POSSIBILITY OF STRUCTURE IN PROJECTILE FRAGMENTATION IN HIGH ENERGY HEAVY ION REACTIONS

BY

MEHRZAD PEYROW

A thesis submitted to the Faculty of Graduate Studies and Research of McGill University, in partial fulfilment of the requirements for the degree of Master of Science.

Department of Physics
McGill University

© Montreal, Quebec

Structure In Projectile Fragmentation In High Energy Heavy Ion Reactions.

ABSTRACT

A study is made to show that a quantum mechanical analogue of the Goldhaber model predicts a structure in the momentum distribution of α's fragmenting from the projectile ^{6}Li. This structure should be measurable.

RESUME

Nous démontrons qu'une modèle quantique analogue au modèle de Goldhaber prédit une structure précise pour la distribution des impulsions des particules α obtenues par fragmentation des projectiles de ^{6}Li. Cette structure devrait être, en principe, mesurable.

ACKNOWLEDGEMENT

I would like to take this opportunity to thank my thesis director, Professor S. Das Gupta for his constant encouragement and his great patience. Evenmore, I am particularly grateful to him because he has known how to pass his enthusiasm of this field of research on to me.

FIGURE CAPTIONS

Fig. 1 Participants and spectators. Certain part A_2 overlaps with a certain part B_2; they are the participants. Parts A_1 and B_1 are the spectators.

Fig. 2 The probability distribution $\mathcal{P}(P)$ in the frame of the projectile. The normalisation is such that $\mathcal{P}(0) = 1$.

Fig. 3 For ^{6}Li projectile at 200 MeV/nucleon in the lab, a theoretical computation of $\frac{d^3\sigma}{dE_L\, d\Omega_L}(P_L)$ for the α's where the angles of measurements are 0^o, 2^o and 4^o.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

This thesis will deal with fragmentation of projectiles in high energy heavy ion collisions. Although the concepts were first developed for collisions in Bevalac energy region (250 MeV/nucleon to 2.1 GeV/nucleon) these seem to have validity also in significantly lower energy regime. This has been verified experimentally.

A concept frequently used in relativistic heavy ion collisions (RHIC) is that of spectators and participants. For a given impact parameter, a certain fraction of the projectile will overlap with another certain fraction of the target. These parts are directly involved in the collision and form the participants. The parts that do not overlap are spectators. Thus there will be a projectile like spectator and a target like spectator. The projectile like spectator will fly off in the forward direction. The existence of these projectile like spectators has been verified in counter type experiments, as well as in streamer chamber experiments. They have a velocity spread around the original beam velocity. A quantum mechanical calculation of this velocity spread is the subject of this thesis. We will deal with the concept of participants and spectators in the next chapter in greater detail; in the remainder of this introduction we will very briefly review the past history of the subject of velocity distribution of fragments in RHIC. The review will be brief; it does not aim at being complete.

Rather accurate measurements of the momentum distribution of projectile like fragments were made in 1975 by Greiner et al [1]. Theoretical predictions were made by Feshbach and Huang [2]. An elegant and simple model for the velocity distribution was proposed by Goldhaber [3] in 1974. This work, by far, has proved to be the most important theoretical development. The work presented in the thesis, is a quantum mechanical extension of the Goldhaber model, applied to a specific case. Goldhaber model is a simple Fermi gas model. Hüfner and Abul-Magd [4] used Glauber type approach to treat fragmentation and found the limits in which Goldhaber results are recovered. Bertsch [5] pointed out that inclusion of antisymmetry in the Goldhaber model will lead to a reduction of the velocity spread. McVoy and Nemes [6] have treated fragmentation in the local momentum plane wave approximation and obtained Goldhaber's results. In the 5th high energy heavy ion study at Berkeley, Wong [7] proposed a basic distribution model applicable to both the peripheral fragmentation and the central collision processes.

The width of the momentum distribution at 213 MeV/nucleon was measured by Vyogi et al [8] at 213 MeV/nucleon. Lower incident energy data at 86 MeV/nucleon were obtained by Mougey [9] et al at CERN. Some other relevant experiments can be found in references 10, 11 and 12.

The plan of the thesis is as follows. In Chapter II we describe the Goldhaber model. Chapter III is the major part of the thesis. Here we develop a quantum mechanical extension of the Goldhaber model and apply it to fragmentation of ^{6}Li into α particle. We find that a structure in the momentum distribution is predicted. While rather accurate experiments are needed to establish this structure, the possibility of measuring the structure cannot be ruled out. In particular, in the proposed heavy ion facility MARIA at the University of Alberta, Edmonton, such experiments are entirely feasible with the high beam current.

Summary and discussions are presented in Chapter IV. Some mathematical details are relegated to the Appendix.

CHAPTER II

PARTICIPANTS AND SPECTATORS; THE GOLDHABER MODEL

Consider the following collision of two heavy ions (Fig. 1). For a given impact parameter b, a fraction A_2 will collide with a certain fraction B_2. If the energy of collision is high (Bevalac energies), then the fact that A_1 was attached to A_2 is entirely incidental. The binding energy is 8 MeV/nucleon and this is negligible compared to the energy of collision. Thus A_1 will fly off with very nearly the original velocity v .

What happens to A_2 and B_2 is another story. This is the region of violent collision. These may form a fireball, two fireballs or firestreaks. Most of the efforts in RHIC have been towards understanding this region of violent collision. This, however, does not concern us directly in this thesis and we merely refer to a review article [13].

The process of separation of A_1 from A_2 is called abrasion. This takes place so fast that what one sees is the momentum distribution of A_1 in the nucleus A. Let us elaborate on this point further. It is convenient to go to the frame of the projectile. In this frame $\vec{P}_A = 0$. Now let us imagine for a moment that nucleons inside the nucleus are all frozen. Then after abrasion we will have $\vec{P}_{A1} = \vec{P}_{A2} = 0$.

However, if the nucleons inside the nucleus are not frozen, i.e., they have Fermi motion, then, after abrasion all we are guaranteed is that $\vec{P}_{A1} + \vec{P}_{A2} = 0$. In general $\vec{P}_{A1}$ is not zero (although $\langle \vec{P}_{A1} \rangle = 0$) and the problem is to find this distribution.

