STUDY OF A NEW TYPE OF BI-DIRECTIONAL AMPLIFIER FOR USE IN TELEPHONE NETWORKS

by

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TABLE OF CONTENTS

ABSTRACT

Telephone repeaters are bi-directional amplifiers which are inserted in a transmission line to compensate for losses. Because of their bi-directional nature special care must be taken to match these amplifiers to the line in order to assure stability and to limit the echo signal. The two basic types of repeaters presently in use are the hybrid and the negative impedance repeaters. In this study a new arrangement, called the bridge compensated repeater, is suggested and analyzed. After developing an analytical method by which the performance of repeaters of various types can be compared, the three systems are analyzed and it is proved that the bridge compensated system possesses some desirable properties as compared to existing systems. The basic problem in designing a bridge compensated system is the design of a two-port network which must satisfy a specification related to its real and imaginary parts simultaneously. A computer method for designing such networks is developed and some results are presented. A circuit for the bridge compensated repeater is described and measuring techniques are developed to compare the accuracy of calculated and measured values. This technique is demonstrated on a simple system in which case the agreement between measured and calculated values is found to be very good.

1. INTRODUCTION

The basic function of the telephone system is to establish a bi-lateral communication channel between two subscribers. If the subscribers to be interconnected are within reasonable distance of each other, a two-wire transmission line can be used to establish the desired connection, (Fig. 1-1), but as the distance between the two parties increases, the attenuation of the line becomes so high that amplifiers must be inserted to maintain a certain level of transmission.

The insertion of amplifiers into the two-wire system of Fig. 1-1 poses a problem because of the bi-lateral nature of the system, which becomes unstable if the arrangement of Fig. 1-2 is used. Solutions presently in use for solving this problem are outlined below. In this introductory section only qua~~n~~litative description of these systems will be attempted, but more comprehensive treatment will be provided in later chapters.

A fully four wire system is shown on Fig. 1-3. In this arrangement, separate cable pairs are used to provide transmission in opposite directions. In most cases, however, it is uneconomical to use the arrangement of Fig. 1-3 because for a large percentage of calls made by a subscriber (local calls), two-wire connection without any amplifier will suffice and therefore the second pair of wires would not be utilized to its full extent.

Fig. 1-4 illustrates an arrangement which we can call a hybrid system [1,2,17] because some of the connections, (the ones where amplification is required), are maintained on a partly two-wire, partly four wire basis. Local calls use a two-wire path. The four-wire and two-wire sections are joined together by means of hybrid transformers (H_1) and (H_2). (See Appendix I for the basic operating principle of the hybrid transformer.) Signals originating from subscriber (A) are amplified by amplifiers ($A_{f1}, \ldots. A_{fn}$) and reach subscriber (B) via hybrid transformer (H_2) which also prevents these signals from entering the "Backward path" of amplifiers ($A_{b1}, \ldots. A_{bn}$). Similar reasoning applies to signals originating at the (B) side.

In practice, however, the return losses of the hybrid transformers are finite, which is mostly due to the fact that compensating impedances Z_{n1} and Z_{n2} are not exactly equal to the characteristic impedances of their associated transmission lines. This deviation from the ideal operating condition has two important effects:

a) It limits the amplification obtainable by this arrangement, because the system becomes unstable at higher gain values (singing).

b) Some part of the signal is returned to the subscriber from which it originated, delayed by the "round trip delay time" and produces an echo type effect. The larger the delay time, the more disturbing is this echo and tables are available specifying the maximum permissible echo level as a function of the round trip delay time.[11]

In some cases (especially when cable cost is high) it is desirable to maintain the entire connection on a two-wire basis even if amplifiers must be used. Fig. 1-5 illustrates an arrangement which can be called the <u>two-wire hybrid system</u>. The bi-lateral amplifiers consisting of two hybrid transformers and two unilateral amplifiers are called <u>hybrid repeaters</u>. The operating principle of this configuration is identical to the system illustrated on Fig. 1-4, the only difference being that at the expense of some hybrid transformers the second cable pair is saved.

It is shown in Appendix IV that a lattice network (such as illustrated on Fig. 1-6a) can provide bi-lateral amplification if Z_a and Z_b are negative impedances. Such an arrangement is called a <u>negative impedance repeater</u> [18,19] and for economic reasons it is usually realized in a form shown on Fig. 1-6b. A two-wire system using negative impedance repeaters is illustrated on Fig. 1-6c. As will be shown in later chapters, this system tends to be unstable and produces echo effects if the image impedance of the repeater does not match the characteristic impedance of the line.

The <u>object of this study</u> is a new bi-directional amplifier which differs very little in appearance from the arrangements using hybrid transformers, but promises some advantages as far as economy is concerned if compared to the systems described above. This network will be referred to as the <u>bridge compensated repeater</u>. The basic operating principle of this arrangement is illustrated on Fig. 1-7a.

A signal originating from source (A) enters the repeater at terminals 1 - 1' and is applied to terminal (a_1) of differential amplifier (A_1). The signal applied to terminal (b_1) of the (A_1) amplifier is not affected by this input signal, because it is assumed that the output impedance of amplifier (A_2) is small compared to the value of Z_1. Thus the signal entering terminals 1 - 1' is amplified by (A_1) and the output signal of the (A_1) amplifier appears at terminals 2 - 2' reduced by the ratio $\dfrac{Z_{L2}}{Z_{L2} + Z_2}$ if it is assumed that the output impedance of (A_1) is small compared to the value of Z_2. If $e_B = 0$ and the condition

$$G_2 = \frac{Z_{L2}}{Z_{L2} + Z_2} \tag{1-1}$$

is satisfied, then equally large signals appear at both input terminals of differential amplifier (A_2) and therefore the output signal of this amplifier is not affected.

If $e_B \neq 0$ then due to the assumption that the output impedance of (A_1) is small compared to the value of Z_1, the signal applied to input terminal (b_2) of amplifier (A_2) is not affected, but the signal appearing at terminal (a_2) of (A_2) will contain a component proportional to e_B, which component will be amplified and will appear at terminals 1 - 1'. By similar reasoning as given before, it can be proved that if

$$G_1 = \frac{Z_{L1}}{Z_{L1} + Z_1} \tag{1-2}$$

then the output signal of (A_1) is not affected by the signals originating from source (B). Therefore bi-directional amplification is achieved, while the stability of the system is assured.

It is worth-while to mention that the only difference between the bridge compensated and hybrid repeaters is that, in the case of the bridge compensated repeater, the hybrid coil is replaced by a bridge circuit as shown on Fig. 1-7b.

As it will be shown later, the basic advantages of the bridge compensated system will arise from the fact that it requires only that the transfer function of a two-port network be realized, rather than driving point impedance, as is the case in all other arrangements.

2. TRANSMISSION LINES USED IN TELEPHONE NETWORKS

In the previous chapter it was shown that all repeater arrangements require some form of compensation for the characteristic impedance of the transmission line to which they are connected. This is achieved by the compensating network in the hybrid arrangement, by the image impedance in the negative impedance repeaters, and by the transfer function G in the bridge compensated system. Therefore, it is necessary to summarize some properties of the transmission lines considered in the following text.

The properties of uniform (or smooth) transmission lines are described in great detail in the standard literature.[1,6,8] They can be characterized by their primary constants R, G, L, and C which give the values of distributed resistance, conductance, inductance and capacitance per unit length, (usually per mile). While in cables used in practice, L and C are not frequency dependent, ~~but~~ the values of R and G depend on frequency as illustrated by the table of Appendix II. Another set of parameters used to describe uniform transmission lines are their secondary constants:

Propagation constant:

$$P = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (2\text{-}1)$$

and characteristic impedance:

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (2\text{-}2)$$

Both these quantities are complex numbers and the table of Appendix II gives the values of these at different frequencies for the cable selected as an example.

It can be shown that for distortionless transmission along a uniform transmission line the condition

$$\frac{RC}{LG} = 1 \qquad (2\text{-}3)$$

must be satisfied. This ratio for the cable represented by the table of appendix II at 1000 cps is found to be :

$$\frac{RC}{LG} = 7.1 \times 10^3 \qquad (2\text{-}4)$$

which shows that the primary constants of a practical cable are intrinsically ill-proportioned.

To correct this situation inductors are sometimes inserted at equal distances into such transmission lines to increase L and thereby achieve a better $\frac{RC}{LG}$ ratio. These lines are called <u>loaded lines</u> and the inductors are referred to as loading. [1,8,13,14,15,16] Fig. 2-1 shows a loaded line terminated in a "σ end section".

The differences between non-dissipative uniform and loaded lines are outlined below. These differences become even more pronounced if losses are considered. [13]

a) The characteristic impedance of a uniform line is resistive and is not dependent on frequency, while for loaded structures the characteristic impedance depends on the "end section" terminating the line. It is frequency dependent and complex, (see Appendix III).

b) A uniform line has a transmitting band extending from zero to infinite frequencies, while a loaded structure possesses an infinite sequence of alternate transmitting and stopping bands. The frequency which separates the lowest transmitting and stopping bands is called the <u>critical frequency</u> of the loaded line. (For the line taken as an example in Appendix III, the value of this is 4850 cps.)

3. QUANTITATIVE EVALUATION OF REPEATER PERFORMANCE (DEFINITIONS)

In order to be able to evaluate the performance of a repeater and also to compare different structures, one has to define some quantitative indicators of merit by which a certain system can be judged. These quantities will be referred to as merit indicators.

It was pointed out in previous sections that the introduction of repeaters in a transmission system produces some undesirable side effects such as the tendency to sing and echo. The quantities to be defined as merit indicators will have to indicate how much penalty must be paid in order to obtain a certain increase in transmission level. These quantities have to be defined in a manner that takes into account economic factors as well, because if such factors are disregarded, any system can be designed to meet a certain specification.

Before these merit indicators can be defined, some more basic definitions are needed. Fig. 3-1 illustrates a transmission line (either loaded or unloaded) represented as a cascade connection of identical passive two-ports (N_i) which are not necessarily symmetrical. [8] Using the notation shown on this figure we can write

$$V_i = V_{if} + V_{ib} \tag{3-1}$$

$$I_i = I_{if} - I_{ib} \tag{3-2}$$

where

$$\frac{V_{if}}{I_{if}} = Z_{of} \quad \text{and} \quad \frac{V_{ib}}{I_{ib}} = Z_{ob} \tag{3-3}$$

Subscripts "f" and "b" refer to waves travelling in forward (left to right) and backward directions respectively. Z_{of} and Z_{ob} are the characteristic impedances of the structure in forward and in backward directions respectively and are not dependent on the value of "i".

If the line is infinite and a signal source is connected to the input of network (N_0) then $V_{ib} = 0$ and also $I_{ib} = 0$ and the power entering network N_i:

$$P_{if} = \text{Re } V_{if} I_{if}^{*} = \left| \frac{V_{if}}{Z_{of}} \right|^2 \text{Re } Z_{of} \tag{3-4}$$

Fig. 3-2a illustrates the same structure as Fig. 3-1 but a repeater has been inserted between networks N_{i-1} and N_i. Fig. 3-2b gives the equivalent circuit of Fig. 3-2a. where Z_i is the input impedance measured between terminals (a-a') of the repeater. For this arrangement

$$\frac{V'_i}{I'_i} = \frac{V'_{if} + V'_{ib}}{I'_{if} - I'_{ib}} = Z_i \tag{3-5}$$

and from this and equations (3-3)

$$\frac{V'_{ib}}{V'_{if}} = \frac{Z_{ob}}{Z_{of}} \cdot \frac{Z_i - Z_{of}}{Z_i + Z_{ob}} \tag{3-6}$$

The ratio of the power travelling in the backward direction (reflected power) to the power associated with the forward travelling wave (incident power) is defined as <u>echo</u> (E_f).

Thus:

$$E_f \equiv \frac{P'_{ib}}{P'_{if}} = \left|\frac{V'_{ib}}{V'_{if}}\right|^2 \left|\frac{Z_{of}}{Z_{ob}}\right|^2 \frac{ReZ_{ob}}{ReZ_{of}} \qquad (3\text{-}7)$$

and

$$E_f = \left|\frac{Z_i - Z_{of}}{Z_i + Z_{ob}}\right| \frac{ReZ_{ob}}{ReZ_{of}} \qquad (3\text{-}8)$$

The term: $\dfrac{Re\ Z_{ob}}{Re\ Z_{of}}$ is irrelevant as far as repeater performance is concerned because it is a function of the transmission line only. *

Therefore, the <u>reflection</u> (R) is defined here as:

$$R_f = \frac{Z_i - Z_{of}}{Z_i + Z_{ob}} \qquad (3\text{-}9)$$

and will be regarded as being representative of the echo properties of a repeater.

Because the transmission line following the repeater is considered to be infinite

$$\frac{V_{ia}}{I_{ia}} = Z_{of} \qquad (3\text{-}10)$$

and the power entering network (N_i) is:

$$P'_{if} = \left|\frac{V_{ia}}{Z_{of}}\right|^2 ReZ_{of} \qquad (3\text{-}11)$$

(*) It can also be proved [13] that in the case of non-dissipative loaded lines $Z_{1-\sigma} = Z^*_{\sigma}$ which gives $\dfrac{ReZ_{ob}}{ReZ_{of}} = 1.$ This relation is approximately true in the case of dissipative structures, as illustrated in Appendix III.

The insertion power gain is defined as:

$$GP_f = \frac{\text{Power entering network } (N_i) \text{ with repeater}}{\text{Power entering network } (N_i) \text{ without repeater}} \tag{3-12}$$

hence:

$$GP_f = \left|\frac{V_{ia}}{V_i}\right|^2 \tag{3-13}$$

The insertion voltage gain is defined in an analogous manner:

$$A_f = \frac{V_{ia}}{V_i} \tag{3-14}$$

It is worth-while to note that in this case

$$GP_f = \left|A_f\right|^2 \tag{3-15}$$

It will be found convenient to define the nominal terminating impedances Z_{n1} and Z_{n2} such that if the repeater is terminated by these impedances at terminals (a-a') and (b-b') respectively, then it will remain stable at any values of insertion gain, and the values of reflection at both terminals are independent of the value of insertion gain.

The matching factors M_1 and M_2 are defined as:

$$M_1 = \frac{Z_{n1} - Z_{ob}}{Z_{n1} + Z_{ob}} \tag{3-16}$$

and

$$M_2 = \frac{Z_{n2} - Z_{of}}{Z_{n2} + Z_{of}} \qquad (3\text{-}17)$$

The nominal insertion gain (A_n) is the value of the insertion gain if the repeater is terminated by its nominal terminating impedances. Thus:

$$A_{nf} = A_f \quad \text{if } M_1 = M_2 = 0 \qquad (3\text{-}18)$$

All quantities which were defined on the preceding pages in the forward direction can be defined in the same manner in the reversed direction. Symbols E_b, R_b, GP_b, A_b and A_{nb} will be used to designate these quantities.

The merit indicator quantities will be defined as follows: *

1. Singing Limit (L_s) is the geometrical mean value of M_1 and M_2 at which the system becomes unstable for given values of A_{nf} and A_{nb}

$$L_s = \sqrt{M_1 M_2} = g(A_{nf}, A_{nb}) \qquad (3\text{-}19)$$
$$\text{for } |A_f| = \infty \text{ and } |A_b| = \infty$$

2. Echo limit (L_{ef} or L_{eb}) gives the value of matching factor M_2 or M_1 which corresponds to a certain reflection $R_f = K_1$ or $R_b = K_2$ if the matching factor at the opposite end is known to be $M_1 = m_1$ or $M_2 = m_2$, for different values of A_{nf} and A_{nb}. Thus:

(*) In the definitions to follow, K_1, K_2, K_3 are real constants while m_1, m_1', and m_2' are complex constants.

$$L_{ef} = M_2 = H(A_{nf}, A_{nb}) \text{ for } R_f = K_1 \text{ and } M_1 = m_1 \qquad (3\text{-}20a)$$

$$L_{eb} = M_1 = H'(A_{nf}, A_{nb}) \text{ for } R_b = K_2 \text{ and } M_2 = m_2 \qquad (3\text{-}20b)$$

3. Distortion limit (L_{df}, L_{db}) is the geometrical mean value of M_1 and M_2 for which A_f or A_b will differ from their respective nominal values by a given ratio (K_3). It will be proved later that $L_{df} = L_{db}$, therefore only one distortion limit (L_d) is defined.

$$L_d = \sqrt{M_1 M_2} = j(A_n) \quad \text{for} \quad A = K_3 \cdot A_n \qquad (3\text{-}21)$$

4. Gain limit (L_{gf}, or L_{gb}) is the maximum nominal gain value in forward or backward direction for a system with known M_1 and M_2 which are permissible for specified levels of echo. (R_f and R_b)

$$L_{gf} = \left|A_{nf}\right|_{max} = k(\left|R_f\right|, \left|R_b\right|) \qquad (3\text{-}22a)$$

$$L_{gb} = \left|A_{nb}\right|_{max} = k'(\left|R_f\right|, \left|R_b\right|) \qquad (3\text{-}22b)$$

$$\text{both for } M_1 = m_1' \text{ and } M_2 = m_2'$$

If the functions defined above are known for a given system, all questions related to the performance of this system can be answered. They can be used e.g. to determine the required M_1 and M_2 values to assure that a certain echo level is not exceeded. Quantities M_1 and M_2 on the other hand are directly related to the cost of the system, because the more accurate is the compensation that is required, the more expensive the system will be.

In the next chapters, the various systems based on these merit indicator quantities will be analyzed. In order to do this, the following functions must be derived for the systems considered:

$$A_f = A_f\,(A_{nf}, A_{nb}, M_1, M_2) \tag{3-23}$$

$$R_f = R_f\,(A_{nf}, A_{nb}, M_1, M_2) \tag{3-24}$$

and a similar set for A_b and R_b.

These calculations are given in Appendicies IV, V, and VI for the negative impedance, bridge compensated and hybrid arrangements respectively and the results are discussed in the next sections.

4. MERIT INDICATOR FUNCTIONS FOR VARIOUS TYPES OF REPEATERS

As is evident from the treatment given in Appendicies IV, V, and VI, the reflection and insertion gain functions of each arrangement can be written in a common form:

$$A = P \frac{1}{1 - K^2} \tag{4-1}$$

and

$$R = Q \frac{K}{1 - K^2} \tag{4-2}$$

(See equations AIV-14 and 23, AV-41 and 43, AV-52 and 53, where P, Q, and K are complex quantities.)

For the negative impedance repeater:

$$K = (A_n M)^2 \tag{4-3}$$

while for the bridge compensated and hybrid* structures:

$$K = A_{fn} A_{bn} M_1 M_2 f_{c1} f_{c2} f_s \tag{4-4}$$

Values of P and Q for the various arrangements can be found, but these will be immaterial for the considerations to follow.

(*) As shown in Appendix VI the hybrid repeater can be represented by the same equivalent circuit as the bridge compensated arrangement and therefore the same formulas apply.

From (4-1) and (4-2) the absolute value of the insertion gain and reflection functions can be written as:

$$|A| = |P|\left|\frac{1}{1 - K^2}\right| \tag{4-5}$$

$$|R| = |Q|\left|\frac{K}{1 - K^2}\right| \tag{4-6}$$

In order to derive some general results for the variation of $|A|$ and $|R|$ with the circuit parameters, the properties of functions:

$$T(K) = \left|\frac{1}{1 - K^2}\right| \tag{4-7}$$

and

$$S(K) = \left|\frac{K}{1 - K^2}\right| \tag{4-8}$$

must be analyzed. Figures 4-1 and 4-2 show the plot of these functions on the complex K plane. Figures 4-3 and 4-4 in turn give the plots of T and S against $|K|$ with (Arg K) as a parameter.

From these plots it can be seen that:

a) For small values of $|K|$ ($|K| < 0.2$), T changes very slowly while S rises linearly with $|K|$. The value of S in this region is independent of (Arg K).

b) For intermediate values of $|K|$ ($0.2 < |K| < 0.5$), the variation of T with $|K|$ is more pronounced and very much dependent on (Arg K); the optimum value being Arg K = $\pi/4$, in which case T remains almost constant within the region. The S function in this region becomes dependent on (Arg K) most rapidly rising at Arg K = 0 and keeping its linear trend for Arg K = $\pi/4$.

c) In <u>general</u> it can be said that if Arg K = 0, then both T and S change most rapidly with $|K|$. The best properties of S are obtained if Arg K = $\pi/2$, and Arg K = 33° gives the best T variation in the region where $|K| \leq 1$.

In designing a repeater it would be desirable to take advantage of all these properties. In practice however, it is almost impossible to design a compensating network and amplifier transfer function to a specification which includes both $|K|$ and Arg K. (It must be kept in mind that K is a composite function of many variables.)

The best practical approach is to design the network according to a $|K|$ specification and assure that Arg K = 0 which, according to the previous argument, provides for the worst possible condition. Therefore in the following arguments, it will be assumed that K is real.

No such problems arise with respect to quantities P and Q. It is obvious that $|A|$ and $|R|$ do not depend on the arguments of P and Q and therefore they can be replaced by real quantities equal to their respective absolute values.

It follows directly from the previous arguments that in equations (AIV-14 and 23), (AV-41 and 43) and (AV-52 and 53), which will form the basis of the calculations to follow each complex quantity can be replaced by its absolute value. <u>The absolute value signs will be omitted in the following discussions, but it is understood that the symbols designate absolute values</u>.

The <u>singing limit</u> as defined by eqn. (3-19) is given by the condition:

$$1 - K^2 = 0 \qquad (4\text{-}9)$$

For the negative impedance repeater

$$L_s = \frac{1}{A_n} \tag{4-10}$$

and for the bridge compensated and hybrid arrangements:

$$L_s = \sqrt{M_1 M_2} \frac{1}{\sqrt{A_{fn} A_{bn}} \sqrt{f_{c1} f_{c2} f_s}} \tag{4-11}$$

which can be rewritten with the following definitions:

$$\begin{aligned} A_n &= \sqrt{A_{fn} A_{bn}} \\ q &= f_{c1} f_{c2} f_s \end{aligned} \tag{4-12}$$

as:

$$L_s = \frac{1}{A_n \sqrt{q}} \tag{4-13}$$

The <u>echo limit</u> for the negative impedance repeater can be obtained from eqn. (AIV-14):

$$R = \frac{MA_n^2}{1 - (MA_n)^2} \tag{4-14}$$

(The negative sign has been omitted, because only absolute values are considered.)

From eqn. 4-14:

$$M^2 + \frac{1}{R} M - \frac{1}{A_n^2} = 0 \tag{4-15}$$

Taking into account the positive root only:

$$M = \frac{1}{2R}\left(\sqrt{1 - \frac{2R}{A_n}} - 1\right) \qquad (4\text{-}16)$$

and because in all practical cases $\left(\frac{2R}{A_n}\right)^2 \ll 1$

$$L_e = \frac{R}{A_n^2} \qquad (4\text{-}17)$$

Using eqn. (4-12) and with the definitions:

$$p = \sqrt{\frac{M_1}{M_2}}\; f_{c1}^2 f_{c2} f_s f_s''$$

$$M = \sqrt{M_1 M_2} \qquad (4\text{-}18)$$

eqn. (AV-52) yields:

$$R_f = \frac{A_n^2 Mp}{1 - A_n^2 M^2 q} \qquad (4\text{-}19)$$

for the bridge compensated and hybrid repeaters, which can be written as:

$$M^2 + \frac{p}{qR_f} M - \frac{1}{qA_n^2} = 0 \qquad (4\text{-}20)$$

Assuming that $\left(\frac{R_f}{A_n}\right)^2 \ll 1$

$$L_{ef} = \frac{R_f}{pA_n^2} \qquad (4\text{-}21)$$

and a similar expression can be found for the "backward" echo limit

$$L_{eb} = \frac{R_b}{p' A_n^2} \qquad (4\text{-}22)$$

where

$$p' = \sqrt{\frac{M_2}{M_1}}\; f_{c1} f_{c2}^{2} f_s f_s'' \qquad (4\text{-}23)$$

The <u>distortion limit</u> function can be obtained from eqn. (AIV-23) for the negative impedance repeater. (See also definition eqn. 3-21.)

$$\left|\frac{A}{A_n}\right| = \frac{1}{1 - (MA_n)^2} \qquad (4\text{-}24)$$

from which:

$$L_d = \sqrt{\frac{1}{AA_n}}\;\sqrt{\frac{A - A_n}{A_n}} \qquad (4\text{-}25)$$

Introducing the notation:

$$\%\Delta A = \frac{A - A_n}{A_n} \qquad (4\text{-}26)$$

eqn. 4-25 becomes:

$$L_d = \frac{1}{A_n}\sqrt{\frac{\%\Delta A}{1 + \%\Delta A}} \qquad (4\text{-}27)$$

To derive the distortion limit function for the bridge compensated and hybrid structures eqn. (AV-41) is rewritten using definitions (4-12 and 4-18):

$$\left|\frac{A_f}{A_{fn}}\right| = \frac{q}{1 - (A_n M)^2 q} \qquad (4\text{-}28)$$

and:

$$L_{df} = \frac{1}{qA_n}\sqrt{\frac{\%\Delta A_f + (1 - q)}{1 + \%\Delta A_f}} \qquad (4\text{-}29)$$

where

$$\%\Delta A_f = \frac{A_f - A_{fn}}{A_{fn}} \qquad (4\text{-}30)$$

An expression exacty like (4-29) can be derived for the backward distortion limit L_{db}, if $\%\Delta A_f$ is replaced with $\%\Delta A_b$ in this equation. Due to the fact, however that these quantities are independent variables of their respective functions, a common expression for L_{df} and L_{db} is obtained:

$$L_d = \frac{1}{qA_n}\sqrt{\frac{\%\Delta A + (1 - q)}{1 + \%\Delta A}} \qquad (4\text{-}31)$$

where $\%\Delta A$ stands for $\%\Delta A_f$ if L_{df} has to be calculated, and for $\%\Delta A_b$ if L_{db} is desired.

The last merit indicator function is the <u>gain limit</u>, which can be directly obtained from eqn. (4-16) or (4-17) for the negative impedance repeater as:

$$L_g = \sqrt{\frac{R}{M}} \qquad (4\text{-}32)$$

For the bridge compensated and hybrid repeaters, equations (4-21) and (4-22) yield:

$$L_{gf} = \sqrt{\frac{R_f}{p\,M}} \qquad (4\text{-}33)$$

and

$$L_{gb} = \sqrt{\frac{R_b}{p'M}} \qquad (4\text{-}34)$$

The results of the previous calculations are summarized in the table of Fig. 4-5.

For $p = p' = q = 1$, the merit indicator functions are exactly the same for all three arrangements. If $q < 1$, then the bridge compensated and hybrid structures result in more desirable L_s and L_d functions, while for $q > 1$, the negative impedance repeater exhibits better properties as far as the singing limit and distortion limit are concerned. For $p < 1$, the echo limit and gain limit functions of the bridge compensated and hybrid structures are more advantageous, while for $p > 1$, the negative impedance repeater provides better properties.

In practice, the deviation of p, p', and q from unity is relatively small.*

The four merit indicator functions for the negative impedance repeater (and hence for the bridge compensated and hybrid structures for $p = p' = q = 1$) are plotted on figures 4-6 and 4-7.

(*) Actually as pointed out in Appendix V, the values of f_{c1} and f_{c2} can be taken as 1 for all practical applications, while it can be seen that if $f_s' < 1$, then $f_s'' > 1$, and vice versa, so these advantages cannot be regarded as very decisive.

5. COMPARISON OF DIFFERENT SYSTEMS

In the previous chapter the merit indicator functions for the various arrangements were derived. It was shown that with proper selection of p, p', and q values, the bridge compensated and hybrid repeaters can exhibit more desirable properties than their negative impedance counterpart.

It was also pointed out that this advantage cannot be utilized in large extent, because in practice these values are very near to unity and furthermore some of the parameters determining these values are dependent on the transmission system into which the repeater is inserted and therefore cannot be chosen freely by the designer. These factors can be taken into account when dealing with a given system, but in general the comparison cannot be based on these arguments.

In this section it will be assumed that the merit indicator functions are the same for all arrangements and comparison made on this basis. This means that the required compensation "M", to meet a certain insertion gain and reflection specification is the same for all types of repeaters.

The three systems will be compared on the basis of how economically a certain compensation can be obtained.

a) <u>Negative impedance</u> repeaters can operate only if inserted into a symmetrical structure. Therefore if they are used in conjunction with asymmetrical transmission media (e.g. loaded lines), so-called "building-out" networks are required to make this system look symmetrical if viewed from the repeater terminals. [19] (See Fig. 5-1.)

