BREAKDOWN OF LIQUID DIELECTRICS

B R E A K D O W N OF L I Q U I D D I E L E C T R I C S

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ABSTRACT.

This thesis studies the breakdown characteristics of an askarel (Monsanto 7030). A set of breakdown experiments is reported, using pure Nickel sphere electrodes of 5 mm. diameter with a 200 gap.

Following careful experimental procedures, a number of series of observations have resulted.

These results were statistically analysed to find the form of the probability distribution which governs the breakdown event. The extreme value distributions were used. Their properties have first been studied and a numerical method has been developed to find the relevant parameters of both the Weibull distribution and the Gumbel I distribution from the measured data.

The analysis of the observations proves that extreme value techniques may be regarded as being valid to analyse breakdown data.

The statistical analysis of the results suggests that two different mechanisms are at work and indications were sought towards a physical explanation of these two mechanisms.

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LIST OF SYMBOLS.

E	voltage gradient
$E\{x\}$	expected value of x
erf	error function
$f(x)$	probability density function of x
$g(x)$	function of x
$g_n(x)$	probability density function associated with the n^{th} flaw
ln	natural logarithm
$P(x)$	cumulative probability function of x
$R(x)$	reliability at x
S	size dependent constant
t	time
t_f	formative time-lag
t_s	statistical time-lag
V	voltage
$w(x)$	$\int_{x_o}^{x} z(x)\,dx$
$z(x)$	intensity function at x : two formulae for z(x) are currently used, Weibull distribution: $z(x) = M x^{\alpha} = (\frac{x}{b})^{\alpha} = S(\frac{x}{d})^{\alpha}$ Gumbel I : $z(x) = k e^{ax} = e^{a(x-x_m)} = S e^{a(x-x^*)}$
$\Gamma(x)$	gamma function of x
γ	Euler's constant (0.5772...)
ε	error term
σ	standard deviation

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I INTRODUCTION

This study deals with the breakdown behaviour of liquid dielectrics. The subject is not new, in fact the search of the breakdown strength of a liquid, is as old as the use of liquids as an insulation of high voltages. The fact that progress has been difficult and slow is due to the great difficulties to find and control the dominant experimental factors and is also due to a few misconceptions of the phenomenon of breakdown.

Originally it was believed, implicitly or explicitly that an intrinsic value could be found for the breakdown strength. This means that breakdown strength is a bulk property of the liquid, and that it is characteristic for the liquid as a whole, as well as for every small volume, if chosen large enough to avoid the influence of the molecular structure. It implies that the liquid is absolutely uniform on the macromolecular scale and that breakdown involves a uniform partial failure of the liquid in the entire volume, which is under breakdown stress. The breakdown is then a failure of the physical structure itself and its causes should be traced down to the physical equilibrium of the liquid.

Given these assumptions it must be possible to designate a unique value to the breakdown strength, allowing only small deviations due to experimental errors, following a Gaussian probability distribution. This is the case for liquid properties such as visco-

sity, acidity or resistivity.

The Gaussian distribution of experimental errors implies among others two important properties. The first one says that estimators for the mode and the mean of the distribution give the same value. It has been shown that this is not true and that the distribution of breakdown values is negatively skewed (Ref. 1.1). The second property follows from the concept of standard deviation and infers that refining the test sample and the experimental conditions will lower the standard deviation. Again this is not true and even the opposite has been stated (Ref. 1.5).

A different approach to the breakdown process, which is also theoretically very attractive, uses the "weakest link" concept. This states that a breakdown originates in a weak spot, which can be a foreign matter, but which also can be a deficiency of the material itself. Both assumptions have their own advocates. Kok (Ref. 1.3) studied the influence of impurity particles and in a broad sense polymers of the dielectric itself. Krasucki (Ref. 1.4) studied the growth and the subsequent ionization of microbubbles of vapour within the liquid under influence of an electric field. Both authors took a deterministic point of view in the evaluation of their observations.

Our point of view is a probabilistic one. We assume that a certain number of weak spots, either particles or vapour bubbles, exist within the liquid and that each of them has a certain probability of causing a breakdown.

Under this assumption it appears to be perfectly logical to look for a breakdown probability which is composed out of a certain number of "partial" probabilities corresponding to the different possible causes of breakdown inception.

Probability functions of this nature fall within the family of extreme value distributions. It may be mentioned that extreme value techniques have been long used in the study of fatigue of metals, even to the point that an important distribution has been given the name of Weibull, in recognition of the Swedish engineer who worked out the extreme value theory of fatigue problems.(Ref. 1.2).

In fact the same two classes of deficiencies, which have been mentioned earlier for liquids, can be found in solids as well, namely the existence of particles of foreign material within the test sample and structural deficiencies (cavities, dislocations).

The aim of the present investigation, is the study of the breakdown behaviour of a dielectric, assuming that it follows extreme value statistics. The test liquid was an experimental sample of askarel (Monsanto 7030).

II WEAKEST LINK CONCEPT

No matter what the exact mechanism of breakdown is, it is clear that there is a certain randomness in the location of the triggering event, whether it is within the bulk of the sample or in the electrode region.

The system under test can thus be thought of, as being composed out of a large N number of possible breakdown spots, only one of which will eventually start the total failure. The probability function for the whole system therefore must account for the influence of all weak spots.

Secondly because breakdown changes the system drastically, it is a one-time event and only the probability of having the first event is important. The probability distribution must thus be an extreme value distribution.

2.1 Assumptions on which the weakest link concept is based.

2.1.1 A certain number of flaws exist, or are created within the material under investigation.

2.1.2 These flaws are spread in a random way throughout a large part of the material.

2.1.3 A breakdown is possible for a certain range of experimental values.

2.1.4 Each of the flaws can be a cause of breakdown.

The next three assumptions are not necessary to build up a theory but give more insight into the composition of the statistical distribution.

2.1.5 Flaws act independently of one another. This assumption introduces a very small error when the distances between flaws are large, compared with their region of influence and when the time intervals between breakdowns are large enough to ensure the return to normal of the test sample.

2.1.6 The probability that a certain predetermined flaw will cause breakdown is very small. In other words no dominant weaknesses are available.

2.1.7 The probability of being the cause of a breakdown is the same for each of the flaws.

Assumption 7 leads to very attractive formulae but its benefit is perhaps only of an academic nature.

2.2 A first approach.

Define $R(x)$ as the probability of survival up to x. This function is called the reliability. x is the independent variable, which can be time, voltage, stress. $P(x)$ is the cumulative distribution function for the probability of breakdown up to x.

Clearly $R(x) = 1 - P(x)$

The reliability $R(x)$ cannot increase with x (corrolary on the axioms of probability (Ref. 2.4) p.26)

$$dR(x) = -\zeta(x)\,dx \qquad \text{with } \zeta(x) \geqslant 0$$

It is logical to assume that for equal flaw activity, the chances of breakdown will be larger for larger $R(x)$. It is therefore convenient to define the following relation:

$$\zeta(x) = z(x) \cdot R(x)$$

Define: intensity function

$z(x)$ is called the intensity function. It is a measure for the activity of flaws within the sample.

$$dR(x) = - z(x) \cdot R(x)\,dx$$

or $$\frac{1}{R(x)} \cdot \frac{dR(x)}{dx} = - z(x)$$

and $$\ln R(x) = -\int_{x_o}^{x} z(x)\,dx$$

Define: $$\int_{x_o}^{x} z(x)\,dx = w(x)$$

$w(x)$ has little physical meaning, therefore it is not mentioned explicitly in any of the important text books or articles. It gives the cumulative effect of the flaw activity up to x.

Plots of - w(x) versus x, have been used very frequently under the name of "Laue plots" (Ref. 2.1)

The relations for R(x), P(x), and f(x) follow immediately :

$$R(x) = \exp\left\{- w(x)\right\}$$

$$P(x) = 1 - \exp\left\{- w(x)\right\}$$

$$f(x) = \frac{dP}{dx} = - \frac{d}{dx} \exp\left\{- w(x)\right\} = z(x) \cdot \exp\left\{- w(x)\right\}$$

$$f(x) = z(x) \cdot R(x)$$

Note: $R(x_{max}) = 0$ implies $\lim_{x \to x_{max}} w(x) = \infty$

or $\int_{x_o}^{x_{max}} z(x)\, dx = \infty$

This development was quite general, but it is powerful in defining relations between the degree of flaw activity and the probability of breakdown.
More precise information is gained from a second analysis.

2.3 A second approach.

Let there be a large number of flaws N(x) within the material, all of which can cause a breakdown. Say the probability that the flaw designated by the arbitrary order num-

ber n, causes a breakdown within the interval dx is $g_n(x) \cdot dx$ and that the reliability up to x is R(x).

By assumption 4 the probability of breakdown in the interval dx can be calculated to be

$$dP(x) = R(x) - R(x) \cdot \prod_{n=1}^{N(x)} (1 - g_n(x)\, dx)$$

and because $dP(x) = - dR(x)$

$$dR(x) = R(x) \cdot \prod_{n=1}^{N(x)} (1 - g_n(x)\, dx) - R(x)$$

$g_n(x)$ is small by assumption 6 and a fortiori $g_n(x)\, dx$

Thus $\prod_{n=1}^{N(x)} (1 - g_n(x)\, dx) = 1 - \sum_{n=1}^{N(x)} g_n(x)\, dx$

with negligible error

and $$dR(x) = R(x) - \sum_{n=1}^{N(x)} g_n(x) \cdot R(x)\, dx - R(x)$$

or $$dR(x) = - \sum_{n=1}^{N(x)} g_n(x) \cdot R(x)\, dx$$

A comparison with previous analysis shows

$$z(x) = \sum_{n=1}^{N(x)} g_n(x)$$

If condition 7 is also met $g_n(x) = g(x)$

and $z(x) = N(x) \cdot g(x)$

The last relation is conceptually very powerful because it makes a separation between the specific properties of the flaws ($g(x)$) and the size - or the rate of growth - of the weak point population ($N(x)$). The factor $N(x)$ is immediately linked to the size effect which is one of the basic properties of extreme value statistics.

2.4 Properties of $z(x)$.

2.4.1 $$z(x)\,dx = \frac{R(x) - R(x + dx)}{R(x)}$$ (Ref. 2.4) p.248

$z(x)\,dx$ is the ratio between the expected number of failures in the interval $(x, x + dx)$ divided by the expected number of good components at x.

$z(x)\,dx$, is the probability of failure in the interval $(x, x + dx)$ provided the system did not fail before x.

2.4.2 Define ξ , moment of failure

$f(\xi)$ probability density function of ξ

then $z(x)\,dx = f(x / \xi \geqslant x)$ at the point $\xi = x$

Thus $z(x)$ equals the numerical value of the conditional density function $f(x / \xi \geqslant x)$ for $\xi = x$.

However as x varies, the condition $\{\xi \geqslant x\}$ changes.

Therefore $z(x)$ is not, as has sometimes been stated, the conditional density function (Ref. 2.4) p.110.

2.4.3 It is clear from the structure of the relations, which have been found between R and z, that z is very insensitive to changes in R, while at the other hand R is very sensitive to changes in z. z gives a measure of the flaw activity within the sample. Thus R is very sensitive to changes in the flaw activity. This should explain why, small mostly uncontrolled or even unknown differences in experimental techniques, give widely differing mean values of breakdown.

2.4.4 It is quite important to realize that z(x) is not a probability function. As a consequence none of the properties of probability functions applies to this function. It is conceptually more correct to visualize z(x) as a function which represents some physical realities, concerning weak spots in the test sample.

2.4.5 Much work has been done by statisticians on extreme value problems, already quite some time ago. Gumbel wrote a very good handbook on the subject (Ref. 2.2) Weibull used extreme value principles since 1939, to explain the fatigue strength of metals. He relates his studies on statistics in a very comprehensive textbook (Ref. 1.2).

2.5 Some implications of the definition of z.

2.5.1 Some authors have worked on a deterministic model of breakdown. Among them Krasucki has developed a model based on the breakdown inception within a vapour bubble (Ref. 1.4). The theory gives an order of magnitude estimate of the breakdown value for very pure liquids (n-hexane). It also predicts time-lags in some experimental ranges. It assumes however, that vapour bubbles of a certain minimum size are produced in large quantities, almost instantaneously when swit-ching on the high voltage source. The generation of microbubbles however is in itself a random process which can be studied by extreme value techniques. The non-normality of the breakdown distribution proves that the random part is very important.

2.5.2 It is relevant to consider three regions for $z(x)$ which are defined in reliability theory (Ref. 2.3) p. 17, (Ref.2.5) Ch.1; before attempting to find analytic approximations for the intensity function. A first region describes the early failures caused by "constructional defects" and is characterized by $z(x) = \exp\left\{-g(x)\right\}$. Breakdown times during the conditioning period must be described by this function.

Brignell (Ref.1.5) has successfully applied the concept of "constructional defects" to the conditioning period.

A second region deals with failures which are randomly distributed $z(x) = K$. This model could well apply to $z(t)$ because there is no reason to believe that the chances of breakdown within a certain time interval will be larger than in any other.

The third region describes the "exhaustion" failures.

$$z(x) = \exp \left\{ g(x) \right\}$$

Failures in this group are caused by deterioration of the test object. The formula probably applies to the breakdown probability as a function of voltage. The existence of wear-out failures shows that extrapolation of results must be done with extreme care.

III LITERATURE

Despite the limited number of publications concerned exclusively with the statistical nature of dielectric breakdown, quite a number of researchers have used extreme value theory to explain their experiments. Therefore a close look at the literature will substantiate this viewpoint. This will be done in chronological order, putting together piece by piece the elements of the present theory.

During the review, the probability functions in time space or in voltage space will not be treated seperately, because the axioms which substantiate the theories are the same for both probability functions. Emphasis will be placed on the manner in which previous authors came to their conclusions, more than on the experimental results themselves. For the same reasons, reports of experiments on both gaseous and solid dielectrics are included. To contrast with the behaviour of dielectrics, articles dealing with mechanical strength are also mentioned. At the end, of this chronological review, the conclusions will be summarized and some results pertinent to the current experiments will be listed in Appendix I.

3.1.1 M. von Laue (Ref.2.1) (1925)

Von Laue in his remarks on the measurements of K.Zuber (Ref.3.1) was perhaps the first to realize that breakdown is principally a random process, which obeys a special class of probability functions. Theory was developed to find the distribution of time-lags registered when measuring the breakdown strength of an air gap.

He assumes that a constant D.C.voltage is applied over N identical spark gaps. After a time t no flash-over will have occurred in N.V(t) of the spark gaps. The number of sparks in the interval t to t + dt will therefore be N.dV(t). The cause of these flash-overs may have been the injection of an electron or anion in βN.V(t) dt of the spark gaps. These electrical charges will have a probability p(t) of causing a spark. The basic relation results:

$$N.dV(t) = -p(t) \cdot \beta N \cdot V(t)\, dt$$

and

$$\ln V(t) = -\beta \int_0^t p(t)\, dt$$

or

$$V(t) = \exp\left\{-\beta \int_0^t p(t)\, dt\right\}$$

It follows that V(t) is the reliability R(t) and βp(t) the intensity function z(t).

p(t) is independent of t if the probability that a certain electron causes a breakdown is unaffected by previous sparks. In that case we find the familiar relationship $z(t) = \beta\, p$ = constant.

The name of von Laue is also connected with a special kind of plot widely used in reliability statistics, namely the "Laue plot". This is a plot of $\ln \frac{n(t)}{N}$ against time, with n the number of breakdowns up to time t and N the total number of breakdowns. As can be seen

$$\ln R = \ln \frac{n(t)}{N} = \ln V(t) = -\beta \int_0^t p(t)\, dt.$$

ln R is represented by - w in the notation used in this report. Therefore Laue plots are plots of - w against the variable.

3.1.2 B. Epstein (Ref.3.2) (1947) and H. Brooks.

The authors have brought the ideas developed by Gumbel and Weibull to the field of electrical engineering. They wrote a very comprehensive article on the subject of extreme value statistics, applying these ideas to the dielectric strength of paper capacitors. They relate the dielectric strength to the distribution of the size of conducting particles. Stating that the strength of a capacitor is the strength of its weakest point, they directly relate the expected size of the largest particle to the breakdown probability. They also state that the probability of finding large particles will increase with the volume of the capacitor and therefore the dielectric strength must decrease.

The authors use $\lambda e^{-\lambda x}$ as the distribution of particle sizes in the capacitor paper and find for the distribution of the largest values, out of a sample size n

$$g_n(x) = n\lambda e^{-\lambda x}(1 - e^{-\lambda x})^{n-1}$$

The limit of this distribution for $n \to \infty$ is the double exponential probability function, known as Gumbel I.

3.1.3 A. Palmgren (Ref.3.3) (1954)

The "lifetime" of a bearing is discussed in this paper. It is assumed that the expected lifetime of a bearing follows a Weibull distribution i.e.

$$z = \left(\frac{L}{b}\right)^{\alpha} \quad \text{and}$$

$$R = \exp\left\{-\frac{b}{\alpha+1}\left(\frac{L}{b}\right)^{\alpha+1}\right\}$$

The applicability of the Weibull distribution points towards an aging effect (see 2.5.2). The nominal lifetime is defined as the lifetime which is reached by 90% of the bearings. The same definition is given by the standards of the International Standards Office (ISO). It is interesting that extreme value theory was already used in a standard for bearings, while the application of weak link statistics was only being attempted for dielectric breakdown.

3.1.4 R.F. Saxe and T.J. Lewis (Ref.3.4) (1954)

The authors link up with the ideas of von Laue. They substantiate his theory by assuming that electrons are injected at a rate I (electrons per second) into the spark gap. Each of these electrons has a probability W of causing a breakdown. The prabability is therefore related to the product W I.

A comparison with the formula of von Laue shows that W corresponds to the p(t) of von Laue and I to the β. Thus z = W I. W must be field dependent, while I is assumed to be constant. That means that the emission of electrons from cosmic radiation, as well as the cathode emission are constant with respect to time.

Under these conditions z(t) is constant.

3.1.5 K.H. Weber and H.S. Endicott (Ref.1.1) (1955)

Weber and Endicott were the first to systematically use extreme value theory in their work. They used a set of four Rogowski profile brass electrodes. The electrodes had diameters of respectively 76mm, 51mm, 25.4mm, and 17.8mm, the spacing in each instance being 1.9mm. The applied voltage was 60 cycle a.c. continuously rising at the standard rate of 3 kV per second.

The authors applied the Gumbel I distribution to their data i.e. $z = e^{a(V - Vm)}$ (Vm is called the modal voltage).

They calculated "a" by the method of moments. By comparing the results which were obtained with the 4 different electrode sizes, they confirmed the size relationship, which was predicted by Gumbel and which is fundamental to all extreme value statistics. Theory predicts a linear increase of z with size and the results of the experiments conform very well with this theory.

The size effect found by Weber is an area effect. He did not find a dependence on the electrode distance, when the spacing was changed from 1.9mm to 6.35mm. Logically the breakdown event must therefore be electrode dominated, which means that, either the emission of electrons, or the dielectric effects at electrode asperities, and eventually the hydrodynamic effects at the electrode-liquid interface are responsible for the triggering event.

The maxima out of groups of m subsequent experimental data exhibit a size effect of the same nature. This effect is also detected. In a recalculation of the data of group 4 (see section 5.4.5) a standard deviation was found of 554V between the theoretical curve and the experimental data. This is only 1.15% of the average breakdown value, while the data are only listed at 1 kV intervals. The closeness of the fit therefore suggests that only one mechanism is responsible for the breakdown.

3.1.6 T.J. Lewis (Ref.3.5) (1956)

Lewis states in this article that, for insulating oils, the definition of I (which he defined as the rate of electron generation within the gap, in his previous text) must probably be broadened to include initiations by ionic impurity, dust particles and ionisation in gas bubbles.

In paragraphs 8, 9 and 10 Lewis calculates the effect of the sample size on the accuracy of the experimental results. He calculates that the discrepansies between measured results can be accounted for by statistical randomness, allowing errors of as much as 26% when the sample size is 10.

3.1.7 K.H. Weber and H.S. Endicott (Ref.3.7) (1956)

The authors corroborate their experimental results, reported in earlier papers (1.1). This time they used an electrode with a diameter of 457mm. together with the previously used 4 electrodes. The area effect proves again that the weak link concept is valid.

The double exponential distribution is compared with the Weibull distribution. Both of these give acceptable results for all experimental values. However, the Weibull distribution does not fit in well with the minima of groups size effect for one point of the data.