If we have A nucleons then $\vec{P}_A = 0$ means $\sum_{i=1}^{A} \vec{P}_i = 0$

Therefore

$$\left(\sum_i \vec{P}_i \right)^2 = \sum_{i=1}^{A} P_i^2 + \sum_{i \neq j} \vec{P}_i \cdot \vec{P}_j = 0 \tag{2.1}$$

There are A terms in $\sum_{i=1}^{A} P_i^2$; A(A-1) terms in $\sum_{i \neq j} \vec{P}_i \cdot \vec{P}_j$

Thus define average values through

$$\langle P^2 \rangle = \frac{1}{A} \sum P_i^2 \tag{2.2}$$

$$\langle \vec{P}_i \cdot \vec{P}_j \rangle = \frac{1}{A(A-1)} \sum \vec{P}_i \cdot \vec{P}_j \tag{2.3}$$

Hence eq. (2.1) can be rewritten as

$$A \langle P^2 \rangle + A(A-1) \langle \vec{P}_i \cdot \vec{P}_j \rangle = 0$$

which gives

$$\langle \vec{P}_i \cdot \vec{P}_j \rangle = -\frac{\langle P^2 \rangle}{A-1} \qquad (2.4)$$

Now let us consider picking out n nucleons out of A (in Fig. 1, $A_1 = n$). Obviously $\langle \vec{P}_n \rangle = 0$ but what about $\langle P_n^2 \rangle$?

If we want to choose n nucleons out of A nucleons we can choose it $\binom{A}{n}$ ways. For example, for $n = 3$, we can have

$$(\vec{P}_1 + \vec{P}_2 + \vec{P}_3)^2$$

or $(\vec{P}_1 + \vec{P}_2 + \vec{P}_4)^2$

or $(\vec{P}_2 + \vec{P}_3 + \vec{P}_4)^2$ etc. It is then clear, that, for example, P_1^2 occurs $\binom{A-1}{n-1}$ times, $\vec{P}_1 \cdot \vec{P}_2$ occurs $\binom{A-2}{n-2}$ times. We can then write

$$\binom{A-1}{n-1}\sum P_i^2 + \binom{A-2}{n-2}\sum \vec{P}_i \cdot \vec{P}_j = \binom{A}{n}\langle P_n^2\rangle \tag{2.5}$$

We now write $\sum P_i^2 = A\langle P^2\rangle$ (eq. (2.2))

$\sum \vec{P}_i \cdot \vec{P}_j = A(A-1)\langle \vec{P}_i \cdot P_j\rangle = A\langle P^2\rangle$ (eq. (2.3) and (2.4)). The algebra now can be worked out to give

$$\langle P_n^2\rangle = \frac{K(A-K)}{A-1}\langle P^2\rangle \tag{2.6}$$

For a Fermi gas $\langle P^2\rangle = \frac{3}{5}\langle P_F^2\rangle$ where P_F = Fermi momentum is of the order of $\approx$ 230 Mev/C for nuclei.

Equation (2.6) gives the average value of the square of momentum P_n^2. At this stage it is assumed that the distribution of momentum $\mathcal{P}(P_n)$ is Gaussian

$$\mathcal{P}(P_n) = \frac{1}{(2\pi\sigma^2)^{\frac{3}{2}}} e^{-\frac{P_n^2}{2\sigma^2}} \tag{2.7}$$

Then $\langle P_n^2\rangle = 3\sigma^2$

$$\therefore \sigma^2 = \frac{1}{5} P_F^2 \frac{n(A-n)}{A-1} \tag{2.8}$$

Equations (2.7) and (2.8) are the final equations of the Goldhaber model. We note that all possible partitions of n nucleons from A nucleons have been allowed for; thus the n nucleons come out in all possible internal states of excitation. Indeed in all experiments reported so far there is no selectivity as to the quantum state of the fragment. For example, if ^{16}O fragments into ^{12}C, the detector does not care whether the ^{12}C nucleus is in the ground state or any of the particle stable excited states. It is possible to devise experiments which allow for selectivity for a particul ar quantum state, but such experiments have not yet been done. Secondly, the model for the nucleus is a Fermi gas model. Note also that a Gaussian distribution is assumed; once this is assumed $\langle P_n^2 \rangle$ allows one to fix the parameter of the Gaussian once for all. There is a universality in equations (2.7) and (2.8) since P_F stays fairly constant with mass number A.

The Goldhaber model has been widely used in all experimental analyses [1, 8, 9, 10, 11, 12] and in many theoretical investigations [4, 5, 6, 7]. One of its virtues is its simplicity. In the next chapter we consider a quantum mechanical extension of the Goldhaber model.

CHAPTER III

A QUANTUM MECHANICAL EXTENSION OF THE GOLDHABER MODEL AND ITS APPLICATION TO ^{6}Li

Basically, the Goldhaber model asks: what is the momentum distribution of n nucleons in a nucleus which has A nucleons? Let the ground state wavefunction of A nucleons be $\mathcal{A}\Psi_0(A)$. Then it is always possible to write

$$\mathcal{A}\Psi_0(A) = \sum_{i,j} \mathcal{A}\left\{ \Psi_i(n)\, \Psi_j(A-n)\, f_{ij}(\vec{n}) \right\} \qquad (3.1)$$

In eq. (3.1) $\mathcal{A}\Psi_0(A)$ is a properly antisymmetrised ground state wavefunction. $\mathcal{A}$ is an antisymmetriser. The ground state wavefunction is an "intrinsic" wavefunction. The usual shell model wavefunctions are not intrinsic wavefunctions since they contain a centre of mass motion for the whole nucleus. Usually this centre of mass motion is of a simple kind, namely, it is in the ground state of a three dimensional harmonic oscillator. It is obvious that since we are delaing with an overall mass motion of n nucleons in a nucleus of A nucleus, we should start with wavefunctions which have no spurious overall centre of mass motion for the A nucleons. In a similar fashion $\Psi_i(n)$ is an internal wavefunction for n nucleon, $\Psi_j(A-n)$ for A-n nucleons; $f_{ij}(\vec{\pi})$ is relative motion between n nucleons and

A-n nucleons. Here $\vec{r}$ is defined as

$$\vec{r} = \frac{1}{n}\sum_{p=1}^{n} \vec{r}_p - \frac{1}{A-n}\sum_{p=n+1}^{A} \vec{r}_p \tag{3.2}$$

The relative wavefunction $f_{ij}(\vec{r})$ can be Fourier expanded

$$f_{ij}(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int a_{ij}(\vec{k})\, e^{i\vec{k}\cdot\vec{r}}\, d^3k \tag{3.3}$$

$$a_{ij}(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int e^{-i\vec{k}\cdot\vec{r}} f_{ij}(\vec{r})\, d^3r \tag{3.4}$$

Here k is the relative momentum

$$\vec{k} = \frac{(A-n)\vec{k}_n - n\,\vec{k}_{A-n}}{A} \tag{3.5}$$

In the rest frame of the A nucleons $\vec{k}_n + \vec{k}_{A-n} = \vec{k}_A = 0$,
thus $\vec{k}$ also is the momentum of the n-nucleons in the rest frame of the parent nucleus. This is, of course, the quantity we are after.

