on Worst Case Performance 44

4 Performance of Selected Codes **47**

5 Conclusions **66**

Bibliography **70**

Appendix A **75**

Appendix B **81**

List of Figures

2.1 The block diagram of a system using DPSK commutation signaling. . 7

2.2 The DPSK commutation signaling modulator. 8

2.3 The DPSK commutation signaling demodulator. 9

4.1 Probability of error as a function of the normalized delay for a binary codebook of length 7 and size 8. 49

4.2 Probability of error as a function of the normalized delay for a binary codebook of length 5 and size 3. 50

4.3 Probability of error as a function of the normalized delay for a binary codebook of length 6 and size 5. 51

4.4 Probability of error as a function of the normalized delay for a binary codebook of length 7 and size 8. 52

4.5 Probability of error as a function of the normalized delay for a binary codebook of length 8 and size 12. 53

4.6 Probability of error as a function of the normalized delay for a binary codebook of length 9 and size 16. 54

4.7 Probability of error as a function of the normalized delay for a four phase codebook of length 4 and size 3. 55

4.8 Probability of error as a function of the normalized delay for a four phase codebook of length 5 and size 5. 56

4.9 Probability of error as a function of the normalized delay for a four phase codebook of length 6 and size 8. 57
4.10 Probability of error as a function of the normalized delay for a four phase codebook of length 7 and size 12. 58
4.11 Probability of error as a function of the normalized delay for a four phase codebook of length 8 and size 16. 59
4.12 Cellular configuration with reuse factor of three. 65

List of Tables

4.1 The average multipath interference parameter I_{av} and the normalized maximum multipath interference parameter I_{max} for different binary codebooks. 63

4.2 The average multipath interference parameter I_{av} and the normalized maximum multipath interference parameter I_{max} for different four phase codebooks. 64

Chapter 1

Introduction

Due to the increasing demand for personal communication services in recent years, broadband indoor wireless communications has received extensive attentions[1, 2]. One of the advantages of wireless systems is the mobility that it allows, i.e., users can move freely within the coverage area of the system. Another advantage is the tetherless access to communication services. However, a wireless indoor radio system has to operate over multipath fading channels[3]. The multipath propagation is mainly due to the radio wave reflections by walls, floor, ceiling, objects and people. The fading phenomenon is primarily a result of the time variations in the phases of radio waves from different paths. These components may add constructively or destructively depending on their phases, leading to amplitude variations in the received signal. One characteristic of a multipath fading channel is the multipath delay spread, which is the time delay between the first and last significant multipath components. Over a multipath fading channel, the transmission rate is limited by the intersymbol interference (ISI) due to the multipath propagation. When the information rate R [bit/sec] is larger than the reciprocal of the multipath delay spread Δ, i.e., when R $\Delta > 1$, severe ISI can occur because the channel's response to previous symbol may still be "ringing" during the current symbol's reception. Consider the 20–60 GHz band for broadband indoor wireless communications. Advantages of using this millimeter-wave band compared to the conventional UHF bands are the availability of high information transport capability and the possibility of frequency re-use between neighboring rooms due to the severe attenuation of most inner walls at this millimeter-wave band. Moreover, the band around 60 GHz is especially advantageous with respect to frequency re-use because of the specific attenuation due to atmospheric oxygen of about 15dB/km. The wideband propagation characteristics at or near 60 GHz has been measured, and the values of the root-mean-square delay spread were found to be in the range of 13–98ns[4, 5]. Suppose the transmission rate R is 155.52 Mb/sec, which corresponding to the SONET OC-3 bit rate. 1/R is about 6ns, one order of magnitude smaller than the worst-case delay spread of 98ns. Even if a four carrier scheme is considered, each carrier has to support a transmission rate of about 40 Mb/sec, and $1/R \sim 25$ns is still significantly smaller than the worst-case delay spread, leading to severe ISI.

To overcome the ISI, several techniques can be used:

1) Adaptive equalization–The idea of equalization is to compensate for the channel distortion by employing a filter with adjustable parameters. These parameters are adjusted to reduce an error criterion. For multipath fading channels, adaptive equalization, where the filter parameters are updated on a periodic basis, is necessary since the channel characteristics is time dependent. Adaptive equalization techniques require no extra bandwidth and have been investigated extensively[6, 7]. However, equalization usually leads to complex receiver implementation. Furthermore, for broadband indoor wireless communications, equalization is computational intensive, and thus demands large power consumption. This may limit its use for portable applications.

2) Anti-multipath modulations–Some anti-multipath modulation schemes have

been proposed in which a redundant phase or amplitude transition is imposed on a conventional modulation technique. These schemes include double phase shift keying (PSK)[8], Manchester-coded PSK[9], return-to-zero PSK[10], and PSK with varied phase[11, 12]. However, these techniques can only cope with multipath delay spreads of less than one symbol duration. With commutation signaling technique introduced in [13], it is possible to cope with much larger multipath delay spreads at the expense of bandwidth. In fact, the amount of multipath delay spread that commutation signaling can cope with depends only on the allowable bandwidth spreading. The concept of commutation signaling is to switch among a number of different signaling alphabets in such a way that any signal that is still "ringing" in the multipath channel will not be used until it fades away. Assume that we have N distinct M-ary signal sets and the signals from different sets have good cross-correlation properties such that the interference between them is very small. When N signal sets are used cyclically and N $> R\Delta/log_2 M$, two transmitted signals from the same set are separated in time by NT = N$(log_2 M)/R$. Since NT $> \Delta$, there is no interference between them. We know that the interference between signals from different sets is very small. Therefore, multipath induced ISI is significantly reduced and it is possible to cope with a multipath delay spread of up to N symbol duration. This is the concept of MxN commutation signaling. The total number of the signals required is MN. If we take N = $\lceil R\Delta/log_2 M \rceil$, where $\lceil x \rceil$ denotes the smallest integer not less than x, then we have

$$MN = M\lceil R\Delta/log_2 M \rceil \tag{1.1}$$

It can be seen that the number of the required signals increases only linearly with the multipath delay spread. Moreover, the signal-to-noise ratio (SNR) performance is determined by each of the signal set. Therefore, we have a separation between the anti-multipath capability which is determined by the number of the signal sets, and the SNR efficiency which is determined by the signal constellation in each signal set. It has been reported[14] that with a receiver that employs matched filtering with equal weight square-law RAKE combining, the commutation signaling technique not only reduce the effects of multipath induced ISI, but also makes it possible to use simple RAKE receiver to exploit the inherent time diversity of multipath.

It is seen that if bandwidth spreading is impossible, adaptive equalization has to be employed to overcome ISI. Since the 20–60 GHz millimeter-wave band offers more bandwidth than the lower frequency bands that are used (or considered to be used) for indoor wireless systems, bandwidth expansion is possible and commutation signaling technique seems to be an attractive candidate for combating multipath induced intersymbol interference. In this work, we will examine a communication system over a two-wave Rayleigh fading channel, using differential binary phase-shift-keyed (DBPSK) commutation signaling. The multiwave Rayleigh model has been widely employed to model multipath fading channels[15, 16]. It appears that the two-wave Rayleigh fading model preserves the essential nature of multipath fading channels, while still being amenable to analysis[16, 17]. Our emphasis is on the construction of commutation signaling codebooks. Commutation signaling sequences (or codewords) are bandwidth spreading sequences similar to the signature sequences used in direct sequence spread spectrum multiple access(DS/SSMA) communication systems, except that the length of commutation signaling sequences has to be quite short due to bandwidth limitation. Bandwidth spreading sequences with good cross-correlation properties have been studied extensively[18–24], but the emphasis is on signature sequences longer than 50. The research on short bandwidth spreading sequences is very limited.

The signature sequence construction methods in [19] can be used for short bandwidth spreading sequences, and it is shown that the average performance of those short sequences is quite good: better than that of the random sequences. However, concerning a systematic approach on construction of short sequences (also called codes) with good worst case performance, no publication has been found. In this work, analysis has been performed to reveal which code parameters characterize the worst case performance and code construction methods have been developed based on the worst case performance criterion. It is shown that codes constructed using the methods not only ensure the worst case performance, but also give average performance that is often better than that of the codes constructed based on the average performance criterion in Ref. [19].

The thesis is organized as following: In chapter 2, the system model is described first, then the error probability performance of the system is considered and the method of computing the probability of error is presented. Because the computation of the probability of error is complicated, it is very difficult to see its dependency on the cross-correlation properties of the commutation signaling codes. Therefore, the probability of error is not an appropriate performance measure for the construction of the commutation signaling codes. As an alternative approach, the signal-to-noise ratio will be considered as the performance measure in chapter 3, and the system using DBPSK commutation signaling is analized to reveal which commutation signaling code parameters characterize the system performance. Based on this analysis, short commutation signaling codes with length less than or equal to 9 are constructed. In chapter 4, the probability of error as a function of inter-wave delay will be presented for different codebooks. The anti-multipath capability and bandwidth spreading for different codebooks will be compared. Finally, conclusions are drawn in chapter 5. The good codebooks we have constructed will be listed in appendix A, and the software used to construct the commutation signaling codebooks are described and presented in appendix B.

Chapter 2

System Analysis

2.1 System Model

The block diagram of the communication system that we will consider is shown in Figure 2.1. The binary source data is differentially encoded and then multiplied by a commutation signaling codeword. This waveform is then converted to passband (see fig. 2.2). The commutation signaling codewords are used in sequence. The channel is modeled as a two-wave Rayleigh fading channel. The commutation signaling demodulator (see fig. 2.3) includes a bank of filters matched to the commutation signals, and followed by square-law devices. The differential decoding operation can also be implemented by a delay and multiply operation, and the outputs of this operation are combined by the RAKE processor with activated taps corresponding to the channel multipath delays.

Consider a polyphase commutation signaling codeword $s^{(i)}$ with length L, the corresponding signal is

$$s_i(t) = \sum_{l=0}^{L-1} s_l^{(i)} p_{T_c}(t - lT_c) \tag{2.1}$$

where T_c denotes the chip duration and $s_l^{(i)}$ is the l-th element of the polyphase

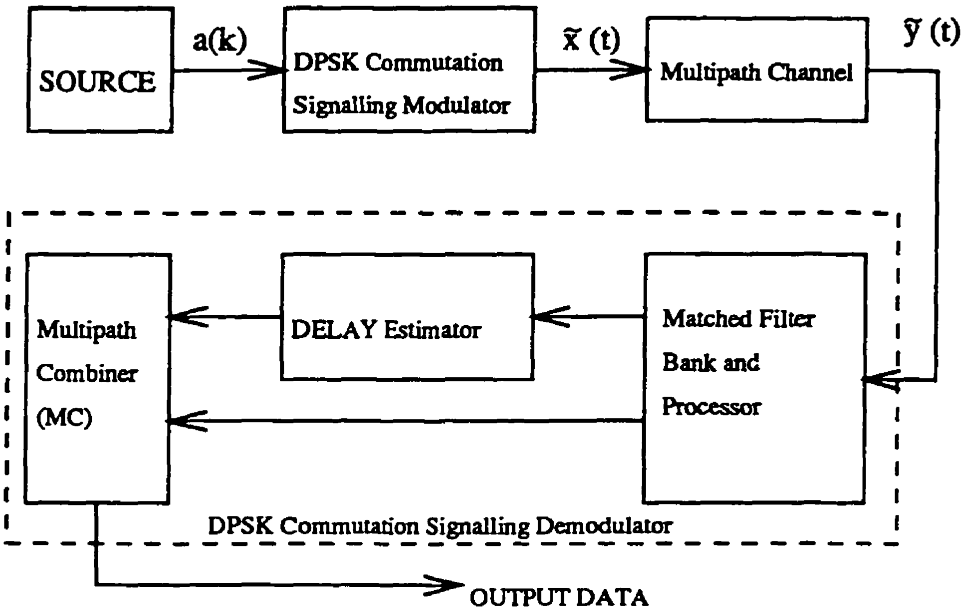

Figure 2.1: The block diagram of a system using DPSK commutation signaling.

codeword (or sequence) $s^{(i)}$. For a M-ary phase codeword, $s_l^{(i)} \in \{exp(j2\pi n/M)|n = 0, 1, \cdots M-1\}$. $p_\tau(t)$ is a unit rectangular pulse of duration τ

$$p_\tau(t) = \begin{cases} 1 & 0 \le t \le \tau \\ 0 & elsewhere \end{cases} \tag{2.2}$$

Let $a(k)$ denotes the binary (± 1) data sequence. The transmitted signal for phase shift keying (PSK) is

$$x(t) = Re\{\sum_{i=-\infty}^{\infty} a(i)s_i(t - iT)e^{j\omega t}\} \tag{2.3}$$

where Re($\cdot$) denotes the real part of ($\cdot$) and ω is the carrier frequency. T is the symbol period and T = LT_c. When differential PSK (DPSK) is employed, the transmitted signal can be written as

$$x(t) = Re\{\sum_{i=-\infty}^{\infty} s_i(t - iT)e^{j(\omega t + \sum_{l=-\infty}^{i} \theta_l)}\} \tag{2.4}$$

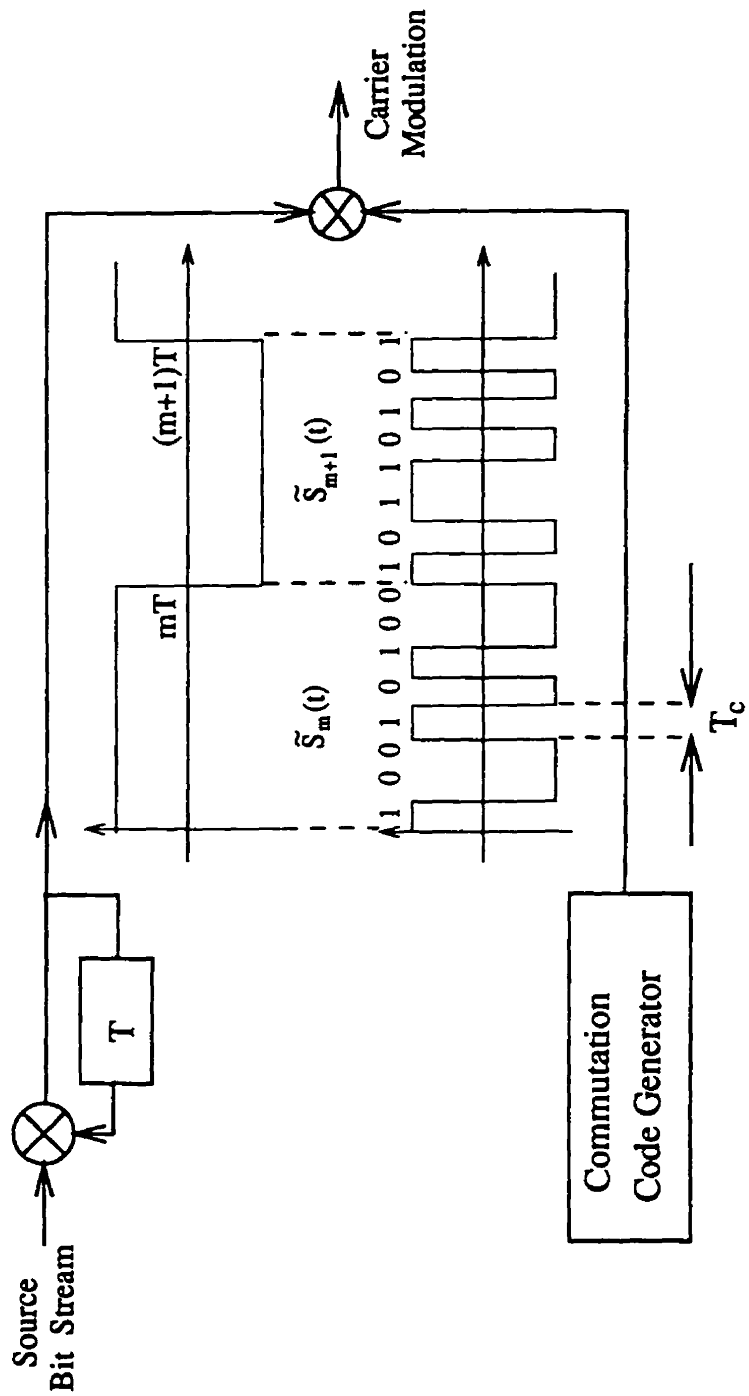

Figure 2.2: The DPSK commutation signaling modulator.

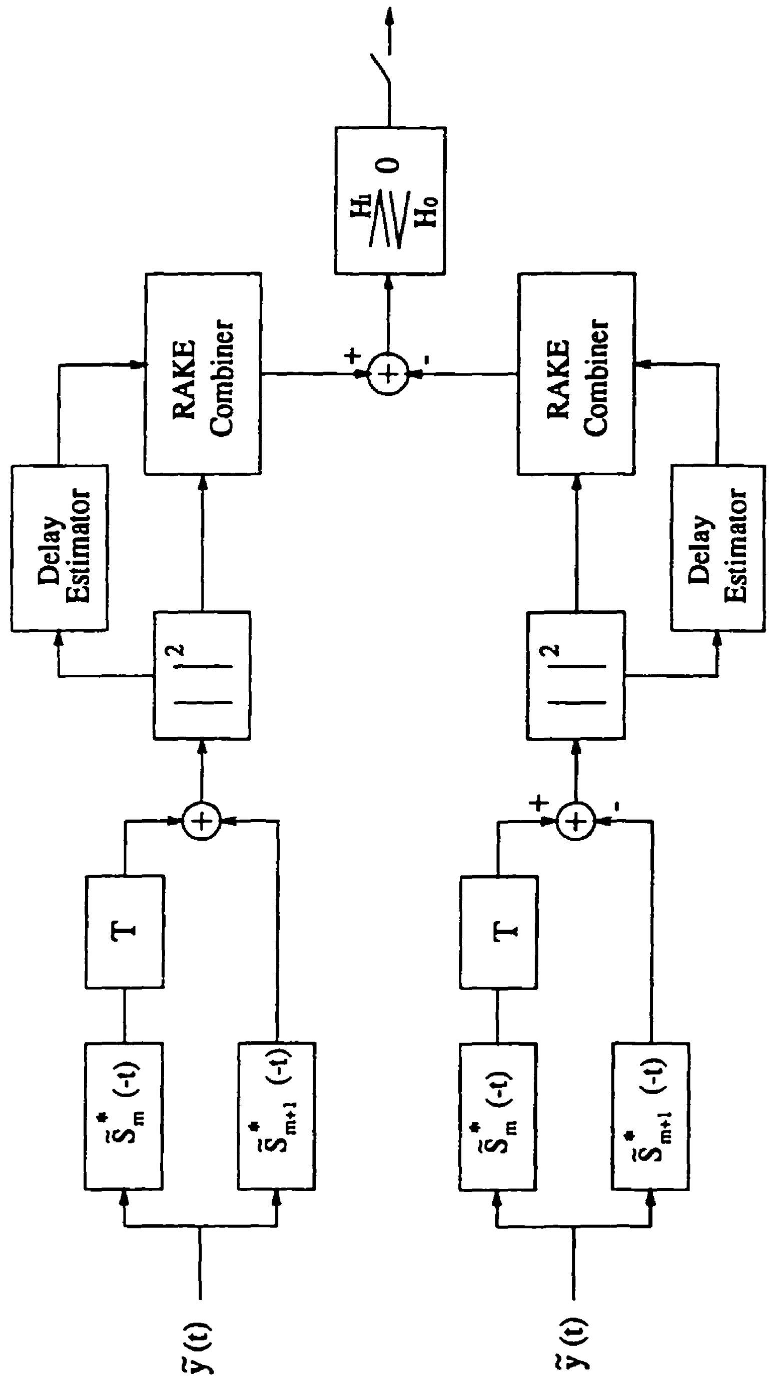

Figure 2.3: The DPSK commutation signaling demodulator.

where $\theta_l \in \{0, \pi\}$ is the phase shift corresponding to the source data.

Assuming that the codeword $s^{(i)}$ is from a codebook of size N (that is, the codebook consists of N codewords), and the codewords are cycled sequentially through the codebook. This means

$$s^{(i)} = s^{(i \pm kN)} \tag{2.5}$$

for any integer k. Therefore, we have

$$s_i(t) = s_{i \pm kN}(t) \tag{2.6}$$

Using the complex envelop representation,

$$x(t) = Re\{\tilde{x}(t)e^{j\omega t}\} \tag{2.7}$$

where $\tilde{x}$(t) is the complex envelop, for DPSK, we have

$$\tilde{x}(t) = \sum_{i=-\infty}^{\infty} s_i(t - iT)e^{j\phi_i} \tag{2.8}$$

where $\phi_i = \sum_{l=-\infty}^{i} \theta_l$.

The delayed waveform, that is, $x(t)$ delayed by τ, is

$$z(t) = x(t - \tau) = \sum_{i=-\infty}^{\infty} s_i(t - \tau - iT)cos(\omega(t - \tau) + \sum_{l=-\infty}^{i} \theta_l) \tag{2.9}$$

Denoting the complex envelop of $z(t)$ as $\tilde{z}$(t), we have

$$\tilde{z}(t) = \sum_{i=-\infty}^{\infty} s_i(t - \tau - iT)e^{j(\phi_i - \omega\tau)} \tag{2.10}$$

The complex envelop of the output of the two-wave Rayleigh fading channel is

$$\tilde{y}(t) = A_1\tilde{x}(t) + A_2\tilde{z}(t) + \tilde{n}(t) \tag{2.11}$$

The path gains A_1 and A_2 are independent, zero mean, complex Gaussian random variables whose real and imaginary parts are independent and identically distributed. Thus, we have

$$E[A_i^* A_i] = \sigma_i^2 \quad E[A_i A_i] = 0 \quad i = 1, 2 \tag{2.12}$$

where σ_i^2 is twice the variance of the real or imaginary parts of A_i. In (2.11), the factor $e^{-j\omega\tau}$ in $\tilde{z}(t)$ can be absorbed into A_2. This new random variable satisfies (2.12) if it is still denoted as A_2. $\tilde{n}(t)$ is the complex envelop of the white Gaussian noise n(t), which has a double sided power spectral density of $N_0/2$. Then, we have

$$E[\tilde{n}(t)\tilde{n}(t')] = 0 \quad E[\tilde{n}(t)\tilde{n^*}(t')] = 2N_0\delta(t-t') \tag{2.13}$$

For a multipath channel modeled as a linear filter with the complex, lowpass impulse response expressed as

$$\tilde{h}(t) = \sum_{i=0}^{\infty} \alpha_i e^{j\theta_i}\delta(t-\tau_i) \tag{2.14}$$

where α_i, θ_i and τ_i are the path gain, phase shift and time delay of the ith resolvable path, it is shown that the optimal generalized likelihood receiver consists of a matched filter and a square-law equal weight RAKE combiner[14]. In the case of two-wave Rayleigh fading channel, this receiver was shown to be suboptimal[17]. Since the optimal receiver is complex to implement, and the difference in performance between the optimal and suboptimal receivers, in the binary signaling case, is small at high SNR[25], we will consider the simple suboptimal receiver here.

Assuming that a codebook has N commutation signal codewords, the receiver contains matched filters with the impulse response $h_i(t) = s_i^*(-t)$, $i = 0, 1, \cdots, N-1$ (the symbol $*$ denotes complex conjugation). For the matched filter with $h_0(t) = s_0^*(-t)$, the decision variable is

$$d_0 = |\int \tilde{y}(t)s_0^*(t)dt|^2 + |\int \tilde{y}(t)s_0^*(t-\tau)dt|^2 \tag{2.15}$$

For the differential encoding, the signal transmitted over the interval [0, 2T] is

$$u_0(t) = s_0(t) + s_1(t-T)e^{j\theta_1} \tag{2.16}$$

and the decision variable is

$$d_0 = |\int \tilde{y}(t)u_0^*(t)dt|^2 + |\int \tilde{y}(t)u_0^*(t-\tau)dt|^2 \tag{2.17}$$

For binary DPSK, suppose that the signal transmitted under the other hypothesis is

$$v_0(t) = s_0(t) + s_1(t-T)e^{j\theta_1'} \tag{2.18}$$

and denoting

$$d_0' = |\int \tilde{y}(t)v_0^*(t)dt|^2 + |\int \tilde{y}(t)v_0^*(t-\tau)dt|^2 \tag{2.19}$$

The detector makes its decision based on $s = d_0 - d_0'$. If $s \geq 0$, it decides that the transmitted message is corresponding to θ_1, otherwise, the transmitted message is corresponding to θ_1'.