The authors conclude therefore that all other properties being the same, and no theoretical reason being prevalent, the double exponential distribution can be chosen as the best fitting.

3.1.8 B.W. Ward and T.J. Lewis (Ref.3.8) (1959)

A theory for the breakdown time-lag is founded on the following assumptions:

a) Electrons are ejected randomly from the cathode with a mean rate I per second.

b) Each electron has a probability W of initiating a breakdown process.

c) There is a formative time-lag t_f. The formative time-lag corresponds to the time needed for a spark to form, after the initiating event has happened. The statistical time-lag is the time passing before the event happens.

$$R(t) = \exp\left\{-W I (t - t_f)\right\}$$

3.1.9 T.J. Gallagher and T.J. Lewis (Ref.3.10) (1964)

In this article the statistical theory for time-lags is applied to liquid argon. However for the breakdown strength averages, are calculated, assuming normal distribution.

3.1.10 A.J. Beddow and J.E. Brignell (Ref. 3.11) (1965)

The time-lag is composed of a statistical time-lag and a formative time-lag. The statistical time-lag is distributed as

$f(t_s) = U(t_s) \cdot \mu e^{-\mu t_s}$ where $U(t_s)$ is a unit step function.

The formative time-lag is normally distributed with a mean t_o and a standard deviation σ. The combined distribution is given by the convolution of the two densities.

$$f(t) = \frac{\mu}{2} \left[1 + \mathrm{erf}\left(\frac{x}{2k} - k \right) \right] e^{(k^2 - x)}$$

with $k = \frac{\mu\sigma}{\sqrt{2}}$

and $x = \mu (t - t_o)$

The distribution reduces to the negative exponential distribution for large x

$f(t)_{t \gg t_o} \mu e^{-x}$

3.1.11 J.E. Brignell (Ref.1.5) (1966)

Brignell was the first to describe the breakdown event as a weak link phenomenon, both in time space and in voltage space. In voltage space he chooses $z = e^{(aE - b)}$ (Gumbel I) and in time space he uses the convolution already given in his earlier article (Ref. 3.11).

In his discussion Brignell states that the physical fact defining z(x) might not be unique. He considers for instance the formation of particle bridges in the spark gap at low stress levels. He made no experiments however to find the intensity function related to these particle bridges.

Note: Brignell's function notations are inconsistent with the notations of most other authors. He writes P(x) for the probability density function f(x) and R(x) for the cumulative distribution P(x), while $\psi(x)$ corresponds to z(x) the intensity function. He writes reliability (usually written R(x)) as 1 - R(x).

3.1.12 P. Dokopoulos (Ref.3.13) (1968)

Dokopoulos wrote a clear report on the breakdown probability of dielectrics in general. He deals very extensively with the size effect and gives expressions for the mean value, and the standard deviation of the Weibull distribution. He performed experiments

on transformer oil (5mm. spacing) and on an epoxyresin and found a size effect which fitted the Weibull distribution in both cases. He only worked in voltage space.

3.1.13 J.M. Oudin, Y. Rérolle and H. Thévenon (Ref.3.14) (1968)

The authors deal with the breakdown probability of polyethylene insulated high-voltage cables. The article is based on the following three assumptions:

1) Breakdown is a phenomenon characterized by a random variable in 2 dimensions (time to breakdown and electrical stress).
2) Breakdown has its origin in a very small region of the insulator.
3) $w(t) = Ct^a E^b$

To compare two samples of different size the following relationship is used: $N_1 t_1^a E_1^b = N_2 t_2^a E_2^b$ (N_1 and N_2 are size factors)

Graphs of ln w against ln t, give straight lines when the Weibull distribution is valid. Therefore these graphs were used by the authors to find a and b for polyethylene cables. They found a clear discontinuity suggesting that two entirely different mechanisms cause breakdown in cables.

3.1.14 S. Palmer and W.A. Sharply (Ref.3.18) (1969)

The article by Palmer and Sharply is interesting because it is quite unusual and disagrees very strongly with the other authors working in the field of breakdown experimts. Palmer and Sharply fit their observations to a normal distribution and find a very good agreement. (Weibull has shown that the extreme value distributions can be approximated by normal distributions, except for the values of high and low breakdown probability. (Ref. 1.2) p.225.

Experiments are reported which show the dependence of the mean failure stress on the oil volume. The relationship is of the form: $E = A - B \ln S$

S being the physical size of the sample, in this case the size is represented by the volume. This last relationship supports the assumption that breakdown failures are extreme value distributed, in contradiction with Palmer and Sharply's theory.

3.1.15 E. Simo (Ref.3.19) (1969)

The results of a very large number of breakdown data obtained on transformer oil (2 mm. spacing) are reported. Extreme value theory fits the data very well. A volume dependence has not been found, while breakdown shows a dependence on the area of the electrodes.

3.1.16 J. Artbauer and J. Griaç (Ref.3.20) (1970)

The authors do not use the double exponential probability distribution on theoretical grounds. In fact the double exponential distribution gives a small but non-zero probability of breakdown for negative values of the variable. For this reason the Weibull distribution was preferred. The Weibull distribution also predicts better the volume dependence, for the experiments on polyethylenes, which are reported in this article.

3.1.17 E. Occhini (Ref.3.21) (1971)

Occhini uses the adapted Weibull distribution as presented by Oudin and co-authors and applies it to large series of tests on samples of high voltage cable. Experiments are in good agreement with the theory.

3.1.18 H. Luy and F. Oswald (Ref.3.22) (1971)

Measurements on polyethylene are reported. Volume dependence is measured both with and without pressure applied to the test sample and it is stated that the volume dependence is negligible when pressure is applied.

3.2 Conclusions

3.2.1 z(t)

The time delay,, before breakdown occurs, is composed of two distinct parts. A first part, called "statistical time-lag" is a random variable connected with the waiting time before the breakdown initiating event takes place. A second part called "formative time-lag" is equal to the time necessary to accomplish the breakdown, once the initiating event has happened. Most studies only dealt with the first group, which generally was assumed to have a constant intensity function.

The existence of the formative time-lag has been mentioned as early as 1956 by Crowe (Ref.3.6) but has been extensively studied by Brignell and his co-workers (Ref.3.11), (Ref.3.12), (Ref.1.5) and (Ref.3.15). Beddow came to the following results for n-hexane (Ref. 3.15)

$$z(t_s) = \mu \text{ (constant)}$$

$$\text{or } f(t_s) = \mu e^{-\mu t_s} \quad \text{for} \quad 10^{-4}\text{s} < t < 10^{-3}\text{s}$$

$$\text{and } f(t_f) = \frac{1}{\sigma \sqrt{2}} \exp\left\{-\frac{(t_f - t_o)^2}{2\sigma^2}\right\} \text{ for } 10^{-8}\text{s} < t < 3.10^{-7}\text{s}$$

(t_o average value of the formative time-lag, σ standard deviation about the mean).

In between these two regions the real delay-time $t = t_f + t_s$ must be calculated. The corresponding distribution $f(t)$ is the convolution of the two previous distributions (appendix II).

$$f(t) = \frac{\mu}{2}\left[1 + \operatorname{erf}\left(\frac{t - t_o}{\sqrt{2}\,\sigma} - \frac{\sigma\mu}{\sqrt{2}}\right)\right] \cdot \exp\left\{\frac{\sigma^2\mu^2}{2} - \mu(t - t_o)\right\}$$

for $2.10^{-6}s < t < 3.10^{-5}s$ the following approximation is valid:

$$\operatorname{erf}\left(\frac{t - t_o}{\sqrt{2}\,\sigma} - \frac{\sigma\mu}{\sqrt{2}}\right) \longrightarrow 1$$

$$\frac{\sigma^2\mu^2}{2} \longrightarrow 0$$

$$f(t) = U(t - t_o) \cdot \mu e^{-\mu(t - t_o)}$$

Some authors working with solids assume a Weibull distributed time-lag (Ref.3.14), (Ref.3.20), (Ref.3.21)

$$z(t) = \left(\frac{t}{t_s}\right)^{\alpha}$$

Lewis and co-workers (Ref.3.4), (Ref.3.5), (Ref.3.8), (Ref. 3.9) have tried to find a physical model for the function $z(t)$. They assume $z(t) = W\,I$.

W is the probability that an electron in the dielectric is the direct cause of a breakdown. I is the number of electrons injected per second.

Lewis mentioned in (Ref.3.5) that the definition of I must probably be broadened to include impurities and vapour bubbles but he never applied his suggestion.

A comparison with the general form for z (Ref.2.3) gives

$$z(t) = N(t) \cdot g(t) = W\,I$$

It shows that Lewis made explicitly or implicitly following assumptions:

1) The free electrons are weak spots of the dielectric or are the origin of them.
2) The number of weak spots is proportional to the number of free electrons in the dielectric.
3) Each free electron acts independently of the other free electrons.

Strength of the theory:

- The relation between the intensity function and the rate of injection of electrons into the dielectric, qualitatively explains the influence of the electrodes on the breakdown time-lag.
- Lewis separates in his analysis electrode- and liquid-controlled breakdowns, on the base of W being liquid-controlled and on the base of I being electrode-controlled.

Weakness:

- The theory accounts insufficiently for the existence of particle and vapour-activity in the liquid.

3.2.2 z(E)

Weber (Ref.1.1) chose $z(E) = e^{a(E - E_m)}$. He compared his data with the three extreme value distributions suggested by Gumbel (Ref.2.2) p.159 and found the best agreement with Gumbel I (double exponential). In a later article (Ref.3.7) the Weibull distribution was used as well and found to fit the data as well as the double exponential, except in one respect. It was found that Gumbel I better fits the minima of groups size effect. This difference is however insignificant.

The Weber choice has been corroborated by Brignell (Ref.1.5) and Epstein (Ref.3.2), but sufficient proof has never really been given.

The Weibull distribution has been used by most authors working with solids (Ref.3.13), (Ref.3.14), (Ref.3.20), (Ref.3.21).

Dokopoulos (Ref.3.13) also used a Weibull distribution to explain his experiments on transformer oil, and corroborated his choice by considering the size effect in the same way as Weber did earlier.

It must be concluded that sufficient proof has not been given to choose one or the other of the possible extreme value distributions. A physical explanation has not yet been tried for this intensity function.

3.2.3 z(t,E)

Only a few authors have attempted to find the two-dimensional probability distribution of breakdown and, if acceptable analytic functions are available in the two previous cases, this is not true for $z(t,E)$.

Brignell worked out an unrealistic model for the impulse strength of dielectrics (Ref.1.5).

Gzowski (Ref.3.17) tried a formula which gave a numerical correction to the time-lag distribution found by Lewis.

This latter model cannot in general be applied, because it is not possible to calculate $f(t)$ or $f(E)$ from his formula.

Oudin (Ref.3.14) proposed a formula, not for $z(t,E)$ but for $w(t,E)$. It is a generalization of the Weibull distribution $w(t,E) = CE^b t^a$. At the same time he uses $w(t) = Kt^a$.
If one calculates $f(t)$ from $f(E,t)$ one finds that $f(t)$, as given by Oudin and co-authors, is in reality the conditional probability

$$f(t / E \leq E_{max})$$

and is given by

$$f(t / E \leq E_{max}) = Ca\, E_{max}^b\, t^{a-1} \exp\left\{- CE_{max}^b\, t^a\right\}$$

The following relation is clear $K = CE_{max}^b$

$f(t)$ as calculated from $\int_0^\infty f(t,E)\, dE$ gives $f(t) = 0$

The formulae of Oudin and co-authors are thus not entirely consistent with probability theory.

3.3 Some properties of the extreme value distribution.

3.3.1 Exponential distribution:

$$z(x) = \lambda$$

$$w(x) = \int_0^x \lambda \, dx = \lambda x$$

$$R(x) = e^{-\lambda x}$$

$$f(x) = \lambda e^{-\lambda x}$$

$$\bar{x} = E\{x\} = \int_0^\infty \lambda x e^{-\lambda x} dx = \frac{1}{\lambda}$$

$$\sigma^2 = E\{x^2\} - E^2\{x\}$$

$$E\{x^2\} = \int_0^\infty \lambda x^2 e^{-\lambda x} dx = \frac{2}{\lambda^2}$$

$$\sigma = \frac{1}{\lambda}$$

probable value of x

$$x(P) = \frac{1}{\lambda} \ln \frac{1}{1 - P} = \bar{x} \ln \frac{1}{1 - P}$$

The standard deviation is hence equal to the mean value. They are both equal to the inverse of the intensity function. It follows immediately that a small intensity function will correspond to a large mean value and a large standard deviation about the mean.

3.3.2 Weibull distribution:

$$z(x) = S\left(\frac{x}{d}\right)^{\alpha} \quad \text{where S is a size dependent constant.}$$

$$w(x) = \frac{Sd}{\alpha+1}\left(\frac{x}{d}\right)^{\alpha+1}$$

$$f(x) = z(x) \cdot e^{-w(x)}$$

$$dw(x) = z(x)\,dx$$

$$\text{say } \frac{\alpha+1}{S} d^{\alpha} = k$$

$$x = (kw)^{\frac{1}{\alpha+1}}$$

$$z(x) = \frac{S}{d^{\alpha}}(kw)^{\frac{\alpha}{\alpha+1}}$$

$$\text{limits } x\,(0,\infty)$$

$$w\,(0,\infty)$$

$$\bar{x} = \int_0^{\infty} x \cdot z(x) \cdot e^{-w(x)}\,dx = \int_0^{\infty} x \cdot e^{-w}\,dw$$

$$\bar{x} = k^{\frac{1}{\alpha+1}} \int_0^{\infty} w^{\frac{1}{\alpha+1}} e^{-w}\,dw = k^{\frac{1}{\alpha+1}} \cdot \Gamma\left(\frac{\alpha+2}{\alpha+1}\right)$$

$$\sigma^2 = E\left\{x^2\right\} - E^2\left\{x\right\}$$

$$E\left\{x^2\right\} = \int_0^\infty x^2 e^{-w}\, dw = k^{\frac{2}{\alpha+1}} \int_0^\infty w^{\frac{2}{\alpha+1}} e^{-w}\, dw$$

$$= k^{\frac{2}{\alpha+1}} \cdot \Gamma\left(\frac{\alpha+3}{\alpha+1}\right)$$

$$\sigma = k^{\frac{1}{\alpha+1}} \left[\Gamma\left(\frac{\alpha+3}{\alpha+1}\right) - \Gamma^2\left(\frac{\alpha+2}{\alpha+1}\right)\right]^{\frac{1}{2}}$$

$$\text{or } \sigma = \bar{x} \cdot \frac{\left[\Gamma\left(\frac{\alpha+3}{\alpha+1}\right) - \Gamma^2\left(\frac{\alpha+2}{\alpha+1}\right)\right]^{\frac{1}{2}}}{\Gamma\left(\frac{\alpha+2}{\alpha+1}\right)}$$

$$x(P) = \left[k \ln \frac{1}{1-P}\right]^{\frac{1}{\alpha+1}} = \frac{\bar{x}}{\Gamma\left(\frac{\alpha+2}{\alpha+1}\right)} \left[\ln \frac{1}{1-P}\right]^{\frac{1}{\alpha+1}}$$

The mean value, the standard deviation and the value with probability P show the same dependence on the size.

3.3.3 Double exponential distribution (Gumbel I)

$$z(x) = S\, e^{a(x - x^*)}$$

$$w(x) = \frac{S}{a} e^{a(x - x^*)}$$

$$f(x) = z(x) \cdot e^{-w(x)} \qquad dw(x) = z(x)\, dx$$

$$x = x^* + \frac{1}{a} \ln \frac{aw}{S}$$

$$z(x) = aw$$

limits $x\ (-\infty , +\infty)$

$w\ (0 , \infty)$

$$\bar{x} - x^* = \int_0^\infty (x - x^*)\, e^{-w}\, dw$$

$$= \frac{1}{a} \ln \frac{a}{S} \int_0^\infty e^{-w}\, dw + \frac{1}{a} \int_0^\infty \ln w\, e^{-w}\, dw$$

$$\bar{x} = x^* + \frac{1}{a} \ln \frac{a}{S} - \frac{\gamma}{a} \qquad \text{with } \gamma = 0.5772 \ldots \text{ Euler's constant}$$

$$\sigma^2 = E\left\{x^2\right\} - E^2\left\{x\right\} = E\left\{(x - x^*)^2\right\} - E^2\left\{x - x^*\right\}$$

$$E\left\{(x - x^*)^2\right\} = \int_0^\infty \frac{1}{a^2}\left[\ln \frac{aw}{S}\right]^2 \cdot e^{-w}\, dw$$

$$= \frac{1}{a^2}\int_0^\infty \left[(\ln \tfrac{a}{S})^2 + 2 \ln \tfrac{a}{S} \cdot \ln w + (\ln w)^2\right] \cdot e^{-w} dw$$

$$= \frac{1}{a^2}\left[(\ln \tfrac{a}{S})^2 - 2\gamma \cdot \ln \tfrac{a}{S} + \gamma^2 + \frac{\pi^2}{6}\right]$$

$$\sigma = \frac{\pi}{a\sqrt{6}}$$

$$x(P) = x^* + \frac{1}{a}\ln \frac{a}{S} + \frac{1}{a}\ln\ln \frac{1}{1 - P} = \bar{x} + \frac{\gamma}{a} + \frac{1}{a}\ln\ln \frac{1}{1 - P}$$

The standard deviation is independent of the size but proportional to the inverse of a. It can again be seen that a slowly rising intensity function gives a large standard deviation.

The mean value is inversily proportional to the logarithm of the size.

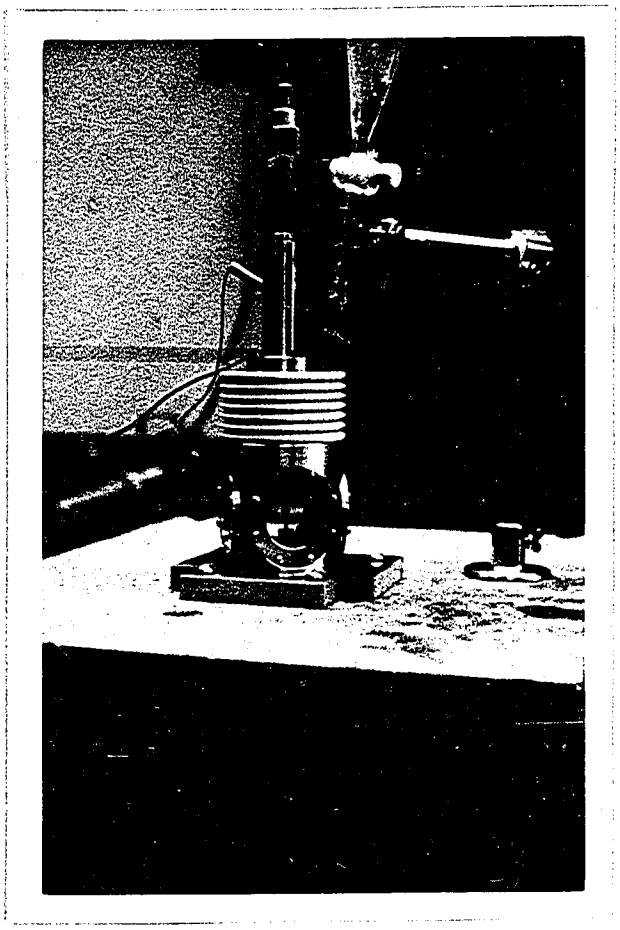
IV EXPERIMENTAL WORK

Many investigators have realized that automatic measurements and a severe control of the experimental parameters are necessary to give reliable data for breakdown measurements, particularly of dielectric liquids. (Ref.1.5), (Ref.3.1), (Ref.4.3).

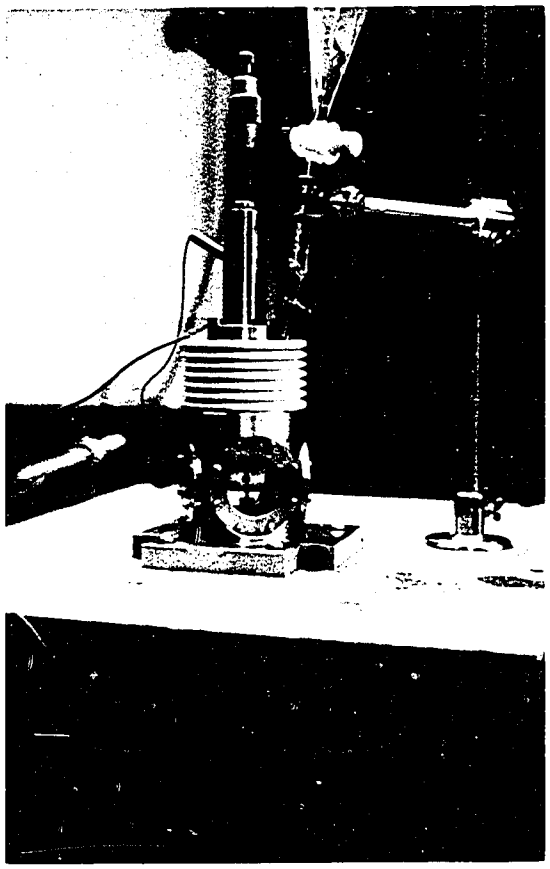
Originally many researchers assumed that the normal distribution represents the statistical spread of breakdown values. This assumption implies that refining the test conditions, should give a better defined mean value with a smaller standard deviation.

