Stochastic Short-term Production Scheduling Accounting for Fleet Allocation, Operational Considerations, Blending Restrictions and Future Multi-element Ore Control Data

by

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Abstract

The mine planning process requires optimization of production scheduling for the deposit being mined, so as to guaranty the best net present value in the evaluation of an open pit mine. This production schedule identifies the total amount of material to be hauled per period, the location of extraction and the destination, which include the processing plant, leach plant, stockpile or waste dumps. Mine production scheduling may be long-term or short-term based on the time period considered and the final objective. The optimization for long-term mine production scheduling is responsible for maximizing total discount cash flows and giving operational guidance for annual production; whereas, the optimization of the short-term mine production scheduling generates a sequence of depletion of any long-term annual production under current operating conditions and constraints. The optimization goal of short-term production scheduling is to minimize the mining cost expected from a mine while satisfying operational constraints, such as mining slope, grade blending, metal production, mining capacity and processing capacity; however some parameters may be uncertain, such as metal quality and fleet parameters. The concept of stochastic programming is used for optimization models that include uncertainty in their parameters as opposed to the traditional deterministic optimization models that are formulated assuming certainty in their inputs. Real situations show the optimization inputs are not certain and may change over time when additional information becomes available.

Traditional short-term production planning is carried out by two sequential optimizations, production schedule is defined at the first step and the available fleet is evaluated for this schedule as a second step, however; the fleet availability, hauling time and mining considerations do not influence the schedule decision. In addition, the fleet optimization algorithms do not consider uncertainty in their parameters and do not take into account the local mineralization of the deposit because a single possibly misleading total aggregated block tonnage is linked to each sector to be mined. The local mineralization or local scale variability

between blocks assists in the blending process and metal quality control; however, the traditional short-term production scheduling is based on exploration drilling or a sparse data ore body model, while in practice grade control data or close spacing blasthole drilling classify the material as ore and waste because their short-scale information is not available at the time of the monthly short-term planning. The local variability is relevant in the short-term production scheduling to define the destination of the material.

The short-term mine production scheduling in this thesis is developed as a single formulation where mining considerations, production constraints, uncertainty in the orebody metal quantity, as well as fleet parameters, are evaluated together to define a well informed sequence of mining that results in high performance at the mine operation. The formulation is implemented at a multi-element iron mine and the resulting monthly schedules show lower cost, minable patterns and, efficient fleet allocation, that ensures a higher and less variable utilization of the fleet over the conventional schedule approach.

Uninformed and ultimately costly decisions can be taken because of imperfect geological knowledge or information effect. The orebody uncertainty may be updated by simulated future ore control data to account for local scale grade variability, and the information used to discriminate ore and waste in practice. Multi-element orebody uncertainty models are updated based on the correlation of exploration data and past ore control data, this orebody uncertainty is then used to optimize the short-term production scheduling that leads to better performance in terms of matching ore quality targets and delivering recoverable reserves.

Future work may focus on including multiprocessing streams and additional sources of uncertainty to the short-term production scheduling formulation; and on advancing computational aspects to generate mine production schedules more efficiently in reasonable computing time. Parallelization and efficient heuristic methods may be options.

Résumé

Le processus de planification d'une mine à ciel ouvert, dont le gisement est en exploitation, nécessite l'optimisation du programme des activités de production afin d'assurer sa meilleure valeur actuelle nette. Le programme des activités de production identifie la quantité totale de matière première à transporter par période, les sites d'extraction et la destination du matériel extrait : l'usine de transformation, l'usine de lixiviation, la réserve du matériel à traiter ou la décharge du matériel traité (stérile). Le programme de production d'une mine peut se faire à long ou à court terme selon l'horizon de temps choisi et l'objectif final visé. L'optimisation du programme à long terme des activités de production permettra de maximiser les flux monétaires actualisés et de fournir des encadrements opérationnels pour le niveau de production annuelle, alors que l'optimisation du programme à court terme des activités de production génère une prévision de la diminution graduelle de la production annuelle selon les conditions et les contraintes des opérations courantes. L'objectif d'optimisation du programme à court terme est de minimiser les coûts d'opération attendus d'une mine en tenant compte de contraintes, tel que la pente de talus, le mélange de matériel, la production de métaux ainsi que la capacité de production et de traitement de la mine. Cependant, certains paramètres tels que la qualité du métal de base et les paramètres définissant la flotte minière peuvent être incertains. Le concept de la programmation stochastique est utilisée pour modéliser l'optimisation de la planification en y incluant un niveau d'incertitude, en opposition aux modèles traditionnels déterministes dont tous les paramètres sont connus. Des situations réelles montrent clairement que les paramètres d'optimisation de la production ne reposent pas sur la certitude et qu'en ce sens ils peuvent changer dans le temps advenant la disponibilité de nouvelles informations.

La planification traditionnelle à court terme des activités de production est assurée par deux optimisations séquentielles. Le programme des activités de production est établi à la première étape et la disponibilité de la flotte est évaluée à la seconde étape. Il faut bien voir cependant

que la disponibilité de la flotte ainsi que le temps de transport et les considérations générales de l'exploitation minière n'influencent pas les décisions de programmation. De plus, les algorithmes d'optimisation de la flotte ne considèrent pas l'incertitude comme un de leurs paramètres et ne prennent pas en compte la minéralisation locale du gisement parce que le résultat agrégé prévu du tonnage global pourrait facilement être trompeur compte tenu qu'il est considéré comme un seul grand bloc lié à chaque secteur à être minés. La minéralisation locale, aussi appelée la variabilité de l'échelle entre les blocs de matière première, aide dans le processus de gestion des mélanges et le contrôle de la qualité des métaux. Cependant, la planification à court terme des activités de production est basée sur les forages d'exploration ou sur un modèle de corps minéralisé qui s'appuie sur des données ayant un certain degré d'incertitude. Alors qu`en pratique le contrôle de qualité du minerai ou la courte distance entre les trous de forage pré-dynamitage classifie le matériel comme économique ou stérile étant donné que l'information à courte échelle n'est pas disponible au moment de la préparation de la planification à court terme des activités de production. La variabilité locale est importante dans la planification à court terme pour définir la destination du matériel.

Le modèle du programme à court terme des activités de production d'une mine tel que proposé dans cette thèse présente une formulation où les considérations minières, les contraintes de production, l'incertitude liée à la quantité de métal présente dans le corps minéralisé ainsi que les paramètres de la flotte minière sont évalués ensembles afin d'obtenir une séquence bien documentée des activités minières qui favorise une performance optimisée des opérations minières. La formulation a été implantée dans une mine de fer avec multiéléments et les résultats de la planification mensuelle ont démontré des coûts moindres et une utilisation efficiente de la flotte minière qui assurent une utilisation élevée et moins variable de la flotte en comparaison de l'approche d'un programme conventionnelle.

Des décisions mal documentées et coûteuses peuvent être prise à cause de connaissances géologiques imparfaites ou d'autres facteurs informationnels. L'incertitude concernant le corps minéralisé peut être mise à jour en simulant des données futures de contrôle des métaux afin de tenir compte de la variabilité du niveau de contrôle dans la qualité du métal présent et de l'information utilisée pour mieux faire la différence entre minéral économique et stérile. Les modèles d'incertitude des corps minéralisés a multiéléments sont mis à jour en se basant sur la corrélation des données d'exploration et les données d'exploration et les données passées de contrôle de métal. Ces incertitudes liées aux corps minéralisées permettent alors d'optimiser le programme à court-terme des activités de production qui mène à une meilleure performance en termes d'atteinte d'objectifs quant à la qualité du métal dans le minerai et de la réserve de matériel utilisable.

Dans de futures recherches pourraient se concentrer sur l'inclusion de flux de multitraitement et de nouvelles sources d'incertitudes à la formulation du programme de la planification court-terme des activités de production et la proposition de nouveaux aspects computationnels afin de pouvoir produire une planification de la production minière plus efficiente dans des délais et coûts raisonnables. La parallélisassions et les méthodes heuristiques pourraient être des options pour résoudre le programme en temps raisonnable.

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Contributions of Authors

The author of this thesis is the primary author. Professor Roussos G. Dimitrakopoulos was the supervisor of the author's Master of Engineering degree and is included as co-author of Chapter 2 and Chapter 3.

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Chapter 1 Introduction

A relevant task of mine planning is the optimization of long-term and short-term production schedules subject to physical constraints, orebody quality and quantity constraints as well as operational constraints. The fluctuations in the orebody quality and other constraint parameters make it difficult to forecast the production schedules perfectly; however, stochastic programming formulations can find optimal solutions to model accounting for uncertain input parameter and variables. Uncertainty in mineral deposits is present in both cases of sparse exploration drillholes and dense production blastholes as the materials being mined change in grade and metal content due to the natural spatial mineral deposits. At the time of short-term production scheduling there is additional information about the operation considerations that is not available in early long-term evaluations. The hauling distance, the availability of fleet and their respective uncertainty influence the production schedule given that more details for short-term production scheduling may efficiently forecast the monthly or daily production

1.1 Statement of the problem

The conventional short-term mine production scheduling is optimized by a two stage approach; the first stage considers constraints similar to as the long-term production scheduling to identify the sectors, destination, production tonnage and quality to be mined monthly or weekly instead of yearly. The location of these sectors, the kind of material to be extracted and the hauling distance to their destination are used as input parameters in the second stage where the available fleet is evaluated. This two stage approach at short-term production scheduling is impractical given that the hauling distance and fleet parameters does not assist in the decision of which sector to be mined per period, the blending process and matching quality requirements.

The short-term mine production scheduling will require several input parameters; such as orebody model, production targets, fleet parameters, mining cost and operational considerations. The performance of the short-term production scheduling is affected when these input parameters change overtime. Mineral deposits are uncertain because data scarcity makes local assessment of grades and material types a challenge. Geostatistical simulation constructs a set of realizations that provide an evaluation of global and local uncertainty in the orebody; however, orebody uncertainty based on sparse exploration data does not provide enough geological knowledge for short-term production scheduling. This geological knowledge based only on exploration data will not assist properly in segregating ore from waste given that a monthly or weekly period of short-term production schedule could be located between two sparse exploration holes.

The operational considerations are quite relevant at the time of mining because the size of the equipment and accessibility restrictions may require realistic extraction patterns that allow the fleet to work efficiently. The traditional short-term production scheduling that accounts for only slope constraints provides unfeasible extraction patterns that are fixed after the optimization is solved. Arbitrary modifications of the extraction patterns after the optimization may not guaranty a match to production targets as expected.

1.2 Goal and objectives

The short-term production scheduling must have the ability to integrate, production targets, operational considerations, possible orebody quality fluctuations and fleet parameter fluctuations into their formulation in order to deliver a well informed production schedule that will have high performance at the time of the production and ensure the efficient utilization of the fleet.

The goal of this thesis is to propose a single formulation that accounts for traditional production schedule constraints, fleet allocation, operational considerations, and uncertainty in multi-element deposits and fleet parameter uncertainty so as to decide which sector to be mined per period. To achieve this goal the following objectives are set:

1. Review the literature relating to short-term production scheduling, stochastic production scheduling and approaches to forecast future ore control data.
2. Propose a single stochastic mixed integer formulation that integrates traditional constraints of production scheduling, fleet allocation, mining width constraints, mining direction constraints and account for uncertainty in their parameters to optimize the short-term mine production scheduling. Implement the proposed stochastic short-term production scheduling formulation at multi-element iron deposit and analyse the results.
3. Update the orebody uncertainty with possible future multi-element ore control data to be accounted for in the stochastic production scheduling. An approach based on minimum/maximum autocorrelation factor is proposed to forecast future ore control data of multi-element deposits based on past blastholes drilling information. Analyse these results and document the value of stochastic production scheduling solution accounting for exploration data versus future ore control data.
4. State conclusions and recommend future work.

1.3 Thesis outline

Chapter One introduces the papers on open pit mine production scheduling, reviews the current approaches of short-term production scheduling and includes the concept and methods of forecasting future ore control data.

Chapter Two proposes a stochastic short-term production scheduling formulation that accounts for sources of uncertainty related to orebody quality and fleet parameters. This formulation includes fleet allocation constraints and operational considerations used to decide the sectors to mine per period. An implementation at a multi-element iron deposit and comparison against conventional deterministic approaches demonstrates the performance of this approach.

Chapter Three proposes the use of simulated future multi-element ore control data to update the orebody uncertainty that will be used in the stochastic short-term production scheduling.

The implementation in a multi-element iron deposit demonstrates the influence of possible local grade variability in the monthly short-term production that takes into account fleet allocation.

Chapter Four gives the conclusions and the recommended future work.

1.4 Literature review

The evaluation of a mine project requires carrying out an accurately open pit mine planning process which includes optimizing the production schedule. The optimization of mining project's long-term production schedule is responsible for projecting an acceptable net present values for the asset. Given large data sets, multiple constraints, and uncertain parameters, finding the most profitable production schedule is a complex task. The goal is to maximize the discounted net revenue expected from a mine while satisfying operational constraints such as mining slope, grade blending, metal production, mining capacity and processing capacity. Many methodologies are available to optimize the mine production schedule, specifically mathematical models such as linear program is discussed by Johnson(1969), mixed integer programming combined with linear programming by Gershon(1982), mixed integer programming with the application of Lagrangian and sub-gradient methods by Dagdelen(1985), a probabilistic approach in complex multi-element deposits that minimizes the risk of deviation from production targets was developed by Ramazan and Dimitrakopoulos (2004), simulated annealing to handle the uncertainty of assigning a period of extraction to a block to minimize the risk of deviation from production target by Godoy and Dimitrakopoulos (2004), and a stochastic integer optimization that maximizes the discounted cash flow and minimize the deviation from production targets accounting for the geological risk discounted rate by Ramazan and Dimitrakopoulos (2007). Dealing with uncertainty in the mine production schedules is a research topic that have been researched for the past 10 years; however, there is still room to improve the current techniques and make them researches more viable to implement at a mine.

The decision-making process in surface mining projects requires an adequate assessment of orebody risk in the pit design while generating a life of mine production schedule. A profitable mining sequence over the life of a mine evaluates both the economic outcome of a project and the technical plan to be executed from mine development to mine closure. The effect of orebody risk on the process is also relevant Ravenscroft (1992), Dowd (1997) and Rendu (2007). At an early stage of the project, the repayment of development capital is critical. The orebody uncertainty is a major contribution causing failure to meet expectations Vallee (2000).

1.4.1 Short-term production scheduling

Short-term mine production scheduling generates a sequence of extraction within an annual production area and may be seen as an operational guidance to match the objective of a long-term plan under current operating conditions and constraints. The short-term planning aspect outlines extraction stages in terms of months, weeks or days. The optimization of short-term scheduling is subject to the life-of-mine or long-term scheduling (Gershon 1982). In the technical literature, several papers related to short-term production scheduling and fleet allocation were published, the first group of papers outline general concepts about short-term production scheduling, the second group of papers consider real-time fleet allocation, the third group of papers formulate deterministic short-term production schedule which consider the total fleet capacity as the only limit on production per period, and the remaining papers are not related among them and are reviewed independently.

Wilke and Reimer (1977), Wilke and Woehrle (1979), Kahle and Scheafter (1979), Fytas and Calder (1986) and Schleifer (1996) state general concepts related to short-term production scheduling. The sequence of extraction on a daily, weekly or monthly basis follows guidelines given by the long-term production scheduling. However, the decisions at the operational level are also influenced by external and internal aspects. The external influences include the market demand and the properties of the deposit, while the internal influences are technical measures. The short-term production schedule is set up to accommodate changes within these external variables that can occur when additional data is acquired. As well, different production strategies could be evaluated by changing the internal variables as mining direction to guarantee the optimal production. The short-term production scheduling involves more details to guarantee feasibility where the operational constraints and the efficient use of available resources are relevant. This production scheduling is implemented to outline production progressions while considering the allocation of resources which match the current available fleet capacity, that is, the short-term production scheduling algorithm must take into account the mining case parameters, layout, situation and operational rules. The objective function is based on minimizing the deviations from a long-term yearly production plan, which is considered optimal in economic terms. The physical constraints are considered because some extraction patterns described by the production plan may not be mineable even though they belong to optimal solutions. Issues arise because of limited cut widths that decrease production stripping requirements which imply costly mining operations, and low equipment efficiency and utilization. The quality constraints are required because the irregularity of the orebody leads to a blending process to match quality feed target and to ensure a homogenous concentrator feed. To maximize the fleet utilization, each block is associated with a loading factor (hour/ton) limited to the shovel capacity and in the same way each block is associated with a transport factor (hour/ton) limited to truck capacity.

Alarie and Gamache (2002) and Souza, et al. (2010) present the real-time fleet allocation approach. A dispatching system in open-pit mines considers different strategies because the transportation may represent 60% of operating cost. The solution strategies used in truck dispatching systems for open pit mines are linked with dispatching problems to improve productivity and reduce operating cost. The ideal dispatching system should be based in the multistage approach. At the upper stage, linear programming may be used to define optimal mining rates taking into account blending requirements, maximum digging rate at each shovel, and pit configuration. At the lower stage, these methods attempt to match flow rates by assigning trucks to the shovels. This multistage approach may consider quality requirements at the crusher, stripping ratios and shovel capacity into the dispatching system to improve the quality of the assignments hourly. The dispatching system concept is able to adapt to changes in the mine status in real time; however, it does not define extraction patterns to mine monthly. The second paper by Souza, et al. proposes a model to optimize the hourly production by minimizing the number of trucks is presented. The goal is to determine the extraction rate of each pit where the production targets and the quality requirements are satisfied. Because of the dynamic allocations of the trucks, the hybrid algorithm combines characteristics of greedy randomized adaptive search procedure and general variable neighborhood search. The shortcoming of these algorithms is that the whole tonnage of every pit are seen as a single macro block where the short-scale variability of the grade is lost and the one hour production and dynamic allocation of the fleet is only related to the dispatch system.

Vargas et al. (2008) and Eivazy and Askari-Nasab (2012) propose a deterministic optimization model for short- term open pit mine production scheduling. Vargas et al. present a formulation that accounts for quality constraints, geometrical constraints, mill capacity and mine capacity. Additional to these components Eivazy and Askari-Nasab formulation accounts for multi-destination, blending stockpiles and how the decision on ramps and their objective function minimize the mining, processing, waste rehabilitation, re-handling and hauling costs. The mining blocks are aggregated as irregular macro blocks called an operational unit resource (MRU) based on the mineralogy or geology similitude to reduce the number of variables at the optimization model. The spatial arrangement of these MRUs is connected through a graph to define the horizontal precedence, vertical precedence and extraction progress control from the access road. The deterministic optimization is solved many times as the number of geological resource realizations to provide a probability map of each block mined at a certain period; however, a production scheduling based on the probability of individual blocks at certain periods may provide extraction patterns that do not respect the precedence conditions and the loss of selectivity because of the aggregation of blocks could mislead mining decisions at short-term production scheduling. The ore body uncertainty is not considered properly inside the short-term formulations to deliver a robust plan, this is a drawback of Vargas et al. formulation. The drawback of Eivazy and Askari-Nasab's (2012) formulation is related to the hauling process which evaluates possible different ramps or hauling distance without considering the availability of the fleet and uncertainty in the metal contain for blending process.

L'Heureux et al. (2013) present a deterministic mixed integer programming model for short-term planning in open-pit mines. The sequence of mining of this model considers operational activities, such as drilling, blasting, transportation, ore processing capacity, the availability and the locations of shovels and drills. The drawback of this formulation is that the mined blocks by day are aggregated regular blocks. The definition of sectors to mine is usually linked to irregular patterns because of the local scale grade variability of the orebody and quality requirements.

Kumral and Dowd (2002) formulate the short-term production scheduling algorithm based on two stages. An initial sub-optimal solution is generated by Lagrangean parameterization at the first stage and the multi-objective annealing is applied to improve the sub-optimal schedules at the second stage. The annealing parameter considers, starting temperature, number of iterations for each temperature, temperature decrement, termination criterion such as no improved new solution for *n* consecutive temperatures and maximum allowable number of blocks for swapping from one period to another. The methodology provides a near optimal schedule; however, relevant operational considerations and hauling process are not considered in the formulation.

Topal and Ramazan (2010) (2012) present a model to minimize the operating costs of the trucks since these equipments represent the largest portion of the fleet at open pit mines. From operating costs, the maintenance costs have a significant proportion and changes non-linearly depending on the road conditions, truck age and truck types. This model considers the truck maintenance cost as a stochastic parameter because of relevant uncertainty in the data, in the scheduling and in the available fleet for matching annual production targets. The approach provides a maintenance cost distribution of the optimized equipment schedule minimizing the cost. Tables are generated with many maintenance costs given deterministic historical maintenance cost and different simulations for each type truck by bins (operating hours). Information about the possible risks of a higher cost as well as the potential of a lower cost is provided for decision making; however, the shovel availability is a variable that may also be considered in the evaluation because it impacts directly on the truck hour calculation.

1.4.2 Simulated future ore control data for short-term production scheduling

Historically short-term production scheduling used the ore body model based on exploration data, with this sequence then adjusted with ore control data; thus, the expected extraction patterns to be mined periodically are modified to meet production targets. These last changes make the original production schedules sub optimal. Besides accounting for operational consideration, the short-term production scheduling should account for additional information collected at the time of production. Ore control data is usually not available ahead of a production period; however, previous mined out data will be useful to simulate this information at a sector where only an orebody model based on exploration data is available. The main assumption will be that the past data from mined out parts must be geologically similar to the sector being scheduled. In the literature, there are methods proposed on stochastic generation, given past blasthole drilling data; however, all of them are for a single element, and a multi-element deposits and the case of accounting for spatial correlation between multi-elements have not been explored to date.

Guardiano et al. (1997) propose a method to measure the efficiency of both estimation and mining in situ reserves. Conditional simulation is used to validate production schedules and recoverable reserves prior to production. The factor two in this method is the relationship of the actual in situ quantities which is the simulation based on exploration data at small grid and the room of mine quantities which is the inventory inside ore body contour based on future grade control data. This future grade control data is sampling from the simulation map based on exploration data plus an additional Gaussian error. The variance of this Gaussian error corresponds to the difference between exploration data nugget effect variogram and grade control data nugget effect variogram (Knudsen 1992). The Gaussian distribution error used to simulate the future grade control data is the same for every location to be evaluated which is not practical and arbitrary for ore deposits.

Khosrowshahi et al. (2007) propose a simulation of the chain of mining to identify errors in every stage of the mining process to forecast the recoverable reserves during mining. Sampling and assaying errors of precision, mining selectivity and movement due to blasting are incorporated to assess several chains of mining which are evaluated to determine the parameters that may match the current mining performance of the mine. A grade control model based on future ore control data was simulated based on sampling error. Two sampling errors due to the shape of the blasthole cone and the impacts of the blasthole subdrill were used to define distributions of the errors. The drawback of this study is associated with the criteria of using the same local normal distribution error through the domain to simulate future ore control data.

Journel and Kyriakidis (2004) show that the difference between ore control data and exploration data which are called error are not constant through the deposit. In a sector where only exploration data is available their error could be forecasted given historical information from similar operations. The local error is considered Gaussian and not stationary because a corrected local conditional error mean and a corrected local conditional variance error are distribution parameters at each neighborhood. The local mean of each neighborhood is multiplied by a corrected factor called relative deviation parameter to obtain their error mean and this error mean is multiplied by a spatial coefficient of variation to obtain their variance error. The deviation parameter and the spatial coefficient are taken from a table which accounts for the behavior of historical ore control data. An error is drawn from the local error distribution calculated and added to the local mean to generate the ore control mean at each location. The drawback is associated with the assumption that the local errors are Gaussian and independent. Indeed, the spatial continuity of the means is ignored in this approach.

Peattie (2007), Peattie and Dimitrakopoulos (2013) present an approach based on the spatial error. The spatial error is the difference of estimation based on exploration data and ore control at block scale. The spatial variability of the error is modeled and error simulation is performed at grid sector where only orebody exploration data is available. The ore control data map is calculated from the orebody based on exploration data base plus the error simulated. Multi-elements are not considered at the evaluation and the variogram model of blocks grade may provide a reduced variability because of volume variance concepts. These are the drawbacks of this approach.

The exploration data and ore control data could be evaluated together by the use of co-simulation which requires the spatial correlation of each data and spatial cross correlation of both data, e.g., three directional variograms of the ore control data for a single element, three directional variograms of exploration data for the same element and their three directions cross-variograms are required. The future ore control data could be carried out by performing co-simulation, Jewbali (2006), and Dimitrakopoulos and Jewbali (2013) present an approach to forecast ore control data of a single element base of the spatial correlation between ore control data and exploration of the mined sector or past data. A pseudo cross variogram is used to evaluate the cross spatial variability of two data that are not at the exact same location. This pseudo cross variogram is used at the ore control sector where only exploration data is available to perform co-simulation conditioning to the available data. This simulation will provide short-scale orebody information in the form of high density future ore control data. Then, a conditional simulation by successive residuals may permit updating the orebody model using the simulated future ore control data. Finally, a stochastic programming mine scheduling formulation that maximizes NPV and minimizes deviation from expected production targets is optimized. The NPV result is higher than conventional stochastic production based on exploration data and the agreement of short-term to long-term production schedules may lead to a higher probability of meeting production targets and increased productivity. The technique seems reasonable for a single element; however, in the presence of multi-elements, the technique will not be straightforward. The future ore control data is not the only parameters used to synchronize short-term and long-term production schedules. The short-term production scheduling requires additional operational consideration and fleet equipment parameters instead of using only long-term production scheduling constraints.

1.4.3 Principal components and minimum/maximun autocorrelation factors

The simulation of future ore control data for multi-element deposits proposed in this thesis is based on multivariate minimum/maximum autocorrelation factors (MAF) technique. MAF was developed by Switzer and Green (1984) and Desbarats and Dimitrakopoulos (2000) use this concept to simulate multi-variate regionalized distributions. The main idea is to de-correlate the variables to simulate these variables independently, and then to re-correlate. The sets of uncorrelated transformed variables or uncorrelated factors could be obtained based on eigenvector-eigenvalues decomposition of the variance–covariance matrix between variables. These transformed variables or principal components or orthogonal factors can be shown to extract successively a maximal fraction of the total variance of the variables (Wackernagel 2003). The first principal components explain most of this variance and a solid representation of the data is achieved; however, the principal components analysis is a naive approach because it ignores spatial structure of the data. The minimum/maximum autocorrelation factor is a principal components-based approach that besides the non spatial statistic of the lag-zero, the variance-covariance matrix accounts for the spatial structure of the data (Desbarats and Dimitrakopoulos 2000)

Chapter 2 Stochastic Short-Term Production Scheduling

Traditional short-term planning is carried out by two sequence optimizations. Initially the production schedule is defined followed by the available fleet being evaluated for this schedule, however; the fleet availability, hauling time and mining considerations do not influence the schedule decision. In addition, the fleet optimization algorithms do not consider the uncertainty in their parameters and do not take into account the short-scale mineralization of the deposit because a single misleading aggregated block tonnage is linked to each sector to be mined. The short-scale grade variability between blocks assists in the blending process and metal quality control at multi- elements deposits.