Assuming such a symmetrical system, the compensation is achieved by realizing two negative impedances such that:

$$Z_{line} = \sqrt{Z_a Z_b} \tag{5-1}$$

Where Z_{line} is the characteristic impedance of the transmission media. Z_a and Z_b are realized by means of negative impedance converter networks as illustrated on Fig. 5-2, [19,20] where Z_A and Z_B are passive impedances. Therefore the stability and accuracy of compensation depends on the stability and accuracy of active circuit parameters μ and ν. Using the symbols defined in Fig. 5-2, one can write eqn. (5-1) as follows:

$$Z_{line} = \sqrt{\mu \nu \; Z_A Z_B} \tag{5-2}$$

As shown in Appendix IV, the nominal insertion gain of the negative impedance repeater depends on the value of "N" which is given as:

$$N = \sqrt{\frac{Z_a}{Z_b}} = \sqrt{\frac{\mu Z_A}{\nu Z_B}} \tag{5-3}$$

Therefore, to adjust the repeater for a given insertion gain value, the ratio $\frac{\mu}{\nu}$ has to be changed. This adjustment however, must be made so that the value of $\mu \nu$ is kept constant, in order to keep the value of the image impedance unchanged, (eqn. 5-2). Therefore, this adjustment is critical.

b) The hybrid repeater requires the realization of a driving point impedance function in order to provide proper compensation. (See Appendix I). The stability of compensation depends only on passive networks and its accuracy is not affected by gain adjustments. It must be noted however, that the compensation properties of this system depend in a great extent on the values of Z_s and Z_r, (see Fig. AI-1) if the ideal condition of $Z_{line} = Z_n$ is not met. Therefore, when designing the system, three driving point impedance functions must be considered to meet a certain specification.

c) In the case of the bridge compensated structure, compensation can be achieved in two ways:

1) Selecting $Z_{ob} = Z_1$ and $Z_{of} = Z_2$ (see Fig. AV-1) and thereby having $G_1 = G_2 = 1/2$. This means that two driving point impedance functions must be realized. This case provides the same economy as the hybrid arrangement, with the exception that no other factors have to be taken into account as in the hybrid repeater and therefore only one impedance function per termination must be considered rather than three.
2) It is possible to design transfer functions G_1 and G_2 to provide for the required compensation.

In practice is is desirable to use the combination of these two methods i.e. to select relatively simple Z_1 and Z_2 impedances that approximately satisfy the conditions stated in (1) and then design G_1 and G_2 to make the compensation meet the actual specification.

The fact that one has the freedom to realize a transfer function instead of driving point impedance functions will improve the economy of the design in most cases, and this property can be found only in the bridge compensated repeater. Neither the negative impedance nor the hybrid repeater provides this additional degree of freedom.

It might be possible e.g. in some cases to design a pure RC compensating network for line impedances which have inductive components. This property is a desirable one because RC networks are more easily reproduced in mass production, their size is usually smaller and non-linearities are less pronounced than in the case of networks containing inductances. The methods, limitations, and other aspects concerning the design of compensating networks will be discussed in greater detail in later sections.

Another advantage of the bridge compensated (and hybrid) repeater as compared to negative impedance repeaters is that the insertion gain can be adjusted to different values in forward and backward directions. This property is important when more repeaters have to be cascaded in a particular transmission path. As shown on Fig. 5-3, the bridge compensated arrangement can assure a more uniform signal level distribution along the line then the negative impedance repeater. In some cases this can lead to actual saving of one or more repeaters, for example where, because of crosstalk considerations, the signal level on the line must be kept below a specified maximum at any point. [19]

All the arguments above indicate that the new bridge compensated arrangement does promise some advantages as compared to presently used systems. The actual value of these advantages however, can be evaluated only if all design parameters are known.

6. COMPENSATING NETWORK DESIGN (GENERAL CONSIDERATIONS)

It was mentioned in the preceeding section that it might be possible to use pure RC compensating networks even if the line impedance function could not be reproduced without the use of inductors. In this chapter, the conditions which must be satisfied in order to be able to obtain such networks, will be investigated.

As shown in the introductory chapter, the transfer function of the compensating network has to satisfy the following condition: (eqn. 1-2)

$$G_1 = \frac{Z_{L1}}{Z_{L1} + Z_1} \tag{6-1}$$

Similar condition applies for G_2. Both impedances Z_1 and Z_{line} are known to be positive real functions and can be represented by the ratio of two polynomials:

$$Z_1 = \frac{p(s)}{q(s)} \tag{6-2}$$

and

$$Z_{line} = \frac{n(s)}{d(s)} \tag{6-3}$$

Thus:

$$G = \frac{n(s)q(s)}{n(s)q(s) + p(s)d(s)} = \frac{A(s)}{B(s)} \tag{6-4}$$

In order that "G" be realizable, A(s) must have only positive coefficients and must not be higher in degree than B(s). This condition is evidently satisfied. In addition B(s) must have only negative real roots, which is not necessarily true for any selection of positive real Z_1 and Z_{line} functions.[3]

Furthermore, because Z_1 must also be RC-realizable, p(s) and q(s) must satisfy the conditions imposed on RC-realizable driving point impedance functions.

In order to prove that any positive real Z_{line} can be compensated for by a pure RC network, it must be shown that for any such function there can be found a corresponding RC realizable function such that:

$$D(s) = n(s)q(s) + p(s)d(s) \qquad (6\text{-}5)$$

has only negative real roots.

In general, it was not possible to find any way to prove or disprove this statement. This is due to the lack of theorems related to the actual number of real roots of a polynomial of higher order. There are numerous methods for finding the real roots of a polynomial if the coefficients are specified, but in general the problem becomes very complicated even at the third degree.[5,19]

However, it is possible to show that for any Z_{line} function for which either n(s) or d(s) has only negative real roots, there corresponds a pure RC realizable transfer function. This category of functions include all LR realizable driving point impedance functions and also some RLC realizable ones. (RC realizable Z_{line} functions are of course included.)

To prove the preceding statement, consider the following:

1. If Z_{line} is positive real and n(s) has only negative real roots, then eqn. (6-5) can be written as:

$$D(s) = t(s) + u(s) \qquad (6\text{-}6)$$

where

$$
\left.
\begin{aligned}
t(s) &= n(s)q(s) \\
&= k_1(s+a_1)(s+a_2) \quad \ldots\ldots(s+a_n) \\
\text{and} \qquad u(s) &= p(s)d(s) \;=\; k_2\, d(s)
\end{aligned}
\right\} \qquad (6\text{-}7)
$$

if it is assumed that

$$
\left.
\begin{aligned}
&q(s) = k_1 \geq 0 \quad \text{and} \quad p(s) = k_2 \geq 0 \\
&\text{while} \quad a_1 < a_2 < \;\ldots\ldots\ldots\; < a_n
\end{aligned}
\right\} \qquad (6\text{-}8)
$$

This selection of $q(s)$ and $p(s)$ assures that Z_1 can be realized by a single resistor.

The condition that $D(s)$ must have only negative real roots means that equation

$$t(s) \quad = \quad -u(s) \qquad (6\text{-}9)$$

must have only negative real solutions.

2. Because of the conditions imposed by positive reality on Z_{line} if $s = 0$ then

$$-u(0) \leq 0 \qquad (6\text{-}10)$$

$$t(0) \geq 0 \qquad (6\text{-}11)$$

and

$$\left.\frac{d\ t(s)}{ds}\right|_{s=0} \geq 0 \qquad (6\text{-}12)$$

3. It is known that the degree of $u(s)$ can only be $(n-1)$, n, or $(n+1)$. [3]

4. Because both $t(s)$ and $u(s)$ are finite for any finite values of "s", it is always possible to select k_1 and k_2 such that for any interval $a_i < s < a_{i+1}$ the condition:

$$\text{Max value of } |(-u)| < \text{Max value of } |t| \tag{6-13}$$

is satisfied, where the maximum values refer to the maximum value of these functions within the interval in question. It is also possible to assure that there exists a negative "s" value (s_1) such that:

$$|s_1| > a_n \tag{6-14}$$

and

$$|-u(s_1)| < |t(s_1)| \tag{6-15}$$

5. If conditions as stated in (4) are satisfied, then the two functions $-u(s)$ and $t(s)$ intersect "n" times between $s = 0$ and $s = s_1$.
6. If the degree of $d(s)$ is the same or smaller than the degree of $n(s)$ the "n" intersections provide the "n" required negative real solutions of eqn. (6-9).
7. If the degree of $d(s)$ is $(n + 1)$, then at infinity, the absolute value of $-u(s)$ is larger than that of $t(s)$ which results in an additional intersection of $-u(s)$ and $t(s)$ between $s = s_1$ and $s = \infty$, furnishing the required $(n + 1)$th solution.
8. Similar reasoning applies if it is assumed that $d(s)$ has only negative real roots.

7. TRANSFER FUNCTION DESIGN. (COMPUTER PROGRAMME)

As shown in the preceding section, in general it could not be proved whether it was possible to find a pure RC compensating network for any line impedance functions. Even if it was possible to prove that such a network can be found, the actual design of this network would be very laborious. On the other hand, even if there were cases when it was theoretically impossible to find an RC-realizable network, the question might arise as to what is the best accuracy with which the desired function can be approximated by an RC-realizable transfer function.

In order to provide an answer for the above questions, a computer programme was developed to find the best fitting RC-realizable transfer function for a given condition. Exactly what is meant by "best fitting" will be defined in more concrete terms later in this section.

There are basically two different methods to find the optimum approximating function for a given specification. [4]

1. Find an approximating function without considering whether it actually corresponds to a realizable network and later apply the necessary tests to determine whether this function can be realized.
2. Assume at the beginning a general form of the approximating function which is known to be realizable, (RC realizable in the present case) and during the optimization process consider all necessary constraints which are imposed on the parameters to assure realizability.

The second method is straight forward, and the computer programme is based on this approach.

The computer programme requires as data:

a) Description of the line impedance function specified by the values of "r" and "x" at selected frequencies where:

$$Z_{line} = r + jx \qquad (7\text{-}1)$$

b) Description of Z_1 (or Z_2) which can be specified in the same manner as Z_{line}, or by giving a resistor value (R) and a capacitance value (C) connected in series or parallel.

c) The highest degree of transfer functions to be tried.

d) Some other data which will be mentioned later, (mostly related to accuracy requirements).

From this data the real and imaginary parts of the desired transfer function (G_d) are calculated at the frequencies where the line impedance was specified.

$$G_d = \frac{Z_{line}}{Z_{line} + Z_1} = g + jh \qquad (7\text{-}2)$$

For the case where Z_{line} is the characteristic impedance of the full-section loaded line described in Appendix III, the solid curves of Fig. 7-1 give the real and imaginary parts of this line impedance. Z_1 was selected to be 900Ω in series with a 2.16μF condenser and the dotted lines give its real and imaginary parts on the same figure. Fig. 7-2 is the plot of the real and imaginary parts of the desired transfer function which results from the just described selection of Z_{line} and Z_1. If Z_1 is a pure resistive impedance of 900Ω, then the plot of Fig. 7-3 results for the desired transfer function.

Next the most simple form of RC realizable transfer functions is assumed:

$$G(s) = A_1 \tag{7-3}$$

and the best value for A_1 is found by the so-called "hill climbing" technique. (See Appendix VII.) The "best value" of A_1 is meant to be "the best value of A_1 with respect to some criterion". The selection of this criterion will be dealt with later. If the accuracy of the approximation obtained does not meet the specified requirement, (as set by a data card), the transfer function of the next higher complexity is tried.

It would be an obvious choice to select:

$$G(s) = \frac{A_1}{(s + a_1)} \tag{7-4}$$

as the next function to try. This is not the case however, for reasons discussed below.

The actual sequence in which transfer functions of various complexity are tried is specified by the table of Fig. 7-4. The reason behind this particular sequence is found in the method by which the initial values for parameters for each trial are selected.

The programme is arranged so that the optimized values of the parameters of a certain transfer function are used as the initial values for some of the subsequent trials. The arrows on Fig. 7-4 indicate the way these initial values are transferred. E.g. the optimized values of $G_{0,2}(s)$ are used for the initial values for transfer functions $G_{2,2}(s)$ and $G_{0,4}(s)$. For definition of $G_{m,n}(s)$ see eqn. (7-5).

This pattern of initial value transfer is justified by the following reasoning.

Consider a transfer function:

$$G_{m,n}(s) = \frac{A_1(1 + A_2 s + A_3 s^2 \; \dots\dots + A_{m+1} s^m)}{(s + a_1)(s + a_2) \; \dots\dots \; (s + a_n)} \qquad (7\text{-}5)$$

where $a_1 < a_2 < \dots\dots\dots < a_n$

and $m \leq n$

Assume that the optimum values of parameters of this function are $(A_{1o}, A_{2o}, \dots\dots A_{(m+1)o}, a_{1o} \dots a_{no})$. Augmenting this function by adding a new term in the numerator gives:

$$G_{(m+1),n} = \frac{A_1(1 + A_2 s \; \dots\dots \; A_{m+1} s^m + A_{m+2} s^{m+1})}{(s + a_1) \; \dots\dots\dots\dots\dots \; (s + a_n)}$$

(7-6)

If the initial value of A_{m+2} is zero, then it is obvious that the optimum initial values for the rest of the parameters are the same as their optimized values for $G_{m,n}(s)$. This type of initial value transfer is indicated on Fig. 7-4 by horizontal arrows.

Another way to increase the complexity of $G_{m,n}(s)$ is to add a term to the denominator obtaining:

$$G_{m,(n+1)} = \frac{A_1(1 + A_2 s \,\ldots\ldots + A_{m+1} s^m)}{(s + a_1)\,\ldots\ldots\ldots\,(s + a_n)\,(s + a_{n+1})} \tag{7-7}$$

In this case there is no logical relation in the general case between the optimized parameters of $G_{m,n}(s)$ and the new transfer function. E.g. if $a_{n+1} = 0$, then, after substituting $s = j\omega$:

$$\begin{aligned} G_{m,(n+1)}(j\omega) &= \frac{1}{j\omega}\left(\mathrm{Re}\, G_{m,n}(j\omega) + j\mathrm{Im}\, G_{m,n}(j\omega)\right) \\ &= \frac{1}{\omega}\,\mathrm{Im}\, G_{m,n}(j\omega) - j\frac{1}{\omega}\,\mathrm{Re}\, G_{m,n}(j\omega) \end{aligned} \tag{7-8}$$

Consider the case however, when $G_{m,(n+2)}(s)$ is selected as the next function to try. In this case, if initially $a_{n+1} = a_{n+2} = 0$, then the following relation results:

$$G_{m,(n+2)}(j\omega) = \frac{-1}{\omega^2}\left(\mathrm{Re}\, G_{m,n}(j\omega) + j\mathrm{Im}\, G_{m,n}(j\omega)\right) \tag{7-9}$$

This relation shows that there is a very close relation between the best initial values of $G_{m,(n+2)}$ and the optimized values of $G_{m,n}$, which can be exploited, namely, at $\omega = 1$,

$$-G_{m,n}(j\omega)\Big|_{\omega=1} = G_{m,(n+2)}(j\omega)\Big|_{\omega=1} \qquad (7\text{-}10)$$

which means that the fitting at the middle of the band is not changed. (Note that the calculations are performed by using normalized frequency variable such that $\omega = 1$ at the middle of the frequency band considered.)

The only deviation from the above reasoning is the case when after the optimum value of A_1 for

$$G_{0,0}(s) = A_1 \qquad (7\text{-}11)$$

is found, the initial value for a_1 in transfer function:

$$G_{0,1}(s) = \frac{A_1}{(s + a_1)} \qquad (7\text{-}12)$$

is selected to be unity, because from

$$G_{0,1}(j\omega) = \frac{A_1 a_1}{\omega^2 + a_1^2} - j\omega \frac{A_1}{\omega^2 + a_1^2} \qquad (7\text{-}13)$$

at $\omega = 1$, it follows that

$$\frac{1}{2} \operatorname{Re} G_{0,0}(j\omega)\Big|_{\omega=1} = \operatorname{Re} G_{0,1}(j\omega)\Big|_{\omega=1} \qquad (7\text{-}14)$$

provided $a_1 = 1$. No such relation exists between imaginary parts due to the fact that $\operatorname{Im} G_{0,0}(j\omega) = 0$. This relation is utilized in the programme and the corresponding initial value transfer is indicated by an arrow between steps "1" and "4" on Fig. 7-3.

As is explained in Appendix VII, the parameter-optimizing programme requires a single merit number to be associated with any particular selection of parameter values. This merit number is referred to as performance indicator (P). The programme decides which particular selection of parameters is more desirable on the basis of the value of "P" associated with a particular set of parameter values. The method by which this performance indicator is calculated if referred to as the performance criterion.

There are many ways of defining a performance criterion, but only two will be discussed here. [4] Probably the most commonly used performance criterion is the mean square error, which in the present case will provide the following expression for P:

$$P_{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(g(\omega_i) - \mathrm{Re}G(\omega_i)\right)^2 + \left(h(\omega_i) - \mathrm{Im}G(\omega_i)\right)^2 \qquad (7\text{-}15)$$

where $g(\omega_i)$ is the real value of the desired transfer function at frequency ω_i.

$\mathrm{Re}G(\omega_i)$ is the real value of the approximating transfer function at frequency ω_i.

$h(\omega_i)$ and $\mathrm{Im}G(\omega_i)$ are the corresponding imaginary values.

The summation extends to all frequencies $\omega_1, \omega_2 \ldots \omega_n$ where the Z_{line} impedance function is specified.

One can also use the maximum error criterion which gives for P:

$$P_{MXE} = \text{Max. value of} \sqrt{\left(g(\omega_i)-\mathrm{Re}G(\omega_i)\right)^2 + \left(h(\omega_i)-\mathrm{Im}G(\omega_i)\right)^2} \qquad (7\text{-}16)$$

where the maximum refers to the fact that P_{MXE} must be evaluated at that frequency which results in the maximum P value. It is worth-while mentioning that any power of "P" can be defined as a valid performance criterion.

The computer programme developed can be set up by data card to use either of the mean square error or the maximum error criterion. It must be pointed out however, that for the purpose of this study, the maximum error criterion is more suitable because singing properties of the repeater depend on the maximum deviation rather than on average of the deviations at various frequencies.

Another factor to be considered is that the maximum error criterion is bound to produce a predictable result, namely a so-called equal ripple or Chebyshev approximation. There might be more than one optimum sets of parameters, but these are all equally desirable. (This theorem is proved in Chebyshev's original paper and is referred to in Tuttle's book. [4]) The mean square error criterion on the other hand, can produce a number of local extrema which are not equally desirable.

The parameter-optimizing programme (see Appendix VII) cannot, in general, find the best optimum if local optimums exist, unless the programme is made uneconomically lengthy. This consideration also gives preference to the maximum-error performance criterion.

It is an easy matter to modify the present programme so that after finding the optimum values of parameters based on the maximum-error criterion, a second search should start to optimize the mean square error subject to a constraint that in the meantime the maximum error should not exceed a specified value.

The actual performance criterion used in the selected approach differs slightly from those given by equations 7-15 and 7-16. This means that weighing factors ($W_1 \ldots W_n$) are specified for each frequency point, by which the corresponding term must be degraded. Thus, the following modified mean square error and maximum error criterions are obtained:

$$P'_{MSE} = \frac{1}{n} \sum_{i=1}^{n} W_i \left\{ \left(g(\omega_i) - ReG(\omega_i) \right)^2 + \left(h(\omega_i) - ImG(\omega_i) \right)^2 \right\} \tag{7-17}$$

and

$$P'_{MXE} = \text{Max. value of } \sqrt{W_i \left(g(\omega_i) - ReG(\omega_i) \right)^2 + \left(h(\omega_i) - Im(\omega_i) \right)^2} \tag{7-18}$$

The reason behind introducing these weighing factors is that errors at different frequencies are not equally significant for two reasons:

1) The amplification of the amplifiers used is not constant within the frequency band considered. Therefore, for both singing and echo considerations higher deviations can be tolerated where the amplification has a lower value.

2) The distribution of speech power over the frequency range is not uniform and therefore for echo properties, (power returned to originating side), poorer compensation at frequencies where the speech power density is lower can be tolerated. [10]

Another factor to be considered in developing the optimizing method is the constraints imposed on the parameters in order to assure that the function is RC realizable. The constraints considered in the computer programme are as follows:

1. Values a_1, a_2, ... a_n must be positive and real.
2. $a_i \neq a_j$ if $i \neq j$
3. All values of A_2, A_3 .... A_{m+1} must be positive and real.
4. A_1 must be real, but no other restrictions apply.

The first three constraints do not need any further explanation; they follow from standard criterion for RC-realizable transfer functions.[3]

Constraint (4) however, is not usually acceptable in this loose wording for passive RC realizable two-port networks. It is usually required that A_1 be positive and real. Furthermore, the Fialkow-Gerst condition [3,12] requires that the absolute value of A_1 be smaller than a certain maximum which depends on the values of the other parameters (A_1 ... A_{m+1}, a_1 ... a_n).

In the present case, these additional constraints on A_1 were dropped for two reasons:

1. If the optimum value of A_1 is negative then $-G(s)$ can be realized instead of $G(s)$. This means that in the scheme of Fig. 7-b, the input signals to the amplifiers must be added, rather than subtracted, which is not a difficult task to perform.
2. If the absolute value of A_1 exceeds the limit imposed by the Fialkow-Gerst condition, it is always possible to select a suitable value "k" such that $G'(s) = kG(s)$

is a realizable transfer function. By multiplying both sides of eqn. (6-1) by "k", the condition

$$G'(s) = k \frac{Z_{line}}{Z_{line} + Z_1} \tag{7-19}$$

is obtained, and can now be satisfied.

8. TRANSFER FUNCTION DESIGN. (EXAMPLE)

The results of an actual design based on the method described in the previous section are plotted on Figures 8-1, 8-2, 8-3, and 8-4. In all cases, Z_{line} was assumed to be as specified on Fig. 7-1. Figures 8-1 and 8-2 represent calculations where Z_1 was taken to be a 900Ω resistor in series with a 2.16 μF condenser, while in the case of figures 8-3 and 8-4, Z_1 was selected to be a 900Ω pure resistive impedance. The transfer functions on these figures are based on a normalized frequency $f_n = \frac{f}{f_o}$ where $f_o = 1000$ c.p.s.

As can be seen, the best approximation is obtained by the transfer function plotted on Fig. 8-1. The function of Fig. 8-2 is comparable in the accuracy of approximation to the one of Fig. 8-1, but the corresponding transfer function is of a higher degree and thus not so desirable.

The importance of selecting a proper Z_1 function is evident if the results of figures 8-1 and 8-2 are compared to the results illustrated on figures 8-3 and 8-4.

In all of these calculations the following weighing factors were used:

f^{cps}	100	200	300	500	1000	1300	2000	2500	3000
W	.1	.3	.6	.8	1.0	1.0	0.5	0.3	0.2

which accounts for the fact that the deviation at 3kc. is approximately five times bigger than at 1kc.

As was pointed out previously, the transfer function of Fig. 8-1 provides the best result and therefore, this will be analyzed in the following.

The realization of such a function is no problem. The network of Fig. 8-5 exactly reproduces the function given in Fig. 8-1. It will be of some interest to investigate the quality of this approximation in terms of the merit indicator functions defined in section (3).

First, the value of "M" must be calculated for the frequencies of concern. Using eqn. (AV-23),

$$(G - G_d) = \frac{Z_{line}}{Z_{line}+Z_1} - \frac{Z_{nl}}{Z_{nl} + Z_1} \qquad (8\text{-}1)$$

is obtained, which can be written as:

$$(G - G_d) = -\frac{Z_1(Z_{line}+Z_{nl})}{(Z_{line}+Z_1)(Z_1 + Z_{nl})} \cdot \frac{(Z_{nl} - Z_{line})}{(Z_{nl} + Z_{line})} \qquad (8\text{-}2)$$

Assume that:

$$\frac{Z_1(Z_{line} + Z_{nl})}{(Z_1+Z_{line})(Z_1+Z_{nl})} = \frac{2\, Z_1 \cdot Z_{nl}}{(Z_1 + Z_{nl})^2} \qquad (8\text{-}3)$$

which is justified because of the close approximation shown on Fig. 8-1. It can be assumed that

$$Z_{line} \approx Z_{nl} \qquad (8\text{-}4)$$

when evaluating the above term.

Substituting (8-3) in (8-2), and with the definition of eqn. 3-16, one gets:

$$|M| = |G - G_d| \frac{2|Z_1| \cdot |Z_{n1}|}{|Z_1 + Z_{n1}|^2} \qquad (8\text{-}5)$$

The $|G - G_d|$ term can be obtained from Fig. 8-1 for the desired frequencies. (Actually, this must be calculated from the original computer figures which are accurate to the fifth figure, due to the fact that the difference of relatively large numbers are involved.)

The term $\frac{2|Z_1||Z_{n1}|}{|Z_{n1}+Z_1|^2}$ can be evaluated in the same manner, hence the value of M can be calculated. The values obtained are listed in the table below:

f^{cps}	M
100	0.0816
200	0.01
300	0.024
500	0.030
1000	0.035
1300	0.059
2000	0.092
2500	0.158
3000	0.195

Using these values, one can refer to Fig. 4-6 and find that for frequencies below 2000 cps nominal insertion gain values up to "10" can be allowed without the risk of singing, while for 2500 cps A_n = 6.5 and for 3000 cps, A_n = 5.0 will make the repeater unstable.

From the slope of the L_s curve at the points of interest, it is possible to estimate how critical these values are to variation in the values of $|M|$.

It is also possible to see that at $f = 1000$ cps, the maximum insertion gain value is $A_n = 2.5 = 8$db if the maximum allowed reflection value is 0.2.

Using this nominal insertion gain value, the echo conditions below 1000 cps will not be impared, while at higher frequencies either the nominal insertion gain value must be smaller or the echo specification relaxed. It is logical to specify a higher echo level at higher frequencies, because only a small fraction of speech power is carried in this frequency domain. [10]

By the method outlined above, the desired frequency characteristic of the amplifiers can be determined. The property that the frequency characteristic of the repeater can be easily selected and adjusted to a required specification independent of matching and compensating properties is a feature of the bridge compensated repeater not exhibited by the negative impedance structure.

(Note that in the treatment above it was assumed that the same "M" values are applicable for both terminations of the repeater. A further improvement of behaviour is obtained if the two compensating networks are designed such that the maximum deviations do not coincide thus making the M_1M_2 product more uniform along the frequency range. By specifying proper weighing factors in the computer programme for the design of these networks this effect can easily be exploited.)

9. CIRCUIT REALIZATION AND MEASURING TECHNIQUES

Fig. 9-1 illustrates one possible circuit arrangement to realize the bridge compensated repeater.

As can be seen, signals originating from line (A) enter the repeater at terminals (1-1') from where they are applied to the base of transistor (T_{1f}) via resistor (R_{1b}) and condenser (C_f). The direct coupled two stage amplifier consisting of transistors (T_{1f}) and (T_{2f}) amplifies the signal applied to the base of (T_{1f}) and feeds it to output terminals (2-2') via the (Z_2') impedance. The gain of the amplifier can be adjusted by potentiometer (P_{ef}). The bias conditions of the transistors are stabilized by the negative feedback between the collector and the base of transistor (T_{1f}) and by the improved I_{CO} stability of direct coupled transistor amplifiers. [23] The signals on the collector of transistor (T_{2f}) are 180° out of phase, therefore, the resulting signal at the base of transistor (T_{1b}) is zero if:

$$G_2 = Q \frac{Z_{of}}{Z_{of} + Z_2} \qquad (9\text{-}1)$$

where Q is a real multiplier which can be set by potentiometer (P_{cf}) to compensate for the various settings of potentiometer (P_{ef}). Z_2 is the series combination of the output impedance of the amplifier and Z_2'. Similar considerations apply to signals travelling in the opposite direction.

In the previous chapters, design formulas were developed for the bridge compensated repeater circuit relating the actual values of insertion gains (A_f and A_b) and reflections (R_f and R_b) to the nominal insertion gain values (A_{fn} and A_{bn}) and to the values of matching factors (M_1 and M_2). In an actual system, A_f, A_b, R_f and R_b can be measured very conveniently by the method outlined below if Z_{ob} and Z_{of} are known impedances and the values of e_1 and e_2 are also available. In order to prove that the relations which were developed in the previous sections are true, some way of measuring A_{fn}, A_{bn}, M_1 and M_2 must be found. If these values can be measured, then it is possible to compare the measured values of A_f, A_b, R_f and R_b to the calculated figures which result from substituting the measured values of A_{fn}, A_{bn}, M_1 and M_2 into the relations obtained in the previous sections.