Quite the contrary was found. In search for an explanation, different breakdown mechanisms were studied. As has been seen, the extreme value statistics give a logical explanation for this phenomenon. This theory says that the better the purification of the oil and the better the preparation of the electrodes, the less flaws are to be expected and the larger will be the spread in breakdown values.

It has also been mentioned earlier (section 2.4.3) that the reliability is very sensitive to changes in the flaw activity. Therefore measurements must be performed with extreme care to be consistent. Realizing that the human factor often disturbs the quality of experimental results, through unknown and uncontrolled influences, several researchers have designed automatic measurements (Ref.1.1), (Ref.3.1), (Ref.3.5), (Ref.3.7), (Ref.4.1)



A view of the test-cell during the experiments.



A view of the test-cell during the experiments.

4.1 Experimental apparatus.

4.1.1 Test-cell.

The test-cell had a stainless-steel body. The inside space was basically cylindrical with a diameter of $1\frac{1}{4}$ inch and a length of $3\frac{1}{2}$ inch, allowing a volume of liquid of approximately 30 ml. At both ends an insulating spacer of glass-loaded teflon was screwed on to the body. Four quartz windows allowed observation of the electrodes. All sealing rings were teflon. The test-cell was evacuated by an Edwards rotary mechanical pump. The test-cell could be separated from the vacuum system by a glass separating flask.

4.1.2 Electrodes.

The electrodes were pure nickel spheres of approximately 5mm. diameter, and had a $\frac{1}{8}$inch diameter stem. They were slid into an electrode holder and clamped with a set screw.

The electrodes were polished by a soft cotton buffing wheel until no scratches were visible under a x 100 microscope. Then they were polished with felt using tooth-paste as a polishing compound. Thereafter the electrodes appeared perfectly shiny under the microscope. They were first boiled in water and later in acetone, before being put into the test-cell.

The electrode spacings used were 150 μ, 200 μ, and 300 μ. It is well known that spheres give a good approximation to uniform fields, if the radius is much larger than the distance between the spheres. The degree of non-uniformity can be calculated by the formula:

$$\frac{E_{max}}{E_{av}} = \frac{1}{4}\left[k + \sqrt{k^2 + 8}\right]$$

$$\text{with } k = \frac{X}{r} + 1$$

X gap spacing

r radius of the sphere (Ref.4.4) p.6

The following results were obtained:

gap distance (in μ)	150	200	300
E_{max} / E_{av}	1.020	1.027	1.041

The degree of this non-uniformity was not negligible but should not disturb the breakdown statistics. In addition this investigation was aimed more at the nature of the statistical distribution of breakdown values than at the exact value of the breakdown voltage. The electrode distance was adjusted by means of an "Etalon" screwmicrometer, adjustable to within 2 μ.

4.1.3 High-voltage source.

The main lines of the design of the high-voltage D.C. source have been conceived by Dr. F. Spitzer. It is a development of the source designed by him in his research at London University (Ref.4.3).

The major improvement was the incorporation of a choice between 11 different rates of voltage application.

The final design and the construction of this source have been performed by S.F. Luk, as his student project, in the summer of 1970 (Ref.4.6).

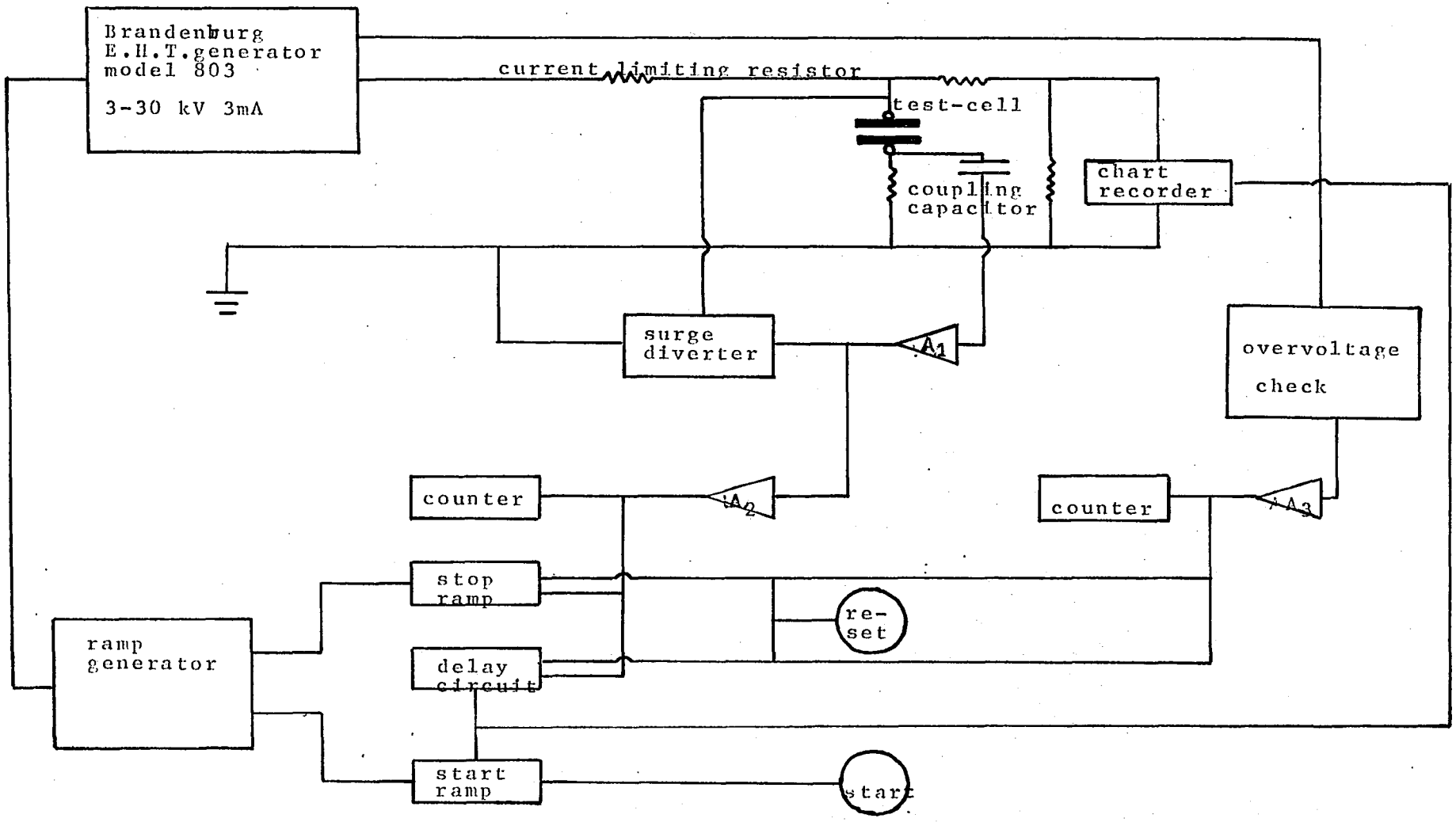
The heart of the high-voltage supply was a "Brandenburg" E.H.T. generator, model 803 (3 to 30 kV at 3 m A). It was normally set at a certain minimum D.C. level (4 kV in these experiments). A ramp generating circuit was coupled to the control circuit of the Brandenburg generator. When the "start" button of the ramp generator was pushed, a ramp was applied to the control of the H.V. generator, which in turn amplified the ramp voltage and applied it to the test-cell. When breakdown occurred a pulse was generated across a resistor between the cathode of the test-cell and ground. This pulse was applied to the ramp generator and fulfilled three functions.

It triggered a thyratron to divert the spark energy from the test-cell, it stopped the ramp and set the voltage back to the original level (4 kV in our experiments), it started a time-delay adjustable from 5 sec to 200 sec. After this time-delay a new ramp was automatically applied. The ramp generator also included a compa-

rator circuit, which stopped the ramp at a preset voltage, if breakdown had not occurred. It reset the ramp generator and started the time-delay. A current limiting resistor of 10 MΩ connected the high voltage supply to the anode of the test-cell. A potentiometric divider applied the anode voltage to a chart-recorder. To calibrate the chart-recorder the voltage was measured directly at the anode of the test-cell with a high-voltage meter. The chart-recorder readings were taken at several voltage levels. After controlling the linearity of the recorder circuit, the ratio was calculated to be 1mm $\hat{=}$ 278.2V.

A block diagram of the whole circuit is given in Fig. 1.

FIG·I BLOCK DIAGRAM OF THE ELECTRICAL TEST CIRCUIT



4.1.4 Askarel.

Askarels are a class of synthetic, non-flammable liquids widely used for capacitor impregnation and transformer insulation. Chemically these liquids are poly-chloro-diphenyls.

The degree of hydrogen-chlorine substitution is reflected in their physical properties: namely viscosity and thermal characteristics. A mixture of askarels of different saturation is used with a slight amount of additives. These additives, called scavengers, neutralize the free chlorine radicals generated by ionisation.

The askarel used in these experiments was a sample of Monsanto 7030 (Lot KM 2636), prepared for experimental use by Monsanto. It contained 70% Aroclor 1254 (penta-chloro-diphenyl)

30% Trichlorobenzene

0.2% Phenoxy Propane Oxide.

The table below shows the relevant characteristics of this liquid compared with other transformer insulation.

Table I

Liquid Dielectric Properties (ASTM Specifications)

	Kinematic Viscosity $m^2/s \times 10^{-6}$	Density kg / m^3	Dielectric Strength (kV)	Dielectric Constant	
Monsanto 7030	9 - 10	1518 - 1528	35	4.2 - 4.5	(Ref.4.7)
Transformer Askarel	5 - 6	1563 - 1528	35	3.8 - 4.2	(Ref.4.7)
Transformer Oil	9 - 10	852 - 882	47.5 - 42.5	2.2 - 2.3	(Ref.4.2)

4.2. Statistics.

4.2.1 Choice of measurements.

Breakdown is a random event both in voltage and in time space. Therefore measurements must be made so that a probability of breakdown can be attached to every point of the voltage versus time space. Because a series of measurements, which scans all points of space is hardly conceivable, a compromise has to be made. Since the ramp rates available from the H.V.supply cover an important part of the voltage versus time plane fairly well, we have used every ramp an equal number of times.

The probability density with respect to voltage can be obtained as the marginal distribution on the voltage axis, while the density with respect to time is obtained in the same way on the time axis.

It has been mentioned earlier (sec.4.1.3) that the Brandenburg generator could not supply voltages from 0 Volts up to maximum, therefore only part of the (t,V) plane is covered and this will cause an error in evaluating the data. If the probability functions involved were known, this error could be calculated. The two marginal density distributions will be calculated to judge the importance of the error.

$$f(V) = \int_0^\infty f(t,V)\, dt$$

Time-lags are measured from 0 up to t_{max}, the distribution can therefore be obtained. The density will only be valid, however for $V \geq V_{min}$. (V_{min} = 4 kV in this case).

$$f(t) = \int_0^{\infty} f(t,V)\, dV$$

The integral can not be calculated because only part of its range is measured. The error can be judged from:

$$f(t) = \int_0^{V_{min}} f(t,V)\, dV + \int_{V_{min}}^{V_{max}} f(t,V)\, dV + \int_{V_{max}}^{\infty} f(t,V)\, dV$$

$\int_{V_{max}}^{\infty} f(t,V)\, dV = 0$, because the voltage range was not restricted upwards.

$\int_0^{V_{min}} f(t,V)\, dV$ remains as an error term.

Two test runs were made on transformer oil using a 150 μ spacing. No breakdowns occurred lower than 5 kV and thus the error was not important. The spread in data was much larger in the measurements on askarel. During the evaluation of the data, it was realized that the error term was important. Therefore the probability density distribution with respect to time could not be evaluated.

4.2.2 Test sequence.

We know that the value of the breakdown voltage depends upon the number of breakdowns previously obtained, on the same test sample, due to decomposition. Beyer has shown that breakdown conditioning exists (Ref.4.5).

We also know that electrodes are damaged by breakdowns, although it is an effect which is difficult to control. To eliminate these influences and others which are perhaps not marked enough to be known, we took care not to perform every set of measurements with exactly the same sequence of ramp rates. On the other hand to obtain measurements which are statistically consistent, a set of twenty-two random numbers was generated. The eleven different voltage slopes were assigned numbers from these 22 (2 numbers for one slope).

24 measurements were made per run, on the base of this sequence, so that a rotation resulted. The test sequence used for the first run was:

1/ 0.5/ 3/ 1.5/ 0.5/ 0.375/ 0.25/ 12/ 0.25/ 1.5/ 6/ 6/
0.75/ 1/ 3/ 0.75/ 30/ 12/ 0.3/ 0.3/ 0.375/ 30/ 1/ 0.5/

given in kV / s.

The second run began with 3 kV / s whereafter 1.5 kV / s and so on.

4.2.3 Influence of the sample size on the expected breakdown value.

As already mentioned, 24 breakdowns were obtained on the same sample with the same electrodes. The number 24 was chosen as a compromise of two conflicting demands. Firstly the number of experiments on the same sample should be as large as possible, to give enough results to make a good statistical analysis possible. Secondly the number of breakdowns must be kept small to avoid excessive damage on the electrodes. A few test runs showed that 24 breakdowns left the electrodes practically unchanged, while 40 breakdowns pitted the electrode surface.

To obtain sufficient data for statistical evaluation the results were evaluated together. The total number of breakdowns with an electrode spacing of 200 μ was 456. 3 runs were performed with a 300 μ spacing, giving 72 data.

Lewis calculated (Ref.3.5) the influence of the sample size on the expected accuracy of the breakdown probabilities. He obtained the following relations:

Probability $\left(\left|\frac{n}{N} - p\right| < \eta\right) \geqslant \alpha$ is approximately equivalent to:

$$\operatorname{erf}\left\{\frac{N\eta + \frac{1}{2}}{Np\,(1-p)^{\frac{1}{2}}}\right\} \geqslant \frac{1}{2}(\alpha + 1)$$

with n number of breakdowns in N trials

p probability for breakdown

η deviation

α confidence level

erf error function

η, the expected deviation of the measured probability, can thus be calculated with a confidence level α. The whole treatment is made on the assumption that the deviations from the theoretical value are normal. This assumption is in the strict sense only meaningfull for small deviations.

Choosing a confidence level $\alpha = 0.95$, the argument of the error function can be calculated.

$$\frac{N\eta + \frac{1}{2}}{Np(1-p)^{\frac{1}{2}}} = 1.96$$

The value of η to be expected is given by:

$$N\eta \leq 1.96 \left[N p(1-p)\right]^{\frac{1}{2}} - \frac{1}{2}$$

In the following table the maximum deviations to be expected with a 95% probability for different numbers of trials, are listed for

$p = 0.5$
$p = 0.95$
$p = 1 - N^{-1}$ (which corresponds to the highest value measured)

Table II in %

N	24	48	72	96	240	456
p =0.5	18	13	11	9.4	6.12	4.5
p =0.95	6.7	5.1	4.3	3.9	1.96	1.9
$p = 1-N^{-1}$	6	3	2	1.5	0.62	0.32

The expected deviation is largest for the mean value and quite considerable when the quantity of data is less than 100. At the high values of breakdown probability (or for the highest recorded values of breakdown) the expected deviation becomes rapidly smaller.

4.3 Experimental Work

4.3.1 Preparation of the test-cell.

Each part of the test-cell was boiled separately in acetone. The cell was assembled and evacuated for half an hour at 1 mm Hg. Then a cleaning followed with:

Trichloroethylene ($CHCl-CCl_2$)

and 2 - Propenal ($CHCO-CH_3$)

Afterwards the cell was left empty and evacuated for at least 15 minutes. Then 3 flushes with chloroform ($CHCl_3$) followed. Finally after a thorough evacuation the test-cell was rinsed with some of the askarel.

During the experiments some doubts arose concerning the effectiveness of the cleaning procedure. We noticed that heavy carbons and presumably polymerisation products were deposited on the electrodes. Therefore other cleaning agents were used from the 12th run on. Firstly a mixture of 10% aniline ($C_6H_5-NH_2$) and 90% toluene ($C_6H_5-CH_3$) was introduced. This mixture is known to be a very good solvent of heavy hydrocarbons. The cell was cleaned out with toluene to remove the remaining aniline as well as the askarel. Finally trimethylpentane ($(CH_3)_3 C C H_2 C H(CH_3)_2$) was used because of its excellent

properties as a solvent and its high vapour pressure. An evacuation period followed before new askarel was introduced.

4.3.2 Introduction of the askarel and conditioning.

The askarel was evacuated after its introduction into the test-cell to avoid the possible influence of air on the breakdown characteristics of the liquid. It has been reported (Ref.3.15) that breakdowns obtained in this manner, were lower in value than when the experiments were done under atmospheric pressure, but the elimination of external influences was pursued more than high breakdown values. The modest evacuation to 1mm Hg avoided boiling the trichlorobenzene which is the lightest component of the askarel sample.

At the same time the askarel was stress conditioned by applying 4 kV at a gap of 1000 μ initially.

After at least half an hour the gap was gradually reduced until the final distance was reached. In this manner the dielectric was evacuated and conditioned at the same time. It took about 90 minutes until no accidental sparks occurred and the liquid showed a continuous laminar motion. The dielectric was maintained 5 cm above the electrodes, therefore the hydrostatic pressure at the electrode region could be estimated at 750 N/m^2 or 5.6 mm Hg. (For transformer oil these values are 430 N/m^2 or 3.2 mm Hg.)

The total pressure at the electrodes can be estimated at 7 mm Hg (5 mm Hg respectively).

The temperature of the dielectric was maximum 30,2 °C and minimum 23 °C during these experiments.

4.3.3 Experiments

The first two experimental runs were performed on a general purpose transformer oil, with the gap distance set at 150 µ. The same dielectric sample and the same electrodes were used for both runs.

The twenty subsequent runs were done on askarel (Monsanto 7080) with a gap spacing of 200 µ. Again the first two runs were performed with the same electrodes and the same liquid. After this set of 20, another 3 runs used a 300 µ spacing, again with askarel as a dielectric. The sequence of ramp voltages was described under 4.2.2.

The time delay between successive voltage application was 200 s. (Spitzer (Ref.4.3) used 220 s.)

From the third run on, the cathode was cleaned every 6 breakdowns. Initially the cathode was cleaned in Chloroform, later in a mixture of aniline and toluene, followed by trimethylpentane, as explained in section (4.3.1).

The cathode was then replaced and the cell was evacuated and conditioned for at least twenty minutes.

4.4 Observations

4.4.1 Gassing

At the beginning of the conditioning period fast-growing vapour bubbles appeared between the electrodes. These emanated mainly from the anode and always appeared in groups. The smallest which were seen, were as small as 0.1 mm. Because this is the limit of visibility it may be suggested that the bubbles were initially much smaller.

The difference in the appearance of gas bubbles with or without voltage application was marked so that a voltage induced mechanism must be assumed.

Krasucki discussed the formation of microbubbles on surface asperities and on impurity particles under electrode stress (Ref.14)

In our experiments the effect was very pronounced because pumping continued during the conditioning period.

4.4.2 Effects of breakdowns on the electrodes.

The first two runs were performed on transformer oil, as mentioned in section (4.3.3). The same oil and the same electrodes were used for the two sets of observations. As could be expected the breakdown values for the second of those sets were in general lower

and the spread in these values was smaller than for the first. The electrode surfaces were then roughened while no marked damage was detected after the first 24 breakdowns.

The same trends were noticed for the first two sets in askarel and again the breakdown values for the second set were significantly lower. After these two sets a heavy layer of carbon and gel (presumably polymerised or decomposed askarel) coated the cathode. The thickness of this layer was estimated at 50 μ by comparison through a microscope with the size of the electrode.