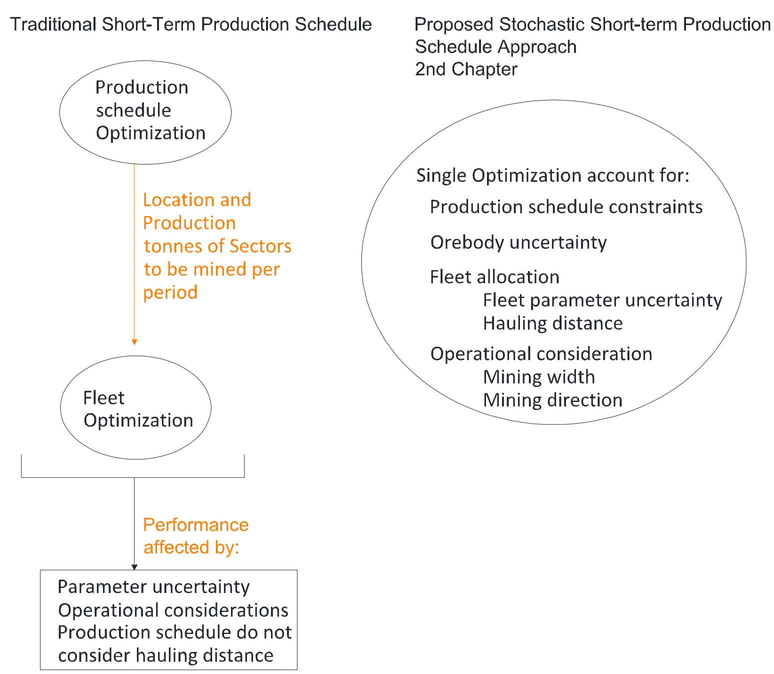


Figure 1: Schematic presentation of proposed stochastic short-term production scheduling approach

A short-term production schedule model is proposed herein as a single formulation where the mining considerations, the production constraints, the uncertainty in the orebody metal quantity, as well as in the fleet parameters, are evaluated together to define a well informed sequence of mining that results in high performance at the mine operation with an efficient fleet allocation that ensures their high utilization. Application at a multi-element iron ore mine shows lower objective function cost and a higher and less variable utilization of the fleet through the monthly schedule than with the conventional approach.

2.1 Introduction

The uncertainty in mineral deposits is present with sparse holes and dense production holes because the grade changes at large scale and small scale regarding the complexity of the deposit. The mechanical availability of the fleet and the hauling distance fluctuate over time. At the time of short-term production scheduling, additional information about the operation is available. The operational considerations are quite relevant at the time of mining because the size of the equipment and accessibility restrictions to the orebody may require realistic production schedule patterns. This allows the fleet to work efficiently whilst achieving production targets. The patterns of the production schedule accounting for mining considerations and fluctuations in their parameters ensures that the sector scheduled for a period may be mined in the respective period.

The target production must be accomplished every period and the hauling time that a block requires to be sent to the mill/stock or to the dump must be considered because the hauling distance is not part of the long-term formulation. This makes sense since the hauling distances are not available at the time of long-term production scheduling evaluation. The hauling distance changes through the sectors at the mine and the blocks from different sectors may be extracted to match ore quality requirements. The hauling distances are used to evaluate the truck cycle time. These cycle times may change even for the same hauling distance. Instead of using a single average truck cycle time by sector, the model considers a distribution of the possible cycle times by every sectors or mining faces per truck.

In the literature review only three publications were found that address short-term production scheduling and propose optimization models; however, those models do not consider uncertainty in their parameters. Vargas et al. (2008) and Eivazy and Askari-Nasab (2012) propose a deterministic optimization model for short-term open pit mine production scheduling. The first paper formulation accounts for quality constraints, geometrical constraints, mill capacity and mine capacity. Additional to these components the second formulation accounts for multi-destination, blending stockpiles and decision on ramps and their objective function minimizing the mining, processing, waste rehabilitation, re-handling and hauling costs. The mining blocks are aggregated as irregular macro blocks called an operational unit resource (MRU) based on mineralogy or geology similitude to reduce the number of variables at the optimization model. The spatial arrangement of these MRU's is connected through a graph to define the horizontal precedence, vertical precedence and extraction progress control from the access road. The deterministic optimization is solved many times as the number of geological resource realizations to provide a probability map of each block mined at certain period. The third paper from L'Heureux et al. (2013) presents a deterministic mixed integer programming model for short-term planning in open-pit mines. The sequence of mining of this model considers operational activities, such as drilling, blasting, transportation, ore processing capacity, and the availability and the locations of shovels and drills.

The production scheduling based on the probability of individual blocks at certain periods may provide extraction patterns that do not respect the precedence conditions in the first paper from Vargas et al. (2008). The ore body uncertainty may be considered correctly inside the short-term formulations so as to deliver a robust plan. The drawback of the second paper from Eivazy and Askari-Nasab (2012) is related to the hauling process which evaluates the possibility of different ramps or hauling distances without considering the availability of the fleet and uncertainty in the metal contain for blending process. The drawback of the third paper from

L'Heureux et al. (2013) is associated with the arbitrary manner used to represent the production patterns per period. The sectors to be mined by period typically have irregular extraction patterns because of the local scale grade variability of the orebody and the quality requirements.

2.2 Method

The short-term mine production scheduling problem discussed above is formulated as a stochastic integer programming model with recourse (Birge and Louveaux 1997) so as to provide a realistic solution. The latter solution minimizes the total mining cost along with deviations from production targets, considers operational aspects such as mining direction and minimum width, and maximizes fleet utilization. In the formulation presented herein, the first-stage decisions are made before the uncertainty is revealed, then the second-stage decisions or recourse actions are made after uncertainty is considered.

The notation used to formulate short-term scheduling follows. Note that indexes relate to the set of trucks, shovels, sectors, blocks, periods and realizations of uncertain parameters.

j : a sector or bench, where j = 1,...,J

i : an shovel, where i = 1,...,I

k : a block at sector, where k = 1,...,$K(j)$

l: a truck model, where l = 1,…,L

p : a period of a production schedule, where p =1,…,P

ε : an element grade of *k* block that have economical value, where $\varepsilon = 1,\ldots,E$

δ : a deleterious element grade of *k* block, where $\delta = 1,\ldots,D$

s : simulated grade realization or scenario, where s = 1,...,S

α : realization of shovel mechanical availability given historical data, where $\alpha = 1,\ldots, A$

r : truck cycle trip and mechanical availability realization, where *r* = 1,…,R

The parameters used at the fleet allocation, cost and penalties at objective function, production target and multi-element quality and tonnage are explained as follows:

h_{fleet} : fleet operation hours by period p

ι : maximum number of shovels allowed by sector

Q_i^{sh} : hourly production of shovel i.

$\omega_i(\mu_i, \sigma_i)$: mean and standard deviation of historical mechanical availability by shovel i

$a_{ij'}^{p-1}$: binary parameter, if shovel i is or not allocated to sector j' at previous period p*-1*

$c_{j'j}^{ExcM}$ *:* cost of moving shovel from p*-1* allocation sector j' to new allocation sector j

$c^{prodExc-}$: penalty cost for tonnage not produced regarding to the expected productivity

Q_l^{trk} : capacity of truck l

$\phi_{jl}(\mu_{jl}, \sigma_{jl})$: mean and standard deviation of cycle time by truck l at sector j

$\psi_l(\mu_l, \sigma_l)$: mean and standard deviation of historical mechanical availability by truck l

c^{ϕ} : time cycle cost per ϕ units

c^{m-}, c^{m+} : penalty cost for shortage and surplus total mining tonnage respect to the targets

c^{o-}, c^{o+} : penalty cost for shortage and surplus ore mining tonnage respect to the targets

$c^{\varepsilon-}, c^{\varepsilon+}, c^{\delta+}, c^{\delta-}$: penalty cost for deviation from main elements and contaminants limits

$P^{\min}, P^{\max}$: minimum and maximum mining tonnage target

$O^{\min}, O^{\max}$: minimum and maximum ore tonnage target

$G^{\varepsilon-}, G^{\varepsilon+}, G^{\delta-}, G^{\delta+}$: quality or grade requirements for ore tonnage produced

$\%Tol_{o-}, \%Tol_{o+}, \%Tol_{m-}, \%Tol_{m+}, \%Tol_{\varepsilon-}, \%Tol_{\varepsilon+}, \%Tol_{\delta-}, \%Tol_{\delta+}$: allowed percentage of tonnage and grade deviation from targets.

B_{jk} : block tonnage k at sector j

$bh,\ ddh$: Ore control data and exploration data at mined sector A

$BH,\ DDH$: Ore control data and exploration data at not mined sector B

c^{m} : mining cost by B_{jk} unit

$g^{\varepsilon}_{jks}, g^{\delta}_{jks}$: grade block k of main elements and deleterious in scenario s at sector j

O_{jks}: binary parameter flagging the block k at j sector for scenario s that has the minimum quality to be used at the blending process; otherwise, the block is flagged as waste.

ϕ_{rjl} : truck cycle time r of truck l at sector j given cycle time distribution

θ_{rjl} : maximum number of trips of truck l at sector j for cycle hauling realization r and mechanical availability realization r

$$\theta_{jlr} = \frac{\psi_{lr} \times h_{fleet}}{\phi_{jlr}}, \quad \forall r=1,\ldots,R,\ \forall j=1,\ldots,J, \forall l=1,\ldots,L$$

$Q^{sh}_{i\alpha}$: maximum production rate of shovel i per mechanic availability realization α and each realization $\omega_{i\alpha}$ is drawn from the available mechanical availability distribution, and it is

$$Q_{i\alpha}^{sh} = \omega_{i\alpha} \times h_{fleet} \times Q_i^{sh}, \quad \forall \alpha = 1,...,A, \ \forall i = 1,..., I$$

The decision variables used are as follow:

x_{jk}^{p} : binary variable, if block k at sector j is mined or not at period p

e_{ij}^{p} : binary variable, if shovel i is or not allocated to sector j at period p

n_{jilr}^{p} : number of trips of truck l to sector j, shovel i at period p for cycle time realization and mechanical availability realization r

$f_{ji\alpha}^{p}$: deviation of shovel i at sector j from expected shovel production $Q_{i\alpha}^{sh}$

y_{jk}^{p} : number of blocks that were not scheduled at period p to mine block k at sector j to match mining width requirements.

d_{p}^{m-}, d_{p}^{m+} : shortage tonnage to match lower production limit and surplus tonnage to match upper production limit at period p

d_{sp}^{o-}, d_{sp}^{o+} : shortage of ore mining to match lower bound and the surplus to match upper bound at period *p* accounting for grade scenario s

$d_{sp}^{\varepsilon-}, d_{sp}^{\varepsilon+}$ *:* deviation from ε grade targets at period *p* for grade scenario s

$d_{sp}^{\delta-}, d_{sp}^{\delta+}$ *:* deviation from δ deleterious grade targets at period *p* for grade scenario s

2.2.1 Objective function

Decision variables x_{jk}^{p}, y_{jk}^{p} and e_{ij}^{p} are related with the first-stage and remaining decision variables are related with the second-stage. The first-stage decisions include minimizing the costs of extraction of materials, movement of shovels, production shortage, and lacking matching mining width. In the second-stage, these costs are minimized over a range of possibilities of a recourse cost associated with deviations from ore production and quality targets, hauling cost, and lack of mining with maximum shovel productivity. The objective function of the proposed mathematical model is:

$$
\begin{aligned}
Minimize = & \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)} c^{m} B_{jk} x_{jk}^{p}}^{1st} \\
& \overbrace{+\frac{1}{R}\sum_{p=1}^{P}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{i=1}^{I} \phi_{jlr} c^{\phi} n_{jilr}^{p}}^{2nd} \\
& \overbrace{+\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{j'=1}^{J}\sum_{j=1}^{J}\left(c_{j'j}^{ExcM} e_{ij}^{p} a_{ij'}^{p-1}\right)}^{3rd} + \overbrace{\frac{1}{A}\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{\alpha=1}^{A}\left(c^{prodExc-} f_{ji\alpha}^{p}\right)}^{4th} \\
& \overbrace{+\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(c^{smoth-} y_{jk}^{p}\right)}^{5th} \\
& \overbrace{+\frac{1}{S}\left\{\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\varepsilon=1}^{E}\left(c^{\varepsilon-} d_{sp}^{\varepsilon-} + c^{\varepsilon+} d_{sp}^{\varepsilon+}\right) + \sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\delta=1}^{D}\left(c^{\delta+} d_{sp}^{\delta+} + c^{\delta-} d_{sp}^{\delta-}\right)\right\}}^{6th} \\
& \overbrace{+\frac{1}{S}\sum_{s=1}^{S}\sum_{p=1}^{P}\left(c^{o+} d_{sp}^{o+} + c^{o-} d_{sp}^{o-}\right)}^{7th} + \overbrace{\sum_{p=1}^{P}\left(c^{m-} d_{p}^{m-} + c^{m+} d_{p}^{m+}\right)}^{8th}
\end{aligned}
\tag{2.1}
$$

The first component of the objective function is associated with the cost of extracting material from the mine. The second component corresponds to minimizing the hauling cost given the uncertainty in the trucks' hauling time and mechanical availability so as to ensure both optimal allocation and maximum track utilization. The third component is the minimization of cost of the shovel movements among sectors. The fourth component minimizes the lack of production per shovel given uncertainty in its mechanical availability, so as to maximize shovel utilization. The fifth term ensures that the operational considerations are respected by penalizing the lack of mining blocks that match the required mining width. The sixth, seventh and eighth components deal with the minimization of geological risk with respect to the quality and the quantity of ore production, and penalize deviations from production targets, respectively. Note that the first, second, fourth, sixth and seventh components are stochastic and contain decision variables that change, given the corresponding realizations of the fleet parameters or element quality. The deterministic solution would be generated if instead of realizations the related components consider the single corresponding average value.

2.2.1.1 Constraints for Production and fleet allocation

The constraints below link the fleet allocation decision variables with mined block decision variables, to guarantee that the short-term production schedule accounts for fleet allocations and production targets.

$$\sum_{p=1}^{P} x_{jk}^{p} \leq 1, \quad \forall j=1,...,J, \forall k=1,...,K(j) \tag{2.2}$$

Constraint (2.2) ensures that a block of material may be mined once at any period. The block is a selective mining unit that may be mined in one period assuming that the time period may be from weeks to months.

$$\sum_{i=1}^{I} e_{ij}^{p} \leq \iota, \quad \forall p=1,...,P, \ \forall j=1,...,J \tag{2.3}$$

$$\sum_{j=1}^{J} e_{ij}^{p} \leq 1, \quad \forall p=1,...,P, \forall i=1,...,I \tag{2.4}$$

$$x_{jk}^{p} - \sum_{i=1}^{I} e_{ij}^{p} \leq 0, \quad \forall p=1,...,P, \forall j=1,...,J, \forall k=1,...,K(j) \tag{2.5}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \phi_{jlr} \times n_{jilr}^{p} \leq h_{fleet} \times \psi_{lr}, \quad \forall p=1,...,P, \forall l=1,...,L, \forall r=1,...,R \tag{2.6}$$

$$n_{jilr}^{p} - \theta_{jlr} e_{ij}^{p} \leq 0, \ \forall p=1,...,P, \forall r=1,...,R, \forall j=1,...,J, \forall l=1,...,L, \forall i=1,...,I \tag{2.7}$$

$$\sum_{l=1}^{L}\left(Q_l^{truck}\times n_{jilr}^{p}\right)-Q_{i\alpha}^{sh}\times e_{ij}^{p}+f_{ji\alpha}^{p}=0\quad \forall p=1,...,P,$$
$$\forall j=1,...,J,\forall i=1,...,I,\forall \alpha=1,...,A,\forall r=1,...,R \tag{2.8}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\left(Q_l^{truck}\times n_{jilr}^{p}\right)-\sum_{k=1}^{K(j)}\left(B_{jk}\times x_{jk}^{p}\right)=0,\quad \forall p=1,...,P,\forall j=1,...,J,\forall r=1,...,R \tag{2.9}$$

The mining equipment can be placed in a given number of locations. A possible path of the locations of each piece of equipment is provided as part of short-term plan. Shovels are allocated to available sectors or remain in the current sector previously allocated. A sector must be mined at some period and a shovel must be allocated to the sector that has a lower cost of hauling and provides the material to match quality requirements. Constraints (2.3) ensure that each sector is allocated with less equal than ι shovels at sector j per period p. The parameter ι is the maximum number of shovels that can be allocated in each sector. Constraint (2.4) ensures that each shovel i may be assigned to one sector while the cost of movement is minimized in the objective function to prevent excessive shovel movement among sectors. Inequality constraints are used for the fleet allocation because not all the available shovels or trucks are allocated in scenarios where there are more equipments than the production requires in accounting for hauling distance. Constraint (2.5) guarantees that a mining block in sector j is mined only if a shovel is allocated to sector j.

Variable n_{jilr}^{p} decides the optimal number of trips for truck l to sector j and shovel i per period p, thus accounting for fluctuations of truck cycle time and mechanical availability. The number of trips decision variable n_{jilr}^{p} also supports in the allocation of each truck l to shovel i to sector j for mechanical availability and hauling realization r per period p. The formulation considers that a truck can be allocated to more than one shovel at the same sector j or different sectors. Constraint (2.6) limits the number of trips of a truck to its scheduled time per period as the operation progresses by extracting minerals and continuously extending the access. Indeed, the roads change dynamically. This implies uncertainty in the hauling time. The trip cycle time ϕ_{jlr} of truck l to sector j is drawn from distribution *R* times.

The decision variable n_{jilr}^{p} is also subject to the maximum number of trips that a truck l can haul from each sector j. The maximum number of trips θ_{jlr} per truck l is a preprocessed parameter because its components are not decision variables. Then, the number of total trips to each sector is restricted to a maximum number of trips times the e_{ij}^{p} binary decision variable. The decision variable e_{ij}^{p} is relevant in the constraints (2.7) because not all the sectors will be allocated with a shovel and a sector without a shovel cannot have number of trips. Decision variables n_{jilr}^{p} and e_{ij}^{p} are linked. The inequality constraint (2.7) also ensures that only an allocated sector with a shovel is assigned with trucks, and not all trucks are allocated at some scenarios. The link of truck l, shovel i and sectors j in the constraints ensure that all assignment possibilities for the trucks, shovel and sectors are taken into account.

There are capacity limits for each truck Q_{l}^{trk} and shovel Q_{i}^{sh}. The available fleet and their respective capacity are included in the formulation. The production of each shovel assigned to sector j is constrained to the maximum production of each shovel $Q_{i\alpha}^{sh}$. The e_{ij}^{p} binary decision variable helps to formulate the shovel capacity constraints (2.8) because not all of the shovels may be allocated. The lack of expected production by each shovel is stored by the decision variable $f_{ji\alpha}^{p}$, which is minimized at the objective function.

There are J sectors and each sector has $K(j)$ blocks to be evaluated. The tonnage of block k is B_{jk} and each block may be hauled from an in-situ location to a blending area or waste dump taking into account the fleet capacity constraints. The decision variables at operational and production constraints are linked to fleet allocation constraints. Indeed, constraint (2.9) links number of trips n_{jilr}^{p} of truck l from sector j and shovel i given mechanic availability and hauling time realization r with the mined block decision variable x_{jk}^{p}. The hauling tonnage by the trucks from sector j for mechanical availability and hauling time realization r must be equal to scheduled blocks tonnage at sector j.

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}Q_l^{trk}\times n_{jilr}^{p}+d_p^{m-}\geq M^{\min}\quad \forall p=1,...,P,\forall r=1,...,R \tag{2.10}$$

$$0\leq d_p^{m-}\leq \%Tol_{m-}\times M^{\min}\quad \forall p=1,...,P \tag{2.11}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}Q_l^{trk}\times n_{jilr}^{p}-d_p^{m+}\leq M^{\max}\quad \forall p=1,...,P,\forall r=1,...,R \tag{2.12}$$

$$0\leq d_p^{m+}\leq \%Tol_{m+}\times M^{\max}\quad \forall p=1,...,P \tag{2.13}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(O_{jks}\times B_{jk}\times x_{jk}^{p}\right)+d_{sp}^{o-}\geq O^{\min}\quad \forall p=1,...,P,\forall s=1,...,S \tag{2.14}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(O_{jks}\times B_{jk}\times x_{jk}^{p}\right)-d_{sp}^{o+}\leq O^{\max}\quad \forall p=1,...,P,\forall s=1,...,S \tag{2.15}$$

$$0\leq d_{sp}^{o-}\leq \%Tol_{o-}\times O^{\min}\quad \forall p=1,...,P,\forall s=1,...,S \tag{2.16}$$

$$0\leq d_{sp}^{o+}\leq \%Tol_{o+}\times O^{\max}\quad \forall p=1,...,P,\forall s=1,...,S \tag{2.17}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{jks}^{\varepsilon}-G^{\varepsilon-}\right)\times O_{jks}\times x_{jk}^{p}\right)+d_{sp}^{\varepsilon-}\geq 0\quad \forall p=1,...,P,\forall s=1,...,S,\forall \varepsilon=1,...,E \tag{2.18}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{jks}^{\varepsilon}-G^{\varepsilon+}\right)\times O_{jks}\times x_{jk}^{p}\right)-d_{sp}^{\varepsilon+}\leq 0\quad \forall p=1,...,P,\forall s=1,...,S,\forall \varepsilon=1,...,E \tag{2.19}$$

$$0\leq d_{sp}^{\varepsilon-}\leq \%Tol_{\varepsilon-}\times O^{\min}\times G^{\varepsilon-}\quad \forall p=1,...,P,\forall s=1,...,S,\forall \varepsilon=1,...,E \tag{2.20}$$

$$0\leq d_{sp}^{\varepsilon+}\leq \%Tol_{\varepsilon+}\times O^{\max}\times G^{\varepsilon+}\quad \forall p=1,...,P,\forall s=1,...,S,\forall \varepsilon=1,...,E \tag{2.21}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{sjk}^{\delta}-G^{\delta+}\right)\times O_{jks}\times x_{jk}^{p}\right)-d_{sp}^{\delta+}\leq 0 \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \quad (2.22)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{sjk}^{\delta}-G^{\delta-}\right)\times O_{jks}\times x_{jk}^{p}\right)+d_{sp}^{\delta-}\geq 0 \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \quad (2.23)$$

$$0\leq d_{sp}^{\delta+}\leq \%Tol_{\delta+}\times O^{\max}\times G^{\delta+} \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \quad (2.24)$$

$$0\leq d_{sp}^{\delta-}\leq \%Tol_{\delta-}\times O^{\min}\times G^{\delta-} \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \quad (2.25)$$

Production per time period p is constrained to the production targets (2.10). The production includes ore tones plus the waste tones. The ore tonnage is the material that has positive economic value meanwhile the waste tonnage is the material without positive economic value that needs to be extracted to allow access to ore and ensure continuity of ore production in the following periods. The number of trip decision variables and truck capacities are used to calculate the total tonnage extracted per period. The proposed model considers strict constraints for early periods and can be relaxed for the latest periods. To relax the production constraints, the shortage d_{p}^{m-} with respect to the target planned is considered, along with their respective tolerance of deviation (2.11). Traditionally an upper bound is not used in production formulation because the cost of mining will limit overproduction; however, at the current formulation the production must be limited because the capacity shovel constraints maximize the production by sector to increase the utilization of the shovel (2.12). The upper bound limits this maximization to keep close to the production targets. The deviation d_{p}^{m+} with respect to the upper bound total production is penalized in the objective function and their tolerance is considered (2.13).

As a production constraint, the ore tonnage should match the target ore production given by long-term production schedules (2.14, 2.15). The shortage d_{sp}^{o-} respects to the target planned and the surplus d_{sp}^{o+}, respects the upper bound ore processing and are penalized in the objective function. The deviations are limited by a percentage of ore production $\%Tol$ (2.16, 2.17). The upper bound is directly related to the ore tonnage scheduled plus the maximum capacity pile of ore next to the delivering location. The exceeding material from the upper bound may be considered as material that go to stockpile, and its tonnage are penalized by the corresponding re-handled cost.

Ore production must match certain quality constraints, that is, the expected grades or quality of the material at the end of the week or month must fit into specific ranges and this range depends on long-term production schedule specifications. To meet this demand, a block x_{jk}^{p} is mined only if their grade helps to satisfy the required quality given the available fleet. Assuming that the study case has E elements that have economic value and D elements as deleterious elements, 2(E+D) quality constraints are needed to meet quality conditions. The grade of the main commodity for ore tonnage should satisfy the constraints (2.18, 2.19) and the quality deviations have tolerance (2.20, 2.21) to ensure a production schedule with low variable average quality.

Ore production cannot have more than the required limits of contaminants because this contains D deleterious elements. The constraints (2.22, 2.23) ensure that the ore delivered by period given S scenarios of the grades have average grades less than $G^{\delta+}$ and more than $G^{\delta-}$ for deleterious element $\delta = 1,...,D$. The quality deviations related with contaminants are also constrained to tolerance (2.24, 2.25) to ensure production schedule with low variable average quality.

Blending of ore from sectors is carried out based on cutoffs that define the minimum quality that a block k must have to be included in the blending process. If a block k has the chance of being used for blending $O_{jks} = 1$; otherwise, the block k is allocated to the waste dump directly $O_{jks} = 0$. The quality constraints are satisfied when the total ore production meets the required quality conditions set as targets.

2.2.1.2 Constraints for Operational considerations

Operational considerations relate to the size of the equipment and accessibility restrictions that may require feasible, in a mining sense, production schedule patterns that allow the available equipment to work efficiently and streamline movements for safety reasons. The first operation consideration is the mining direction that facilitates access to the sectors to be mined and it is:

$$x_{jk}^{p} - \sum_{\tau=1}^{p} x_{jk'}^{\tau} \leq 0, \quad \forall p=1,...,P, \forall j=1,...,J, \forall k=1,...,K(j), k' \in \Omega_{k'} \tag{2.26}$$

Where $\Omega_{k'}$ is the set of indexes representing blocks that are horizontal predecessors which must be mined before block k to match the mining direction. A sector could be mined following eight directions, as shown in Figure 2.

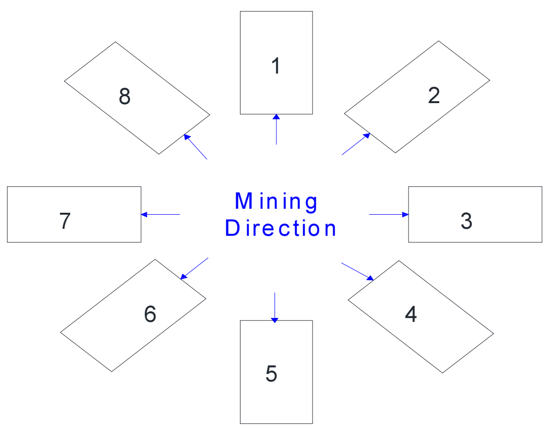


Figure 2: Eight mining directions considered by the formulation

The second operational consideration is the mining width and relates to the minimum width the patterns of a short-term schedule period has that permits fleet access to the orebody and materials that need be extracted. Production schedules without accounting for mining width may deliver schedule patterns with singular blocks of early periods surrounded by blocks from later period, as shown in Figure 3. This production scheduling cannot be implemented as the blocks scheduled for period 1, (blue squares) cannot be mined before some blocks belonging to period 2 (orange squares) are extracted.

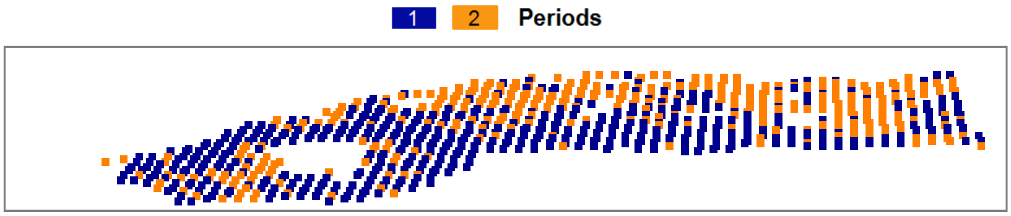


Figure 3: Production schedule without mining width constraints

The following mining width constraints account for feasible extraction patterns and may force the mining of some blocks before a given block k as shown in Figure 3.

$$-2\times\sum_{k'=1}^{\nu}x_{jk'}^{p}-\sum_{k'=1}^{\upsilon}x_{jk'}^{p}\ +\left(2\times\nu+\upsilon\right)\times x_{jk}^{p}-y_{jk}^{p}\leq 0,\quad \forall p=1,\ldots,P,\forall j=1,\ldots,J,\forall k=1,\ldots,K(j) \quad (2.27)$$

The mining width is discretized into υ blocks. To mine a block k, υ blocks may be mined at the same period or have been mined at previous periods. Priority of mining adjacent blocks ν is considered to avoid single blocks from some periods being surrounded by blocks from different periods. Indeed, the blocks ν that surround block k must be mined with twice the priority than the second term at constraints (2.27) to avoid infeasible mining patterns. The adjacent ν blocks belong to the inner window and the υ blocks belong to the outer window in smooth constraints (Dimitrakopoulos and Ramazan 2004).These smooth mining constraints are linked to mining width to provide feasible mining sequences that the fleet requires to operate efficiently. It is important to remark that υ number of blocks that match mining width are variable through the sector. The blocks that are located close to the border will require less υ blocks to be moved because some blocks were already mined or are 'air' (non-physically existing) blocks.