2.2 Performance

Let us define the following complex random variables

$$\begin{aligned} V_1 &= \int \tilde{y}(t)u_0^*(t)dt \\ V_2 &= \int \tilde{y}(t)u_0^*(t-\tau)dt \\ V_3 &= \int \tilde{y}(t)v_0^*(t)dt \\ V_4 &= \int \tilde{y}(t)v_0^*(t-\tau)dt \end{aligned} \tag{2.20}$$

then

$$s = |V_1|^2 + |V_2|^2 - |V_3|^2 - |V_4|^2 \tag{2.21}$$

If we define a random vector $\mathbf{V} = (V_1\ V_2\ V_3\ V_4)^t$ (t denotes simple transposition), s can be written as

$$s = \mathbf{V}^H Q \mathbf{V} \tag{2.22}$$

where H denotes the Hermitian operator (complex conjugation followed by simple transposition) and Q is a Hermitian matrix given by

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2.23}$$

It is seen that s is a quadratic form in the random variables V_1, V_2, V_3 and V_4.

$$\begin{aligned} E[V_1 V_1] &= E[\int \tilde{y}(t)u_0^*(t)dt \int \tilde{y}(t')u_0^*(t')dt'] \\ &= E\{\int \int [A_1\tilde{x}(t) + A_2\tilde{z}(t) + \tilde{n}(t)] \\ &\quad [A_1\tilde{x}(t') + A_2\tilde{z}(t') + \tilde{n}(t')]u_0^*(t)u_0^*(t')dtdt'\} \end{aligned} \tag{2.24}$$

Since A_1, A_2 and $\tilde{n}(t)$ are mutually independent and $E[A_1] = E[A_2] = E[\tilde{n}(t)] = 0$,

$$\begin{aligned} E[V_1 V_1] = \int\int \; & \{E[A_1A_1]\tilde{x}(t)\tilde{x}(t') + E[A_2A_2]\tilde{z}(t)\tilde{z}(t') + E[\tilde{n}(t)\tilde{n}(t')]\} \\ & u_0^*(t)u_0^*(t')dtdt' \end{aligned} \tag{2.25}$$

Using (2.12) and (2.13), we can show that

$$E[V_1 V_1] = 0 \tag{2.26}$$

In fact, it can be shown $E[V_iV_j] = 0$ for $i, j = 1, 2, 3, 4$. This means

$$E[\mathbf{V}\mathbf{V}^t] = 0 \tag{2.27}$$

It has been shown[26, 27] that for a random vector $\mathbf{V}$ which satisfies (2.27), the characteristic function of the quadratic form $s = \mathbf{V}^H Q \mathbf{V}$ is given by

$$\begin{aligned} \Phi(j\omega) &= E[e^{j\omega s}] \\ &= \frac{1}{|I - j\omega\Gamma Q|} exp\{-\mathbf{U}^H\Gamma^{-1}[I - (I - j\omega\Gamma Q)^{-1}]\mathbf{U}\} \end{aligned} \tag{2.28}$$

where $\mathbf{U} = \mathrm{E}[\mathbf{V}]$ is the mean of the complex vector $\mathbf{V}$ and $\Gamma = \mathrm{E}[(\mathbf{V}-\mathbf{U})(\mathbf{V}-\mathbf{U})^H]$ is the covariance matrix. I is the unity matrix. The probability density function of s is given by the Fourier transform of $\Phi(j\omega)$

$$p(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(j\omega)e^{-j\omega s}d\omega \tag{2.29}$$

The cumulative distribution of s is then

$$Pr(s < x) = \int_{-\infty}^{x} p(s)ds = \frac{1}{2\pi}\int_{-\infty}^{x}\int_{-\infty}^{\infty} \Phi(j\omega)e^{-j\omega s}d\omega ds \tag{2.30}$$

Integrating over s and let $z = j\omega$, we have

$$Pr(s < x) = -\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{\Phi(z)}{z}e^{-zx}dz \tag{2.31}$$

where the path of integration is taken to be deformed into the left-half z-plane at all imaginary-axis singularities.

In our case, $\mathbf{U} = \mathrm{E}[\mathbf{V}] = 0$, and $\Phi(z)$ assumes a simple form[26]

$$\Phi(z) = \frac{1}{|I - j\omega\Gamma Q|} = \prod_{n=1}^{4}(1-\lambda_n z)^{-1} \tag{2.32}$$

where λ_n are the eigenvalues of ΓQ. We see that $\Phi(z)$ is the reciprocal of an fourth degree polynomial in z. The zeros of the polynomial lie at the reciprocals of the eigenvalues λ_n. The singularities of $\Phi(z)$ in this case consist of finite number of finite-order poles. Once the eigenvalues of ΓQ are found, $Pr(s < x)$ is easily determined through the use of the residue theorem[26, 27].

$$Pr(s < 0) = -\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{\Phi(z)}{z}dz = -\sum res\{\frac{\Phi(z)}{z}\} \tag{2.33}$$

Next, we will evaluate the elements of covariance matrix Γ. With no loss of generality, it is assumed that $\phi_0 = \sum_{l=-\infty}^{0}\theta_l = 0$. Suppose $0 \le kT \le \tau < (k+1)T \le NT$, the random variables V_1 is

$$V_1 = \int_0^{2T} \tilde{y}(t)u_0^*(t)dt$$

$$
\begin{aligned}
&= \int_0^{2T} [A_1\tilde{x}(t) + A_2\tilde{z}(t) + \tilde{n}(t)][s_0^*(t) + s_1^*(t-T)e^{-j\theta_1}]dt \\
&= A_1 \int_0^{2T} [\sum_{i=-\infty}^{\infty} s_i(t-iT)e^{j\phi_i}][s_0^*(t) + s_1^*(t-T)e^{-j\theta_1}]dt \\
&+ A_2 \int_0^{2T} [\sum_{i=-\infty}^{\infty} s_i(t-\tau-iT)e^{j\phi_i}][s_0^*(t) + s_1^*(t-T)e^{-j\theta_1}]dt \\
&+ \int_0^{2T} \tilde{n}(t)u_0^*(t)dt
\end{aligned} \tag{2.34}
$$

Denoting

$$
E_s = \int_0^T |s_i|^2(t)dt; \quad i = 0, 1, \cdots, N-1. \tag{2.35}
$$

and

$$
\begin{aligned}
a_{12} &= \frac{1}{E_s}\{e^{j\phi_{-(k+1)}} \int s_{N-(k+1)}(t-\tau+(k+1)T)s_0^*(t)dt \\
&+ e^{j\phi_{-k}} \int s_{N-k}(t-\tau+kT)s_0^*(t)dt \\
&+ e^{j(\phi_{-k}-\theta_1)} \int s_{N-k}(t-\tau+kT)s_1^*(t-T)dt \\
&+ e^{j(\phi_{-(k+1)}-\theta_1)} \int s_{N-(k+1)}(t-\tau+(k-1)T)s_1^*(t-T)dt\}
\end{aligned} \tag{2.36}
$$

we have

$$
V_1 = E_s(2A_1 + a_{12}A_2) + \int \tilde{n}(t)u_0^*(t)dt \tag{2.37}
$$

Defining the continuous-time partial cross-correlation functions as[28]

$$
R_{j,i}(\tau) = \int_0^T s_j(t-\tau+T)s_i^*(t)dt \tag{2.38}
$$

$$
\hat{R}_{j,i}(\tau) = \int_0^T s_j(t-\tau)s_i^*(t)dt \tag{2.39}
$$

for $0 \leq \tau < T$. a_{12} can be written as

$$
\begin{aligned}
a_{12} &= \frac{1}{E_s}[e^{j\phi_{-(k+1)}} R_{N-(k+1),0}(\tau') + e^{j\phi_{-k}} \hat{R}_{N-k,0}(\tau') \\
&+ e^{j(\phi_{-k}-\theta_1)} R_{N-k,1}(\tau') + e^{j(\phi_{-(k-1)}-\theta_1)} \hat{R}_{N-(k-1),1}(\tau')]
\end{aligned} \tag{2.40}
$$

with $\tau' = \tau - kT$. It should be noted that the indices i, j of $R_{i,j}(\tau')$ should be understood as i mod. N and j mod. N since $s_i(t) = s_{i\pm kN}(t)$ for any integer k.

V_2, V_3 and V_4 can be evaluated in a similar way and all the results are summarized in the following:

$$
\begin{aligned}
V_1 &= E_s(2A_1 + a_{12}A_2) + \int \tilde{n}(t)u_0^*(t)dt \\
V_2 &= E_s(a_{21}A_1 + 2A_2) + \int \tilde{n}(t)u_0^*(t-\tau)dt \\
V_3 &= E_s A_{32} A_2 + \int \tilde{n}(t)v_0^*(t)dt \\
V_4 &= E_s A_{41} A_1 + \int \tilde{n}(t)v_0^*(t-\tau)dt
\end{aligned} \tag{2.41}
$$

and

$$
\begin{aligned}
a_{12} &= \frac{1}{E_s}[e^{j\phi_{-(k+1)}} R_{N-(k+1),0}(\tau') + e^{j\phi_{-k}} \hat{R}_{N-k,0}(\tau') \\
&+ e^{j(\phi_{-k}-\theta_1)} R_{N-k,1}(\tau') + e^{j(\phi_{-(k-1)}-\theta_1)} \hat{R}_{N-(k-1),1}(\tau')] \\
a_{21} &= \frac{1}{E_s}\{e^{j\phi_{(k+1)}}[R_{0,k+1}(\tau')]^* + e^{j\phi_k}[\hat{R}_{0,k}(\tau')]^* \\
&+ e^{j(\phi_{(k+2)}-\theta_1)}[R_{1,k+2}(\tau')]^* + e^{j(\phi_{(k+1)}-\theta_1)}[\hat{R}_{1,k+1}(\tau')]^*\} \\
a_{32} &= \frac{1}{E_s}[e^{j\phi_{-(k+1)}} R_{N-(k+1),0}(\tau') + e^{j\phi_{-k}} \hat{R}_{N-k,0}(\tau') \\
&+ e^{j(\phi_{-k}-\theta_1')} R_{N-k,1}(\tau') + e^{j(\phi_{-(k-1)}-\theta_1')} \hat{R}_{N-(k-1),1}(\tau')] \\
a_{41} &= \frac{1}{E_s}\{e^{j\phi_{(k+1)}}[R_{0,k+1}(\tau')]^* + e^{j\phi_k}[\hat{R}_{0,k}(\tau')]^* \\
&+ e^{j(\phi_{(k+2)}-\theta_1')}[R_{1,k+2}(\tau')]^* + e^{j(\phi_{(k+1)}-\theta_1')}[\hat{R}_{1,k+1}(\tau')]^*\}
\end{aligned} \tag{2.42}
$$

with $\tau' = \tau - kT$.

Since $E[V_i] = 0$ $(i = 1,2,3,4)$, the elements of the covariance matrix are $\Gamma_{ij} = E[V_i V_j^*]$.

$$
\begin{aligned}
\Gamma_{11} &= E[V_1 V_1^*] \\
&= E\{[E_s(2A_1 + a_{12}A_2) + \int \tilde{n}(t)u_0^*(t)dt] \\
&\quad [E_s(2A_1^* + a_{12}^*A_2^*) + \int \tilde{n}^*(t')u_0(t')dt']\}
\end{aligned} \tag{2.43}
$$

Because A_1, A_2 and $\tilde{n}(t)$ are mutually independent, (2.43) can be simplified to

$$\begin{aligned}\Gamma_{11} &= 4E_s^2 E[A_1 A_1^*] + E_s^2 |a_{12}|^2 E[A_2 A_2^*] \\ &+ \int E[\tilde{n}(t)\tilde{n}^*(t')] u_0^*(t) u_0(t') dt dt' \end{aligned} \quad (2.44)$$

Recalling that $E[A_i A_i^*] = \sigma_i^2$ $(i = 1, 2)$ and $E[\tilde{n}(t)\tilde{n}^*(t')] = 2N_0\delta(t - t')$, we have

$$\begin{aligned}\Gamma_{11} &= 4E_s^2\sigma_1^2 + E_s^2|a_{12}|^2\sigma_2^2 + 2N_0 \int u_0(t)u_0^*(t)dt \\ &= 4E_s^2\sigma_1^2 + E_s^2|a_{12}|^2\sigma_2^2 + 4E_s N_0 \\ &= 4E_s N_0[\frac{E_s}{N_0}\sigma_1^2 + |a_{12}|^2 \frac{E_s}{4N_0}\sigma_2^2 + 1] \end{aligned} \quad (2.45)$$

Setting the signal-to-noise ratio (SNR) $\gamma = \frac{E_s}{2N_0}$, and $\gamma_i = \frac{E_s}{2N_0}\sigma_i^2$ to be the average SNR of the ith path (i=1,2), then

$$\Gamma_{11} = 4E_s N_0[2\gamma_1 + \frac{\gamma_2}{2}|a_{12}|^2 + 1] \quad (2.46)$$

Similarly, we have

$$\begin{aligned}\Gamma_{12} &= E[V_1 V_2^*] \\ &= E\{[E_s(2A_1 + a_{12}A_2) + \int \tilde{n}(t)u_0^*(t)dt] \\ &\quad [E_s(a_{21}^*A_1^* + 2A_2^*) + \int \tilde{n}^*(t')u_0(t' - \tau)dt']\} \\ &= 2E_s^2 a_{21}^*\sigma_1^2 + 2E_s^2 a_{12}\sigma_2^2 + 2N_0 \int u_0^*(t)u_0(t - \tau)dt \end{aligned} \quad (2.47)$$

Defining $b_{12} = \int u_0^*(t)u_0(t - \tau)dt$, then

$$\begin{aligned} b_{12} &= \int u_0^*(t)u_0(t - \tau)dt \\ &= \int [s_0^*(t) + s_1^*(t - T)e^{-j\theta_1}][s_0(t - \tau) + s_1(t - \tau - T)e^{j\theta_1}] \\ &= \int s_0(t - \tau)s_0^*(t)dt + e^{-j\theta_1} \int s_0(t - \tau)s_1^*(t - T)dt \\ &+ e^{j\theta_1} \int s_1(t - \tau - T)s_0^*(t)dt + \int s_1(t - \tau - T)s_1^*(t - T)dt \end{aligned} \quad (2.48)$$

With

$$\int s_1(t-\tau-T)s_0^*(t)dt = 0 \tag{2.49}$$

$$\int s_0(t-\tau)s_0^*(t)dt = \begin{cases} \hat{R}_{0,0}(\tau) & 0<\tau\le T \\ 0 & \tau>T \end{cases} \tag{2.50}$$

$$\int s_1(t-\tau-T)s_1^*(t-T)dt = \begin{cases} \hat{R}_{1,1}(\tau) & 0<\tau\le T \\ 0 & \tau>T \end{cases} \tag{2.51}$$

$$\int s_0(t-\tau)s_1^*(t-T)dt = \begin{cases} R_{0,1}(\tau) & 0<\tau\le T \\ \hat{R}_{0,1}(\tau-T) & T<\tau\le 2T \\ 0 & \tau>2T \end{cases} \tag{2.52}$$

we have

$$b_{12} = \begin{cases} \frac{1}{E_s}[\hat{R}_{0,0}(\tau)+e^{-j\theta_1}R_{0,1}(\tau)+\hat{R}_{1,1}(\tau)] & 0<\tau\le T \\ \frac{1}{E_s}e^{-j\theta_1}\hat{R}_{0,1}(\tau-T) & T<\tau\le 2T \\ 0 & \tau>2T \end{cases} \tag{2.53}$$

and

$$\Gamma_{12} = 4E_sN_0[\gamma_1 a_{21}^* + \gamma_1 a_{12} + \frac{b_{12}}{2}] \tag{2.54}$$

Other elements of the covariance matrix can be evaluated in the similar way and they are shown below.

$$\Gamma_{13} = 4E_sN_0[\frac{\gamma_2}{2}a_{12}a_{32}^*]$$
$$\Gamma_{14} = 4E_sN_0[\gamma_1 a_{41}^* + \frac{b_{14}}{2}]$$
$$\Gamma_{22} = 4E_sN_0[\frac{\gamma_1}{2}|a_{21}|^2 + 2\gamma_2 + 1]$$
$$\Gamma_{23} = 4E_sN_0[\gamma_2 a_{32}^* + \frac{b_{23}}{2}]$$
$$\Gamma_{24} = 4E_sN_0[\frac{\gamma_1}{2}a_{21}a_{41}^*]$$
$$\Gamma_{33} = 4E_sN_0[\frac{\gamma_2}{2}|a_{32}|^2 + 1]$$
$$\Gamma_{34} = 4E_sN_0[\frac{b_{34}}{2}]$$

$$\Gamma_{44} = 4E_s N_0[\frac{\gamma_1}{2}|a_{41}|^2 + 1] \tag{2.55}$$

where

$$b_{14} = \begin{cases} \frac{1}{E_s}[\hat{R}_{0,0}(\tau) + e^{-j\theta_1} R_{0,1}(\tau) - \hat{R}_{1,1}(\tau)] & 0 < \tau \leq T \\ \frac{1}{E_s} e^{-j\theta_1} \hat{R}_{0,1}(\tau - T) & T < \tau \leq 2T \\ 0 & \tau > 2T \end{cases} \tag{2.56}$$

$$b_{23} = \begin{cases} \frac{1}{E_s}\{[\hat{R}_{0,0}(\tau)]^* + e^{-j\theta'_1}[R_{0,1}(\tau)]^* - [\hat{R}_{1,1}(\tau)]^*\} & 0 < \tau \leq T \\ \frac{1}{E_s} e^{-j\theta'_1}[\hat{R}_{0,1}(\tau - T)]^* & T < \tau \leq 2T \\ 0 & \tau > 2T \end{cases} \tag{2.57}$$

$$b_{34} = \begin{cases} \frac{1}{E_s}[\hat{R}_{0,0}(\tau) + e^{-j\theta'_1} R_{0,1}(\tau) + \hat{R}_{1,1}(\tau)] & 0 < \tau \leq T \\ \frac{1}{E_s} e^{-j\theta'_1} \hat{R}_{0,1}(\tau - T) & T < \tau \leq 2T \\ 0 & \tau > 2T \end{cases} \tag{2.58}$$

Since Γ is a Hermitian matrix, the matrix elements that are not listed can be determined from the elements shown above by using $\Gamma_{ij} = \Gamma^*_{ji}$. Here, the singularities of $\Phi(z)/z$ on the left-half z-plane consist of first and second order poles and the residues at those points depend on the ratios (rather than the values) of eigenvalues of matrix ΓQ. Therefore, Γ can be normalized by dividing its elements by $4E_sN_0$. After doing so, it is seen that given $\theta_1, \phi_{-(k+1)}, \phi_{-k}, \phi_{-(k-1)}, \phi_k, \phi_{k+1}$, and ϕ_{k+2}, the conditional error probability $Pr(s < 0)$ depends on the average SNR of the two waves (γ_1 and γ_2) as well as the continuous-time partial cross-correlation functions $R_{i,j}(\tau')$ and $\hat{R}_{i,j}(\tau')$. The conditional error probability can be calculated using the residue method and the probability of error is then obtained by average the conditional error probability over all possible values of $\theta_1, \phi_{-(k+1)}, \phi_{-k}, \phi_{-(k-1)}, \phi_k, \phi_{k+1}$, and ϕ_{k+2}.

Suppose a codebook of size four is used, then $N = 4$. Let us look at the a_{ij} that determine the covariance matrix Γ when inter-wave delay τ is in different range. If $0 \leq \tau \leq T$, we have k=0. $\phi_{-k} = \phi_k = \phi_0 = 0$ and $\phi_{-(k-1)} = \phi_{k+1} = \phi_1 = \theta_1$.

Therefore, we only need to consider $\phi_1, \phi_{-(k+1)} = \phi_{-1}$ and $\phi_{k+2} = \phi_2$. Denoting

$$d_1 = e^{j\phi_{-1}} \quad d_2 = e^{j\phi_1} \quad d_3 = e^{j(\phi_2 - \phi_1)} \tag{2.59}$$

we have

$$\begin{aligned}
a_{12} &= \frac{1}{E_s}[d_1 R_{3,0}(\tau) + \hat{R}_{0,0}(\tau) + d_2 R_{0,1}(\tau) + \hat{R}_{1,1}(\tau)] \\
a_{21} &= \frac{1}{E_s}\{d_2[R_{0,1}(\tau)]^* + [\hat{R}_{0,0}(\tau)]^* + d_3[R_{1,2}(\tau)]^* + [\hat{R}_{1,1}(\tau)]^*\} \\
a_{32} &= \frac{1}{E_s}[d_1 R_{3,0}(\tau) + \hat{R}_{0,0}(\tau) - d_2 R_{0,1}(\tau) - \hat{R}_{1,1}(\tau)] \\
a_{41} &= \frac{1}{E_s}\{d_2[R_{0,1}(\tau)]^* + [\hat{R}_{0,0}(\tau)]^* - d_3[R_{1,2}(\tau)]^* - [\hat{R}_{1,1}(\tau)]^*\}
\end{aligned} \tag{2.60}$$

In this case, the probability of error is the average of the conditional error probability over all possible values of d_1, d_2 and d_3. For $T \le \tau \le 2T$, $k = 1$. Among $\theta_1, \phi_{-(k+1)}, \phi_{-k}, \phi_{-(k-1)}, \phi_k, \phi_{k+1}$, and ϕ_{k+2}, five variables are in dependent, and they are $\phi_{-(k+1)} = \phi_{-2}$, $\phi_{-k} = \phi_{-1}, \phi_k = \phi_1 = \theta_1, \phi_{k+1} = \phi_2$ and $\phi_{k+2} = \phi_3$.

$$\begin{aligned}
a_{12} &= \frac{1}{E_s}[e^{j\phi_{-2}} R_{2,0}(\tau') + e^{j\phi_{-1}} \hat{R}_{3,0}(\tau') \\
&+ e^{j(\phi_{-1}-\theta_1)} R_{3,1}(\tau') + e^{j(\phi_0-\theta_1)} \hat{R}_{0,1}(\tau')] \\
a_{21} &= \frac{1}{E_s}\{e^{j\phi_2}[R_{0,2}(\tau')]^* + e^{j\phi_1}[\hat{R}_{0,1}(\tau')]^* \\
&+ e^{j(\phi_3-\theta_1)}[R_{1,3}(\tau')]^* + e^{j(\phi_2-\theta_1)}[\hat{R}_{1,2}(\tau')]^*\} \\
a_{32} &= \frac{1}{E_s}[e^{j\phi_{-2}} R_{2,0}(\tau') + e^{j\phi_{-1}} \hat{R}_{3,0}(\tau') \\
&- e^{j(\phi_{-1}-\theta_1)} R_{3,1}(\tau') - e^{j(\phi_0-\theta_1)} \hat{R}_{0,1}(\tau')] \\
a_{41} &= \frac{1}{E_s}\{e^{j\phi_2}[R_{0,2}(\tau')]^* + e^{j\phi_1}[\hat{R}_{0,1}(\tau')]^* \\
&- e^{j(\phi_3-\theta_1)}[R_{1,3}(\tau')]^* - e^{j(\phi_2-\theta_1)}[\hat{R}_{1,2}(\tau')]^*\}
\end{aligned} \tag{2.61}$$

with $\tau' = \tau - T$.