To avoid the coating effect, the cathode was cleaned every 6 breakdowns following the second experimental run. At the end of the 24 breakdowns which were performed per run, the electrode damage was very limited, but different in pattern between anode and cathode. The cathode was centrally pitted while the anode damage was diffused.

4.4.3 Appearance of breakdown-sparks.

It was noticed that two distinctly different kinds of sparks were generated during breakdown. The first kind was very short and white and was immediately extinguished by the discharge diverter. The second kind was associated with the rapid growth of a vapour bubble. This spark occurred within the bubble. It had a colour turning from clear blue to violet and was difficult to extinguish.

There were very little indications to clarify the nature of the breakdown initiating event, connected with these two families of sparks.

V EVALUATION OF THE EXPERIMENTAL DATA.

5.1 Use of the experimental results.

No matter what the exact form of the intensity functions is, an estimate for R and w can always be calculated from the experimental results. Therefore the first analysis gave plots of R versus V, the breakdown voltage, and plots of w versus V.

5.1.1 Estimates of R

R is a measure of the number of breakdowns which occur in the interval (x,∞) x being the test variable. An estimate for R is easily obtained by counting the number of breakdowns for a value equal to or larger than x and dividing that number by the total number of tests within the same set of experiments.

In practice the following procedure was followed. The breakdown voltages were sequenced in descending order. An order number was assigned to each of these measurements. The quotient of the sequence number and the total number of measurements will approach the reliability when the number of experiments tends to infinity. (This definition corresponds to the "relative-frequency" definition of probability (Ref.2.4) p.8).

In a real set of experiments the following adjustment will have to be made. The smallest value recorded may not be the smallest value possible and therefore we normalize by N + 1 instead of N (N number of observations). This correction avoids assigning the reliability "1" to the smallest recorded value.

5.1.2 Estimates for w and z.

w is simply found by taking -ln R.

Estimates for z can be found in two different ways:

$$z = -\frac{dw}{dx} = -\frac{d}{dx} \ln R$$

or

$$z = -\frac{1}{R} \cdot \frac{dR}{dx}$$

Analytically the two methods give identical results. Numerically however they will be very differently influenced by the measurement- and estimation errors. The Langrangian method of third degree was used in both cases for the numerical differentiation.

$$\left.\frac{dy}{dx}\right|_i \simeq \frac{1}{\Delta x_i + \Delta x_{i-1}} \left[\frac{\Delta x_{i-1}}{\Delta x_i} y_{i+1} - \frac{\Delta x_i}{\Delta x_{i-1}} y_{i-1} \right] + \frac{\Delta x_i - \Delta x_{i-1}}{\Delta x_i \cdot \Delta x_{i-1}} y_i$$

where $\Delta x_i = x_{i+1} - x_i$

The results obtained by the two calculation methods were very different when the evaluation was done point by point. It was therefore concluded that the numerical errors involved were much too large and the estimate useless. Therefore all the emphasis was focussed towards the use of w.

5.1.3 Advantages and disadvantages of the use of w.

R is very little affected by small changes in the behaviour of the test sample, while w better shows the influences. Therefore plots of w yield much more information about the breakdown event than do plots of R. However a disadvantage to the use of w, is the non-linearity of the logarithm-function. As a result the numerical errors on the values of R will be transformed. A 95% probability interval for the error on w can be calculated from the result of section (4.2.3).

We find:

$$0 \geq \varepsilon(w) \geq -\ln\left\{1 + 1.96\left(\frac{1-R}{NR}\right)^{\frac{1}{2}} - \frac{1}{2NR}\right\}$$

Choosing $N = 24$, estimates of ε can be calculated for different values of R.

$$R = \frac{1}{N} \qquad \varepsilon(w) \geq -0.9$$

$$R = \frac{1}{2} \qquad \varepsilon(w) \geq -0.3$$

$$R = \frac{N-1}{N} \qquad \varepsilon(w) \geq -0.06$$

which shows that the error can be as large as 0.9 for the high values of breakdown.

5.2 Curves obtained.

An initial analysis was made on the results of runs 1, 3, 4, 5 and 6, giving 120 experimental observations. The results of this analysis have been reported in an article by F. Spitzer and the author. (Ref.5.2).

As expected the curve R(V) - V, suggested a smooth inverted S-curve to connect the experimental data. Conversely the w(V) - V curve showed a marked discontinuity slightly above 9 kV (Fig. II). Of course the same discontinuity appears in the R(V) - V curve (Fig. III), but originally the spread of data was too small to allow the recognition of any irregularity. When later the same analysis was performed on the set of 458 measurements obtained in runs 1 to 20 (2 excluded) the discontinuity had disappeared. (Fig. IV)

Thereafter the runs were grouped and curves were produced. Some showed a discontinuity, some did not. Through comparison the following was concluded.

1) A discontinuity in the w-V curve can appear at a breakdown value which is slightly higher than 9 kV.
2) Some of the runs had none or very few values higher than 9 kV. They were associated with very noisy breakdowns with large vapour evolutions (4.4.3 second kind) (runs 3, 4, 5, 7).
3) A second group had a considerable number of large break-

down values and was associated with sharp discharges (4.4.3 first kind) , (runs 14, 15, 17, 18, 20)

4) Many runs could not be classified because of the subjectiveness of the criteria up to this point. (runs 1, 6, 8, 9, 10, 11, 12, 13, 18, 19)

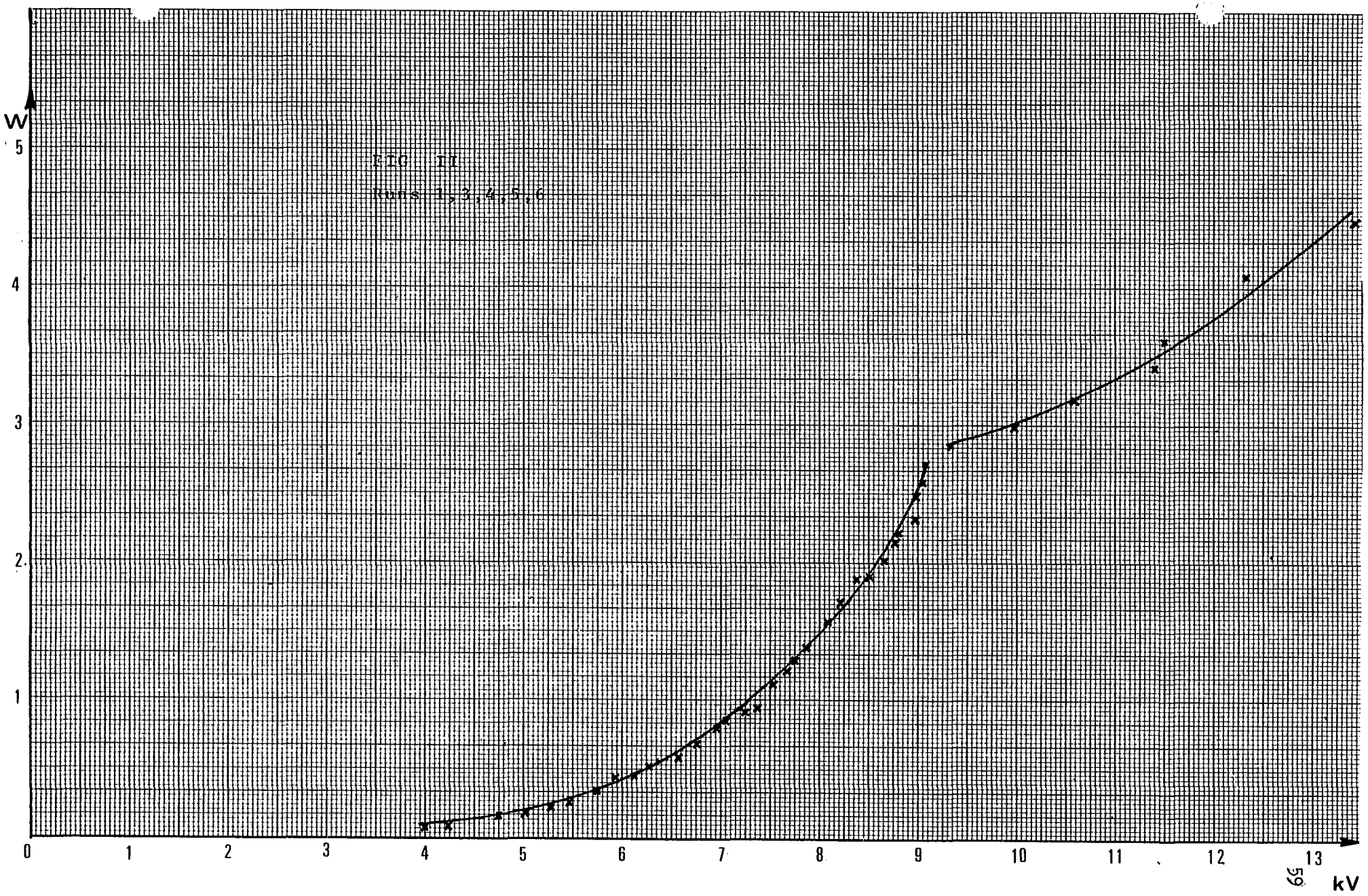
The curves obtained are shown on Fig. V. It can be seen that curves 2) and 3) are both smooth curves. The intensity function associated with curve 2), is much higher in value than the intensity function associated with curve 3).

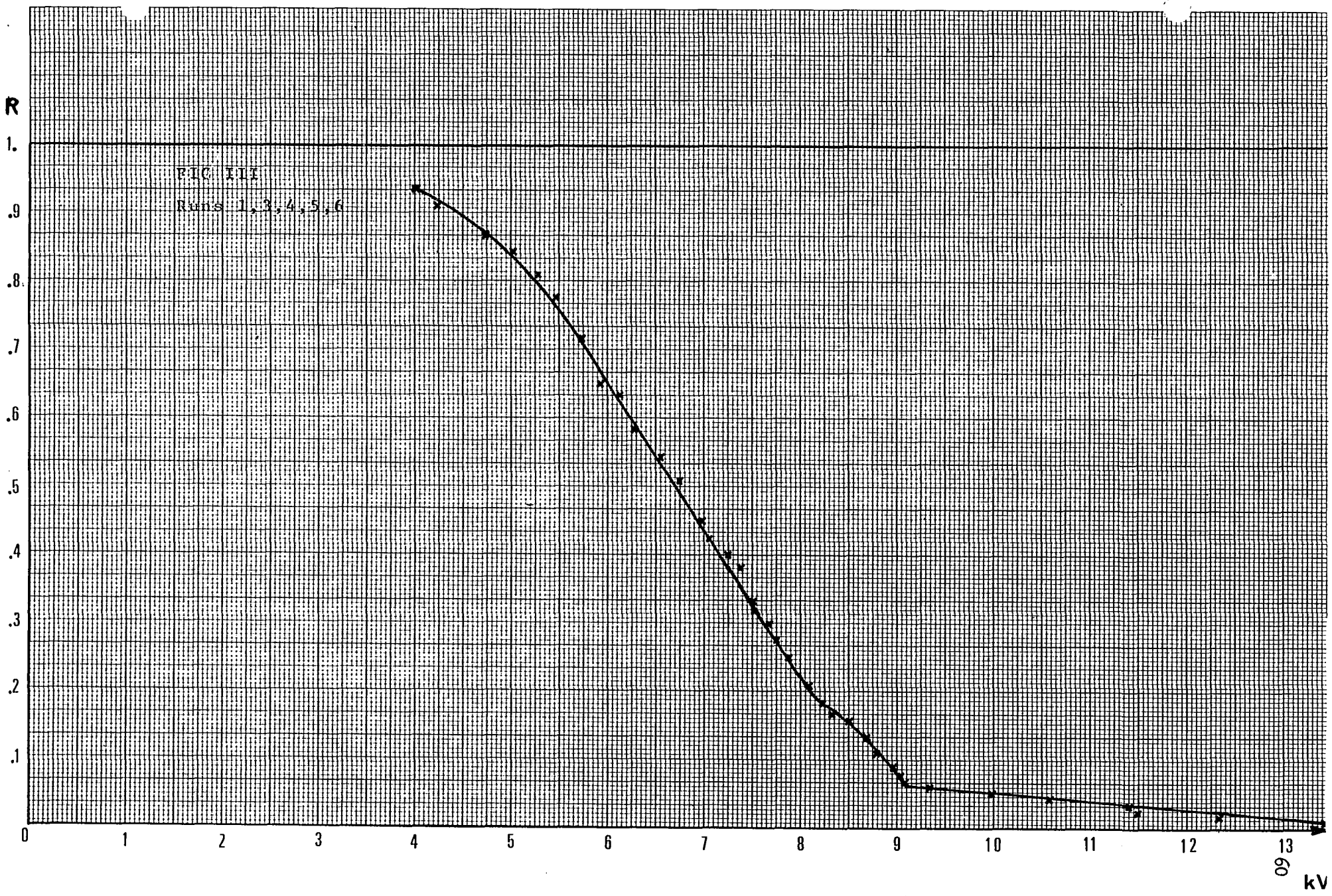
The remaining runs 4) do not form a smooth curve in between the two previous ones, instead their curve tends to curve 3) at the higher values and follows curve 2) at the lower values. The change-over occurs in the region of 9kV. It can be concluded that two distinctly different mechanisms are the cause of breakdowns in askarels.

When a comparable amount of the two families of breakdowns occur in the same set of experiments a discontinuity in the w vs. V curve appears at a breakdown value slightly higher than 9 kV.

A second series of measurements has been made on the same kind of askarel but with a gap spacing of 300 μ. A discontinuity was found at about 9.7 kV (Fig. VI). The breakdown value at the discontinuity was thus slightly higher but not proportional to the electrode distance.

The nature of the electrode gap dependence was not investigated further.


W
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
kV
FIG II
Runs 1, 2, 4, 5, 6


R
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
kV
PIC III
Runs 1,3,4,5,6