Using the notation of Fig. 9-1, the insertion gain values can be obtained as:

$$A_f = \frac{Z_{of} + Z_{ob}}{Z_{of}} \left. \frac{V_{i2}}{e_1} \right|_{e_2=0} \qquad (9\text{-}2)$$

and

$$A_b = \frac{Z_{of} + Z_{ob}}{Z_{ob}} \left. \frac{V_{i1}}{e_2} \right|_{e_1=0} \qquad (9\text{-}3)$$

The reflection values can be obtained from the relation:

$$V_{i1} = \left. \frac{1}{2}e_1 + \frac{1}{2}e_1 R_f \right|_{e_2=0} \qquad (9\text{-}4)$$

which can be solved for R_f and gives:

$$R_f = \left. \frac{2V_{i1} - e_1}{e_1} \right|_{e_2=0} \qquad (9\text{-}5)$$

and similarly:

$$R_b = \left. \frac{2V_{i2} - e_2}{e_2} \right|_{e_1=0} \qquad (9\text{-}6)$$

Unfortunately, the nominal insertion gain values cannot be measured in such a straightforward manner, because this would require that the repeater be terminated by its nominal impedances and therefore the measurements could not be performed with the actual terminations attached to the system. The values of M_1 and M_2 are similarly hard to obtain in an existing system if it is desirable to perform the measurements without disturbing the circuitry. In the following, a technique is developed by which these values can be obtained by measuring voltages at various points in the system.

With the notation defined in Appendix V, section 1, page 71, the relation

$$v_2 = A_1 v_1 F_2 + a_2 e_2 - A_1 v_1 G_2 \qquad (9\text{-}7)$$

is obtained. It is obvious however, that in the arrangement of Fig. 9-1

$$V_{r1} = k_1 v_2 \qquad (9\text{-}8)$$

where k_1 is a proportionality constant which can be determined as follows: if $v_1=0$, (short circuit the corresponding terminals), then from (9-7) and (9-8):

$$V_{r1} = \left. k_1 a_2 e_2 \right|_{v_1=0} \qquad (9\text{-}9)$$

but

$$a_2 e_2 = V_{i2} \tag{9-10}$$

thus

$$k_1 = \left.\frac{V_{r1}}{V_{i2}}\right|_{v_1=0} \tag{9-11}$$

and for later use it is also possible to obtain the following relations:

$$a_2 = \frac{Z_2}{Z_{of} + Z_2} = \left.\frac{V_{i2}}{e_2}\right|_{v_1=0} \tag{9-12}$$

$$F_2 = \left.\frac{e_2 - V_{i2}}{e_2}\right|_{v_1=0} \tag{9-13}$$

and

$$Z_2 = Z_{of} \left.\frac{V_{i2}}{e_2 - V_{i2}}\right|_{v_1=0} \tag{9-14}$$

If $e_2=0$, then from equations (9-7) and (9-8):

$$V_{r1} = k_1 v_2 = k_1 A_1 v_1 (F_2 - G_2) \tag{9-15}$$

and

$$F_2 - G_2 = \left.\frac{V_{r1}}{k_1 A_1 v_1}\right|_{e_2=0} \tag{9-16}$$

but from Fig. AV-1 in Appendix V, page 71 ,

$$A_1 v_1 = \left.\frac{E_2}{F_2}\right|_{e_2=0} \tag{9-17}$$

which in the case of the arrangement of Fig. 9-1 yields:

$$A_1 v_1 = \left.\frac{V_{i2}}{F_2}\right|_{e_2=0} \tag{9-18}$$

and substituting this into (9-16):

$$F_2 - G_2 = \frac{F_2}{k_1} \left.\frac{V_{r1}}{V_{i2}}\right|_{e_2=0} \tag{9-19}$$

is obtained. By similar reasoning the following relations can be proved:

$$k_2 = \left.\frac{V_{r2}}{V_{i1}}\right|_{v_2=0} \tag{9-20}$$

$$a_1 = \left.\frac{V_{i1}}{e_1}\right|_{v_2=0} \tag{9-21}$$

$$F_1 = \left.\frac{e_1 - V_{i1}}{e_1}\right|_{v_2=0} \tag{9-22}$$

$$Z_1 = Z_{ob} \left.\frac{V_{i1}}{e_1 - V_{i1}}\right|_{v_2=0} \tag{9-23}$$

and

$$F_1 - G_1 = \frac{F_1}{k_2} \left.\frac{V_{r2}}{V_{i1}}\right|_{e_1=0} \tag{9-24}$$

It is also possible to calculate the nominal terminating impedances from the relation

$$G_2 = \frac{Z_{n2}}{Z_{n2} + Z_2} \tag{9-25}$$

in which equation only Z_{n2} is not known because G_2 can be obtained from equations (9-16) and (9-13). Thus,

$$Z_{n2} = Z_2 \frac{G_2}{1 - G_2} \tag{9-26}$$

and similarly:

$$Z_{n1} = Z_1 \frac{G_1}{1 - G_1} \tag{9-27}$$

With Z_1, Z_{n1}, Z_2, Z_{n2}, Z_{ob} and Z_{of} known, f_{c1}, f_{c2}, f_s, f_s', f_s'', and f_{ss} can be calculated from their respective definitions given in Appendix V.

The nominal insertion gain values can be obtained as: *

$$A_{fn} = \frac{Z_{n1} + Z_{n2}}{Z_{n2}} \cdot \frac{Z_{ob} + Z_1}{Z_{n1} + Z_1} \left. \frac{V_b}{e_1} \right|_{e_2=0 \text{ and } v_2=0} \tag{9-28}$$

and

$$A_{bn} = \frac{Z_{n1} + Z_{n2}}{Z_{n1}} \frac{Z_{of} + Z_2}{Z_{n2} + Z_2} \left. \frac{V_d}{e_2} \right|_{e_1=0 \text{ and } v_1=0} \tag{9-29}$$

(*) For these relations to hold it is assumed that $R_{1f} = R_{2f}$, and that $R_{1b} = R_{2b}$ within the accuracy of this measurement. These resistors however, must be accurately matched is proper compensation is desired.

These relations hold because, provided that $e_2=0$,

a) If $Z_{of} = Z_{n2}$, then $V_{i2} = V_b$ and $v_2 = 0$.

b) If $Z_{ob} = Z_{n1}$, then the nominal value of $V_{i1}(V_{i1n})$ is $V_{i1n} = \dfrac{Z_1}{Z_{n1}+Z_1} e_1$, but the actual value as measured is: $V_{i1} = \dfrac{Z_1}{Z_{ob}+Z_1} e_1$, therefore a correction factor $\dfrac{Z_{ob}+Z_1}{Z_{n1}+Z_1}$ appears in eqn. (9-28).

Similar proof can be given for eqn. (9-29).

From eqn. (8-2) the values of M_1 and M_2 can be obtained. This equation can be written as:

$$F_1 - G_1 = \frac{Z_1(Z_{ob}+Z_{n1})}{(Z_{ob}+Z_1)(Z_{n1}+Z_1)} M_1 \qquad (9\text{-}30)$$

in which all quantities but M_1 were measured before. Therefore,

$$M_1 = (F_1 - G_1) \frac{(Z_{ob}+Z_1)(Z_{n1}+Z_1)}{Z_1(Z_{ob}+Z_{n1})} \qquad (9\text{-}31)$$

and for M_2:

$$M_2 = (F_2 - G_2) \frac{(Z_{of}+Z_2)(Z_{n2}+Z_2)}{Z_2(Z_{of}+Z_{n2})} \qquad (9\text{-}32)$$

Using the relations derived in this section, it is possible to measure the relevant quantities of an existing system by measuring voltages only, without disassembling the circuit to take impedance measurements. If it is desired to take into account both the absolute value and phase angle of the measured quantities, both absolute value and phase angle of all voltages must be measured. It was pointed out however, in section 4, that it is legitimate to use real quantities instead of complex ones in all the calculations, replacing all quantities with their respective absolute values. Therefore, only the absolute values of the voltages are of interest. This is due to the fact that the design formulas in this case give the worst possible properties of the repeater. If the accuracy of the design formulas have to be proved, the above method cannot be used, since it does not give the exact values, only the worst possible values. Therefore, there are two possible ways to proceed:

a) Measure both phase angle and magnitude of all voltages and use complex arithmetic to calculate the desired values.

b) Build a system which contains only real impedances for the sake of this experiment, in which case the absolute value measurements will result in exact values.

In the case of this study, the second method was used. The system on which the measurements were performed is shown on Fig. 9-2. As can be seen, this system is the same as the one shown on Fig. 9-1, with the exception that G_1 and G_2 have been replaced by potentiometers P_3 and P_4, and Z_1' and Z_2' have been replaced by rheostats RH_1 and RH_2 respectively. During the first set of measurements, the condition $R_{ob} = R_{of} = R_1 = R_2$ was maintained where R_1 is the sum of RH_1 and the output impedance of emitter follower (T_{2b}) and R_2 is the sum of RH_2 and emitter follower (T_{2f}). Various M values were obtained by adjusting potentiometers P_3 and P_4.

At the start, the values of k_1 and k_2 were calculated from measurements based on equations (9-11) and (9-20). Next potentiometers P_3 and P_4 were adjusted to provide complete compensation and the values of A_{fn} and A_{bn} were obtained as defined by equations (9-28) and (9-29) where in this case:

$$\frac{Z_{n1} + Z_{n2}}{Z_{n2}} \quad \frac{Z_{ob} + Z_1}{Z_{n1} + Z_1} = 2 \qquad (9\text{-}33)$$

and

$$V_b = V_{i2} \qquad (9\text{-}34)$$

thus:

$$A_{fn} = 2 \left. \frac{V_{i2}}{e_1} \right|_{e_2=0} \qquad (9\text{-}35)$$

and

$$A_{bn} = 2 \left. \frac{V_{i1}}{e_2} \right|_{e_1=0} \qquad (9\text{-}36)$$

After these initial measurements, various settings of potentiometers P_3 and P_4 were tried, making two sets of measurements at each particular setting, one with e_1=0 and another with e_2=0. In the case of the arrangement considered, eqn. (9-31) can be written as:

$$M_1 = 2(F_1 - G_1) \qquad (9\text{-}37)$$

and with eqn. (9-24), this gives:

$$M_1 = \frac{1}{k_2} \left. \frac{V_{r2}}{V_{i1}} \right|_{e_1=0} \qquad (9\text{-}38)$$

and

$$M_2 = \frac{1}{k_2} \frac{V_{r1}}{V_{i1}} \Bigg|_{e_2=0} \qquad (9\text{-}39)$$

Some of the results of these measurements made at 1000 cps, are listed in Table 9-1.

TABLE 9-1

$k_1 = 9.1$; $k_2 = 10.2$; $A_{fn} = 4.84$; $A_{bn} = 4.22$

		Measured Values				Calculated Values			
M_1	M_2	A_f	A_b	R_f	R_b	A_f	A_b	R_f	R_b
0	0.02	4.84	4.22	0.4	0	4.84	4.22	0.413	0
0	0.05	4.84	4.22	1.0	0	4.84	4.22	1.02	0
0	0.16	4.84	4.22	3.0	0	4.84	4.22	3.02	0
0	0.44	4.84	4.22	9.0	0	4.84	4.22	8.95	0
0	0.75	4.84	4.22	15.0	0	4.84	4.22	15.03	0
0.098	0.046	5.3	4.6	1.0	2.2	5.3	4.62	1.03	2.2
0.098	0.112	6.3	5.4	3.0	2.56	6.25	5.41	2.94	2.58
0.098	0.258	10.	9.0	11.4	4.1	10.01	8.75	11.2	4.16

The calculated values were based on equations (AV-41 and 43), and (AV-52 and 53) given in Appendix V. In the case of these measurements, the values of f_{c1}, f_{c2}, f_s and f_s'' are all unity, because $Z_{ob} = Z_{of}$ and $Z_{n1} = Z_{n2}$ and also $Z_1 = Z_{n1}$ and $Z_2 = Z_{n2}$. The measured results and the calculated values show very good agreement as can be seen from the table above.

Another set of measurements was made without attempting to satisfy any symmetry conditions. The condition $R_{ob} = 1000\Omega$ and $R_{of} = 1500\Omega$ was selected while rheostats (RH_1 and RH_2) and potentiometers (P_3 and P_4) were set to some arbitrary position. The following readings were obtained:

a) $e_1 = 0.1$ V and $e_2 = 0$ V

$V_{i1} = 0.13$ V; $V_{i2} = 0.35$ V; $V_{r1} = 0.12$ V; $V_{r2} = 0.61$ V

b) $e_1 = 0$ V $e_2 = 0.1$ V

$V_{i1} = 0.28$ V; $V_{i2} = 0.47$V; $V_{r1} = 0.5$ V; $V_{r2} = 0.76$ V

c) $e_1 = 0$ V $e_2 = 0.1$ V $v_1 = 0$ V

$V_{i1} = 0.2$ V; $V_{i2} = 0.041$ V; $V_{r1} = 0.35$ V; $V_{r2} = 0$ V;
$V_d = 0.15$ V;

d) $e_1 = 0.1$ V; $e_2 = 0$ V; $v_2 = 0$ V;

$V_{i1} = 0.04$ V; $V_{i2} = 0.24$ V; $V_{r1} = 0$ V; $V_{r2} = 0.41$ V;
$V_b = 0.25$ V;

From this data, the following results can be calculated using equations (9-2), (9-3), (9-5) and (9-6):

$A_f = 5.85$; $A_b = 7.0$; $R_f = 1.6$; $R_b = 8.4$;

From equations (9-11), (9-12), (9-13), (9-14), and (9-16):

$k_1 = 8.6$; $a_2 = 0.41$; $F_2 = 0.59$; $Z_2 = 1040\Omega$;
$G_2 = 0.566$;

and from equations (9-20), (9-21), (9-22), (9-23), and (9-24):

$$k_2 = 10.2; \quad a_1 = 0.4; \quad F_1 = 0.6; \quad Z_1 = 670\Omega;$$
$$G_1 = 0.44;$$

The values of the nominal terminating impedances are obtained from equations (9-26) and (9-27).

$$Z_{n1} = 525\Omega; \qquad Z_{n2} = 1360\Omega;$$

Equations (9-28) and (9-29) yield the values for the nominal insertion gains:

$$A_{fn} = 4.8; \qquad A_{bn} = 5.8;$$

Matching factors M_1 and M_2 are calculated next using equations (9-31) and (9-32).

$$M_1 = 0.315; \qquad M_2 = 0.049;$$

From equations (AV-33), (AV-34), (AV-35), (AV-47), and (AV-50) in Appendix V:

$$f_{c1} = 1.04; \quad f_{c2} = 0.99; \quad f_s = 0.8; \quad f_s' = 1.25;$$
$$f_s'' = 0.83;$$

The values calculated above can be substituted into equations (AV-41) and (AV-43) to obtain the insertion gain values:

$$A_f = 6.1 \qquad \text{and} \qquad A_b = 7.3$$

comparing these values with the measured results, ($A_f = 5.85$; $A_b = 7.0$;)

shows that the deviation is less than 10%. This deviation can be justified if the accuracy of the instruments used for this measurement is taken into account.

For the reflection values, equations (AV-52) and (AV-53) give:

$$R_f = 2.2 \qquad \text{and} \qquad R_b = 9.1$$

The value of R_b is in good agreement with the measured result, ($R_f = 1.6$; $R_b = 8.4$), but R_f deviated considerably from its measured value. This is due to the fact that equation (AV-41) does not take into account the reflection which would be present even at $M_2 = 0$ due to the fact that $Z_{ob} \neq Z_1$. If this factor is taken into account, the accuracy improves.

CONCLUSION

On the preceding pages, the new bridge compensated repeater was compared with the systems presently in use. It was found that this arrangement provides more degree of freedom to achieve a certain level of matching specified by matching factors M_1 and M_2 than any other system, which property can be utilized to decrease the cost of the repeater.

With the introduction of the merit indicator functions a method was established by which the desired values of the matching factors can be determined to meet given insertion gain and reflection specifications. A procedure based on a computer programme was developed for the design of the compensating network required to provide the calculated values of M_1 and M_2 and a technique was described by which a given system can be tested without disturbing the circuitry of of the system.

Finally, the measured results were compared with the calculated values for a simple system and the agreement was found to be very good.

APPENDIX I

Operating principle of a hybrid transformer.

The four port network inside the dotted lines on fig AI-1 illustrates the simplest form of a hybrid transformer.

It can be seen that if $Z_n = Z_L$ then signals applied to terminals (2-2') will give rise to equally large currents I_1 and I_2 and therefore no part of this signal will be transmitted to terminals (3-3'), and the signal power will be divided equally between impedances Z_n and Z_L.

If the condition $Z_n = Z_L$ is not satisfied then some portion of the signal applied to terminals (2-2') will be received at terminals (3-3'). The quality of isolation between terminal pairs (2-2') & (3-3') is given by the return loss (R) which is defined as:

$$R\ (db) = 10 \log \frac{\text{Power received at terminals } 3\text{-}3'}{\text{Power applied to terminals } 2\text{-}2'}$$

assuming that $e_L = 0$.

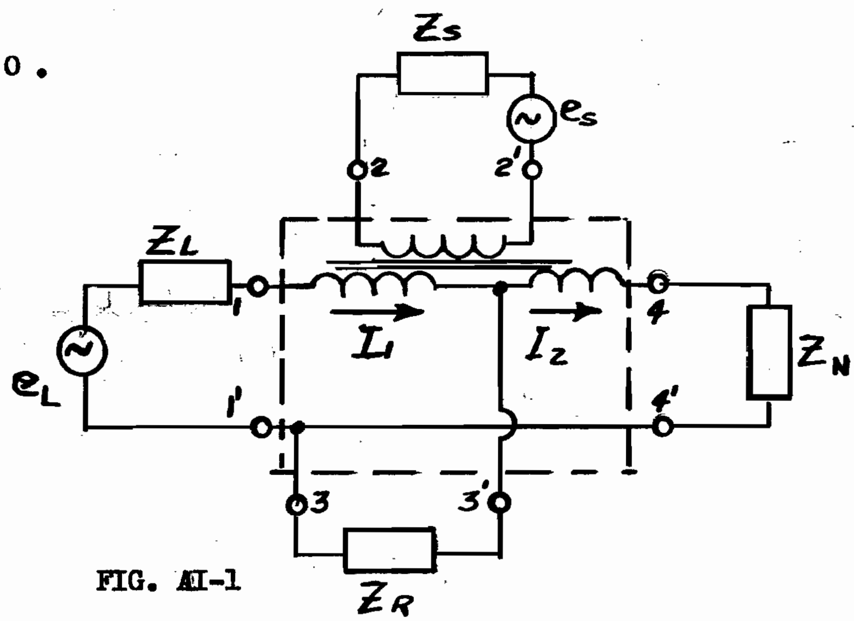
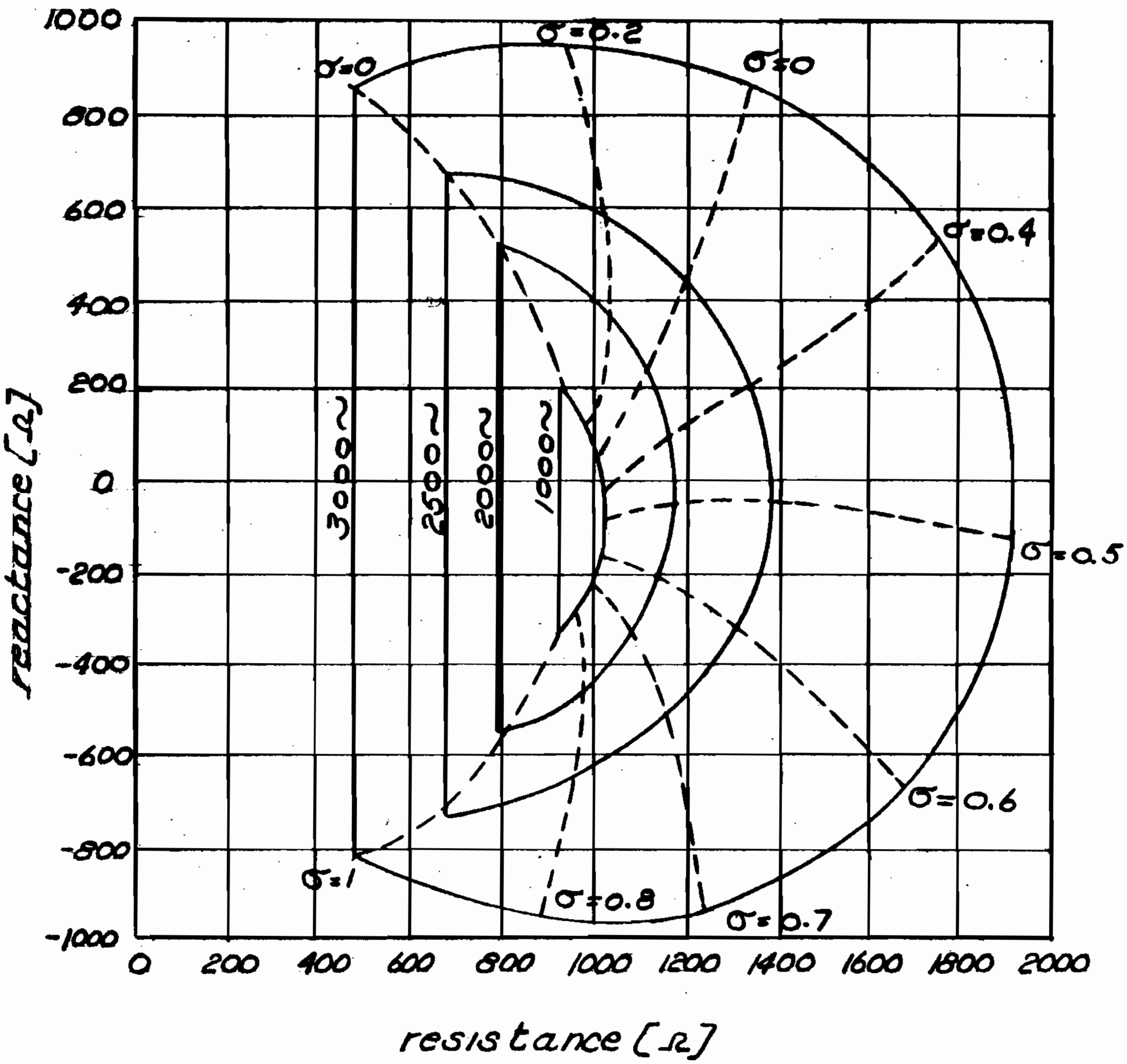


FIG. AI-1

FREQUENCY [cps]	PRIMARY CONSTANTS $L = .777\ mH$ $C = .084\ \mu F$ $R\ [\Omega]$	$G\ [\mu\mho]$	SECONDARY CONSTANTS PROPAGATION CONST. (P) ATTENUATION $Re\,P = \alpha\ [db]$	PHASE SHIFT $Arg\,P = \beta\ [rad]$	CHARACTERISTIC IMP. (Z_o) $Re(Z_o) = r\ [\Omega]$	$Im(Z_o) = x\ [\Omega]$
50	85.0	0.05	.2908	.0335	1272	−1266
100	85.0	0.12	.4107	.0474	901	− 894
200	85.0	0.22	.5791	.0673	639	− 630
500	85.1	0.47	.9076	.1073	470	− 395
1000	85.2	1.30	1.266	.1540	292	− 275
2000	85.4	3.30	1.743	.2242	213	− 189
3000	85.6	5.90	2.078	.2827	179	− 150

PRIMARY AND SECONDARY CONSTANTS
OF A UNIFORM TRANSMISSION LINE
(SAMPLE)

APPENDIX III



CHARACTERISTIC IMPEDANCE CHART
FOR A LOADED LINE
(SAMPLE)

APPENDIX IV.

Reflection and insertion gain functions for negative impedance repeaters.

1. Definitions

The lattice structure of Fig. AIV-1 illustrates the equivalent lattice network of a negative impedance repeater. Due to the fact that all existing negative impedance repeaters are symmetrical structures, the lattice network representation is not restricting universality.

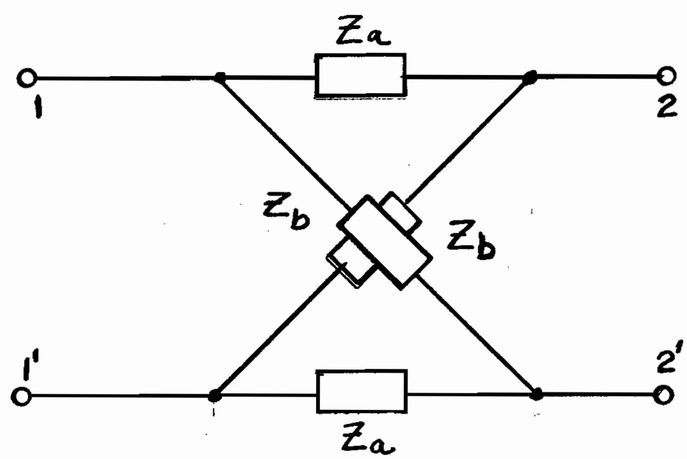


FIG. AIV-1

The image impedance of this network is known to be:

$$Z_I = \sqrt{Z_a \cdot Z_b} \qquad \text{(AIV-1)}$$

The "bridge ratio" (N) is defined as:

$$N = \sqrt{\frac{Z_a}{Z_b}} \qquad \text{(AIV-2)}$$

It will be found convenient to use the "T-equivalent" of the lattice network in our calculations, which is shown on Fig. AIV-2.

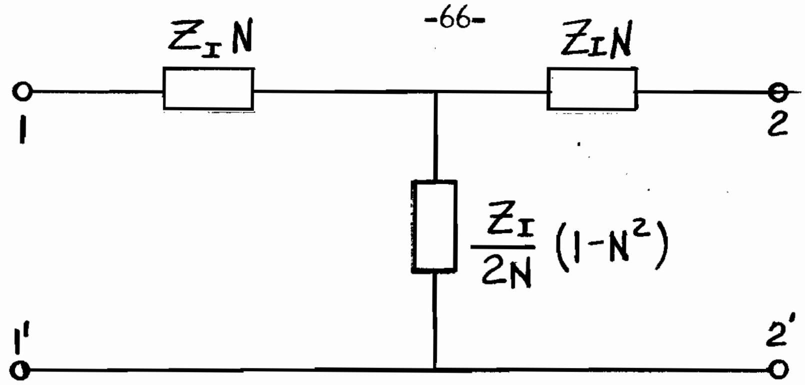


FIG. AIV-2

2. <u>Input impedance of the terminated lattice.</u>

The input impedance of the network illustrated on Fig. AIV-2 if terminated by an impedance "Z" at terminals (2-2') is:

$$Z_i = NZ_I + \frac{(NZ_I + Z)\ \frac{Z_I}{2N}\ (1-N^2)}{NZ_I + Z + \frac{Z_I}{2N}\ (1-N^2)} \qquad \text{(AIV-3)}$$

and from this

$$Z_i = Z_I \frac{2NZ_I + (N^2 + 1)\ Z}{(N^2 + 1)Z_I + 2NZ} \qquad \text{(AIV-4)}$$

can be obtained.

3. <u>Reflection</u>

As defined by equation (3-9) the reflection is:

$$R = \frac{Z_i - Z_{of}}{Z_i + Z_{ob}} \qquad \text{(AIV-5)}$$

Assuming that the repeater is terminated by identical impedances (Z) at each port;

$$Z_{of} = Z_{ob} = Z \tag{AIV-6}$$

This is no restriction of generality, because this kind of repeater has to be inserted in symmetrical structures.