Define

$$d_1 = e^{j\phi_{-2}} \quad d_2 = e^{j\phi_{-1}} \quad d_3 = e^{j\theta_1}$$
$$d_4 = e^{j\phi_2} \quad d_5 = e^{j(\phi_3-\theta_1)} \tag{2.62}$$

we can write

$$\begin{aligned}
a_{12} &= \frac{1}{E_s}[d_1 R_{2,0}(\tau') + d_2\hat{R}_{3,0}(\tau') + d_2 d_3 R_{3,1}(\tau') + d_3\hat{R}_{0,1}(\tau')] \\
a_{21} &= \frac{1}{E_s}\{d_4[R_{0,2}(\tau')]^* + d_3[\hat{R}_{0,1}(\tau')]^* + d_5[R_{1,3}(\tau')]^* + d_4 d_3[\hat{R}_{1,2}(\tau')]^*\} \\
a_{32} &= \frac{1}{E_s}[d_1 R_{2,0}(\tau') + d_2\hat{R}_{3,0}(\tau') - d_2 d_3 R_{3,1}(\tau') - d_3\hat{R}_{0,1}(\tau')] \\
a_{41} &= \frac{1}{E_s}\{d_4[R_{0,2}(\tau')]^* + d_3[\hat{R}_{0,1}(\tau')]^* - d_5[R_{1,3}(\tau')]^* - d_4 d_3[\hat{R}_{1,2}(\tau')]^*\}
\end{aligned} \tag{2.63}$$

The conditional error probability has to be averaged over five independent data symbols d_1, d_2, d_3, d_4 and d_5. For $2T \le \tau \le 3T$, $k = 2$. The seven phase shifts $\theta_1, \phi_{-(k+1)}, \phi_{-k}, \phi_{-(k-1)}$, ϕ_k, ϕ_{k+1}, and ϕ_{k+2} are independent of each other. Using the following notations

$$d_1 = e^{j\phi_{-(k+1)}} = e^{j\phi_{-3}} \quad d_2 = e^{j\phi_{-k}} = e^{j\phi_{-2}}$$
$$d_3 = e^{j\phi_{-(k-1)}} = e^{j\phi_{-1}} \qquad d_4 = e^{j\theta_1} \qquad d_5 = e^{j\phi_k} = e^{j\phi_2}$$
$$d_6 = e^{j\phi_{(k+1)}} = e^{j\phi_3} \quad d_7 = e^{j\phi_{(k+2)}} = e^{j\phi_4} \tag{2.64}$$

we have

$$\begin{aligned}
a_{12} &= \frac{1}{E_s}[d_1 R_{1,0}(\tau') + d_2\hat{R}_{2,0}(\tau') + d_2 d_4 R_{2,1}(\tau') + d_3 d_4\hat{R}_{3,1}(\tau')] \\
a_{21} &= \frac{1}{E_s}\{d_6[R_{0,3}(\tau')]^* + d_5[\hat{R}_{0,2}(\tau')]^* + d_7 d_4[R_{1,0}(\tau')]^* + d_6 d_4[\hat{R}_{1,3}(\tau')]^*\} \\
a_{32} &= \frac{1}{E_s}[d_1 R_{1,0}(\tau') + d_2\hat{R}_{2,0}(\tau') - d_2 d_4 R_{2,1}(\tau') - d_3 d_4\hat{R}_{3,1}(\tau')] \\
a_{41} &= \frac{1}{E_s}\{d_6[R_{0,3}(\tau')]^* + d_5[\hat{R}_{0,2}(\tau')]^* \\
&- d_7 d_4[R_{1,0}(\tau')]^* - d_6 d_4[\hat{R}_{1,3}(\tau')]^*\}
\end{aligned} \tag{2.65}$$

with $\tau' = \tau - 2T$. Now, the probability of error is an average of conditional error probability over seven independent data. Finally, When $3T \leq \tau \leq 4T$, $k = 3$. Let $\tau' = \tau - 3T$, a_{ij} can be written as

$$\begin{aligned}
a_{12} &= \frac{1}{E_s}[d_1 R_{0,0}(\tau') + d_2\hat{R}_{1,0}(\tau') + d_2 d_4 R_{1,1}(\tau') + d_3 d_4 \hat{R}_{2,1}(\tau')] \\
a_{21} &= \frac{1}{E_s}\{d_6[R_{0,0}(\tau')]^* + d_5[\hat{R}_{0,3}(\tau')]^* + d_7 d_4[R_{1,1}(\tau')]^* + d_6 d_4[\hat{R}_{1,0}(\tau')]^*\} \\
a_{32} &= \frac{1}{E_s}[d_1 R_{0,0}(\tau') + d_2\hat{R}_{1,0}(\tau') - d_2 d_4 R_{1,1}(\tau') - d_3 d_4 \hat{R}_{2,1}(\tau')] \\
a_{41} &= \frac{1}{E_s}\{d_6[R_{0,0}(\tau')]^* + d_5[\hat{R}_{0,3}(\tau')]^* - d_7 d_4[R_{1,1}(\tau')]^* - d_6 d_4[\hat{R}_{1,0}(\tau')]^*\} \qquad (2.66)
\end{aligned}$$

where $d_1, d_2, d_3, d_4, d_5, d_6$ and d_7 are defined by

$$\begin{aligned}
&d_1 = e^{j\phi_{-(k+1)}} = e^{j\phi_{-4}} \quad d_2 = e^{j\phi_{-k}} = e^{j\phi_{-3}} \\
&d_3 = e^{j\phi_{-(k-1)}} = e^{j\phi_{-2}} \qquad d_4 = e^{j\theta_1} \qquad d_5 = e^{j\phi_k} = e^{j\phi_3} \\
&d_6 = e^{j\phi_{(k+1)}} = e^{j\phi_4} \quad d_7 = e^{j\phi_{(k+2)}} = e^{j\phi_5} \qquad (2.67)
\end{aligned}$$

Again, when calculating the probability of error, the conditional error probability is averaged over seven independent data.

Chapter 3

Code Construction

3.1 Code Design Rules

Ideally, the commutation signaling codes should be constructed to minimize the conditional error probability given $\theta_1, \phi_{-(k+1)}, \phi_{-k}, \phi_{-(k-1)}, \phi_k, \phi_{k+1}$, and ϕ_{k+2}. However, it is shown that a close form for the conditional error probability can not be derived for the system we considered here, and the computation of the conditional error probability is quite complicated. Therefore, it is very difficult to see how the conditional error probability depends on the correlation properties of a code and which code parameters have the strongest effect on the conditional error probability. Due to this problem, we have taken an alternative approach where the signal-to-noise ratio is used as a performance measure for the purpose of code construction[29]. The system is analized in terms of average performance and worst case performance. The error probability results for codebooks constructed here will be presented in Chapter 4.

3.1.1 Average Signal-to-Noise Ratio

Concerning the average performance, one of the important measures is the average signal-to-noise ratio[29]. It should be noted that the two integrals in the decision variable d_0, i.e., V_1 and V_2, are the outputs of the matched filters. Recalling

$$V_1 = 2E_s A_1 + a_{12} E_s A_2 + \int \tilde{n}(t) u_0^*(t) dt \tag{3.1}$$

It can be seen that the first term is the desired signal. The second term is the interference due to the multipath, and the last term is a Gaussian random variable due to the noise. Similarly, the three terms in V_2

$$V_2 = 2E_s A_2 + a_{21} E_s A_1 + \int \tilde{n}(t) u_0^*(t-\tau) dt \tag{3.2}$$

may be interpreted in the same way. Denoting the two multipath interference terms in V_1 and V_2 as I_{01} and I_{02} respectively, we have

$$\begin{aligned} I_{01} &= A_2[e^{j\phi_{-(k+1)}} R_{N-(k+1),0}(\tau') + e^{j\phi_{-k}} \hat{R}_{N-k,0}(\tau') \\ &+ e^{j(\phi_{-k}-\theta_1)} R_{N-k,1}(\tau') + e^{j(\phi_{-(k-1)}-\theta_1)} \hat{R}_{N-(k-1),1}(\tau')] \end{aligned} \tag{3.3}$$

$$\begin{aligned} I_{02} &= A_1\{e^{j\phi_{(k+1)}} [R_{0,k+1}(\tau')]^* + e^{j\phi_k} [\hat{R}_{0,k}(\tau')]^* \\ &+ e^{j(\phi_{(k+2)}-\theta_1)} [R_{1,k+2}(\tau')]^* + e^{j(\phi_{(k+1)}-\theta_1)} [\hat{R}_{1,k+1}(\tau')]^*\} \end{aligned} \tag{3.4}$$

where $\tau' = \tau - kT$ and $kT \leq \tau \leq (k+1)T$.

In this approach, the time delay and the data symbols are treated as mutually independent random variables. It is assumed that the time delay τ is uniformly distributed in the range [0, NT) and θ_l takes on the values of 0 and π with equal probability. This implies that ϕ_i takes on the values of 0 and π with equal probability. The multipath induced interferences can then be treated as additional noise, thus the new noise components become

$$n_1 = I_{01} + \int \tilde{n}(t) u_0^*(t) dt \tag{3.5}$$

$$n_2 = I_{02} + \int \tilde{n}(t)u_0^*(t-\tau)dt \tag{3.6}$$

Since

$$E[e^{j\theta_l}] = 0 \quad E[e^{j\phi_i}] = 0 \quad E[\tilde{n}(t)] = 0 \tag{3.7}$$

we have

$$\begin{aligned} E[I_{01}] = E[I_{02}] &= 0 \\ E[\int \tilde{n}(t)u_0^*(t)dt] = E[\int \tilde{n}(t)u_0^*(t-\tau)dt] &= 0 \end{aligned} \tag{3.8}$$

therefore, the means of n_1 and n_2 are zero. The variance of n_1 is

$$\begin{aligned} var(n_1) &= E[n_1 n_1^*] \\ &= E[I_{01}I_{01}^* + I_{01}\int \tilde{n}^*(t)u_0(t)dt + I_{01}^*\int \tilde{n}(t)u_0^*(t)dt + \int \tilde{n}(t)u_0^*(t)dt \int \tilde{n}^*(t')u_0(t')dt'] \\ &= E[I_{01}I_{01}^*] + \int\int E[\tilde{n}(t)\tilde{n}^*(t')]u_0^*(t)u_0(t')dtdt' \end{aligned} \tag{3.9}$$

Recalling that $E[\tilde{n}(t)\tilde{n}^*(t')] = 2N_0\delta(t-t')$, we have

$$var(n_1) = var(I_{01}) + 2N_0\int u_0(t)u_0^*(t)dt = var(I_{01}) + 4N_0E_s \tag{3.10}$$

where E_s is defined as

$$E_s = \int_0^T |s_i|^2(t)dt; \;\; i = 0, 1, \cdots, N-1. \tag{3.11}$$

Similarly, the variance of n_2 is given as

$$var(n_2) = var(I_{02}) + 4N_0E_s \tag{3.12}$$

Now, let us look at the $var(I_{01})$ and $var(I_{02})$.

$$var(I_{01}) = E[I_{01}I_{01}^*] \tag{3.13}$$

The expectation is with respect to time delay τ and data symbols. Since data symbols are mutually independent,

$$E[e^{j(\phi_i-\phi_j)}] = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{3.14}$$

τ and data symbols are independent and τ is uniformly distributed in the range [0, NT), thus

$$\begin{aligned} var(I_{01}) &= \frac{|A_2|^2}{NT} \sum_{k=0}^{N-1} \int_0^T \{|R_{N-(k+1),0}(\tau')|^2 + |\hat{R}_{N-k,0}(\tau')|^2 \\ &+ |R_{N-k,1}(\tau')|^2 + |\hat{R}_{N-(k-1),1}(\tau')|^2\} d\tau' \end{aligned} \tag{3.15}$$

Similarly, we have

$$\begin{aligned} var(I_{02}) &= \frac{|A_1|^2}{NT} \sum_{k=0}^{N-1} \int_0^T \{|R_{0,k+1}(\tau')|^2 + |\hat{R}_{0,k}(\tau')|^2 \\ &+ |R_{1,k+2}(\tau')|^2 + |\hat{R}_{1,k+1}(\tau')|^2\} d\tau' \end{aligned} \tag{3.16}$$

Consider the first integral V_1

$$\begin{aligned} V_1 &= 2E_sA_1 + a_{12}E_sA_2 + \int \tilde{n}(t)u_0^*(t)dt \\ &= 2E_sA_1 + n_1 \end{aligned} \tag{3.17}$$

The signal-to-noise ratio (SNR) is determined by signal power and the variance of the noise.

$$(SNR)_1 = \frac{|2E_sA_1|^2}{var(n_1)} = \frac{|2E_sA_1|^2}{var(I_{01}) + 4N_0E_s} \tag{3.18}$$

Similarly, the signal-to-noise ratio for the second integral V_2 is given by

$$(SNR)_2 = \frac{|2E_sA_2|^2}{var(n_2)} = \frac{|2E_sA_2|^2}{var(I_{02}) + 4N_0E_s} \tag{3.19}$$

It can be seen that the desired signal (numerators in (3.18) and (3.19)) and the second terms of $var(n_1)$ and $var(n_2)$ are not dependent on the commutation signaling codes, therefore, concerning the effects of commutation signaling codes on the average signal-to-noise ratio, $var(I_{01})$ and $var(I_{02})$ are the only parameters to be considered.

Next, following a similar procedure as in [30], we will establish the relationship between the continuous-time partial cross-correlation functions $R_{j,i}(\tau)$, $\hat{R}_{j,i}(\tau)$ and the discrete aperiodic cross-correlation functions $C_{j,i}(l)$ and $C_{j,i}(l-L)$, which are defined as

$$C_{j,i}(l) = \begin{cases} \sum_{n=0}^{L-l-1} s_n^{(j)}[s_{n+l}^{(i)}]^* & 0 \le l \le L-1 \\ \\ \sum_{n=0}^{L+l-1} s_{n-l}^{(j)}[s_n^{(i)}]^* & 1-L \le l < 0 \\ \\ 0 & |l| \ge L \end{cases} \tag{3.20}$$

Recalling

$$\hat{R}_{j,i}(\tau) = \int_0^T s_j(t-\tau)s_i^*(t)dt \tag{3.21}$$

$$s_i^*(t) = \sum_{n=0}^{L-1}[s_n^{(i)}]^* p_{T_c}(t-nT_c) \tag{3.22}$$

$$s_j(t-\tau) = \sum_{m=0}^{L-1} s_m^{(j)} p_{T_c}(t-\tau-mT_c) \tag{3.23}$$

Assume that $lT_c \le \tau < (l+1)T_c$, let $u = \tau - lT_c$, thus $\tau + mT_c = (l+m)T_c + u$ and

$$\begin{aligned} \hat{R}_{j,i}(\tau) &= \int_0^T s_j(t-\tau)s_i^*(t)dt = \int_\tau^T s_j(t-\tau)s_i^*(t)dt \\ &= \sum_{n=0}^{L-1}\sum_{m=0}^{L-1} s_m^{(j)}[s_n^{(i)}]^* \int_\tau^T p_{T_c}(t-nT_c)p_{T_c}(t-(m+l)T_c-u)dt \end{aligned} \tag{3.24}$$

The integral is not zero only when $n = m+l$ or $n = m+l+1$. Since n can not take on values larger than $L-1$, the summation over m will be from 0 to $L-l-1$ for $n = m+l$ and be from 0 to $L-l-2$ for $n = m+l+1$.

$$\begin{aligned} \hat{R}_{j,i}(\tau) &= \sum_{m=0}^{L-l-1} s_m^{(j)}[s_{m+l}^{(i)}]^* \int_{(m+l)T_c}^{(m+l+1)T_c} p_{T_c}(t-(m+l)T_c)p_{T_c}(t-(m+l)T_c-u)dt \\ &+ \sum_{m=0}^{L-l-2} s_m^{(j)}[s_{m+l+1}^{(i)}]^* \end{aligned}$$

$$\int_{(m+l+1)T_c}^{(m+l+2)T_c} p_{T_c}(t-(m+l+1)T_c)p_{T_c}(t-(m+l)T_c-u)dt \tag{3.25}$$

It is clear that

$$C_{j,i}(l) = \sum_{m=0}^{L-l-1} s_m^{(j)}[s_{m+l}^{(i)}]^* \tag{3.26}$$

$$C_{j,i}(l+1) = \sum_{m=0}^{L-l-2} s_m^{(j)}[s_{m+l+1}^{(i)}]^* \tag{3.27}$$

$$\int_{(m+l)T_c}^{(m+l+1)T_c} p_{T_c}(t-(m+l)T_c)p_{T_c}(t-(m+l)T_c-u)dt = T_c - u \tag{3.28}$$

$$\int_{(m+l+1)T_c}^{(m+l+2)T_c} p_{T_c}(t-(m+l+1)T_c)p_{T_c}(t-(m+l)T_c-u)dt = u \tag{3.29}$$

Thus, we have

$$\begin{aligned}
\hat{R}_{j,i}(\tau) &= C_{j,i}(l)(T_c-u) + C_{j,i}(l+1)u \\
&= C_{j,i}(l)T_c + [C_{j,i}(l+1) - C_{j,i}(l)]u \\
&= C_{j,i}(l)T_c + [C_{j,i}(l+1) - C_{j,i}(l)](\tau - lT_c) \qquad (3.30)
\end{aligned}$$

similarly, it can be shown that

$$R_{j,i}(\tau) = C_{j,i}(l-L)T_c + [C_{j,i}(l+1-L) - C_{j,i}(l-L)](\tau - lT_c) \tag{3.31}$$

for $lT_c \le \tau < (l+1)T_c$.

Now, writing

$$\begin{aligned}
&\sum_{k=0}^{N-1}\int_0^T \{|R_{N-(k+1),0}(\tau')|^2 + |\hat{R}_{N-k,0}(\tau')|^2\}d\tau' \\
= \;&\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\int_{lT_c}^{(l+1)T_c} \{|R_{N-(k+1),0}(\tau')|^2 + |\hat{R}_{N-k,0}(\tau')|^2\}d\tau' \qquad (3.32)
\end{aligned}$$

Substituting (3.30) and (3.31) into the right side of (3.32), we obtain

$$
\begin{aligned}
& \sum_{k=0}^{N-1} \int_0^T \{|R_{N-(k+1),0}(\tau')|^2 + |\hat{R}_{N-k,0}(\tau')|^2\} d\tau' \\
= \;& \frac{T_c^3}{3} \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} \{|C_{N-(k+1),0}(l-L)|^2 + |C_{N-k,0}(l)|^2 \\
+ \;& |C_{N-(k+1),0}(l+1-L)|^2 + |C_{N-k,0}(l+1)|^2 \\
+ \;& Re\{C_{N-(k+1),0}(l-L)[C_{N-(k+1),0}(l+1-L)]^* \\
+ \;& C_{N-k,0}(l)[C_{N-k,0}(l+1)]^*\}\}
\end{aligned} \tag{3.33}
$$

Since N codewords $s^{(i)}$, $i = 0,1, \cdots, N-1$, cycles sequentially with index i, the subscripts j and i of $C_{j,i}(l)$ should be understood as j mod. N and i mod. N. With this in mind, it is easy to show

$$
\sum_{k=0}^{N-1} C_{N-(k+1),0}(l-L)[C_{N-(k+1),0}(l+1-L)]^* = \sum_{k=0}^{N-1} C_{k,0}(l-L)[C_{k,0}(l+1-L)]^* \tag{3.34}
$$

$$
\sum_{k=0}^{N-1} C_{N-k,0}(l)[C_{N-k,0}(l+1)]^* = \sum_{k=0}^{N-1} C_{k,0}(l)[C_{k,0}(l+1)]^* \tag{3.35}
$$

With (3.34) and (3.35), (3.33) can be simplified to

$$
\begin{aligned}
& \sum_{k=0}^{N-1} \int_0^T \{|R_{N-(k+1),0}(\tau')|^2 + |\hat{R}_{N-k,0}(\tau')|^2\} d\tau' \\
= \;& \frac{T_c^3}{3} \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} \{|C_{k,,0}(l-L)|^2 + |C_{k,0}(l)|^2 + |C_{k,0}(l+1-L)|^2 + |C_{k,0}(l+1)|^2 \\
+ \;& Re\{C_{k,0}(l-L)[C_{k,0}(l+1-L)]^* + C_{k,0}(l)[C_{k,0}(l+1)]^*\}\}
\end{aligned} \tag{3.36}
$$

Using the following identity

$$
\sum_{l=0}^{L-1} [|C_{k,,0}(l-L)|^2 + |C_{k,0}(l)|^2] = \sum_{l=0}^{L-1} [|C_{k,0}(l+1-L)|^2 + |C_{k,0}(l+1)|^2] \tag{3.37}
$$

we have

$$
\sum_{k=0}^{N-1} \int_0^T \{|R_{N-(k+1),0}(\tau')|^2 + |\hat{R}_{N-k,0}(\tau')|^2\} d\tau'
$$

$$
\begin{aligned}
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{k,0}(l-L)|^2+|C_{k,0}(l)|^2] \\
&+ Re[C_{k,0}(l-L)C^*_{k,0}(l+1-L)+C_{k,0}(l)C^*_{k,0}(l+1)]\} \qquad (3.38)
\end{aligned}
$$

Similarly, it can be shown that

$$
\begin{aligned}
&\sum_{k=0}^{N-1}\int_0^T\{|R_{N-k,1}(\tau')|^2+|\hat{R}_{N-(k-1),1}(\tau')|^2\}d\tau' \\
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{k,1}(l-L)|^2+|C_{k,1}(l)|^2] \\
&+ Re[C_{k,1}(l-L)C^*_{k,1}(l+1-L)+C_{k,1}(l)C^*_{k,1}(l+1)]\} \qquad (3.39)
\end{aligned}
$$

Substituting (3.38) and (3.39) into (3.15), we obtain

$$
\begin{aligned}
var(I_{01}) &= \frac{|A_2|^2T_c^2}{3NL}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{k,0}(l)|^2+|C_{k,0}(l-L)|^2+|C_{k,1}(l)|^2+|C_{k,1}(l-L)|^2] \\
&+ Re[C_{k,0}(l)C^*_{k,0}(l+1)+C_{k,0}(l-L)C^*_{k,0}(l+1-L) \\
&+ C_{k,1}(l)C^*_{k,1}(l+1)+C_{k,1}(l-L)C^*_{k,1}(l+1-L)]\} \qquad (3.40)
\end{aligned}
$$

Following similar procedures, we have

$$
\begin{aligned}
&\sum_{k=0}^{N-1}\int_0^T\{|R_{0,k+1}(\tau')|^2+|\hat{R}_{0,k}(\tau')|^2\}d\tau' \\
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{|C_{0,k+1}(l-L)|^2+|C_{0,k+1}(l+1-L)|^2+|C_{0,k}(l)|^2+|C_{0,k}(l+1)|^2 \\
&+ Re[C_{0,k+1}(l-L)C^*_{0,k+1}(l+1-L)+C_{0,k}(l)C^*_{0,k}(l+1)]\} \\
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{|C_{0,k}(l-L)|^2+|C_{0,k}(l+1-L)|^2+|C_{0,k}(l)|^2+|C_{0,k}(l+1)|^2 \\
&+ Re[C_{0,k}(l-L)C^*_{0,k}(l+1-L)+C_{0,k}(l)C^*_{0,k}(l+1)]\} \\
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{0,k}(l+1)|^2+|C_{0,k}(l+1-L)|^2] \\
&+ Re[C_{0,k}(l-L)C^*_{0,k}(l+1-L)+C_{0,k}(l)C^*_{0,k}(l+1)]\} \qquad (3.41)
\end{aligned}
$$

$$
\sum_{k=0}^{N-1}\int_0^T\{|R_{1,k+2}(\tau')|^2+|\hat{R}_{1,k+1}(\tau')|^2\}d\tau'
$$

$$
\begin{aligned}
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{|C_{1,k+2}(l-L)|^2+|C_{1,k+2}(l+1-L)|^2+|C_{1,k+1}(l)|^2+|C_{1,k+1}(l+1)|^2 \\
&+ Re[C_{1,k+2}(l-L)C^*_{1,k+2}(l+1-L)+C_{1,k+1}(l)C^*_{1,k+1}(l+1)]\} \\
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{|C_{1,k}(l-L)|^2+|C_{1,k}(l+1-L)|^2+|C_{1,k}(l)|^2+|C_{1,k}(l+1)|^2 \\
&+ Re[C_{1,k}(l-L)C^*_{1,k}(l+1-L)+C_{1,k}(l)C^*_{1,k}(l+1)]\} \\
&= \frac{T_c^3}{3}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{1,k}(l+1)|^2+|C_{1,k}(l+1-L)|^2] \\
&+ Re[C_{1,k}(l-L)C^*_{1,k}(l+1-L)+C_{1,k}(l)C^*_{1,k}(l+1)]\} \qquad (3.42)
\end{aligned}
$$

Then the variance of the multipath induced interference in V_2 can be written as

$$
\begin{aligned}
var(I_{02}) &= \frac{|A_1|^2T_c^2}{3NL}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{0,k}(l+1)|^2+|C_{0,k}(l+1-L)|^2 \\
&+ |C_{1,k}(l+1)|^2+|C_{1,k}(l+1-L)|^2] \\
&+ Re[C_{0,k}(l)C^*_{0,k}(l+1)+C_{0,k}(l-L)C^*_{0,k}(l+1-L) \\
&+ C_{1,k}(l)C^*_{1,k}(l+1)+C_{1,k}(l-L)C^*_{1,k}(l+1-L)]\} \qquad (3.43)
\end{aligned}
$$

Replacing $L-l-1$ with l and noticing that $C_{j,i}(l) = C^*_{i,j}(-l)$, we have

$$
\begin{aligned}
var(I_{02}) &= \frac{|A_1|^2T_c^2}{3NL}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{k,0}(l)|^2+|C_{k,0}(l-L)|^2+|C_{k,1}(l)|^2+|C_{k,1}(l-L)|^2] \\
&+ Re[C_{k,0}(l)C^*_{k,0}(l+1)+C_{k,0}(l-L)C^*_{k,0}(l+1-L) \\
&+ C_{k,1}(l)C^*_{k,1}(l+1)+C_{k,1}(l-L)C^*_{k,1}(l+1-L)]\} \qquad (3.44)
\end{aligned}
$$

It is seen that except the factor $|A_i|^2$, the variances of I_{01} and I_{02} are the same. Since we are interested in the influence of the commutation signaling codes on the system performance, the factors $\frac{|A_i|^2T_c^2}{3}$ (i = 1,2) in (3.40) and (3.44) can be ignored and the multipath induced interference for the filter matched to $u_0^*(-t)$ (actually for the filters matched to $s_0^*(-t)$ and $s_1^*(-t)$) can be characterized by

$$
I_0 = \frac{1}{NL}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{k,0}(l)|^2+|C_{k,0}(l-L)|^2+|C_{k,1}(l)|^2+|C_{k,1}(l-L)|^2]
$$

$$
\begin{aligned}
&+ \quad Re[C_{k,0}(l)C^*_{k,0}(l+1) + C_{k,0}(l-L)C^*_{k,0}(l+1-L) \\
&+ \quad C_{k,1}(l)C^*_{k,1}(l+1) + C_{k,1}(l-L)C^*_{k,1}(l+1-L)]\}
\end{aligned} \tag{3.45}
$$

In general, the multipath interference for the filter matched to $u^*_m(-t)$, $m = 0, 1 \cdots, N-1$, can be characterized by

$$
\begin{aligned}
I_m &= \frac{1}{NL}\sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\{2[|C_{k,m}(l)|^2 + |C_{k,m}(l-L)|^2 + |C_{k,m+1}(l)|^2 + |C_{k,m+1}(l-L)|^2] \\
&+ \quad Re[C_{k,m}(l)C^*_{k,m}(l+1) + C_{k,m}(l-L)C^*_{k,m}(l+1-L) \\
&+ \quad C_{k,m+1}(l)C^*_{k,m+1}(l+1) + C_{k,m+1}(l-L)C^*_{k,m+1}(l+1-L)]\}
\end{aligned} \tag{3.46}
$$

We can also consider the average of I_m over all the matched filters in the receiver, which is given by

$$
I_{av} = \frac{1}{N}\sum_{m=0}^{N-1} I_m \tag{3.47}
$$

It should be pointed out that the summations in (3.46) contains terms with zero inter-wave delay ($\tau = 0$), which corresponding to the desired signal. Since this contribution is a constant independent of m, it will not be subtracted from I_m. This facilitates the computation of I_{av} when we consider the code construction later.