The expansion implied in eq. (3.1) is well known in theories of nuclear reactions and in cluster models of nuclei [14]. Eq. (3.1) is very often the starting point of resonating group calculations. Some comments are in order about the function $f_{ij}(\vec{r})$. The function $\psi_i(m)\psi_j(A-m)f_{ij}(\vec{r})$ is not antisymmetrised. The antisymmetriser $\mathcal{A}$ has very non trivial consequences in eq. (3.1). Different choices of $f_{ij}(\vec{r})$ can lead to the same final function $\mathcal{A}\,\psi_0(A)$. There is therefore some ambiguity in defining $f_{ij}(\vec{r})$ although there is no such ambiguity in the final antisymmetrised wavefunction.

The expansion (3.1) is mathematically non-trivial. Therefore here we try a simple case but one which is still experimentally meaningful. We will consider the breakup of ^{6}Li into an α particle and two nucleons. Since α has no particle-stable excited states the summation on i is restricted to the ground state of α. The summation on j is still unstricted since the experiment does not care which final state the two nucleons were left behind. Thus the probability we want (see eq. (2.7)) is

$$\mathcal{P}(\vec{k}) \propto \sum_j \left| \left\langle \mathcal{A}\left\{ {}^6Li\,(\text{intrinsic}) \right\} \right| \phi_\alpha(\text{intrinsic})\,\phi_{2j}(\text{intrinsic})\, e^{i\vec{k}\cdot\vec{r}} \right\rangle \Big|^2 \qquad (3.6)$$

We have made a slight change of notation from the one used in eq. (3.1). In eq. (3.6) it is not necessary to antisymmetrise both the bra and the ket, and we find it mathematically more convenient to antisymmetrise the bra.

We now outline the steps needed to compute eq. (3.6). We take the shell model wavefunction for ^{6}Li to be $\mathcal{A}\left[(1s)^4(1p)^2\ L=0\ S=0\right]$.

We also take the magnetic substate to be $M = 1$. The final answer will not depend upon the magnetic substate. This shell model wavefunction is

$$\mathcal{A}\left[(1s)^4(1p)^2\ L=0\ \ S=1\ ,M=1\right]$$
$$= \mathcal{A}\, e^{-a\sum_{i=1}^{6} r_i^2}\,(x_5x_6 + y_5y_6 + z_5z_6)\,\Psi \tag{3.7}$$

Here Ψ is a spin-isospin wavefunction

$$\Psi = \chi^{(1)}_{\frac{1}{2}\frac{1}{2}}\ \chi^{(2)}_{\frac{1}{2}-\frac{1}{2}}\ \chi^{(3)}_{-\frac{1}{2}\frac{1}{2}}\ \chi^{(4)}_{-\frac{1}{2}-\frac{1}{2}}\ \chi^{(5)}_{\frac{1}{2}\frac{1}{2}}\ \chi^{(6)}_{-\frac{1}{2}\frac{1}{2}} \tag{3.8}$$

The subscripts to χ refer to isospin and spin; i.e., $\chi_{\frac{1}{2}-\frac{1}{2}}$ is a proton with spin down. In equations (3.7) and (3.8) before antisymmetrisation, we singled out particles 1 to 4 being $(1s)^4$.

and particles 5,6 being $(Ip)^2$; of course, after antisymmetrisation, such distinctions cannot be made. Equations (3.7) and (3.8) define the shell model wavefunction for ^{6}Li. We have to extract from it the intrinsic state. Note that in eq. (3.7) $e^{-a\sum_{i=1} r_i^2}$ is already symmetric thus we can take the operator $\mathcal{A}$ past it and are left with

$$^6L_i\,(\text{Shell Model}) = e^{-a\sum r_i^2}\,\mathcal{A}\,(X_5 X_6 + Y_5 Y_6 + Z_5 Z_6)\;\varphi \tag{3.9}$$

In order to extract the centre of mass motion, we now go to the Jacobi coordinates. Define

$$\begin{aligned}
\vec{\tilde{r}}_1 &= \vec{r}_1 - \vec{r}_2 \\
\vec{\tilde{r}}_2 &= \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \\
\vec{\tilde{r}}_3 &= \tfrac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4) \\
\vec{\tilde{r}}_{56} &= \vec{r}_5 - \vec{r}_6 \\
\vec{r} &= \frac{1}{4}\sum_{i=1}^{4}\vec{r}_i - \tfrac{1}{2}(\vec{r}_5 + \vec{r}_6) \\
R &= \frac{1}{6}\sum_{i=1}^{6}\vec{r}_i
\end{aligned} \tag{3.10}$$

The reverse transformations are

$$\vec{r}_6 = \vec{R} - \frac{2}{3}\vec{r} - \frac{1}{2}\vec{\tilde{r}}_{56}$$

$$\vec{r}_5 = \vec{R} - \frac{2}{3}\vec{r} + \frac{1}{2}\vec{\tilde{r}}_{56}$$

$$\vec{r}_4 = \vec{R} + \frac{1}{3}\vec{r} - \frac{3}{4}\vec{\tilde{r}}_3$$

$$\vec{r}_3 = \vec{R} + \frac{1}{3}\vec{r} + \frac{1}{4}\vec{\tilde{r}}_3 - \frac{2}{3}\vec{\tilde{r}}_2$$

$$\vec{r}_2 = \vec{R} + \frac{1}{3}\vec{r} - \frac{1}{2}\vec{\tilde{r}}_1 + \frac{1}{3}\vec{\tilde{r}}_2 + \frac{1}{4}\vec{\tilde{r}}_3$$

$$\vec{r}_1 = \vec{R} + \frac{1}{3}\vec{r} + \frac{1}{2}\vec{\tilde{r}}_1 + \frac{1}{3}\vec{\tilde{r}}_2 + \frac{1}{4}\vec{\tilde{r}}_3$$