The mining width constraints are relaxed because at some locations feasible solutions will require to mine only some υ blocks. The discrete decision variable y_{jk}^{p} will store the lack of mining blocks that match the mining width considerations. This decision variable is penalized and minimized at the objective function.

2.3 Model implementation

The STPS model is solved by IBM ILOG CPLEX v12.4 in a computer with dual-core processor of 2.80GHz and 8GB of RAM for the deterministic formulation and a computer with a dual-core processor of 2.67GHz and 24GB of RAM for the stochastic formulation. The evaluation of the whole periods in one optimization demand unreasonable computing time. Sequential optimization with aggregated continuous periods is implemented to accelerate the solutions time; for example, twelve production periods may be optimized in four sequential optimizations of three periods.

2.4 Application at an iron ore mine

The proposed stochastic short-term production schedule (SSTPS) formulation is applied at an iron deposit. Iron ore deposits are typical examples of a multi-element environment, where the main production objective is to satisfy the customer quality requirement at a lower cost by optimally blending the different sectors of a mine. More specifically, when the iron content is evaluated and must be within customer specified limits there are also specific restrictions on the content of the so-called deleterious elements, such as phosphorous (P), silica (SiO_2), alumina (Al_2O_3) and the water and organic content measured as “loss on ignition” (LOI). These deleterious elements influence the physical and chemical properties of the iron ore product, significantly varies from customer to customer and contractual agreement to be met, and the performance of the process it will be used for. For instance, phosphorous affects steel quality (added cost), high silica and high alumina affect furnace efficiency, and the LOI affect fuel use and water in a hot furnace for steel making.

As noted earlier, the stochastic long-term production scheduling (SLTPS) of a given mine provides the larger scale framework defining the targets production of the short-term production schedule. Figure 4, for example shows the long-term production schedule of the iron mine in this case study and contains five periods (years). The first year (dark blue in the figure) is used herein for short-term production scheduling which is optimized over twelve periods (months).

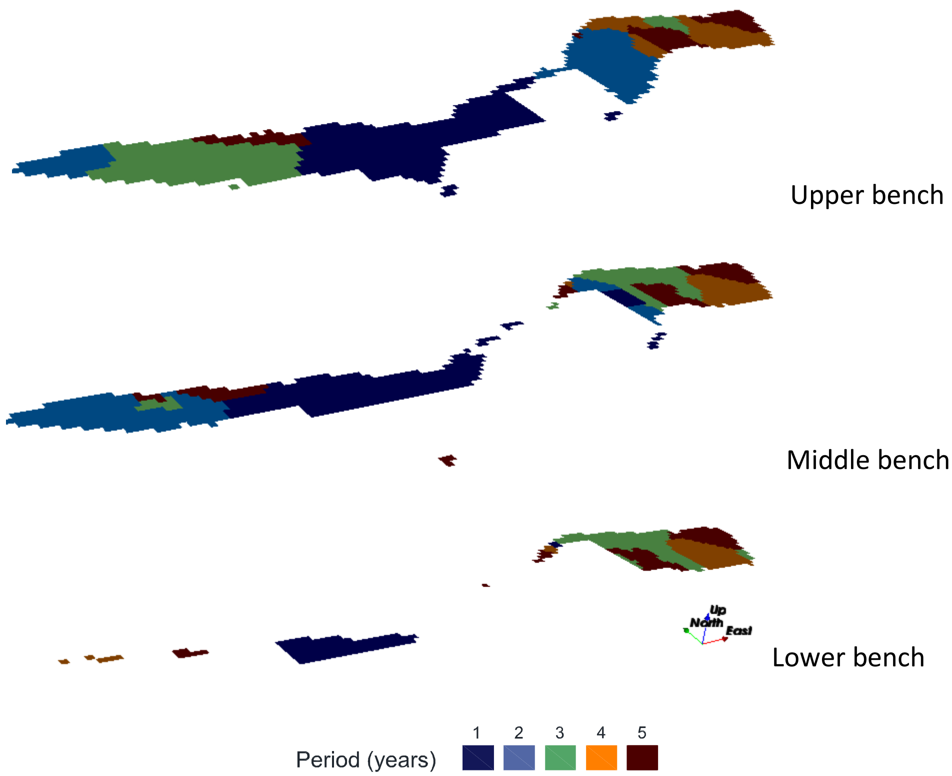


Figure 4: Stochastic long-term mine production schedule, 5 periods located at three benches (upper middle and lower), modified from (Benndorf 2005).

The quality targets and tonnes for the SSTPS and for the first year of production considered herein are given, see Table 1.

Table 1: First year production quantity and quality requirements.

Period	**Ore Tonage**	**Fe2O3(%)**	**P (%)**	**SiO2 (%)**	**Al2O3 (%)**	**LOI (%)**
1	14,000,000	57.1- 59.4	0.032 - 0.038	4.6 - 5.2	0.9 - 1.05	9.5 - 11

Note: Ore/Waste cut-off grade is Fe>= 56%

From the first year tonnage in Table 1; the mine must produce iron ore of about 1.16 millions of Iron tonnes each month. The average grade of the related elements per month may be in the intervals of the first year long-term ore quality given; however, the spatial variability of these grades varies when monthly increments are considered and along the mining direction and operational mining width. Ore quality intervals correspond to the upper bound and lower bound per element over the total year.

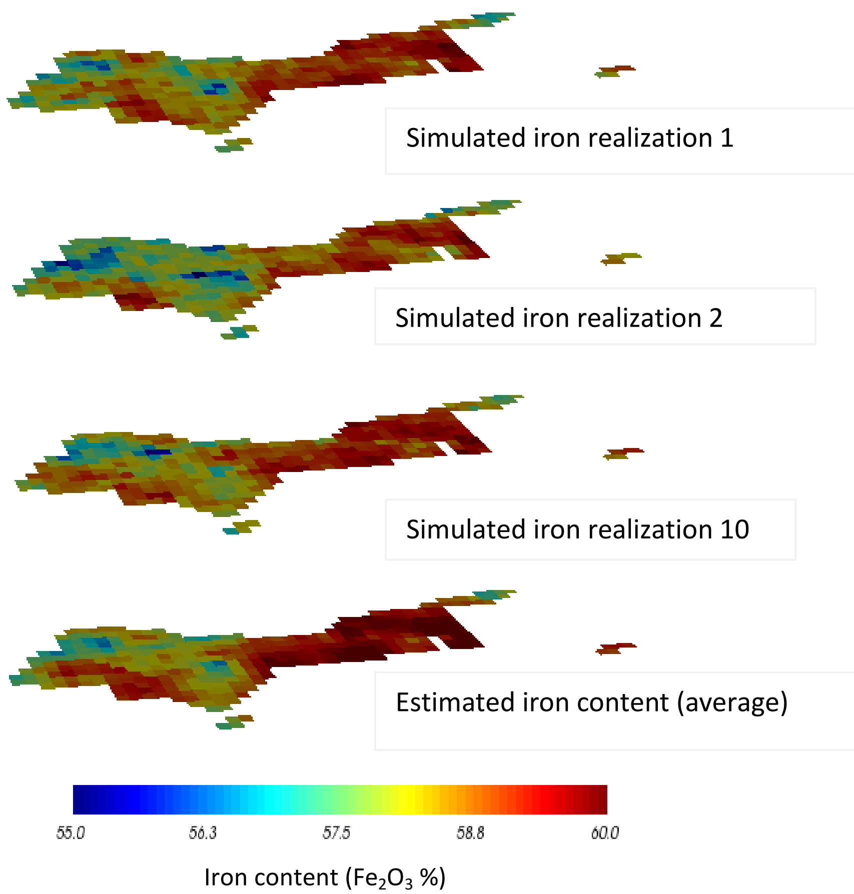


Figure 5: Iron ore content within the sector to be mined in the first year of production (Figure 4); 3 stochastically simulated realizations and the deterministic estimate for the upper bench (extraction units of 25x25x12 m^3).

The iron ore may be extracted from blocks of 25x25x12 m^3 located at three consecutive mining benches of 12m height. For this case study, ten equally probable scenarios of iron content, phosphorous, silica, aluminum and LOI are used to quantify the joint uncertainty in the characteristics of the iron ore deposit considered and are the input to the SSTPS formulation proposed in the previous section. The simulated scenarios available were provided and generated using the stochastic simulated technique detailed in Boucher and Dimitrakopoulos (2009, 2012). The area considered is bounded by the limits of the given volume of production in the long-term first year production schedule provided. Figure 5 shows 3 scenarios of iron ore content as well as the corresponding conventional and single estimated (average) representation of iron content (Fe_2O_3 %) for the upper bench. In total, 734 blocks from 3525 to 21150 tonnes, with Fe_2O_3 from 54.59% to 60.63%, P from 0.02% to 0.04%, SiO_2 from 3.10% to 8.58%, Al_2O_3 from 0.53% to 1.88% and LOI from 8.75% to 11.75% are available.

In addition to the uncertainty of the materials being extracted addressed above, the parameters related to the mining fleet available are given, so as to allocate efficiently and maximize the utilization of this fleet. The fleet size, mechanical availability and hauling time from the orebody to the various destinations are parameters used to allocated shovels and trucks at the related mine sectors. For this case study two shovels and ten trucks are the available fleet. The hourly productivity of each shovel fluctuates between 1180 and 1400 tonnes. The shovel model, digging rate and mechanical availability parameter distributions are given (Table 2) along with the track model, capacity and mechanical availability parameter distribution per truck (Table 3).

Table 2: Shovel model and mechanical availability parameter distribution

			Mechanical Availability (%)	
Model	**Shovel (*i*)**	**Production (Tonnes/hour)**	**Mean**	**Std.Dev.**
HS6020	1	1180	83	4.5
HS6030	2	1400	83	4

Table 3: Truck model and mechanical availability parameter distribution

Model	Truck (l)	Tonnes	Mechanical Availability (%)	
			Mean	**Std.Dev.**
Cat785D_501	1	136	83	5
Cat785D_502	2	136	83	4
	:	:	:	:
Cat785D_510	8	136	83	4
Cat77G_511	9	100	83	5
Cat77G_512	10	100	83	5

Short-term evaluation has the advantage of accounting for additional short-term information such as the hauling distance that is available at the short-term evaluation. This supports the allocation of trucks because the past records of speed per truck, truck hauling time per sector in a mine and blending pad location are available. Additionally, the parameter distribution of the time that spends l truck from the sector j to the destination is calculated as shown in Table 4.

Table 4: Trucks cycle time and parameter distribution (ϕ_{jlr})

Sector (j)	Truck (l)	Cycle time (minutes)	
		Mean	**Std.Dev.**
1	1	32	2.8
1	:	:	:
1	10	32	3.3
2	1	25	2.6
2	:	:	:
2	10	25	3.1
3	1	20	2.5
3	:	:	:
3	10	20	3

The cycle time ϕ_{jlr} from sector j to destination will be drawn r times from the respective distribution and the maximum trips are calculated given the mechanical availability per truck.

The parameters used to implement the proposed SSTPS formulation proposed herein are given in Table 5.

Table 5: Target month production and parameters.

Production Target	Parameter	Value	Unit	Penalty
	Max production	1,210,000	Tonnes	160
	Min production	1,100,000	Tonnes	160
	Max Ore production	1,210,000	Tonnes	16
	Min Ore production	1,000,000	Tonnes	4
	Allowed deviation tolerance	<=10	%	
Quality Requirement	Iron Ore(Fe2O3)	57.1- 59.4	%	1
	Phosphorous	0.032 - 0.038	%	10
	Silica	4.6 - 5.2	%	10
	Alumina	0.9 - 1.05	%	10
	Loss on ignition	9.5 - 11	%	1
	Allowed deviation tolerance	<=10	%	

Ore Definition	Parameter	Value	Unit
	Fe2O3	>= 56	%

Economic Parameters	Parameter	Value	Unit
	Mining Cost*	40	$/Tonne
	Cycle time Cost	120	$/hour
	Shovel Moving Cost	1000	$/100 meters

* Not include hauling cost

The total tonnage to be mined after twelve months of production is approximately 14 400 000 iron ore tonnes, and given the ore cut-off >=56% Fe_2O_3 almost all the material will be mined as ore. The targets of production and actual ore production are quite similar. Note that a high penalty is applied to the lack of mining from the expected monthly production because all material scheduled for the twelve months must be mined to align short-term production with long-term planning targets.

2.4.1 Short-term scheduling under uncertainty

The uncertainty in the iron grade and deleterious elements, mechanical fleet availability and hauling time are a major source of uncertainty that is incorporated into the production schedule formulation presented in Section 2.2. Figure 6 (left) shows the SSTPS production schedule at the iron ore mine in this application. For reasons of comparison, Figure 6 (right) shows the corresponding deterministic schedule generated from the deterministic equivalent of the SSTPS presented in Section 2.2, based on the average values for all related inputs. It is important to stress that the deterministically generated schedule may not be feasible in the actual presence of uncertainty that is not accounted for but is present.

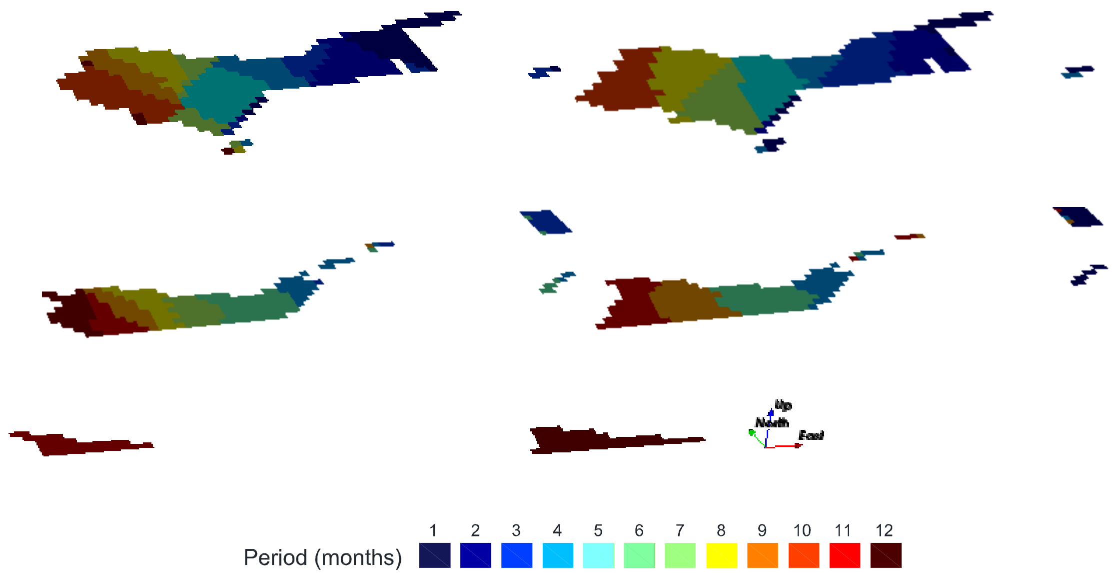
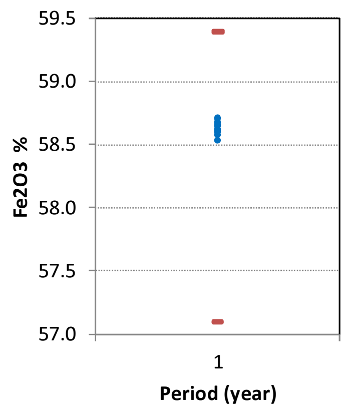


Figure 6: The stochastic short-term schedule(left) and the deterministic schedule (right).

Both production schedules in Figure 6 consider the same operational considerations and allocate similar sectors of the iron ore deposit to be mined until the 5th month of production; then the effect of uncertainty becomes evident as not enough materials are located at the upper bench to match quality requirements and the fleet is moved to lower benches. The quality target production given by the year long-term stochastic schedule is verified for iron as well as each deleterious element in this study case. The percentage of Iron per each scheduled month accounting for uncertainty and without accounting for uncertainty are compared against the expected production targets.



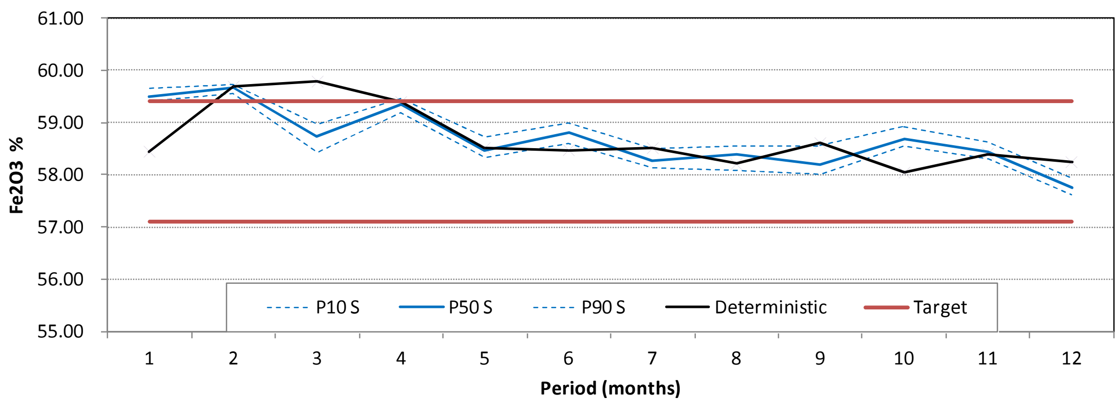


Figure 7: The long-term schedule for iron content considered and year's expectations (top); short-term schedule solution (bottom) (red lines: upper and lower bounds; black line: deterministic solution; blue lines: stochastic solution and risk profile)

Figure 7 shows the stochastic schedule solution (bottom) and the corresponding risk information in terms of P10 and P90. The long term schedule for the corresponding year (Figure 7, top) shows a more narrow space of uncertainty for iron than the twelve months of stochastic short-term schedule risk profile. This is expected because grades of small volumes (monthly production) are more variable that grades of larger volumes (yearly production). The next element to be assessed is phosphorous. Figure 8 (bottom) shows the stochastic schedule solution. The upper bound is satisfied for both the stochastic and deterministic solutions; however, the lower bound is violated for several periods. The phosphorous grade of each scheduled month is closer to the lower bound as the provided long-term schedule for the whole year (Figure 8, top).

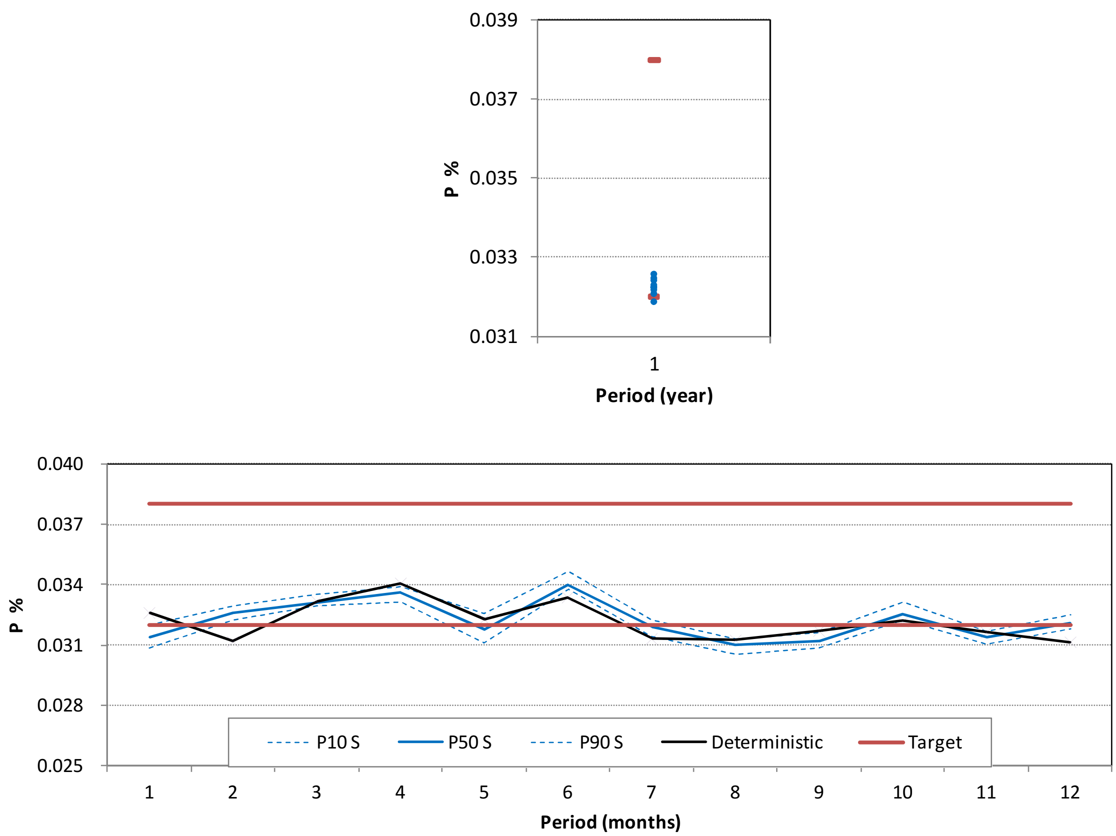


Figure 8: The long-term schedule for phosphorous content considered and year's expectations (top); short-term schedule solution (bottom) (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile)

The upper quality target is satisfied for both the stochastic solution and the deterministic solution; however, the lower quality target is violated for half of the periods. The phosphorous grade of each scheduled month is closer to the lower quality target. The next elements to be verified correspond to the three remaining deleterious elements.

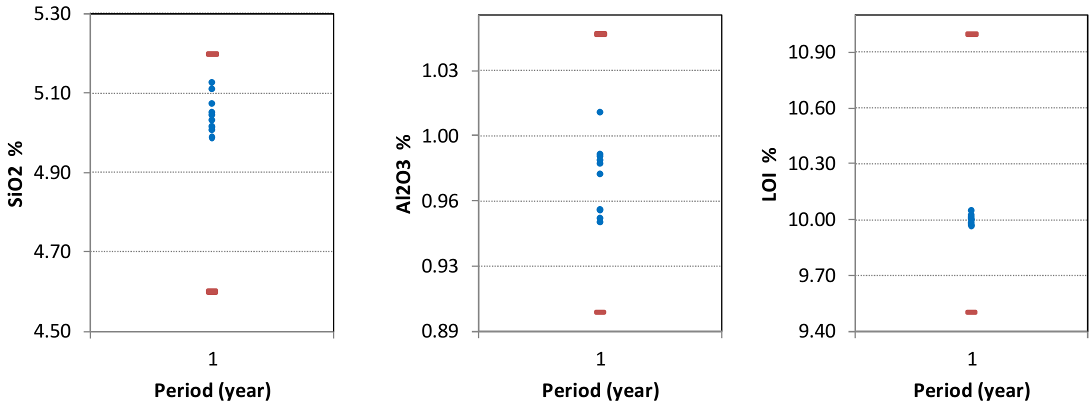
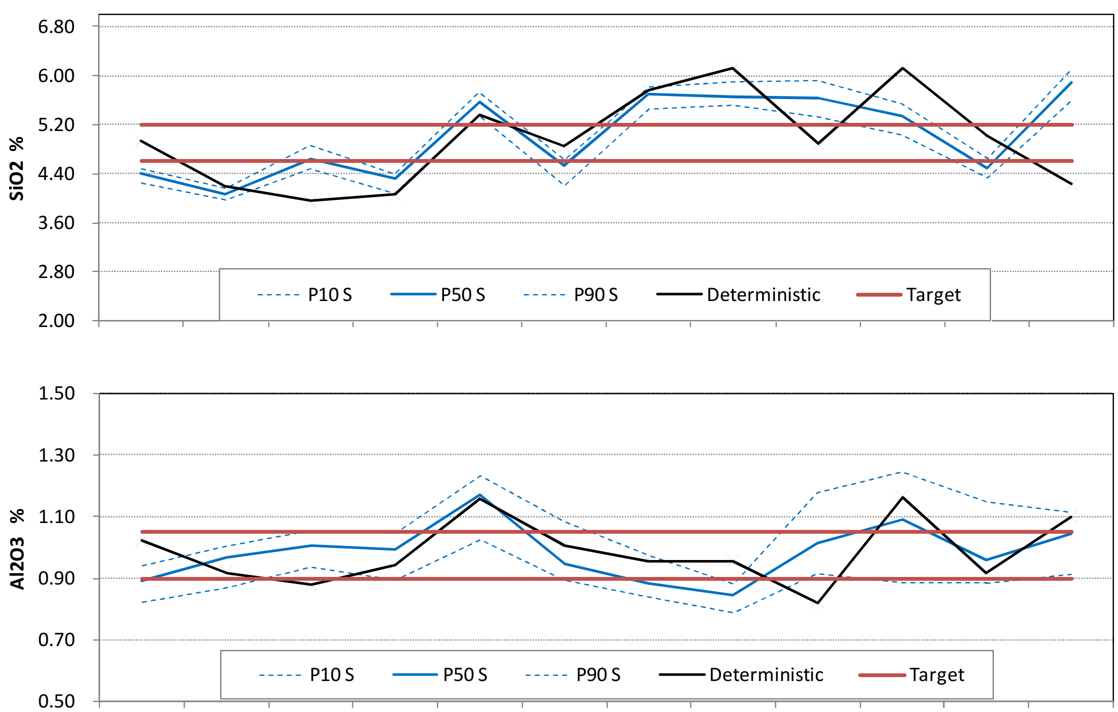


Figure 9: The long-term schedule risk profiles for silica, alumina and LOI (blue points) with lower and upper bound (red line)

The SSTPS solution for silica, alumina and LOI is shown in Figure 10.



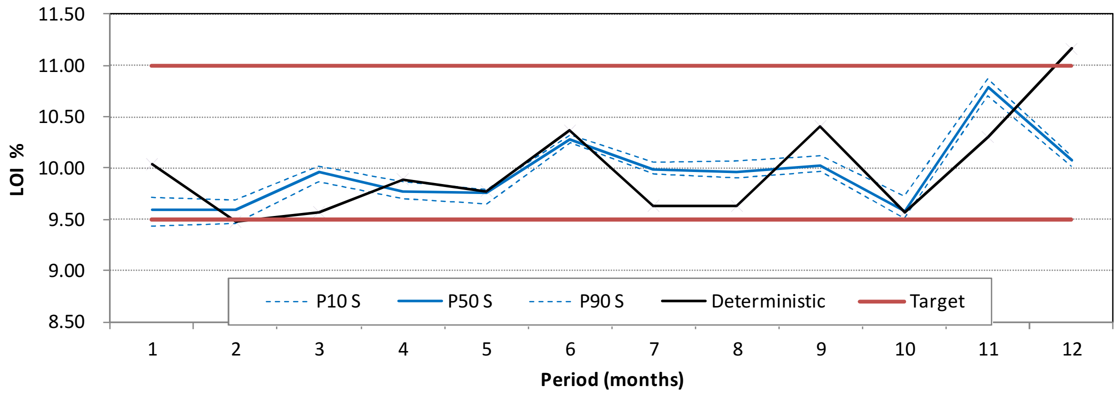


Figure 10: Short-term schedule solution for silica (top), alumina (middle) and LOI (bottom) (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile)

The stochastic short-term schedule risk profiles of the silica and alumina for twelve months coincide with the given stochastic long-term schedule risk profiles of the whole year regarding not matching lower quality targets; however, the upper quality targets are violated by the SSTPS in some periods. The quality of the expected year plan is not reproduced monthly because of the spatial variability of the grade, mining direction and mining width. Tolerance of 10% respect to each bound was applied for the elements; however, the lower quality targets tolerance was relaxed to be able to have feasible solutions for the twelve months.

