THE MINIMUM COST OPTIMAL POWER FLOW PROBLEM SOLVED VIA THE-RESTART
HOMOTOPY CONTINUATION METHOD

by

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ABSTRACT

This thesis is the result of an investigation to assess the potential of the continuation method to solve the minimum cost optimal power flow problem. For this purpose, a restart homotopy continuation method algorithm is developed, which contains in essence two phases.

The pertinent steps of the first phase are as follows:

1. Create a sub-problem of the complete optimal power flow problem by relaxing boundary limits on all functional variables, namely voltages at load buses, line flows and reactive generations.
2. Parameterize a subset of the whole set of controls which comprises initially of tap-changers, phase-shifters, shunt controllers, and the voltages at generation buses.
3. Optimize the resulting problem.

The solution in step (3) is used as an initial starting point in a continuation process, designed to track this solution to the optimal solution of the sub-problem defined in step (1). The tracking is accomplished via a predictor-corrector path following algorithm embodying certain special features, such that the solution accuracy can be improved to any desired degree through a flexible restart feature developed in this study. Within the tracking process only a subset (identified in step 2 above) of the whole set of controls require specific monitoring for break-points. This feature greatly reduces the computational burden. Termination of the first phase marks an operating point in which all controls are strictly feasible.

If, following the termination of the first phase, functional variables previously ignored prove to be within their respective bounds, the solution to the sub-problem becomes the solution to the complete optimal power flow problem. However, should functional variables violate their bounds the second phase of the algorithm is invoked, which in essence creates a new sub-problem by changing the roles of the control and violated dependent variables, such that the newly modified sub-problem maintains the same basic structure as its predecessor.

Phase I is invoked again at this juncture to solve the modified sub-problem. This process is repeated in cycles until the Kuhn-Tucker optimality conditions are satisfied. Simulations suggest that convergence is usually achieved within two or three Phase I/II cycles.

This being a method unique to the minimum cost optimal power flow problem, numerous examples (up to 118 buses) have been tested and compared against the commercial code MINOS. The newly proposed algorithm appears to be faster and more reliable.

RÉSUMÉ

Cette thèse est le fruit d'une étude qui cherche à évaluer l'utilité de la méthode de continuation dans la résolution du problème d'écoulement optimal de puissance ayant comme tâche le coût minimum. Pour cette fin, un algorithme a été développé, basé sur la méthode de continuation dite à départ répété. Cet algorithme est composé essentiellement de deux phases.

La première phase comporte ces trois étapes importantes:

1. Une approximation du problème complet d'écoulement optimal de puissance est créée en déplaçant les limites sur les contraintes fonctionnelles. Les variables affectées sont les tensions de charges, les puissances réactives aux générateurs, et les transits de lignes.
2. Un sous-ensemble des variables commandées est exprimé en fonction d'un paramètre de continuation. Ici, les variables affectées sont les rapports de transformation Variables d'amplitude et d'angle, les compensateurs réactifs, et les tensions aux générateurs.
3. Le problème issu des deux premières étapes est optimisé.

La solution à la troisième étape sert de point de départ dans le processus de continuation, qui cherche à suivre la trajectoire des solutions optimales pour l'ensemble des problèmes intermédiaires définis à la première étape. Un algorithme de type prédicteur - correcteur est utilisé pour suivre ces trajectoires. On y incorpore cependant quelques dispositifs nouveaux. Par exemple, l'accès aux départs répétés permet d'atteindre plus rapidement la précision de calcul voulue. L'identification d'un sous-ensemble des variables commandées potentiellement actives (selon la deuxième étape) permet aussi d'accélérer de beaucoup le calcul, en réduisant le nombre de calculs. Le calcul de la première phase se termine en un point où toutes les contraintes sur les variables commandées sont vérifiées.

Suite au calculs de la première phase, les valeurs des variables dépendantes, jusque-là négligées, sont calculées et comparées à leurs limites. Si toutes ces contraintes sont respectées, la solution de la

première phase coïncide avec celle du problème général. Si par contre une ou plusieurs de ces contraintes ne sont pas respectées, la deuxième phase de l'algorithme est interpellée. Celle-ci crée une nouvelle approximation en échangeant les rôles de quelques variables commandées avec ceux des variables dépendentes affectées. La nouvelle approximation garde essentiellement la même structure que celle de son prédécesseur. La première phase est ensuite répétée avec la nouvelle approximation. Les cycles phase 1 - phase 2 sont répétées jusqu'à temps que les conditions d'optimalité de Kuhn-Tucker soient respectées. Nos essais numériques indiquent que d'habitude cette convergence est atteinte en deux ou trois cycles des phase 1 - phase 2.