(3.11)

Using these transformations, eq. (3.9) becomes

$$^6\mathrm{Li}\ (\text{Shell Model}) = e^{-6aR^2}\, e^{-\frac{1}{2}a\tilde{r}_1^2}\, e^{-\frac{2}{3}a\tilde{r}_2^2}\, e^{-\frac{3}{4}a\tilde{r}_3^2} \times e^{-\frac{1}{2}a\tilde{r}_{56}^2}\, e^{-\frac{4}{3}ar^2}\, \mathcal{A}\left\{(\vec{R}-\tfrac{2}{3}\vec{r})^2 - \frac{1}{4}\tilde{r}_{56}^2\right\}\varphi$$

(3.12)

In the exponential part, the c.m. part is easily seen to separate out but in the remainder part of the above wavefunction the intrinsic and the c.m. appear to be coupled. Actually the operator $\mathcal{A}$ kills all parts which has an R in it. Consider the part $\mathcal{A} R^2 \Psi$. Since $\vec{R}$ is already symmetric we can take $\mathcal{A}$ past R^2 and then Ψ is readily seen to vanish (Ψ is defined in eq. (3.8)). What is less obvious is that $\mathcal{A} \vec{R} \cdot \vec{\tilde{\lambda}} \, \Psi = 0$ but this is shown to be true in the Appendix. We thus see that the c.m. part is a purely simple harmonic motion. Taking this out we have

$$
{}^6Li\,(\text{intrinsic}) = e^{-\frac{1}{2}a\tilde{\lambda}_1^2}\, e^{-\frac{2}{3}a\tilde{\lambda}_2^2}\, e^{-\frac{3}{4}a\tilde{\lambda}_3^2}\, e^{-\frac{1}{2}a\tilde{\lambda}_{56}^2}\, e^{-\frac{4}{3}a\lambda^2} \times
$$

$$
\times \mathcal{A}\left(\frac{4}{9}\lambda^2 - \frac{1}{4}\tilde{\lambda}_{56}^2\right)\Psi \tag{3.13}
$$

We now consider the ket of eq. (3.6). This can be written as Φ_α (intrinsic) Φ_{2j} (intrinsic) $e^{i\vec{k}\cdot\vec{\lambda}}$

$$
= e^{-\frac{1}{2}a\tilde{\lambda}_1^2}\, e^{-\frac{2}{3}a\tilde{\lambda}_2^2}\, e^{-\frac{3}{4}a\tilde{\lambda}_3^2}\, \phi_{2j}(\tilde{\lambda}_{56})\,\Psi\, e^{i\vec{k}\cdot\vec{\lambda}} \tag{3.14}
$$

There is no need to antisymmetrise and there is no loss of generality in taking ψ to be that given by eq. (3.8). In the above equation $\phi_{2j}(\tilde{r}_{56})$ will have to span a complete set of states. We will take a three dimensional harmonic oscillator basis of which the lowest state $(N=0 \;\; \ell=0)$ is

$$\phi_{2,N=0,\ell=0}(\tilde{r}_{56}) = \left(\frac{a}{\pi}\right)^{\frac{3}{4}} e^{-\frac{1}{2}a\tilde{r}_{56}^{2}} \tag{3.15}$$

Another state which has $N=2 \;\; \ell=0$ is

$$\phi_{2,N=2,\ell=0}(\tilde{r}_{56}) = \sqrt{\frac{3}{2}}\left(\frac{a}{\pi}\right)^{\frac{3}{4}}\left(1-\frac{2}{3}a\tilde{r}_{56}^{2}\right) e^{-\frac{1}{2}a\tilde{r}_{56}^{2}} \tag{3.16}$$

It turns out that in the overlap integral of eq. (3.6) only the two states of eqs. (3.15) and (3.16) contribute, the rest give zero. The overlap integral is still somewhat lengthy to calculate and we provide the calculational details in the Appendix. The final answer for the right hand side of eq. (3.6) is

$$\mathcal{P}(k) = c\, e^{-\frac{3k^2}{8a}}\left[1+\frac{27}{8}\left(1-\frac{k^2}{4a}\right)^2\right] \tag{3.17}$$

Several comments on eq. (3.17) are in order. The overall Guassian shape is a direct consequence of using harmonic oscillator wavefunctions for ^{6}Li. We can also understand the numerical factor $\frac{3k^2}{8a}$ in eq. (3.17). This came from the Fourier transform of $e^{-\frac{4}{3}ar^2}$ in eq. (3.12). The factor $\frac{4}{3}$ arises because the reduced mass of an α and two nucleons is $m\frac{4\times 2}{6} = m\frac{4}{3}$. If one uses harmonic oscillator shell model wavefunctions for the ground state, then the relative motion of n nucleons and (A-n) nucleons will always have a Gaussian overall behaviour given by $\exp\left[-\frac{n(A-n)}{A}ar^2\right]$. However, the relative motion is not just a Gaussian (note the other factors involving $\vec{r}$ in equation (3.13). Nonetheless the factor $\frac{3k^2}{8a}$ in eq. (3.17) arose just from the Fourier transform of $\exp\left[-\frac{n(A-n)}{A}ar^2\right]$ for the case $n = 2$, $A = 6$. In a more general case the overlap integral will give rise to $\exp\left[-\frac{k^2}{4a}\frac{A}{n(A-n)}\right]$ and when squared for $P(k)$ the overall mutliplicative factor will be $\exp\left[-\frac{k^2}{2a}\frac{A}{n(A-n)}\right]$ The same factor in the Goldhaber model is (eqs. (2.7) and (2.8)) is $\exp\left[-k^2\frac{5}{2k_F^2}\frac{A-1}{n(A-n)}\right]$ Since $a = \frac{1}{2}\frac{m\omega}{\hbar}$ the factor $\frac{5}{2k_F^2}$ is replaced by $\frac{\hbar}{m\omega}$ in the harmonic oscillator model. If we take $mc^2 = 939$ Mev, $\hbar\omega = 41A^{-1/3}$ Mev, $P_F c = 230$ Mev then $\frac{\hbar}{m\omega}\Big/\frac{5}{2k_F^2} = A^{1/3}\times 0.5497$. For $A = 6$ the two answers are about identical, but we note that in the harmonic oscillator model there is an A dependence left.