Therefore

$$R = \frac{Z_i - Z}{Z_i + Z} \tag{AIV-7}$$

Substituting the value of Z_i from equ. (AIV-4) into (AIV-7) gives:

$$R = \frac{Z_I - Z}{Z_I + Z} \quad \frac{(Z + Z_I)^2 N}{(Z + Z_I)^2 N + ZZ_I(1 - N)^2}$$

(AIV-8)

This can also be written as:

$$R = \frac{Z_I - Z}{Z_I + Z} \left(1 - \frac{(1 - N)^2}{(1 + N)^2} \frac{ZZ_I (1 + N)^2}{(Z + Z_I)^2 N + ZZ_I(1 - N)^2} \right)$$

AIV-9)

According to equation (3-16) the term $\frac{Z_I - Z}{Z_I + Z}$ is the matching factor (M). With this notation the following relation holds:

$$\frac{ZZ_I}{(Z_I + Z)^2} = \frac{1}{4} (1 - M^2)$$

(AIV-10)

Substituting (M) into (AIV-9), and also making use of (AIV-10) the following relation for the reflection is obtained:

$$R = M \left(1 - \frac{(1 - N)^2}{(1 + N)^2} \quad \frac{1 - M^2}{1 + M^2 \quad \frac{(1 - N)^2}{(1 + N)^2}} \right)$$

(AIV-11)

As will be shown later (see equ. AIV-21) the nominal insertion gain of the network as defined by equation (3-18) is:

$$A_n^2 = \frac{(1 - N)^2}{(1 + N)^2} \qquad \text{(AIV-12)}$$

And the final form of the expression for the reflection is:

$$R = M \left(1 - A_n^2 \frac{1 - M^2}{1 - M^2 A_n^2} \right) \qquad \text{(AIV-13)}$$

This expression can be simplified if it is assumed that:

$$M \ll 1 \qquad \text{and} \qquad A_n \gg 1 \qquad \text{but} \qquad |MA_n| < 1$$

and obtain an approximate formula:

$$R \approx \frac{-MA_n^2}{1 - (MA_n)^2}$$

(AIV-14)

4. <u>Insertion Gain.</u>

The insertion voltage gain is defined by equation (3-14). Using the notations shown on Fig. AIV-3, the insertion gain is:

$$A = \frac{E_2}{\frac{1}{2}e} \tag{AIV-15}$$

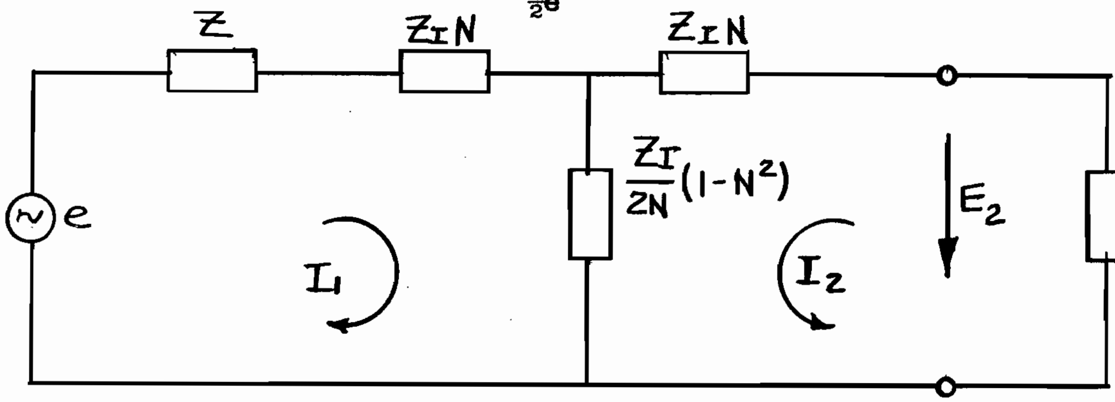


FIG. AIV-3

and it can be easily shown that

$$A = 2Z \frac{b}{a^2 - b^2} \tag{AIV-16}$$

where

$$a = Z + Z_I N + \frac{Z_I}{2N}(1 - N^2) \tag{AIV-17}$$

and

$$b = \frac{Z_I}{2N}(1 - N^2) \tag{AIV-18}$$

Substituting (AIV-17 & 18) into (AIV-16) and performing the possible simplifications the following expression is obtained:

$$A = \frac{1 - N}{1 + N} \quad \frac{ZZ_I (1 + N)^2}{(Z + Z_I)^2 N + ZZ_I(1 - N)^2} \tag{AIV-19}$$

This last equation can be rewritten by the same reasoning as applied in case of equation (AIV-9) and yields

$$A = \frac{1 - N}{1 + N} \quad \frac{1 - M^2}{1 - \left(M \frac{1 - N}{1 + N}\right)^2} \qquad \text{(AIV-20)}$$

From this it can be seen that if $M = 0$ then the nominal gain is

$$A_n = \frac{1 - N}{1 + N} \qquad \text{(AIV-21)}$$

and with this, the insertion gain can be written as:

$$A = A_n \frac{1 - M^2}{1 - (MA_n)^2} \qquad \text{(AIV-22)}$$

An approximate formula can be used in most cases of practical importance, (assuming that $M^2 \ll 1$)

$$A = \frac{A_n}{1 - (MA_n)^2} \qquad \text{(AIV-23)}$$

It can be seen from equation (AIV-21) that if the absolute value of the nominal insertion gain is to be larger than unity then

$\text{Re } N < 0$ must be satisfied. (AIV-24)

From equation (AIV-2) it follows that this condition is satisfied if:

$$\left|(\text{Arg } Z_a - \text{Arg } Z_b)\right| > \pi \qquad \text{(AIV-25)}$$

and hence at least one of the impedances Z_a or Z_b must have negative real part. These types of impedances are called negative impedances.

APPENDIX V.

Reflection and insertion gain functions for Bridge Compensated Repeaters.

1. Definitions

The equivalent circuit of the bridge compensated repeater arrangement is shown on Fig. AV-1. In order to obtain this circuit it is assumed that the output impedances of amplifiers A1 & A2 (Fig. 1 - 7) are small compared to the values of Z_1 & Z_2 respectively.

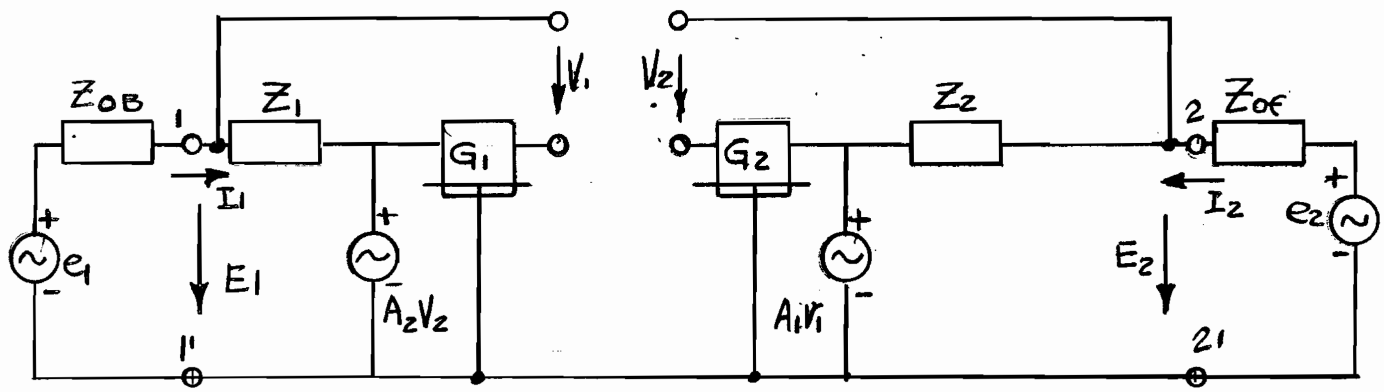


FIG. AV-1.

In the following treatment it will be found convenient to introduce the following notations:

$$A_m = \sqrt{A_1 A_2} \quad \text{(mean value of amplification)} \qquad \text{(AV-1)}$$

$$F_1 = \frac{Z_{ob}}{Z_{ob} + Z_1} \quad \text{(input attenuation at terminals 1-1')} \qquad \text{(AV-2)}$$

$$F_2 = \frac{Z_{of}}{Z_{of} + Z_2} \quad \text{(Input attenuation at terminals 2-2')} \qquad \text{(AV-3)}$$

$$a_1 = \frac{Z_1}{Z_{ob} + Z_1} \quad \text{(Output attenuation at terminals 1-1')} \qquad \text{(AV-4)}$$

$$a_2 = \frac{Z_2}{Z_{of} + Z_2} \quad \text{(Output attenuation at terminals 2-2')} \qquad \text{(AV-5)}$$

$$C_1 = \frac{G_1}{F_1} \quad \text{(Compensation ratio at terminals 1-1')} \qquad \text{(AV-6)}$$

$$C_2 = \frac{G_2}{F_2} \quad \text{(Compensation ratio at terminals 2-2')} \qquad \text{(AV-7)}$$

$$r_1 = \frac{Z_1 - Z_{of}}{Z_1 + Z_{ob}} \quad \text{(Reflection at terminals 1-1', if all active elements in the repeater are disabled. } A_1 = A_2 = 0 \text{)} \qquad \text{(AV-8)}$$

$$r_2 = \frac{Z_2 - Z_{ob}}{Z_2 + Z_{of}} \quad \text{(Same as } r_1 \text{ for terminals 2-2')} \qquad \text{(AV-9)}$$

2. Input Impedance of the Repeater

In the following the equation for the input impedance at terminals 1-1' will be derived. Using the symbols of Fig. AV-1:

$$E_1 = I_1 Z_1 + A_2 v_2 \qquad \text{(AV-10)}$$

$$v_1 = E_1 - A_2 G_1 v_2 \qquad \text{(AV-11)}$$

$$v_2 = A_1 v_1 (F_2 - G_2) \qquad \text{(AV-12)}$$

From these the following expression for the input impedance in forward direction (Z_{if}) is obtained:

$$Z_{if} = \frac{E_1}{I_1} = Z_1 \frac{1 + A_m^2 G_1 (F_2 - G_2)}{1 - A_m^2 (F_2 - G_2)(1 - G_1)} \qquad \text{(AV-13)}$$

The expression for the input impedance in reverse direction can be derived in the same manner:

$$Z_{ib} = Z_2 \frac{1 + A_m^2 G_2 (F_1 - G_1)}{1 - A_m^2 (F_1 - G_1)(1 - G_2)} \qquad \text{(AV-14)}$$

3. <u>Insertion gain.</u>

According to the definition of (3-14) the insertion voltage gain in forward direction (A_f) is given as: (see Fig. AV-1)

$$A_f = \frac{E_2}{\dfrac{Z_{of}}{Z_{of} + Z_{ob}} e_1} = \frac{Z_{of} + Z_{ob}}{Z_{of}} \frac{E_2}{e_1} \qquad \text{(AV-15)}$$

For the system considered the following relations hold:

$$\frac{E_1 - e}{Z_{ob}} + \frac{E_1 - A_2 v_2}{Z_1} = 0 \qquad \text{(AV-16)}$$

$$\frac{E_2}{Z_{of}} + \frac{E_2 - A_1 v_1}{Z_2} = 0 \qquad \text{(AV-17)}$$

$$v_1 = E_1 - A_2 v_2 G_1 \qquad \text{(AV-18)}$$

$$v_2 = E_2 - A_1 v_1 G_2 \qquad \text{(AV-19)}$$

From these one gets:

$$\frac{E_2}{e_1} = A_1F_1F_2 \frac{Z_1}{Z_{ob}} \frac{1}{1 - A_m^2(F_1 - G_1)(F_2 - G_2)}$$

(AV-20)

Substitute now (AV-20) into (AV-15) and also the value for F from equation (AV-2) to obtain for the forward insertion gain:

$$A_f = A_1F_2a_1 \frac{Z_{of} + Z_{ob}}{Z_{of}} \frac{1}{1 - A_m^2(F_1 - G_1)(F_2 - G_2)}$$

(AV-21)

and in similar manner it is possible to derive the formula for the "backward insertion gain" (A_b):

$$A_b = A_2F_1a_2 \frac{Z_{of} + Z_{ob}}{Z_{ob}} \frac{1}{1 - A_m^2(F_1 - G_1)(F_2 - G_2)}$$

(AV-22)

In order to obtain a more meaningful form for these formulas consider the following:

From the definition of the nominal terminating impedances for a repeater (Z_{n1} & Z_{n2}), as given in section 3 the following defining equations can be obtained:

$$G_1 = \frac{Z_{n1}}{Z_{n1} + Z_1} \qquad \text{(AV-23)}$$

and

$$G_2 = \frac{Z_{n2}}{Z_{n2} + Z_2} \qquad \text{(AV-24)}$$

It is obvious that in this case:

$$F_1 - G_1 = 0 \quad \text{and also} \quad F_2 - G_2 = 0 \qquad \text{(AV-25)}$$

which will produce a finite gain for any finite values of A_1 and A_2, as indicated by equations (AV-21 & 22), thus producing a stable system. It is also evident from equations (AV-13 & 14) that in this case:

$$Z_{if} = Z_1 \quad \text{and} \quad Z_{ib} = Z_2 \qquad \text{(AV-26)}$$

and therefore the reflection at the repeater terminals is independent of the insertion gain of the arrangement. Thus Z_{n1} and Z_{n2} are really the nominal terminating impedances of the system.

In order to obtain the nominal insertion gain values substitute

$$Z_{ob} = Z_{n1} \quad \text{and} \quad Z_{of} = Z_{n2} \qquad \text{(AV-27)}$$

into equations (AV-21 & 22). With this substitution the nominal forward insertion gain is:

$$A_{fn} = A_1 \frac{Z_1(Z_{n1} + Z_{n2})}{(Z_{n1} + Z_1)\ (Z_{n2} + Z_2)} \qquad \text{(AV-28)}$$

and the corresponding expression for the nominal backward insertion gain is:

$$A_{bn} = A_2 \frac{Z_2(Z_{n1} + Z_{n2})}{(Z_{n1} + Z_1)(Z_{n2} + Z_2)} \qquad \text{(AV-29)}$$

Using the above relations, the term $A_m^2(F_1 - G_1)(F_2 - G_2)$ in the denominator of equations (AV-21 & 22) can be rewritten as follows:

$$A_m^2(F_1 - G_1)(F_2 - G_2) = A_{fn}A_{bn}\, M_1 M_2\, f_{c1} f_{c2} f_s$$

(AV-30)

where the matching factors M_1 and M_2 are defined in equations (3-16 & 17) in general and in the particular case considered are given as:

$$M_1 = \frac{Z_{n1} - Z_{ob}}{Z_{n1} + Z_{ob}} \qquad \text{(AV-31)}$$

and

$$M_2 = \frac{Z_{n2} - Z_{of}}{Z_{n2} + Z_{of}} \qquad \text{(AV-32)}$$

Furthermore f_{c1} and f_{c2} are factors indicating the degree of deviation of the values of Z_1 and Z_2 from their corresponding Z_{n1} and Z_{n2}. As can be seen from the formulas given below, these factors are rather insensitive to this deviation, because in any practical case the correspondence between Z_{ob} and Z_{n1} on one hand and between Z_{of} and Z_{n2} on the other is fairly good, in which case these factors have a value of unity not dependent on the values of Z_1 and Z_2. The expressions for f_{c1} and f_{c2} are as follows:

$$f_{c1} = \frac{(Z_{ob} + Z_{n1})(Z_1 + Z_{n1})}{(Z_{ob} + Z_1)\, 2Z_{n1}} \qquad \text{(AV-33)}$$

and

$$f_{c2} = \frac{(Z_{of} + Z_{n2})(Z_2 + Z_{n2})}{(Z_{of} + Z_2)\, 2Z_{n2}} \qquad \text{(AV-34)}$$

The last factor in equation (AV-30) is f_s which is an indicator of symmetry. If $Z_{of} = Z_{ob}$ and thus Z_{n1} is equal to Z_{n2}, this factor has a value of unity. The expression for f_s is:

$$f_s = \frac{4Z_{n1}Z_{n2}}{(Z_{n1} + Z_{n2})^2} \qquad \text{(AV-35)}$$

By similar reasoning it is possible to write the miltiplying factor $\alpha = A_1 a_1 F_2 \dfrac{Z_{of} + Z_{ob}}{Z_{of}}$ in equation (AV-21) as:

$$\alpha = A_{fn}\, f_{c1} f_{c2} f_s\, (1 - M_1 M_2 + f_{ss}) \qquad \text{(AV-36)}$$

where

$$f_{ss} = \frac{(Z_{ob} - Z_{of})(Z_{n1} - Z_{n2})}{(Z_{ob} + Z_{n1})(Z_{of} + Z_{n2})} \qquad \text{(AV-37)}$$

which is zero if the system is symmetrical.

In order to obtain equation (AV-36) the following relation was used:

$$1 - M_1 M_2 = \frac{2Z_{n1}Z_{of} + 2Z_{n2}Z_{ob}}{(Z_{n1} + Z_{ob})(Z_{n2} + Z_{of})} \qquad \text{(AV-38)}$$

Substituting equations (AV-30) and (AV-36) into (AV-21) the following expression results for the forward insertion gain:

$$A_f = A_{fn}\,(f_{c1} f_{c2} f_s)\, \frac{1 - M_1 M_2 + f_{ss}}{1 - A_{fn} A_{bn} M_1 M_2 (f_{c1} f_{c2} f_s)}$$

(AV-39)

A good approximation to this formula can be obtained by assuming that

$$M_1M_2 \ll 1 \qquad \text{and that} \quad f_{ss} \ll 1 \tag{AV-40}$$

to get:

$$A_f \approx \frac{A_{fn}(f_{c1}f_{c2}f_s)}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \tag{AV-41}$$

a similar expression for the backward insertion gain is:

$$A_b = A_{bn}(f_{c1}f_{c2}fs) \frac{1 - M_1M_2 + f_{ss}}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \tag{AV-42}$$

and the approximate formula is:

$$A_b \approx \frac{A_{bn}(f_{c1}f_{c2}f_s)}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \tag{AV-43}$$

4. <u>Reflection</u>

According to the definition of equation (3-9) the reflection at the (1-1') terminal pair (R_f) is given as:

$$R_f = \frac{Z_{if} - Z_{of}}{Z_{if} + Z_{ob}} \tag{AV-44}$$

Substitution of the expression for Z_{if} (AV-13) into this relation gives:

$$R_f = \frac{Z_1 - Z_{of}}{Z_1 + Z_{of}} - A_1A_2 \frac{Z_1(Z_{of}+Z_{ob})}{(Z_{ob}+Z_1)^2} \frac{(F_2 - G_2)}{1-A_1A_2(F_1-G_1)(F_2-G_2)} \tag{AV-45}$$

By using equations (AV-28, 29, 30, 31, 32, 33, 34, and 35) this can be rewritten as:

$$R_f = r_1 - A_{fn}A_{bn}M_2(f_{c1}^2 f_{c2} f_s f_s') \frac{1 - M_1^2}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)}$$

(AV-46)

where r_1 is defined by equation (AV-8) and

$$f_s' = \frac{Z_{of} + Z_{ob}}{2Z_{ob}} \qquad \text{(AV-47)}$$

an approximate formula if $M_1^2 \ll 1$ is:

$$R_f \approx r_1 - \frac{A_{fn}A_{bn}M_2(f_{c1}^2 f_{c2} f_s f_s')}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \qquad \text{(AV-48)}$$

Expressions for the reflection at terminals (2-2') can be derived in a similar manner and result in the following expressions:

$$R_b = r_2 - A_{fn}A_{bn}M_1(f_{c1}f_{c2}^2 f_s f_s'') \frac{1 - M_2^2}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)}$$

(AV-49)

where

$$f_s'' = \frac{Z_{of} + Z_{ob}}{2Z_{of}} \qquad \text{(AV-50)}$$

If it is assumed that $M_2 \ll 1$ the following simplified formula results:

$$R_b \approx r_2 - \frac{A_{fn}A_{bn}M_1(f_{c1}f_{c2}^2 f_s f_s'')}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \qquad \text{(AV-51)}$$

Expressions (AV-48) & (AV-51) can be further simplified if only that portion of the reflection which is due to the active part of the network, and is dependent on the insertion gain, is considered. In this case:

$$R_f \approx \frac{A_{fn}A_{bn}M_2(f_{c1}^2 f_{c2} f_s f_s')}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \qquad \text{(AV-52)}$$

and

$$R_f \approx \frac{A_{fn}A_{bn}M_1(f_{c1} f_{c2}^2 f_s f_s'')}{1 - A_{fn}A_{bn}M_1M_2(f_{c1}f_{c2}f_s)} \qquad \text{(AV-53)}$$

APPENDIX VI.

Relation of the parameters of a hybrid and a bridge compensated repeater.

In this section it will be shown that a hybrid repeater can be transformed into the equivalent circuit of the bridge compensated repeater, and therefore the results of appendix V can be used in evaluating the properties of this arrangement.

The aim of the following discussion is to establish the relations required to transform the circuit parameters of a hybrid repeater into the parameters of the equivalent circuit of the bridge compensated structure.

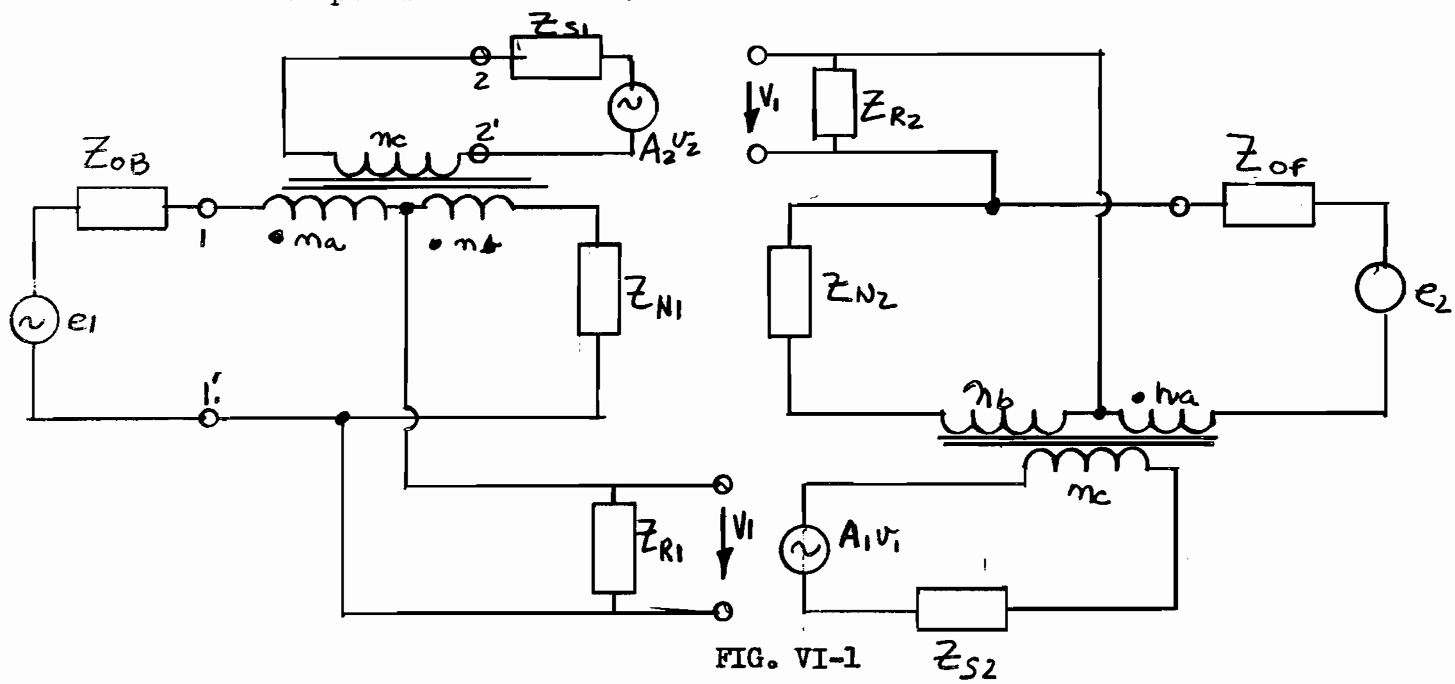


FIG. VI-1

Fig. AV1-1 illustrates the schematic of a hybrid repeater and gives the notation used in the following discussion.

This arrangement can be readily converted into the circuit of Fig. AV1-2.

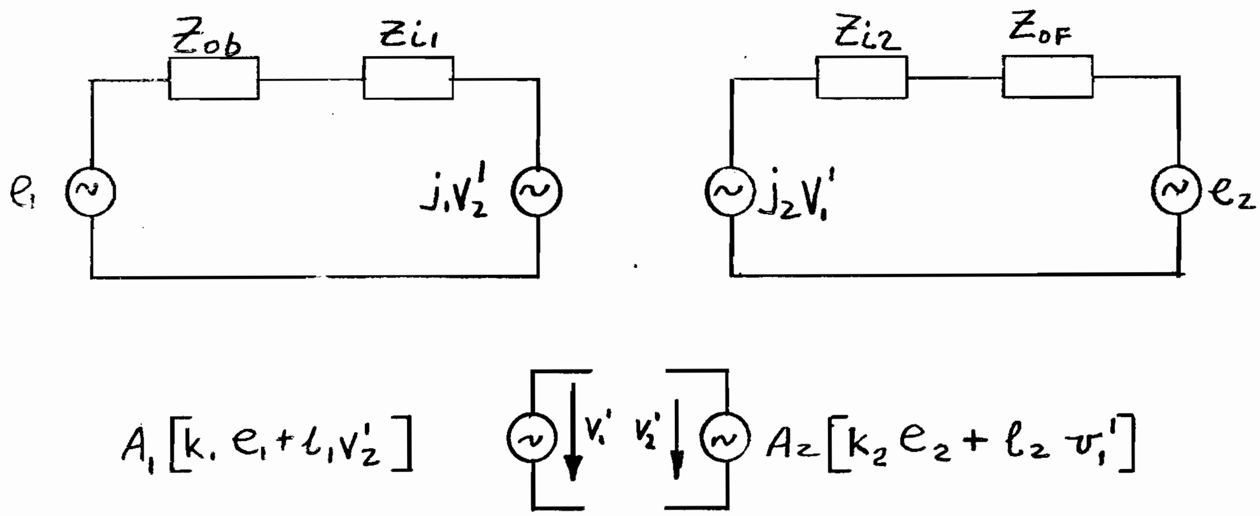


FIG. AV1-2.

Assuming, that the hybrid transformer is symmetrical, i.e. $n_a = n_b$, the following expressions can be derived for j_1, k_1, l_1 and Z_{i1}: (These relations are given without proof. Calculations follow conventional methods [2] but are too lengthy to be reproduced here.)

If:
$$Z_s' = \frac{n_a^2}{n_c^2} Z_s \qquad \text{(AVI-1)}$$

then:
$$Z_{i1} = \frac{4Z_{s1}'Z_{r1} + Z_{n1}(Z_{r1} + Z_{s1}')}{Z_{s1}' + Z_{r1} + Z_{n1}} \qquad \text{(AVI-2)}$$

$$j_1 = \frac{n_a}{n_c} \quad \frac{2Z_{r1} + Z_{n1}}{Z_{s1}' + Z_{r1} + Z_{n1}} \qquad \text{(AVI-3)}$$

$$k_1 = Z_{r1} \frac{2Z'_{s1} + Z_{n1}}{Z'_{s1}(Z_{ob} + 4Z_{r1} + Z_{n1}) + Z_{ob}(Z_{r1} + Z_{n1}) + Z_{r1}Z_{n1}}$$

(AVI-4)

$$l_1 = Z_{r1}\frac{n_a}{n_b} \frac{Z_{ob} - Z_{n1}}{Z'_{s1}(Z_{ob} + 4Z_{r1} + Z_{n1}) + Z_{ob}(Z_{r1} + Z_{n1}) + Z_{r1}Z_{n1}}$$

(AVI-5)

Similar expressions can be obtained for Z_{i2}, j_2, k_2 and l_2.

The arrangement of Fig. AVI-2 can be converted into the circuit shown on Fig. AVI-3 using the following equivalences:

$$A_1 m_1 \frac{Z_{i1}}{Z_{ob} + Z_{i1}} = A_1 k_1 \qquad \text{(AVI-6)}$$

and

$$A_1 m_1 j_1 \left(\frac{Z_{ob}}{Z_{ob} + Z_{i1}} - G_1 \right) = A_1 l_1 \qquad \text{(AVI-7)}$$

which can be solved for the values of m_1 and G_1 to obtain:

$$m_1 = k_1 \frac{Z_{ob} + Z_{i1}}{Z_{i1}} \qquad \text{(AVI-8)}$$

and

$$G_1 = \frac{1}{Z_{ob} + Z_{i1}} \left(Z_{ob} - \frac{l_1}{j_1\, k_1} Z_{i1} \right)$$

(AVI-9)

and similarly for m_2 and G_2.