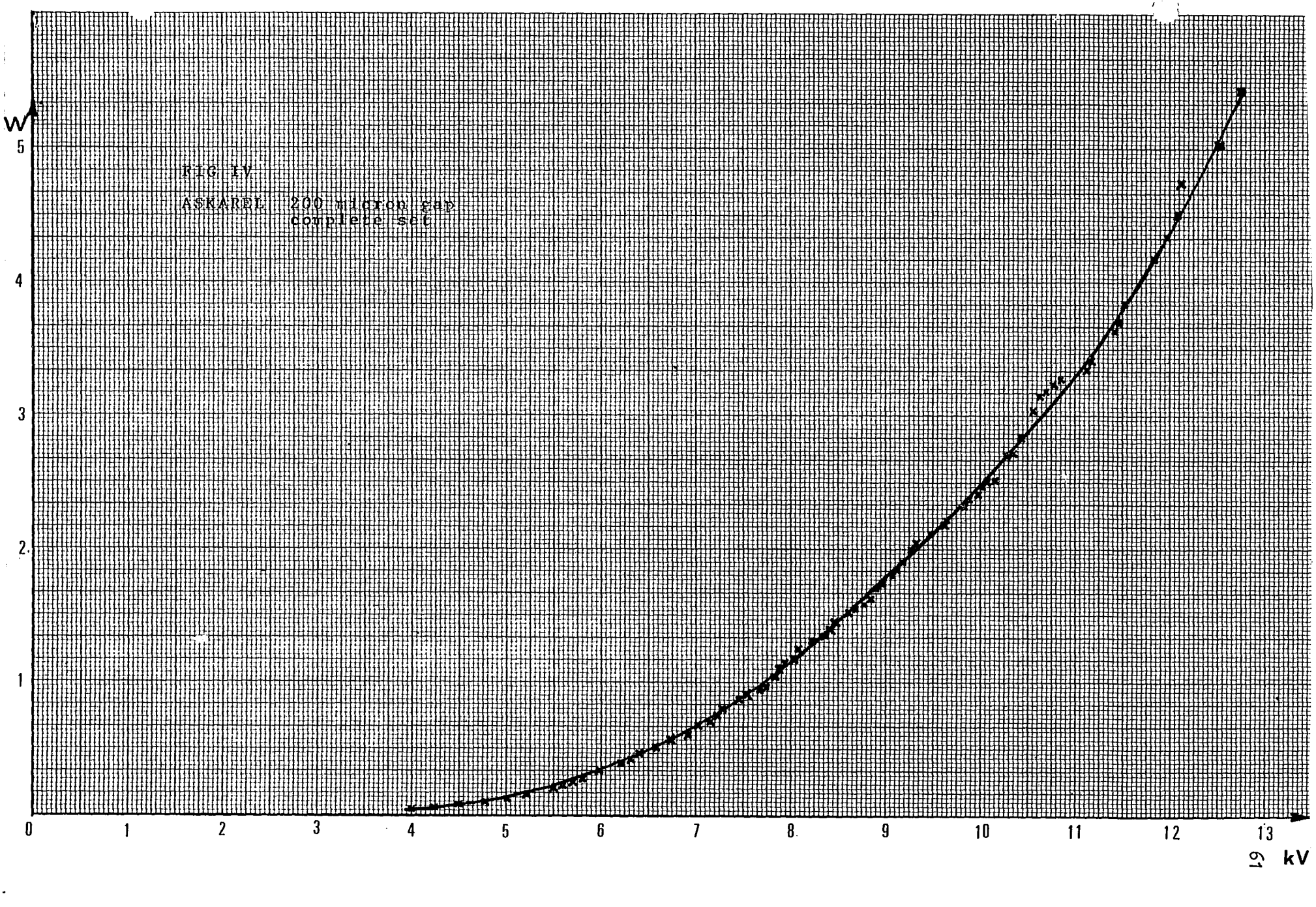


FIG IV

ASKAREL 200 micron gap
complete set

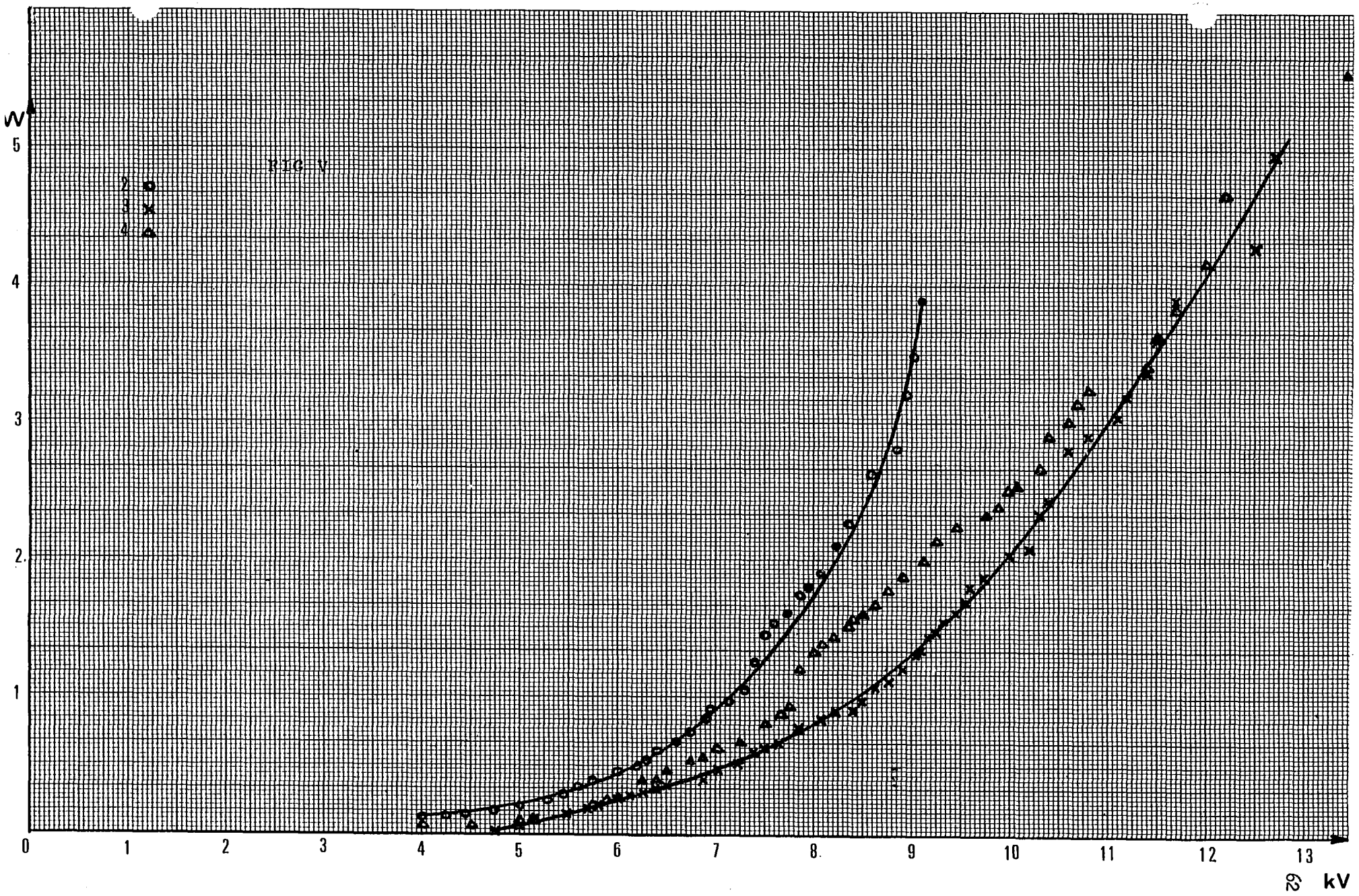


FIG V

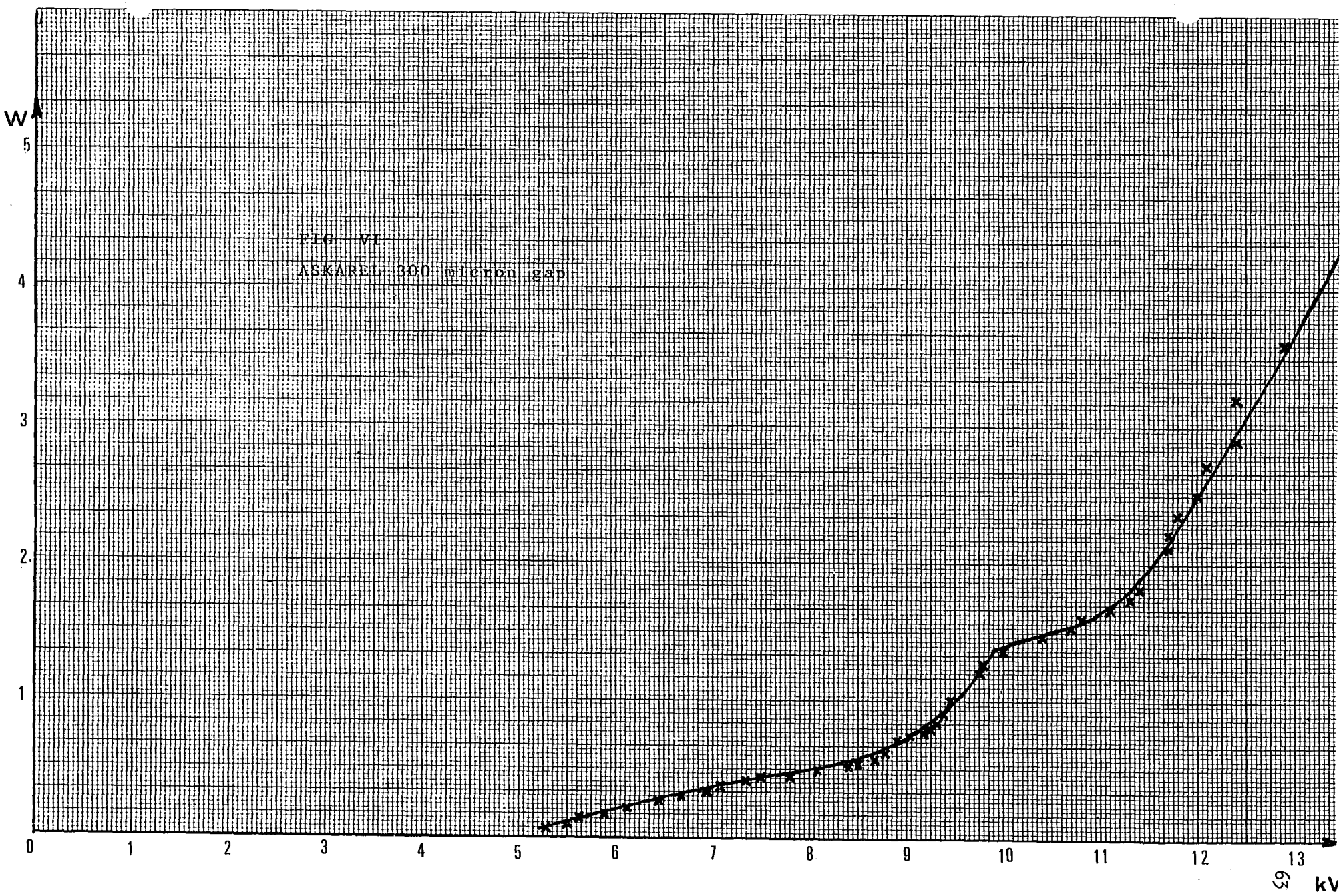


FIG VI

ASKAREL 300 micron gap

5.3 Curve fitting.

It is mathematically cumbersome and numerically very difficult to scan all possible functions of the family

$$f(x) = z(x) \cdot \exp (- w(x)).$$

We will restrict ourselves to the two most publicized groups

$$z(x) = K \cdot e^{ax}$$

and $$z(x) = M \cdot x^{\alpha}$$

A parameter estimation will be performed in the two cases and an attempt will be made to choose the better fitting one.

5.3.1 Maximum-likelihood estimators.

The method of maximum-likelihood estimators will be used to estimate the parameters of the probability distributions. It has been widely applied in the field of statistics and is well documented (Ref.2.5) p.86. The method will be briefly reviewed.

Assume a probability function $f(x,\theta)$ for the random variable x. A parameter to be estimated is θ. The probability that the measured quantity x_i will be equal to the real value for x_i is $f(x_i,\theta) \cdot dx_i$.

Since the measured quantities are mutually independent, their joint density function can be written as the product of the marginal density functions.

Thus $L(x_1, x_2, \ldots, x_N; \theta) \cdot dx_1, dx_2, \ldots, dx_N = \prod_{i=1}^{N} f(x_i, \theta).dx_i$

The joint density function L is called the likelihood function. The most likely value of 0 will be the one which maximizes L. In practice we make use of the one-to-one nature of the mapping of the functions max L and max(ln L).

$$\text{Call } \mathcal{L} = \ln L = \sum_{i=1}^{N} \ln f(x_i,\theta) = \sum_{i=1}^{N} \ln \left\{ z_i R_i \right\}$$

The most likely value for θ will be found to be the solution of the equation $\frac{\partial \mathcal{L}}{\partial \theta} = 0$

$$\text{In our case } \frac{\partial}{\partial \theta} \sum_i \left\{ \ln z_i - w_i \right\} = 0$$

5.3.2 Assume $z = Ke^{ax}$ (double exponential distribution)

$$w = \frac{K}{a} e^{ax}$$

The two parameters "a" and "K" are to be estimated.

$$\frac{\partial \mathcal{L}}{\partial a} = \sum_{i=1}^{N} \left\{ x_i - \frac{K}{a} x_i e^{ax_i} + \frac{K}{a^2} e^{ax_i} \right\} = 0 \quad (I)$$

$$\frac{\partial \mathcal{L}}{\partial K} = \sum_{i=1}^{N} \left\{ \frac{1}{K} - \frac{e^{ax_i}}{a} \right\} = 0 \quad (II)$$

$$\text{or} \quad \sum_{i=1}^{N} e^{ax_i} = \frac{aN}{K}$$

$$\text{and} \quad \sum_{i=1}^{N} w_i = N$$

$$(\text{I}) \longrightarrow \quad \sum_i x_i - \frac{K}{a} \sum_i x_i \, e^{ax_i} + \frac{K}{a^2} \cdot \frac{aN}{K} = 0$$

$$\text{or} \quad \frac{N}{a} = \frac{K}{a} \sum_i x_i \, e^{ax_i} - \sum_i x_i$$

A first estimate for "a" will be obtained when solving

$$\frac{1}{a} = \frac{1}{N} \sum_{i=1}^{N} x_i \, (w_i - 1)$$

It must be remembered however that the values we have for w_i are themselves estimates and not measured values, therefore they can not be relied upon for more than a first approximation. A further estimate will be found by iteration.

$$\frac{1}{a_{j+1}} = \frac{\sum x_i \, e^{a_j x_j}}{\sum e^{a_j x_j}} - \frac{\sum x_i}{N}$$

Finally $K = \dfrac{\hat{a}\, N}{\sum e^{ax_i}}$

The intensity function can be written under a different form

$$z = e^{a(x - x_m)} = K\, e^{ax}$$

$x_m = -\dfrac{1}{a} \ln K$ follows immediately

5.3.3 Assume $z = M x^{\alpha}$ (Weibull distribution)

$$w = \frac{M}{\alpha+1}\, x^{\alpha+1}$$

Solution for M and α:

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \sum_i \left\{ \ln x_i + \frac{M x_i^{\alpha+1}}{(\alpha+1)^2} - \frac{M x_i^{\alpha+1}}{(\alpha+1)} \ln x_i \right\} = 0 \qquad (I)$$

$$\frac{\partial \mathcal{L}}{\partial M} = \sum_i \left\{ \frac{1}{M} - \frac{x_i^{\alpha+1}}{\alpha+1} \right\} = 0 \qquad (II)$$

$$(II) \longrightarrow \quad N \frac{\alpha+1}{M} = \sum_i x_i^{\alpha+1}$$

and again $\sum_i w_i = N$

$$(\mathrm{I}) \longrightarrow \sum_i \ln x_i + \frac{M}{(\alpha+1)^2} \cdot \frac{\alpha+1}{M} \cdot N - \frac{M}{\alpha+1} \sum_i x_i^{\alpha+1} \ln x_i = 0$$

$$\frac{N}{\alpha+1} = \frac{M}{\alpha+1} \sum_i x_i^{\alpha+1} \ln x_i - \sum_i \ln x_i$$

$$\text{or} \quad \frac{N}{\alpha+1} = \sum_i (w_i - 1) \cdot \ln x_i$$

Again a first approximation for α is obtained from

$$\frac{1}{\alpha+1} = \frac{1}{N} \sum_{i=1}^{N} (w_i - 1) \cdot \ln x_i$$

A next estimate will be found by iteration

$$\frac{1}{\alpha_{j+1}+1} = \frac{\sum_i x_i^{\alpha_j+1} \ln x_i}{\sum_i x_i^{\alpha_j+1}} - \frac{\sum_i \ln x_i}{N}$$

$$\text{and} \quad M = \frac{N \cdot (\alpha+1)}{\sum_i x_i^{\alpha+1}}$$

A second form for z is: $z = \left(\frac{x}{b}\right)^{\alpha} = M x^{\alpha}$ thus $b = M^{-\frac{1}{\alpha}}$

Note: b and x_m must have the same numerical value, indeed for $z = 1$, we find in the Weibull case $x = b$ and in the double exponential case $x = x_m$

5.3.4 Comparison of the functional approximations with the measured values.

As outlined before (sec. 5.1.1) an estimate for R can be found by dividing a sequence number by a number representing the number of observations from which the data are taken. Say "A", is the numerical value for this number.

$$R_{ic} = \frac{i}{A}$$

$$\sum_{i=1}^{N} R_{ic} = \frac{1}{A} \sum_{i=1}^{N} i = \frac{N(N+1)}{2A}$$

$$\text{Thus } A = \frac{N(N+1)}{2 \sum_{i=1}^{N} R_{ic}}$$

The two function parameters can be estimated through previous methods. Thereafter R_{ic} can be calculated and used to find A. Ideally A must be equal to N. When the set of measurements which is studied is complete, A must be close to N. Then the calculation of A gives a test on the validity of the approximations.

The standard deviation provides a second control.

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - x_{ic})^2$$

x_i is a measured value

x_{ic} is calculated knowing R_i

5.3.5 In section (3.3) a parameter for the size effect was introduced in the general form of z. The relation with the parameters found here is simple.

$$\text{Gumbel I: } z = K e^{ax} = e^{a(x - x_m)} = S e^{a(x - x^*)}$$

$$\text{thus} \qquad K = S e^{-ax^*} = e^{-ax_m}$$

$$\text{Weibull: } z = M x^{\alpha} = \left(\frac{x}{d} \right)^{\alpha}$$

$$\text{and} \qquad M = S d^{-\alpha} = b^{-\alpha}$$

The influence of the electrode size, the electrode spacing and the liquid volume have not been measured. Therefore the size effect can not be calculated and the general expression will not be used. The expression with x_m and b is most attractive but has "a priori" no physical meaning.

5.3.6 Rate of convergence

The iterative methods used to calculate "a" and "α" are one-point iteration methods. They can both be characterized by the iteration formula

$$\frac{1}{k_{j+1}} = F(k_j) - C$$

Say $F(k_j) - C = F(k) + \varepsilon_j G(k_j) - C$

with $G(k_j) \simeq \left.\frac{\partial F}{\partial k}\right|_{k_j}$ for ε_j small enough

$$F(k_j) - C = \frac{1}{k} + \varepsilon_j G(k_j)$$

$$k_{j+1} = \frac{k}{1 + \varepsilon_j k G(k_j)}$$

$$\varepsilon_{j+1} = k_{j+1} - k = \frac{-\varepsilon_j k^2 G(k_j)}{1 + \varepsilon_j k G(k_j)} \simeq -\varepsilon_j k^2 G(k_j)$$

Under the condition that $G(k_j)$ is continuous, which is true for the functions used here, a number G can be chosen so that

$G \geqslant G(k_j)$ for any k_j used in the iteration and $|\varepsilon_{j+1}| \leq |\varepsilon_j| \cdot k^2 G$

The iteration will converge when $k^2G < 1$

A calculation of a and G on the data of runs 17 to 20 gives

$$a^2G < 0.83$$

The data of runs 3 to 7 give

$$a^2G < 0.9$$

During the iteration the calculated approximation oscillated around the final value. Therefore as a refinement, the new approximation was taken as the average of the calculated estimate and the previous estimate.

Applying Aitken's method after 5 approximations immediately gave a very good estimate, but convergence when taking averages was already so good that there was no reason to complicate the iteration by the use of Aitken's method. About 10 iterations were needed to find 5 decimal figures.

The new iteration formula is given by

$$k_{j+1} = \frac{k_j + k'_{j+1}}{2}$$

with $$k'_{j+1} = k - \varepsilon_j k^2 G(k_j)$$

thus $$k_{j+1} = \frac{1}{2}\left[k_j + k - \varepsilon_j k^2 G(k_j)\right]$$

$$k_{j+1} = \frac{1}{2}\left[k + \varepsilon_j + k - \varepsilon_j k^2 G(k_j)\right]$$

$$k_{j+1} = k + \frac{\varepsilon_j}{2}\left[1 - k^2 G(k_j)\right]$$

and $$k_{j+1} - k = \frac{\varepsilon_j}{2}\left[1 - k^2 G(k_j)\right]$$

or $$\varepsilon_{j+1} < \frac{\varepsilon_j}{2}$$

After N iterations $$\varepsilon_N < 2^{-N} \varepsilon_1$$

Convergence will be fast and very stable for $0 \leq k^2 G \leq 1$ Convergence is possible even for negative values of G provided $-1 < k^2 G$, but the speed will decrease.

In practice the first estimate, obtained by using w, gave only a very rude approximation. Occasionally it could not even be used to start the iteration because it gave a negative value as a second estimate. In fact the iterations could be started by using any reasonable positive number.

5.4 Results.

5.4.1 For every individual run, the parameters of the Weibull distribution as well as the double exponential distribution were calculated. At the same time A was estimated and the standard deviation between the measured and the calculated breakdown voltage was given.

Fairly large discrepansies appeared in the results. Statistically the number of 24 values was too small to give a good agreement between different sets. Physically, the existence of two mechanisms (see 5.2) of breakdown, with the possibility that neither of the two was predominant, made any good approximation impossible.

The results are listed in Appendix III.

The smallest standard deviation occurred in the runs 2, 9, 3, 7, 5 and 4, for which we know fairly well from the experiments and the graphs of section (5.2) that only the first mechanism was active. For the same runs the difference between V_m and b was small and the calculated A was fairly close to N, the number of breakdowns.

Despite the discrepansies an attempt was made to find runs with a similar characteristic. They were plotted on a: a versus V_m, and on a: α versus b plot, respectively shown in Fig. VII and VIII.

Two zones were formed with following limiting values:

for the first zone, in the a vs. V_m plane $9.63 < V_m < 11.514$

$0.388 < a < 0.749$

in the α vs. b plane $9.818 < b < 12.348$

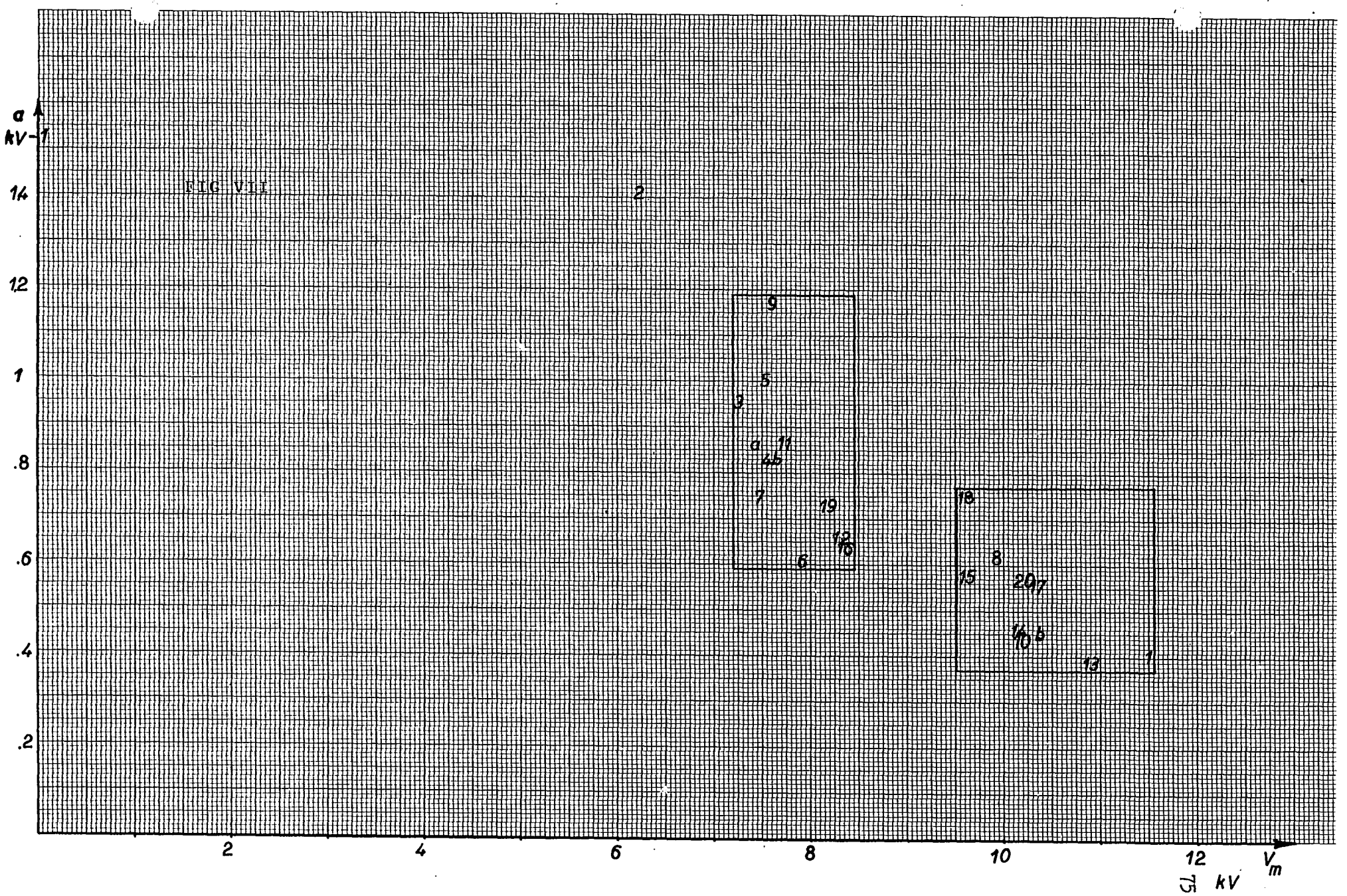
$2.073 < \alpha < 5.174$

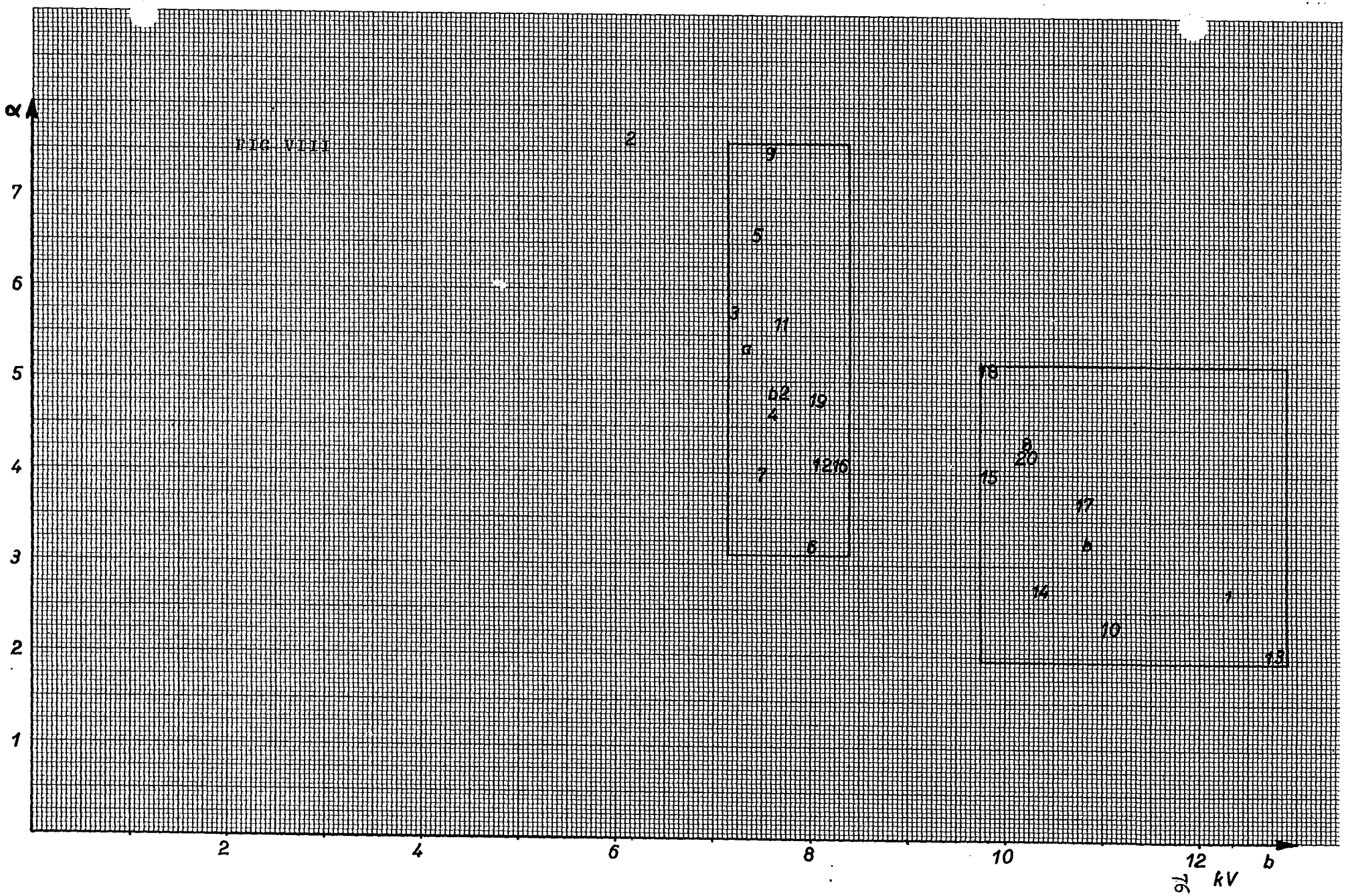
the second zone is described by $7.286 < V_m < 8.369$

$0.602 < a < 1.167$

or $7.226 < b < 8.306$

$3.224 < \alpha < 7.54$


a
kV⁻¹
14
12
1
.8
.6
.4
.2
2
4
6
8
10
12
V_m
kV
FIG VII


α
1
2
3
4
5
6
7
2
4
6
8
10
12
b
kV
FIG VIII

There is a clear gap of 1.26 kV in the values for b and V_m, while a corresponding gap does not exist for a and α. In addition the division into two groups corresponding to V_m and b, respects the division of(sec.5.2).

So we came to the following division:

a) Runs corresponding to the first group(noisy breakdowns) 3,4,5,6,7,9,11,12,16,19.

b) Runs corresponding to the second group (sharp discharges) 1,8,10,13,14,15,17,18,20.

The results are listed in Table III.

Table III

Group	$a(kV^{-1})$	$V_m(kV)$	σ_a (V)	A_a	N
a	0.911	7.404	135	107.5	109
b	0.497	10.341	330	134.5	138
	α	b(kV)	σ_α (V)	A_α	N
a	5.35	7.381	68	109.2	
b	3.26	10.804	196	136.6	138

Group "a" gave an excellent fit with standard deviations which are perfectly justifiable.

A part of the error term causing a difference between the calculated and the measured values is due to the reading error on the chart-recorder output. The standard deviation of a reading error can be calculated to be equal to the range of the error divided by the square root out of 12. By this rule a standard deviation of 80 V must be expected when the reading accuracy of the plots is 1mm.

Considering that this is a minimum estimate, counting only with one source of error, the results of group "a" are very satisfactory.

The standard deviations of group "b" were larger. No better could be expected, because group"b" clearly must have breakdowns of both the first and the second kind.

As w vs. V plots show a discontinuity at 9.18 kV, the calculations were repeated on the subsets of group "b", and the results are given in Table IV p.79.and p. 80, of which the explanation of the different combinations listed, is given on page 81.

Table IV

Runs	$a(kV^{-1})$	$V_m(kV)$	$\sigma_a(V)$	A_a	N
b_2	0.814	7.825	222	74.4	75
3,5	0.974	7.435	215	41.5	42
4,7	0.796	7.548	233	44.9	45
7,9	0.881	7.577	157	46.4	46
4,11	0.842	7.698	164	45.8	46
6,7	0.656	7.735	295	45.4	46
6,12,19	0.660	8.156	407	67.1	69
12,16	0.656	8.337	496	45	46
b_1	0.834	11.694	296	27.8	27
17,20	0.561	10.287	244	45.8	46
14,20	0.471	10.270	540	45.3	46
8,20	0.590	10.121	417	43.9	44
8,15	0.596	9.856	341	46.4	48
15,18	0.633	9.728	218	47	47
7,18	0.558	9.319	325	48.9	49

Table IV (Continued)

Runs	α	b(kV)	σ_α(V)	A_α	N
b_2	4.730	7.880	189	75.6	75
3, 5	6.136	7.364	162	41.9	42
4, 7	4.436	7.588	208	45.5	45
7, 9	5.105	7.599	149	47.2	46
4, 11	5.083	7.677	104	46.5	46
6, 7	3.549	7.784	213	45.9	46
6, 12, 19	3.985	8.166	280	67.9	69
12, 16	4.128	8.278	357	45.4	46
b_1	8.227	11.700	259	28.3	27
17, 20	3.990	10.530	209	46.6	46
14, 20	3.135	10.546	419	45.7	46
8, 20	4.312	10.253	299	44.5	44
8, 15	4.133	10.044	223	47.1	48
15, 18	4.380	9.913	194	48	47
7, 18	3.193	9.858	303	49.7	49

Group b_1 has only breakdown values larger than 9.18 kV, while group b_2 contains only breakdown values smaller than 9.18 kV. This subdivision yielded good results. The parameters for group b_2 agreed very well with the parameters from group a. The group b_1 gave poor results because, implicitly it was assumed that the reliability, below the discontinuity point was equal to 1. This is only true as an approximation.

Some calculations were made after combining the data of runs of which the parameters were only slightly different.The standard deviation was then smaller and the new parameters were averages of the previous ones. As a last control combinations were made of runs with similar a and α, and very different V_m and b. (runs 7, 18) The standard deviations were now much larger and the new parameters were entirely different from the previous obtained sets.

As a contrast a combination with largely different a and α, but similar V_m and b, again gave averages as results, (runs 7, and 9, 15 and 18). Also the standard deviations decreased.

Finally the calculations were repeated for all measurements made with a 200 micron electrode spacing. The results were poor. A subdivision of these observations again largely improved the results. The subset with the breakdown values which are higher than the discontinuity had only a few data more than the previously studied subset b_1, the parameter estimation therefore gave nearly the same results. The values smaller than the discontinuity gave (a = 0.813 and V_m = 7.81) and (α = 4.868 and b = 7.82).

Again the parameters agree very well with the results of group a and group b_2. The smallest standard deviation of all was found for group a.

Table V

Group	$a(kV^{-1})$	$V_m(kV)$	$\sigma_a(V)$	A_a	N
200 μ	0.555	9.607	523	218.8	193
V< 9.18kV	0.813	7.815	206	187.5	192
V> 9.18kV	1.093	10.938	502	329.8	55
	α	b(kV)	$\sigma_\alpha(V)$	A_α	N
200 μ	3.850	9.606	377	219.8	193
V< 9.18kV	4.868	7.821	143	190	192
V>9.18kV	11.205	10.877	394	330.2	55

It can be concluded that the parameters for breakdowns associated with noisy discharges can be estimated by: ($a = 0.9\ kV^{-1}$, $V_m = 7.4kV$) and ($\alpha = 5.3$, $b = 7.4$ kV). The breakdowns associated with sharp discharges are characterized by: ($a = 0.83\ kV^{-1}$, $V_m = 11.7$ kV) and ($\alpha = 8.2$, $b = 11.7$kV). The second of these groups is not as well defined as the first group.

The results of runs 21 to 23 (300 μ spacing) were treated the same way, and are given in table VI. It is impossible to draw conclusions from the comparison of these results with the results of the 200 micron spacing experiments, without knowing the size-effect parameter.

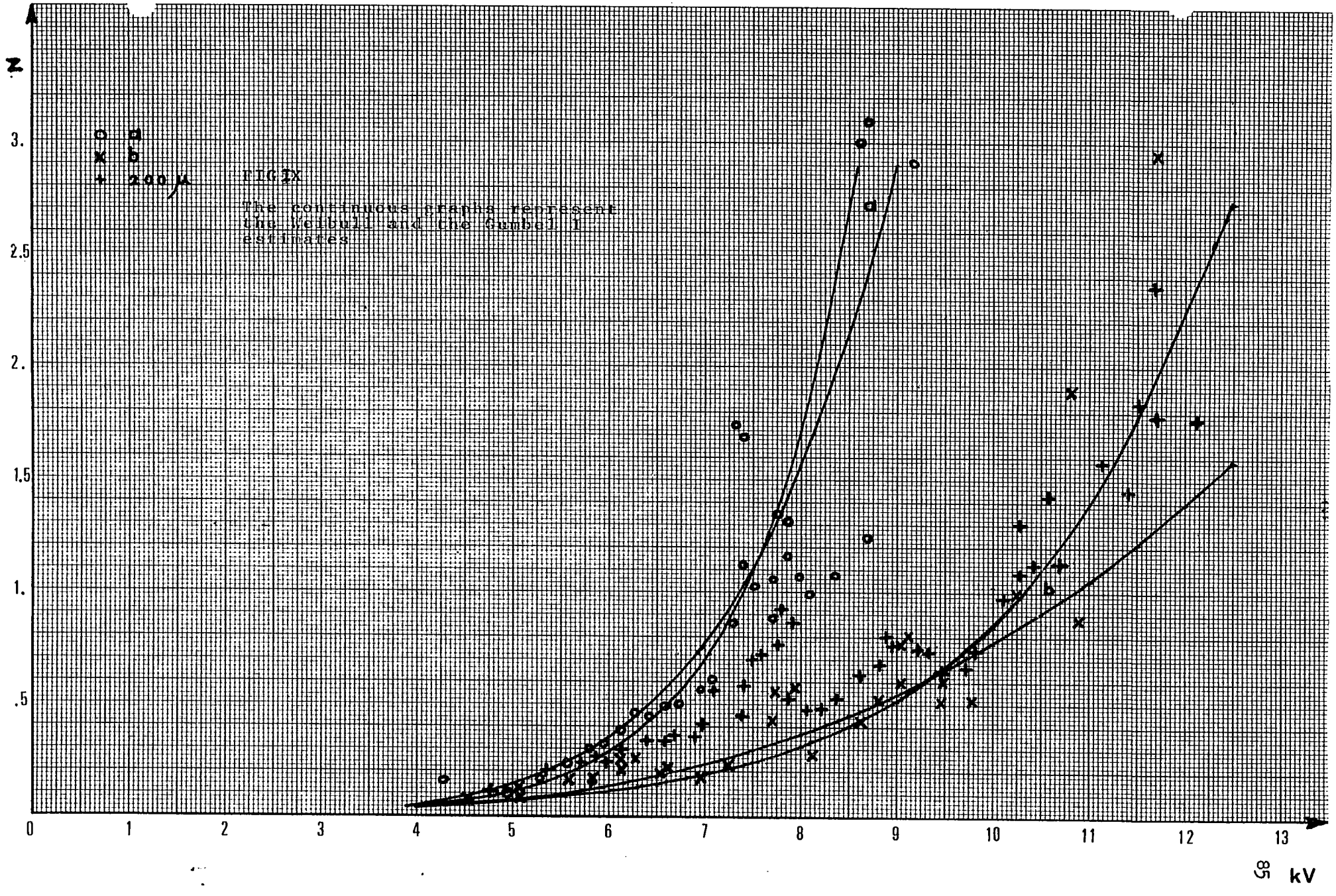
The relevant evaluated parameters with their ratios are listed in Table VI, on p. 84.

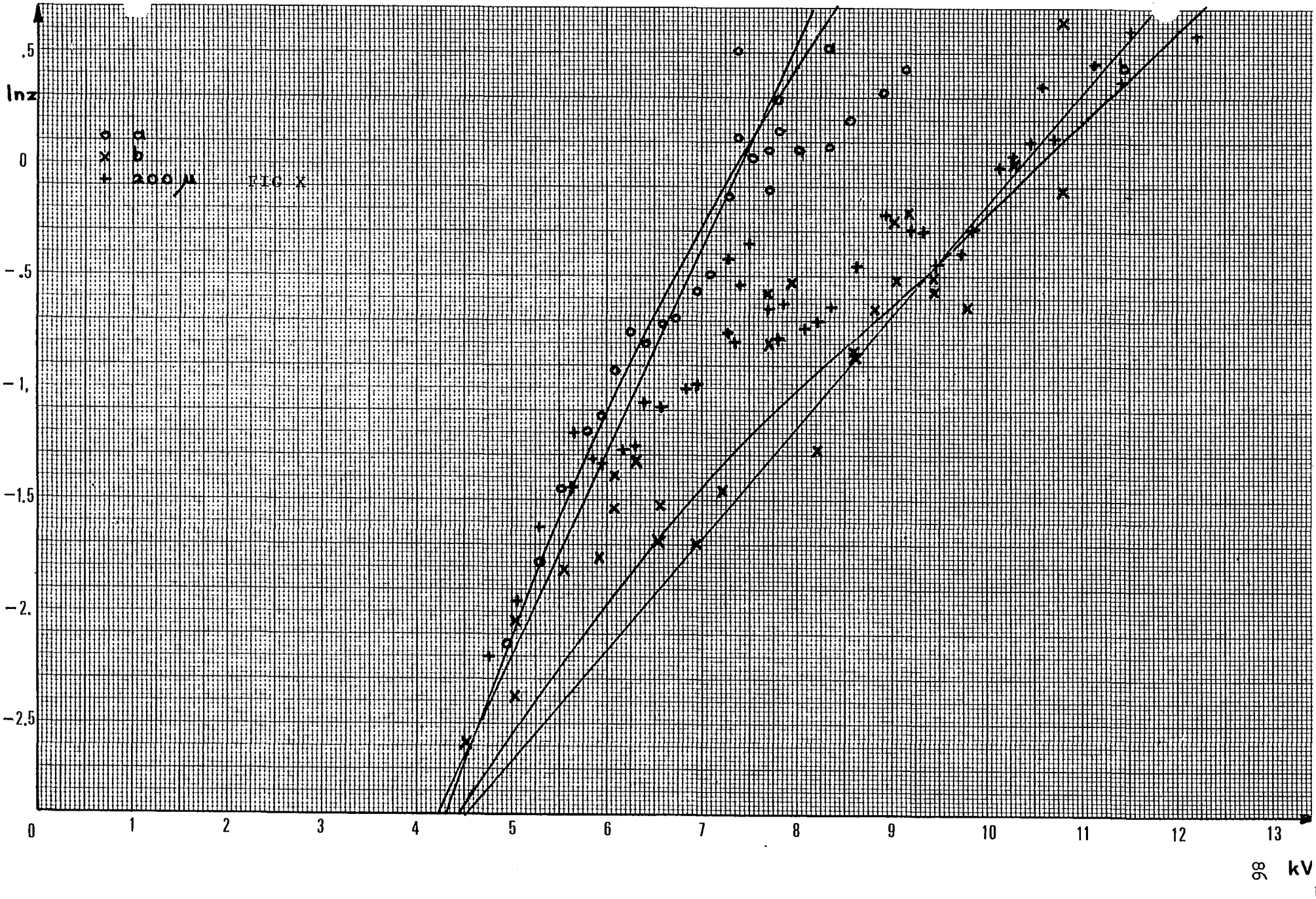
5.4.2 z vs. V

The point by point calculation of $z(x)$ gave no valuable results. (sec.5.1.2). For comparison with the obtained results every n^{th} point was used, where n varied from 5 to 30. The results were still poor, but conform within wide limits with the calculations. (Fig. IX, p.85 and Fig. X, p.86)

Table VI

		200 μ	300 μ	ratio
	Discontinuity	9.2 kV	9.6 kV	1.05
Complete	V_m	9.77 kV	11.42 kV	1.17
	b	9.89 kV	12.08 kV	1.22
	a	0.49 kV^{-1}	0.47 kV^{-1}	0.96
Group	α	3.22	3.38	1.05
	$a V_m$	4.8	5.4	1.13
	$b^{-\alpha}$	0.32×10^{-3}	0.22×10^{-3}	0.69
Above	V_m	11.69 kV	11.92 kV	1.02
	b	11.70 kV	11.87 kV	1.02
Discon-	a	0.83 kV^{-1}	1.06 kV^{-1}	1.28
	α	8.23	11.82	1.34
tinuity	$a V_m$	9.72	12.7	1.3
	$b^{-\alpha}$	0.16×10^{-8}	0.20×10^{-12}	1.22×10^{-4}
Lower	V_m	7.81 kV	8.20 kV	1.06
than	b	7.82 kV	8.27 kV	1.06
discon-	a	0.81 kV^{-1}	0.79 kV^{-1}	0.97
tinuity	α	4.87	4.83	0.99
	$a V_m$	6.35	6.48	1.02
	$b^{-\alpha}$	0.45×10^{-4}	0.37×10^{-4}	0.825


z
o a
x b
+ 200 µ
FIG IX
The continuous graphs represent the Weibull and the Gumbel I estimates
3.
2.5
2.
1.5
1.
.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
95 kV


lnz
.5
0
-.5
-1.
-1.5
-2.
-2.5
a
b
200 µ
FIG X
0
1
2
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5
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7
8
9
10
11
12
13
kV

5.4.3 Reliability paper.

It is possible to design probability paper on which the variables plotted against a suitable transformation of the reliability appear on a straight line. The normal probability paper is well known.

Reliability paper has been used by previous authors (Ref.1.1) (Ref.3.19) to show that the breakdown values fit the double exponential distribution. Here it is preferred to use more objective criteria(parameter calculation) and only use reliability paper as another representation of the data. In (sec.3.3.2) the expected value for the Weibull reliability was calculated:

$$V(R) = \left[k \ln \frac{1}{R}\right]^{\frac{1}{\alpha+1}}$$

or $$\ln V(R) = \frac{1}{\alpha+1} \ln k + \frac{1}{\alpha+1} \ln \ln \frac{1}{R}$$

Thus a plot of $\ln V(R)$ against $\ln \ln \frac{1}{R}$ is a straight line, with slope $\frac{1}{\alpha+1}$, if the distribution is being obeyed.