The quantity $\mathcal{P}(k)$ of eq. (3.17) is plotted in Fig. 2. We note that there is a structure; there is a dip at $P \approx 200$ MeV/c. This is a new feature and comes from the polynomial multiplying the exponential factor in eq. (3.17). The precise numbers appearing in the polynomial come out of calculations (Appendix) but here we note that contributions are from only two values of j (eq. (3.6)). The two wavefunctions ϕ_{2j} that contribute are written down in eqs. (3.15) and (3.16). To understand why there is structure, one should look at ^{6}Li (intrinsic) as written down in Eq. (3.13). The polynomial there is $\mathcal{A}\left(\frac{4}{9} r^2 - \frac{1}{4}\tilde{r}_{56}^2\right)\Psi$. Strange as it may seem, after antisymmetrisation $-\frac{1}{4}\tilde{r}_{56}^2$ contribute exactly in the same fashion as $\frac{4}{9} r^2$ does (see Appendix; such peculiarities are well known in the cluster model [14]). Thus the relative motion between α and d is not a $N=0$ $\ell=0$ harmonic oscillator but, loosely speaking, it is a $N=2$ $\ell=0$ mode. Thus the Fourier transform will exhibit structure.

Eq. (3.17) allows us to write

$$\frac{d^3\sigma}{d^3p}(p) = A\, e^{-\frac{3k^2}{8a}}\left[1 + \frac{27}{8}\left(1 - \frac{k^2}{4a}\right)^2\right] \qquad (3.18)$$

Where A is a constant. This equation is valid in the frame of the projectile. We want to convert this to an expression for $\frac{d^3\sigma}{dE_L d\Omega_L}(P_L)$, a quantity which was, for example, measured in ref. 9. The subscript L refers to lab measured quantities. Let Θ_L be the direction of observation, P_L be the momentum; $\beta = \frac{v}{c}$ be the beam velocity; $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. Then

$$P_{\parallel} = \gamma (E_L - \beta P_L \cos\Theta)$$

$$P_{\perp} = (P_L)_{\perp}$$

$$P = \sqrt{P_{\parallel}^2 + P_{\perp}^2} \qquad (3.19)$$

Use now the relativistic equalities

$$E_L \frac{d^3\sigma}{d^3P_L}(P_L) = E \frac{d^3\sigma}{d^3P}(P)$$

and

$$\frac{d^3\sigma}{dE_L d\Omega_L} = P_L E_L \frac{d^3\sigma}{d^3P_L}$$

to obtain finally

$$\frac{d^3\sigma}{dE_L d\Omega_L}(P_L) = P_L E \frac{d^3\sigma}{d^3P}(P) \tag{3.20}$$

The relationship between P_L and P is given by eq. (3.19).

In Fig. 3 we plot $\frac{d^3\sigma}{dE_L d\Omega_L}$ where θ is at 0^o, 2^o and 4^o. For illustration, assume that the incident beam is at 200 MeV/nucleon. At 0^o degree where the fragmentation cross-section is highest, the predicted dip comes at values of momentum where the cross-section is below an order of nagnitude of the maximum cross-section. At 4^o, the value of the cross-section at the dip is comparable to the value at the maximum, but the cross-section at any momentum is small. Thus, in general, rather precise measurements are needed to find the structure. For example, the error bars in the experiments of ref 8 will completely hide the structure. However future machines being planned will have beam intensities two orders of magnitude higher (for example, in the proposed MARIA project at University of Alberta, Edmonton) and then such experiments for low cross-sections are quite feasible.

CHAPTER IV

SUMMARY AND DISCUSSION

We have done a quantum mechanical calculation based on the Goldhaber model and found that the momentum distribution of α's fragmenting from ^{6}Li is predicted to have some structure. Will the structure be there in other fragmentations and if so, why have they not been observed?

In answer to the last part, measurements have not been accurate enough to measure structure even if it were present. The key questions asked so far have been the widths of the momentum distribution rather than the detailed measurements of shapes. But aside from that, structure may be washed out, even in theory, in other fragmentations. In order to appreciate this, let us refer back eq. (3.6). There is no summation, on various quantum states of the α because α has only one particle stable state. For other fragmentation processes (for example, ^{16}O fragmenting into ^{15}N), there would be summation over the particle stable state of the fragment and this has the effect of wiping out structure. For the case of α fragmenting from ^{6}Li, in eq. (3.6) j runs over two states (these states are written down in eqs. (3.15) and (3.16). If we were so lucky as to have only one state, namely the one of eq. (3.15), we would have, in theory, not a dip but an actual zero and the prediction would be more interesting. The more summation there is, the more unlikely it is to get any structure.

Notice that we have made no mention of the actual nucleon-nucleon interaction which causes fragmentation to take place. The assumption is the same as in the Goldhaber model: namely, that abrasion takes place so fast that nucleons do not readjust. One gets the momentum distribution that was there before the two nuclei actually hit each other. The problem has been looked at in time independent formalism by McVoy and Nemes [6]. They find that nucleon-nucleon interaction should affect the overall cross-section but not affect the shape. If so, Then our calculations are still valid.

Thus the only reason that the structure may get wiped out is that there may be other competing processes that dominate where the fragmentation cross-section is low. For example, one such possibility could be that ^{6}Li is first excited and then boil off an α One could think of other alternatives, however, unlikely. Only detailed experiments could tell.