3.1.2 Worst Case Performance

In the previous section, the average performance has been considered in terms of average SNR. It should be noted that this SNR is a statistical average with respect to the time delay and data symbols. It is possible that the probability of error are quite large at certain time delays even the average SNR is high. For some applications, it is desirable to find commutation signaling codes for which the probability of error is acceptable for all time delays. In this case, the maximum value of the error probability needs to be considered.

It has been shown that the decision variable d_0 contains two integrals which are outputs of the matched filters.

$$V_1 = 2E_sA_1 + I_{01} + \int \tilde{n}(t)u_0^*(t)dt \tag{3.48}$$

$$V_2 = 2E_sA_2 + I_{02} + \int \tilde{n}(t)u_0^*(t-\tau)dt \tag{3.49}$$

where $2E_sA_1$ and $2E_sA_2$ are desired signals and I_{01}, I_{02} are multipath induced interference. The two integrals are Gaussian random variables due to noise. It is clear that the desired signals are not dependent on the commutation signaling codes. The noise terms are integration of commutation signaling codewords multiplied by a white Gaussian process, therefore, the commutation signaling codes have little effects on these terms. It is expected that I_{01} and I_{02} are the key parameters that characterizing the effects of commutation signaling codes on the conditional error probability of the system. For equal weight combining, I_{01} and I_{02} are equally important and it is reasonable to assume that the maximum conditional error probability is essentially dependent on $\max(|I_{01}| + |I_{02}|)$. Because the probability of error is an average of the conditional error probability, the maximum probability of error does not only depends on $\max(|I_{01}| + |I_{02}|)$. However, it is expected that $\max(|I_{01}| + |I_{02}|)$ is one of the key parameters that determine the maximum probability of error. Defining

$$\begin{aligned} f_{01} &= e^{j\phi_{-(k+1)}} R_{N-(k+1),0}(\tau') + e^{j\phi_{-k}} \hat{R}_{N-k,0}(\tau') \\ &+ e^{j(\phi_{-k}-\theta_1)} R_{N-k,1}(\tau') + e^{j(\phi_{-(k-1)}-\theta_1)} \hat{R}_{N-(k-1),1}(\tau') \end{aligned} \tag{3.50}$$

$$\begin{aligned} f_{02} &= e^{j\phi_{(k+1)}} [R_{0,k+1}(\tau')]^* + e^{j\phi_k} [\hat{R}_{0,k}(\tau')]^* \\ &+ e^{j(\phi_{(k+2)}-\theta_1)} [R_{1,k+2}(\tau')]^* + e^{j(\phi_{(k+1)}-\theta_1)} [\hat{R}_{1,k+1}(\tau')]^* \end{aligned} \tag{3.51}$$

we have

$$max(|I_{01}| + |I_{02}|) = max(|A_2||f_{01}| + |A_1||f_{02}|) \tag{3.52}$$

Since A_1 and A_2 are two independent, identically distributed random variables, and they do not depends on the commutation signaling codes it is reasonable to ignore A_1, A_2 and only to consider $\max(|f_{01}| + |f_{02}|)$.

From (3.30) and (3.31), it is seen that $R_{j,i}(\tau)$ and $\hat{R}_{j,i}(\tau)$ are linear functions of τ when $0 \le lT_c \le \tau < (l+1)T_c \le \mathrm{T}$ for $l = 0,1,\cdots,L-1$. Therefore, $|f_{01}|+|f_{02}|$ can take on maximum value only when τ is a multiple of T_c. For $\tau = lT_c, l = 0,1,\cdots,L-1$, $R_{j,i}(\tau) = C_{j,i}(l-L)T_c$ and $\hat{R}_{j,i}(\tau) = C_{j,i}(l)T_c$. Then

$$\begin{aligned} f_{01} &= e^{j\phi_{-(k+1)}}C_{N-(k+1),0}(l-L) + e^{j\phi_{-k}}C_{N-k,0}(l) \\ &+ e^{j(\phi_{-k}-\theta_1)}C_{N-k,1}(l-L) + e^{j(\phi_{-(k-1)}-\theta_1)}C_{N-(k-1),1}(l) \end{aligned} \quad (3.53)$$

$$\begin{aligned} f_{02} &= e^{j\phi_{(k+1)}}C^*_{0,k+1}(l-L) + e^{j\phi_k}C^*_{0,k}(l) \\ &+ e^{j(\phi_{(k+2)}-\theta_1)}C^*_{1,k+2}(l-L) + e^{j(\phi_{(k+1)}-\theta_1}C^*_{1,k+1}(l) \end{aligned} \quad (3.54)$$

and

$$\begin{aligned} max(|f_{01}|+|f_{02}|) &= T_c max\{|C_{N-(k+1),0}(l-L) + C_{N-k,0}(l) + C_{N-k,1}(l-L) \\ &+ C_{N-(k-1),1}(l)| + |C^*_{0,k+1}(l-L) + C^*_{0,k}(l) \\ &+ C^*_{1,k+2}(l-L) + C^*_{1,k+1}(l)|\} \end{aligned} \quad (3.55)$$

In general, for the filter matched to $u^*_m(-t)$, $m = 0,1,\cdots,N-1$, the code parameter that characterizes the influence of the multipath induced interference on the maximum conditional error probability is given by

$$\begin{aligned} max(|f_{m1}| &+ |f_{m2}|) = T_c max\{|C_{N-(k+1)+m,m}(l-L) + C_{N-k+m,m}(l) \\ &+ C_{N-k+m,m+1}(l-L) + C_{N-(k-1)+m,m+1}(l)| + |[C_{m,m+k+1}(l-L)]^* \\ &+ [C_{m,m+k}(l)]^* + [C_{m+1,m+k+2}(l-L)]^* + [C_{m+1,m+k+1}(l)]^*|\} \end{aligned} \quad (3.56)$$

For binary codes,

$$max(|f_{m1}| + |f_{m2}|) = T_c max\{|C_{N-(k+1)+m,m}(l-L)| + |C_{N-k+m,m}(l)|$$

$$+ \quad |C_{N-k+m,m+1}(l-L)| + |C_{N-(k-1)+m,m+1}(l)| + |C_{m,m+k+1}(l-L)|$$
$$+ \quad |C_{m,m+k}(l)| + |C_{m+1,m+k+2}(l-L)| + |C_{m+1,m+k+1}(l)|\} \qquad (3.57)$$

Again, the subscripts i, j of $C_{i,j}(l-L)$ and $C_{i,j}(l)$ should be understood as i mod. N and j mod. N.

Concerning the worst case performance, that is, the maximum conditional error probability, commutation signaling code should be selected to minimize $max(|f_{m1}| + |f_{m2}|)$ for $0 \leq m, k \leq N-1$, $0 \leq l \leq L-1$ ($l \neq 0$ for $k=0$).

3.2 Code Construction

3.2.1 Code Construction Based on Average Performance

Based on the analysis of the average performance, it can be concluded that the commutation signaling code $\{s^{(0)}, s^{(1)}, \cdots, s^{(N-1)}\}$ should be constructed to minimize the largest I_m (defined in (3.46)) for $m = 0, 1, \cdots, N-1$. Similar to the performance analysis on direct-sequence spread-spectrum multiple-access (DS/SSMA) systems[19], we can also consider the average of I_m, that is, I_{av} as an alternative performance measure. Next, we will focus on the minimization of I_{av}. However, the bounds of I_m will be considered whenever it is possible.

Average Performance for Random Code

Before commutation signaling codes are constructed, it is useful to look at the average performance for random code. This can be done by calculating the expectation value of I_m and I_{av} assuming that the codewords $s^{(m)}$, $m = 0, 1 \cdots, N-1$, are random sequences. In this report, only two phase (or binary) and four phase codewords are considered. For binary codewords, $s_j^{(m)}$ (j-th element of the codeword $s^{(m)}$) is assumed to be independent and identically distributed random variables with

$Pr\{s_j^{(m)} = 1\} = Pr\{s_j^{(m)} = -1\} = \frac{1}{2}$ for $j = 0,1 \cdots, L-1$. While for four phase codewords, $s_j^{(m)}$ is supposed to be independent and identically distributed random variables with $Pr\{s_j^{(m)} = 1\} = Pr\{s_j^{(m)} = -1\} = Pr\{s_j^{(m)} = i\} = Pr\{s_j^{(m)} = -i\} = \frac{1}{4}$ for $j = 0,1 \cdots, L-1$.

Define

$$\begin{aligned} r_{k,m} &= \sum_{l=0}^{L-1} \{2[|C_{k,m}(l)|^2 + |C_{k,m}(l-L)|^2] \\ &+ Re[C_{k,m}(l)C_{k,m}^*(l+1) + C_{k,m}(l-L)C_{k,m}^*(l+1-L)]\} \end{aligned} \tag{3.58}$$

then I_m can be expressed as

$$I_m = \frac{1}{NL} \sum_{k=0}^{N-1} (r_{k,m} + r_{k,m+1}) \tag{3.59}$$

Using the following identities

$$\sum_{l=0}^{L-1} [|C_{k,m}(l)|^2 + |C_{k,m}(l-L)|^2] = \sum_{l=1-L}^{L-1} |C_{k,m}(l)|^2 \tag{3.60}$$

$$\sum_{l=0}^{L-1} [C_{k,m}(l)C_{k,m}^*(l+1) + C_{k,m}(l-L)C_{k,m}^*(l+1-L)] = \sum_{l=1-L}^{L-1} C_{k,m}(l)C_{k,m}^*(l+1) \tag{3.61}$$

$r_{k,m}$ can be written as

$$r_{k,m} = \sum_{l=1-L}^{L-1} \{2|C_{k,m}(l)|^2 + Re[C_{k,m}(l)C_{k,m}^*(l+1)]\} \tag{3.62}$$

For binary case, the expectation of $r_{k,m}$ for $m \neq k$ has been shown to be $2L^2$[31]. Here, we will consider four phase codewords. In our case, k may be equal to m. Thus, the expectation of $r_{k,m}$ is different from that obtained in the binary case.

Recalling that $C_{k,m}(l)$ is given by

$$C_{k,m}(l) = \begin{cases} \sum_{n=0}^{L-l-1} s_n^{(k)} [s_{n+l}^{(m)}]^* & 0 \leq l \leq L-1 \\ \\ \sum_{n=0}^{L+l-1} s_{n-l}^{(k)} [s_n^{(m)}]^* & 1-L \leq l < 0 \\ \\ 0 & |l| \geq L \end{cases} \tag{3.63}$$

Assume that $k \neq m$, for $l \leq 0$, we have

$$\begin{aligned} E[|C_{k,m}(l)|^2] &= E\{C_{k,m}(l)[C_{k,m}(l)]^*\} \\ &= \sum_{n=0}^{L-l-1}\sum_{i=0}^{L-l-1} E\{s_n^{(k)}[s_i^{(k)}]^*\}E\{s_{i+l}^{(m)}[s_{n+l}^{(m)}]^*\} \\ &= \sum_{n=0}^{L-l-1} E\{s_n^{(k)}[s_n^{(k)}]^*\}E\{s_{n+l}^{(m)}[s_{n+l}^{(m)}]^*\} = L - l \end{aligned} \tag{3.64}$$

$$E\{C_{k,m}(l)[C_{k,m}(l+1)]^*\} = \sum_{n=0}^{L-l-1}\sum_{i=0}^{L-l-2} E\{s_n^{(k)}[s_i^{(k)}]^* s_{i+l+1}^{(m)}[s_{n+l}^{(m)}]^*\} = 0 \tag{3.65}$$

For $l < 0$, we can show that

$$E[|C_{k,m}(l)|^2] = L + l \tag{3.66}$$

$$E\{C_{k,m}(l)[C_{k,m}(l+1)]^*\} = 0 \tag{3.67}$$

When $k = m$, if $l \neq 0$, the results above are still true. However, if $l = 0$,

$$E[|C_{k,m}(l)|^2] = \sum_{n=0}^{L-1}\sum_{i=0}^{L-1} E\{s_n^{(k)}[s_n^{(k)}]^*\}E\{s_i^{(k)}[s_i^{(k)}]^*\} = L^2 \tag{3.68}$$

But still

$$E\{C_{k,m}(l)[C_{k,m}(l+1)]^*\} = 0 \tag{3.69}$$

In summary, we have

$$E[|C_{k,m}(l)|^2] = \begin{cases} L - |l| & l \neq 0 \text{ or } k \neq m \\ L^2 & l = 0 \text{ and } k = m \end{cases} \tag{3.70}$$

$$E[C_{k,m}(l)C_{k,m}^*(l+1)] = 0 \tag{3.71}$$

It then follows

$$E[r_{k,m}] = \begin{cases} 2L^2 & k \neq m \\ 4L^2 - 2L & k = m \end{cases} \tag{3.72}$$

Similarly, we have

$$E[r_{k,m+1}] = \begin{cases} 2L^2 & k \neq m+1 \\ 4L^2 - 2L & k = m+1 \end{cases} \tag{3.73}$$

With the expectations of $r_{k,m}$ and $r_{k,m+1}$, the expectation value of I_m can be evaluated

$$\begin{aligned} E[I_m] &= \frac{1}{NL}\sum_{k=0}^{N-1}\{E[r_{k,m}] + E[r_{k,m+1}]\} \\ &= \frac{1}{NL}\{E[r_{m,m}] + \sum_{k=0,k\neq m}^{N-1} E[r_{k,m}] + E[r_{m+1,m+1}] + \sum_{k=0,k\neq m+1}^{N-1} E[r_{k,m+1}]\} \\ &= \frac{1}{NL}[2(4L^2 - 2L) + 2(N-1)(2L^2)] \\ &= 4L + \frac{4L-4}{N} \end{aligned} \tag{3.74}$$

This is the expectation value of I_m for random four phase codeword. Since the result is not dependent on m, we have

$$E[I_{av}] = 4L + \frac{4L-4}{N} \tag{3.75}$$

It should be point out that the expectation values of I_m and I_{av} for random two phase codewords are the same.

Construction of Binary Codes

We have found that the code parameters to be minimized (I_m and I_{av}) are similar to those for a DS/SSMA system[19]. Therefore, commutation signaling codes can be constructed in a way similar to that used to construct signature sequences for DS/SSMA systems. Here, only two phase (binary) codes are considered.

First, we will rewrite the expression of I_m using the following identities[21]

$$\sum_{l=1-L}^{L-1} C_{k,m}(l)C_{k,m}(l+1) = \sum_{l=1-L}^{L-1} C_{k,k}(l)C_{m,m}(l+1) \tag{3.76}$$

With (3.76), we obtain

$$
\begin{aligned}
r_{k,m} &= \sum_{l=1-L}^{L-1} \{2|C_{k,m}(l)|^2 + Re[C_{k,m}(l)C^*_{k,m}(l+1)]\} \\
&= \sum_{l=1-L}^{L-1} [2C^2_{k,m}(l) + C_{k,m}(l)C_{k,m}(l+1)] \\
&= \sum_{l=1-L}^{L-1} [2C_{k,k}(l)C_{m,m}(l) + C_{k,k}(l)C_{m,m}(l+1)] \qquad (3.77)
\end{aligned}
$$

$$
\begin{aligned}
r_{k,m+1} &= \sum_{l=1-L}^{L-1} \{2|C_{k,m+1}(l)|^2 + Re[C_{k,m+1}(l)C^*_{k,m+1}(l+1)]\} \\
&= \sum_{l=1-L}^{L-1} [2C^2_{k,m+1}(l) + C_{k,m+1}(l)C_{k,m+1}(l+1)] \\
&= \sum_{l=1-L}^{L-1} [2C_{k,k}(l)C_{m+1,m+1}(l) + C_{k,k}(l)C_{m+1,m+1}(l+1)] \qquad (3.78)
\end{aligned}
$$

Substituting (3.77) and (3.78) into (3.59), I_m can be written as

$$
\begin{aligned}
I_m &= \frac{1}{NL} \sum_{l=1-L}^{L-1} [2C_{m,m}(l) + C_{m,m}(l+1) \\
&+ 2C_{m+1,m+1}(l) + C_{m+1,m+1}(l+1)][\sum_{k=0}^{N-1} C_{k,k}(l)] \qquad (3.79)
\end{aligned}
$$

Next, two construction methods will be presented. For the first construction method, N binary codewords $s^{(k)}, k = 0, 1, \cdots, N-1$, of length $L = N-1$ are constructed from an m-sequence of period L and an arbitrary sequence of length L. Let u be the m-sequence and v be the arbitrary sequence, then the codewords $s^{(k)}, k = 0, 1, \cdots, N-1$ are defined by

$$
s^{(k)} = (v_0 u_k, v_1 u_{k+1}, \cdots, v_{L-1} u_{k+L-1}) \quad 0 \le k \le N-2 \qquad (3.80)
$$

and

$$
s^{(N-1)} = (v_0, v_1, \cdots, v_{L-1}) \qquad (3.81)
$$

For $0 \leq l \leq L-1$, we can write

$$C_{k,k}(l) = \sum_{j=0}^{L-l-1} s_j^{(k)} s_{j+l}^{(k)} = \begin{cases} \sum_{j=0}^{L-l-1} v_j u_{k+j} v_{j+l} u_{k+j+l} & k = 0, 1, \cdots, N-2 \\ \\ \sum_{j=0}^{L-l-1} v_j v_{j+l} & k = N-1 \end{cases} \tag{3.82}$$

then we have

$$\begin{aligned} \sum_{k=0}^{N-1} C_{k,k}(l) &= \sum_{k=0}^{N-2} \sum_{j=0}^{L-l-1} v_j u_{k+j} v_{j+l} u_{k+j+l} + \sum_{j=0}^{L-l-1} v_j v_{j+l} \\ &= \sum_{j=0}^{L-l-1} v_j v_{j+l} \sum_{k=0}^{L-1} u_{k+j} u_{k+j+l} + \sum_{j=0}^{L-l-1} v_j v_{j+l} \\ &= C[v](l)[1 + \theta[u](l)] \end{aligned} \tag{3.83}$$

where $C[v](l)$ is the aperiodic auto-correlation function of the sequence v and $\theta[u](l)$ is the even correlation function of the sequence u defined by

$$\theta[u](l) = C[u](l) + C[u](l-L) \tag{3.84}$$

It is easy to show that (3.83) is also valid for $1 - L \leq l < 0$. Since u is an m-sequence,

$$\theta[u](l) = \begin{cases} -1 & 0 < |l| < L-1 \\ L & l = 0 \end{cases} \tag{3.85}$$

We see that

$$\sum_{k=0}^{N-1} C_{k,k}(l) = \begin{cases} 0 & 0 < |l| < L-1 \\ NL & l = 0 \end{cases} \tag{3.86}$$

Substituting (3.86) into (3.79), we have

$$I_m = 4L + C_{m,m}(1) + C_{m+1,m+1}(1) \tag{3.87}$$

Now I_{av} can be evaluated easily since $\sum_{m=0}^{N-1} C_{m,m}(1) = \sum_{m=0}^{N-1} C_{m+1,m+1}(1) = 0$

$$I_{av} = 4L \tag{3.88}$$

which is smaller than the expectation value for the random code.

If v is one period of an m-sequence of period L, and u, v form a preferred pair of m-sequence, then each codeword $s^{(k)}$ consists of one period of a Gold sequence[23, 24]. In this special case, bounds on I_m can be obtained. Since

$$C_{m,m}(1) + C_{m,m}(1-L) = \theta_{m,m}(1) \tag{3.89}$$

It is known that for a Gold sequence, $\theta_{m,m}(1)$ can only take one of the three values: $-t(n), -1$, and $t(n)-2$, where $t(n) = 1 + 2^{\lfloor (n+2)/2 \rfloor}$ and $n = log_2(L+1)$, $\lfloor \alpha \rfloor$ denotes the integer part of α. This indicates

$$-t(n) \leq \theta_{m,m}(1) \leq t(n) - 2 \tag{3.90}$$

It is clear that $C_{m,m}(1-L)$ must have value 1 or -1. From (3.89), we can see

$$-2^{\lfloor (n+2)/2 \rfloor} - 2 \leq C_{m,m}(1) \leq 2^{\lfloor (n+2)/2 \rfloor} \tag{3.91}$$

Similarly, we can show

$$-2^{\lfloor (n+2)/2 \rfloor} - 2 \leq C_{m+1,m+1}(1) \leq 2^{\lfloor (n+2)/2 \rfloor} \tag{3.92}$$

Combining (3.87) with (3.91) and (3.92), we obtain the bounds for I_m

$$4L - 2(2^{\lfloor (n+2)/2 \rfloor} + 2) \leq I_m \leq 4L + 2^{\lfloor (n+2)/2 \rfloor + 1} \tag{3.93}$$

With the second method, commutation signaling code $\{s^{(0)}, s^{(1)}, \cdots, s^{(N-1)}\}$ of size N and length L are constructed from an m-sequence of period N where L and N are relatively prime and $L < N$. Suppose $\mathbf{x}$ is a vector of length N given by $\mathbf{x} = (x_0, x_1, \cdots, x_{N-1})$ and x is an (infinitely long) m-sequence of period N obtained by repeating the vector $\mathbf{x}$ over and over again. Let the codeword $s^{(k)}$ be defined as

$$s^{(k)} = (x_{kL}, x_{kL+1}, \cdots, x_{(k+1)L-1}) \tag{3.94}$$

Since N and L are relatively prime, there are only N distinct codewords defined by (3.94) and $s^{(k)} = s^{(k+N)}$. Furthermore, the starting elements of the N codewords, $x_{kL}, k = 0, 1, \cdots N-1$, are just the elements $x_k, k = 0, 1, \cdots N-1$, in a different order. This can be shown as following. First, we notice that the index of every x_{kL} modulo N, that is, kL mod. N must be the index of some x_k. On the other hand, since N and L are relatively prime, there exist integers a and b so that $1 = aL + bN$. It follows that $k = kaL$ mod. N, which indicates that every index of x_k is a multiple (modulo N) of the index of the starting element for some codeword.