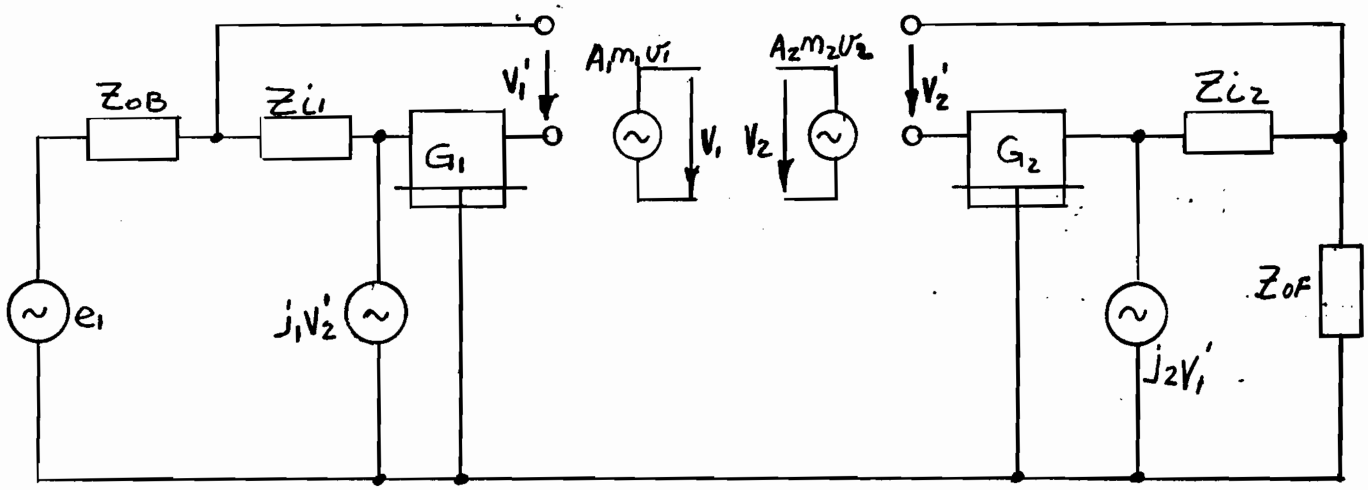


FIG. AVI-3

It is obvious that the circuit of Fig. AV I-3 can be redrawn as shown on Fig. AVI-4 which has the same structure as the equivalent circuit of the bridge compensated repeater.

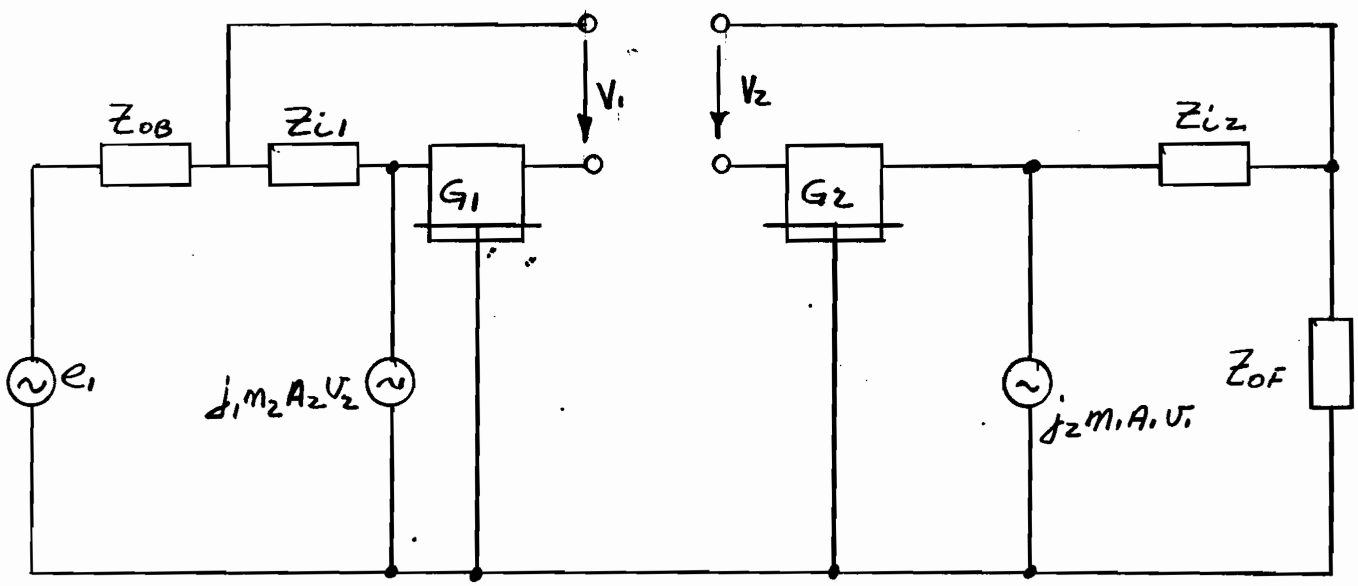


FIG. AVI-4

APPENDIX VII

Parameter Optimization by Hill-Climbing Technique.

In order to find the extrema of a function:

$$P = P\,(x_1,\ x_2 \ldots x_n) \qquad \text{(AVII-1)}$$

the following "n" equations have to be simultaneously satisfied:

$$\frac{\partial P}{\partial x_1} = 0 \quad \vdots \quad \frac{\partial P}{\partial x_n} = 0 \qquad \text{(AVII-2)}$$

Whether this equation system can be solved analytically depends on the form of function $P\,(x_1, \ldots x_n)$

In the case where (P) is defined by equation 7-17 or 7-18, an analytical solution of finding the extrema does not exist because (P) itself is not expressed in analytical form. ($g(\omega)$ and $h(\omega)$ are defined at a set of frequency points and are not given in analytical form).

One technique which can be used to obtain at least one extremum of this function is the so called hill climbing technique, which can be conveniently programmed on a digital computer.

The first step is to assume a set of initial conditions:

$$x_1 = X_1\ ;\ \ x_2 = X_2\ ;\ \ldots\ldots\ ;\ \ x_n = X_n \qquad \text{(AVII-3)}$$

and evaluate P_1 :

$$P_1 = P(X_1, X_2 \ldots X_n) \qquad \text{(AVII-4)}$$

then change some of the variables such that:

$$x_1 = X_1';\ \ x_2 = X_2';\ \ \ldots\ \ x_n = X_n \qquad \text{(AVII-5)}$$

and evaluate:

$$P_2 = P(X_1', \ldots X_n') \qquad \text{(AVII-6)}$$

Comparing the values of P_1 and P_2, the programme decides which set of parameters is more desirable.

Subsequent changes to parameters are made on the basis of the P values obtained in previous selections. This process of trying new sets of parameter values is continued, until all changes result in less desirable P values than the best found up to this point. In this case the set of $(x_1, x_2 \ldots x_n)$ values, which resulted in the most favourable P value is considered the location of the desired extremum.

It must be noted, that the extremum thus located is only one of the many possible extrema, and is not necessarily the best. In the case where it is suspected that more desirable extrema exists, it is possible to start with a set of different initial values and repeat the process.

There is no guarantee however that the best possible extremum is found unless a complete search of the whole domain of parameters is made, which in most cases is economically unfeasible.

From what was mentioned before it is evident, that the logic, according to which subsequent sets of parameter values are tried, is a very decisive factor in obtaining a fast converging and effective technique.

Many possible logical patterns (strategies) can be devised, but only two will be discussed here.

1. Gradient Strategies.

In this kind of technique, small incremental changes (probing steps) to $x_1, x_2 \ldots x_n$ are made to evaluate the approximate values of

$$\frac{\partial P}{\partial x_1} , \quad \frac{\partial P}{\partial x_2} \quad \ldots \quad \text{and} \quad \frac{\partial P}{\partial x_n}$$

at a particular point, then a larger change follows (adjusting step) in the direction of the gradient thus identified.

Dependent on the various modifications of this technique the size of the adjustment step can be made constant, or proportional to some power of the absolute value of the gradient.

Another option of this technique is that after the adjustment step has been successful, one can recalculate the gradient at the new point and proceed in this new direction, or can proceed in the direction used before, as long as P keeps on improving.

2. Univariate Strategies.

These techniques are based on changing only one variable at a time. They usually differ from gradient strategies by omitting probing steps in the sense that there are no incremental steps preceeding one or more larger steps.

An approach is to try to change a certain variable and make changes to it until better and better P values are obtained, than to try to change the next variable etc.

The amount by which the variables are changed can be fixed, or proportional to some power of the slope of P taken in the direction of the variable being changed. It can also be made to be a function of some other criterion.

These strategies were analyzed before deciding on the one which is used in this project by using a two variable performance criterion developed by Mr. J. Michaelski (McGill University) for the purpose of the study of various strategies.

The function in question is known to have one extremum only. It is defined by the equation:

$$P(x_1,x_2) = \sin^2 2\theta \; e^{-0.5\left(\frac{\sqrt{x_1^2+x_2^2} - \sqrt{\frac{1}{1.77\cos^2\theta + 6.25\sin^2\theta}}}{0.07}\right)^2} \qquad \text{(AVII-7)}$$

where

$$\theta = \tan^{-1}\frac{x_2}{x_1} \qquad \text{(AVII-8)}$$

The constant P contours are plotted on Fig. AVII-1, this plot was also provided by Mr. J. Michaelski (See page 119)

It was found that strategies based on the gradient methods are not as efficient in this particular case as univariate strategies, because of the essential number of probing steps required to implement an adjusting step. The ratio by which they reduced the necessary number of adjusting steps did not seem to warrant the additional number of steps required for probing.

Another factor which was considered in deciding against using a gradient strategie was that the particular P function to be optimized depends on the parameters in the form given in equation (7-5).

In this case if during the search, or because of the selection of initial values, we get the condition:

$$a_i = a_j \qquad \text{where} \quad i \neq j \tag{AVII-9}$$

then from this point on, due to the symmetry of P with respect to a_i and a_j

$$\frac{\partial P}{\partial a_i} = \frac{\partial P}{\partial a_j} \tag{AVII-10}$$

and the search will consider only equal values of a_i and a_j which first of all is not usually the optimum, second it will result in an unrealizable transfer function. Therefore, special measures should be provided in the case of gradient strategies to compensate for this behaviour which was not worth while in the view that these strategies did not promise much more efficient convergence, (actually from preliminary study they were worse than their univariate counterpart).

Univariate strategies on the other hand have the disadvantage, that they might proceed along a direction which has a very low gradient, while there is another direction available which would produce faster convergence.

To compensate for this fault, the strategy finally developed, limits the maximum number of successful steps in a certain direction as specified by a data card.

Another problem which arises is that the value of P is more sensitive to changes in one variable than in some others, therefore different values of adjustment steps have to be used in different directions. It must be kept in mind, however, that due to the complexity of the P function, the optimum values for these adjustment steps in various directions are not known. Therefore, some provision must be made to adjust these values on an adaptive basis by the computer programme.

To illustrate how the selected strategy works, the trajectories of the optimizing processes performed on the performance criterion specified in equation (AVII-7) are plotted on Fig's AVII-2 and AVII-3 for the two univariate strategies tested in this study.*

The plot of Fig. AVII-2 corresponds to the early version of the strategy, which did not have the adaptive property, while Fig. AVII-3 corresponds to the strategie which adapts the step size to the requirements in various directions.

It is worth while noting that after the initial "learning steps" the second strategy uses larger steps in the x_1 direction than in the x_2 direction, while the first strategy keeps the size of the steps in both directions always at the same value.

This feature, though it does not seem to be very important in the case of the particular performance criterion selected for this example, is very useful in the case when applied to the actual performance criterions of equations 7-17 & 7-18 because the sensitivities in various directions differ by a much larger factor in this case.

(*) See Figures on pages 120 and 121

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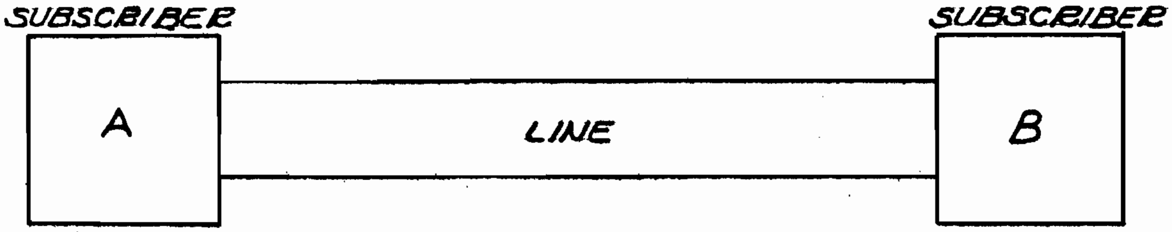
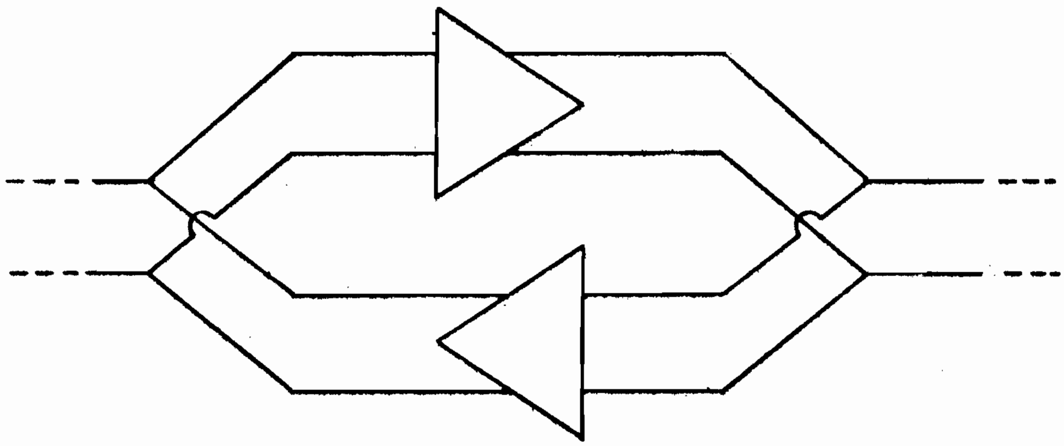


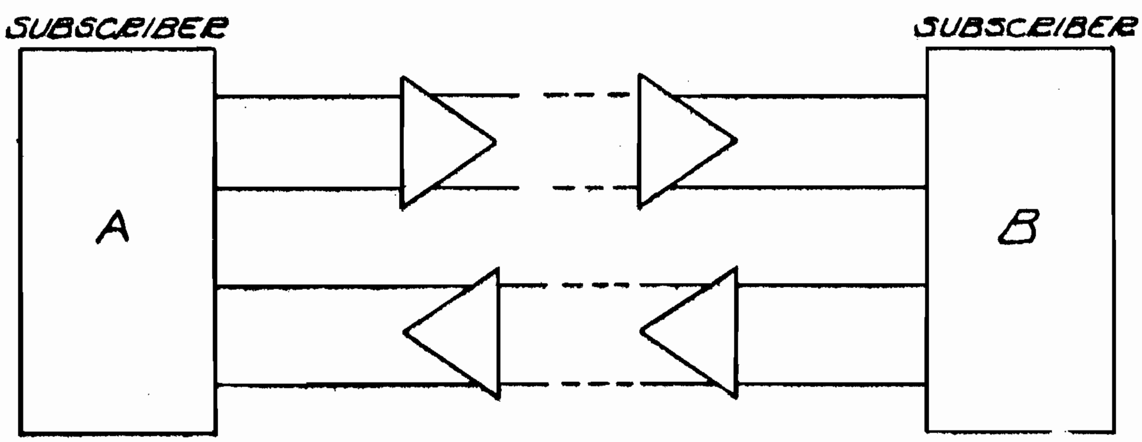
TWO-WIRE CONNECTION BETWEEN SUBSCRIBERS A & B

FIG. 1-1



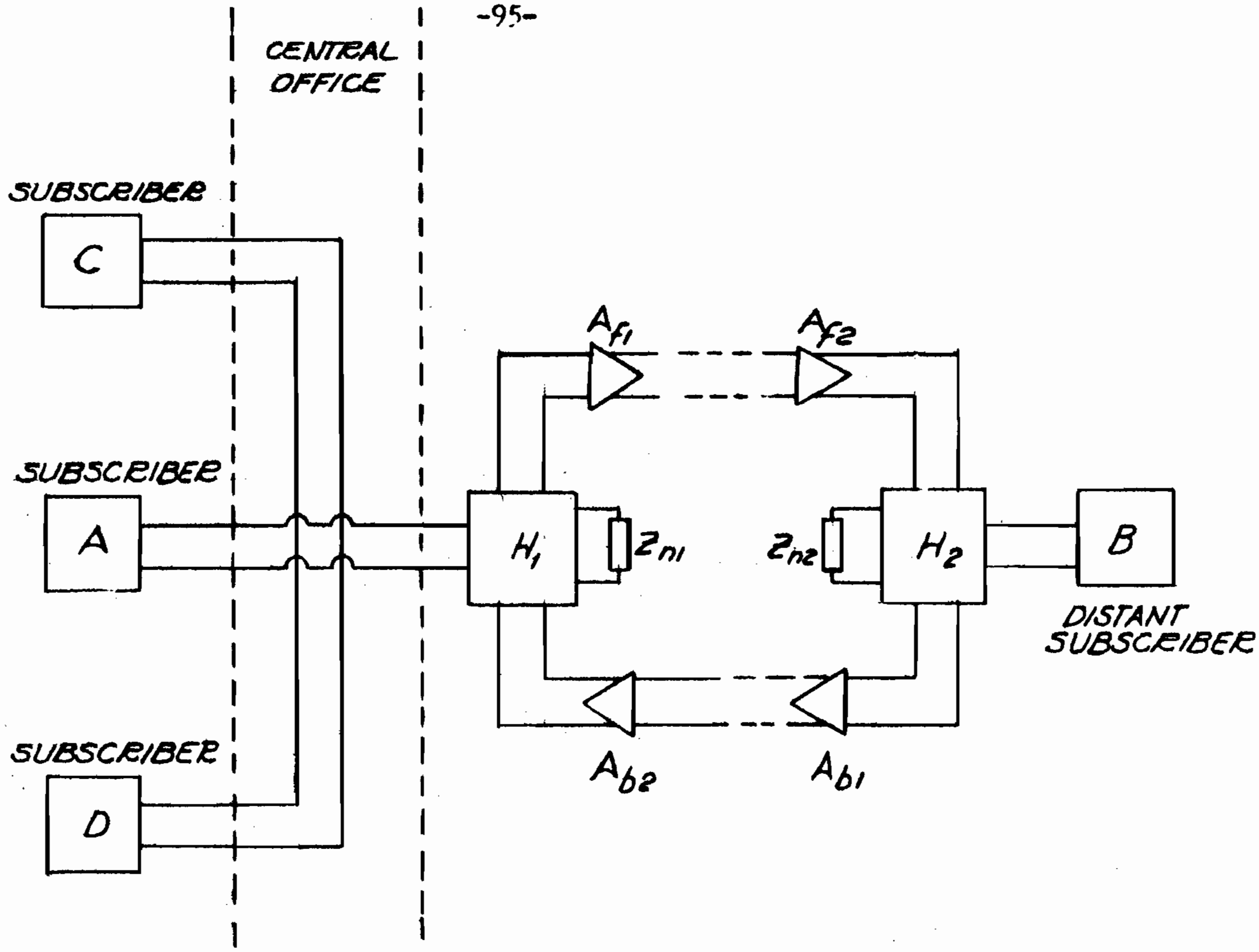
UNSTABLE CONFIGURATION OF TWO AMPLIFIERS

FIG. 1-2

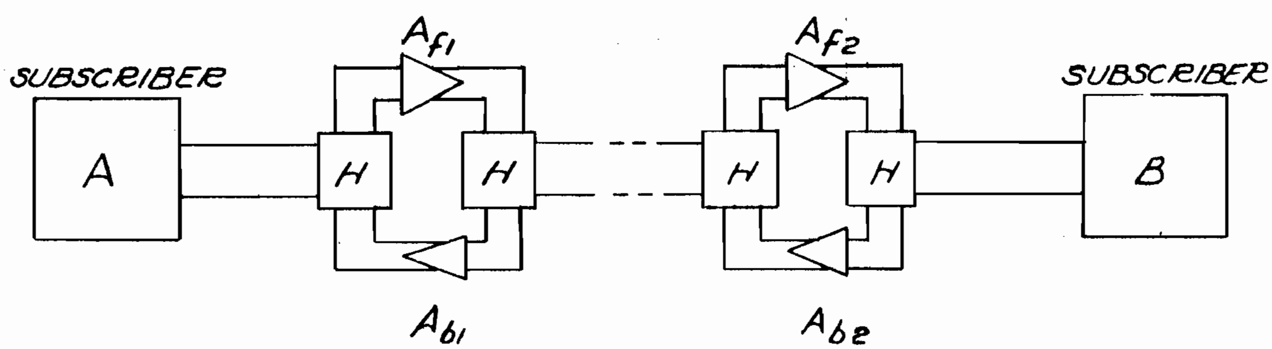


FULLY FOUR-WIRE SYSTEM

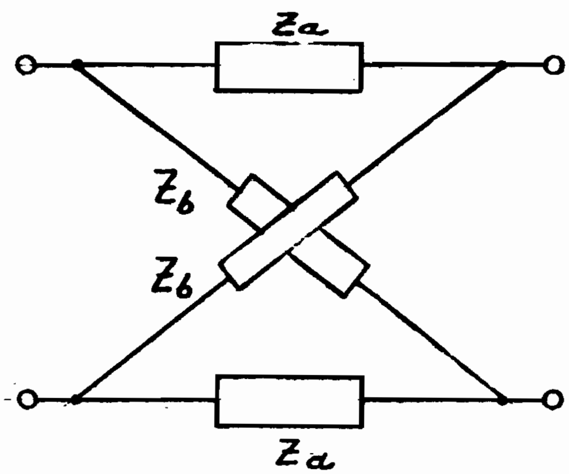
FIG. 1-3



FOUR-WIRE HYBRID SYSTEM
FIG. 1-4



TWO-WIRE HYBRID SYSTEM
FIG. 1-5



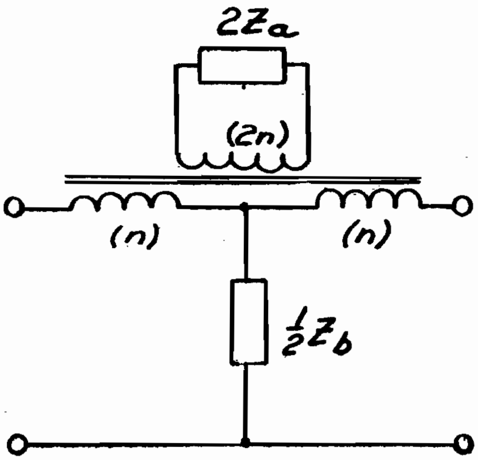
$$Z_I = \sqrt{Z_a Z_b}$$

$$G = 10 \log \frac{1-N}{1+N}$$

$$N = \sqrt{\frac{Z_a}{Z_b}}$$

LATTICE NETWORK REPRESENTATION OF A NEGATIVE IMPEDANCE REPEATER

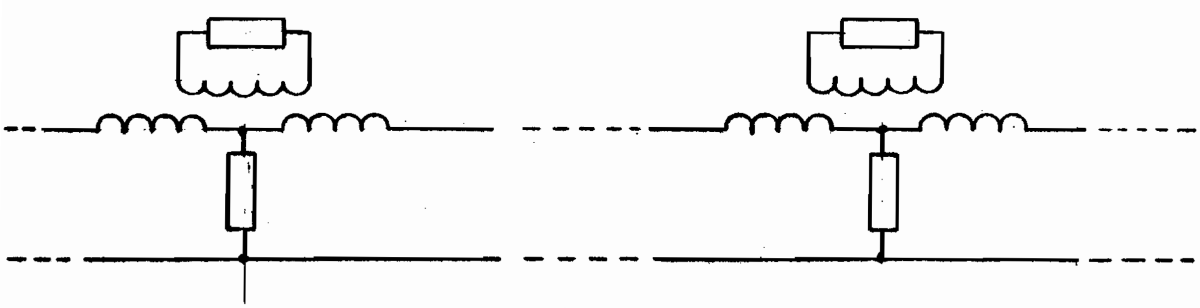
FIG. 1-6a



NOTE: THE SAME CONSIDERATIONS THAT ARE VALID FOR DERIVING THE RELATIONS FOR THE HYBRID TRANSFORMER IN APPENDIX VI CAN BE USED TO PROVE THAT THIS CIRCUIT IS EQUIVALENT TO THE LATTICE OF FIG. 1-6a.

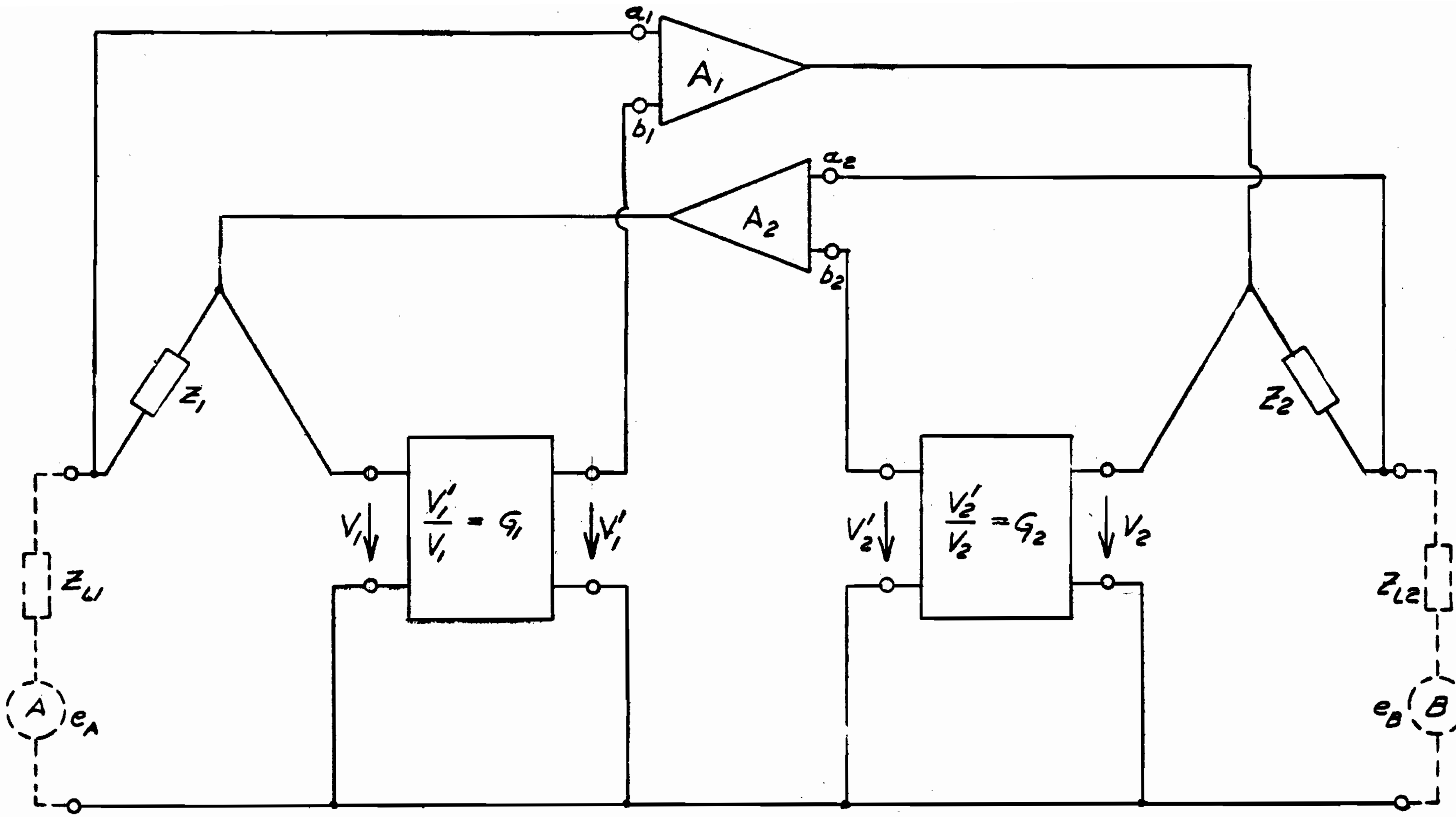
BRIDGED-T EQUIVALENT OF THE LATTICE OF FIG. 1-6a

FIG. 1-6b



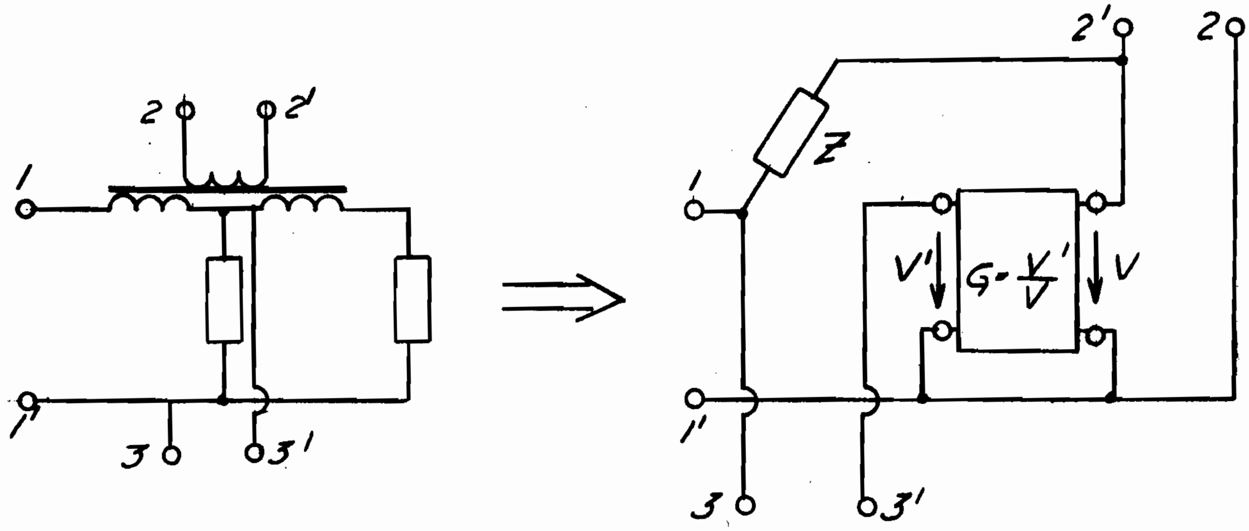
TRANSMISSION SYSTEM USING NEGATIVE IMPEDANCE REPEATERS

FIG. 1-6c



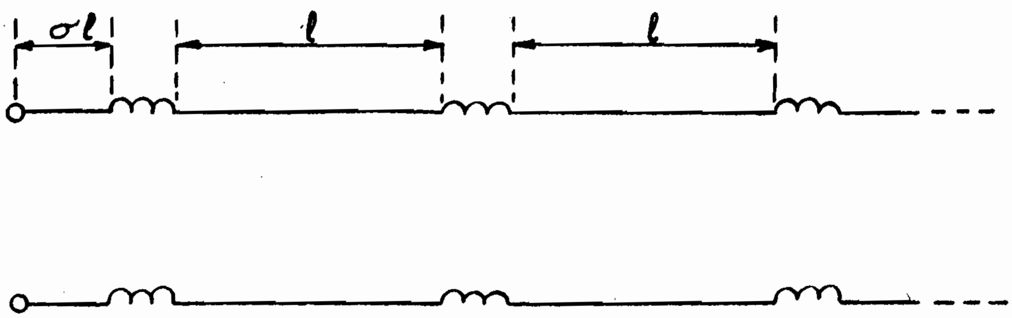
SCHEMATIC OF THE BRIDGE COMPENSATED REPEATER.