The ore tonnage of the stochastic production schedule is compared against the target production and the deterministic short-term schedule.

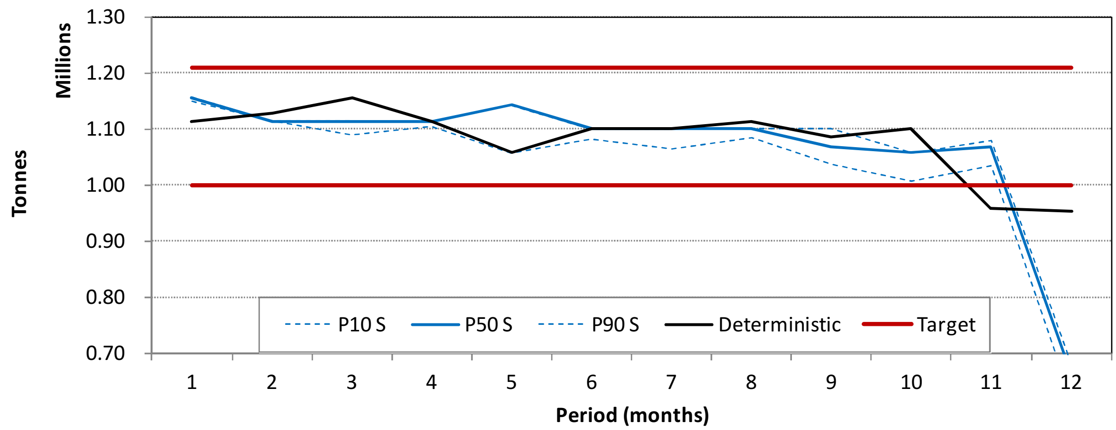
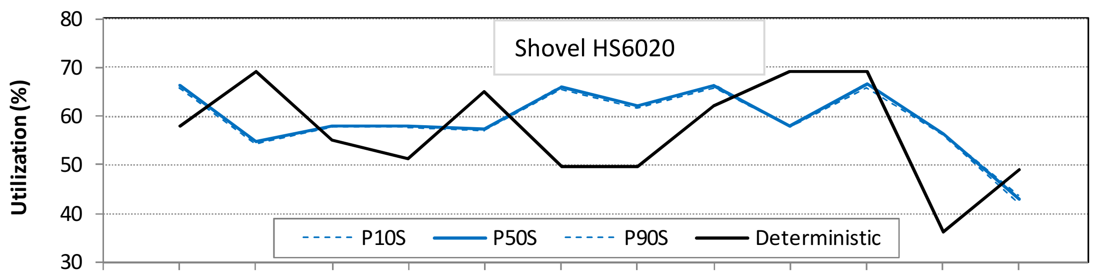


Figure 11: The short-term schedule for ore tonnes considered and year's expectations (top); short-term schedule solution (bottom) (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile)

The stochastic production schedule shows lower variability until the eleven month than the deterministic schedule. The material scheduled in the last month does not match lower bound ore tonnage because the remaining material to be scheduled does not have enough ore tonnage with the quality required.

Maximum expected shovel production is planned given mechanical availability and scheduled time. The lack of matching this expected production is penalized to maximize the utilization of the shovels.



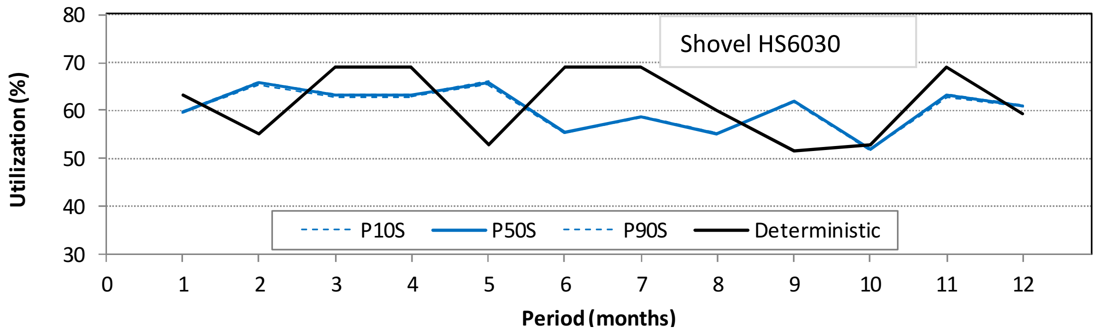


Figure 12: Utilization risk profiles of shovels for SSTPS on blue lines

The shovel utilization accounting for uncertainty at the SSTPS solution results in a higher and less variable than the shovel utilization of the deterministic STPS solution. From the STPS solution, the shovel with a historically high production was allocated preferentially to a sector that ensures its better utilization and the shovel with a historically low production to sectors with high production uncertainty.

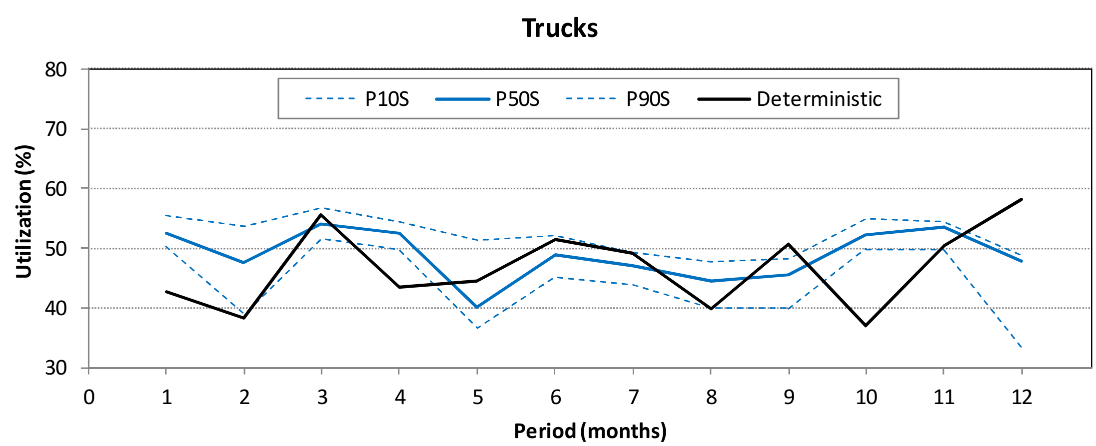


Figure 13: Utilization risk profiles of trucks for SSTPS on blue lines

The utilization of each shovel and the trucks are not exactly proportional because the trucks can be assigned to many shovels per period meanwhile the shovel is assigned to a sector and their movement between sectors are restricted by the cost associated. For example the small shovel HS6020 (Figure 12, upper) is allocated to the sector that has less available material to be mined making its utilization low at the final periods; however, the availability of the trucks are not affected because the trucks can be allocated to different sectors as shovels are allocated for each period. This ensures high utilization of the trucks, as shown in Figure 13. In some periods not all the trucks need to be allocated to match production targets. Considering that both schedules match production targets, the SSTPS shows a more efficient allocation or high utilization than the deterministic STPS because it allocates a less number of trucks, as shown in Figure 14.

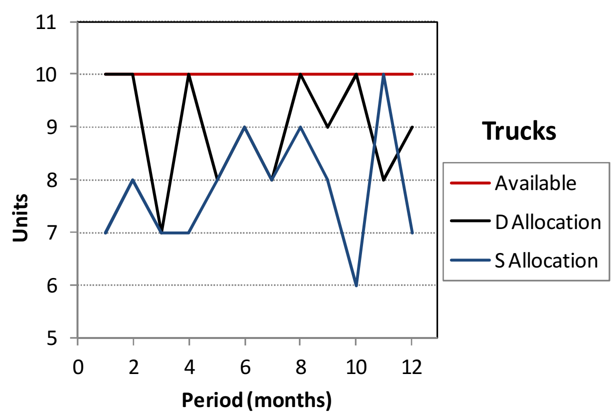


Figure 14: Available trucks in red line, number of trucks allocated accounting for uncertainty on blue line and without accounting for uncertainty on black line

The stochastic formulation provides a well informed schedule because it accounts for possible fluctuations of the grades and fleet parameters. From Figure 12 and Figure 13, the utilization of the fleet is shown as less variable through the periods when the uncertainty is considered. Implementation of the proposed SSTPS formulation also provides information about:

- Detailed production by periods
- Expected allocation, production and utilization of the fleet by period
- The fleet needed at each period to match production targets
- The report used to calculate risk profiles of the deterministic and stochastic production schedule

Additional to the high utilization of the fleet, the stochastic short-term production schedule solution shows feasible extraction patterns that have higher performance at the time of production because the operational considerations are accounted for.

2.4.2 Operational considerations

The further analysis evaluates the functionality of the proposed SSTPS formulation accounting for operational considerations. The operational considerations such as mining width and mining direction are evaluated individually and compared to production schedules where only slope constraints are considered. The production schedule in Figure 15 accounts for fleet allocation and slope constraints, neither mining directions constraints nor mining width constraints are taken into account.

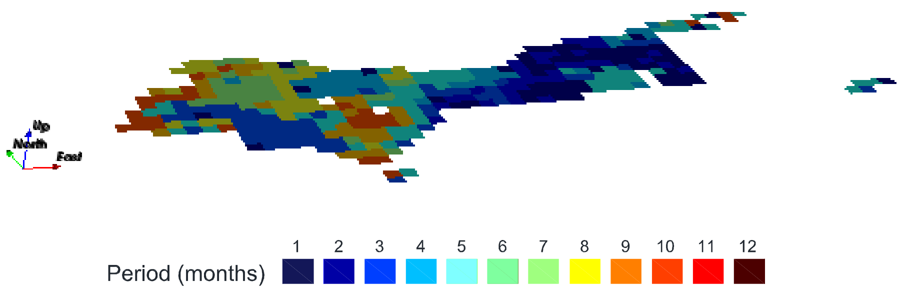


Figure 15: Production schedule without operational considerations, upper bench

As expected, the previous schedule respects the production targets and the utilization of the fleet is maximized and satisfies slope constraints but the patterns are unfeasible. There are blocks of earlier periods surrounded by blocks of later periods. This production schedule will have low performance at the time of mining because some blocks will not able to be mined in the scheduled period. To improve these patterns, additional operative considerations are incorporated into the formulation.

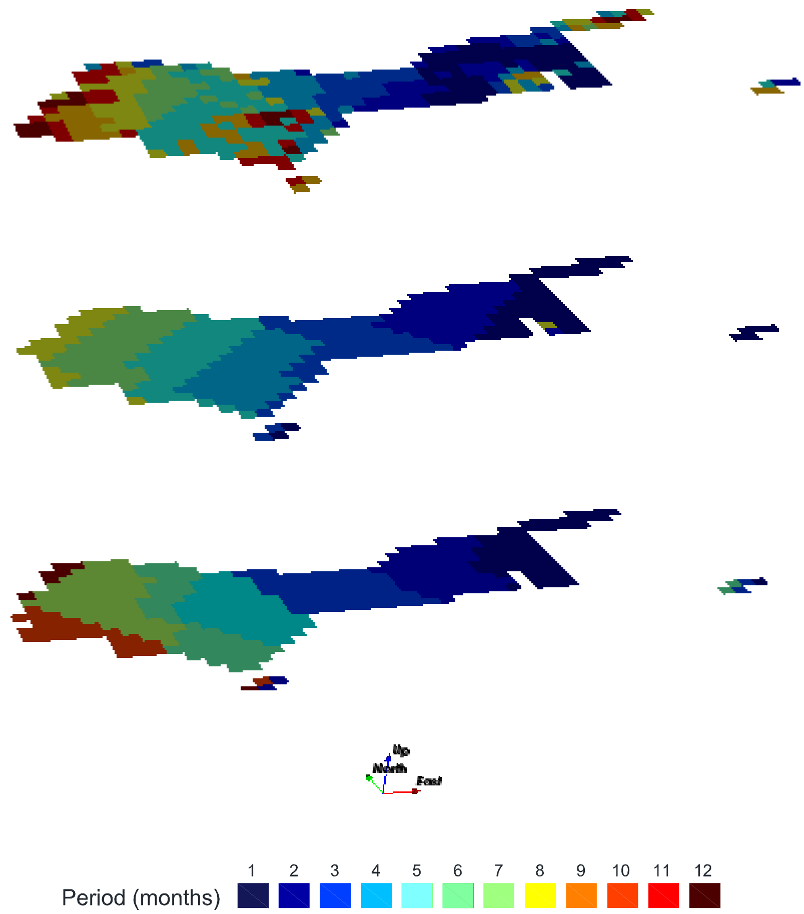


Figure 16: The upper bench of the production schedule accounting for mining width constraints (top), mining direction constraints (middle) and both mining direction and mining width constraints (bottom).

The production schedule (Figure 16, top) with high penalty cost to match mining width does not sufficiently improve the extraction patterns. At the short-term production schedule time, additional information about the feasible directions of mining may be available. Mining direction constraints are incorporated instead of mining width constraints where the upper bench is mined following an East to West direction. Figure 16 (middle) shows more feasible patterns per period ignoring mining direction constraints; however, this could still be improved by using mining width and mining directions simultaneously to better define the patterns of the monthly periods as shown in Figure 16 (bottom).

2.4.3 Sources of uncertainty in the mine production schedule

The proposed formulation of the stochastic short-term mine production schedule herein is influenced by the uncertainty of four parameters simultaneously: such as orebody uncertainty, shovel mechanical availability uncertainty, trucks mechanical availability uncertainty and truck cycle time uncertainty. However, the influence of each parameter at the time is studied in this section, that is, sensitivity analysis or parametric analysis will support the determination of the influence of each parameter in the objective function and in the production schedule patterns.

2.4.3.1 Effect of geological uncertainty

The decision of mining a block in a certain period is influenced by possible fluctuations in its quality. The influence of the orebody uncertainty in the production schedule patterns is compared with the deterministic production schedule patterns (where the orebody grade comes from an estimated model) as shown in Figure 17.

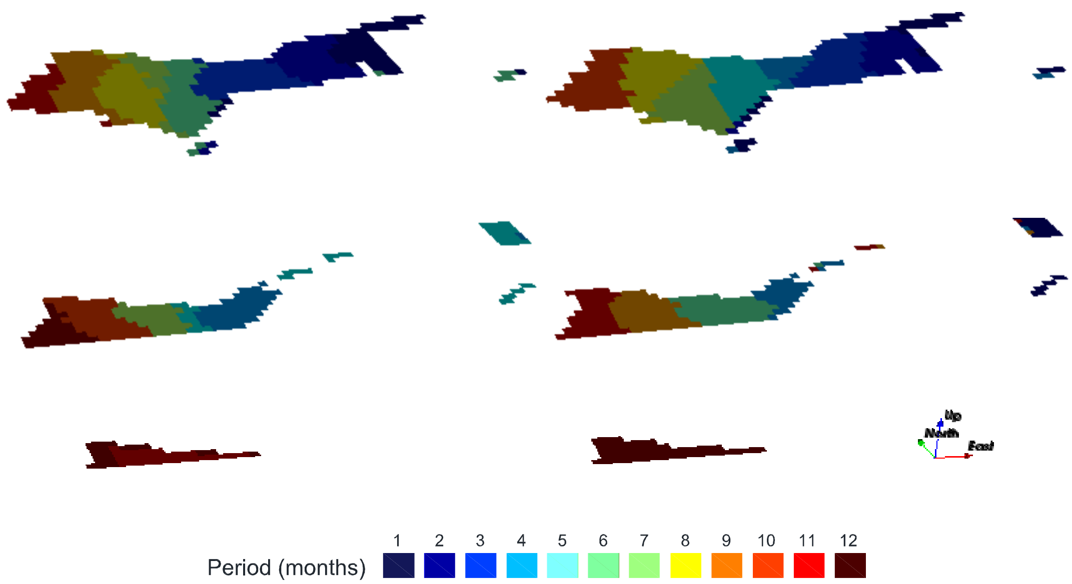


Figure 17: The short-term production schedule that account for orebody uncertainty (left) and the deterministic one with the estimated ore body model (right).

The production schedules illustrated previously are similar until period three, after which the orebody uncertainty influences in the decision of mining different sectors compared with the deterministic production schedules with estimated ore body model. Risk profiles of the production schedule accounting for orebody uncertainty are computed given ten realizations. The Fe_2O_3 required was >= 57% and <= 59.4% and both deterministic STPS solution and stochastic STPS solution satisfy this condition quality for most of the periods as shown in Figure 18 and very similar to stochastic STPS that accounts for four sources of uncertainty in Figure 7; however, the extraction patterns are different.

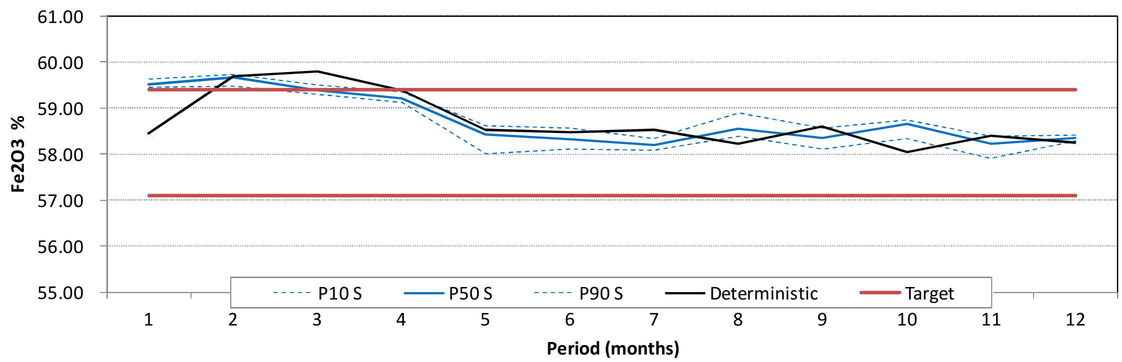


Figure 18: The short-term schedule solution for iron (red lines: upper and lower bounds; black line: deterministic solution; blue lines: stochastic solution and risk profile).

The multi-element deposit considers phosphorous, silica, alumina and loss on ignition, the monthly average grade of most of these elements match reasonably the upper bounds and have problems matching the lower bounds. To avoid unfeasible solutions, the tolerance of lower bound deviation was not used since there is not enough material to be blended to match quality requirements for the twelve periods; however, the monthly average of the silica shows quite different results, the silica upper bound cannot be satisfied from period six to nine and this solution cannot be improved even the silica deviations are penalized ten times more than the iron element; therefore, the average quality of one year of production schedule cannot be expected to be reproduced when it is divided into short-term periods. Since the short-term production scheduling must respect operational considerations and the grade is variable throughout the deposit, limited flexibility is given and some STPS periods may not match quality targets.

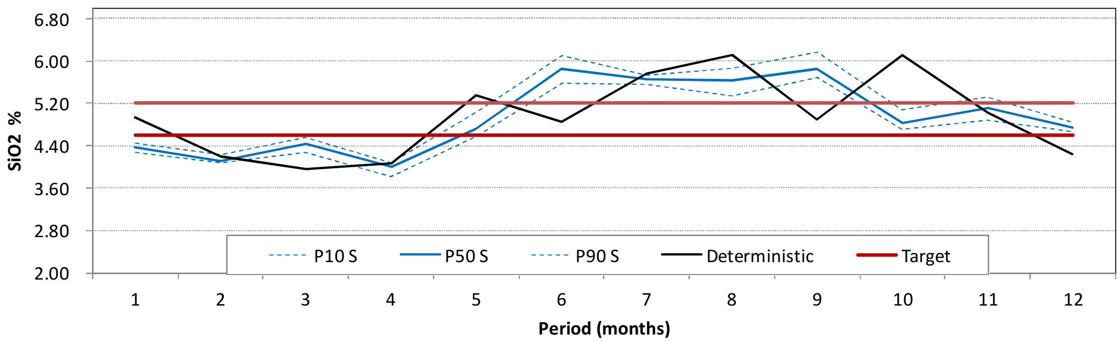


Figure 19: The short-term schedule solution for silica (red lines: upper and lower bounds; black line: deterministic solution; blue lines: stochastic solution and risk profile)

The monthly ore tonnages of the production schedule that accounts for orebody uncertainty hardly match the production targets. If we recall, the last period of ore tonnage of the production schedule that accounts for four sources of uncertainty, see Figure 11, did not match production targets, and the first eleven months are even less variable than the last production schedule as shown in Figure 20. This difference is due to the influence of the fleet parameter uncertainty.

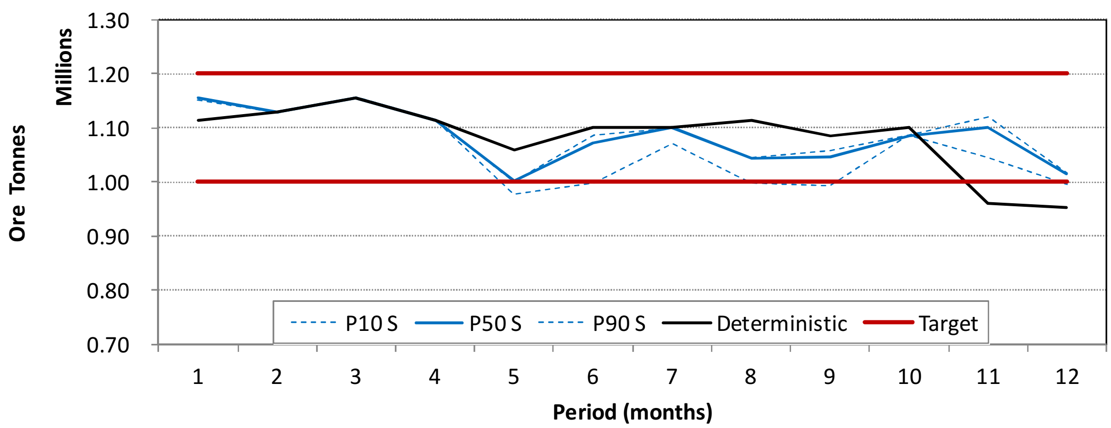


Figure 20: The short-term schedule solution (red lines: upper and lower bounds; black line: deterministic solution; blue lines: stochastic solution and risk profile)

In summary, the production schedule accounting for orebody uncertainty satisfies more efficiently the quality targets than the deterministic solution. False expectations could be generated by the deterministic solution because the orebody may behave differently from the estimated ore body model at the time of the production.

2.4.3.2 Effect of shovel mechanical availability uncertainty

The influence of the shovel mechanical availability uncertainty in the short-term production scheduling was compared with deterministic scheduling where the mechanical availability comes from the average values of historical data. The shovel HS6030 and shovel HS6020 were originally located at the upper bench and at the middle bench respectively. Then, the schedule accounting for only mechanical availability fluctuations showed that the shovel HS6020 is moved to upper bench and stays there for the four first periods because the material evaluated at the upper bench guarantees better utilization of this shovel under uncertainty. Indeed, the production of the first four periods is planned at the upper bench.

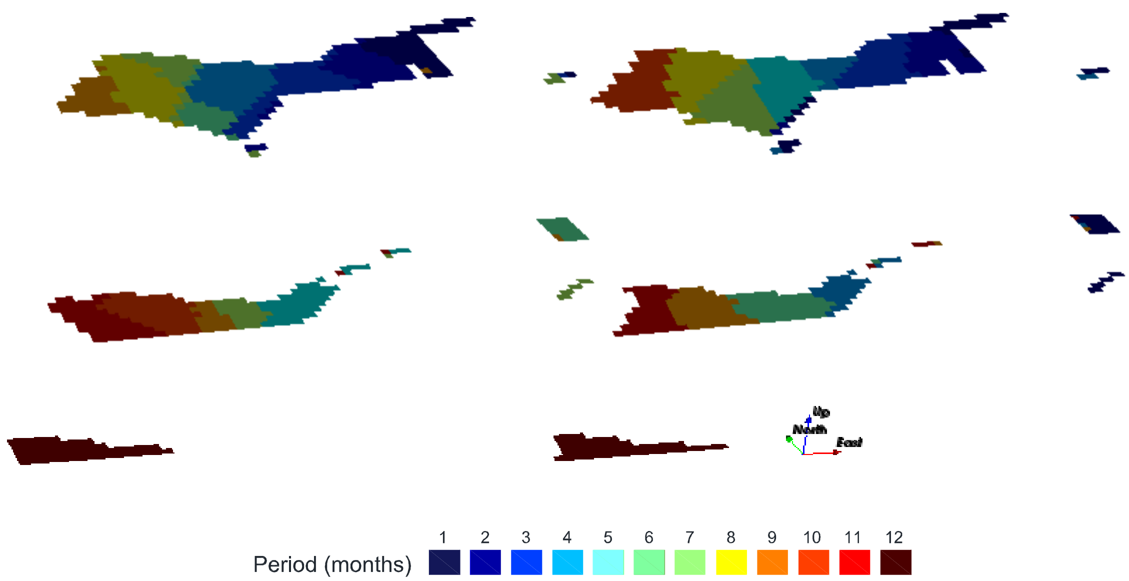


Figure 21: STPS solution that account for shovel parameter uncertainty on the left and deterministic STPS solution on the right.

The utilization of the shovels accounting for average mechanical availability which is deterministic and for uncertainty in the mechanical availability changes through the twelve months. Slightly less variable and higher utilization is observed when the production schedule accounts for the mechanic availability uncertainty source.

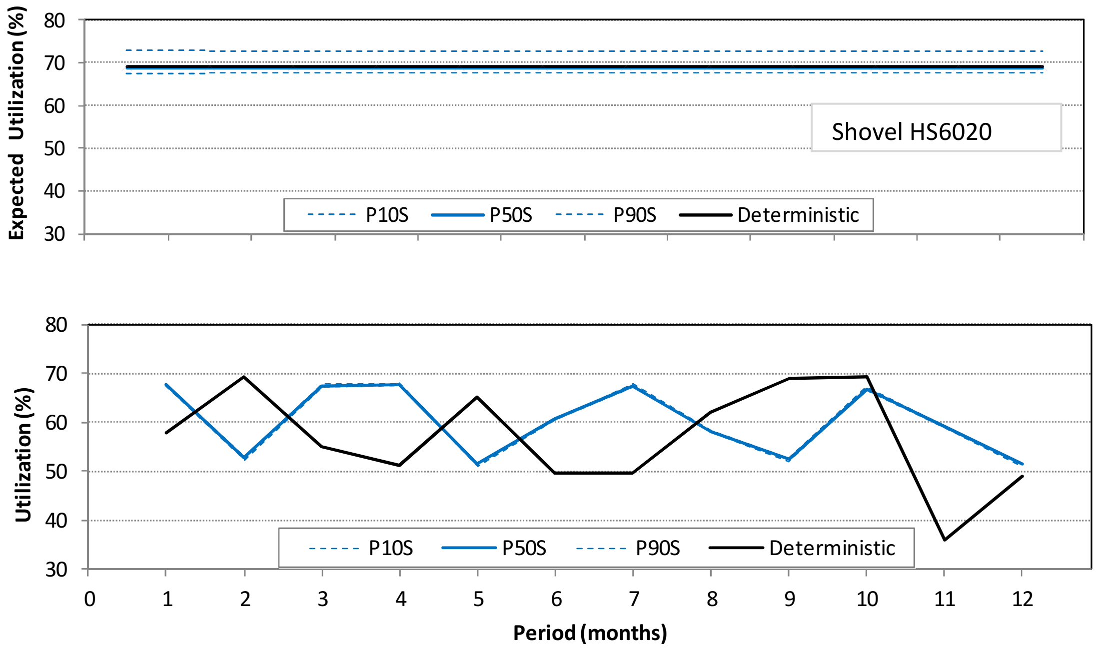
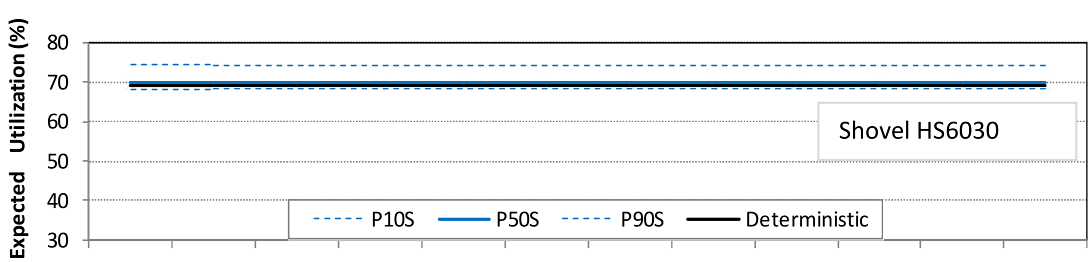


Figure 22: Expected utilization of shovel HS6020 (top) and the utilization after the production scheduling is solved (bottom).