Based on the above arguments, we have for $0 \leq l \leq L-1$

$$\begin{aligned}
\sum_{k=0}^{N-1} C_{k,k}(l) &= \sum_{k=0}^{N-1} \sum_{n=0}^{L-l-1} s_n^{(k)} s_{n+l}^{(k)} = \sum_{n=0}^{L-l-1} \sum_{k=0}^{N-1} x_{kL+n} x_{kL+n+l} \\
&= \sum_{n=0}^{L-l-1} [\sum_{k=0}^{N-1} x_{k+n} x_{k+n+l}] = (L-l)\theta[\mathbf{x}](l) \qquad (3.95)
\end{aligned}$$

where $\theta[\mathbf{x}](l)$ is the even correlation function of the vector $\mathbf{x}$.

similarly, we can show that for $1 - L \leq l < 0$

$$\sum_{k=0}^{N-1} C_{k,k}(l) = (L+l)\theta[\mathbf{x}](l) \qquad (3.96)$$

Thus, for $0 \leq |l| \leq L-1$, we have

$$\sum_{k=0}^{N-1} C_{k,k}(l) = (L-|l|)\theta[\mathbf{x}](l) \qquad (3.97)$$

Substituting (3.97) into (3.79), we obtain

$$\begin{aligned}
I_m = \frac{1}{NL} \sum_{l=1-L}^{L-1} \quad & [2C_{m,m}(l) + C_{m,m}(l+1) + 2C_{m+1,m+1}(l) + C_{m+1,m+1}(l+1)] \\
& (L-|l|)\theta[\mathbf{x}](l) \qquad (3.98)
\end{aligned}$$

Using (3.97), it is easy to show

$$I_{av} = \frac{1}{N^2 L} \sum_{l=1-L}^{L-1} \{4(L-|l|)\theta[\mathbf{x}](l) + 2(L-|l+1|)\theta[\mathbf{x}](l+1)\}(L-|l|)\theta[\mathbf{x}](l) \quad (3.99)$$

Since $\theta[\mathbf{x}](0) = N$ and $\theta[\mathbf{x}](-l) = \theta[\mathbf{x}](l)$, we can write (3.99) as

$$\begin{aligned} I_{av} &= \frac{4}{N^2 L}[N^2 L^2 + NL(L-1)\theta[\mathbf{x}](1)] \\ &+ \frac{4}{N^2 L}\sum_{l=1}^{L-1}[2(L-l)^2\theta[\mathbf{x}]^2(l) + (L-l-1)(L-l)\theta[\mathbf{x}](l+1)\theta[\mathbf{x}](l)] \end{aligned} \quad (3.100)$$

Because x is an m-sequence of period N, we have $\theta[\mathbf{x}](l) = -1$ for $0 < l < N$, then (3.100) simplifies to

$$I_{av} = 4L - \frac{4(L-1)(N-L+1)}{N^2} \quad (3.101)$$

It can be seen that I_{av} is even smaller than that obtained in the first method, and of course it is smaller than the expectation value for the random sequences. However, bound on I_m can not be established. Moreover, for a codebook constructed in this way, the multipath interference is large at some delays. Look at the N codewords we have: $(x_0, x_1, \cdots, x_{L-1})$, $(x_1, x_2, \cdots, x_L)$, $(x_2, x_3, \cdots, x_{L+1})$, $\cdots$, $(x_{N-2}, x_{N-1}, \cdots, x_{N+L-3})$, $(x_{N-1}, x_N, \cdots, x_{N+L-2})$. It is clear that the first $L-1$ elements of a codeword are identical to the last $L-1$ elements of another codeword (for example, x_1 to x_{L-1} are the last $L-1$ elements of the codeword $(x_0, x_1, \cdots, x_{L-1})$ and the first $L-1$ elements of the codeword $(x_1, x_2, \cdots, x_L)$). Therefore, no matter how we order these codewords in a codebook, at certain delays, the cross-correlation between two codewords is large due to the $L-1$ same elements in the two codewords, leading to large multipath interference.

For these two construction methods, the size and the length of the codebook are very limited. With the first method, the length L of the codewords can only be $2^n - 1$, where n is an integer, and the size N of the codebook has to be $L+1$. For the second method, the size N of a codebook must be $2^n - 1$ with n being an integer, and the length L can not be larger than N. Furthermore, N and L need to be relatively prime. In both cases, the length L has to be smaller than size N.

3.2.2 Code Construction Based on Worst Case Performance

It has been shown that to construct a commutation signaling code $\{s^{(0)}, s^{(1)}, \cdots, s^{(N-1)}\}$ that minimizes the maximum conditional error probability, the code parameter $max(|f_{m1}| + |f_{m2}|)$, which characterizes the maximum interference due to multipath, should be minimized for $0 \leq m, k \leq N-1$, $0 \leq l \leq L-1$ ($l \neq 0$ for $k=0$). In fact, we may neglect the constant T_c in (3.56), thus $max(|f_{m1}| + |f_{m2}|)$ only contains the aperiodic crosscorrelations between the commutation signaling codewords.

$$\begin{aligned} max(|f_{m1}| + |f_{m2}|) &= max\{|C_{N-(k+1)+m,m}(l-L) + C_{N-k+m,m}(l) \\ &+ C_{N-k+m,m+1}(l-L) + C_{N-(k-1)+m,m+1}(l)| + |C_{m,m+k+1}(l-L) \\ &+ C_{m,m+k}(l) + C_{m+1,m+k+2}(l-L) + C_{m+1,m+k+1}(l)|\} \end{aligned} \tag{3.102}$$

In this work, only the two phase and four phase codes are considered. The codewords $\{s^{(0)}, s^{(1)}, \cdots, s^{(N-1)}\}$ of length L are selected from all possible codewords of length L and they have to satisfy the following two conditions (for the two phase case, the codewords may also be selected from subsequences of long m-sequences). First, $max(|f_{m1}| + |f_{m2}|)$ can not be larger than a bound we set when $0 \leq m, k \leq N-1$, and $0 \leq l \leq L-1$ ($l \neq 0$ for $k=0$). The bound depends on the length of the codewords and the maximum conditional error probability. Second, $|C_{N-(k+1)+m,m}(l-L) \pm C_{N-k+m,m}(l)|$, $|C_{N-k+m,m+1}(l-L) \pm C_{N-(k-1)+m,m+1}(l)|$, $|C_{m,m+k+1}(l-L) \pm C_{m,m+k}(l)|$ and $|C_{m+1,m+k+2}(l-L) \pm C_{m+1,m+k+1}(l)|$ can not be larger than another bound smaller than L. This is to ensure that each codeword is different from any segment of length L from the long sequence $\{s^{(0)}, s^{(1)}, \cdots, s^{(N-1)}, s^{(0)}\}$ (except itself, of course). Let the codeword $s^{(m)} = (s_0^{(m)}, s_1^{(m)}, \cdots, s_{L-1}^{(m)})$ where $s_i^{(m)}$ can be ± 1 or $\pm i$. For binary (or two phase) case, the second condition eliminates the possibility that any segment of length L is the same as $-s^{(m)} = (-s_0^{(m)}, -s_1^{(m)}, \cdots, -s_{L-1}^{(m)})$. While for four phase case, it eliminates the possibility that any segment of length L is the same as $\pm[s^{(m)}]^* = (\pm[s_0^{(m)}]^*, \pm[s_1^{(m)}]^*, \cdots, \pm[s_{L-1}^{(m)}]^*)$.

It should be emphasized that although $max(|f_{m1}| + |f_{m2}|)$ is the code parameter that determines the maximum conditional error probability, the second condition is necessary. Firstly, since the probability of error is an average of the conditional error probability, the maximum probability of error does not only depends on the maximum conditional error probability, it also depends on the distribution of the conditional error probability. By setting a limit to the individual terms in f_{m1} and f_{m2}, it is aimed to minimize the non-maximum conditional error probability, thus, to minimize the maximum probability of error. Secondly, with the second condition, if we have a codeword $s^{(m)}$, it is not possible for $-s^{(m)}$ (two phase case) or $\pm[s^{(m)}]^*$ (four phase case) to be another codeword. Therefore, we can work directly on codebooks of length L instead of working on differential codebooks of length $L-1$[32]. It also should be point out that the probability of error of the system depends on the order of the codewords in the commutation signaling codebook. With our construction procedure, the codewords and the ordering of them can be considered at the same time.

The codes are constructed by computer programs that implement above design rules. Consider the construction of a binary codebook with length L and size N from a m-sequence $\mathbf{x}$ of period P. Let $\mathbf{x} = (x_1, x_2, \cdots, x_{P-1}, x_P)$. We begin the construction by setting $s^{(0)} = (x_1, x_2, \cdots, x_L)$, then $|C_{0,0}(l-L) \pm C_{0,0}(l)|$ is tested for $l = 1, 2, \cdots, L-1$. If they are not above the bound we set, we have the first codeword and start to search second codeword by assigning $(x_{L+1}, x_{L+2}, \cdots, x_{2L})$ to $s^{(1)}$. This time, the two conditions have to be checked for all possible m, k and l. If one condition is not satisfied at certain m, k or l, $s^{(1)}$ will be re-assigned as $(x_{L+2}, x_{L+3}, \cdots, x_{2L+1})$ and the conditions will be tested again. Otherwise, we have $s^{(1)} = (x_{L+1}, x_{L+2}, \cdots, x_{2L})$ and $s^{(2)}$ will be searched. The process continues until N codewords are selected or one period of the m-sequence has been searched. In a more general method, all binary or four-phase words are considered. Here, a codeword is not selected from subsequences of a long m-sequence, instead, it is selected from all possible words for a given length. The same conditions are tested and the construction process is terminated when N codewords are selected or all possible words are searched.

Chapter 4

Performance of Selected Codes

As we mentioned before, the average SNR that we considered as a measure of the average performance is a statistical average over the time delay and data symbols. Consequently, the codes constructed based on the average performance may not ensure an acceptable probability of error for all time delays. In fact, we have calculated the probability of error for a codebook constructed based on the average performance using the first method. The codebook is obtained from a preferred pair of m-sequences of period 7. One period of the m-sequences are given as u = (1 1 0 0 1 0 1) and v = (1 1 0 1 0 0 1). In this case, the codewords are Gold sequences, and we know that the average multipath interference parameter I_{av} for the codebook is $4L = 28$ (L is the length of the codebook). The maximum multipath interference parameter, $max(|f_{m1}| + |f_{m2}|)$, is also calculated in order to compare with that of the codebooks constructed based on the worst case performance. Since the possible maxima of $|C_{i,j}(l)|$ and $|C_{i,j}(l - L)|$ are $L - l$ and l respectively, the possible maximum of $|f_{m1}| + |f_{m2}|$ is $4L$. To facilitate comparison between codes with different lengthes, $max(|f_{m1}| + |f_{m2}|)$ will be normalized by dividing it by $4L$ and will be referred to as I_{max}. For this codebook of length 7, we have $I_{max} = 0.71$. The error probability for the codebook is shown in Fig.4.1. It can be seen that although the probability of error is below 5x10^{-4} for most values of delay, the maximum error probability is much higher and reachs 5x10^{-3}. Since we are mainly interested in the codes for which the maximum probability of error is below a given limit, we will only consider the codes constructed based the worst case performance in the following.

Binary commutation signaling codes of length 5, 6, 7, 8, and 9 (with size of 3, 5, 8, 12 and 16 respectively) are generated using computer programs outlined before. For four phase codes, codebooks with the same size of the binary codebooks have been obtained, but the length of the codes are reduced by 1 comparing with the binary codebooks. That is, the four phase codebooks are of length 4 and size 3, length 5 and size 5, length 6 and size 8, length 7 and size 12, length 8 and size 16. The bit error probability as a function of the normalized delay for different codes are presented here. For most of the codes, we have used an average SNR of 23 dB and equal average SNR in different waves, which implies an average SNR of 10 dB for each of the two waves.

Fig.4.2 shows the probability of error for a binary codebook of length 5 and size 3. The normalized maximum multipath interference parameter I_{max}=0.6, in contrast to that of 0.71 for the codebook constructed based on the average performance, while the average multipath interference parameter I_{av} is 20.356, which is 4.07L, very close to 4L achieved by the codebook constructed based on the average performance. It is seen that when the inter-wave delay is between 10% to 285% of the symbol duration, the probability of error never exceeds 3x10^{-4} for SNR = 23dB. For the ideal differentially coherent detection of DPSK with equal weight two-fold diversity, the probability of error is about 0.8x10^{-4} at the SNR of 23dB, and the error rate of 3x10^{-4} reflects a 3dB degradation[33]. This means that this codebook can be used for a delay spread of nearly three times the symbol duration with a performance degradation of not more than 3dB. The probability of error at SNR of 18dB and 13dB are also shown in the same figure. It is seen that when SNR decreases by 5dB, the probability of

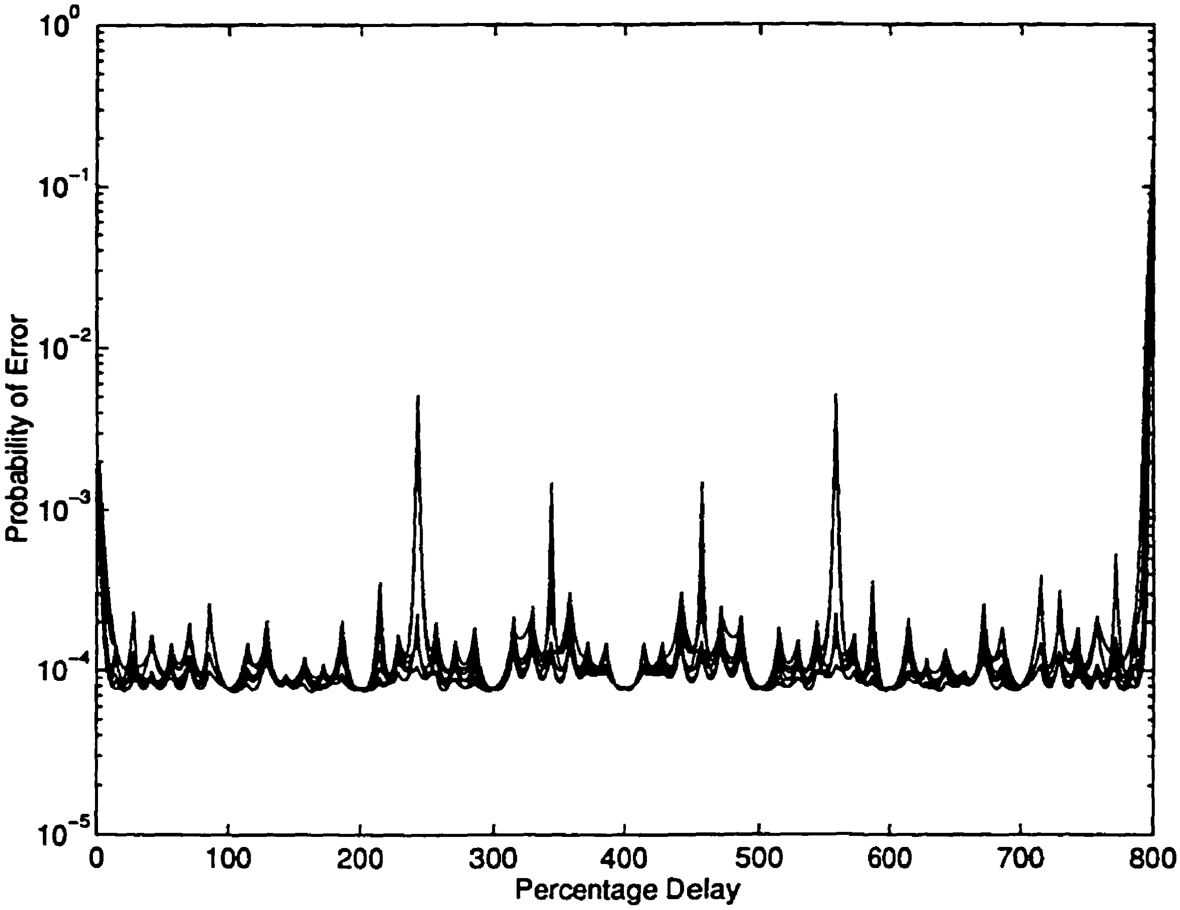

Figure 4.1: Probability of error as a function of the normalized delay for a binary codebook of length 7 and size 8. The codebook is constructed based on the average performance. SNR = 23dB. Codebook: [0001100, 0100010, 1111110, 1000111, 0110101, 1010000, 0011011, 1101001]

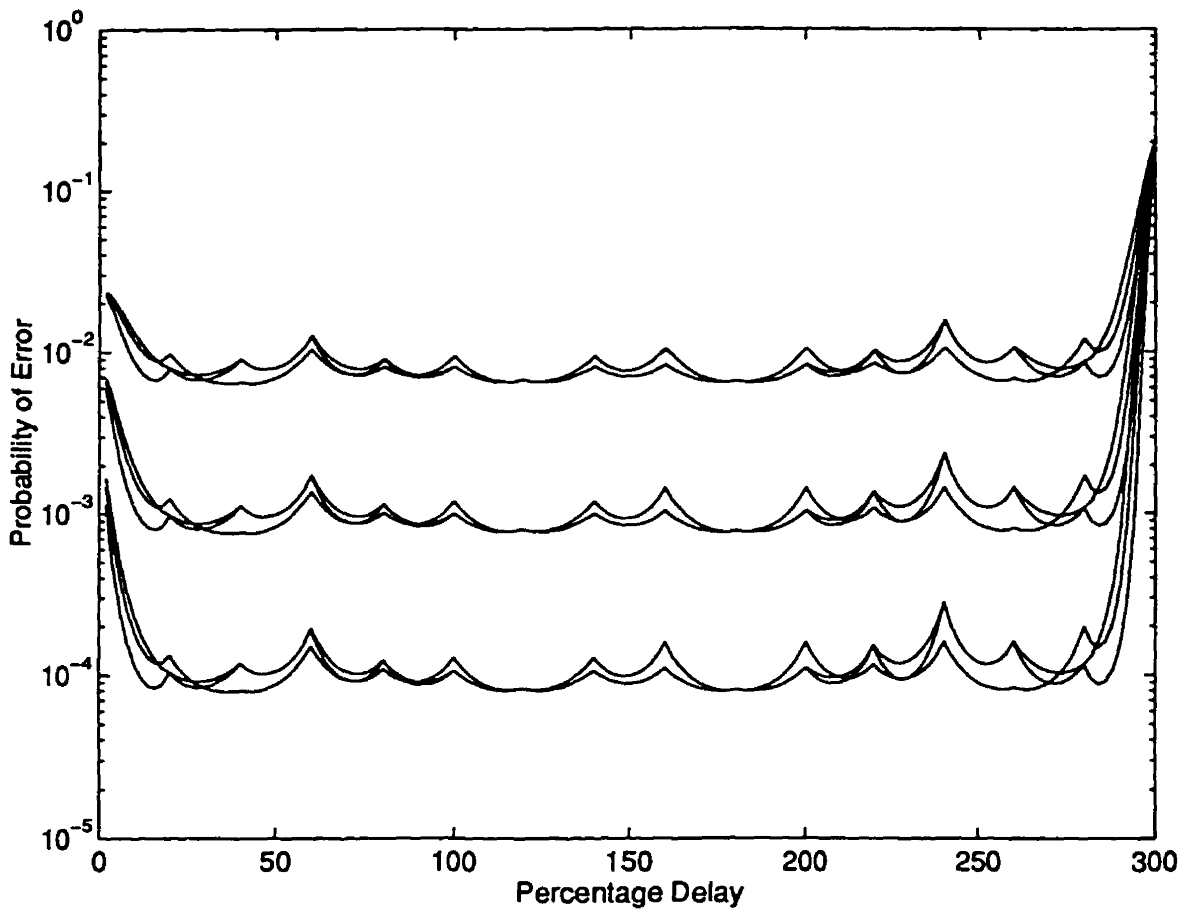

Figure 4.2: Probability of error as a function of the normalized delay for a binary codebook of length 5 and size 3. SNR is 13dB, 18dB and 23dB for the curves on the top, in the middle and at the bottom respectively. Codebook: [11110, 00110, 00101]

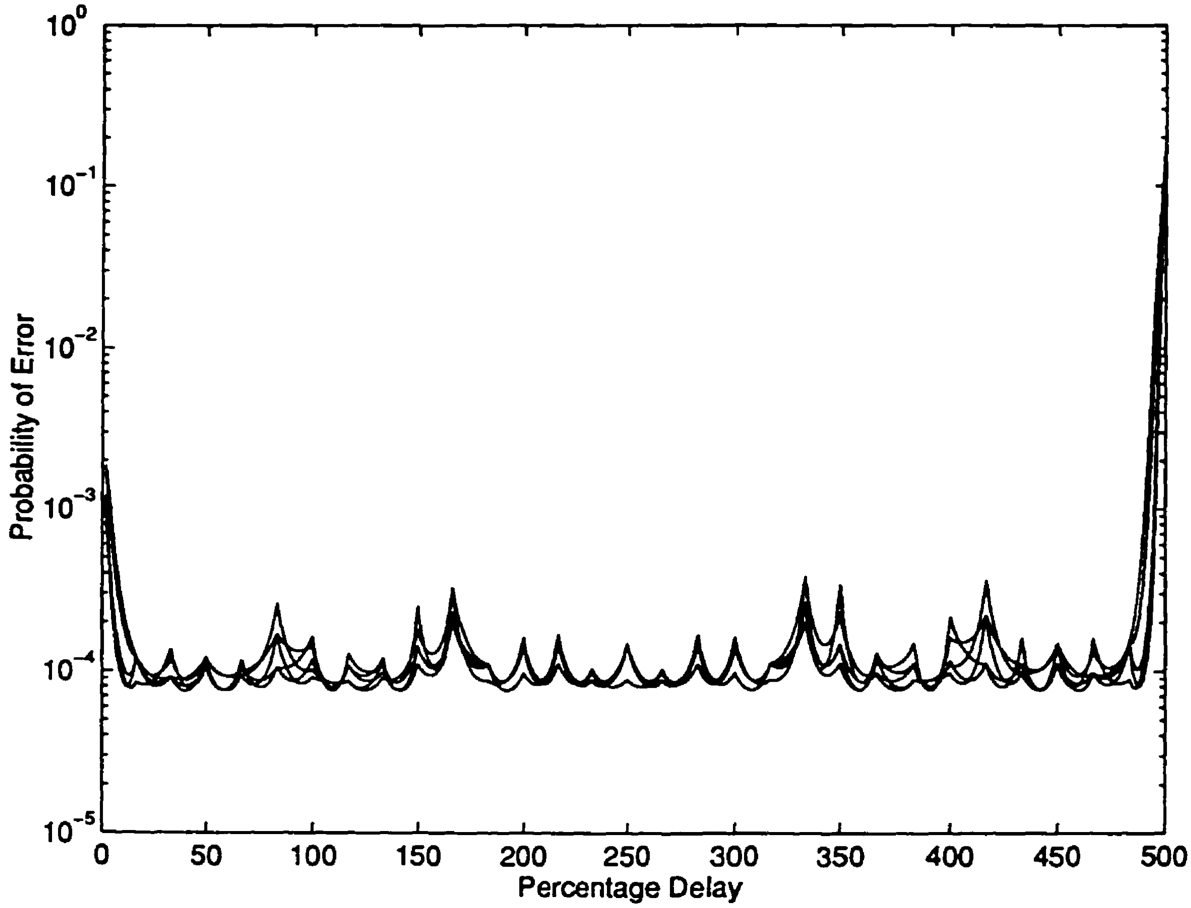

Figure 4.3: Probability of error as a function of the normalized delay for a binary codebook of length 6 and size 5. SNR is 23dB. Codebook: [111000, 110010, 011101, 101001, 111101]