FIG. 1-7a



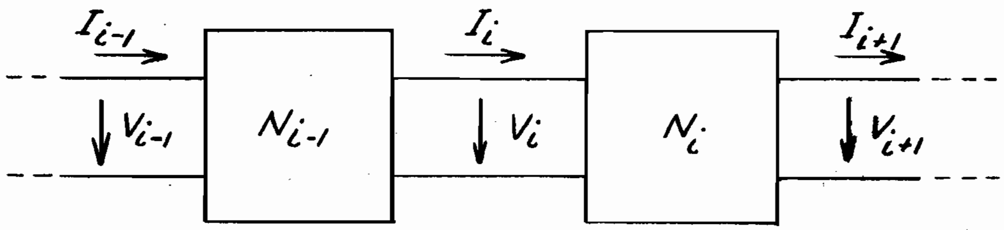
BRIDGE EQUIVALENT OF THE HYBRID TRANSFORMER USED IN THE BRIDGE COMPENSATED REPEATER

FIG. 1-7b



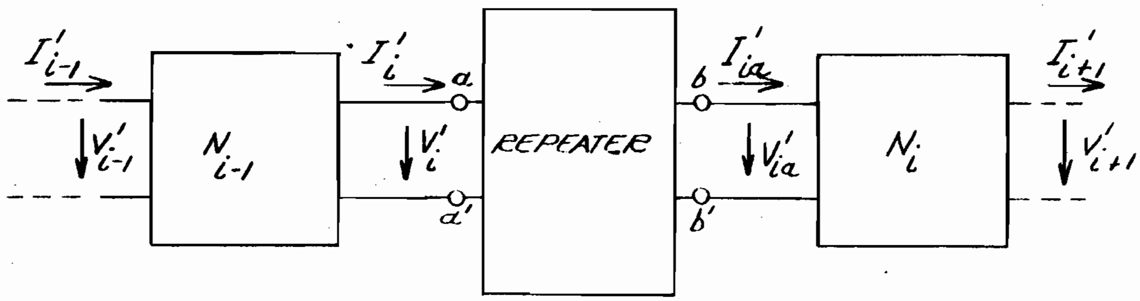
LOADED LINE TERMINATED IN A „σ" END SECTION

FIG. 2-1



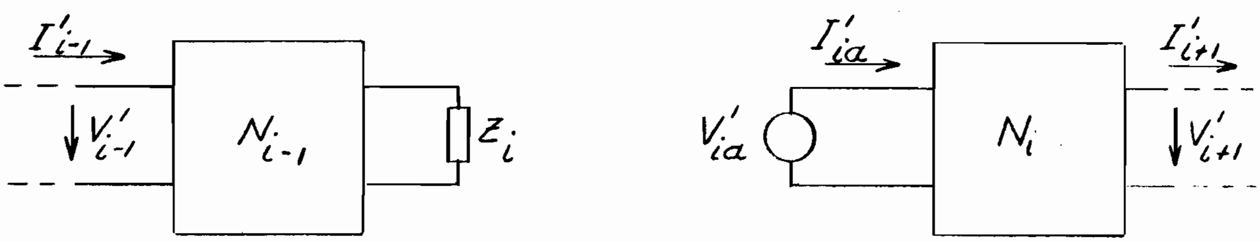
TRANSMISSION LINE REPRESENTED AS THE CASCADE CONNECTION OF IDENTICAL TWO-PORT NETWORKS

FIG. 3-1



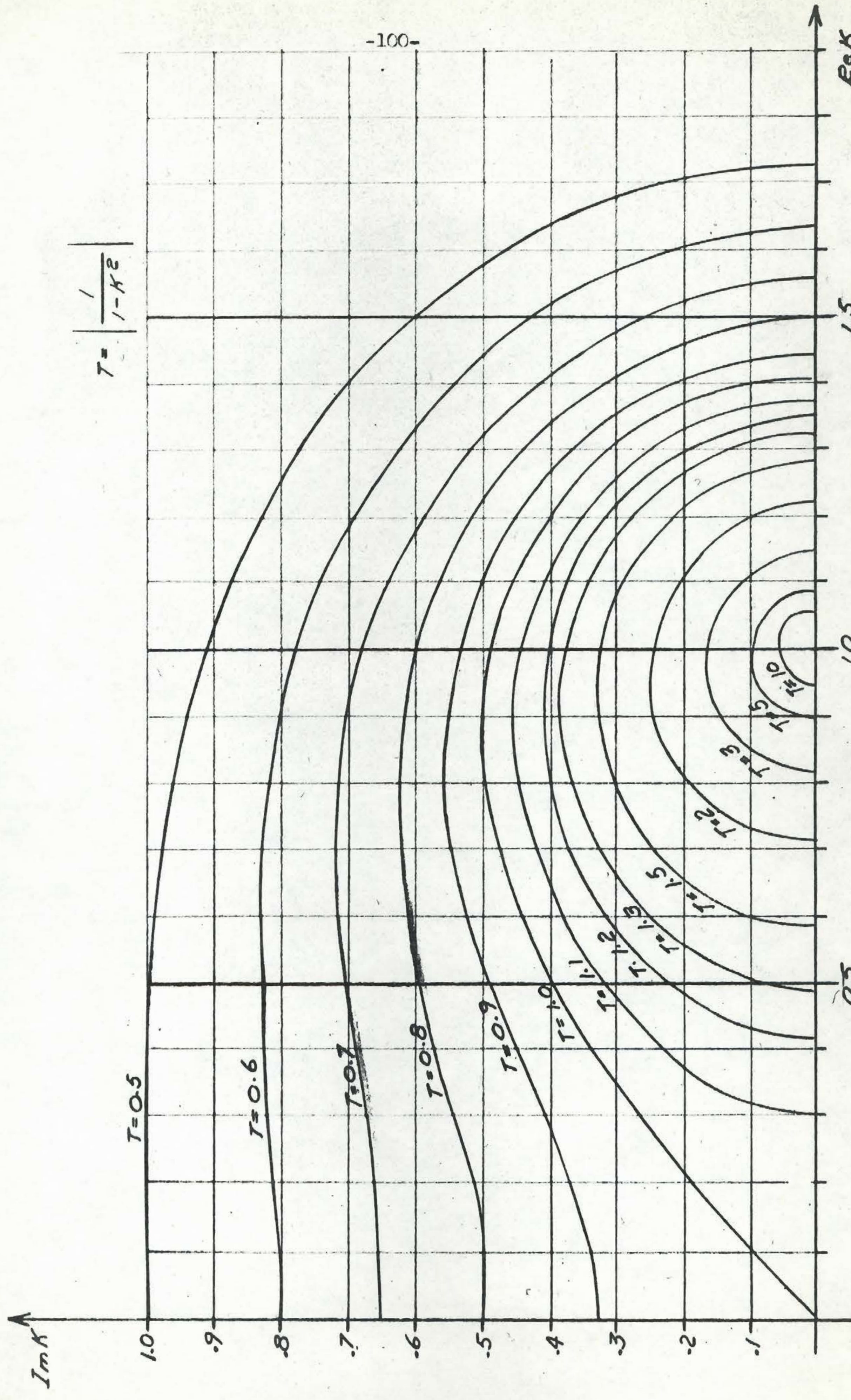
REPEATER INSERTED IN THE TRANSMISSION LINE OF FIG. 3-1

FIG. 3-2a



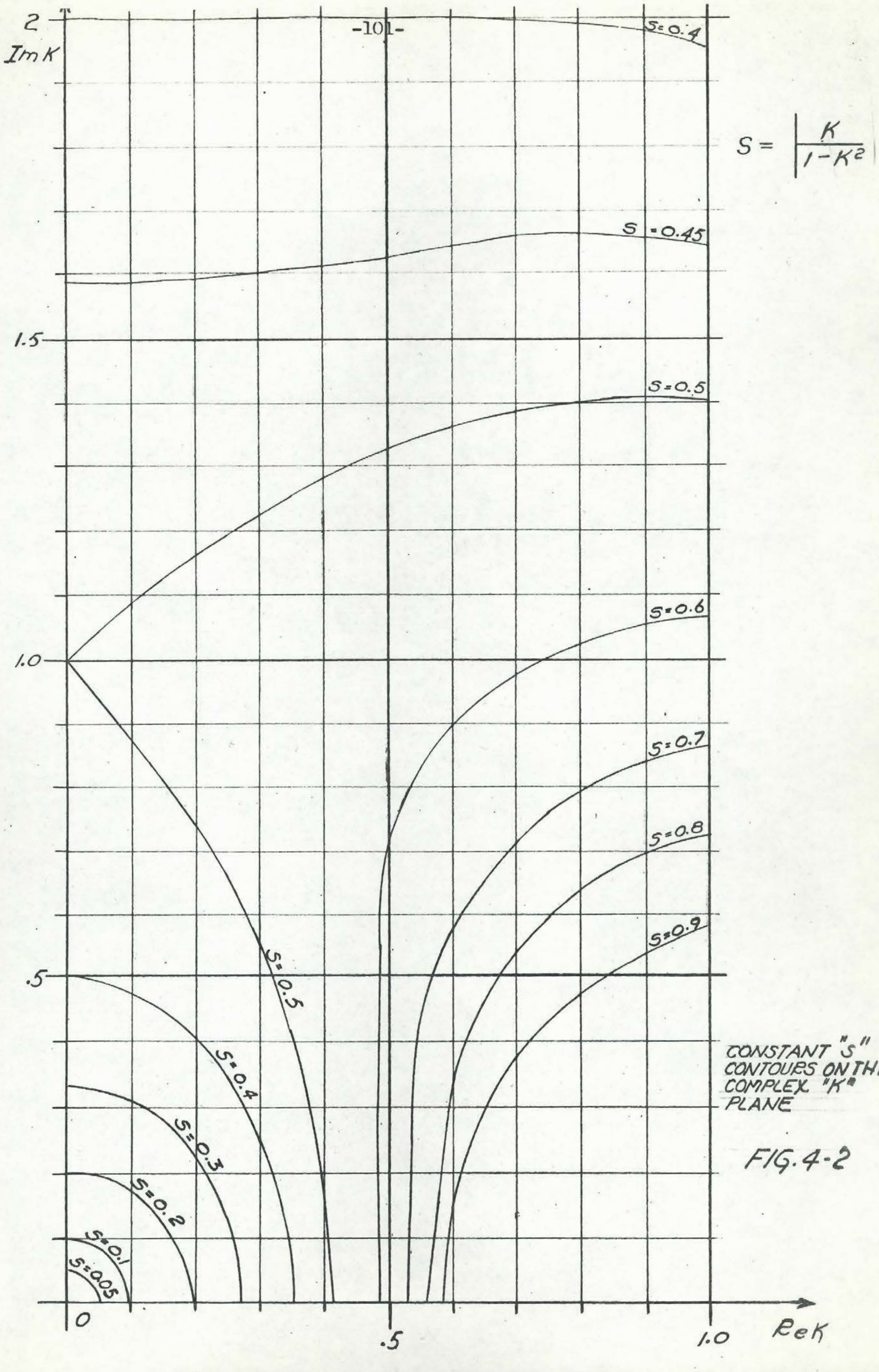
EQUIVALENT CIRCUIT OF FIG. 3-2a

FIG. 3-2b



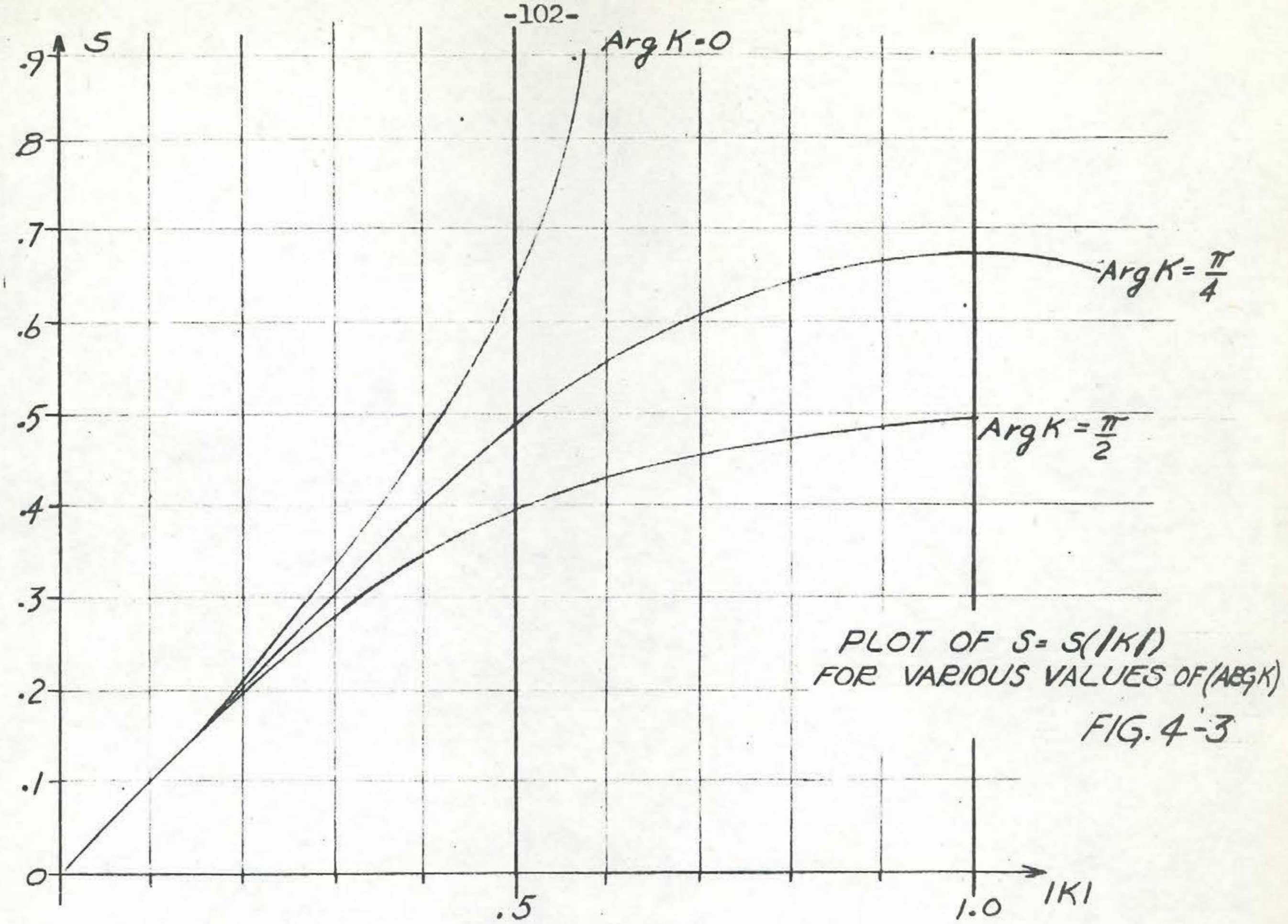
CONSTANT „T" CONTOURS ON THE COMPLEX „K" PLANE

FIG. 4-1

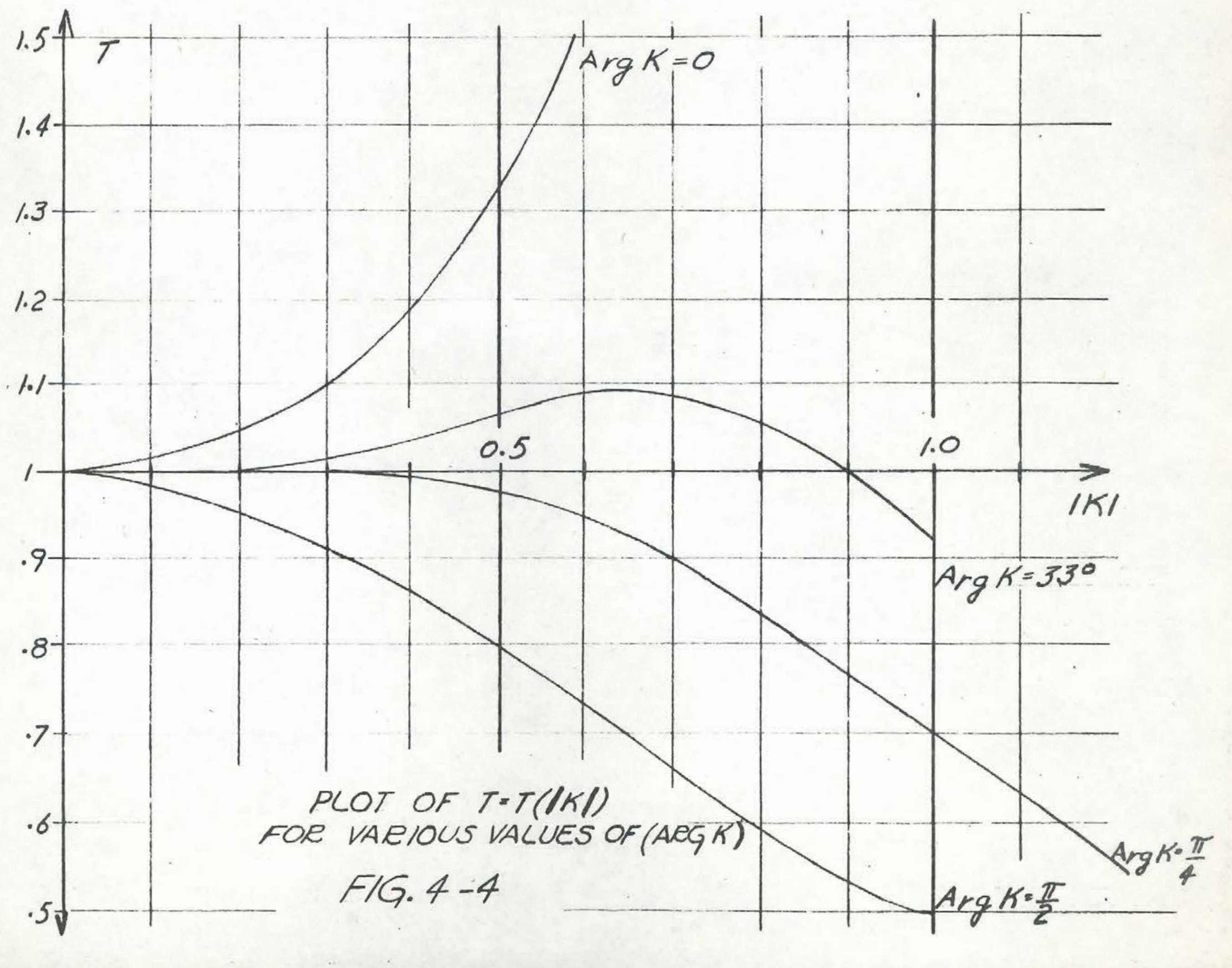


CONSTANT "S" CONTOURS ON THE COMPLEX "K" PLANE

FIG. 4-2



PLOT OF S = S(|K|)
FOR VARIOUS VALUES OF (ARG K)
FIG. 4-3



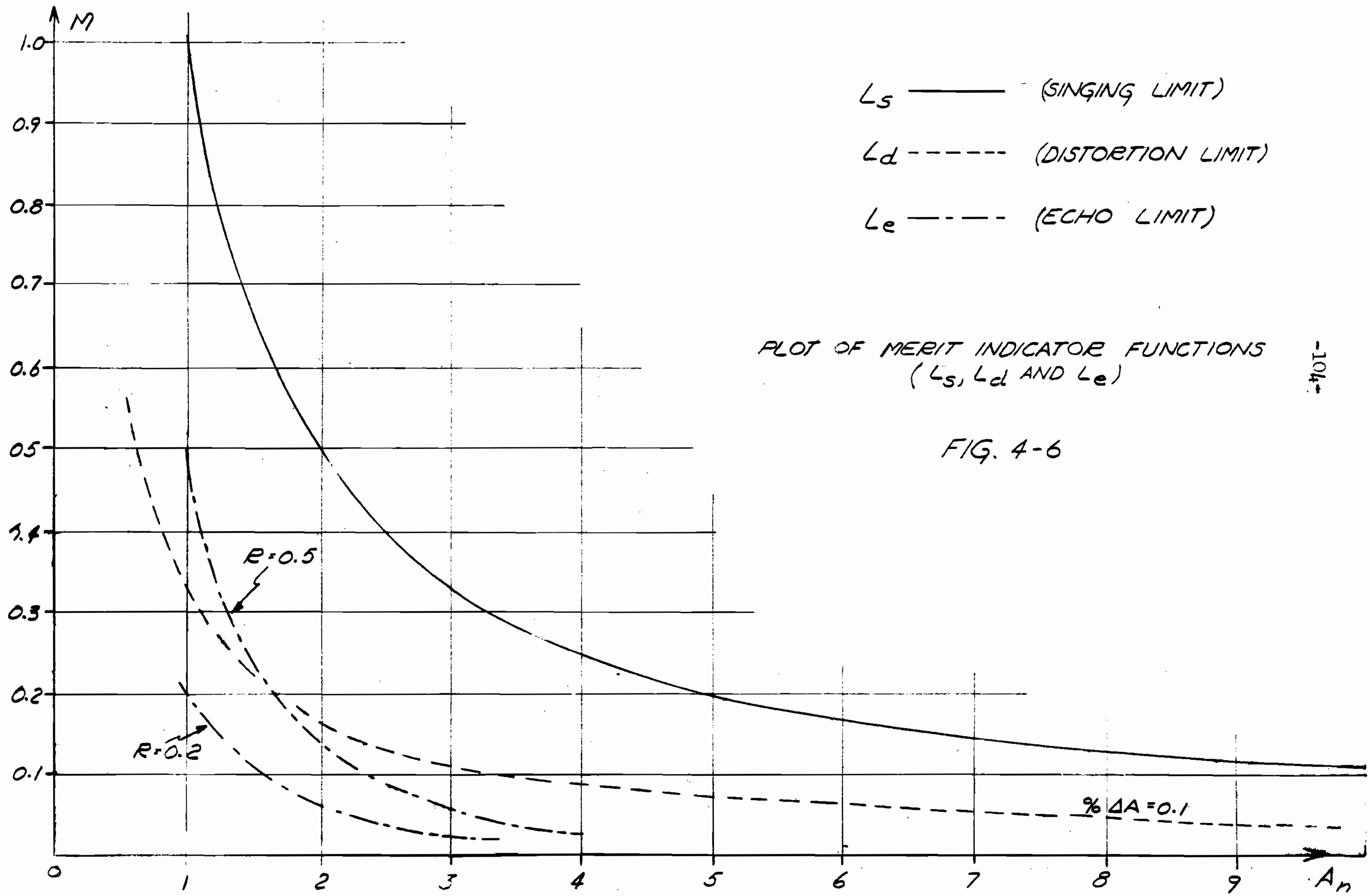
PLOT OF T = T(|K|)
FOR VARIOUS VALUES OF (ARG K)
FIG. 4-4

	NEGATIVE IMPEDANCE REPEATER	BRIDGE COMPENSATED AND HYBRID REPEATERS
L_s	$\dfrac{1}{A_n}$	$\dfrac{1}{A_n\sqrt{q}}$
L_e	$\dfrac{R}{A_n^2}$	$L_{ef} = \dfrac{R_f}{pA_n^2}$; $L_{eb} = \dfrac{R_b}{p'A_n^2}$
L_d	$\dfrac{1}{A_n}\sqrt{\dfrac{\%\Delta A}{1+\%\Delta A}}$	$\dfrac{1}{q A_n}\sqrt{\dfrac{\%\Delta A + (1-q)}{1+\%\Delta A}}$
L_g	$\sqrt{\dfrac{R}{M}}$	$L_{gf} = \sqrt{\dfrac{R_f}{pM}}$; $L_{gb} = \sqrt{\dfrac{R_b}{p'M}}$

(*) $q = f_{c1}\, f_{c2}\, f_s$; $p = \sqrt{\dfrac{M_1}{M_2}}\, f_{c1}^2\, f_{c2}\, f_s\, f'_{s'}$; $p' = \sqrt{\dfrac{M_2}{M_1}}\, f_{c1}\, f_{c2}^2\, f_s\, f''_s$; $A_n = \sqrt{A_{fn} A_{bn}}$; $M = \sqrt{M_1 M_2}$