The expected utilization is calculate from mechanical availability and hours scheduled by period. Indeed, deterministic expected utilization comes partially from the average mechanical availability historical data and the expected utilization accounting for fluctuations comes from the mechanical availability drawn from its distribution.



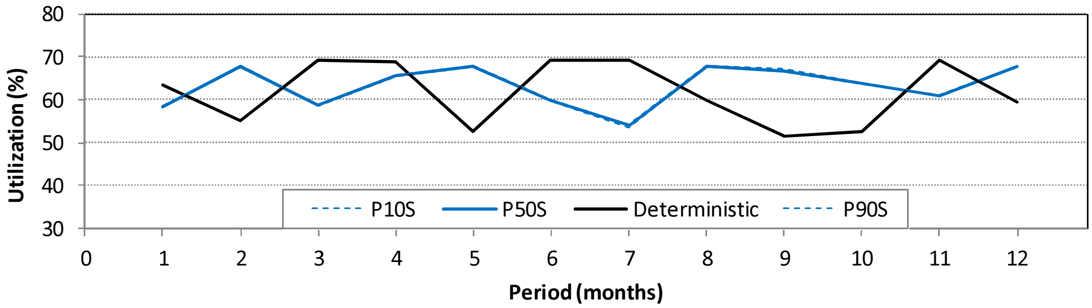


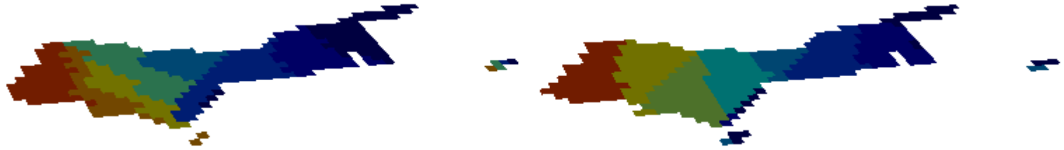
Figure 23: Expected utilization of shovel HS6030 (top) and the utilization after the production scheduling is solved (bottom).

The utilization of the HS6030 shovel accounting for mechanical availability uncertainty in the STPS formulation provides better utilization than using a single average mechanical availability because the shovel that has more capacity is allocated to places that guarantee higher utilization.

The shovel HS6030 has around 10% more capacity than the HS6020. The big shovel presents a consistent utilization through the periods; otherwise this happens with the small shovel that has low utilization in the last period.

2.4.3.3 Effect of truck mechanical availability

The mechanical availability of the trucks and hauling distance has influence in the decision of mining blocks in certain period. The trucks with high availability usually are assigned to a bench allocated with shovels that have less hauling time and thus more chance to reach the target ore tones that satisfy quality constraints. The influence of the truck mechanical availability and the hauling time fluctuations in the STPS is evaluated. These results are compared with the deterministic production schedule where fluctuation in the truck parameters is not considered.



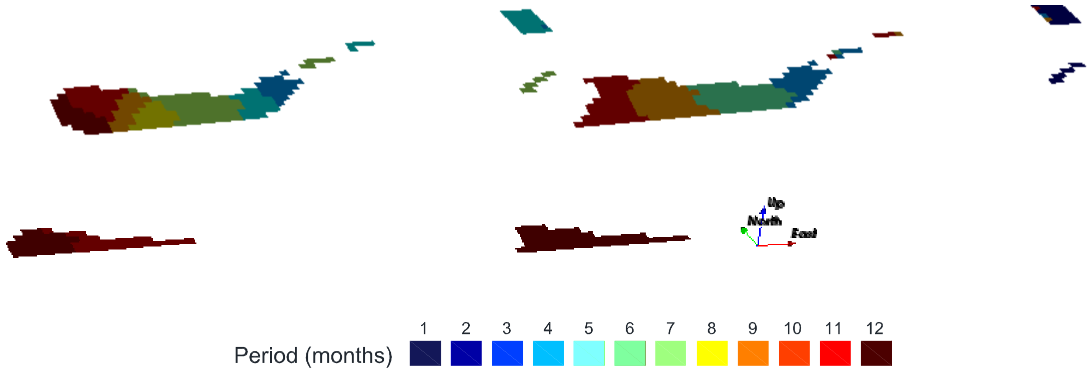


Figure 24: Short-term production schedule that account for truck parameter uncertainty (left) and the deterministic one that does not consider any uncertainty (right).

A lower utilization of the trucks means that there are more trucks allocated than necessary for production schedule need by period. The high cost of hauling demands to have an optimal allocation of trucks by period.

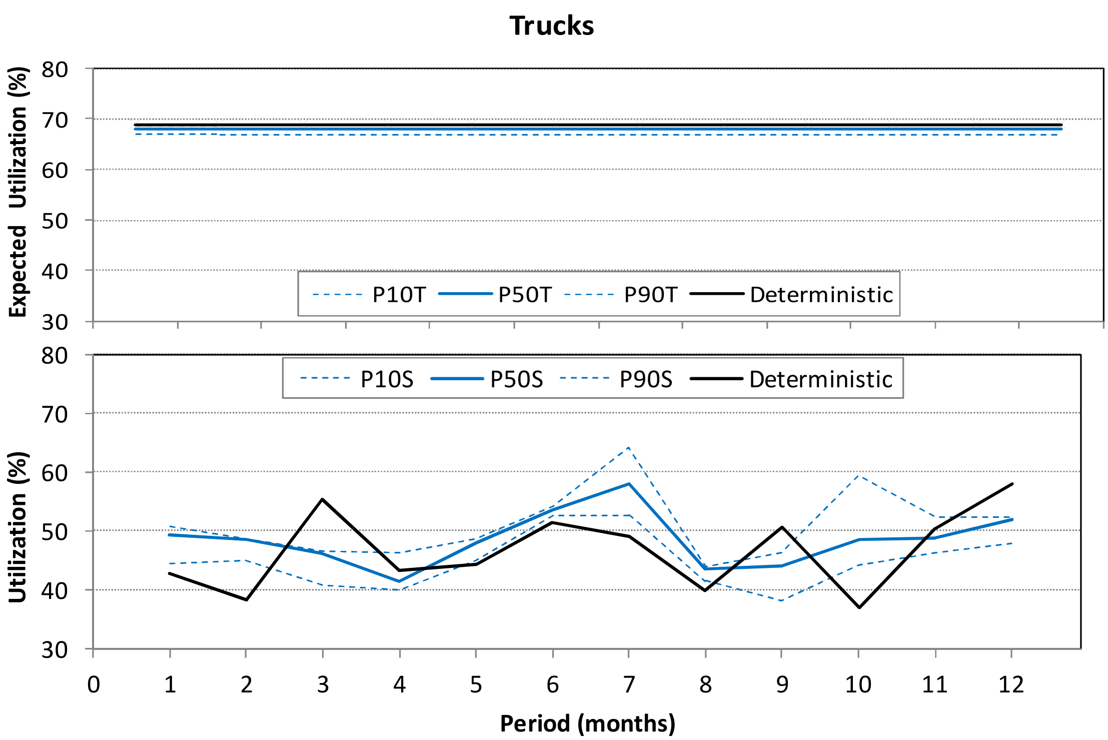


Figure 25: Expected utilization of trucks (top) and the utilization after the production scheduling is solved (bottom).

The fleet allocation accounting for deterministic truck parameters is more inefficient than accounting for possible fluctuations of mechanical availability and hauling time.

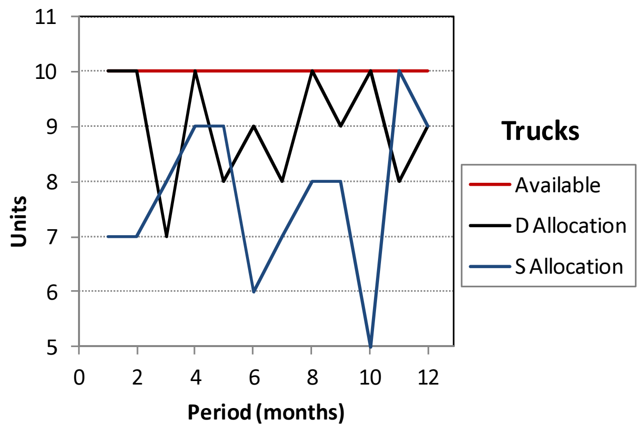


Figure 26: The total trucks allocated using a deterministic and stochastic formulation

The short-term production schedule accounting for single average mechanical availability and hauling time allocates more trucks with lower utilization, see Figure 26. The short-term production schedule accounting for trucks parameter uncertainty provides less variable and higher truck utilization.

2.4.4 Influence of the components in the objective function

The objective function consider some terms associated with operating fleet cost, mining cost, and penalty cost to penalize deviation from production target, expected fleet utilization and mining width. The penalties cause some terms to have more priority than the others because the optimization preferably minimizes the components that have high value. Figure 27 shows the comparison of minimized costs associated to each component after the optimization is solved.

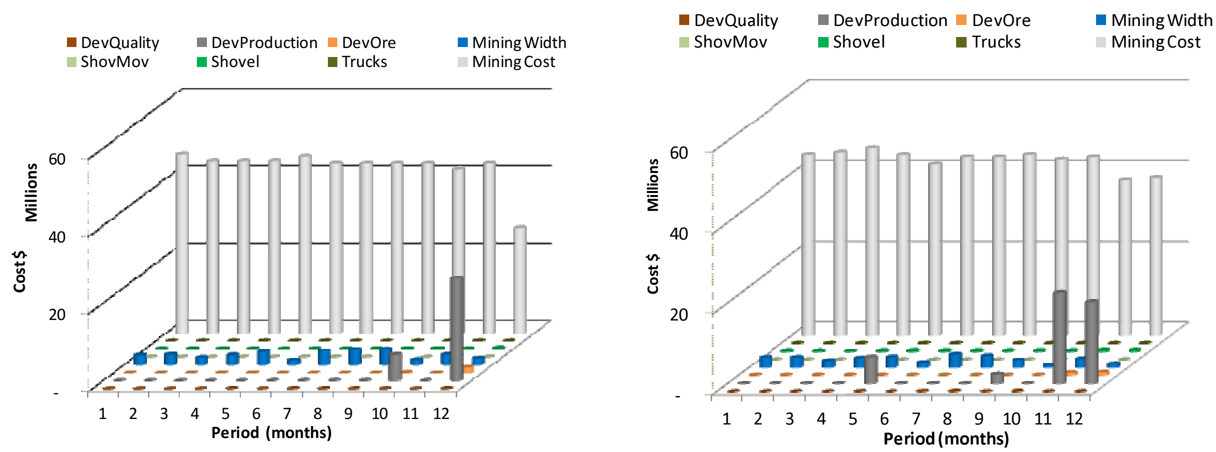


Figure 27: Minimized cost associate to objective function terms, stochastic STPS solution (left) and deterministic STPS solution (right).

The stochastic short-term production schedule solution shows less cost through the terms in the objective function than the deterministic schedule solution. From the formulation, the minimized cost means that the plan guarantees the minimum deviation from the production targets, such as tonnage and quality, maximum utilization of the fleet, minimum cost of production extraction and a better match of the mining width considerations. Indeed, the best production schedule may be the one that obtains the lower minimized cost.

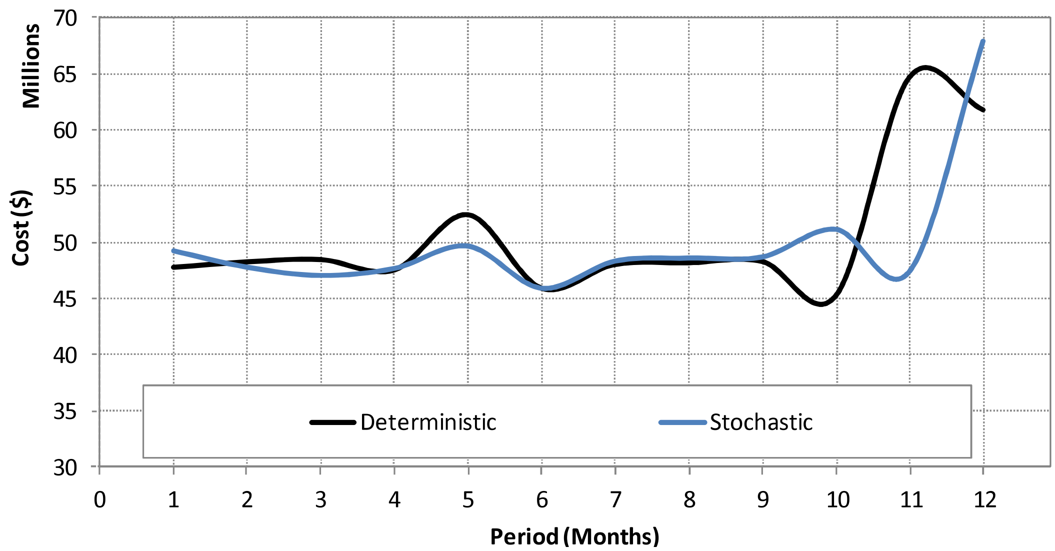


Figure 28: Testing the schedule performance, the deterministic STPS solution where all parameters are single estimated (black line) and the stochastic STPS solution accounting for four source of uncertainty (blue solid line).

The stochastic STPS solution provides less variable cost through the periods than the deterministic solution that does not consider uncertainty in their formulation.

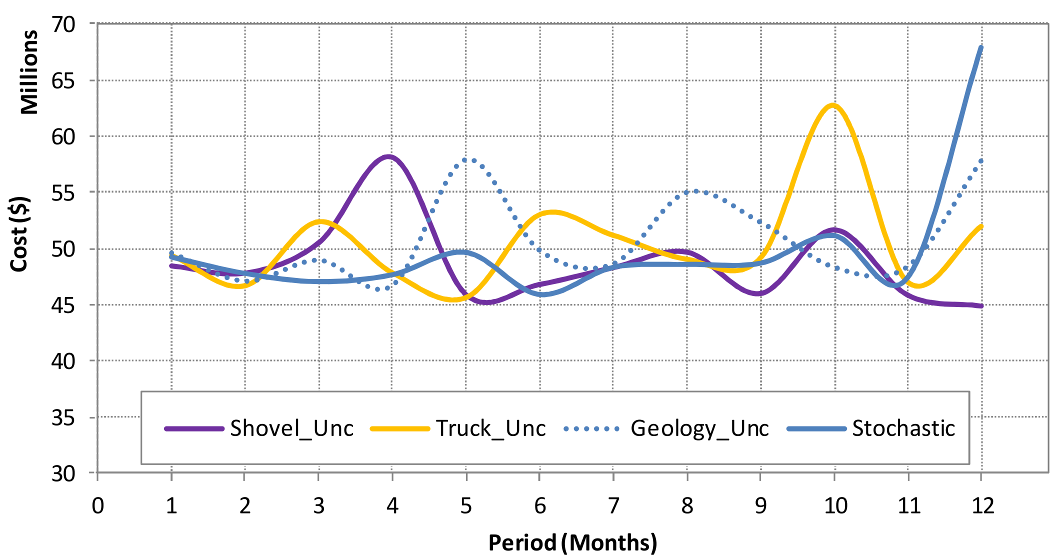


Figure 29: Testing the schedule performance, the stochastic STPS solution accounting for four source of uncertainty (blue solid line), the STPS solution accounting for orebody uncertainty

(blue dash line), the STPS solution accounting truck parameters uncertainty (orange line) and the STPS accounting for shovel parameters uncertainty (purple line).

Figure 29 shows that the mechanical availability shovel source of uncertainty has more influence in the stochastic solution. The stochastic STPS solution provides a less variable cost through the periods than the solution of the other schedules; however, the last period does not find enough resources to guarantee high utilization of the fleet and match the production target under uncertainty.

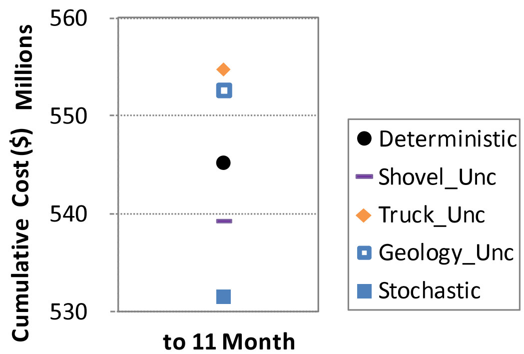


Figure 30: Cumulative cost of the short-term production schedules

The implementation of the proposed stochastic formulation in an iron ore deposit provides an improvement cost of about fifteen million CAD dollars less than the deterministic or production schedule that ignores parameter uncertainty. The deterministic STPS formulation cannot minimize in the same range as the stochastic STPS does because the uncertainty in the parameter is not accounted for. The uncertainties in the mechanical availability, in the orebody model and in the hauling time give more feasible solutions in the solution space to choose the best solution.

2.5 Summary and conclusions

The formulation proposed for stochastic short-term production scheduling obtains the solution with the lower cost in the application at a multi-element ore iron mine; however, the robustness of this formulation is based on the idea that their schedule is a well informed plan because it accounts for operations considerations, possible fluctuations of the orebody metal quality and fluctuations of the fleet parameters to decide which sector to be mined per period.

Chapter 3 Stochastic Short-Term Production Scheduling with simulated Future Ore Control Data

3.1 Introduction

Short-term production scheduling usually uses orebody models based on exploration data and ignores available past ore control data. The past ore control data could be used to define waste and ore more efficiently where only sparse exploration data is available. Short-scale information of the deposit is important at the time of the mine operation because a detailed short-term plan should be based on an accurate orebody evaluation that leads to better performance in terms of matching ore quality targets.

The optimization goal of short-term production scheduling is to minimize the mining cost and satisfy operational constraints such as mining slope, grade blending, metal production, mining capacity and processing capacity; however, some parameters may be uncertain, such as metal quality and fleet parameters. Stochastic programming is an approach for modeling optimization models that include uncertainty in their parameters as opposed to the traditional deterministic optimization models that are formulated assuming certainty in their input parameters. Real situations show the optimization parameters are not certain and may change over time when additional information could be available. The stochastic short-term production scheduling herein proposes to update the orebody uncertainty with future multi-element ore control data while taking mining considerations and uncertainty in the fleet parameters into their formulation.

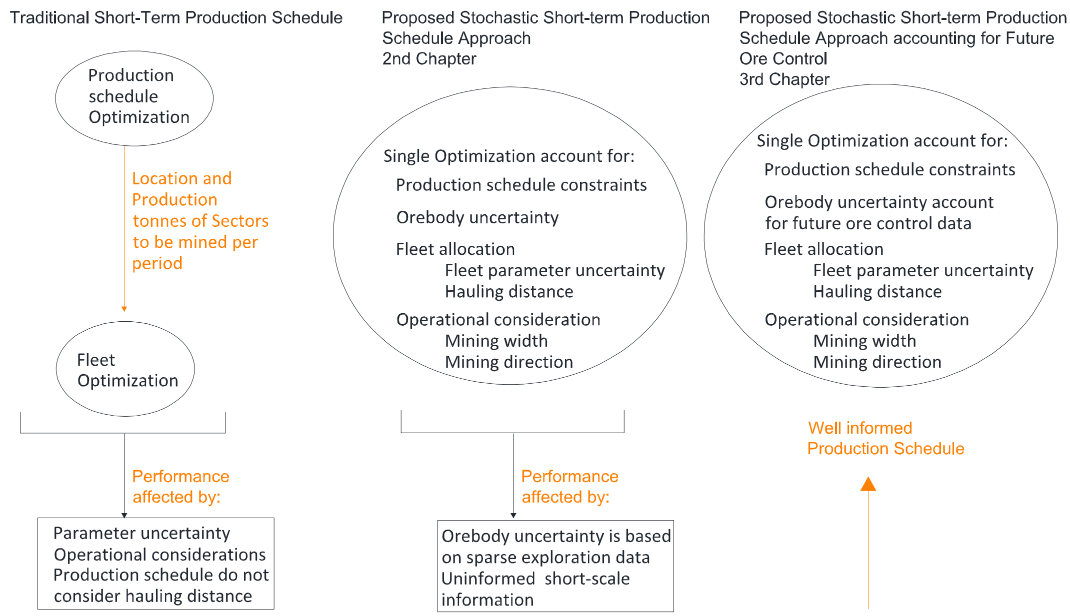


Figure 31: Schematic presentation of proposed stochastic short-term production scheduling approach

Two kinds of data are usually available at the mine production time, such as exploration data and ore control data. The exploration data have sparse drillholes information used to define orebody model and provide information about the metal quality that is used in long-term production scheduling; otherwise, the ore control data have dense blastholes information used to make polygons for daily production and provide information about the short-scale information that assists in the daily production scheduling. These ore control data are usually not available ahead of a production period; however, previous mined out data may be used to simulate the short-scale information of a similar geologic domain where only sparse exploration data is available. The main assumption will be that the past data from mined out parts must be geologically similar to the sector being scheduled. In the literature, there are methods proposed for generating stochastically future data, given past blasthole drilling data; however, all of them are for a single element. In practice, several deposits of copper or iron contain more than a single element to be evaluated. Future data for multi-element deposits have not been explored to date.

Guardiano et al. (1997) propose a method to measure the efficiency of both estimation and mining in situ reserves. Conditional simulation was used to validate production scheduling and recoverable reserves prior to production through factors. One factor in this method is the relationship of the actual in situ quantities which is the simulation based on exploration data at small grid and the room of mine quantities which are the inventory inside ore body contour based on future grade control data. This future grade control data is sampling from the simulation map based on exploration data plus an additional Gaussian error. The variance of this Gaussian error corresponds to the difference between exploration data nugget effect variogram and grade control data nugget effect variogram (Knudsen 1992). This technique is arbitrary and not practical for an ore deposit because the Gaussian distribution error used to simulate the future grade control data is the same for every location to be evaluated. Khosrowshahi et al. (2007) propose a simulation of the chain of mining to identify errors in every stage of the mining process to forecast the recoverable reserves during mining. Sampling and assaying errors of precision, mining selectivity and movement due to blasting are incorporated into the evaluation of several chains of mining to determine the parameters that may match the current mining performance of the mine. A grade control model is based on future grade control data and this future data is simulated based on sampling error distribution. Two sampling errors due to the shape of the blasthole cone and the impacts of the blasthole subdrill were used to define distributions of the errors. The drawback of this study is the same as Guardiano et al. (1997) because a local normal distribution error is used to simulate future ore control data through the domain. Journel and Kyriakidis (2004) show that the difference between ore control data and exploration data are not constant errors through the deposit. In a sector where only exploration data is available, their error could be forecasted given historical information from the same deposit or similar operations. The local distributions of errors are calculated from correspondence tables that take into account for historical ore control data behavior. An error is drawn from the local error distribution calculated and added to the local mean to generate the ore control datum mean at each location. The drawback is associated with the assumption that the local errors are Gaussian and independent. Indeed, the spatial continuity of the means and errors are ignored in this approach. To overcome this problem Peattie (2007), Peattie and Dimitrakopoulos (2013) present an approach based on the spatial variability of the error. The errors are calculated from the difference of estimation based on exploration data and estimation based on ore control at block scale. The spatial variability of these errors is modeled to simulate the error at the same grid where only an orebody model based on exploration data is available. The ore control data map is calculated from the orebody model available plus the error simulated. The idea of using the spatial variability of the error from historical data to forecast error seems reasonable; however, the variogram of the errors are calculated based on the block values which may provide a reduced spatial variability because of volume variance concept, and the approach is implemented only for a single element.

The mine production scheduling may account for short-scale information to efficiently match their production targets. The short-scale information before a production period may be defined by the orebody simulation based on the simulated future ore control data. Jewbali (2006), Dimitrakopoulos and Jewbali (2013) present an approach to forecast ore control data of a single element base on the spatial correlation between ore control data and exploration of a mined out sector. A pseudo cross variogram is used to evaluate the cross spatial variability of two data that are not at the exact same location. This pseudo cross variogram is used at the ore control sector where only exploration data is available to perform co-simulation conditioning to the available data. This simulation will provide short-scale orebody information in the form of high density future ore control data. Then, a conditional simulation by successive residuals may permit updating the orebody model using the simulated future ore control data. Finally, a stochastic programming mine scheduling formulation that maximizes NPV and minimizes deviation from expected production targets is optimized. The NPV becomes higher than conventional stochastic production based on only exploration data and the agreement of short-term to long-term production schedules may lead to a higher probability of meeting production targets and increased productivity. However, the future ore control data is not the only parameters used to synchronize short-term and long-term production schedules, and many mine deposits need the evaluation of more than a single element. The short-term production scheduling requires additional operational consideration and fleet equipment parameters instead of accounting only for long-term production scheduling constraints. This approach was implemented for a single element where a covariance matrix of primary data and secondary data and use of a pseudo cross variogram is required; however, the approach is not straightforward in the presence of multiple elements. There are techniques available that simulate multi-variables, such as MAF and Stepwise that require de-correlating the variables to simulate them independently and avoid the cumbersome techniques that require cross-variograms. Minimum/Maximum Autocorrelation Factor (MAF) (Desbarats and Dimitrakopoulos 2000) will be used to simulate spatial multi-element errors at the sector where the orebody model is based only on exploration data. The MAF is a principal components-based approach that besides the non spatial statistic of the lag-zero variance-covariance matrix accounts for the spatial relationship of the data.

3.2 Method

The proposed approach will have higher probability to meet production targets and increase productivity at multi-element deposits because it considers possible short-scale information that better assists in the classification of the material. The short-term production scheduling requires a precise local classification of waste and ore that cannot be given by an orebody model based on sparse exploration data.

The proposed formulation considers two stages; the future data is simulated in the first stage and stochastic mine production scheduling is optimized in the second stage as shown in Figure 32.

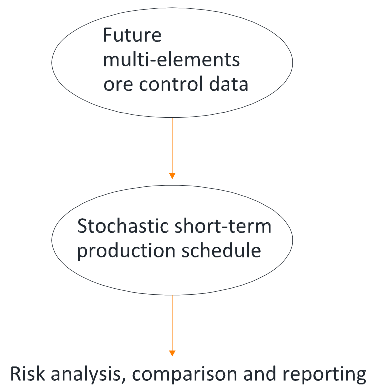


Figure 32: Sketch of proposed stochastic short-term production schedule approach

To test the outcomes of this approach, a comparison among traditional deterministic approaches and short-term production scheduling with and without accounting for future ore control data and risk analysis evaluation are carried out.