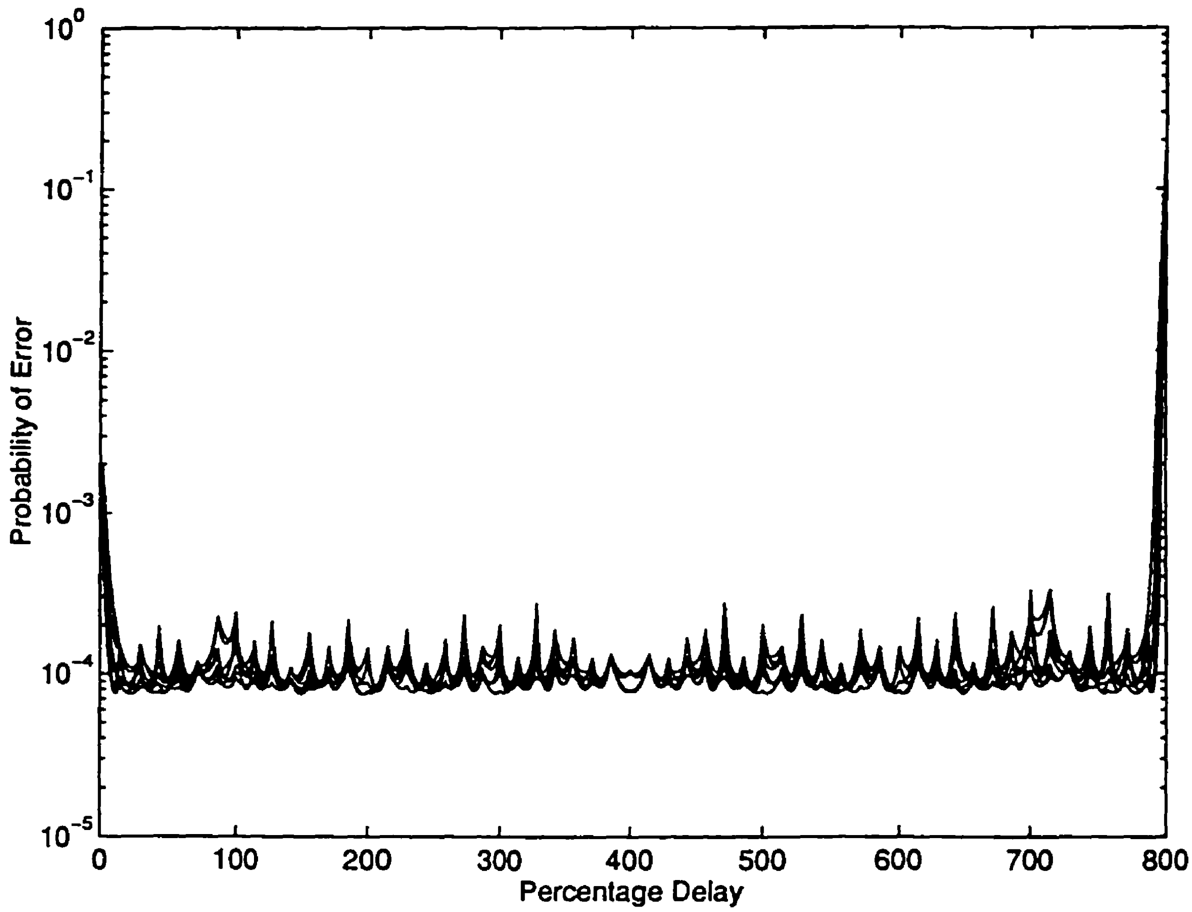

Figure 4.4: Probability of error as a function of the normalized delay for a binary codebook of length 7 and size 8. SNR is 23dB. Codebook: [1100010, 1100001, 0111001, 1001010, 1000001, 1111110, 1011101, 0100110]

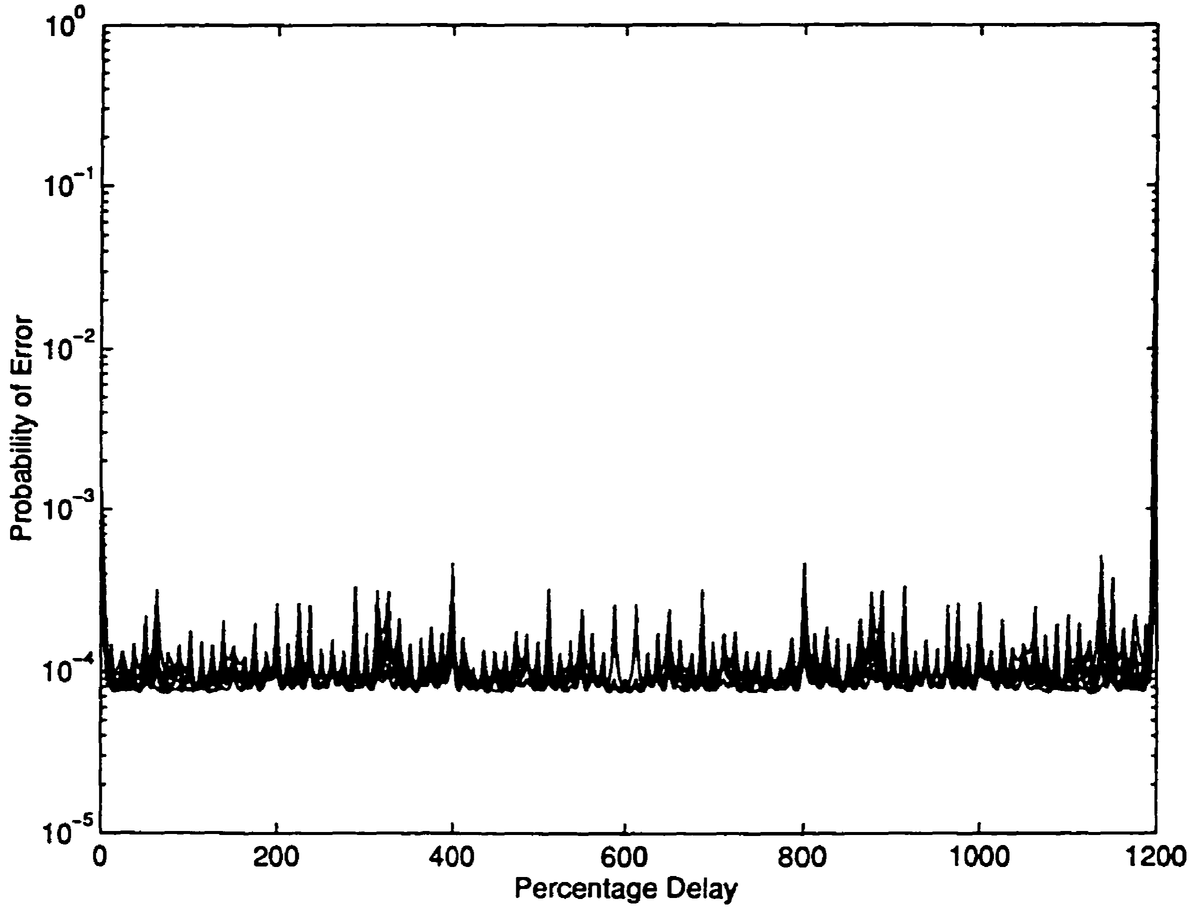

Figure 4.5: Probability of error as a function of the normalized delay for a binary codebook of length 8 and size 12. SNR is 23dB. Codebook: [11100100, 01100111, 11000101, 10000111, 11100101, 01010010, 00110010, 11111101, 11011010, 01110111, 10111110, 00001010]

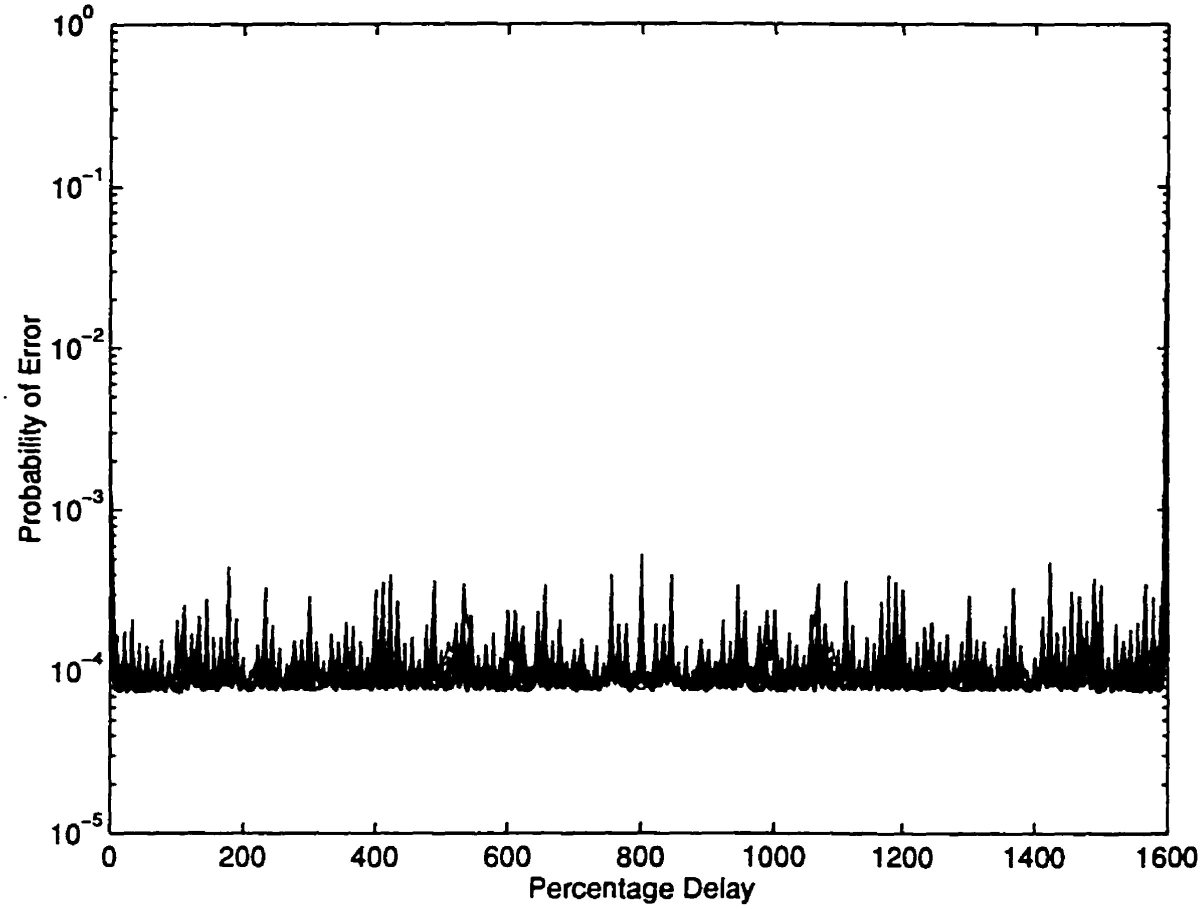

Figure 4.6: Probability of error as a function of the normalized delay for a binary codebook of length 9 and size 16. SNR is 23dB. Codebook: [101001101, 111110111, 001110100, 010011110, 101010100, 000101011, 010000000, 100110000, 011010100, 100111011, 001010010, 000011000, 111001000, 000111011, 011110000, 011011011]

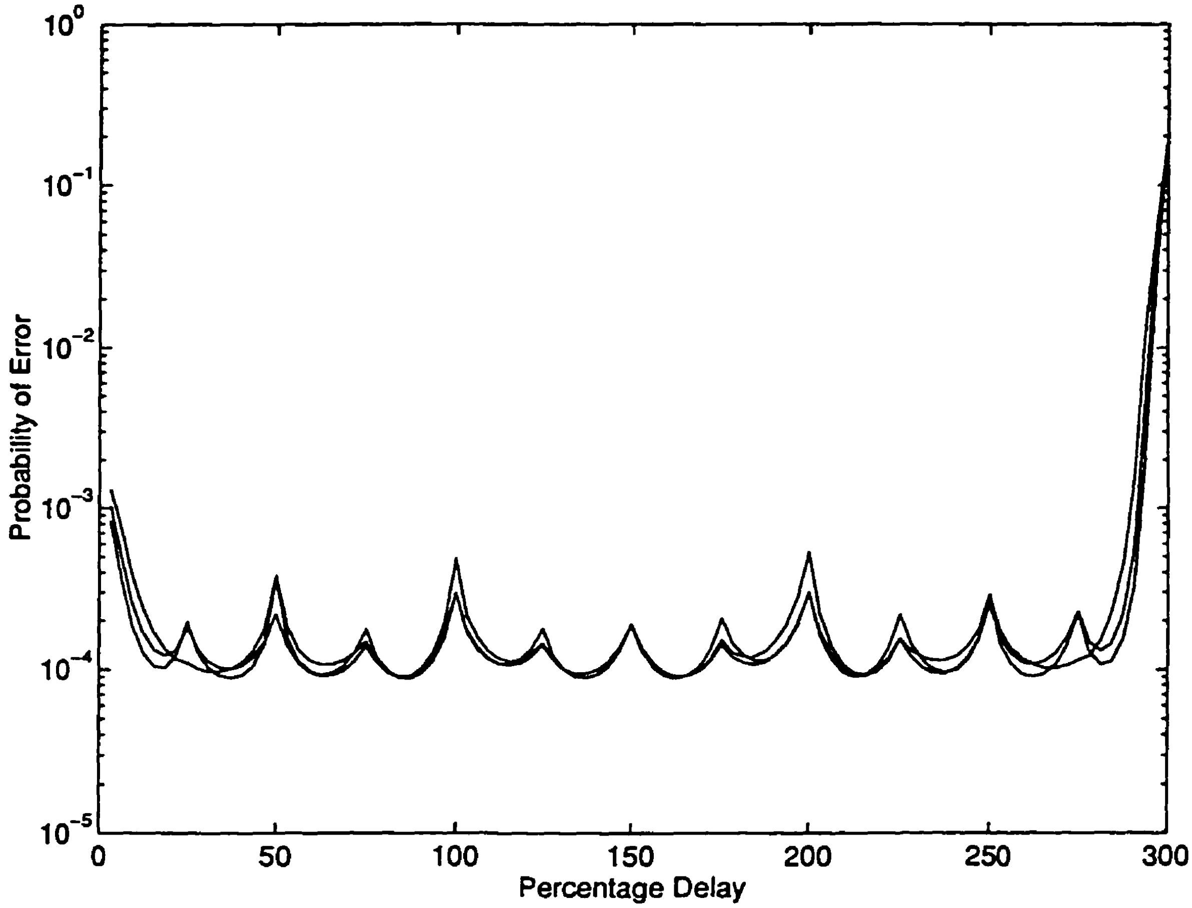

Figure 4.7: Probability of error as a function of the normalized delay for a four phase codebook of length 4 and size 3. SNR is 23dB. Codebook: [1132, 3132, 3303]

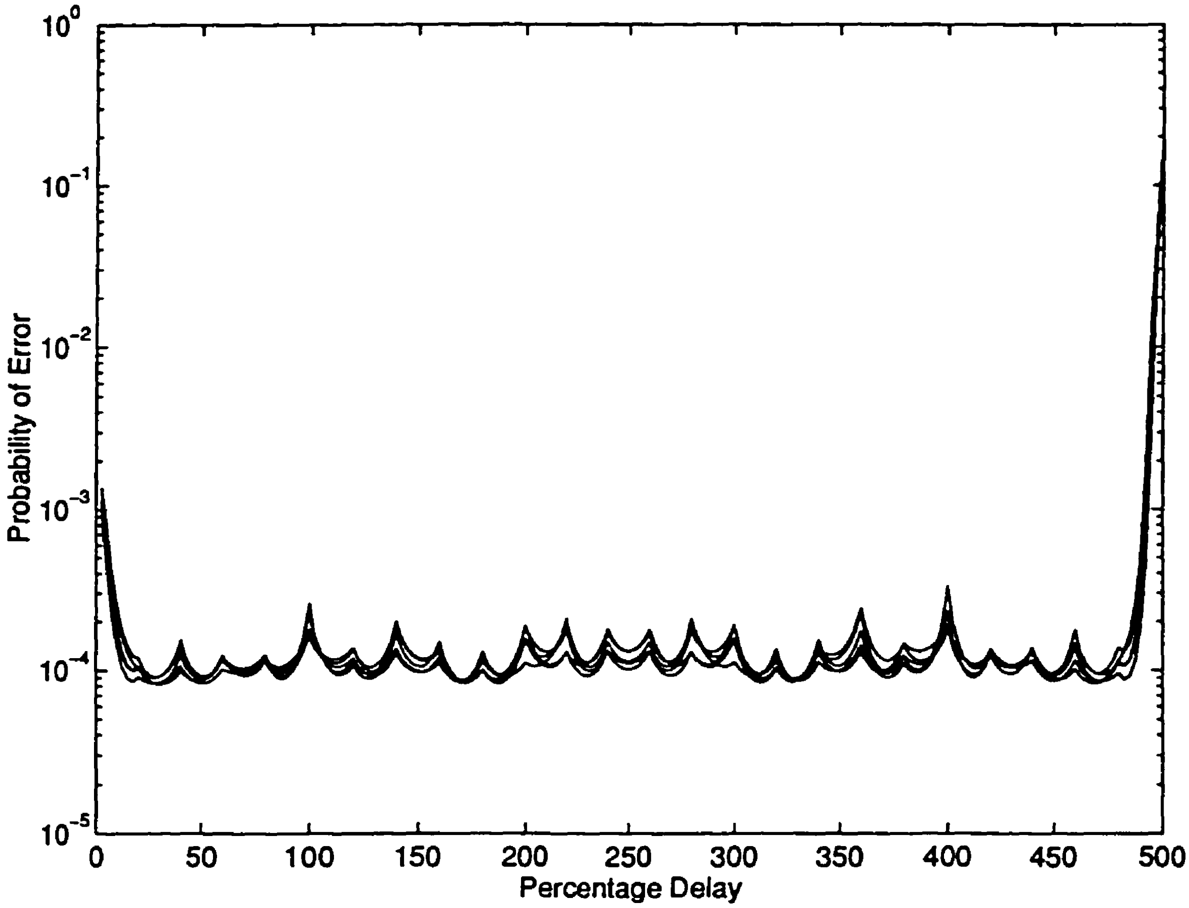

Figure 4.8: Probability of error as a function of the normalized delay for a four phase codebook of length 5 and size 5. SNR is 23dB. Codebook: [13212, 00312, 23312, 30032, 32000]

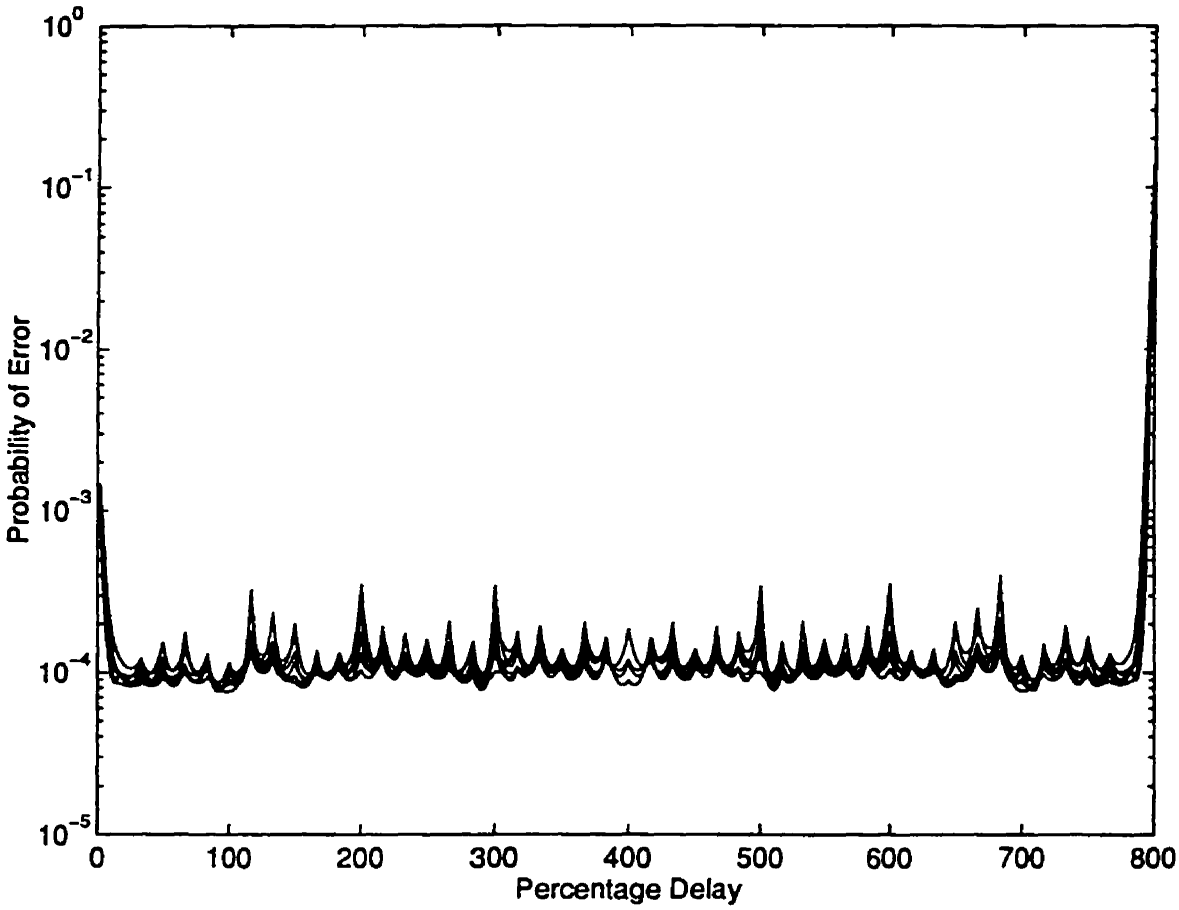

Figure 4.9: Probability of error as a function of the normalized delay for a four phase codebook of length 6 and size 8. SNR is 23dB. Codebook: [110103, 320000, 002010, 113000, 101200, 331310, 110200, 311030]

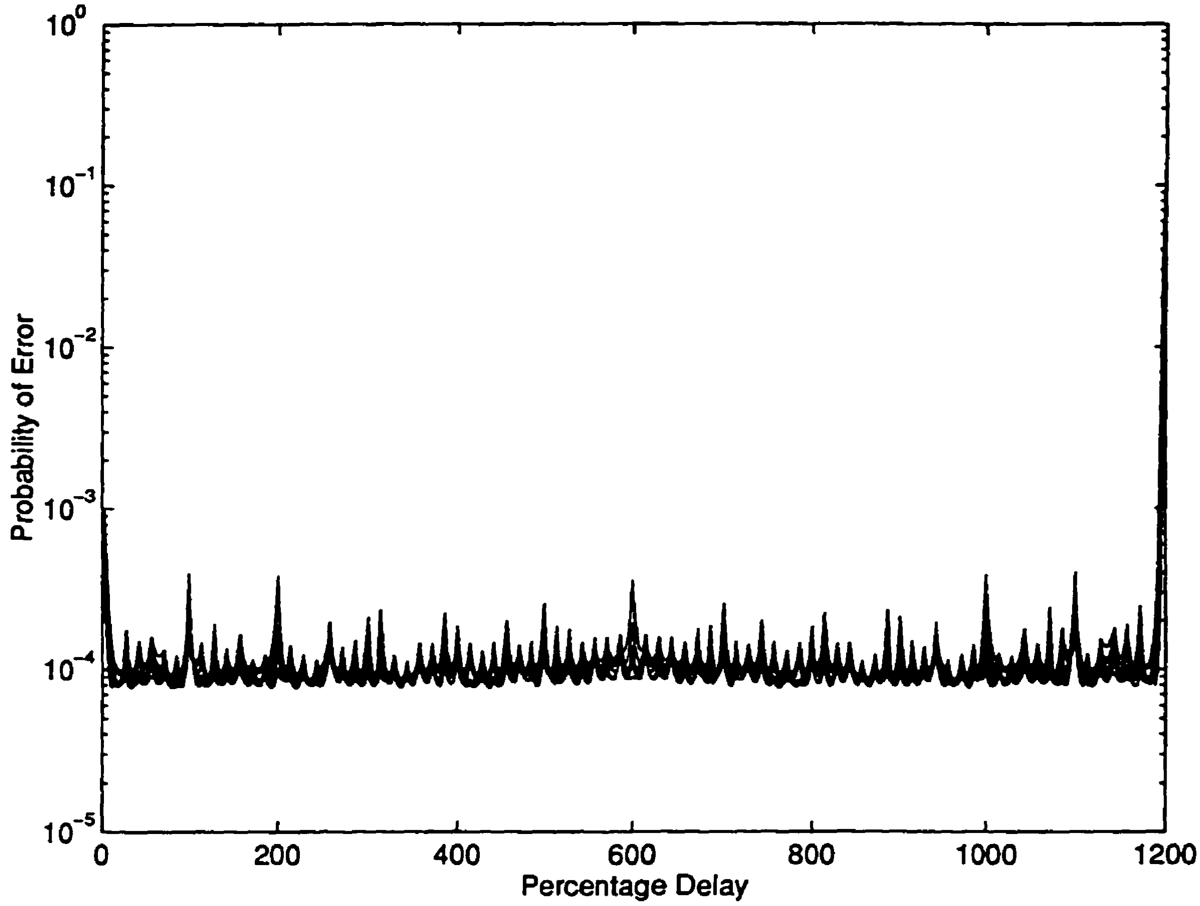

Figure 4.10: Probability of error as a function of the normalized delay for a four phase codebook of length 7 and size 12. SNR is 23dB. Codebook: [3330030, 0201030, 0011030, 2211030, 1311030, 2132030, 1232030, 2003030, 2001130, 3311130, 0132130, 0200301]