APPENDIX

We start with eq. (3.12). We have to consider

$$\left\{ (\vec{R} - \tfrac{2}{3}\vec{r})^2 - \frac{1}{4} \tilde{r}_{66}^2 \right\} \Psi \tag{A-1}$$

where Ψ is the spin-isospin wavefunction of eq. (3.8).

$$\Psi = \chi^{(1)}_{\frac{1}{2}\frac{1}{2}} \chi^{(2)}_{\frac{1}{2}-\frac{1}{2}} \chi^{(3)}_{-\frac{1}{2}\frac{1}{2}} \chi^{(4)}_{-\frac{1}{2}-\frac{1}{2}} \chi^{(5)}_{\frac{1}{2}\frac{1}{2}} \chi^{(6)}_{-\frac{1}{2}\frac{1}{2}} \tag{A-2}$$

It is obvious that the R^2 term drops out after antisymmetrisation; $\vec{R}$ is symmetric thus A can be taken past it to act on Ψ and this will give zero. The term linear in $\vec{R}$ can be written as

$$-\frac{4}{3} \vec{R} \cdot \mathcal{A} \left\{ \vec{r}\, \Psi \right\}$$

The identity operation, P(1,5), P(3,6) and P(1,5) P(3,6) all leave Ψ invariant. However, $\vec{r}$ from eq. (3.10) is given by

$$\vec{r} = \frac{1}{4} \sum_{i=1}^{4} \vec{r}_i - \frac{1}{2} (\vec{r}_5 + \vec{r}_6) \tag{A-3}$$

Thus, for example, under interchange of 1 and 5, $\vec{r}$ goes to

$$\vec{r} \Rightarrow \frac{1}{4}(\vec{r}_5 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4) - \frac{1}{2}(\vec{r}_1 + \vec{r}_6) \qquad \text{(A-4)}$$

One can readily verify that

$$\left[1 - P(1,5) - P(3,6) + P(1,5)P(3,6)\right]\vec{r}\,\Psi = 0$$

A little thinking shows that this is enough to conclude that $\mathcal{A}\vec{r}\Psi = 0$
It is not necessary to write out each term.

The overlap integral that one has is then (see eqs. (3.13) and (3.14))

$$\begin{aligned}\text{overlap} = \int \cdots \int & d^3\tilde{r}_1\, d^3\tilde{r}_2\, d^3\tilde{r}_3\, d^3 r\, d^3\tilde{r}_{56} \\ & e^{-\frac{1}{2}a\tilde{r}_1^2}\, e^{-\frac{2}{3}a\tilde{r}_2^2}\, e^{-\frac{3}{4}a\tilde{r}_3^2}\, e^{-\frac{1}{2}a\tilde{r}_{56}^2}\, e^{-\frac{4}{3}ar^2} \\ & \mathcal{A}\left\{\left[\frac{4}{9}r^2 - \frac{1}{4}\tilde{r}_{56}^2\right]\Psi^\dagger\right\} \\ & e^{-\frac{1}{2}a\tilde{r}_1^2}\, e^{-\frac{2}{3}a\tilde{r}_2^2}\, e^{-\frac{3}{4}a\tilde{r}_3^2}\, \phi_{2j}(\tilde{r}_{56})\,\Psi\, e^{i\vec{k}\cdot\vec{r}}\end{aligned} \qquad \text{(A-5)}$$

The antisymmetriser acts upon $\psi^{\dagger}$ but not upon ψ. One can see from eq. (A-2) that the only permutations that do not lead to zero because of orthogonality of spins are 1 - P(1,5) - P(3,6) + P(1,5) P(3,6). We have to find the effects of these permutations on both $\vec{\mathcal{R}}$ and $\vec{\mathcal{R}}_{56}$. For example, let us again find the effect of P(1,5) on $\vec{\mathcal{R}}$. We go back to eq. (A-4) and make use of eq. (3.11) to obtain that

$$\vec{\mathcal{R}} \Rightarrow \frac{1}{4}\vec{\mathcal{R}} + \frac{3}{8}\vec{\tilde{\mathcal{R}}}_{56} - \frac{3}{8}\vec{\tilde{\mathcal{R}}}_1 - \frac{1}{4}\vec{\tilde{\tilde{\mathcal{R}}}}_2 - \frac{3}{16}\vec{\tilde{\mathcal{R}}}_3 \tag{A-6}$$

For $\tilde{\mathcal{R}}_{56}$, the use of eqs. (3.10) and (3.11) lead to the results that

$$P(1,5)\,\tilde{\mathcal{R}}_{56} = \vec{\mathcal{R}} + \frac{1}{2}\vec{\check{\mathcal{R}}}_1 + \frac{1}{3}\vec{\tilde{\check{\mathcal{R}}}}_2 + \frac{1}{4}\vec{\check{\mathcal{R}}}_3 + \frac{1}{2}\vec{\tilde{\mathcal{R}}}_{56} \tag{A-7}$$

The procedure should now be clear.

It would seem that after antisymmetrisation there will be many terms in the overlap integral of eq. (A-5). However, one immediately notices that many of these terms are of the type

$$\int \vec{r}_1 \cdot \vec{r}_2 \; e^{-a r_1^2} e^{-b r_3^2} \; d^3 r_1 \, d^3 r_2 = 0$$

The final overlap calculation reduced to calculating

$$\text{overlap} = \int \cdots \int e^{-\frac{1}{2} a \tilde{r}_1^2} e^{-\frac{2}{3} a \tilde{r}_2^2} e^{-\frac{3}{4} a \tilde{r}_3^2} e^{-\frac{1}{2} a \tilde{r}_{56}^2} e^{-\frac{4}{3} a r^2} \times$$
$$\times \left(r^2 - \frac{1}{4} \tilde{r}_{56}^2 - \frac{2}{9} \tilde{r}_2^2 + \frac{1}{16} \tilde{r}_3^2 \right) \times$$
$$\times \, e^{-\frac{1}{2} a \tilde{r}_1^2} e^{-\frac{2}{3} a \tilde{r}_2^2} e^{-\frac{3}{4} a \tilde{r}_3^2} \phi_i (\tilde{r}_{56}) \, e^{i \vec{k} \cdot \vec{r}}$$
$$d^3 \tilde{r}_1 \, d^3 \tilde{r}_2 \, d^3 \tilde{r}_3 \, d^3 r \, d^3 \tilde{r}_{56} \qquad \text{(A-8)}$$

The $\phi_i (\tilde{r}_{56})$ is a complete set of states. If we take the harmonic oscillator basis then it is obvious that only two i's contribute. These two wavefunctions are given in eqs. (3.15) and (3.16). The overlaps are calculated for each. The sum of the squares of the overlap is given in eq. (3.17).