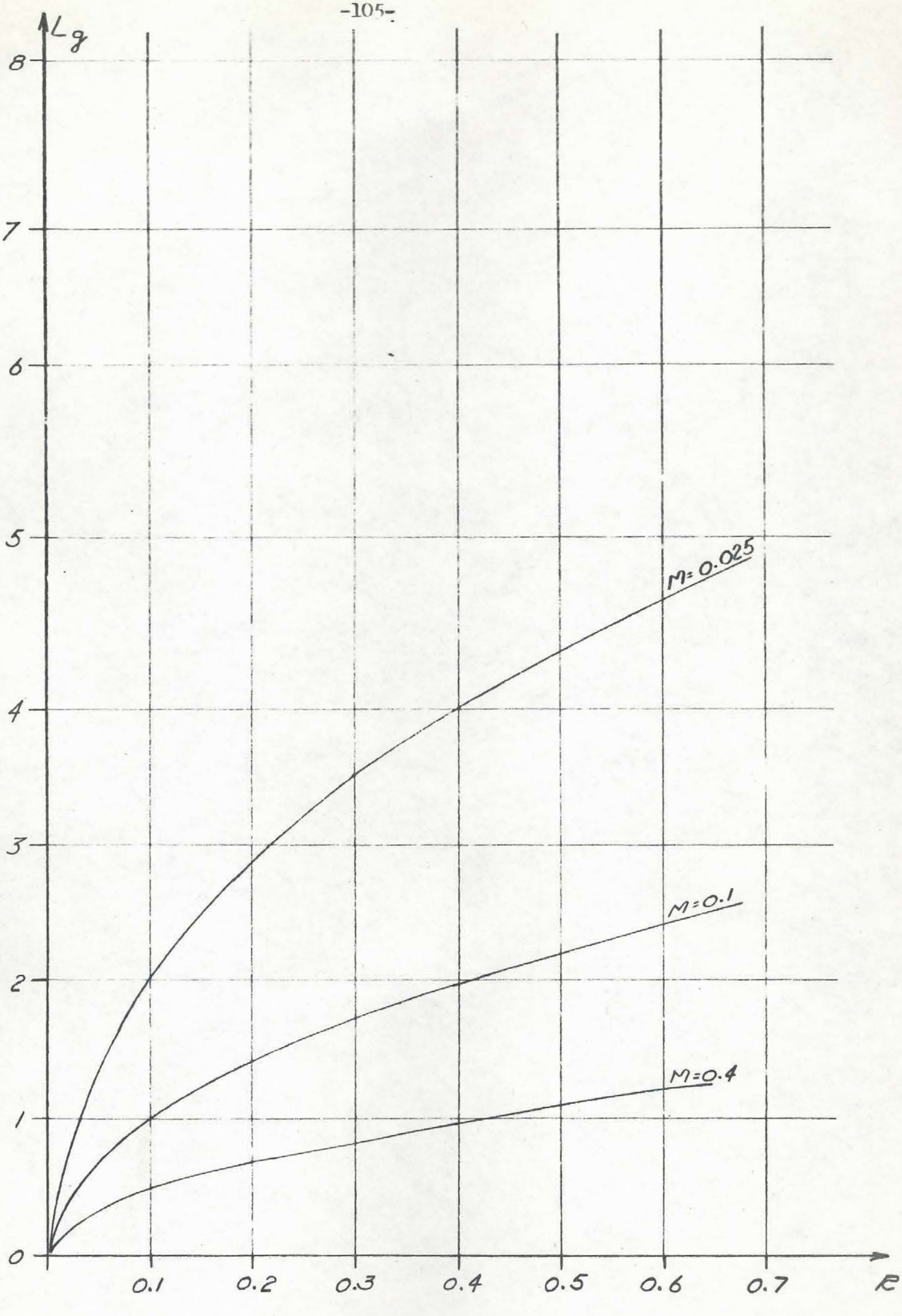
TABLE OF MERIT INDICATOR FUNCTIONS

FIG. ~~4~~ 5



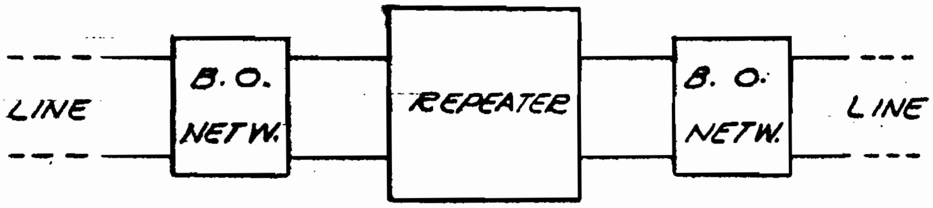
PLOT OF MERIT INDICATOR FUNCTIONS
(L_s, L_d AND L_e)

FIG. 4-6



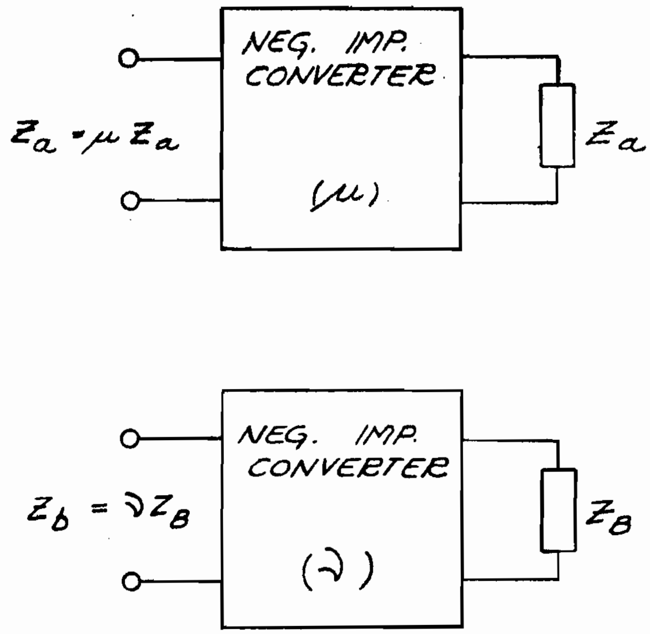
PLOT OF MERIT INDICATOR FUNCTION (L_g)

FIG. 4-7



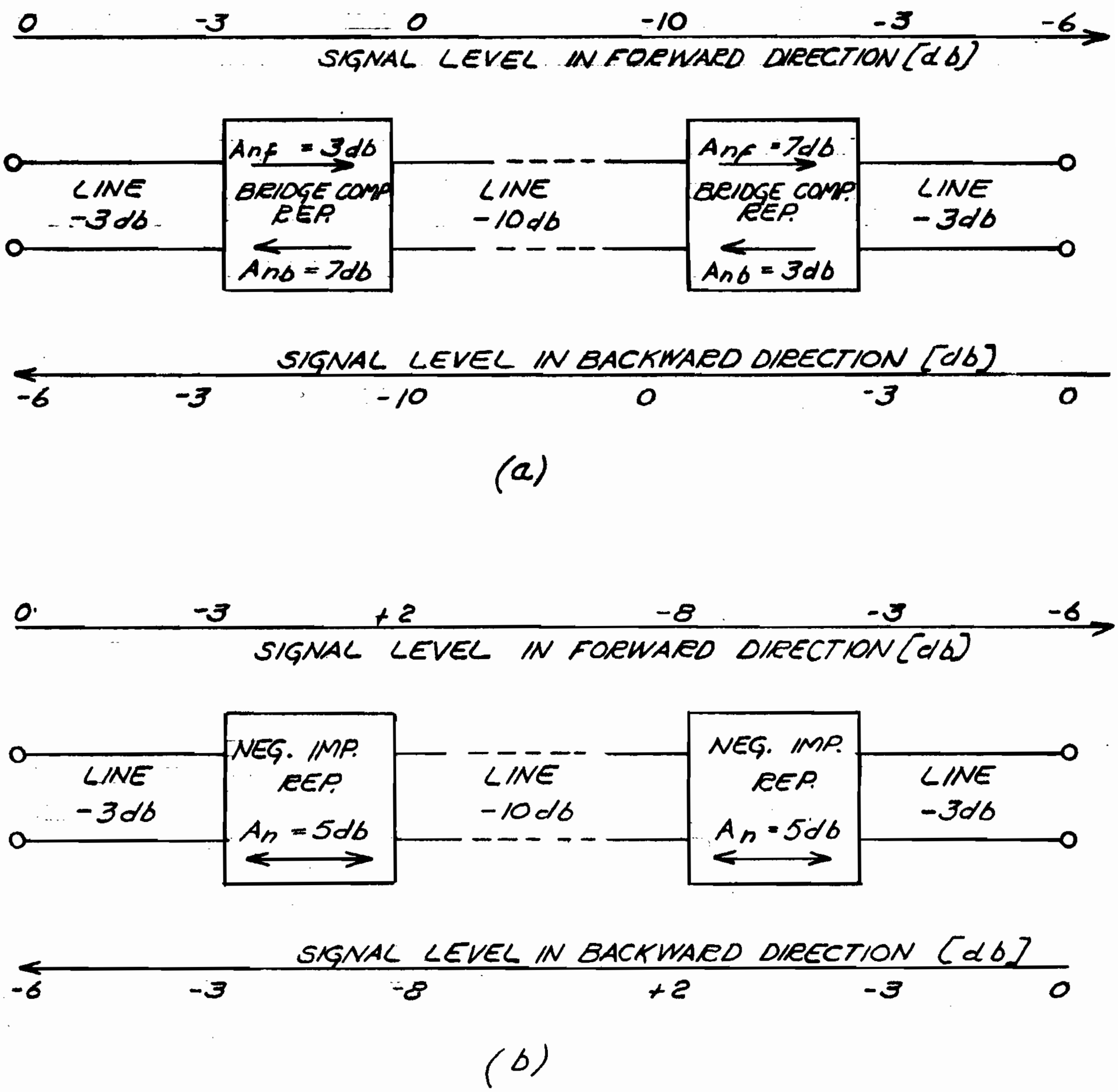
NEGATIVE IMPEDANCE REPEATER WITH BUILDING-OUT NETWORKS

FIG. 5-1



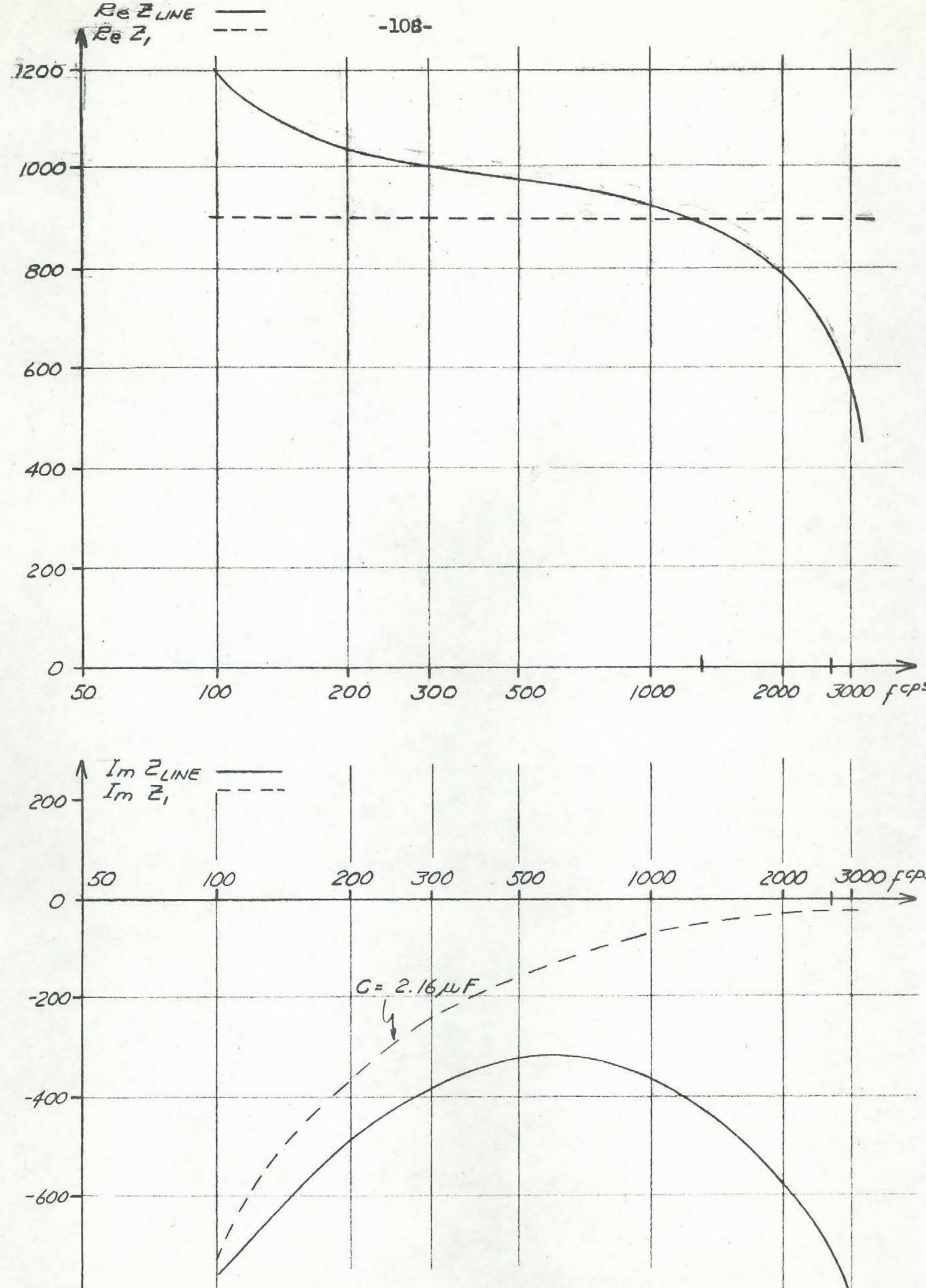
PRINCIPLE OF REALIZING NEGATIVE IMPEDANCES USED IN NEGATIVE IMPEDANCE REPEATERS.

FIG. 5-2



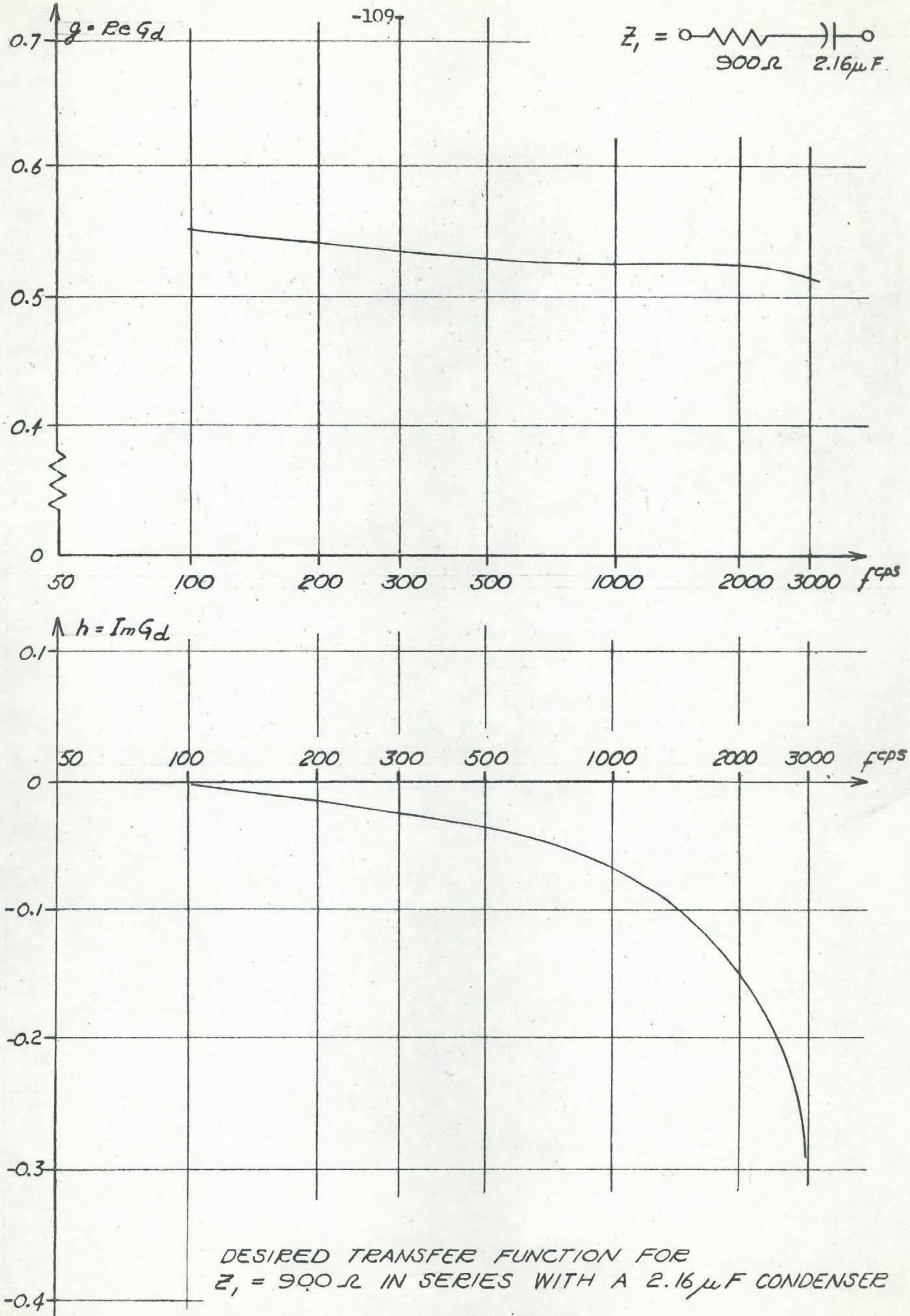
SIGNAL LEVEL DISTRIBUTION ALONG A TRANSMISSION PATH WITH (a) BRIDGE COMPENSATED AND (b) NEGATIVE IMPEDANCE REPEATERS.

FIG. 5-3



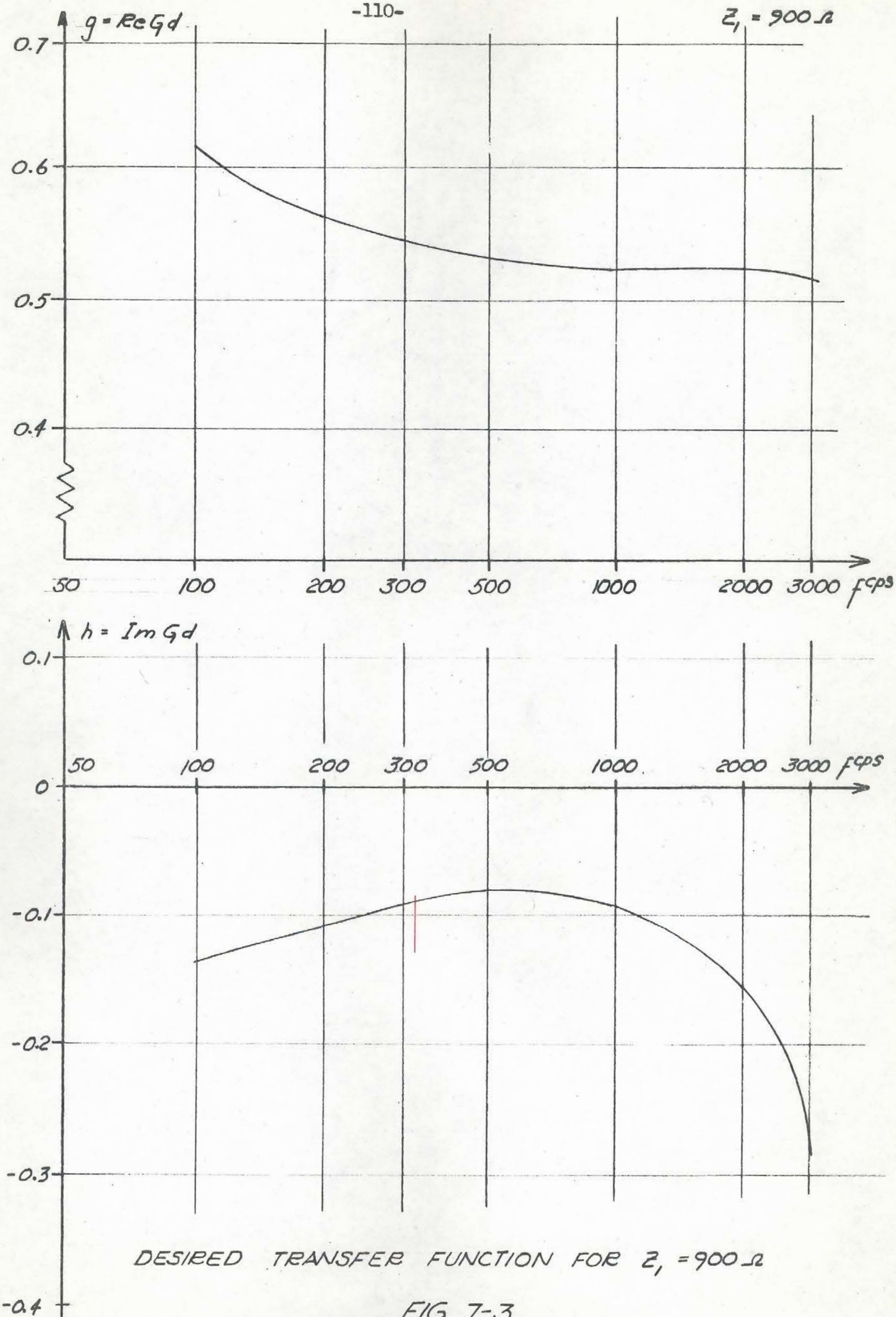
LINE IMPEDANCE & Z_1 FUNCTIONS FOR THE EXAMPLE OF SECTION 7. [Z_1 IS 900Ω IN SERIES WITH A 2.16 μF CONDENSER

FIG. 7-1



DESIRED TRANSFER FUNCTION FOR
Z_1 = 900 Ω IN SERIES WITH A 2.16 μF CONDENSER

FIG. 7-2



DESIRED TRANSFER FUNCTION FOR $Z_1 = 900\,\Omega$

FIG. 7-3

DEGREE OF DENOMINATOR (n) \ DEGREE OF NUMERATOR (m)	0	1	2	3	4	5
0	1	X	X	X	X	X
1	4	7	X	X	X	X
2	2	10	12	X	X	X
3	5	8	14	16	X	X
4	3	11	13	18	19	X
5	6	9	15	17	20	21

NOTES

1.) GENERAL FORM OF $G(s)$

$$G_{m,n}(s) = \frac{A_1(1 + A_2 s + \dots + A_{m+1} s^m)}{(s+a_1)(s+a_2)\dots(s+a_n)}$$

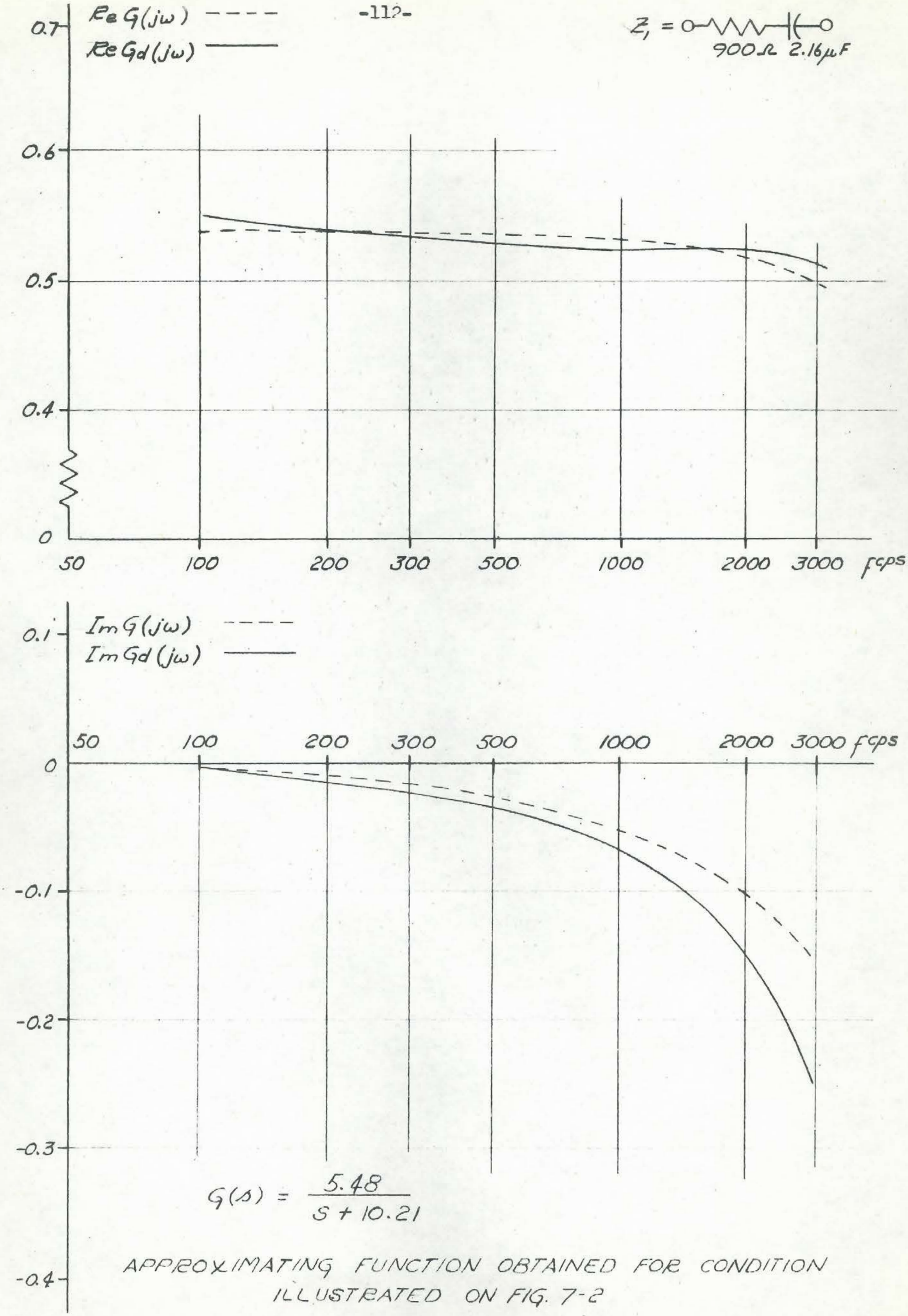
2.) NUMBERS IN CIRCLE INDICATE THE SEQUENCE IN WHICH SUBSEQUENT "m" & "n" VALUES ARE SELECTED

3.) X INDICATE NON-REALIZABLE FUNCTIONS.

4.) ARROW INDICATE THE PATTERN OF INITIAL VALUE TRANSFER.

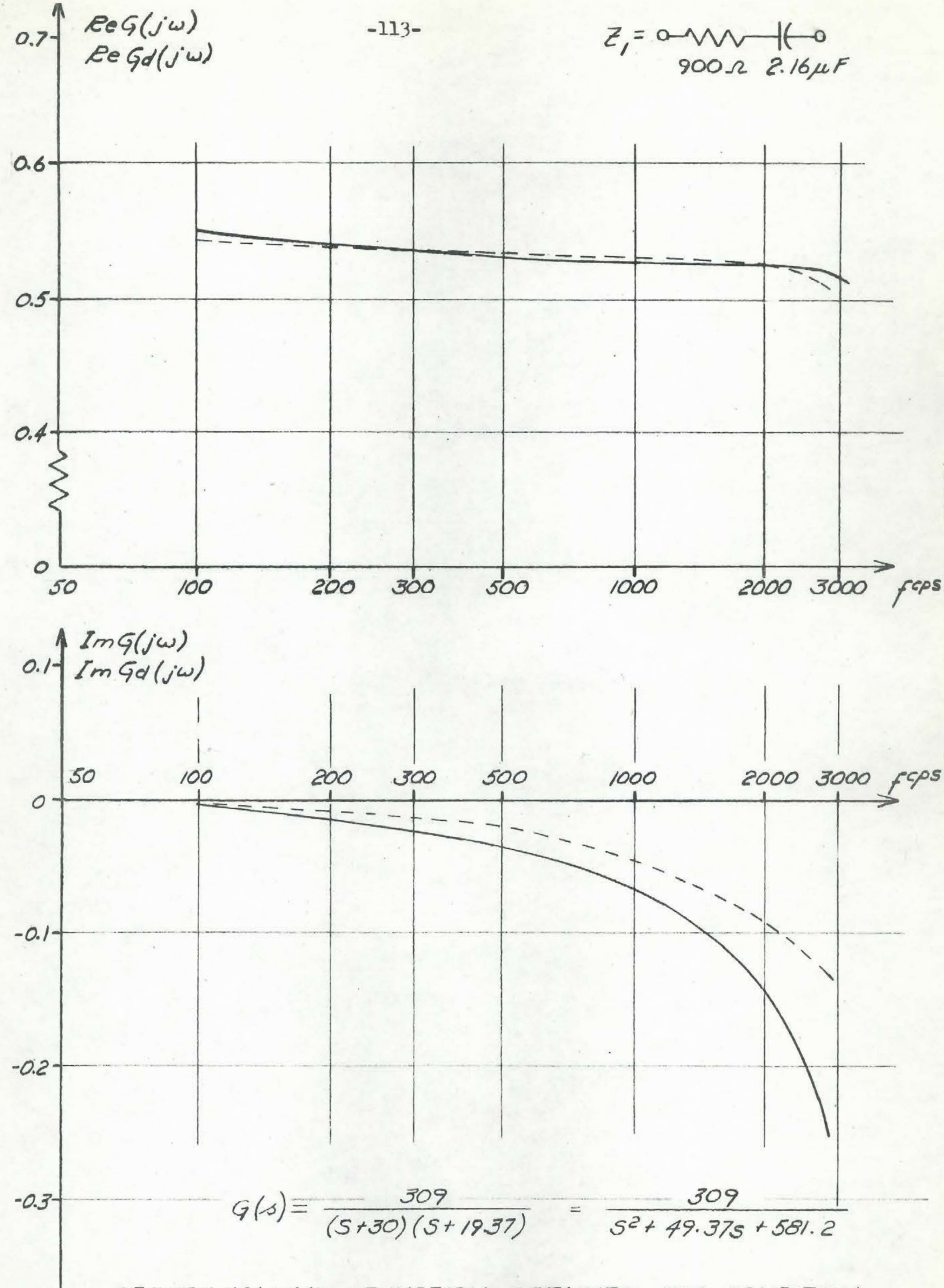
SEQUENCE OF TRIALS & PATTERN OF INITIAL VALUE TRANSFER FOR TRANSFER FUNCTION DESIGN.