3.2.1 Stage 1: Future multi-element ore control data

The multivariate technique Minimum/Maximum Autocorrelation Factor assists to simulate multi-element deposits that consider the spatial relationship of the geological attributes or elements. The samples from earth science describe many geological attributes which exhibit spatial variability. The parametric models such as Lognormal, Hyperbolic and Fractal do not effectively describe the multivariate distributions. To lead with multivariate distributions, geostatistic simulations approach may require dimension reduction and de-correlation to simulate independently the variables and avoid the cumbersome techniques that require cross-correlations (Desbarats and Dimitrakopoulos 2000).

The MAF (Switzer and Green 1984) procedure set in a geostatistical context considers the multivariate observation vector as $Y(\mathbf{x}) = (Y_1(\mathbf{x}), \ldots, Y_q(\mathbf{x}))^T$ and their q orthogonal lineal combinations or Minimum/Maximum autocorrelated factors which correspond to $F_i(\mathbf{x}) = a_i^T Y(\mathbf{x}),\ i = 1, \ldots, q$. The multivariable $Y(\mathbf{x})$ is de-correlated as follows:

- Decompose the variance-covariance symmetric matrix of $Y(\mathbf{x})$. Their spectral decomposition HDH^T provides orthonormal eigenvectors matrix H and diagonal matrix of eigenvalues D.
- Compute the conventional principal components factors as $W^T = HD^{-1/2}$
- Calculate the variogram matrix $\Gamma_Y(\Delta)$ and their spectral decomposition into orthonormal eigenvectors C matrix and eigenvalues Λ diagonal matrix.
- Then, the MAF transformation matrix is $HD^{-1/2}C$
- The MAF factors will be $F(\mathbf{x}) = Y(\mathbf{x})\overbrace{HD^{-1/2}}^{W^T}C$

The MAF non-correlated factors $F(\mathbf{x})$ are simulated independently where their spatial relationship is modeled and sequential Gaussian simulation is carried out. The output of the simulations is re-correlated using the inverse of the MAF transformation matrix $(W^T C)^{-1}$.

This thesis proposes an approach to joint simulate future multi-element ore control data based on this Minimum/Maximum Autocorrelation Factors. The approach considers the spatial relationship of the errors between exploration data and ore control data from mined sector in order to forecast ore control data where only exploration data is available. The mined sector (A) must have similar geology as the sector (B) where it is required to simulate future ore control data.

The joint simulation of errors requires map of errors per element from the mined out sector A and to then de-correlate these errors. Next, the spatial correlation of these de-correlated errors is modeled to be used in the joint simulation at the grid of sector B. The simulated maps of errors are added to the orebody realizations at sector B to forecast the map of the future ore control data. The details of the approach are as follows:

- Calculate the error e_{bh-ddh} between exploration data and ore control data per element.
- Standard normal score transformation per error for sector A, $Y_A(\mathbf{x})$.

- MAF transformation $F(\mathbf{x}) = Y_A(\mathbf{x})W^T C$ is executed to de-correlate error variables of sector A to permit simulating the error of the elements independently.
- The variogram $\gamma^A_{F(\mathbf{x})}(\mathbf{h})$ of the MAF non-correlated factors $F(\mathbf{x})$ are modeled
- Unconditional simulation is performed for each MAF at sector B using the modeled variograms $\gamma^A_{F(\mathbf{x})}(\mathbf{h})$ of mined sector A.
- The inverse of the MAF transformation matrix $(W^T C)^{-1}$ is applied to MAF variables unconditional simulation at sector B to recover the correlation among errors.
- The normal score simulated errors are back transformed into the original space.
- Error maps are added to the available realizations based on exploration data at sector B

The approach requires modeling variograms as the number of elements is evaluated. The sketch is as follows:

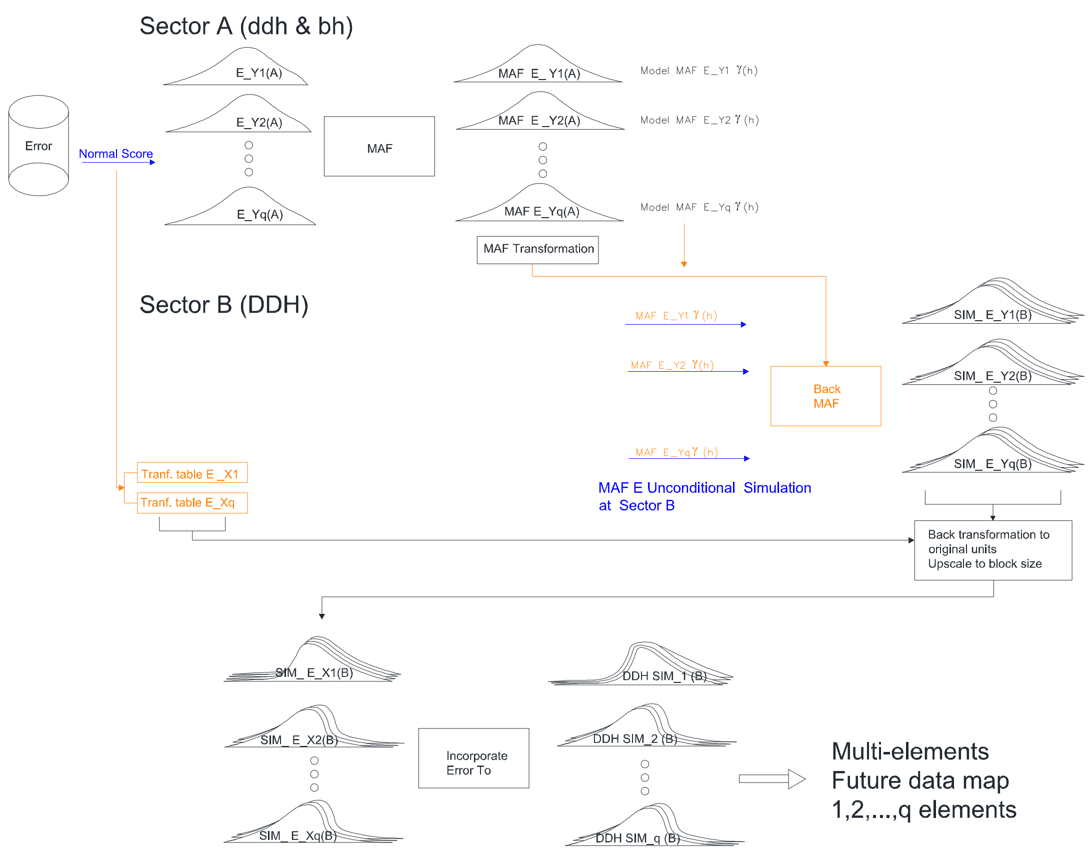


Figure 33: Simulated future multi-element ore control data sketch, second alternative

The MAF transformation permits the resulting MAF non-correlated factors to be simulated independently. For the simulation, the sector *B* is discretized by nodes and each node is simulated in Gaussian space, conditioned to previous nodes simulated but is not conditioned to the neighborhood data. These nodes are simulated sequentially following a path that changes from realization to realization. Once all the MAF non-correlated factors are simulated, back MAF transformation is applied to recover the correlation among variables. These variables are the errors and the simulation of multi-element map errors is added to the multi-element simulation at sector *B* to obtain the maps of the future ore control data. This approach is simple to implement because the future ore control data with multi-elements could be simulated without the tedious task of modeling many cross-variograms.

3.2.2 Stage 2: Stochastic short-term production scheduling

The formulation of the stochastic short-term production scheduling considers eight components in the objective function. The first, fourth, fifth and eighth components depend on deterministic parameters. The cost of extracting every tonne of the pit is reduced directly in the first term, and this cost does not include hauling cost. The hauling cost is covered and minimized in the second component in order to efficiently allocate the trucks. This component considers the possible fluctuations of two parameters: cycle trip and mechanic availability of each truck. The fourth component considers uncertainty in the mechanic availability of each shovel and the loading cost is indirectly reduced where the lack of mining expected digging rate is minimized to maximize the utilization of the shovels. The third component avoids inefficient excessive shovel movements and the fifth component avoids unrealistic short-term production patterns.

$$
\begin{aligned}
Minimize = & \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)} c^{m} B_{jk} x_{jk}^{p}}^{1st} \\
& + \overbrace{\frac{1}{R}\sum_{p=1}^{P}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{i=1}^{I} \phi_{jlr} c^{\phi} n_{jilr}^{p}}^{2nd} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{j'=1}^{J}\sum_{j=1}^{J}\left(c_{j'j}^{ExcM} e_{ij}^{p} a_{ij'}^{p-1}\right)}^{3rd} + \overbrace{\frac{1}{A}\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{\alpha=1}^{A}\left(c^{prodExc-} f_{ji\alpha}^{p}\right)}^{4th} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(c^{smoth-} y_{jk}^{p}\right)}^{5th} \\
& + \overbrace{\frac{1}{S}\left\{\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\varepsilon=1}^{E}\left(c^{\varepsilon-} d_{sp}^{\varepsilon-} + c^{\varepsilon+} d_{sp}^{\varepsilon+}\right) + \sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\delta=1}^{D}\left(c^{\delta+} d_{sp}^{\delta+} + c^{\delta-} d_{sp}^{\delta-}\right)\right\}}^{6th} \\
& + \overbrace{\frac{1}{S}\sum_{s=1}^{S}\sum_{p=1}^{P}\left(c^{o+} d_{sp}^{o+} + c^{o-} d_{sp}^{o-}\right)}^{7th} + \overbrace{\sum_{p=1}^{P}\left(c^{m-} d_{p}^{m-} + c^{m+} d_{p}^{m+}\right)}^{8th}
\end{aligned}
\tag{3.1}
$$

Components six and seven account directly for the updated orebody uncertainty; however, the short-scale information of the multi-elements may influence indirectly the rest of the decision variables of others components. Where the index j represents a sector or bench i represents an shovel, k represents a block at sector, l represents a truck model, p represents a period of the production schedule, ε represents an element grade of k block, δ represents an deleterious element grade of k block, s represents a grade realization or scenario, α represents a shovel mechanical availability realization and r represents a truck cycle trip realization and a mechanical availability realization.

B_{jk} represents the block tonnage k at sector j and ϕ_{rjl} represent the truck cycle time r of truck l at sector j given cycle time distribution. The costs at objective function are explained as follows:

$c_{j'j}^{ExcM}$ *:* cost of moving shovel from period *p-1 and* sector *j'* to new allocation sector *j*

$a_{ij'}^{p-1}$: binary parameter, if shovel i is or not allocated to sector j' at previous period p-*1*

$c^{prodExc-}$: penalty cost for tonnage not produced according to the expected productivity

c^{ϕ} : cost per cycle time ϕ units

c^{m-}, c^{m+} : penalty cost for shortage and surplus total mining tonnage respect to the targets

c^{o-}, c^{o+} : penalty cost for shortage and surplus ore mining tonnage respect to the targets

$c^{\varepsilon-}, c^{\varepsilon+}, c^{\delta+}, c^{\delta-}$: penalty cost for deviation from grade limits

c^{m} : mining cost by B_{jk} unit

The decision variables choose the optimal mined blocks by period, the allocation of the fleet, the number of trips and the deviation from the targets after the model is optimized.

x_{jk}^{p} : binary variable, if block k at sector j is mined or not mined at period p

e_{ij}^{p} : binary variable, if shovel i is or not allocated to sector j at period p

n_{jilr}^{p} : number of trips of truck l to sector j, shovel i at period p for cycle time realization and mechanical availability realization r

$f_{ji\alpha}^{p}$: deviation of shovel i at sector j from expected shovel production $Q_{i\alpha}^{sh}$

y_{jk}^{p} : number of blocks that were not scheduled at period p to mine block k at sector j to match mining width requirements.

d_{p}^{m-}, d_{p}^{m+}: shortage tonnage to match lower production limit and surplus tonnage to match upper production limit at period p

d_{sp}^{o-}, d_{sp}^{o+} : shortage of ore mining to match lower bound and the surplus to match upper bound at period p accounting for grade scenario s

$d_{sp}^{\varepsilon-}, d_{sp}^{\varepsilon+}$ *:* deviation from ε grade targets at period p for grade scenario s

$d_{sp}^{\delta-}, d_{sp}^{\delta+}$ *:* deviation from δ deleterious grade targets at period p for grade scenario s

The x_{jk}^{p}, y_{jk}^{p} and e_{ij}^{p} decision variables are scenario-independent because these they will be the same for each realization of the multi-elements and mechanical availability; otherwise, the deviations, lack of expected shovel production $f_{ji\alpha}^{p}$ and truck number of trips n_{jilr}^{p} are considered as recourse decision variables and depend on the source of uncertainties and scenario-independent decision variables. The recourse cost incurred whenever the stochastic constraints are violated. The constraints that consider the updated orebody uncertainty based on future ore control data are the set related to quality constraints and ore production constraints.

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(O_{jks}\times B_{jk}\times x_{jk}^{p}\right)+d_{sp}^{o-}\geq O^{\min}\quad \forall p=1,...,P,\forall s=1,...,S \tag{3.2}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(O_{jks}\times B_{jk}\times x_{jk}^{p}\right)-d_{sp}^{o+}\leq O^{\max}\quad \forall p=1,...,P,\forall s=1,...,S \tag{3.3}$$

$$0\leq d_{sp}^{o-}\leq \%Tol_{o-}\times O^{\min}\quad \forall p=1,...,P,\forall s=1,...,S \tag{3.4}$$

$$0\leq d_{sp}^{o+}\leq \%Tol_{o+}\times O^{\max}\quad \forall p=1,...,P,\forall s=1,...,S \tag{3.5}$$

The previous constraints are related to the ore production targets where the upper bound and lower bound are defined by any period of long-term production schedule. The deviations from these targets are penalized; however, the deviations have to be less than a percentage of the ore production targets to deliver low variable ore tonnage per period.

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{jks}^{\varepsilon}-G^{\varepsilon-}\right)\times O_{jks}\times x_{jk}^{p}\right)+d_{sp}^{\varepsilon-}\geq 0 \quad \forall p=1,...,P,\forall s=1,...,S,\forall\varepsilon=1,...,E \tag{3.6}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{jks}^{\varepsilon}-G^{\varepsilon+}\right)\times O_{jks}\times x_{jk}^{p}\right)-d_{sp}^{\varepsilon+}\leq 0 \quad \forall p=1,...,P,\forall s=1,...,S,\forall\varepsilon=1,...,E \tag{3.7}$$

$$0\leq d_{sp}^{\varepsilon-}\leq \%Tol_{\varepsilon-}\times O^{\min}\times G^{\varepsilon-} \quad \forall p=1,...,P,\forall s=1,...,S,\forall\varepsilon=1,...,E \tag{3.8}$$

$$0\leq d_{sp}^{\varepsilon+}\leq \%Tol_{\varepsilon+}\times O^{\max}\times G^{\varepsilon+} \quad \forall p=1,...,P,\forall s=1,...,S,\forall\varepsilon=1,...,E \tag{3.9}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{sjk}^{\delta}-G^{\delta+}\right)\times O_{jks}\times x_{jk}^{p}\right)-d_{sp}^{\delta+}\leq 0 \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \tag{3.10}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(B_{jk}\times\left(g_{sjk}^{\delta}-G^{\delta-}\right)\times O_{jks}\times x_{jk}^{p}\right)+d_{sp}^{\delta-}\geq 0 \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \tag{3.11}$$

$$0\leq d_{sp}^{\delta+}\leq \%Tol_{\delta+}\times O^{\max}\times G^{\delta+} \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \tag{3.12}$$

$$0\leq d_{sp}^{\delta-}\leq \%Tol_{\delta-}\times O^{\min}\times G^{\delta-} \quad \forall p=1,...,P,\forall s=1,...,S,\forall\delta=1,...,D \tag{3.13}$$

Where $O^{\min},O^{\max}$ represents the minimum and maximum ore tonnage target, $G^{\varepsilon-},G^{\varepsilon+},G^{\delta-},G^{\delta+}$ represents quality or grade requirements for ore tonnage produced, $g_{jks}^{\varepsilon},g_{jks}^{\delta}$ represents grade block k of main elements and deleterious in scenario s at sector j and O_{jks} represents binary parameters that flag the block k at j sector for scenario s which has the minimum quality to be used at the blending process; otherwise, the block is flag as waste. The previous eight set of constraints are related to the ore quality targets, the upper bound and lower bound are given as the ore tonnage. The deviations from these quality targets are penalized; however, these deviations must be less than a percentage of the ore quality targets in order to prevent excessive monthly quality fluctuations.

3.3 Application at an iron ore deposit

The implementation of the proposed stochastic short-term mine production schedule formulation accounting for future ore control data is the goal of this case study. Indeed, the future ore control data is simulated, the orebody uncertainty is updated with simulated future ore control data, and the influence of short-scale information in the mine production scheduling and in the fleet allocation are evaluated. This iron mine deposit should satisfy the customer quality requirement at lower cost by optimal blending from the different sectors. There are critical geochemical parameters where the iron content is evaluated. This evaluation is subject to Fe_2O_3 limits and deleterious elements restrictions, such as phosphorous content, silica content, alumina content and the water and organic content measured as loss on ignition that influence the physical and chemical properties of the product and the performance of the process.

The stochastic long-term production scheduling (SLTPS) is a relevant input parameter used to define the target production of the short-term production scheduling (SSTPS). From the given long-term production schedule of five periods (years), the sector that correspond to the first period will be used to schedule the short-term mining sequence, this period or annual production is optimized into short time periods of twelve months and the quality targets and tonnage of the SSTPS are given, see Table 6.

Table 6: First year of stochastic LTPS values copied from (Benndorf 2005) thesis.

Period	**Ore Tonage**	**Fe2O3(%)**	**P (%)**	**SiO2 (%)**	**Al2O3 (%)**	**LOI (%)**
1	14,000,000	57.1- 59.4	0.032 - 0.038	4.6 - 5.2	0.9 - 1.05	9.5 - 11

Note: Ore/Waste cut-off grade is Fe>= 56%

The monthly short-term production schedule must have an ore production around 1.16 millions of iron tonnes. The average grade of the elements per month may be in the intervals of the first year long-term ore quality given. In practice, the spatial variability of the grade, the mining direction and mining width make these quality targets hard to match on a monthly basis;

however, the total annual production should satisfy these upper and lower bounds per element. The twelve months production may be extracted from 734 blocks of 25 meters by 25 meters of 3525 to 21150 tonnes, located at three consecutive benches of twelve meters height each. The orebody uncertainty based on exploration data is an input data for the sector to be extracted (Sector B). S = 10 equally probable scenarios of the multi-element grades are considered and filtered inside the limits of the given long-term first year production schedule.

Additionally, two sets of data are provided from another sector that has similar geological domains and was mined out (sector A), for exploration data and ore past control data. The information of the exploration data contains 922 drillholes the depth of which fluctuate from 6 meters to 108 meters and has the median space distance of 48 meters. The information of ore control data has 18 181 blastholes with lengths of 6 m and the median space distance of approximately 15 meters.

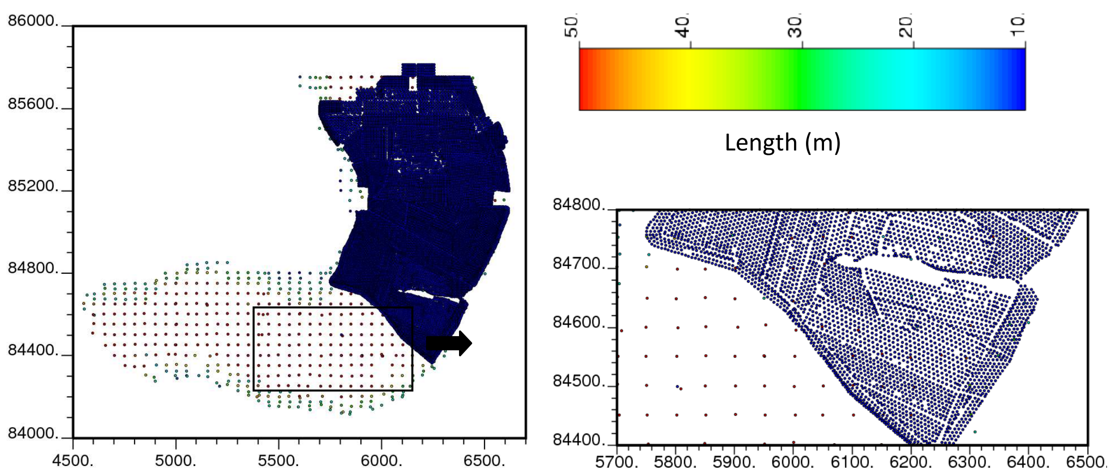


Figure 34: Location of drillholes (sparse data) and blastholes (dense data) at sector *A*

Besides the input data sets used to simulate future ore control data, the parameters related to the fleet are required. The size of the fleet, mechanical availability and hauling time from in situ orebody to destination are parameters used to allocated shovels and trucks at orebody sectors. For this case study i *=2* shovels and *l=10* trucks are given as a fleet. The hour productivity or digging rate of each shovel fluctuates between 1 180 and 1 400 tonnes and each truck can haul from 100 to 136 tonnes each trip. The mechanical availability of the fleet is considered as an uncertain parameter into the short-term production schedule formulation. The distribution of mechanical availability per truck and per shovel is given as input parameters from past data. Short-term production scheduling has the advantage of having some relevant information related to daily production that long-term production scheduling does not have. The hauling distance and the historical speed per truck are important parameters that are available and support in the allocation of the trucks, the truck hauling time for each sector to blending pad location or another destination may be calculated and are considered as uncertain parameters into the short-term production schedule formulation. The parameter distribution of the cycle time per truck is calculated. The cycle time ϕ_{jlr} from sector j, truck model l to destination will be drawn r times from their respective distribution and finally the target production used at the short-term constraints formulation are as follows:

Table 7: Target month production and parameters.

Production Target	Parameter	Value	Unit	Penalty
	Max production	1,210,000	Tonnes	160
	Min production	1,100,000	Tonnes	160
	Max Ore production	1,210,000	Tonnes	16
	Min Ore production	1,000,000	Tonnes	4
	Allowed deviation tolerance	<=10	%	
Quality Requirement	Iron Ore(Fe2O3)	57.1- 59.4	%	1
	Phosphorous	0.032 - 0.038	%	10
	Silica	4.6 - 5.2	%	10
	Alumina	0.9 - 1.05	%	10
	Loss on ignition	9.5 - 11	%	1
	Allowed deviation tolerance	<=10	%	

Ore Definition	Parameter	Value	Unit
	Fe2O3	>= 56	%

Economic Parameters	Parameter	Value	Unit
	Mining Cost*	40	$/Tonne
	Cycle time Cost	120	$/hour
	Shovel Moving Cost	1000	$/100 meters

* Not include hauling cost

The total tonnage to be mined after 12 months of production must be 14 441 925 and from this consists of 14 000 000 iron ore tonnage, according to the provided table of LTPS. Since the cut-off of mining is >=56% Fe_2O_3, almost all the material will be mined as ore. The targets of production and ore production are quite similar. High penalty is applied to any lack of mining in the expected monthly production because all material scheduled for the 12 months must be mined in order to align the short-term production with the long-term planning expectations.

3.3.1 Joint-simulation of future ore control data

The mined out sector A has exploration data and past ore control data, used to joint-simulate the future ore control data at sector B where only an orebody model based on exploration data is available. The six meters exploration data composites may be located at different locations than the dense ore control data the height of which is six meters; however, some composites of the exploration data may coincide at the same locations of the ore control data or be close enough to be considered “co-located”. The ore control data that intersects or are located <=1 meter close to the exploration data location is filtered, 411 “co-located” samples are found.

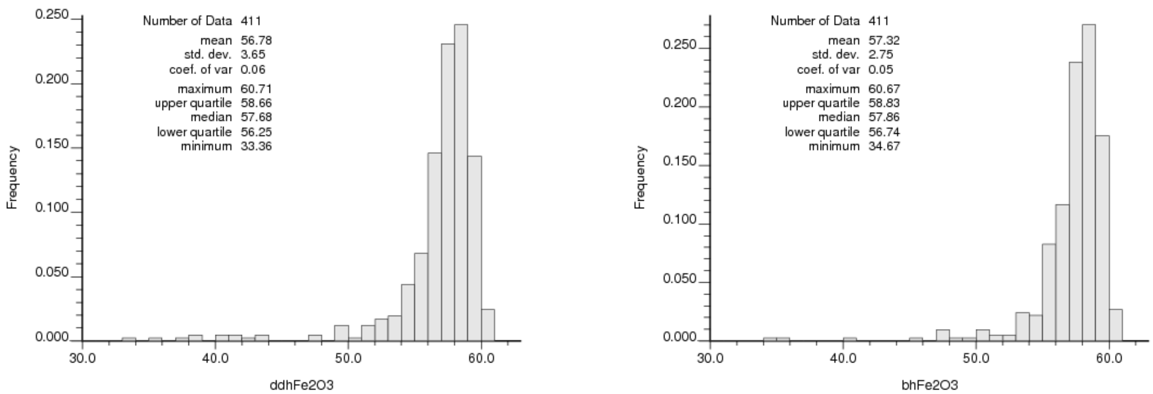


Figure 35: Iron histogram of exploration data (left) and ore control data (right)

The previous distributions show only the Fe_2O_3 % data of the mined out sector *A* where exploration data and ore control data is available. The exploration data contains Fe_2O_3 from 33.36% to 60.71%, P from 0.018% to 0.043%, SiO_2 from 1.86% to 21.51%, Al_2O_3 from 0.23% to 16.61% and LOI from 9.07% to 12.69%. On the other hand, the ore control data contains grades of Fe_2O_3 that fluctuate from 34.67% to 60.67%, P from 0.016% to 0.05%, SiO_2 from 1.9% to 23.01%, Al_2O_3 from 0.26% to 14.23% and LOI from 9.37% to 11.84%. The spatial location of the co-located data are illustrated, see Figure 36.

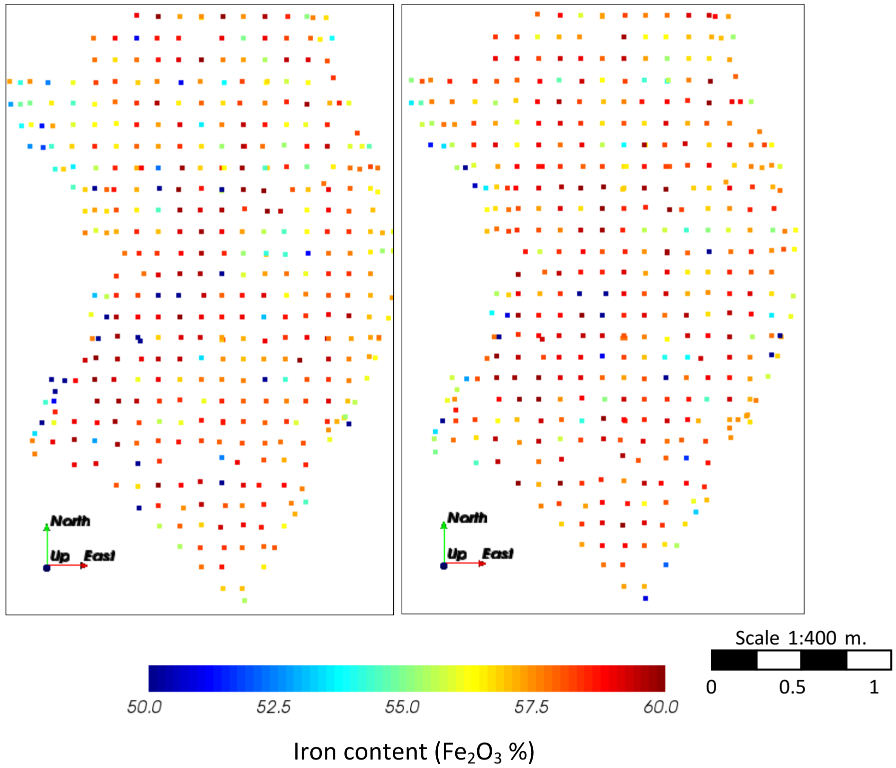


Figure 36: Co-located iron element of exploration data (right) and ore control data (left)

The difference between ore control data and exploration data per element is modeled and used to simulate future ore control data at similar geological domains. These differences will be named errors in this approach. The univariate distributions of these differences or errors are illustrated, see Figure 37. The source of these errors are variable, for instance, the collar coordinates of the exploration data is usually well measured; however, the survey angle that defines the location of each sample through the drillhole could be slightly miss-measured, then as the length of this drillhole increases, the error become higher because the samples are miss-located. In consequence, the exploration data may differ from the ore control data at closest location. The other source of difference between both data is the quality of sampling; the core sample of exploration data have more precise information about the structures, mineralogy, grade and alterations; otherwise, the sampling of blasthole piles for ore control data cannot provide precise samples because the reverse circulation drilling may contaminate the samples and the quality of the collection of samples is subject to the team expertise and time sampling available(Pitard 2008).