REFERENCES

1. D.E. Greiner, P.J. Lindstrom, H.H. Heckman, B. Cork, F.S. Bieser, Phys. Rev. Lett. 35 (1975) 152.
2. H. Feshbach and K. Huang, Phys. Lett. 47B (1973) 300.
3. A.S. Goldhaber, Phys. Lett. 53B (1974) 306.
4. A. Abul-Magd and J. Hüfner, Zeit. Physik A, 277 (1976) 379.
5. G.F. Bertsch, Phys. Rev. Lett. 46 (1981) 472.
6. K.W. McVoy and C. Nemes, Zeit. Physik A, 295 (1980) 177.
7. C.Y. Wong, invited talk presented at the 5th High Energy Heavy Ion Study, May 18-22, 1981, LBL 21652.
8. Y.P. Vyogi, T.J.M. Symons, P. Doll, D.E. Greiner, H.H. Heckman, D.L. Hendrie, P.J. Lindstrom, D.K. Scott, K. Van Bibber, G.D. Westfall, Phys. Rev. Lett. 42 (1979) 33.
9. J. Mougey, R. Rost, M. Buenerd, A.J. Cole, C. Guet, D. Lebrun, J.M. Loiseaux, P. Martin, M. Maurel, E. Monnand, H. Nifenecker, P. Perrin, J. Pinston, C. Ristori, P. de Saintignon, F. Schussler, L. Carlen, B. Jackobson, A. Oskarsson, I. Otterlund, B. Schroder, H.A. Gustafsson, T. Johnansson, H. Ryde, J.P. Bondorf, O.B. Nielsen and G. Tibelli, Phys. Lett. 105B (1981) 25.
10. K. Van Bibber, D.L. Hendrie, D. K. Scott, H.H. Weiman, L.S. Schroeder, J.V. Geaga, S.A. Pessin, R. Truehaft, Y.J. Grossiond, J.O. Rasmussen and C.Y. Wong, Phys. Rev. Lett. 43 (1979) 840.
11. Ch. Egelhaaf, G. Bohlen, H. Fuchs, A. Gamp, H. Homeyer and H. Kluge, Phys. Rev. Lett. 46 (1981) 813.
12. F. Saint Laurent, M. Conjeaud, R. Dayras, S. Hanar, H. Oeschler and C. Volant, Phys. Lett. 110B (1982) 372.