FIG. 7-4



APPROXIMATING FUNCTION OBTAINED FOR CONDITION ILLUSTRATED ON FIG. 7-2

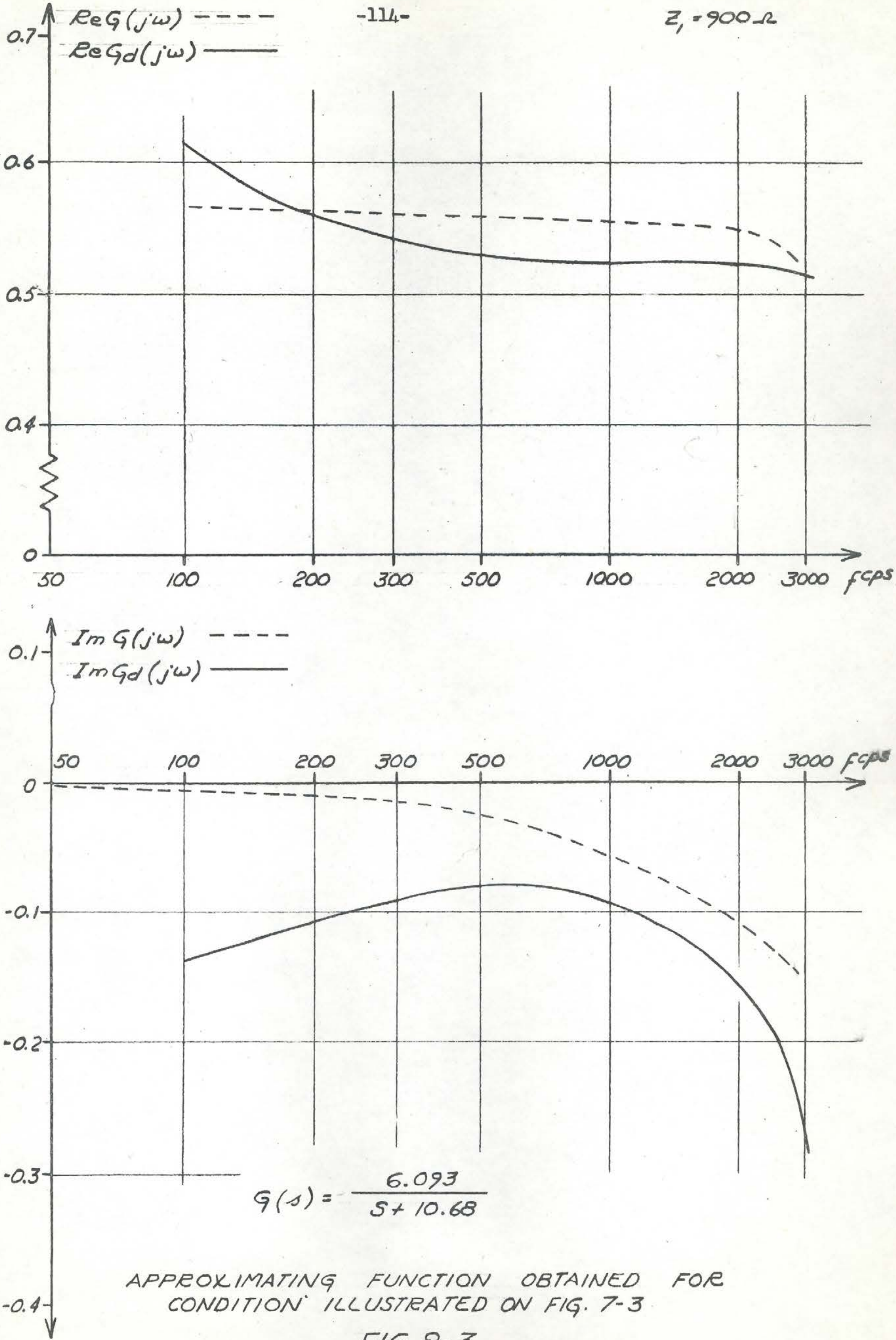
FIG. 8-1



$$G(s) = \frac{309}{(S+30)(S+19.37)} = \frac{309}{S^2 + 49.37s + 581.2}$$

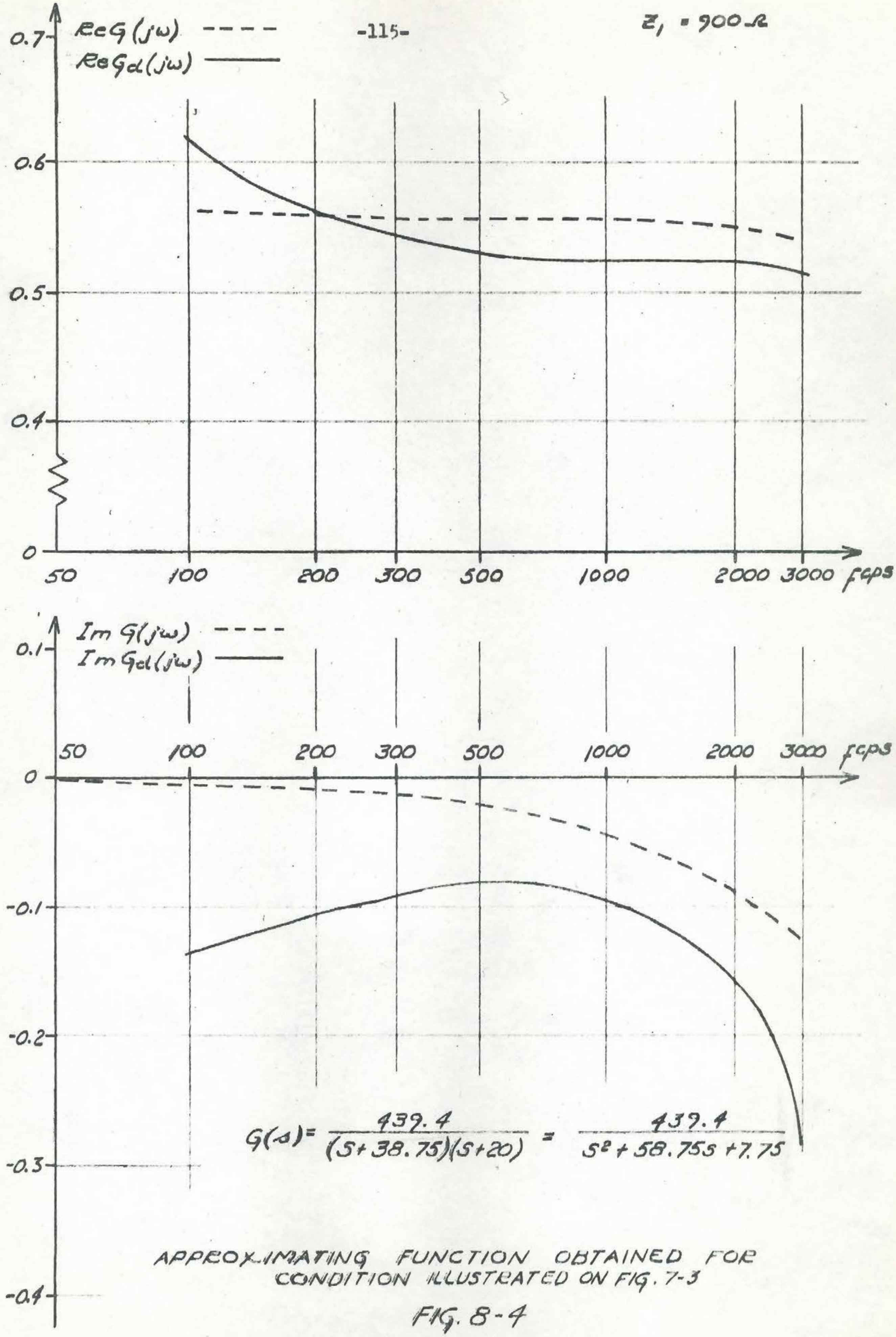
APPROXIMATING FUNCTION OBTAINED FOR CONDITION ILLUSTRATED ON FIG. 7-2

FIG. 8-2



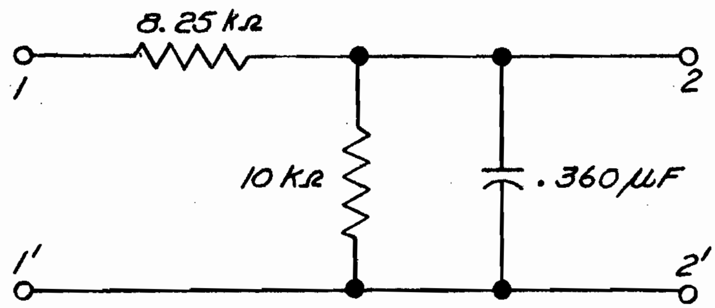
APPROXIMATING FUNCTION OBTAINED FOR CONDITION ILLUSTRATED ON FIG. 7-3

FIG. 8-3



APPROXIMATING FUNCTION OBTAINED FOR CONDITION ILLUSTRATED ON FIG. 7-3

FIG. 8-4

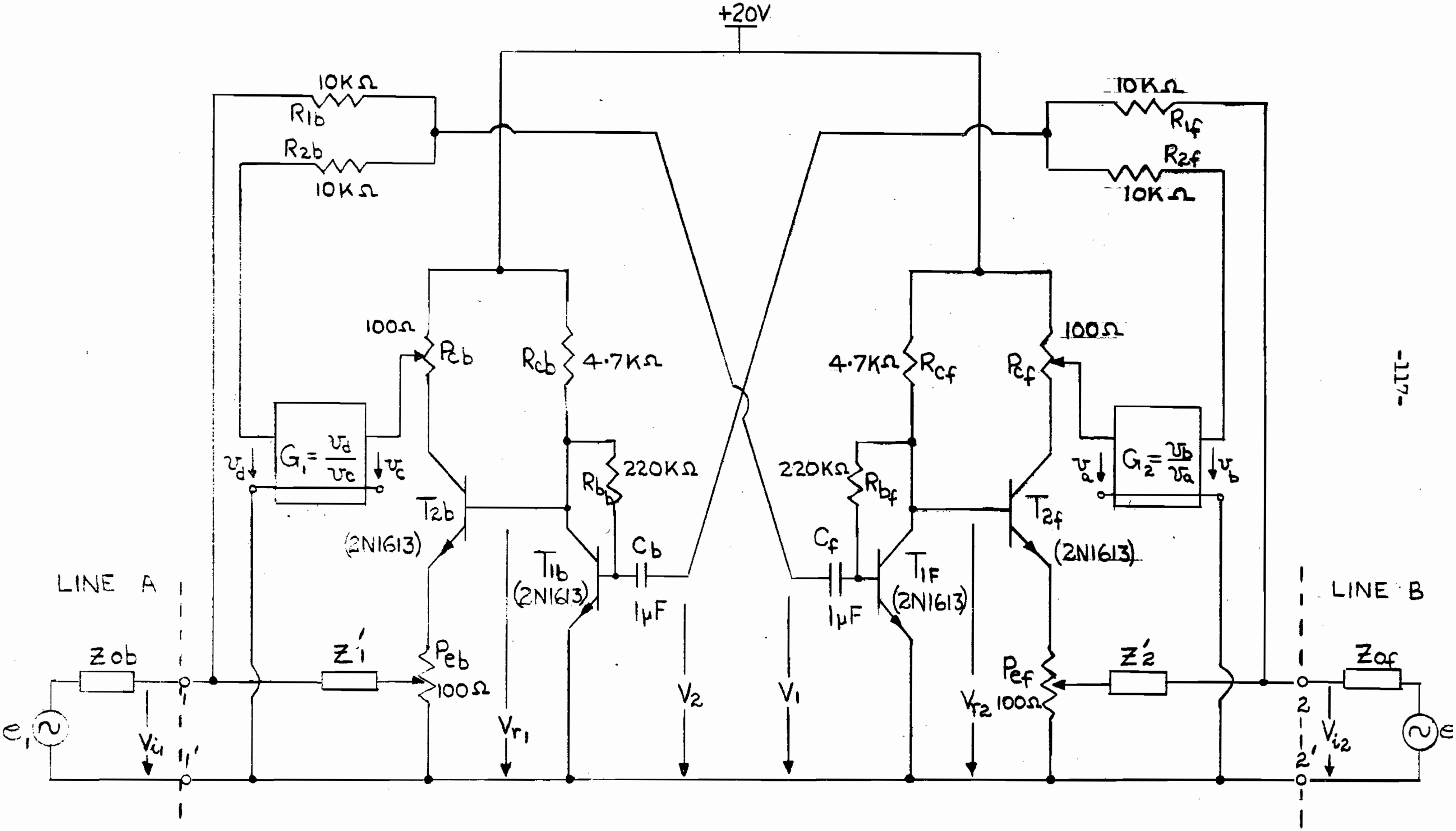


COMPENSATING NETWORK CORRESPONDING

TO $G(s) = \dfrac{5.48}{s + 10.21}$

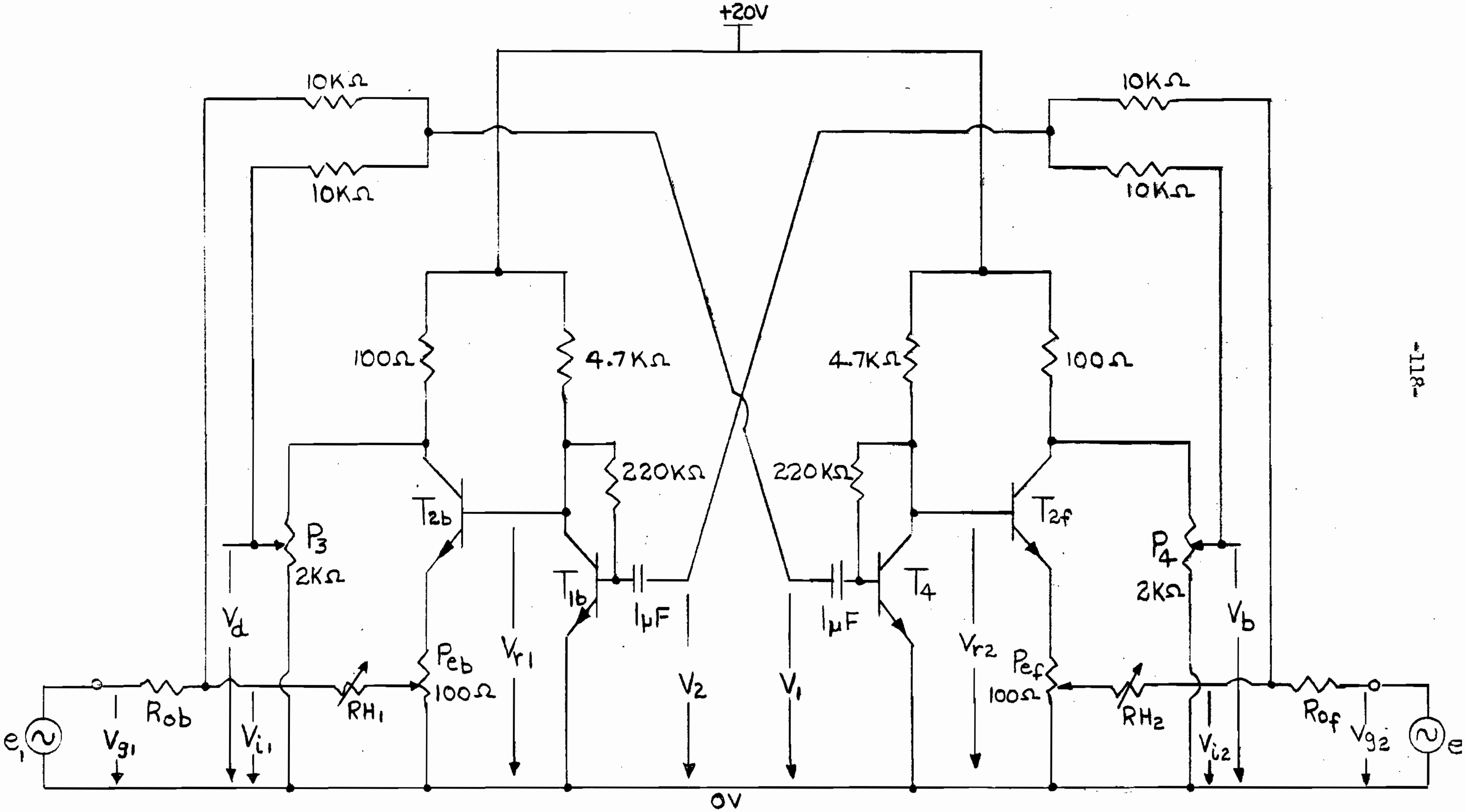
[NOTE: $\omega_n = \frac{\omega}{2\pi 10^3}$; WHERE ω_n IS THE NORMALIZED FREQUENCY]

FIG. 8-5



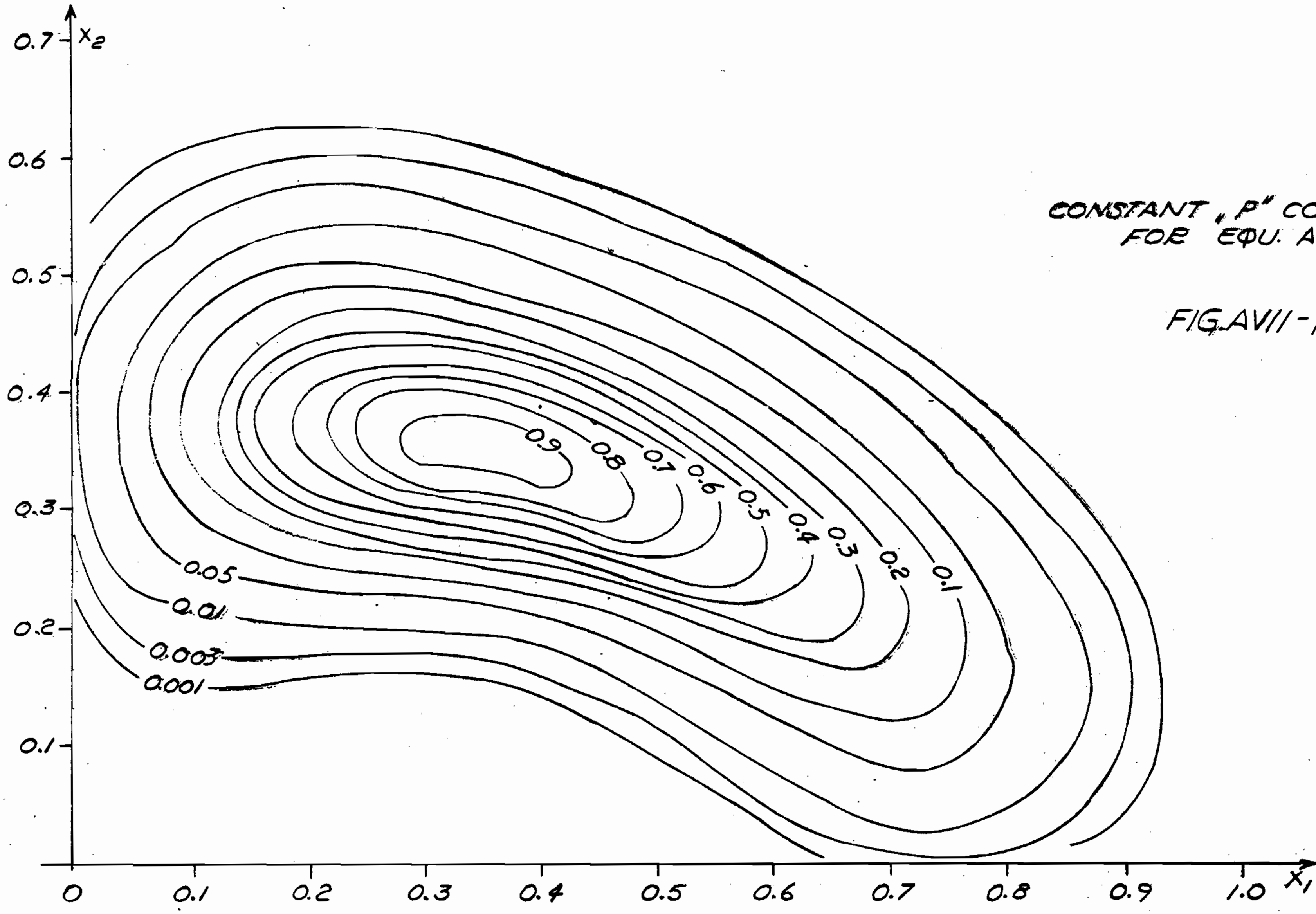
CIRCUIT REALIZATION OF THE BRIDGE COMPENSATED REPEATER

FIG. 9-1



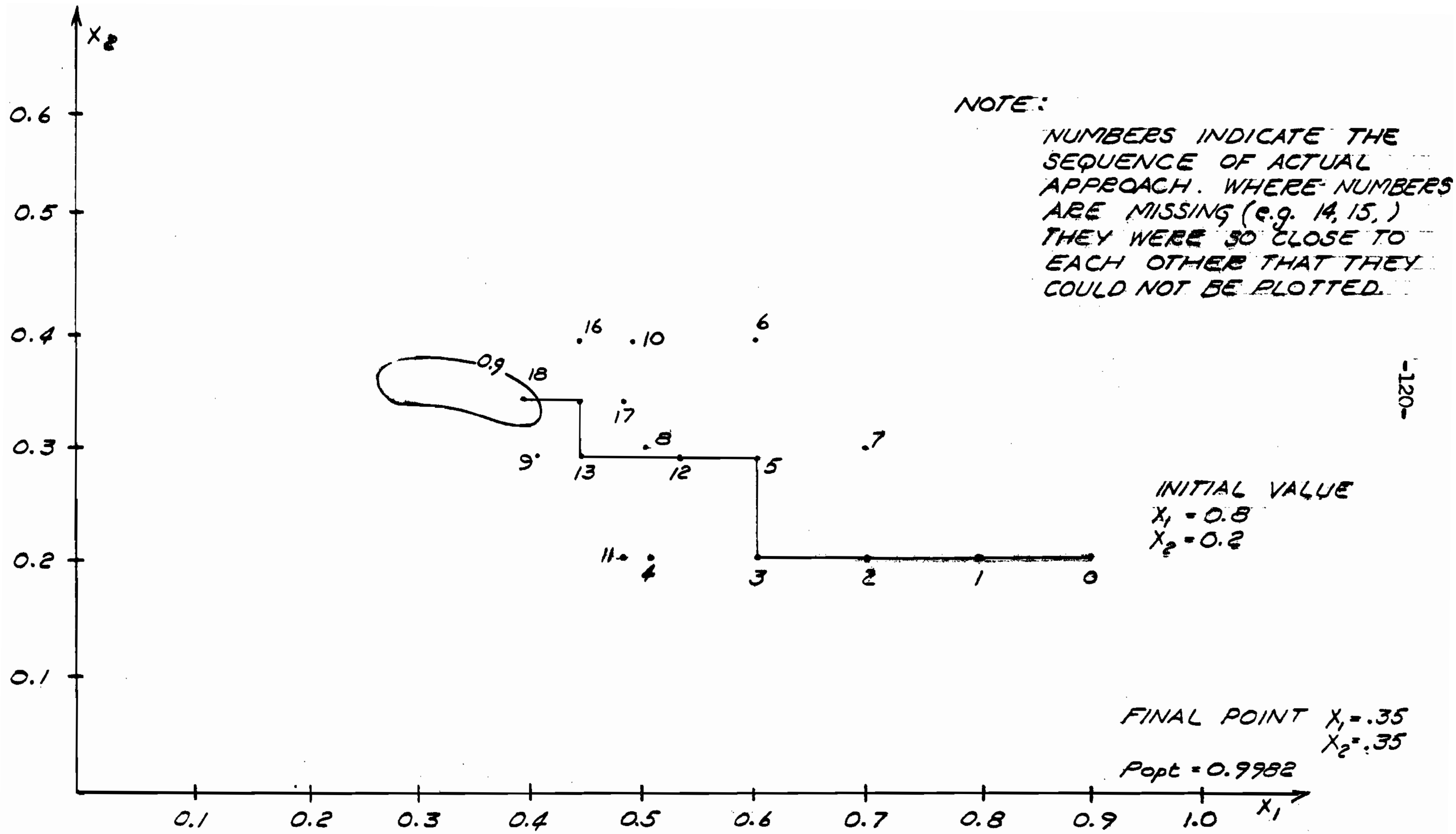
SET-UP USED FOR THE MEASUREMENT DESCRIBED IN SECTION 9.

FIG. 9-2



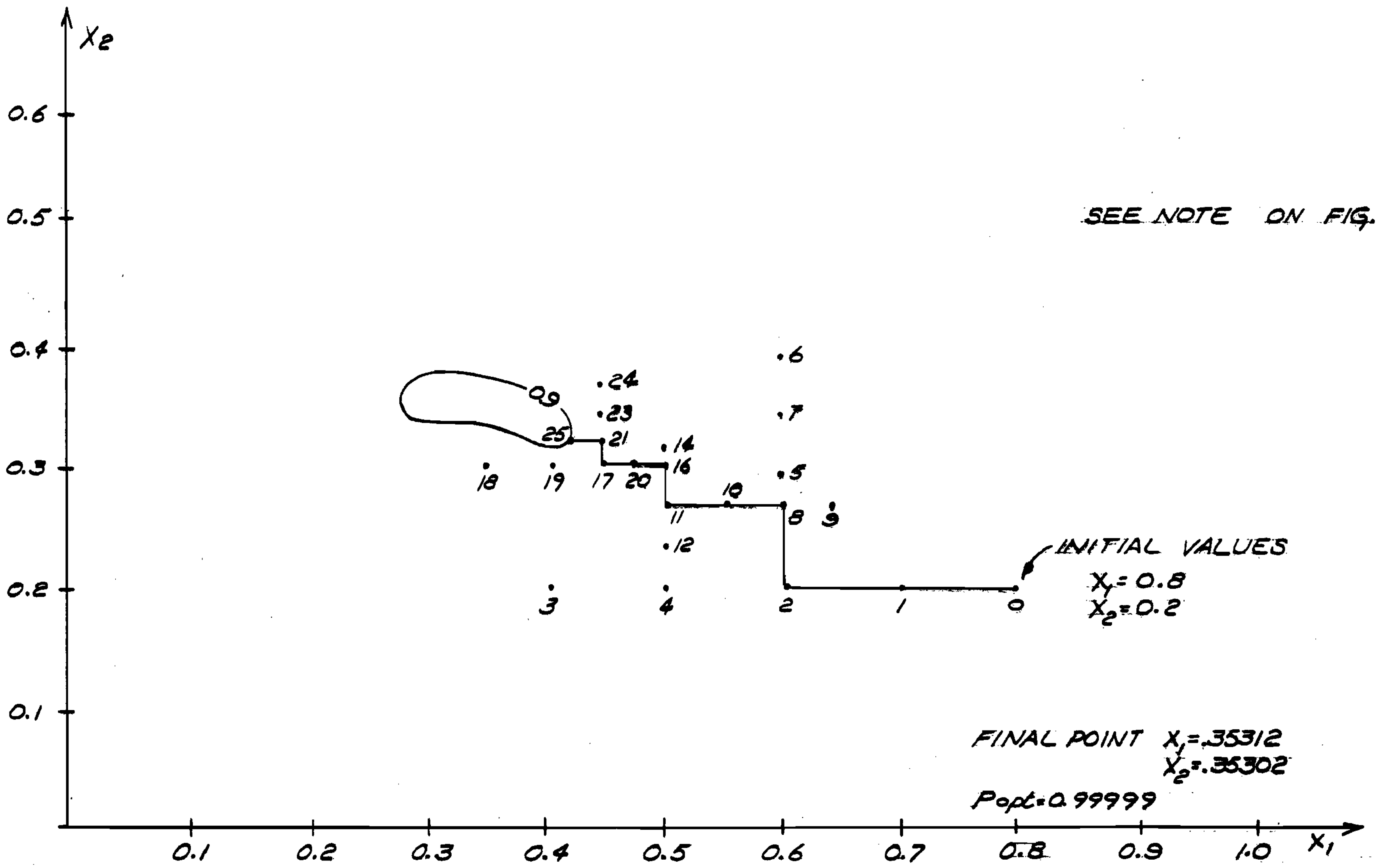
CONSTANT „P" CONTOURS
FOR EQU. AVII-7

FIG. AVII-1



TRAJECTORY OF OPTIMUM SEARCHING USING NON-ADAPTIVE STRATEGY

FIG. AVII-2



TRAJECTORY OF OPTIMUM SEARCHING USING ADAPTIVE STRATEGY.

FIG. AVII-3