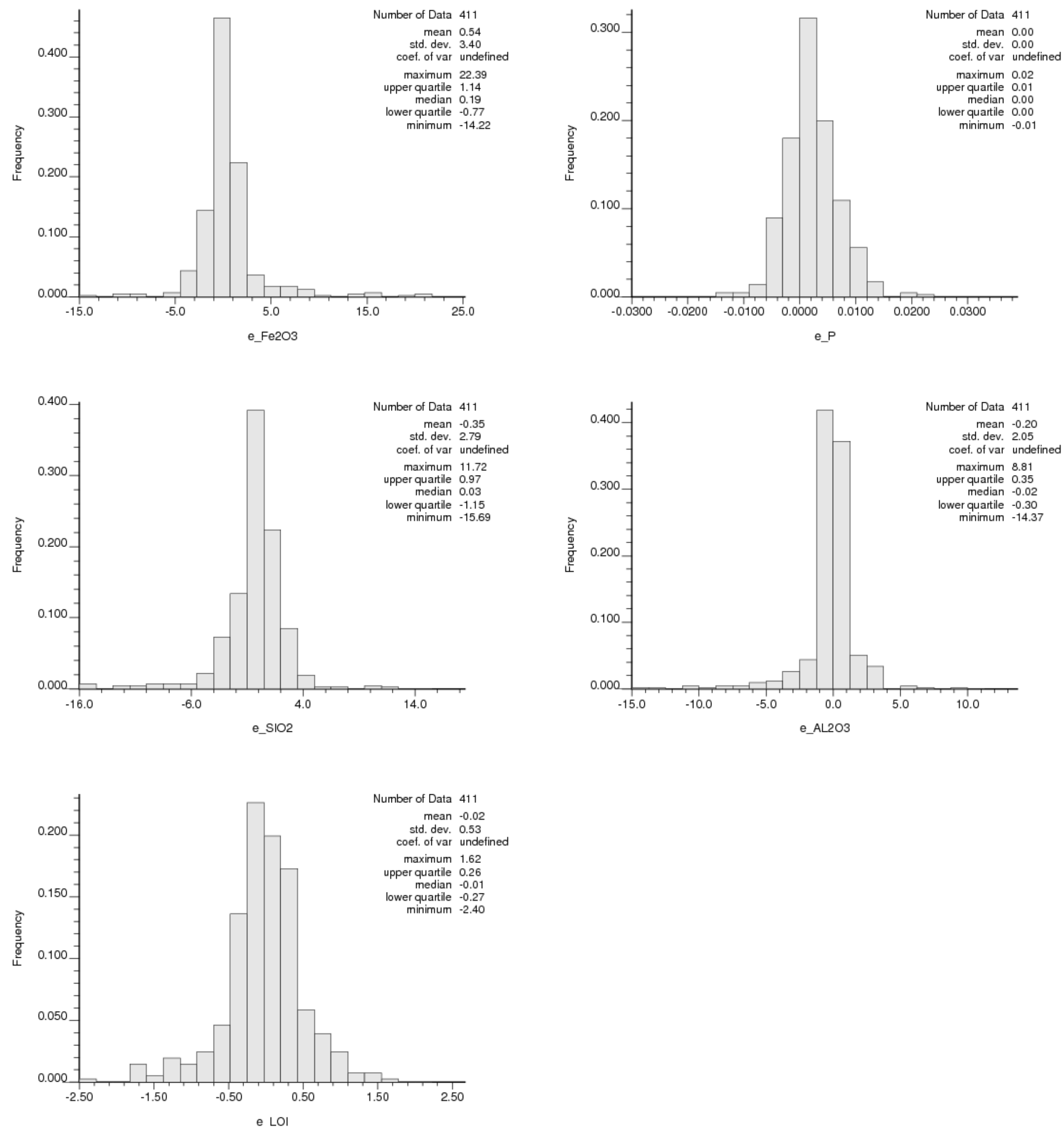


Figure 37: Univariate distribution of errors at mined out sector A

The errors are transformed into standard normal score and their correlation matrix for lag zero show that only three sets of errors have reasonable correlation. These correspond to Fe_2O_3,

SiO_2 and Al_2O_3 elements; otherwise, the set of errors that correspond to P and LOI elements show low correlation between them as well as to the previous set of elements.

Table 8: Correlation matrix of the multi-element error.

	NS e_Fe2O3	**NS e_P**	**NS e_SiO2**	**NS e_Al2O3**	**NS e_LOI**
NS e_Fe2O3	1	0.1757	-0.9135	-0.7373	-0.1677
NS e_P	0.1757	1	-0.2859	0.0596	0.0756
NS e_SiO2	-0.9135	-0.2859	1	0.4941	-0.092
NS e_Al2O3	-0.7373	0.0596	0.4941	1	0.278
NS e_LOI	-0.1677	0.0756	-0.092	0.278	1

The next figure shows the illustration of the correlation between elements, the shadow part of Table 8 is considered by the scatter plot graphs which are in original units.

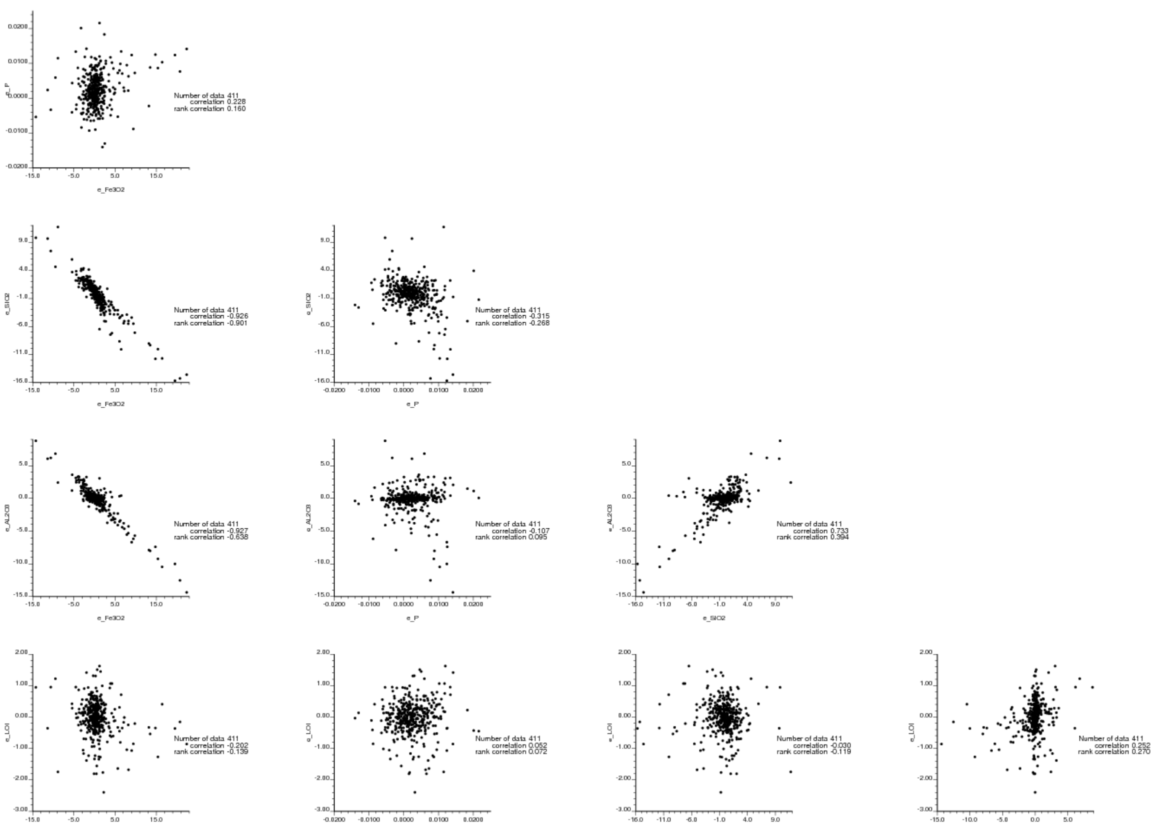


Figure 38: Scatter between errors

The scatter plot maps show a negative slope or negative correlation between Fe_2O_3 and the deleterious elements SiO_2 and Al_2O_3, that is, more iron quality by tonnage imply less silica and alumina quality by tonnage. The MAF procedure de-correlates this normal score errors at all lags and the back MAF transformation matrix is used to re-correlate these errors. Then, the simulation of these MAF non-correlated factors are performed independently. The lag used is 48 meters which corresponds to the median spacing among the data location. The spatial correlation of the MAF non-correlated factors is modeled to perform joint simulation at sector B where only exploration data is available. The set of simulations per error element recover their correlation by using the back MAF transformation matrix. The scatter plots of the errors at sector B after the joint simulation are as follows:

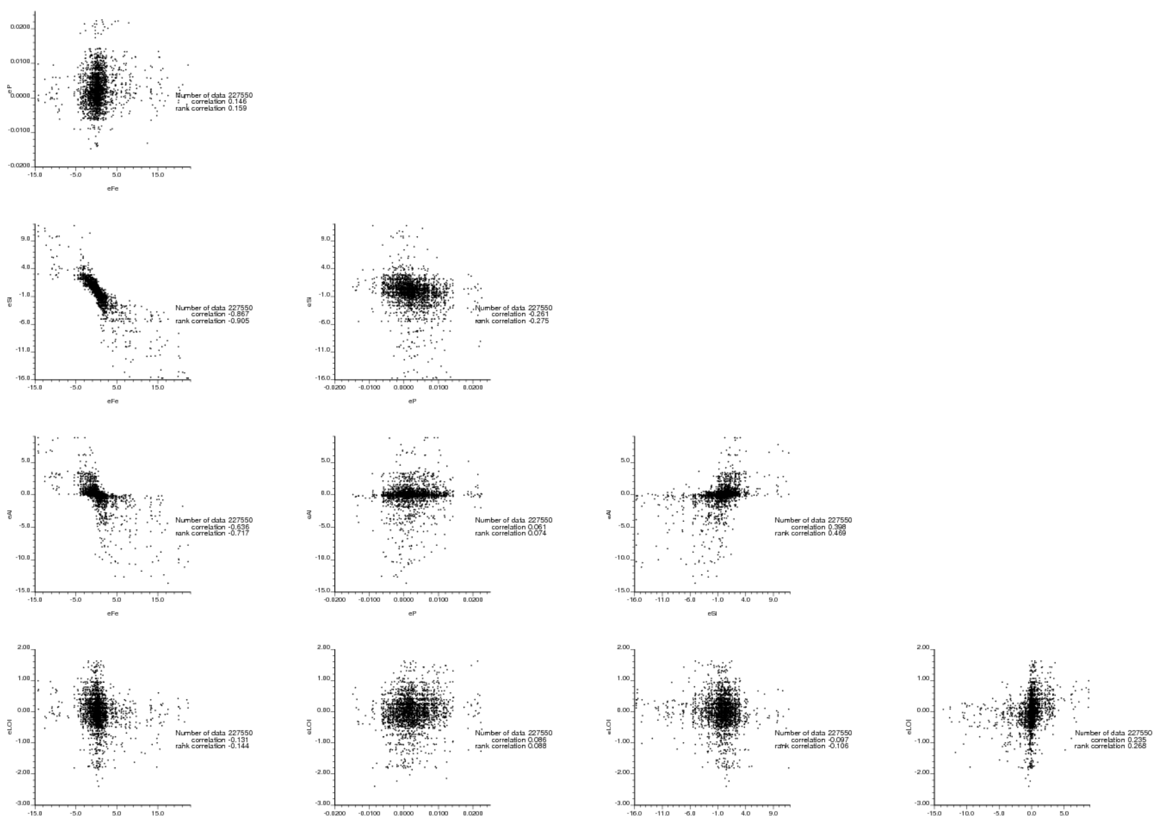


Figure 39: Scatter between errors corresponding to realization 1

The comparison of the input scatter plot between the errors of mined sector A, see Figure 38, and the scatter plot between the joint simulation errors of sector B, see Figure 39, show that the input correlation between errors are reproduced by the MAF simulation. Indeed, the spatial correlation between multi-element errors of exploration data and ore control data of historical data was used to generate the possible error that may happen in the sector B where only exploration data is available.

The joint simulation of errors at nodes is scaled up to the block size. These errors maps are added to the simulation based on exploration data used to generate the map of possible ore control data at sector B per element and update the orebody uncertainty. The simulation based on exploration data and the updated orebody simulation that considers the correlation of the errors of mined sector are compared. The future ore control data orebody model shows higher entropy than exploration data orebody model.

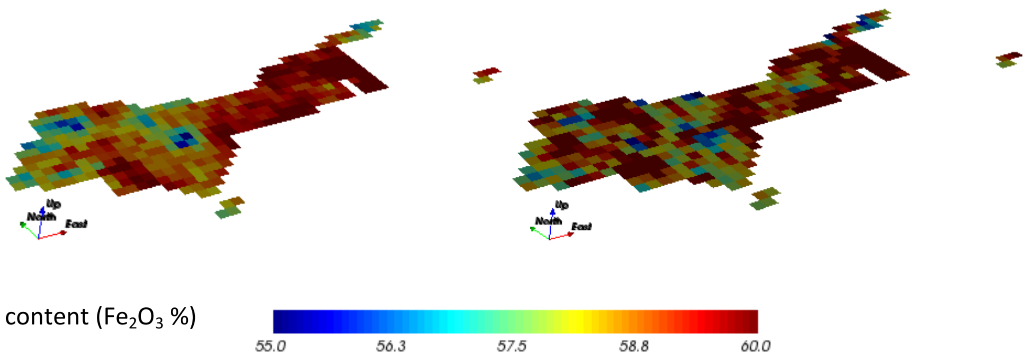


Figure 40: Realization 1 upper bench Fe_2O_3 map of sector *B* base of exploration data (right) and future ore control data map (left)

The mean of the Iron increases slightly from 58.58 to 58.85 % and the future iron ore control data distribution is wider than the available simulation based on exploration data.

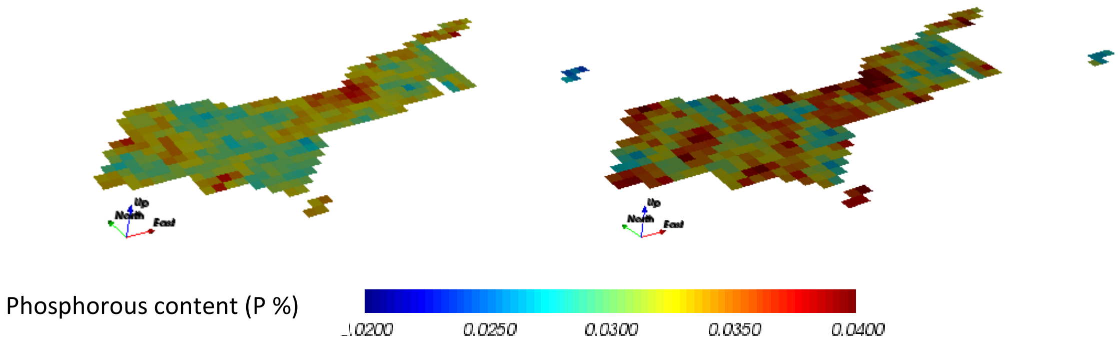


Figure 41: Realization 1 upper bench phosphorous map of sector B base of exploration data (right) and future ore control data map (left)

The mean of the phosphorous increases from 0.032% to 0.034% and the resulting future phosphorous ore control data distribution obtains a longer right tail of 0.047% than 0.038% for the available simulation based on exploration data.

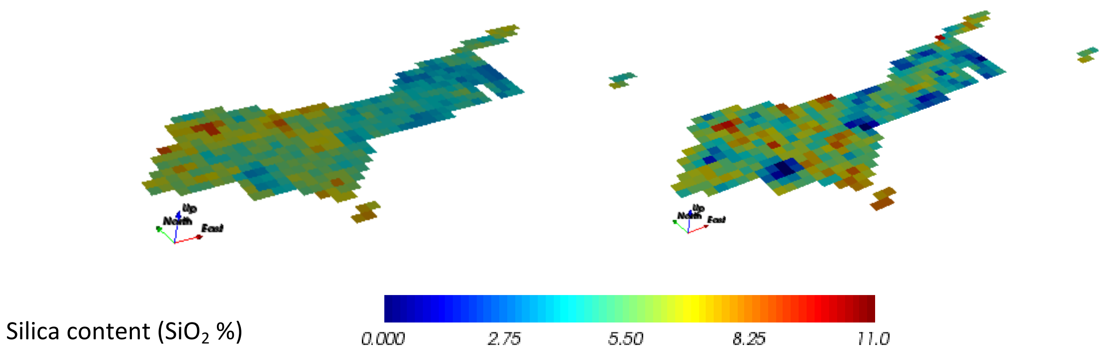


Figure 42: Realization 1 upper bench SiO_2 map of sector B base of exploration data (right) and future ore control data map (left).

The silica grade mean decreases from 5.12% to 5.03% and the silica future ore control data distribution becomes wider than the available simulation based on exploration data.

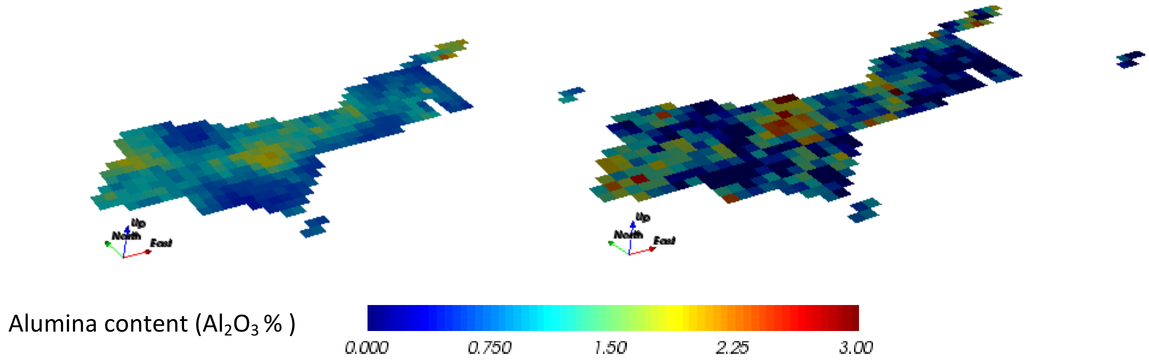


Figure 43: Realization 1 upper bench Al_2O_3 map of sector B base of exploration data (right) and future ore control data map (left).

The alumina grade mean decreases from 0.96 to 0.91% and the alumina future ore control data distribution becomes wider than the available simulation based on exploration data.

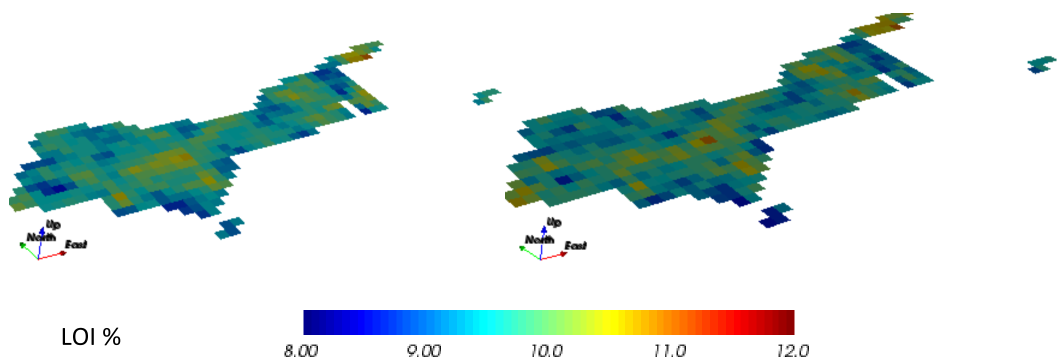


Figure 44: Realization 1 upper bench LOI map of sector B base of exploration data (right) and future ore control data map (left)

The grade mean of the loss of ignition increases from 9.99% to 10.02% and their distribution becomes wider than the available simulation based on exploration data.

3.3.2 Simulated future multi-element ore control data in stochastic short-term production scheduling and risk analysis

The short-term production scheduling proposed incorporates operational considerations in order to deliver recoverable reserves for each period. Besides these considerations, the orebody model used in the short-term production evaluation must take into account for grade control process prior to production. The ore control process in term of orebody quality is the grade provided by ore control data which have short-scale information since the information is dense in comparison to the sparse exploration data that provide information of the deposit at a large scale.

The short-term production scheduling must consider relevant short-scale information to efficiently match target production. The orebody uncertainty based on sparse exploration data is updated using future multi-element ore control data. Then, 10 simulations of the future multi-element ore control data and the e-type of these realizations per element are the input data for the monthly production scheduling. The influence of possible short-scale information in the stochastic short-term production scheduling is evaluated.

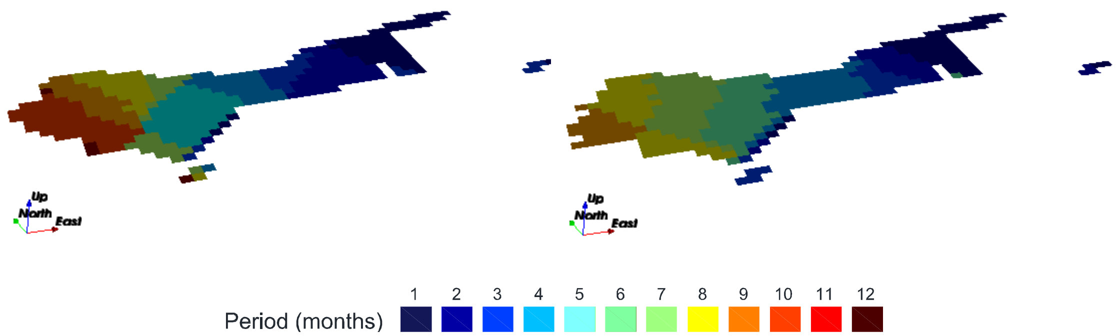


Figure 45: Stochastic production schedule upper bench accounting for ore body uncertainty based on exploration data (left) and based on future ore control data (right)

The patterns of the production schedule based on exploration data orebody uncertainty is quite different from the production schedule patterns based on the future ore control data orebody uncertainty, that is, a change in the quality of the multi-elements made a relevant impact in the decision of which sector to mine per month, as expected.

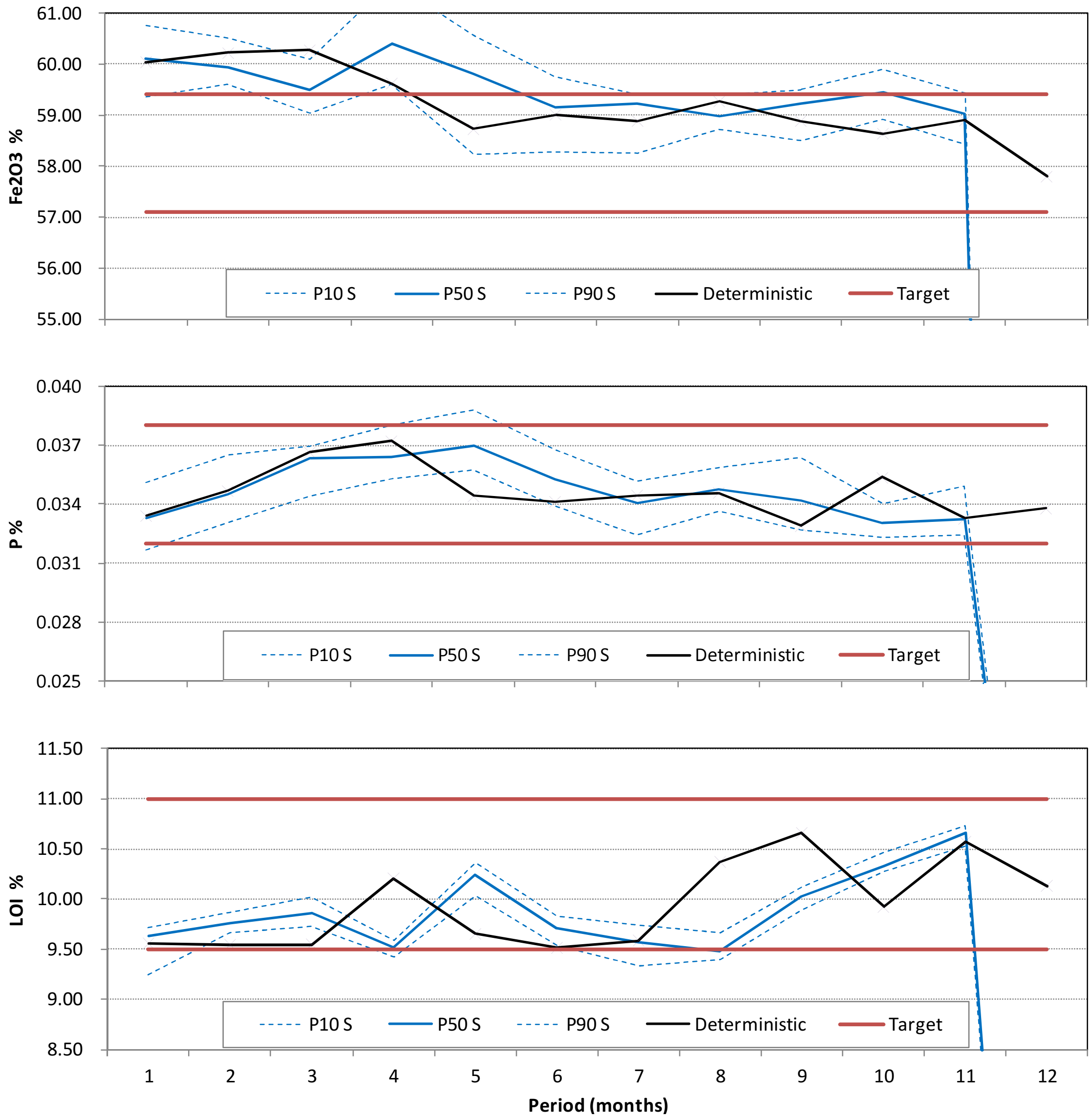


Figure 46: Short-term schedule solution for iron (top), phosphorous (middle) and LOI (bottom) where (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile)

It is important to remark that the stochastic production scheduling considers four source of uncertainty: orebody uncertainty, shovel mechanic availability uncertainty, truck mechanic availability uncertainty and hauling time uncertainty. The iron monthly average of the six first months does not match the upper bound; however, the remaining six months do match the upper bound. The other two elements of phosphorous and loss on ignition satisfy both upper bounds and lower bounds. Variable production schedule monthly average grade is observed for all the elements because the future ore control data map simulations show an increase of entropy or spatial variability of the grades.