In (sec.3.3.3) the corresponding value for the double exponential distribution was obtained:

$$V(R) = V^* + \frac{1}{a} \ln\left[\frac{a}{S} \ln \frac{1}{R}\right]$$

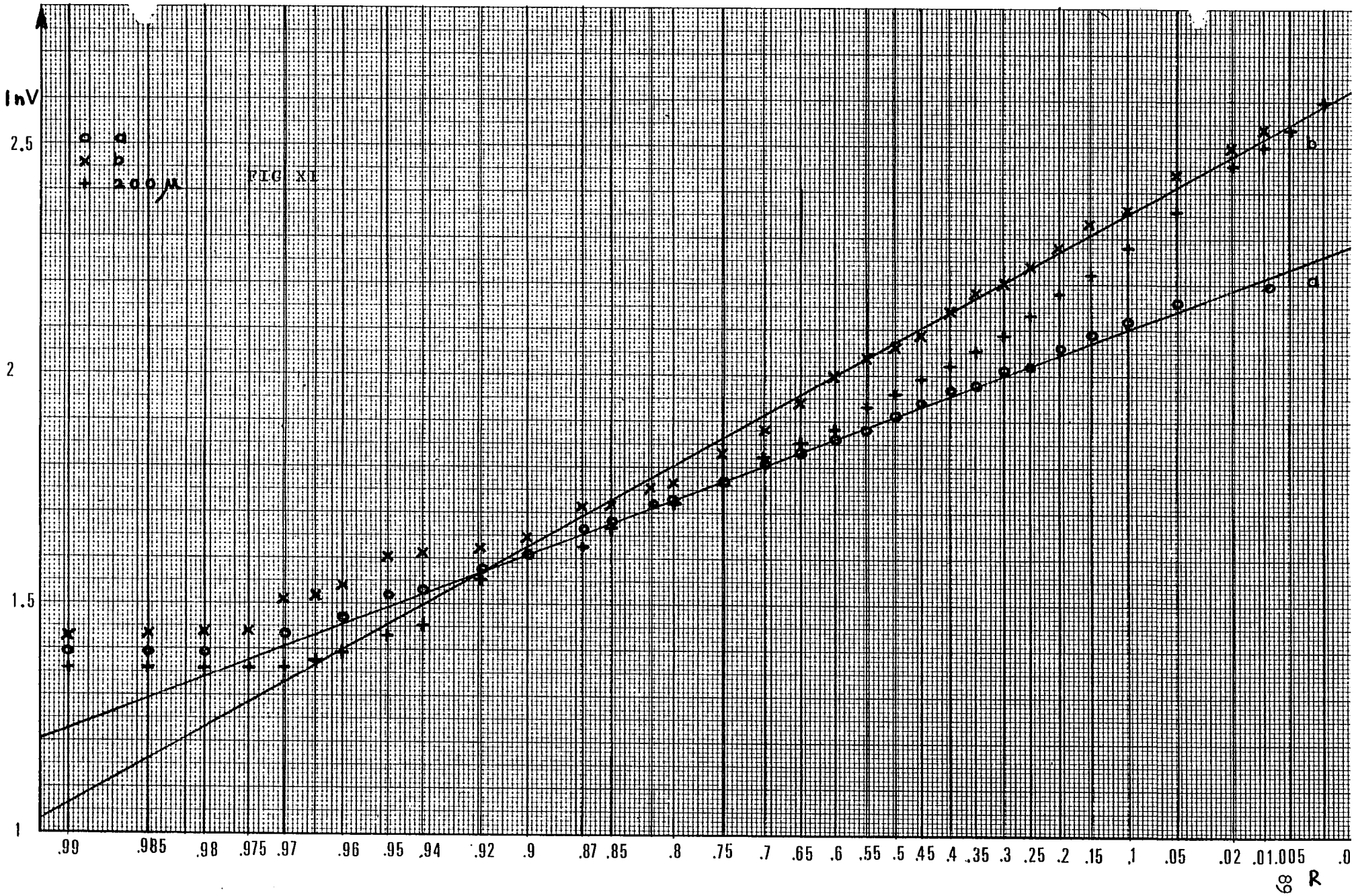
or $$V(R) = V_m + \frac{1}{a} \ln\left[a \ln \frac{1}{R}\right]$$

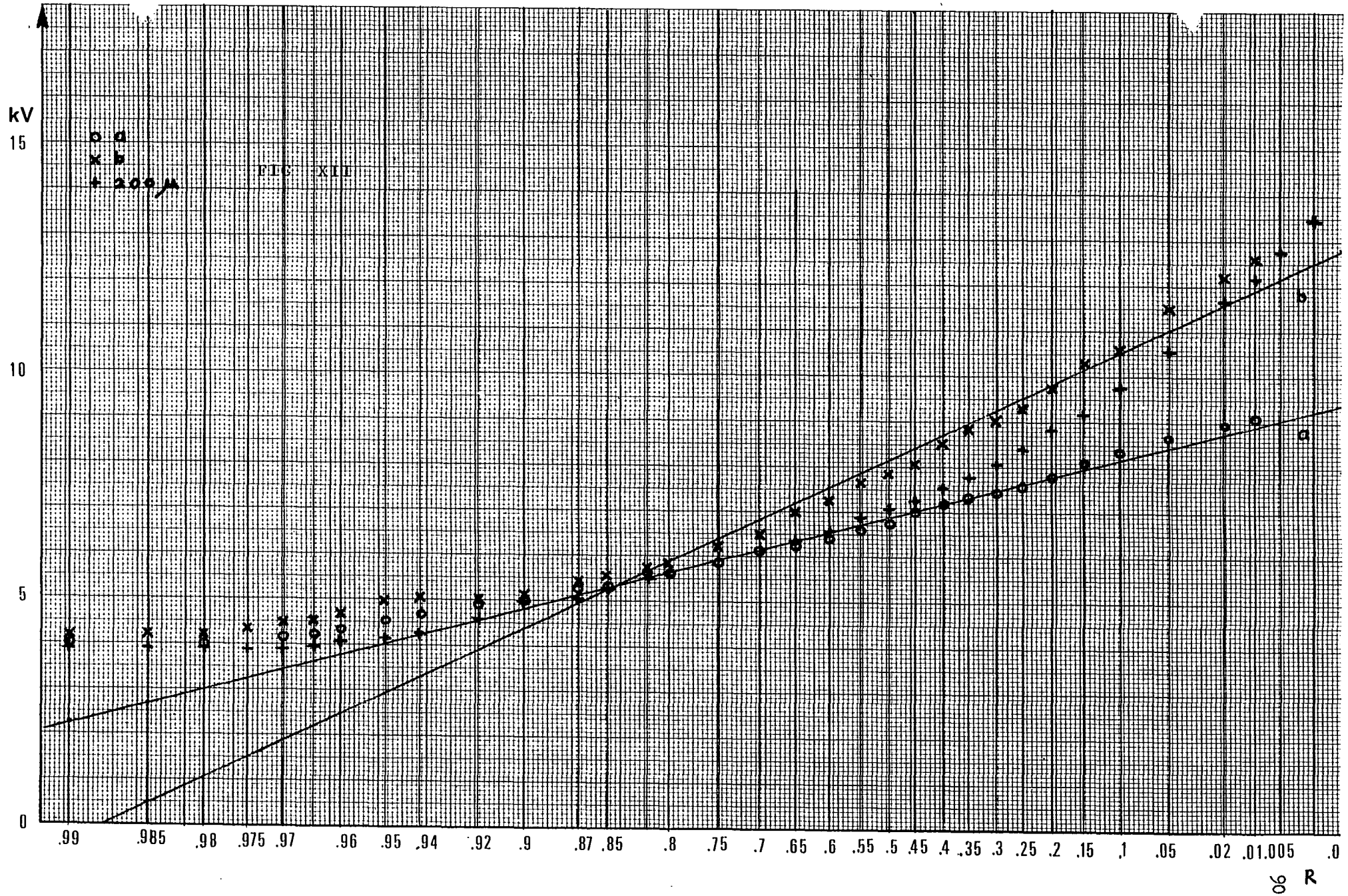
and $$V(R) = V_m + \frac{1}{a} \ln a + \frac{1}{a} \ln \ln \frac{1}{R}$$

Here a plot of $V(R)$ against $\ln \ln \frac{1}{R}$ is a straight line with slope $\frac{1}{a}$, if this distribution is being obeyed.

The reliability plots are handy in practical studies for a first evaluation of the results. In the present research this tool has not been used because fitting a straight line to a set of data is a very subjective operation.

Fig. XI, p.89 and Fig. XII, p.90 give the results of the two main categories of probability functions.


lnV
2.5
2
1.5
1
o a
× b
+ 200 M
.99 .985 .98 .975 .97 .96 .95 .94 .92 .9 .87 .85 .8 .75 .7 .65 .6 .55 .5 .45 .4 .35 .3 .25 .2 .15 .1 .05 .02 .01 .005 .0
R
b
a
FIG XI

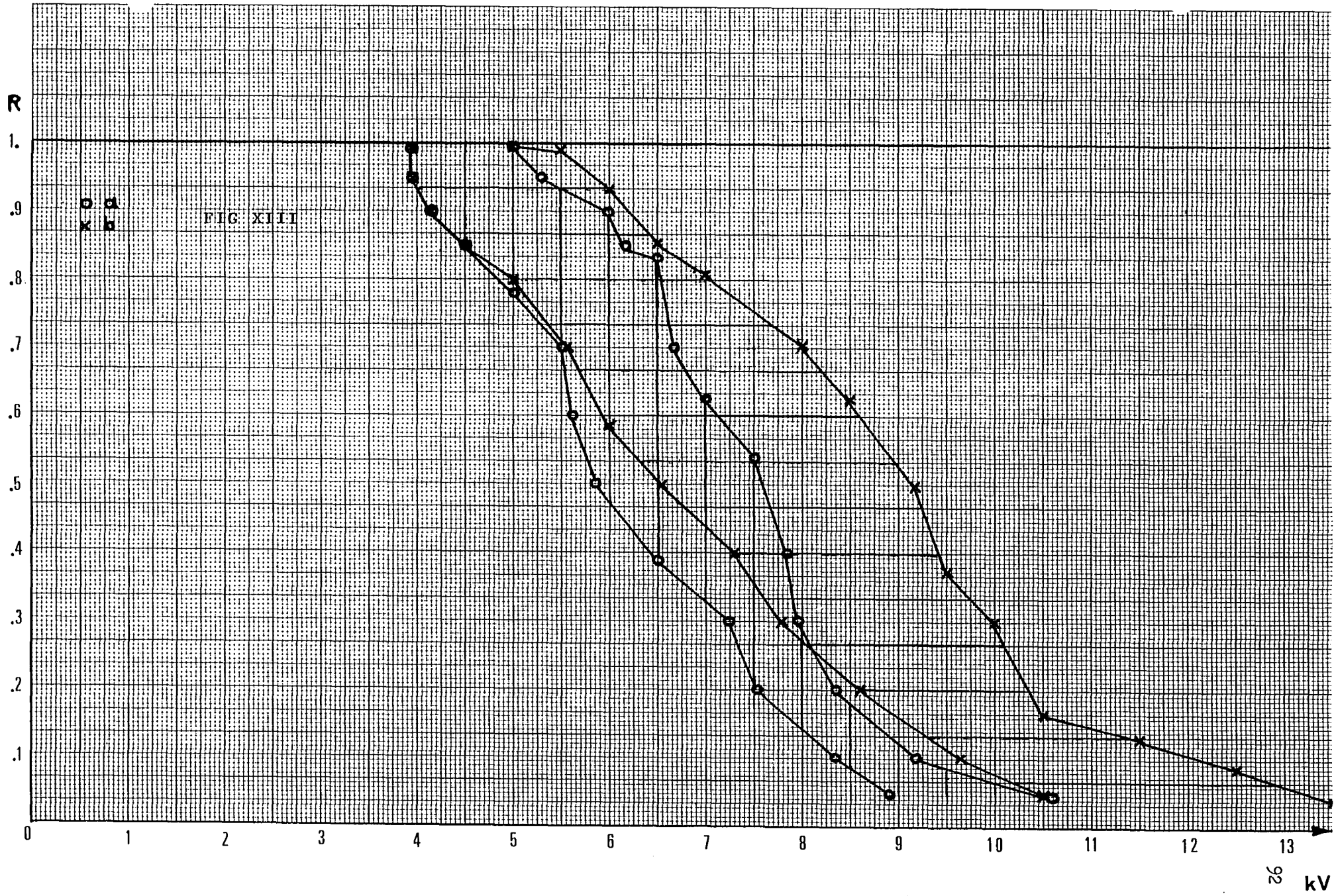

kV
15
10
5
0
o a
× b
+ 200 µ
FIG XIII
.99 .985 .98 .975 .97 .96 .95 .94 .92 .9 .87 .85 .8 .75 .7 .65 .6 .55 .5 .45 .4 .35 .3 .25 .2 .15 .1 .05 .02 .01 .005 .0
R
a
b

5.4.4 Maxima-minima Curves.

In the previous chapters we have tried to find the probability distributions associated with the breakdown of askarels.

A conservative approach would be to plot the reliabilities of all 19 runs performed on askarel at 200 μ electrode spacing and to draw the maximum and the minimum curve.

The data of both group a, and group b, were separately plotted this way. The w vs. V diagram is given in Fig. XIII, p.92.


R
1.
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.8
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.6
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.3
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0
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2
3
4
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8
9
10
11
12
13
kV
a
b
FIG XIII

5.4.5 Data by Weber and Endicott. (Ref.1.1)

These authors have listed all their measured data in their article. This gave an opportunity to check the calculation method of this report against their results.

Groups 1 and 4 of their article have been recalculated, (electrode sizes 17.8 mm and 76 mm. diameter.)

Weber and Endicott only calculated the Gumbel I parameters. The current results differ slightly from theirs. They used mean value and standard deviation to calculate their parameters. The standard deviation between the calculated values and the measured values was computed for the data of group 4, once using the parameters found by Weber and Endicott and once using the parameters found by the current method. The new method yields parameters with a better fit.

With $a = 0.3558\ kV^{-1}$ a standard deviation of 727 V was obtained, with $a = 0.302\ kV^{-1}$ - as found by Weber and Endicott - the standard deviation was 841 V. (The calculation using Weber's "a" is indicated by W 4 b, in Table VII, p.94.

The expected value for the standard deviation is $\frac{E}{\sqrt{12}} = 290$ V. To show the dependence on the sample size and on the individual values of the data, the calculations were also made on subgroups formed by group 1 vertical of this article: first, second and third column respectively (see p. 373 of Ref. 1.1).

Table VII

	$a\ (kV^{-1})$	V_m (kV)	σ_a (V)	A_a	N
W 1.1	0.258	61.416	500	68	67
W 1.2	0.277	60.149	656	68	67
W 1.3	0.285	59.879	1273	69.6	66
W 1	0.265	60.669	360	135.7	134
W 4	0.356	48.242	727	137.5	133
W 4 b	0.302	49.128	841	144.1	133
	α	b (kV)	σ_α (V)	A_α	N
W 1.1	13.093	62.257	547	68.6	67
W 1.2	13.818	60.910	657	68.6	67
W 1.3	13.847	60.849	1432	70.7	66
W 1	13.206	61.536	524	137.2	134
W 4	14.433	48.730	889	139.4	133

It can be seen that the parameters of the subgroups do not differ much from the parameters of the complete group, while the standard deviations for the subgroups are larger than the standard deviation for the complete group.

5.4.6 z(t)

It has been mentioned in section 4.2.1 that the probability function associated with time, could not be evaluated accurately.

However an estimate could be obtained. Assuming that time-delays are exponentially distributed, λ can be evaluated in two different ways. It was shown in section (3.3.1) that λ is the inverse of the mean value as well as the inverse of the standard deviation.

The parameter λ was calculated for the time-delays of the runs of group a and group b separately. The results are given in Table VIII.

Table VIII

	Group a	Group b
$\lambda = \frac{1}{t_{mean}}$	0.257	0.180
$\lambda = \frac{1}{\sigma_t}$	0.268	0.192

These parameters are only given as estimates.

5.5 Comments.

5.5.1 Choice of intensity function.

It has clearly been shown that the extreme value approach is valuable to explain the breakdown phenomena observed in askarel.

It is not clear however which intensity function should be chosen, since both Weibull and the double exponential distributions fit the data with reasonable accuracy. The Weibull distribution, when applied to askarel very often has smaller standard deviations, as well as maximum deviations. By contrast, our calculations on Weber' and Endicott's data resulted in a smaller standard deviation for the Gumbel I distributions. Our data from run 2 (dirty askarel and used electrodes)and from run 25(dirty oil and used electrodes) also gave a slightly smaller or equal standard deviation for the Gumbel I distribution.

Although these indications are not significant, they are in our opinion not less significant than the arguments of Weber and Dokopoulos, who both based their choice on the minimum of groups size effect.

Finally no conclusion can be drawn whether to choose Gumbel I or Weibull. The possibility exists that each of the different mechanisms (at least 2) involved, have different intensity functions and that the resultant probability density must be calculated.

It is also possible that the intensity function is more complicated than has been assumed and that the two distributions considered here are only good approximations, within the experimental range.

Under the assumption that two different, mutually independent, classes of events can cause breakdown, the following model can be built.

Assuming the probability of not having a breakdown caused by events of the group 1 is R_1, while the probability of not having a breakdown caused by events of the group 2 is R_2.

Then the combined reliability is $R = R_1 \cdot R_2$

$$w = w_1 + w_2$$

and $z = z_1 + z_2$ follow immediately.

The exact separation is in general very difficult, especially when the two intensity functions have different analytic formulations.

5.5.2 Examination of the assumption that the breakdown values are normally distributed.

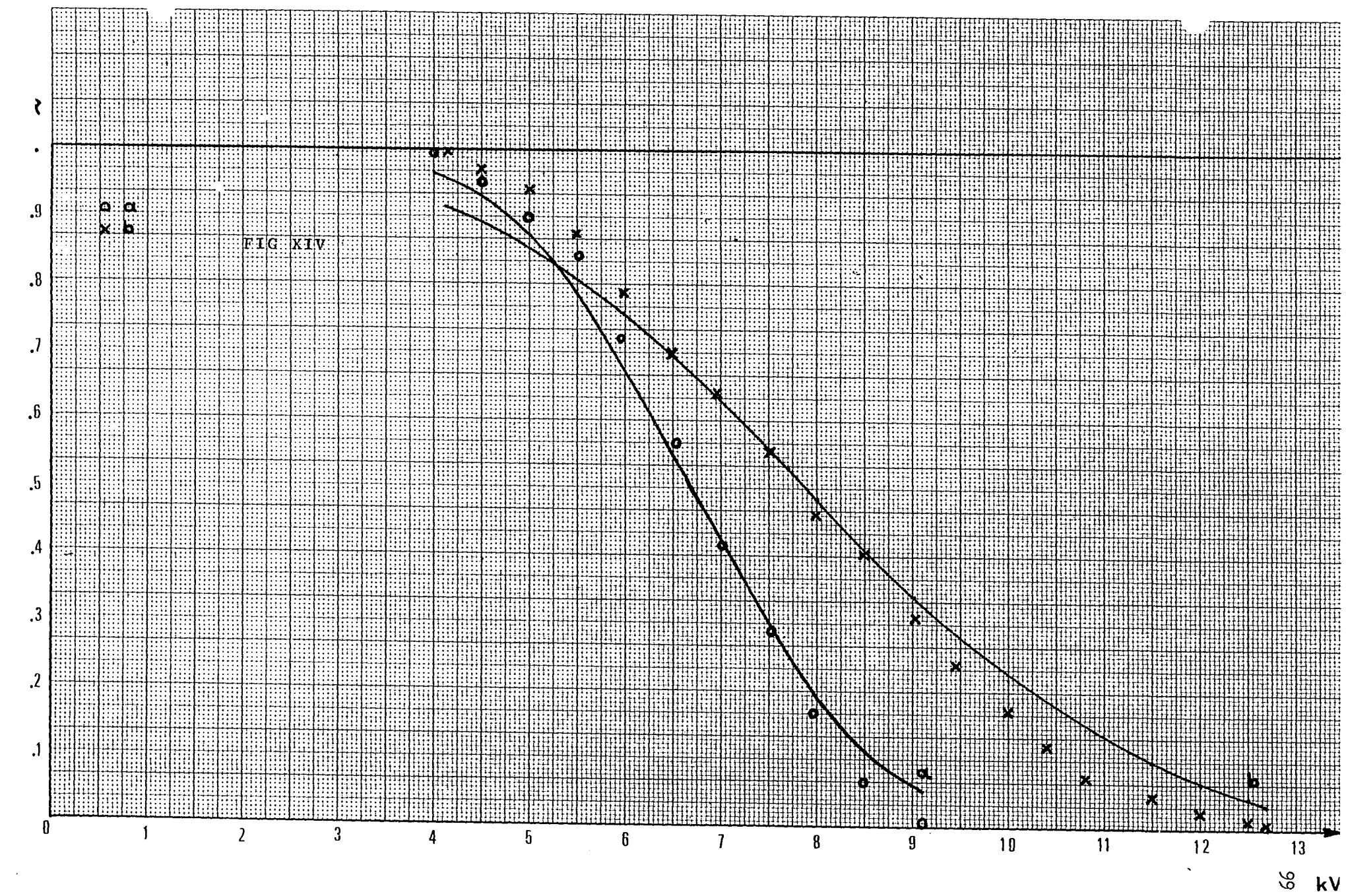
The mean value and the standard deviation have been calculated for the results of groups a and b, and the results are listed in Table IX p.98.