13. S. Das Gupta and A.Z. Mekjian, Physics Reports 72 No. 3 (1981).

14. K. Wildermuth and W. McClure, Cluster representations of Nuclei (Springer, New York, 1966).

FIGURE 1

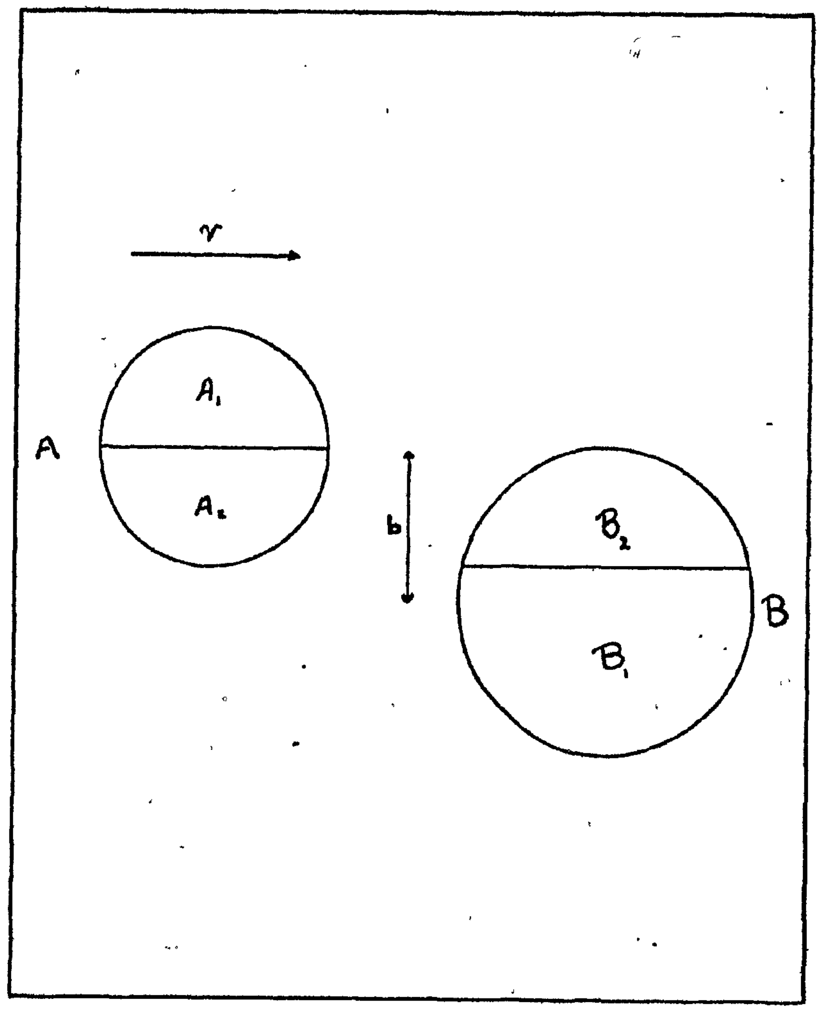


FIGURE 2

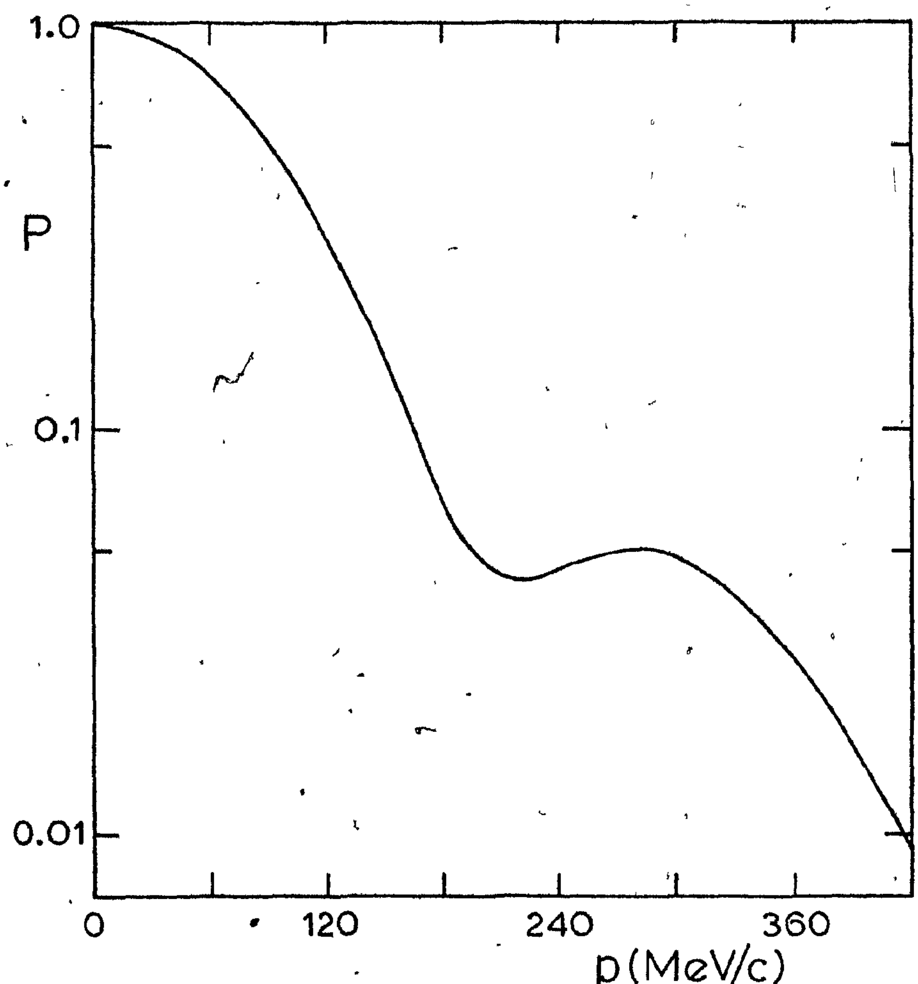
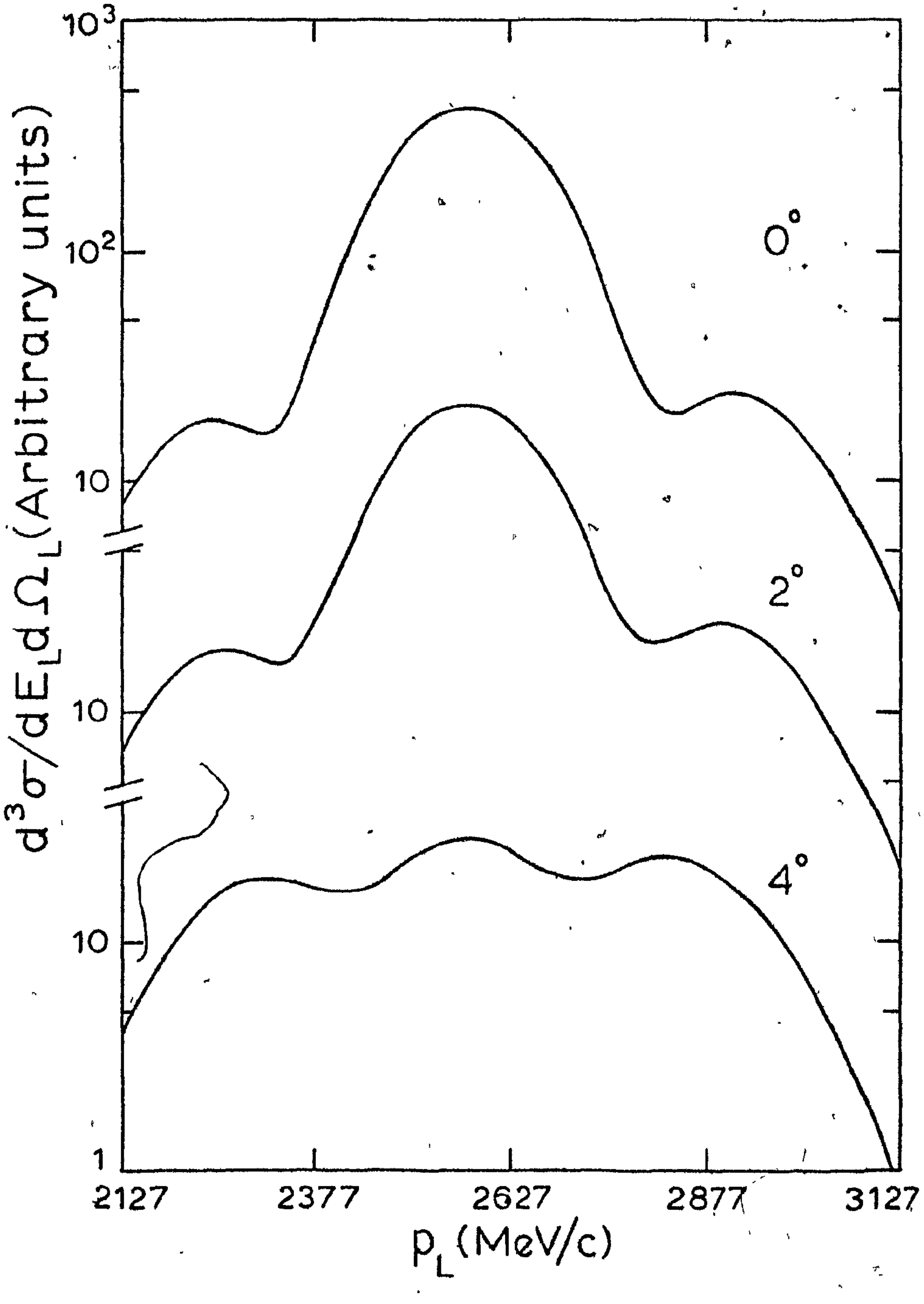


FIGURE 3


10^3
10^2
10
10
10
1
0°
2°
4°
2127
2377
2627
2877
3127
p_L(MeV/c)
$d^3\sigma/dE_L d\Omega_L$(Arbitrary units)