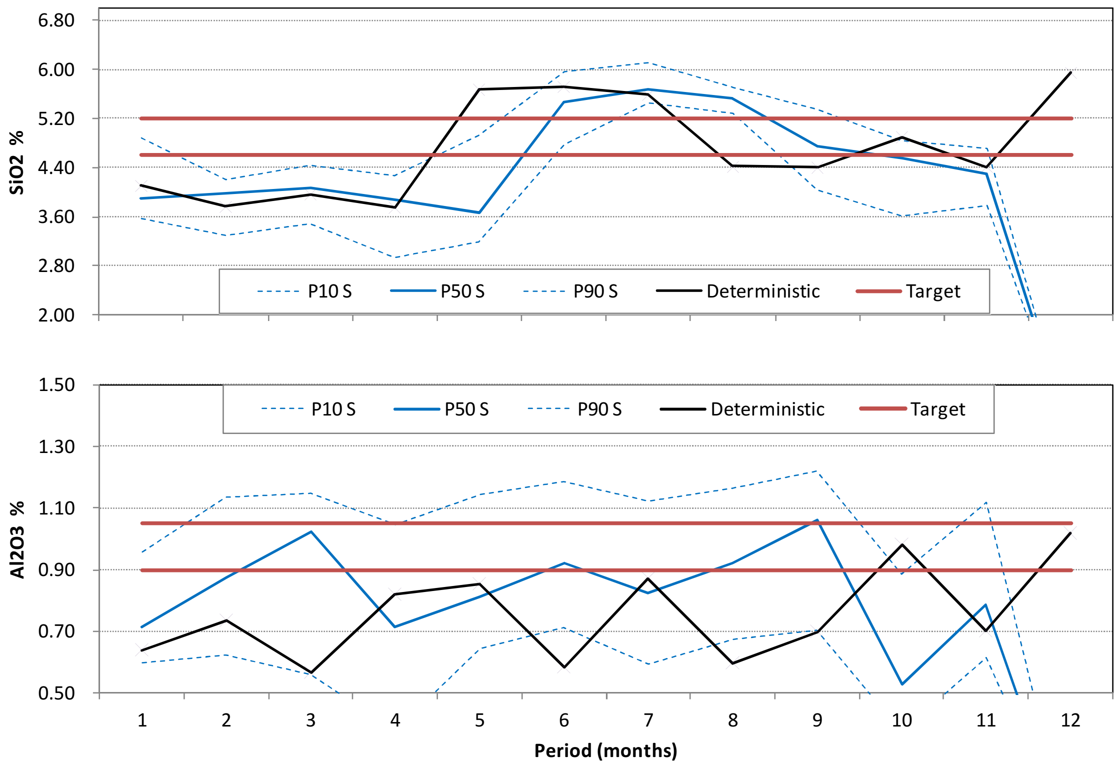


Figure 47: Short-term schedule solution for silica (top) and alumina (middle) where (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile)

The silica and alumina quality of the monthly production schedule barely match their quality targets. The same effect was observed by the production schedule based on exploration data orebody uncertainty with the same elements; however, the average grade of the monthly production taking into account future data is more variable.

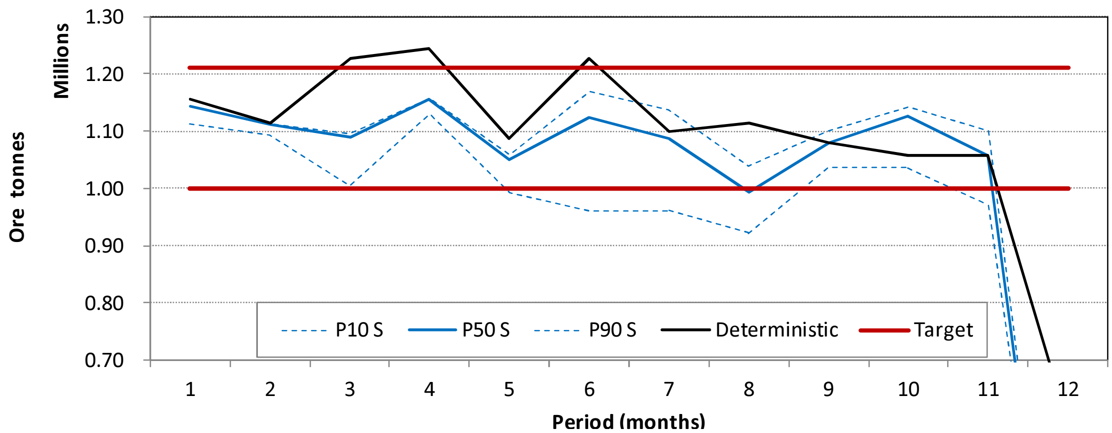
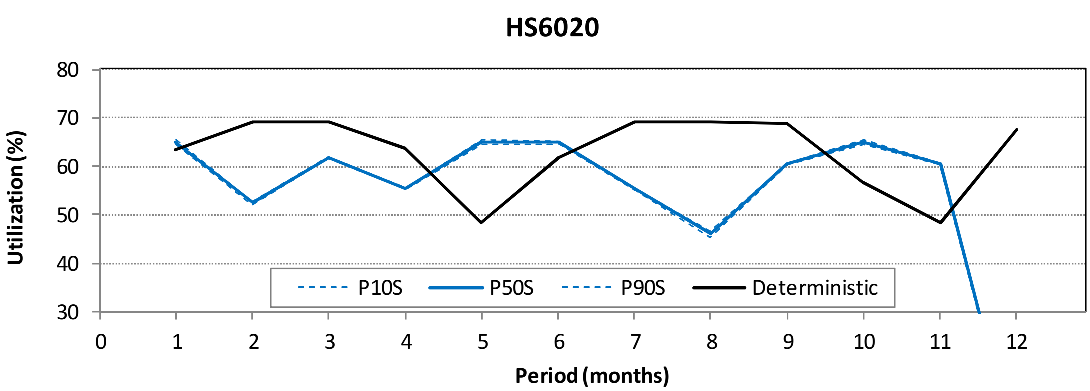


Figure 48: Short-term schedule solution for ore tonnes (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile)

The monthly ore tonnage of the stochastic production schedule accounting for future ore control data orebody uncertainty and fleet parameters source of uncertainty are less variable than the deterministic one that does not consider any source of uncertainty. The possible lack of matching quality targets in some periods is expected since the upper and lower bounds provided by the long-term production schedule was determined based on the exploration of sparse data orebody uncertainty where the short-scale information was not taken into account.

The utilization of the fleet when the source of uncertainty related to the multi-element grade considers the future ore control data is showed in Figure 49.



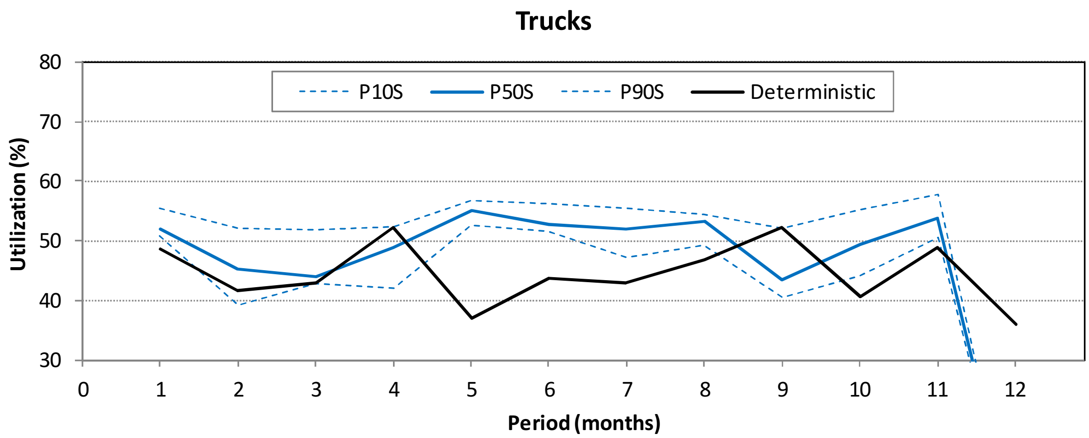


Figure 49: The risk profiles of the stochastic production schedule fleet utilization (blue) and the deterministic production schedule fleet utilization (black)

The utilization of the trucks and shovels accounting for future ore control data orebody uncertainty and fleet parameters source of uncertainty is slightly higher and less variable than the utilization of the deterministic production schedule that does not account for any source of uncertainty. When the local grade variability of the multi-elements is higher, the stochastic production schedule formulation allocates more efficiently than the deterministic one which allocates one or three more trucks per month with lower utilization. The allocation of an optimal number of trucks will reduce the overall mining cost.

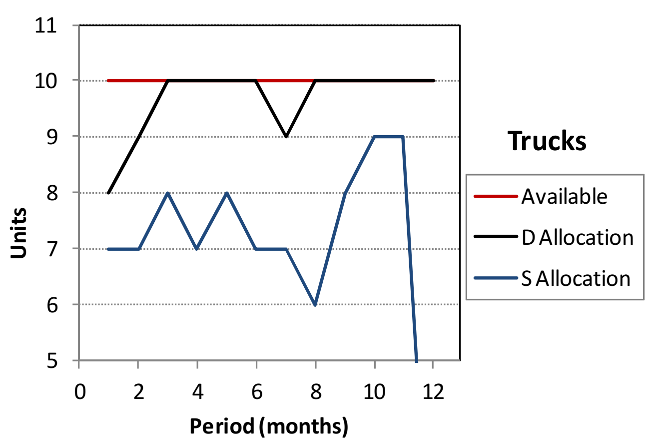


Figure 50: Available trucks (red), number of trucks allocated accounting for four sources of uncertainty (blue) and without accounting for uncertainty (black)

The fleet utilization of the stochastic production schedule which account for future ore control data orebody uncertainty is higher and less variable than the stochastic production schedule fleet utilization that accounts for exploration data orebody uncertainty.

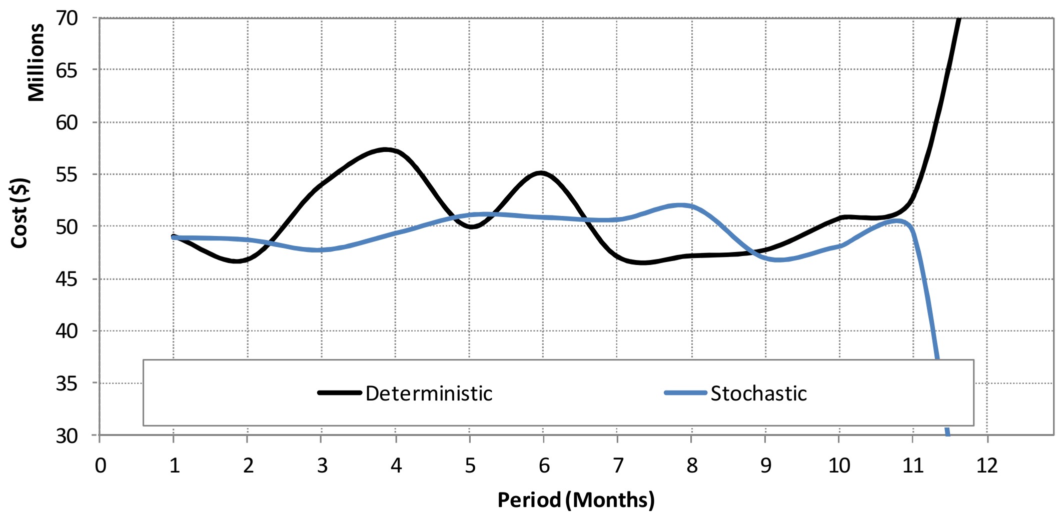


Figure 51: Testing performance of the stochastic short-term production scheduling that account for future short-scale information (blue) against the production scheduling without any source of uncertainty (black).

The stochastic production scheduling seems more robust than the deterministic one in handling high local variability of the multi-elements that was provided by the simulation of the future ore control data. Less variable monthly cost is observed.

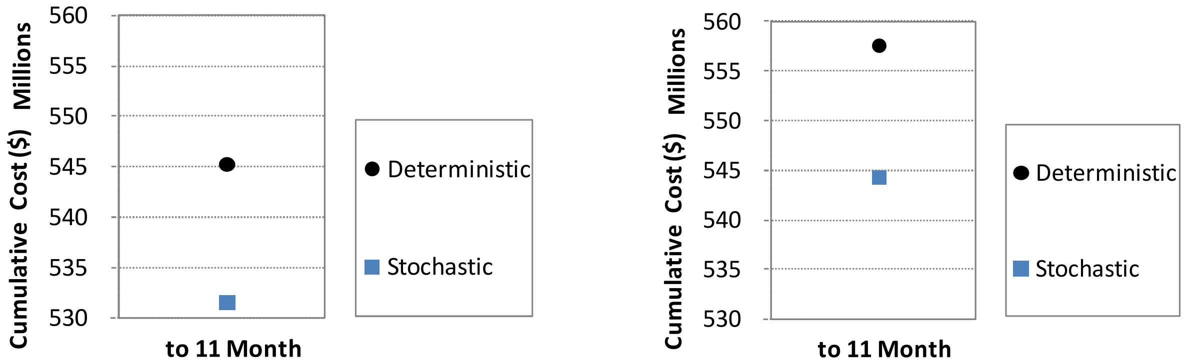


Figure 52: Cumulative cost of the short-term production scheduling that account for orebody uncertainty based on exploration data (left) and orebody uncertainty based on future ore control data (right).

The cumulated cost of the stochastic monthly production schedule is around 15 million CAD dollars less than production schedule without any source of uncertainty. This proportion of difference was also observed for stochastic production schedule that accounts for orebody uncertainty based on exploration data. The orebody uncertainty that accounts for future short-scale information of the multi-elements deals to a higher production schedule cost because the quality and tonnage targets are hardly matched, increasing the influence of the penalty cost per month in the seventh and eighth components of the objective function.

3.4 Summary and conclusions

The orebody uncertainty is updated by simulated future ore control data to account for short-scale information. The alternative, based on the errors between exploration data and historical ore control data used to simulate future ore control data at a similar domain where only exploration data is available, was implemented. The updated orebody uncertainty was used then to optimize the short-term production to better match the ore quality targets at the time of production.

The local spatial variability or entropy of the future ore control data orebody model maps is higher than the exploration data orebody model maps. When the spatial variability of the multi-element grade is higher, the proposed stochastic production scheduling approach delivers high fleet utilization, more efficient truck allocation and low variable production monthly average grade, ore tonnage and cost than the deterministic production scheduling that does not account for any source of uncertainty.

Chapter 4 Conclusions and Future Work

4.1 Conclusions

The formulation of the stochastic mixed integer programming model proposed in this thesis for short-term mine production scheduling is developed as a single formulation where mining considerations, production constraints, uncertainty in the orebody metal quantity as well as fleet parameters are evaluated together to define a well informed sequence of mining that has high performance at the mine operation. The quality of material may influence also in the allocation of fleet. Indeed, the optimization allocate the fleet to sectors that ensure the accomplishment of the production target, match the quality conditions, maximize fleet utilization, respect the operational considerations and accounts for uncertainty in the parameters. The components of the objective function are in terms of costs where the minimized total cost implies that the plan guarantees the minimum deviation from production target, maximum utilization of the fleet, minimum cost of production extraction and better match of mining width requirements. At the time of short-term production scheduling, additional information related to the operational restrictions, such as mining width and mining directions, are available. These additional physical constraints were implemented to deliver feasible production schedule patterns that will have better performance during the operations.

The orebody uncertainty is updated under the concept of future ore control data. The future multi-element ore control data orebody uncertainty is used to perform the stochastic short-term production scheduling properly instead of using simulation base of exploration data. The stochastic production scheduling that accounts for future ore control data orebody uncertainty shows more variability in quality and tonnage per month than the one that accounts for exploration data orebody uncertainty. This is expected since the future ore control simulation map shows more entropy or spatial variability of the grade. The ore quality targets are matched for the iron, phosphorous and loss on ignition elements in most of the periods; however, the quality targets of silica and alumina are barely matched in most of the periods. The fleet utilization of the stochastic production scheduling that accounts for future ore control data orebody uncertainty is higher and less variable than the stochastic production schedule fleet utilization that accounts for exploration data orebody uncertainty.

Risk profiles shows that the monthly ore production tonnage given grade fluctuations are less than the production tonnage using the single orebody estimation. The deterministic solution could overestimate the ore production per period because the uncertainty of the orebody is ignored in the formulation of the model. The utilization of the fleet accounting for uncertainty is less variable and higher than versions not accounting for uncertainty in their parameters. Wrong expectations could mislead mine short-term production scheduling because of overestimation of the ore tones. The risk of losses during the production process is present and must be identified at the planning stage so as to elaborate contingency plans.

4.2 Future Work

The optimization of the stochastic short-term production schedule model is complex and computationally expensive to solve as the size of the deposit and the fluctuations of the parameters increase. The sequential optimization of aggregated consecutive periods or time-frame window methodology used to reduce the time to reach a feasible solution may deliver a global sub optimal solution. Advance computational aspects may be researched to evaluate the whole periods in one optimization in a reasonable computing time. Parallelization and efficient heuristic methods may be an option.

The mill, processing plant or blending pad have maximum tonnages capacity and quality targets to satisfy. Because of the spatial variability of the grade and the requirement to match quality targets, there may be some ore blocks that cannot be blended with blocks from different sectors at the time that are extracted and need to be mined to access the blocks that help to match quality constraints. These ore blocks are hauled to temporal stock piles and re-handled in the blending of later monthly production scheduling, that is, the spatial variability of the grade may demand the use of temporal stockpiles to deliver feasible solutions and low variable feed alimentation of the mill.

Beside the uncertainty in the orebody, the proposed short-term production scheduling approach accounts for uncertainty in regards to the hauling and loading process which represents from 30% to 60% of most of the cost of extracting a tonne from the pit; however, there are other sources of uncertainty that may be incorporated, such as the operating efficiency of the fleet because the historical idle time and the delay time may be incorporated.

Some mines have more than one process and every process has quality constraints to be accomplished. The short-term production scheduling may be expanded to account for multiple processes or multiple qualities requirements.

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Appendix A

The MAF (Switzer and Green 1984) procedure set in geostatistical context is reviewed. The multivariate observation vector is defined as $Y(\mathbf{x}) = (Y_1(\mathbf{x}),...,Y_q(\mathbf{x}))^T$ and their q orthogonal lineal combinations or minimun/maximum autocorrelated factors correspond to $F_i(\mathbf{x}) = a_i^T Y(\mathbf{x}),\ i = 1,...,q$. Where each transform $F_i(\mathbf{x})$ exhibit higher spatial correlation than any previous determined transforms $F_j(\mathbf{x})$. The spatial correlation between MAF $F_i(\mathbf{x})$ and $F_i(\mathbf{x}+\mathbf{h})$ space by a lag $\mathbf{h} = \Delta$ is represented by $\rho_i(\Delta)$ where the vector of coefficients $a_i,\ i = 1,...,q$ are calculated as follows:

$$\begin{aligned} \rho_i(\Delta) &= \min_a \left\{ corr\left(a^T Y(\mathbf{x}), a^T Y(\mathbf{x}+\Delta)\right)\right\} \\ \rho_p(\Delta) &= \max_a \left\{ corr\left(a^T Y(\mathbf{x}), a^T Y(\mathbf{x}+\Delta)\right)\right\} \end{aligned} \qquad \text{(A.1)}$$

Where $\rho_i(\Delta)$ and $\rho_q(\Delta)$ correspond to the minimization and maximization of *a* in the function $corr\left(a^T Y(\mathbf{x}), a^T Y(\mathbf{x}+\Delta)\right)$. The MAF ranking of the transforms is defined by the vector of coefficients a_i and could be also calculated from the left hand eigenvectors of the nonsymmetric matrix $\Pi_\Delta \Pi_0^{-1}$; where Π_Δ is the covariance matrix considering lag Δ distance and Π_0 is the variance-covariance matrix of $Y(\mathbf{x})$

$$\begin{aligned} \Pi_\Delta &= \mathrm{cov}\left[\left(Y(\mathbf{x}) - Y(\mathbf{x}+\Delta)\right), \left(Y(\mathbf{x}) - Y(\mathbf{x}+\Delta)\right)\right] = 2\Gamma_Z(\Delta) \\ \Pi_0 &= \mathrm{cov}\left[Y(\mathbf{x}), Y(\mathbf{x})\right] \end{aligned} \qquad \text{(A.2)}$$

The $\Gamma_Y(\Delta)$ corresponds to the variogram matrix with lag Δ distance.

The multivariate (q dimensional) stationary random function $Y(\mathbf{x})$ is equal to $S(\mathbf{x})+N(\mathbf{x})$, where $S(\mathbf{x})$ and $N(\mathbf{x})$ are uncorrelated signal and noise components which covariance matrices are

$$\begin{aligned}
&\operatorname{Cov}\left[N(\mathbf{x}),N(\mathbf{x})\right]=C_N(0)=\Pi_0\\
&\operatorname{Cov}\left[S(\mathbf{x}),S(\mathbf{x})\right]=C_S(0)=\Pi_1\\
&\operatorname{Cov}\left[Y(\mathbf{x}),Y(\mathbf{x})\right]=C_Y(0)=\Pi_0+\Pi_1=\Pi
\end{aligned}\tag{A.3}$$

The noise identifies the low-number variates that have minimal spatial autocorrelation and the signal identifies the high numbered variates that have maximal spatial autocorrelation. An intrinsic or linear coregionalization models could represent the spatial correlation structures of $S(\mathbf{x})$ and $N(\mathbf{x})$. Then, the multivariate $Y(\mathbf{x})$ is represented by two-structure lineal model of coregionalization when the lag distance $\mathbf{h}>0$ of the spatial covariance matrices, the scalar spatial correlation $\rho_1(\mathbf{h})$ is greater than $\rho_0(\mathbf{h})$ for all $\mathbf{h}$.

$$\begin{aligned}
&\operatorname{Cov}\left[N(\mathbf{x}),N(\mathbf{x}+\mathbf{h})\right]=C_N(\mathbf{h})=\Pi_0\rho_0(\mathbf{h})\\
&\operatorname{Cov}\left[S(\mathbf{x}),S(\mathbf{x}+\mathbf{h})\right]=C_S(\mathbf{h})=\Pi_1\rho_1(\mathbf{h})\\
&\operatorname{Cov}\left[Y(\mathbf{x}),Y(\mathbf{x}+\mathbf{h})\right]=C_Z(\mathbf{h})=\Pi_0\rho_0(\mathbf{h})+\Pi_1\rho_1(\mathbf{h})
\end{aligned}\tag{A.4}$$

The variogram matrix for short lag $\mathbf{h}=\Delta$ in term of scalar spatial correlation could be expressed as follows:

$$\begin{aligned}
2\Gamma_Z(\Delta)&=\operatorname{cov}\left[\left(Y(\mathbf{x})-Y(\mathbf{x}+\Delta)\right),\left(Y(\mathbf{x})-Y(\mathbf{x}+\Delta)\right)\right]\\
&=2\left(1-\rho_1(\Delta)\right)\Pi+2\left(\rho_1(\Delta)-\rho_0(\Delta)\right)\Pi_0
\end{aligned}\tag{A.5}$$

The previous equation times Π^{-1} will be expressed in term of the matrix of its left-hand eigenvectors Θ^T and the diagonal matrix of its eigenvalues Λ

$$\begin{aligned}\Theta^T\Gamma_Z(\Delta)\Pi^{-1} &= \left(1-\rho_1(\Delta)\right)\Theta^T + \left(\rho_1(\Delta)-\rho_0(\Delta)\right)\Theta^T\Pi_0\Pi^{-1} \\ &= \frac{\Lambda}{2}\Theta^T\end{aligned} \tag{A.6}$$

Normalize the matrix of eigenvectors Θ^T into $\Theta^T\Pi\Theta = I$. Then, multiply the previous equation by $\Pi\Theta$

$$\frac{\Lambda}{2} = \left(1-\rho_1(\Delta)\right)I + \left(\rho_1(\Delta)-\rho_0(\Delta)\right)\Theta^T\Pi_0\Theta \tag{A.7}$$

The eigenvectors are independent of the lag and Θ^T is used to decorrelated at all **h** the multivariate spatial random function $Y(\mathbf{x})$

$$F(\mathbf{x}) = \Theta^T Y(\mathbf{x}) \tag{A.8}$$

Then variance-covariance matrix of $F(\mathbf{x})$ is an identity matrix I and the spatial correlation matrix of $F(\mathbf{x})$ at lag $\Delta$is

$$Corr\left[F(\mathbf{x}), F(\mathbf{x}+\Delta)\right] = \rho_1(\Delta)I + \left(\rho_1(\Delta)-\rho_0(\Delta)\right)\Theta^T\Pi_0\Theta \tag{A.9}$$

From (A.7) and (A.9)

$$Corr\left[F(\mathbf{x}), F(\mathbf{x}+\Delta)\right] = I - \frac{\Lambda}{2} \tag{A.10}$$

Despite the coregionalization model, the matrix of eigenvalues is diagonal since the elements $F_i(\mathbf{x})$ of $F(\mathbf{x})$ are orthogonal at lag $\Delta \neq 0$ and $\Delta = 0$. $F_i(\mathbf{x})$ is ranked in increasing spatial correlation order due to eigenvalues λ_i at Λ decrease when the index i increase. Then, MAF transform is given by $F(\mathbf{x}) = \Theta^T Y(\mathbf{x})$ where Θ^T is the matrix of normalized eigenvectors of $2\Gamma_Z(\Delta)\Pi^{-1}$.

The matrix of eigenvectors and the matrix of eigenvalues of a symmetric matrix is calculated using spectral decomposition (Switzer and Green 1984). This involves four steps:

- Π is a symmetric matrix and their spectral decomposition $\Pi = HDH^T$, where H correspond to orthonormal eigenvectors matrix and D correspond to diagonal matrix of eigenvalues.
- $W = HD^{-1/2}$ where $W^T \Pi W = I$ to transform or decorrelate variables $V(\mathbf{x}) = W^T Y(\mathbf{x})$.
- The covariance matrix $\left(V(\mathbf{x}) - V(\mathbf{x}+\Delta)\right)$ for a two-structure Lineal corregionalization model is written.

$$\begin{aligned} &Cov\left[\left(V(\mathbf{x}) - V(\mathbf{x}+\Delta)\right), \left(V(\mathbf{x}) - V(\mathbf{x}+\Delta)\right)\right] = \\ &2\left(1-\rho_1(\Delta)\right)I + 2\left(\rho_1(\Delta) - \rho_0(\Delta)\right)W^T \Pi_0 W \end{aligned} \quad \text{(A.11)}$$

- Then, the spectral decomposition of the covariance $\left(V(\mathbf{x}) - V(\mathbf{x}+\Delta)\right)$ into orthonormal eigenvectors *C* matrix and eigenvalues Λ diagonal matrix.

$$\begin{aligned} &\left(1-\rho_1(\Delta)\right)I + \left(\rho_1(\Delta) - \rho_0(\Delta)\right)W^T \Pi_0 W = C\frac{\Lambda}{2}C^T \\ &\left(1-\rho_1(\Delta)\right)I + \left(\rho_1(\Delta) - \rho_0(\Delta)\right)C^T W^T \Pi_0 W C = \frac{\Lambda}{2} \end{aligned} \quad \text{(A.12)}$$

From (A.7) $\Theta^T = C^T W^T$ then $\Theta^T = C^T \overbrace{D^{-1/2} H^T}^{W^T}$ and MAF transform could be expressed as:

$$F(\mathbf{x}) = C^T D^{-1/2} H^T Y(\mathbf{x}) = Y(\mathbf{x}) H D^{-1/2} C \quad \text{(A.13)}$$

When $Y(\mathbf{x})$ is represented by two-structure Lineal model of coregionalization, the MAF decorrelates the transformed variables at all lags. Indeed the simulation of these variables can be evaluated independently.