Table IX

	V_{mean} (kV)	σ (kV)
Group a	6.704	1.039
Group b	7.896	1.93
Group b_2	6.883	1.16

Again a higher standard deviation corresponds to a higher mean value. Using the mean value and the standard deviation, the reliabilities were calculated and plotted together with the measured values in Fig. XIV p.99.

The graphs show that the normal distribution predicts too high reliabilities at high values of the applied voltage and too low reliabilities at low values of the applied voltage.


a a
x b
FIG XIV
1
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12
13
kV

5.5.3 Prediction of reliabilities outside the measured range.

In industrial applications it is important to know the flash-over probability at voltage levels which are generally far below the range used for testing in laboratory conditions.

It has already been mentioned that the predictions made assuming a normal distribution are too pessimistic. The double exponential and the Weibull distribution will now be compared. Let us therefore calculate the voltage corresponding to a 0.999 reliability for the data of group "a" as well as the data of group "b", see Table X, p.101.

Artbauer (Ref.3.20) justifiably rejected the Gumbel I distribution because it allows a non-zero probability of breakdown for negative values of V. It is clear from the example and from theory that the Gumbel I distribution cannot be used to describe the statistics of breakdown voltages at low levels of probability. Therefore the Weibull distribution must be preferred for technical investigations.

Brignell gave an approximation (Ref.1.5) for low probability levels. In the notation of chapter (.2.3) his reasoning goes as follows. Assume that only one flaw causes breakdown (the weakest possible spot within the material), then

$$dP(x) = g_n(x) \cdot dx$$

or $dP(x) = z(x)\,dx$ and $P(x) = \int_x z(x)\,dx = w(x)$

The same relation is found by expanding the function R(x) into a power series.

$$R(x) = e^{-w(x)} = 1 - w(x) + \mathcal{E}(x) \quad \text{and again}$$

$$P(x) \backsimeq w(x)$$

The predictions using this last approximation are also listed in Table X.

Based on the current results the approximation appears to be very good and thus useful.

Table X

	Gumbel I	Weibull	Gumbel I Approximation	Weibull Approximation
a	-0.280 kV	2.429 kV	-0.281 kV	2.429 kV
b	-4.964 kV	1.716 kV	-4.965 kV	1.716 kV

5.5.2 Definition of breakdown strength.

Several methods have been designed to calculate the breakdown strength using a particular set of measurements. The simplest way is to take the mean of the breakdown values. This method is also proposed by the A.S.T.M. tests (Ref.5.1). It is significant that this test method specifies what to do when the spread between the individual measurements is too large to be acceptable. Because of the large spread in data and the extremal nature of breakdown inception the mean value is a very poor specification of the liquid.

As can be seen from the analytic calculation (section 3.3) the mean value is very strongly affected by the individual characteristics of the dielectric. Therefore the mean value can not be used to compare liquids of different composition. That has been realized by several authors who took a reliability point of view. They proposed to define the probability by a certain minimum level, and sometimes also a maximum level to give an indication for the spread in experimental data.

The ISO-standards for bearings simply take the value for $R = 0.9$ as the nominal strength (Ref.3.3).

Brignell (Ref.1.5) proposes from experience to take $E\{R = 0.999\}$, obtained with a slope of 100 V/s, as a measure of the technically maximum attainable stress.

Simo (Ref.3.19) suggested to calculate in addition $E\{R = 0.001\}$ and the mean value to define the breakdown strength

and recommended to take at least 250 measurements.

It is very tempting to associate the definition of breakdown strength to a certain high level of reliability. This is also useful from an engineering point of view.because liquid dielectrics are mostly used in transformers and capacitors, which both have a very high design reliability. Therefore the maximum stress applied to a technically clean dielectric and thus the defined breakdown strength should correspond with a reliability of at least 99.5 %.

Additional parameters are interesting because they give an idea of the spread. The current measurements have shown that a simple definition is not sufficient as long as the theoretical distributions of breakdown strength are not known, especially as long as nothing more is known about the two kinds of breakdowns encountered in these experiments.

Therefore the present author proposes a different kind of measurement of the quality of the dielectric in withstanding high voltages.

The intensity function z and also w are larger for a poor dielectric than for a dielectric of high quality. On the other hand it is a simple matter to find estimates for w, even on a fairly small number (say 40) of measurements. The present author suggests that the manufacturer should provide graphs of w vs. V, obtained with oils of different quality, ranging from laboratory purified to technically dirty. A test on an unknown sample will produce a new set of w vs. V values. Plotting these data on the standard graph will give a qualitative idea of the value of the unknown dielectric.

VI SOME IMPLICATIONS OF THE WEAKEST LINK CONCEPT ON BREAKDOWN THEORIES

6.1 Physical base of the flaw concept.

There is no theoretical reason why the concept of the existence of an intrinsic breakdown strength for askarel should be discarded, but there are many practical reasons why this should be done. The askarel is an industrial mixture of different liquids. Its high viscosity justifies the assumption that impurity particles will be present in the liquid even after careful purification. This is a first source of flaws. If it is assumed that the particles have been removed through extreme purification, a second source of weaknesses will still be present.

Constant degassing of the liquid can be seen during the conditioning period. Therefore there are good reasons to believe that increasing the stress intensifies the formation of micro-bubbles. Those too are weak spots, available in large quantities. They could probably be eliminated by an extreme degassing before performing the experiment or by pressurizing the sample during the experiment. Then come the individual differences between the molecules. Because of the chemical processes themselves, there will be individual molecules which are larger in size than the other ones of the same batch. These are the smallest weaknesses. Considering the high viscosity however,

it is most unlikely that the required level of purification can ever be attained. Therefore the flaw concept is very attractive.

Coelho has written more about the weaknesses of liquid dielectrics and the mechanisms associated with them (Ref.6.4).

Assuming that flaws act independently of one another, the intensity function can be written as:

$z(x) = N(x) \cdot g(x)$ (see section 2.3)

Introducing the size effect gives:

$z(x) = n(x) \cdot S \cdot g(x)$

Where $g(x)$, probability function associated with each one of the flaws, and $n(x)$, number of flaws per unit of size.
S, size of the source of flaws. (S is an area when the breakdown is fully electrode-controlled, while S is a volume when the breakdown is dielectric-controlled.)

Each breakdown model will have to define an $n(x)$ and a $g(x)$ for the kind of flaws considered.

6.2 Particle model.

If $n(r)$ designates the number of particles with radius r per unit volume,
S the volume under stress
$g(r)$ the probability function associated with particles of radius r,
then: $z = \sum_r n(r) \cdot S \cdot g(r)$

Kok (Ref.1.3) assumes that breakdown is caused by the formation of particle bridges within the electrode gap. These bridges are dispersed in the two directions parallel to the electrodes by the thermal energy kT of the particles.

The equilibrium condition is given by:

$$16\,\pi\epsilon_o\,\frac{\epsilon-\epsilon_o}{\epsilon+2\epsilon_o}\,r^3\,E_o^2 = kT$$

Where: ϵ permittivity of the particle

ϵ_o permittivity of the liquid

E_o electric field

k Boltzmann's constant

T temperature in K

This condition defines the minimum radius which a particle should have to be still effective as a source of breakdown

for $r < r_{min}$ $\quad g(r) = 0$

for $r \geqslant r_{min}$ $\quad g(r) = 1$

$$\text{thus } z = \sum_{r_{min}}^{\infty} n(r) \cdot S$$

$$\text{or } z = S\int_{r_{min}}^{\infty} n(r)\,dr$$

with r_{min} defined by the equilibrium formula.

Palmer (Ref.3.18) gives a graph showing the dependence of the particle density on the particle size. The relationship can be approximated by: $n(r) = k\,r^{-m}$

$$\text{Thus} \quad z \triangleq S \int_{r_{min}}^{\infty} k\, r^{-m}\, dr$$

$$z \triangleq S\, k \frac{r_{min}^{1-m}}{m-1} \qquad \text{for} \quad m > 1$$

$$r_{min} \equiv E_o^{\frac{2}{3}}$$

$$\text{and} \quad z \triangleq \frac{S\,C}{m-1} \; E_o^{\frac{2}{3}(m-1)}$$

which corresponds to the Weibull distribution if $m \sim 5$

Filtration will change the relations in the following way:

$$z = S \int_{r_{min}}^{r_{max}} k r^{-m}\, dr$$

$$z = \frac{S\,k}{1-m} \left[r_{max}^{1-m} - r_{min}^{1-m} \right]$$

$$\text{or} \qquad z = \frac{S}{1-m} \left[A - C\, E^{\frac{2}{3}(m-1)} \right]$$

The order of magnitude of r_{min} can be calculated from the equilibrium condition.

For E_o = 4 kV / 200 μ, r_{min} will be 20 Å

Of course, the n(r) used here is only one of the possible functions, as our choice is purely empirical.

6.3 Bubble model.

A second more publicized model assumes that vapour bubbles are formed within the liquid, or at the electrode surfaces. When the applied electric field has a value which is higher than a critical level, the bubble will elongate until it is unstable.

Krasucki built a breakdown model centered on this fact, (Ref.1.4). He stated that a bubble, elongated in the field direction will eventually ionize when the breakdown strength for vapour is reached within the vapour bubble. This ionization will initiate the discharge between the two electrodes.

The critical field is given by:

$$E_o = 1130\sqrt{\frac{1}{\epsilon}\left(P + \frac{2\sigma}{r}\right)}$$

Where E_o : critical field in $\frac{V}{m}$

σ : surface tension in $\frac{N}{m}$

r radius of the bubble in m

ϵ relative permittivity of the bubble

P hydrostatic pressure in $\frac{N}{m^2}$

The model can again be modified in statistical terms. The intensity function is: $z = \sum n(r) \cdot S \cdot g(r)$

$n(r)$ is the number of bubbles within the liquid with radius r.

Frenkel (Ref.6.1) p. 178, gives an expression for this number. He writes:

$$d\,n(r) = N\, e^{-\frac{W(r)}{kT}} \cdot \bar{\Phi}\,(r)\,dr$$

Where: N is the total number of particles per unit volume

$W(r)$ is the energy associated with a vapour bubble of radius r

k Boltzmann's constant

T temperature in K

$\bar{\Phi}\,(r)$ a function of r, Frenkel assumes $\bar{\Phi} = r^s$

$W(r)$ is composed of the surface energy of the liquid-vapour interface, the hydrostatic energy applied to the bubble and the electrostatic energy trying to enlarge the bubble.

Thus $W(r) = \frac{4}{3}\pi r^3 P + 4\pi\sigma r^2 - 2\pi\epsilon_o \frac{\epsilon-\epsilon_o}{\epsilon+2\epsilon_o} r^3 E_o^2$

$$z = \int_{r_{min}}^{\infty} n(r) \cdot S \cdot dr$$

$g(r \geqslant r_{min})$ is again 1 because it is assumed that the existence of a vapour bubble of size $r \geqslant r_{min}$ is a sufficient condition to have breakdown.

r_{min} is defined by E_o

$$z = S\int_{r_{min}}^{\infty} dn(r) = S\,N\int_{r_{min}}^{\infty} e^{-\frac{W(r)}{kT}}\ \bar{\Phi}\,(r)\,dr$$

This integral cannot be further evaluated.

6.4 Electro-hydrodynamic theory.

The measurements on askarel which have been reported showed a marked discontinuity at a value of 9.18 kV. It is suggested that the discontinuity is associated with the change from one breakdown mechanism to an other.

The discharges at low levels could be caused by particle triggering and possibly also by vapour bubbles, while the high level discharges should only be caused by vapour bubbles.

To explain the ineffectiveness of particles at high voltage levels, the electro-hydrodynamic theory could perhaps be helpful.

Ostroumov (Ref.6.2) showed that a liquid under electrostatic stress acquire(movement at a speed which increases with the stress. At a certain level the critical speed is attained and the laminar flow becomes turbulent. It is clear that particle bridging will be impossible in a turbulent movement. The change is associated with a constant, defined by Felici (Ref.6.3).

The constant is given by: $\sqrt{\frac{\epsilon}{\rho}} \cdot \frac{V}{\nu}$

where ϵ is the permittivity of the liquid

ρ is the density of the liquid

V is the voltage across the gap

ν is the kinematic viscosity

The numerical value of this factor is not measured, therefore it is not known if V coincides with the discontinuity in breakdown behaviour. Felici (Ref.6.3) states that the factor has a numerical value between 100 and 1000 for most dielectric liquids.

The corresponding V for askarel (Monsanto 7030) is between 6.2 kV and 62 kV and thus, the order of magnitude is right. During the conditioning period the liquid appeared to have a laminar motion of the order of 10 cm/s.

Spitzer, in a study on the conductivity of transformer oil in the pre-breakdown range, has produced plots of log current against log voltage showing a discontinuity where the change-over occurs from laminar to turbulent flow (Ref.4.3).

Hence the existence of a discontinuity in the reliability plots at 9.18 kV, is not inconsistent with hydrodynamic theory.

VII CONCLUSIONS.

It has been shown that extreme value theory is a useful tool to analyse the breakdown data of askarel. Two different mechanisms, triggering the breakdown event were detected. as a probabilistic model, both the Gumbel I (double exponential) and the Weibull distribution were tried and the relevant parameters were calculated for the two breakdown mechanisms. The Weibull distribution tends to give a better fit with these data.

A short discussion of possible models of breakdown inception shows that the real distribution could be much more complicated.

APPENDIX I

The breakdown parameters for the Gumbel I intensity function, obtained by previous authors are listed below. Where necessary the parameters have first been transformed into the notations of this report:

$$z(x) = e^{a(V-Vm)}$$

or $z(x) = S\, e^{a(V-V^*)}$, where the size effect has been measured.

(Note: $Vm = V^* - \frac{\ln S}{a}$)

1. Data by Brignell (Ref 1.5):

breakdown data for n-hexane

electrodes: Nickel spheres with 1 cm diameter

gap spacing: 100 μ

tests performed with 1.2/50 pulses

results: $a = 1.18\ kV^{-1}$

$Vm = 2.12$ kV

2. Data by Weber and Endicott (Ref. 1.1):

breakdown data for transformer oil

electrodes: brass Rogowski electrodes

gap spacing: 1.9 mm

tests performed with a 60 cycles voltage rising at a rate of 3 kV/s.

Results:

electrode size (cm^2)	2.485	5.08	20.32	45.72
$a(kV^{-1})$	0.257	0.32	0.308	0.302
V*(kV)	29.67	29.24	34.05	33.60

tests performed with a pulse front of 65 kV/s.

electrode size (cm^2)	2.485	5.08	20.32	45.72
$a(kV^{-1})$	0.119	0.097	0.083	0.09
V*(kV)	105.15	95.39	91.79	99.98

3. Data by Simo (Ref.3.19):

breakdown data for transformer oil

uniform field electrodes

tests performed with a ramp voltage of 2 kV/s.

results:

1° electrode size: 7.85 cm^2

spacing: 2 mm

	aluminum		brass		
	I	II	III	IV	V
$a(kV^{-1})$	0.234	0.207	0.231	0.206	0.218
Vm(kV)	67.95	70.97	71.48	71.69	72.62

2° electrode size: 177 cm^2

spacing: 4 mm

brass

	I	II	III	IV
$a(kV^{-1})$	0.142	0.132	0.130	0.118
Vm (kV)	125.35	124.02	125.63	129.19

3° electrode size: 708 cm^2

spacing: 5 mm

brass

	I	II	III	IV
$a(kV^{-1})$	0.085	0.079	0.080	0.092
Vm(kV)	146.68	147.53	147.54	140.81

aluminum

	I	II	III	IV
$a(kV^{-1})$	0.116	0.123	0.107	0.107
Vm(kV)	103.97	106.09	113.68	115.97

APPENDIX II

Calculation of the combined probability of $t = t_f + t_s$

Given: $$f(t_f) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(t_f - t_o)^2}{2\sigma^2}\right\}$$

$$f(t_s) = \mu e^{-\mu t_s}$$

$$f(t) = \int_t \mu e^{-\mu s} \cdot \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(t - s - t_o)^2}{2\sigma^2}\right\} ds$$

$$\mu s + \frac{(t - s - t_o)^2}{2\sigma^2} = \frac{1}{2}\left(\frac{t - s - t_o}{\sigma} - \mu\sigma\right)^2 - \frac{\sigma^2\mu^2}{2} + \mu(t - t_o)$$

say $$\frac{t - t_o - s}{\sigma} - \mu\sigma = \sqrt{2}\, y$$

$$ds = -\sqrt{2}\,\sigma\, dy$$

$$f(t) = \mu \exp\left\{\frac{\sigma^2\mu^2}{2} - \mu(t - t_o)\right\} \int_{\frac{t-t_o}{\sqrt{2}\sigma} - \frac{\mu\sigma}{\sqrt{2}}}^{\infty} e^{-y^2}\, dy$$

$$f(t) = \frac{\mu}{2}\left[1 + \operatorname{erf}\left(\frac{t - t_o}{\sqrt{2}\,\sigma} - \frac{\mu\sigma}{\sqrt{2}}\right)\right] \cdot \exp\left\{\frac{\sigma^2\mu^2}{2} - \mu(t - t_o)\right\}$$

APPENDIX III

Listing of the parameters of the intensity functions used in this research.

Runs: 1,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20 were performed on askarel with a 200 μ gap spacing, taking fresh askarel and new electrodes for each run.

Run 2 used the same askarel and the same electrodes as run 1.

Runs 21, 22, 23 used askarel with a 300 μ gap.

In run 24 a general purpose transformer oil was investigated, the gap spacing being 150 μ. In run 25 the same oil and the same electrodes were used as in run 24.

Run	a(kV^{-1})	V_m(kV)	$\overline{U}_a$(V)	A_a	N
1	0.405	11.514	697	22.9	23
3	0.954	7.286	236	20.3	20
4	0.822	7.589	252	23.6	23
5	0.102	7.542	270	22.0	22
6	0.601	7.939	410	23.0	23
7	0.744	7.485	249	23.3	23
8	0.625	9.950	353	25.4	23
9	1.167	7.631	150	24.9	24
10	0.419	10.207	554	24.8	25
11	0.871	7.788	467	23.0	23
12	0.659	8.305	615	22.7	23
13	0.388	10.939	755	23.8	24
14	0.429	10.182	779	22.9	23
15	0.578	9.721	544	24.0	24
16	0.652	8.368	377	23.3	23
17	0.558	10.351	322	24.5	24
18	0.749	9.630	399	25.5	24
19	0.733	8.196	449	24.1	24
20	0.563	10.218	579	22.3	22
2	1.410	6.21	145	25.7	25
21	0.500	11.829	593	24.2	24
22	0.474	10.518	558	23.8	24
23	0.528	11.302	413	24.7	24
24	1.050	7.614	294	37.6	38
25	3.555	6.156	61	24.5	23

Run	α	b(kV)	σ_α (V)	A α	N
1	2.714	12.348	551	23.2	23
3	5.753	7.226	164	20.6	20
4	4.610	7.641	258	23.9	23
5	6.624	7.463	244	22.2	22
6.	3.224	8.017	328	23.2	23
7	4.001	7.535	224	23.7	23
8	4.331	10.202	312	25.8	23
9	7.540	7.605	160	25.2	24
10	2.346	11.098	433	25.3	25
11	5.620	7.715	294	23.3	23
12	4.123	8.250	521	22.8	23
13	2.073	12.801	623	24.1	24
14	2.753	10.377	707	22.9	23
15	4.025	9.818	355	24.4	24
16	4.133	8.306	304	23.5	23
17	3.737	10.837	385	25.1	24
18	5.174	9.838	773	26.0	24
19	4.821	8.098	338	24.2	24
20	4.262	10.243	448	22.5	22
2	7.632	6.16	142	26.1	25
21	4.043	12.193	413	24.7	24
22	3.168	10.935	559	24.1	24
23	3.794	11.950	410	25.2	24
24	6.890	7.554	237	37.9	38
25	21.454	6.141	68	24.7	23

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