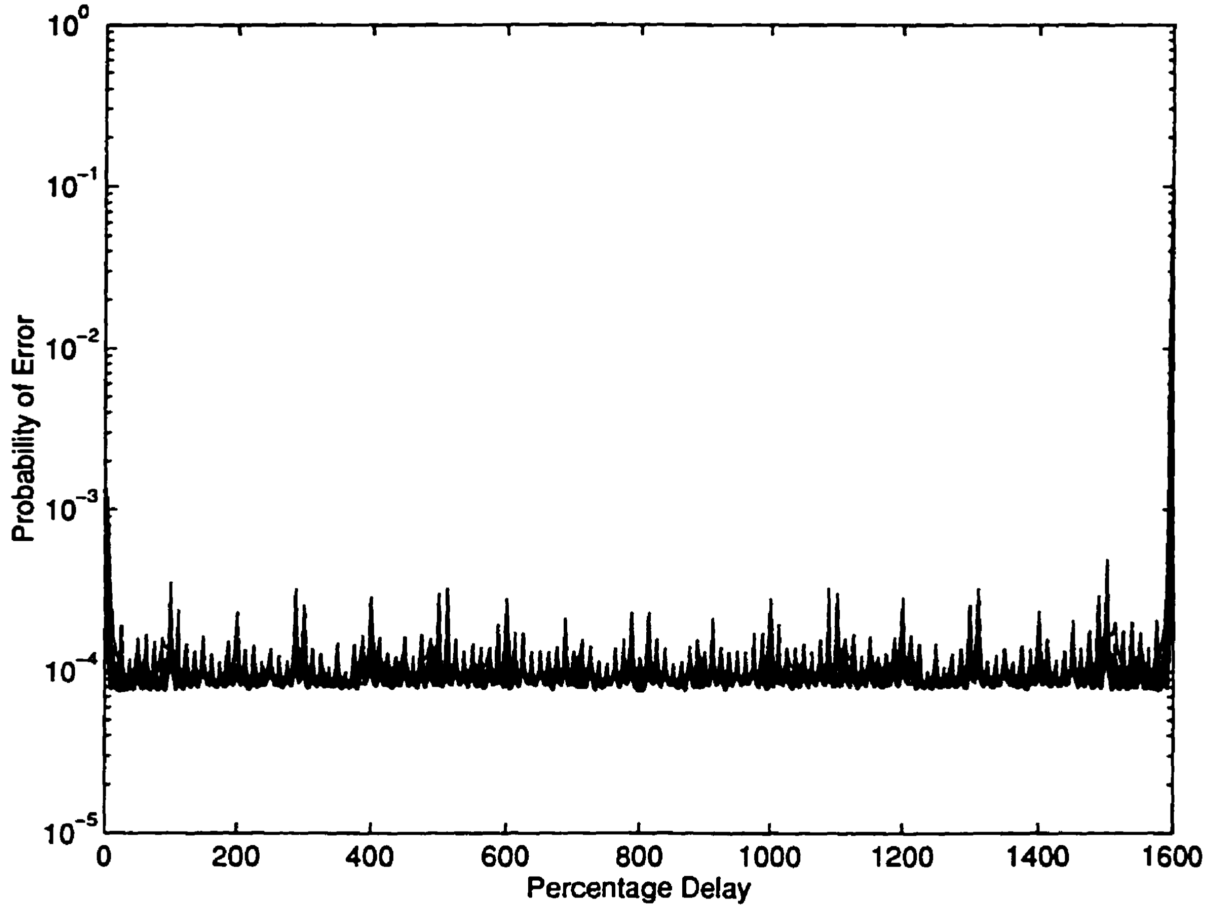

Figure 4.11: Probability of error as a function of the normalized delay for a four phase codebook of length 8 and size 16. SNR is 23dB. Codebook: [30111011, 21110111, 23110311, 10213311, 20210021, 02212121, 13213221, 00312321, 01312031, 22310231, 03312231, 00022331, 23220322, 00232313, 21303130, 33020012]

error increases approximately by an order of magnitude, but the shapes of the error probability curves remain essentially the same.

The performance for a binary codebook of length 6 and size 5 is quite similar to that of the previous codebook (see Fig.4.3). The normalized maximum multipath interference parameter I_{max} is 0.67. The average multipath interference parameter I_{av}=3.95L is a little smaller than that of the previous codebook, and it is much smaller than the expectation value of I_{av} (4.67L) for the random codebook. The error probability is below $3\text{x}10^{-4}$ at SNR of 23dB for most values of the inter-wave delay. Thus, the performance degradation is similar to that of the previous codebook, but the codebook can be used for inter-wave delay of about five times the symbol duration.

The probability of error for a codebook of length 7 and size 8 is shown in Fig.4.4. For this codebook, we have I_{max}=0.64 and I_{av}=4.04L. I_{av} is smaller than the expectation value for the random codebook, which is 4.43L, and is almost the same as that for the codebook constructed based on the average performance ($4.0L$). Comparing the probability of error for the two codebooks designed based on worst case performance and average performance (with the same size and length), it is seen that the maximum probability of error for the worst case performance codebook is only $3.28\text{x}10^{-4}$, much smaller than that for the average performance codebook ($5\text{x}10^{-3}$) at SNR of 23dB. One of the reason for this is that the normalized maximum multipath interference parameter for the worst case performance codebook is smaller than that for the average performance codebook. However, the more important reason is that the second condition used for constructing the worst case performance codebooks makes the non-maximum conditional error probability smaller, and therefore reduces the maximum probability of error, which is an average of conditional error probability.

In Fig.4.5 and Fig.4.6, the probability of error for codebooks of length 8 and size 12, length 9 and size 16 are shown. The normalized maximum multipath interference parameter I_{max} and the average multipath interference parameter I_{av} for these codebooks as well as the other codebooks are listed in Table 4.1. It is seen that for many codebooks, I_{av} is smaller than that of the codebook constructed based on average performance. For almost all of the codebooks, I_{av} is smaller than the expectation value for the random codebook, and the maximum probability of error is less than 5×10^{-4} for SNR = 23dB.

The probability of error for four phase codebooks are shown in Fig.4.7 to Fig.4.11. Since the number of possible four phase words are much larger than that of two phase words for a given length, it is expected that the size of the four phase codebooks is larger than that of two phase (or binary) codebooks. In fact, this is what we obtained. Fig.4.7 shows the probability of error for a four phase codebook of length 4 and size 3. It is seen that for most values of the inter-wave delay, the probability of error is below 3×10^{-4}. This means that the error probability performance is very close to that of the binary codebooks of length 5 and size 3. Although the average multipath interference parameter I_{av}=4.39L is larger than that (4.07L) of a binary codebook of the same size, for which the probability of error is shown in Fig.3, it is smaller than the expectation value of the random codebook (5.00L) and the length of the four phase codebook is decreased by 1 comparing to the binary codebook of the same size.

In Fig.4.8, we plot the probability of error for a four phase codebook of length 5 and size 5. Again, the average multipath interference parameter I_{av}=4.27L is large comparing to that (3.95L) of the binary codebook1 of the same size (see Table 4.1), and is smaller than the expectation value of the random codebook. The maximum probability of error for inter-wave delay between 15% and 485% of the symbol duration is 3.33×10^{-4}. Comparing to the binary codebook of the same size, the length of the four phase codebook is reduced by 1. If codebooks with the same length 5 are compared, we see that the size of the four phase codebook is larger than that of the binary codebook. In fact, the conclusion is true for all codebooks with different lengthes and sizes.

The probability of error for four phase codebooks of length 6 and size 8, length 7 and size 12 as well as length 8 and size 16 are shown in Fig.4.9, Fig.4.10 and Fig.4.11 respectively. The maximum probability of error for all of the three codebooks is less than 5.0x10^{-4}. It can be seen that for all of the binary and four phase codebooks presented here, the error probability performance shows the same features: first, the maximum probability of error is essentially not larger than 5.0x10^{-4}; second, for most values of inter-wave delay, the probability of error is below 3x10^{-4}. Therefore, with these codebooks, the performance degradation with respect to ideal two-fold diversity is essentially not more than 3dB, while the system can cope with a multipath delay spread of up to sixteen times the symbol duration.

From table 4.1 and table 4.2, it is seen that the average multipath interference parameter I_{av} for the four phase codebooks is usually larger than that for the binary codebooks. However, for most four phase codebooks, I_{av} is still smaller than the expectation value of the random codebook. Moreover, the maximum probability of error for the four phase codebooks does not increase comparing to that of the binary codebooks. It should be emphasized that for each given length, the size of the four phase codebooks is always larger than that of the binary codebooks. The increase of codebook size is 2 for length of 5 and 4 for length of 8. This indicates that for the same bandwidth expansion, the anti-multipath capability can be increased by using four phase codes. Or equivalently, we can say that for the same size of codebooks, the length of the four phase codes can be reduced by 1 compared to that of the two phase codes while the system performance in terms of probability of error is essentially the same. This means that with four phase codes, less bandwidth expansion is required to achieve the same anti-multipath capability and error probability performance compared with the binary codes.

A hybrid multiple access scheme using commutation signaling has been proposed

Table 4.1: The average multipath interference parameter I_{av} and the normalized maximum multipath interference parameter I_{max} for different binary codebooks. Codebook1 refers to the first codebook in appendix, for which the probability of error is plotted here. Codebook2, codebook3 and codebook4 refer to the second, the third and the fourth codebooks listed in appendix.

		codebook1	codebook2	codebook3	codebook4	random codebook
L = 5	I_{av}	4.07L	4.07L	4.07L	4.07L	5.07L
N = 3	I_{max}	0.60	0.60	0.60	0.60	-
L = 6	I_{av}	3.95L	4.23L	3.95L	4.20L	4.67L
N = 5	I_{max}	0.67	0.67	0.67	0.67	-
L = 7	I_{av}	4.04L	4.22L	3.74L	3.95	4.43L
N = 8	I_{max}	0.64	0.64	0.64	0.64	-
L = 8	I_{av}	3.94L	3.94L	3.88L	3.95	4.29L
N = 12	I_{max}	0.63	0.63	0.63	0.63	-
L = 9	I_{av}	4.03L	3.98L	4.30L	3.74	4.22L
N = 16	I_{max}	0.67	0.67	0.67	0.67	-

for broadband indoor wireless communications[34]. The intra-cell multiple access is by time division and commutation signaling is employed for combating the multipath induced intersymbol interference (ISI). In order to reduce inter-cell interference, different frequencies and commutation signaling codebooks are assigned to adjacent cells. The frequency bands of adjacent cells can overlap to increase bandwidth efficiency. This combined time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA) scheme is similar to a hybrid intra-cell TDMA/inter-cell CDMA system[35], except that commutation signaling is not used in the latter system. Such a hybrid system combines the high

Table 4.2: The average multipath interference parameter I_{av} and the normalized maximum multipath interference parameter I_{max} for different four phase codebooks. Codebook1 refers to the first codebook in appendix, for which the probability of error is plotted here. Codebook2, codebook3 and codebook4 refer to the second, the third and the fourth codebooks listed in appendix.

		codebook1	codebook2	codebook3	codebook4	random codebook
L = 4	I_{av}	4.39L	4.11L	5.67L	4.83L	5.00L
N = 3	I_{max}	0.56	0.56	0.56	0.56	–
L = 5	I_{av}	4.27L	4.59L	3.69L	4.56L	4.64L
N = 5	I_{max}	0.55	0.55	0.55	0.55	–
L = 6	I_{av}	4.40L	3.93L	4.51L	4.43L	4.42L
N = 8	I_{max}	0.54	0.54	0.54	0.54	–
L = 7	I_{av}	3.97L	4.28L	4.15L	3.86L	4.29L
N = 12	I_{max}	0.55	0.55	0.55	0.55	–
L = 8	I_{av}	4.07L	4.23L	4.03L	4.14L	4.22L
N = 16	I_{max}	0.56	0.56	0.56	0.56	–

intra-cell capacity of TDMA [35, 36] with the multipath interference rejection capability of commutation signaling. Unlike CDMA, power control is not necessary. Also, it does not require complex and power consuming equalization like TDMA. For such a cellular system, at least three different commutation signaling codebooks are required to ensure that different codebooks are assigned to adjacent cells (see Fig.4.12). Here, by different codebooks, we mean that these codebooks share no common codeword. To be more accurate, the cross-correlation between any two codewords from these codebooks is less than the length of the codewords. We have constructed four different codebooks for each given length and these binary and four phase codebooks are listed in appendix A. For binary codebooks, 0 and 1 are used to represent 1 and -1, while for four phase codebooks, $\{0, 1, 2, 3\}$ are corresponding to $\{j^0, j^1, j^2, j^3\}$ with $j = \sqrt{-1}$. The average multipath interference parameter I_{av} and the normalized maximum multipath interference parameter I_{max} for those codebooks are listed in Table 4.1 and Table 4.2.

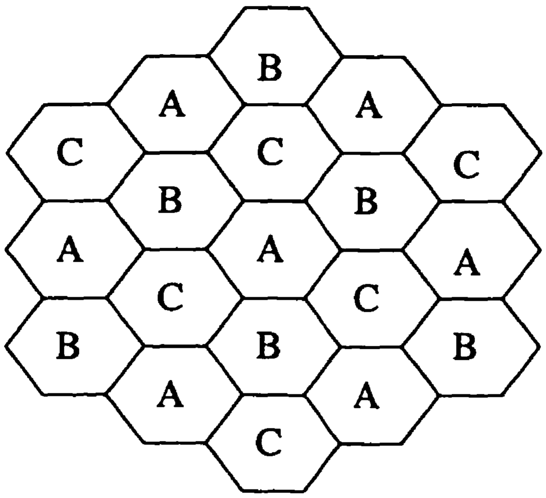

Figure 4.12: Cellular configuration with reuse factor of three. The cells labelled with the same letter will be assigned same commutation signaling codebook.

Chapter 5

Conclusions

Commutation signaling technique can be used for communication systems operating over multipath fading channels to overcome the multipath induced ISI. We have analized a system for a two-wave Rayleigh fading channel in terms of average and worst case performance. Parameters of commutation signaling codes which have the greatest impact on system performance are revealed and binary and four phase commutation signaling codes have been constructed based on these code parameters.

Concerning the average performance, binary codes that give average signal-to-noise ratio better than that for random code can be constructed analytically. However, the sizes and lengthes of those codes are constrained. For one of the two methods considered in the thesis, the length L of the codewords can only be $2^n - 1$, where n is an integer, and the size N of the codebook has to be $L+1$. With another method, the size N of a codebook must be $2^n - 1$ with n being an integer, and the length L can not be larger than N. Furthermore, N and L need to be relatively prime. In both cases, the length L has to be smaller than size N. Another problem associated with those codes is that the maximum probability of error can be quite high at certain delays. For a codebook constructed based on the average performance, it is found that with a SNR of 23dB, the maximum probability of error is about $5x10^{-3}$. Comparing to the probability of error of $8x10^{-5}$ for the ideal differentially coherent detection of DPSK with equal weight two-fold diversity, it is seen that the degradation in performance is significant.

The codes generated based on the worst case performance can ensure an acceptable maximum probability of error for a delay spread which is significantly larger than the symbol duration. The constructed binary codebooks have sizes of 3, 5, 8, 12, and 16 with lengthes of 5, 6, 7, 8, and 9 respectively. At SNR of 23dB, the calculated probability of error for these codebooks shows that although the maximum probability of error is about $5x10^{-4}$, the probability of error is below $3x10^{-4}$ for most values of delays up to N (size of a codebook) times the symbol duration. Since the error rate of $3x10^{-4}$ reflects a 3dB degradation with respect to ideal two-fold diversity, this means that these codebooks can be used to cope with a multipath delay spread of up to N times symbol duration with a performance degradation essentially not more than 3dB. For a similar error probability performance, four phase codebooks have been constructed with sizes of 3, 5, 8, 12, and 16 (lengthes of 4, 5, 6, 7, and 8 respectively). Since the size and the length of a codebook are related to the anti-multipath capability and the bandwidth spreading, it is seen that with the same anti-multipath capability, systems using four phase codebooks require less bandwidth expansion compared to that for binary codebooks. Or equivalently, with the same bandwidth expansion, the anti-multipath capability is increased by using four phase codebooks. Among all codebooks, four phase codebooks of length 4 require least bandwidth expansion, and can be used for inter-wave delay as large as nearly three times the symbol duration for a performance degradation of about 3dB with respect to ideal two-fold diversity. If more bandwidth is available, codebooks of larger length (and bigger size) can be used, and the anti-multipath capability is increased. When codebooks of length 8 are allowed, the system can cope with a inter-wave delay of up to nearly sixteen times the symbol duration for a similar performance degradation.

It should be pointed out that for most codebooks (both binary and four phase) constructed based on the worst case performance, the average performance in terms of average signal-to-noise ratio is better than that for the random code. Furthermore, the average performance for many codebooks constructed based on the worst case performance is even better than that of the binary codebooks constructed according to the average performance.

Commutation signaling codebooks can be used in a combined time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA) scheme. The intra-cell multiple access for the scheme is by time division and commutation signaling is employed for combating the multipath induced intersymbol interference (ISI). To reduce inter-cell interference, different frequencies and commutation signaling codebooks are assigned to adjacent cells. For such a multiple access system, at least three different commutation signaling codebooks are required to ensure that different codebooks are assigned to adjacent cells, and we present here four different codebooks for each given length and size.

In this work, code construction are considered with both worst case and average performance as criteria. Our main interest is in the worst case performance, and in this case, both binary and quaternary codes have been constructed. For the case of average performance, only binary codes are considered due to the limited time. Therefore, there are something not clear about the quaternary codes. It is found that for binary codes constructed based on the average performance criterion, the probability of error can be very large at certain delays. What will be the situation for quaternary codes? It is expected that quaternary codes will show similar behavior as binary codes since, in general, the requirements for good average performance can not ensure good worst case performance. However, to have a precise and convincing answer, more work is needed. Another question arise when comparing binary codes with quaternary codes. If we look at the worst case performance, quaternary codes are shown to be much better than binary codes. Is this true for the average performance?

As a suggestion to future research, we think that it would be interesting to analysis the average performance for quaternary codes and to develop code construction methods. It is certainly useful to compare the average performance of binary codes and quaternary codes.

Bibliography

[1] J.E. Padgett, C.G. Günther, and T. Hattori, "Overview of wireless personal communications" *IEEE Communications Magazine*, vol. 33, pp.28–41, Jan. 1995.

[2] D.C. Cox, "Wireless personal communications: what is it", *IEEE Personal Communications"*, pp. 20–35, April 1995.

[3] H. Hashemi, "The indoor radio propagation channel", *Proc. of the IEEE*, vol. 81, pp.943–968, 1993.

[4] P.F.M. Smulders and A.G. Wagemans, "Wideband indoor radio propagation measurements at 58 GHz", *Electron. Lett.*, vol. 28, pp.1270–1272, 1992.

[5] R. Davies, M. Bensebti, M.A. Beach and J.P. McGeehan, "Wireless propagation measurements in indoor multipath environments at 1.7 GHz and 60 GHz for small cell systems", *Proc. IEEE Veh. Technol. Conf.*, pp.589–593, 1991.

[6] J.G. Proakis, "Adaptive equalization for TDMA digital mobile radio", *IEEE Trans. Vehic. Technol.,* vol. 40, pp.333–341, 1991.

[7] N.W.K. Lo, D.D. Falconer and A.U.H. Sheikh, "Adaptive equalization techniques for multipath fading and co-channel interference", *Proc. IEEE Veh. Technol. Conf.*, pp.653–656, 1993.

[8] S. Ariyavisitakul, S. Yoshida, F. Ikegami and T. takeuchi, "A novel anti-multipath modulation technique DSK", *IEEE Trans. Commun.*, vol. 35, pp.1252–1264, 1987.

[9] S. Yoshida and F. Ikegami, "Anti-multipath modulation technique–Manchester-coded PSK", *Proc. IEEE ICC*, pp.1371–1375, 1987.

[10] S. Ariyavisitakul, S. Yoshida, F. Ikegami, K. Tanaka and T. takeuchi, "A power-efficient linear digital modulator and its application to an anti-multipath modulation PSK-RZ scheme", *Proc. IEEE Veh. Technol. Conf.*, pp.66–71, 1987.

[11] H. Takai, "In-room transmission BER performance of anti-multipath modulation PSK-VP", *IEEE Trans. Vehic. Technol.*, vol. 42, pp.177–185, 1993.

[12] H. Takai, "BER performance of anti-multipath modulation scheme PSK-VP and its optimum phase-waveform", *IEEE Trans. Vehic. Technol.*, vol. 42, pp.625–639, 1993.

[13] G.L. Turin, "Commutation signaling–an antimultipath technique", *IEEE JSAC*, vol. SAC-2, No. 4, pp. 548–562, 1984.

[14] K.M. Cheung, "Generalized likelihood commutation signaling for indoor communications", *M. Eng. Thesis, Department of Electrical Engineering, McGill University*, October 1992.

[15] A.A.M. Saleh and R.A. Valenzuela, "A statistical model for indoor multipath propagation", *IEEE JSAC*, vol. SAC-5, No. 2, pp. 128–137, 1987.

[16] J.E. Mazo, "Exact matched filter bound for two-beam Rayleigh fading", *IEEE Trans. Commun.*, vol. 39, No. 7, pp. 1027–1030, 1991.

[17] M. Alles and S. Pasupathy, "Suboptimum detection for the two-wave Rayleigh-fading channel", *IEEE Trans. Commun.,* vol. 42, pp.2947–2958, 1994.

[18] D.V. Sarwate, and M.B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences", *Proc. IEEE,* vol. 68, pp. 593–619, 1980.

[19] D.V. Sarwate, M.B. Pursley, and T.Ŭ. Basar, "Partial correlation effects in direct-sequence spread-spectrum multiple-access communication systems", *IEEE Trans. Commun.,* vol. 32, pp. 567–573, 1984.

[20] D.V. Sarwate, "Bounds on crosscorrelation and autocorrelation of sequences", *IEEE Trans. Inform. Theory,* vol. IT-25, pp. 720–724, 1980.

[21] M.B. Pursley and D.V. Sarwate, "Performance evaluation for phase-coded spread-spectrum multiple-access communication–Part II: Code sequence analysis", *IEEE Trans. Commun.,* vol. 25, pp. 800–803, 1977.

[22] M.B. Pursley and H.F.A. Roefs, "Numerical evaluation of correlation parameters for optimal phases of binary shift-register sequences", *IEEE Trans. Commun.,* vol. 27, pp. 1597–1604, 1979.

[23] R. Gold, "Optimal binary sequences for spread spectrum multiplexing", *IEEE Trans. Inform. Theory,* vol. IT-13, pp. 619–621, 1967.

[24] R. Gold, "Maximal recursive sequences with 3-valued recursive crosscorrelation functions", *IEEE Trans. Inform. Theory,* vol. IT-14, pp. 154–156, 1968.

[25] M. Alles and S. Pasupathy, "Channel knowledge and optimal performance for two-wave Rayleigh fading channels", *IEEE Trans. Vehic. Technol.,* vol. 43, pp.8–20, 1994.

[26] G.L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables", *Biometrika,* vol. 47, pp. 199–201, 1960.

[27] G.L. Turin, "Some computations of error rates for selectively fading multipath channels", *Proc. Nat. Electron. Conf.,* pp. 431–440, 1959.

[28] F.D. Garber and M.B. Pursley, "Performance of offset quadriphase spread-spectrum multiple-access communications", *IEEE Trans. Commun.,* vol. 29, pp. 305–314, 1981.

[29] M.B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication–Part I: system analysis", *IEEE Trans. Commun.,* vol. 25, pp. 795–799, 1977.

[30] M.B. Pursley, "Spread-spectrum multiple-access communications," in *Multi-User Communication Systems*, G. Longo, Ed. New York: Springer-Verlag, pp. 139–199, 1981.

[31] H.F.A. Roefs and M.B. Pursley, "Correlation parameters of random sequences and maximal length sequences for spread-spectrum multiple access communication", *Proceedings of IEEE Canadian Conference on Communications and Power*, pp. 141–143, 1976.

[32] P.V. Krishnamurthy, M. Alles, Y. Qian and H. Leib, "An anti-multipath transmission technique for indoor wireless communications", *CITR Technical Report, Department of Electrical Engineering, McGill University,* Oct. 1994.

[33] J.G. Proakis, *Digital Communications*, McGraw Hill, New York, 1989.

[34] P.V. Krishnamurthy and H. Leib, "A combined FDMA/CDMA/TDMA technique for cellular wireless communication", *Book of Abstracts for CITR (Cana-*

dian Institute for Telecommunications Research) Research Conference, pp.54, 1995.

[35] M.L. Honig and U. Madhow, "Hybrid intra-cell TDMA/inter-cell CDMA with inter-cell interference suppression for wireless networks", *Proc. IEEE Veh. Technol. Conf.*, pp.309–312, 1993.

[36] A.D. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel", *IEEE Trans. Inform. Theory*, vol. IT-40, pp. 1713–1727, 1994.

Appendix A

List of Good Codebooks

Here we list some good codes with different sizes and lengthes.

Binary codebooks:

- Good codebooks of length 5 and size 3 (no common codeword):

$$\begin{bmatrix} 11110 \\ 00110 \\ 00101 \end{bmatrix}, \begin{bmatrix} 10100 \\ 00011 \\ 11011 \end{bmatrix}, \begin{bmatrix} 10010 \\ 10001 \\ 01111 \end{bmatrix}, \begin{bmatrix} 10110 \\ 01000 \\ 11000 \end{bmatrix}.$$

- Good codebooks of length 6 and size 5 (no common codeword):

$$\begin{bmatrix} 111000 \\ 110010 \\ 011101 \\ 101001 \\ 111101 \end{bmatrix}, \begin{bmatrix} 100001 \\ 011111 \\ 111011 \\ 100101 \\ 101100 \end{bmatrix}, \begin{bmatrix} 100111 \\ 001010 \\ 010001 \\ 000011 \\ 110110 \end{bmatrix}, \begin{bmatrix} 101011 \\ 001111 \\ 000110 \\ 001011 \\ 110111 \end{bmatrix}.$$

- Good codebooks of length 7 and size 8 (no common codeword):

$$\begin{bmatrix} 1100010 \\ 1100001 \\ 0111001 \\ 1001010 \\ 1000001 \\ 1111110 \\ 1011101 \\ 0100110 \end{bmatrix}, \begin{bmatrix} 1100101 \\ 0111011 \\ 0011011 \\ 0110000 \\ 1010110 \\ 0111000 \\ 1010000 \\ 1111100 \end{bmatrix}, \begin{bmatrix} 1101011 \\ 1011000 \\ 0001011 \\ 1100111 \\ 0010000 \\ 1001101 \\ 1101010 \\ 0010010 \end{bmatrix}, \begin{bmatrix} 1011110 \\ 0010001 \\ 1101001 \\ 0011111 \\ 1110101 \\ 0101011 \\ 1011111 \\ 1001100 \end{bmatrix}.$$

- Good codebooks of length 8 and size 12 (no common codeword):

$$\begin{bmatrix} 11100100 \\ 01100111 \\ 11000101 \\ 10000111 \\ 11100101 \\ 01010010 \\ 00110010 \\ 11111101 \\ 11011010 \\ 01110111 \\ 10111110 \\ 00001010 \end{bmatrix}, \begin{bmatrix} 11110010 \\ 00111011 \\ 11000110 \\ 11101001 \\ 11110110 \\ 10110110 \\ 11111110 \\ 10000011 \\ 00101011 \\ 01011100 \\ 11101011 \\ 10101011 \end{bmatrix}, \begin{bmatrix} 11110100 \\ 01100110 \\ 00010101 \\ 11111011 \\ 01101000 \\ 10000100 \\ 01110110 \\ 01011001 \\ 11100111 \\ 00110101 \\ 01011000 \\ 11001001 \end{bmatrix}, \begin{bmatrix} 11111001 \\ 11010011 \\ 01101011 \\ 11101111 \\ 00010011 \\ 00101111 \\ 10101000 \\ 01101100 \\ 01000110 \\ 10001111 \\ 01010110 \\ 01101111 \end{bmatrix}.$$

- Good codebooks of length 9 and size 16 (no common codeword):

$$\begin{bmatrix} 101001101 \\ 111110111 \\ 001110100 \\ 010011110 \\ 101010100 \\ 000101011 \\ 010000000 \\ 100110000 \\ 011010100 \\ 100111011 \\ 001010010 \\ 000011000 \\ 111001000 \\ 000111011 \\ 011110000 \\ 011011011 \end{bmatrix}, \begin{bmatrix} 001100101 \\ 111010101 \\ 000111101 \\ 111101011 \\ 010111011 \\ 100011111 \\ 010010000 \\ 110011000 \\ 011111000 \\ 011000101 \\ 110010011 \\ 101101000 \\ 010010110 \\ 110111010 \\ 000001101 \\ 001100011 \end{bmatrix}, \begin{bmatrix} 101100011 \\ 000110011 \\ 011101011 \\ 001000111 \\ 000011111 \\ 110111111 \\ 011000000 \\ 100010000 \\ 010011000 \\ 101111000 \\ 010100010 \\ 101010010 \\ 011011010 \\ 010111001 \\ 001111101 \\ 110100000 \end{bmatrix}, \begin{bmatrix} 100011000 \\ 010010100 \\ 101110100 \\ 011111100 \\ 100100010 \\ 011000011 \\ 111001011 \\ 101111011 \\ 110101000 \\ 011100010 \\ 100110010 \\ 000010110 \\ 001010101 \\ 111110101 \\ 011010011 \\ 111011011 \end{bmatrix}.$$

Four Phase codebooks:

- Good codebooks of length 4 and size 3 (no common codeword):

$$\begin{bmatrix} 1132 \\ 3132 \\ 3303 \end{bmatrix}, \begin{bmatrix} 2010 \\ 3110 \\ 0230 \end{bmatrix}, \begin{bmatrix} 1011 \\ 3011 \\ 1022 \end{bmatrix}, \begin{bmatrix} 1211 \\ 3211 \\ 3031 \end{bmatrix}.$$

- Good codebooks of length 5 and size 5 (no common codeword):

$$\begin{bmatrix} 13212 \\ 00312 \\ 23312 \\ 30032 \\ 32000 \end{bmatrix}, \begin{bmatrix} 01000 \\ 23200 \\ 12000 \\ 11310 \\ 02300 \end{bmatrix}, \begin{bmatrix} 21300 \\ 33100 \\ 10200 \\ 10120 \\ 03020 \end{bmatrix}, \begin{bmatrix} 12332 \\ 33332 \\ 13223 \\ 20220 \\ 13221 \end{bmatrix}.$$

- Good codebooks of length 6 and size 8 (no common codeword):

$$\begin{bmatrix} 110103 \\ 320000 \\ 002010 \\ 113000 \\ 101200 \\ 331310 \\ 110200 \\ 311030 \end{bmatrix}, \begin{bmatrix} 112100 \\ 101000 \\ 230100 \\ 031000 \\ 020200 \\ 133000 \\ 323100 \\ 230210 \end{bmatrix}, \begin{bmatrix} 120300 \\ 010000 \\ 022000 \\ 131100 \\ 011200 \\ 131210 \\ 313300 \\ 303200 \end{bmatrix}, \begin{bmatrix} 112312 \\ 300022 \\ 323310 \\ 223320 \\ 120321 \\ 232131 \\ 023202 \\ 022212 \end{bmatrix}.$$

- Good codebooks of length 7 and size 12 (no common codeword):

$$\begin{bmatrix} 3330030 \\ 0201030 \\ 0011030 \\ 2211030 \\ 1311030 \\ 2132030 \\ 1232030 \\ 2003030 \\ 2001130 \\ 3311130 \\ 0132130 \\ 0200301 \end{bmatrix}, \begin{bmatrix} 3131333 \\ 1331333 \\ 0002333 \\ 1202333 \\ 3012333 \\ 3332000 \\ 1221100 \\ 2031100 \\ 1302100 \\ 1320010 \\ 1032010 \\ 2310020 \end{bmatrix}, \begin{bmatrix} 0212100 \\ 1112010 \\ 3012210 \\ 3202230 \\ 3231231 \\ 2131302 \\ 2321322 \\ 2230331 \\ 2000311 \\ 3002011 \\ 3333311 \\ 1010110 \end{bmatrix}, \begin{bmatrix} 1330210 \\ 3232210 \\ 1230310 \\ 1130020 \\ 2123320 \\ 0010102 \\ 1322122 \\ 3000013 \\ 1002013 \\ 3212130 \\ 0103203 \\ 3012130 \end{bmatrix}.$$

- Good codebooks of length 8 and size 16 (no common codeword):

$$\begin{bmatrix} 30111011 \\ 21110111 \\ 23110311 \\ 10213311 \\ 20210021 \\ 02212121 \\ 13213221 \\ 00312321 \\ 01312031 \\ 22310231 \\ 03312231 \\ 00022331 \\ 23220322 \\ 00232313 \\ 21303130 \\ 33020012 \end{bmatrix}, \begin{bmatrix} 32232300 \\ 23231010 \\ 00333010 \\ 10330110 \\ 31332210 \\ 02333210 \\ 22331310 \\ 00000120 \\ 01100230 \\ 30203101 \\ 13201011 \\ 02300311 \\ 20112131 \\ 21312212 \\ 13221013 \\ 01030223 \end{bmatrix}, \begin{bmatrix} 23002330 \\ 03022330 \\ 22003330 \\ 01310101 \\ 30320101 \\ 13202101 \\ 03212101 \\ 20320111 \\ 02203111 \\ 10230211 \\ 33111211 \\ 13131211 \\ 03102211 \\ 20010031 \\ 00030031 \\ 00220231 \end{bmatrix}, \begin{bmatrix} 10223202 \\ 33100302 \\ 23110302 \\ 32101302 \\ 11122302 \\ 01132302 \\ 30103302 \\ 10023012 \\ 00033012 \\ 11312112 \\ 10313112 \\ 22201212 \\ 02221212 \\ 10213212 \\ 11031003 \\ 33303003 \end{bmatrix}.$$

Appendix B

Software Description

The software used to construct commutation signaling codes is described here. The program is implemented by MATLAB software package and is named **code.m**. To understand the program, it is necessary to explain some variables we used. Since bi-phase and four phase codebooks are considered, a variable *phase* is used to specify the phase of the codebook in addition to the variables *length, num* that define the length and size of the codebook. As indicated in chapter 3, in constructing the codebooks, there are two conditions that limit the multipath induced interference. In the program, *limit2* represents the limit for maximum multipath induced interference $max(|f_{m1}| + |f_{m2}|)$, and *limit1* is the bound for $|C_{N-(k+1)+m,m}(l - L) \pm C_{N-k+m,m}(l)|$, $|C_{N-k+m,m+1}(l-L) \pm C_{N-(k-1)+m,m+1}(l)|$, $|C_{m,m+k+1}(l-L) \pm C_{m,m+k}(l)|$ and $|C_{m+1,m+k+2}(l - L) \pm C_{m+1,m+k+1}(l)|$. The array *gword* stores the codewords that can not be used in constructing the current codebook. This is useful when constructing several codebooks with no common codeword. In the program, a four phase word is related to a quaternary representation of a decimal number (module *MAXI*, the number of all possible words for a given length). For example, word (1 1 3 2) is viewed as the quaternary representation of decimal number 94. The variables that store this decimal number is named *gene*, which is increased by *step* each time to form the next searched word. *step* and *MAXI* are relatively prime so that after *MAXI* times, all possible words are searched. We start code construction by selecting the first codeword, then the second codeword is searched by testing all possible word. If there exist words that satisfy the two conditions, the first one will be taken as the codeword. The process continues until *num* (the size of the codebook) codewords are obtained or all possible word are tested.

The complete program with comments is given as following.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                           %
%  Copyright (C) 1996  McGill University                                    %
%  Copyright (C) 1996  Canadian Institute for Telecommunications            %
%                      Research  (CITR)                                     %
%  Copyright (C) 1996  Hong Ren                                             %
%                                                                           %
%  This program is protected by copyright and all rights are reserved.      %
%  Reproduction, adaption, or translation without written permission        %
%  is prohibited, except as allowed under the copyright laws.               %
%                                                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                           %
%  This program is used to construct commutation signaling codebooks.      %
%                                                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

length = 5;         %%%    length of a commutation signaling codebook   %%%
num = 5;            %%%    size of the codebook    %%%
phase = 4;          %%%    phase of the codebook   %%%
MAXI = phase^length;     %%%   number of possible words   %%%%
limit1 = 4.0;
limit2 = 11.0;

gword = [
 1   1   1   1   1
];                       %%%   The codewords that can not be used    %%%
d = size(gword);
ge = d(1);          %%%   number of the codewords that can not be used  %%%

code = zeros(num,length);     %%%  to save the codebook     %%%
f_corre = zeros(4,length);    %%%  aperiodic correlations   %%%
s_corre = zeros(4,length);    %%%  aperiodic correlations   %%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%  Select the first codeword     %%%%%%%%%%%%%%%%%%%%%%%

TIMES = 0;
gene = 9;
step = 7;
count = 1;
while count <= 1;

        cflag = 0;
        while cflag < 1;
                n = 1;
                gene = gene + step;
                gene = rem(gene,MAXI);
                ge_loc = gene;
                for i = 1:length;
                        l = length - i;
                        expo = fix(ge_loc/phase^l);
                        code(count,l+1) = (sqrt(-1))^expo;
                        ge_loc = rem(ge_loc,phase^l);
                end;
                TIMES = TIMES + 1;

%%%%%%%%%%   check whether the word can be used  %%%%%%%%%%%%%%%%%%%%%
                while n <= ge;
                        sum = 0;
                        for k = 1:length;
                                sum = sum + gword(n,k) * conj(code(count,k));
                        end;
                        if abs(sum) > length-1;
                                n = ge + 1;
                        elseif  n == ge;
                                n = n + 1;
                                cflag = 2;
                        else
                                n = n + 1;
                        end;
                end;
        end;

%%%%%%%%%%%%%%%%  check whether correlations satisfy the conditions  %%%%%%%%%%%%
        for shift = 1:length-1;
                f_corre(1,shift) = 0;
                f_corre(2,shift) = 0;
                for i = 1:length - shift;
```

```
                        j = i + shift;
                        f_corre(1,shift) = f_corre(1,shift) + code(1,i) * conj(code(1,j));
                end;
                for i = 1:shift;
                        j = i + length -shift;
                        f_corre(2,shift) = f_corre(2,shift) + code(1,i) * conj(code(1,j));
                end;

                test = abs(f_corre(1,shift)) + abs(f_corre(2,shift));
                if test > limit1;
                        shift = length + 1;
                else
                        if shift == length - 1;
                                count = count + 1;
                        end;
                end;
        end;
end;

%%%%%%% open a file to save the codebook and related information %%%%%%%%
fid = fopen('cbook', 'w');

%%%%%%%%%%%%%%%%%%%%%%%%%% search for the other codewords %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
times = 0;
count = 2;
while count <= num;
        cflag = 0;
        while cflag < 1;
                n = 1;
                times = times + 1;
                TIMES = TIMES + 1;
%%%%%% terminate the program if the number of tested words   %%%%%%%%%%%%%%%
%%%%%%%%%%%%%  larger than the number of possible words  %%%%%%%%%%%%%%%%%%%%%
                if times > MAXI;
                        cflag = 2;
                        count = num + 1;
                        mes = 'no code is found'
                end;

                gene = gene + step;
                gene = rem(gene,MAXI);
                ge_loc = gene;
                for i = 1:length;
                        l = length - i;
                        expo = fix(ge_loc/phase^l);
                        code(count,l+1) = (sqrt(-1))^expo;
                        ge_loc = rem(ge_loc,phase^l);
                end;

%%%%%%%%%%   check whether the word can be used  %%%%%%%%%%%%%%%%%%%%%%
                while n <= ge;
                        sum = 0;
                        for k = 1:length;
                        sum = sum + gword(n,k)  * conj(code(count,k));
                        end;
                        if abs(sum) > length-1;
                                n = ge + 1;
                        elseif  n == ge;
                                n = n + 1;
                                cflag = 2;
                        else
                                n = n + 1;
                        end;
                end;
        end;

%%%%%%%%%%%%%%%%%%%%   Check the two conditions    %%%%%%%%%%%%%%%%%%%%

%%%%%  The codewords are shifted cyclicly by the following loop %%%%
        first = 1;
        while first <= count
                second = first + 1;
                if second > count;
                        second = rem(first+1,count);
                end;
                delay = 0;
%%%%%  The real DELAY = delay * length + shift  %%%%%%%%%%%%%%
                while delay <= count-1;
                        f_prev = first + delay;
                        if f_prev > count
```

```
                f_prev = rem(first+delay,count);
        end;
        f_current = f_prev + 1;
        if f_current > count;
                f_current = rem(f_current,count);
        end
        f_next = f_current + 1;
        if f_next > count;
                f_next = rem(f_next,count);
        end;

        s_current = first - delay;
        if s_current <= 0;
                s_current = s_current + count;
        end;
        s_prev = s_current - 1;
        if s_prev <= 0;
                s_prev = s_prev + count;
        end;
        s_next = s_current + 1;
        if s_next > count;
                s_next = rem(s_next,count);
        end;

        if delay < count - 1;
                maxsf = length;
        else
                maxsf = length - 1;
        end;

        shift = 1;
        while shift <= maxsf;
                f_corre(1,shift) = 0;
                f_corre(2,shift) = 0;
                f_corre(3,shift) = 0;
                f_corre(4,shift) = 0;
                s_corre(1,shift) = 0;
                s_corre(2,shift) = 0;
                s_corre(3,shift) = 0;
                s_corre(4,shift) = 0;

                for i = 1:length-shift;
                j = i + shift;
                f_corre(1,shift) = f_corre(1,shift) + code(first,i) * conj(code(f_prev,
j));
                f_corre(2,shift) = f_corre(2,shift) + code(second,i) * conj(code(f_curr
ent,j));
                s_corre(1,shift) = s_corre(1,shift) + code(s_current,i) * conj(code(fir
st,j));
                s_corre(2,shift) = s_corre(2,shift) + code(s_next,i) * conj(code(second
,j));
                end;

                for i = 1:shift;
                j = i + length - shift;
                f_corre(3,shift) = f_corre(3,shift) + code(first,j) * conj(code(f_curre
nt,i));
                f_corre(4,shift) = f_corre(4,shift) + code(second,j) * conj(code(f_next
,i));
                s_corre(3,shift) = s_corre(3,shift) + code(s_prev,j) * conj(code(first,
i));
                s_corre(4,shift) = s_corre(4,shift) + code(s_current,j) * conj(code(sec
ond,i));
                end;

                sum1 = f_corre(1,shift) + f_corre(3,shift);
                sum2 = f_corre(1,shift) - f_corre(3,shift);
                sum3 = f_corre(2,shift) + f_corre(4,shift);
                sum4 = f_corre(2,shift) - f_corre(4,shift);
                sum5 = s_corre(1,shift) + s_corre(3,shift);
                sum6 = s_corre(1,shift) - s_corre(3,shift);
                sum7 = s_corre(2,shift) + s_corre(4,shift);
                sum8 = s_corre(2,shift) - s_corre(4,shift);
                mag13p = abs(sum1 + sum3);
                mag13m = abs(sum1 - sum3);
                mag14p = abs(sum1 + sum4);
                mag14m = abs(sum1 - sum4);
                mag23p = abs(sum2 + sum3);
                mag23m = abs(sum2 - sum3);
                mag24p = abs(sum2 + sum4);
```

```
                                        mag24m = abs(sum2 - sum4);

                                        mag57p = abs(sum5 + sum7);
                                        mag57m = abs(sum5 - sum7);
                                        mag58p = abs(sum5 + sum8);
                                        mag58m = abs(sum5 - sum8);
                                        mag67p = abs(sum6 + sum7);
                                        mag67m = abs(sum6 - sum7);
                                        mag68p = abs(sum6 + sum8);
                                        mag68m = abs(sum6 - sum8);

                                        test1 = max(abs(sum1), abs(sum2));
                                        test2 = max(abs(sum3), abs(sum4));
                                        test3 = max(abs(sum5), abs(sum6));
                                        test4 = max(abs(sum7), abs(sum8));
                                        magm1 = [mag13p mag13m mag14p mag14m mag23p mag23m mag24p mag24m];
                                        test12 = max(magm1);
                                        magm2 = [mag57p mag57m mag58p mag58m mag67p mag67m mag68p mag68m];
                                        test34 = max(magm2);
                                        testall = test12 + test34;
                                        flag = 0;

                                        if test1 > limit1;
                                                flag = 1;
                                        end;
                                        if test2 > limit1;
                                                flag = 1;
                                        end;
                                        if test3 > limit1;
                                                flag = 1;
                                        end;
                                        if test4 > limit1;
                                                flag = 1;
                                        end;
                                        if testall > limit2;
                                                flag = 1;
                                        end;

%%%%%%%%%   Discard the codeword if the conditions are not satisfied     %%%%%%%%%%
                                        if flag >= 1;
                                                shift = length + 1;
                                                delay = count + 1;
                                                first = count + 1;
%%%%%%%%%%%%%  Otherwise, test the conditions for the next shift  %%%%%%%%%%%%%%%%
                                        else
                                                shift = shift + 1;
                                        end;
                                end;

%%%%%%%%%%%%%%   Test the conditions for the next delay   %%%%%%%%%%%%%%
                                delay = delay + 1;
                        end;

%%%%%%%%%%%%   Shift the codewors                  %%%%%%%%%%%%%%
                        first = first + 1;

                end;
                if shift <= length;
                        times = 0;
                        count = count + 1;
                end;
end;

qcd = zeros(num, length);
for i = 1:num;
        for l = 1:length;
                if real(code(i,l)) > 1e-2;
                        qcd(i,l) = 0;
                elseif real(code(i,l)) < -1e-2;
                        qcd(i,l) = 2;
                elseif imag(code(i,l)) > 0;
                        qcd(i,l) = 1;
                else
                        qcd(i,l) = 3;
                end;
        end;
end;

%%%%%%%%%%%%%%%%%%%%%   Save the codebook in a file     %%%%%%%%%%%%%%%%%%%%%%%%%
```

```
for i = 1:num;
        for l = 1:length;
                fprintf(fid, '%1.0f   ', qcd(i,l));
        end;
        fprintf(fid,'\n');
end;
fprintf(fid,'\n');

for i = 1:num;
        for l = 1:length;
                if real(code(i,l)) > 1e-2;
                        fprintf(fid,' 1  ');
                elseif real(code(i,l)) < -1e-2;
                        fprintf(fid,'-1  ');
                elseif imag(code(i,l)) > 0;
                        fprintf(fid, ' j  ');
                else
                        fprintf(fid, '-j  ');
                end;
        end;
        fprintf(fid,'\n');
end;
status = fclose(fid);
```

IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

APPLIED IMAGE . Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989