Cette méthode étant proposée pour la première fois dans le domaine de l'écoulement optimal de puissance, on a cru bon d'effectuer plusieurs essais numériques. Ces essais ont été effectués sur de nombreux réseaux, allant jusqu'à 118 barres. Nos résultats ont été comparés à ceux obtenus avec le programme commercial MINOS. D'après les résultats, notre algorithme semble être plus rapide et plus fiable que MINOS.

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NOMENCLATURE

- a A positive fraction to define robust simplices, or the lower boundary defining a closed interval $[a, b]$, the context being clarified in the text.
- a_i Cost coefficient in $c_i + a_i P_{gi} + kb_i P_{gi}^2$ \$/hr.
- A_F A set, whose elements are the indices corresponding to the components of vector h at a limit, i.e., active.
- b A constant vector of dimension as defined in the text
- b An upper boundary defining the closed interval $[a, b]$.
- b_b Value of b corresponding to the basic variables
- b_i Cost coefficient in $c_i + a_i P_{gi} + kb_i P_{gi}^2$ \$/hr
- b_n Value of b corresponding to the non-basic variables.
- b_s Value of b corresponding to the super-basic variables
- B^j A facet on a simplex.
- B_{ij} Imaginary part of Y_{ij}
- B_{sh} Variable susceptance of a SVC device, or used as a symbol to represent such a device the context being clarified in the text.
- B_{sh} A vector whose elements constitute of shunt controllers.
- B_T Bus Type
1. Slack Bus (V, δ).
 2. Load Bus (P, Q).
 3. Generation Buses; including any bus with synchronous condensers; (PV).
- $C_i(P_{gi})$ Variable operating expenditure of the i th generation unit.
- c_i Cost coefficient in $c_i + a_i P_{gi} + kb_i P_{gi}^2$ \$/hr.
- d^k Step length equal to $\theta^{k+1} - \theta^k$.
- d Direction of descent.
- D Set of disturbance (load) variables.
- D Vector of disturbance variables.
- D_i The i th element of D .
- E The easy-to-solve (deformed) problem.

f Scalar objective function.

f_v, f_u Partitioned components of the row vector $f'(x_k)$. This is defined by $f'(x^k) = (f_v, f_u)$.

$f'(x) = \frac{\partial f(\cdot)}{\partial x^t}$ a row vector.

$f''(x) = \frac{\partial^2 f(\cdot)}{\partial x \partial x^t}$ a matrix.

$\nabla f_u = \frac{df(\cdot)}{du^t}$ objective gradient (total derivative), a row vector.

$F(\cdot)$ A Scalar Objective Function.

$F(x) = 0$; Set of vector equations to be solved.

$F'(x) = \frac{\partial F(x)}{\partial x^t}$ a matrix.

$\underline{F} = [H(y), b^t(y - y^{k+1})]^t = 0$, a vector function by construction to be used in the Newton corrector step.

g Scalar function representing the LF manifold.

g Vector form of the LF equations.

G Set of generation nodes, excluding the slack node.

G A vector function, which approximates H on a simplex.

G_{ij} Real part of Y_{ij}

$g'(u) = \frac{\partial g(u)}{\partial u^t}$ a row vector.

$g'(x) = \frac{\partial g(x, u)}{\partial x^t}$ a matrix.

h (a) Vector form of functional constraints or,

(b) as in (a) and incorporating LF equalities g or,

(c) as in (b) and incorporating inequality limits on x , and u variables.

$h'(x) = \frac{\partial h(x)}{\partial x^t}$ a matrix.

$h_A'(u) = \frac{\partial h_A(x, u)}{\partial u^t}$ a matrix.

h_A A sub vector of h , composed of elements which are at a limit, i.e., active.

h_{AL} Taylor series expansion of vector h_A about a given expansion point.

H The set of functional variables, or the Hessian matrix.

$H(x, \theta)$ Homotopy function.

H^{-1} A continuous path generated by the solution $x(\theta)$ to the homotopy $H(x, \theta) = 0$.

H^{-1} A restart homotopy path.

$H'(x, \theta) = \frac{\partial H(x, \theta)}{\partial (x, \theta)^t}$ a matrix.

$H'(y) = H'(x, \theta)$

$H'_{-i}(y) = H'(y)$, with the i th column removed.

$H'(x) = \frac{\partial H(x, \theta)}{\partial x^t} = H'_{-(n+1)}$ a matrix.

$H'(\theta) = \frac{\partial H(x, \theta)}{\partial \theta}$ a vector.

i Subscript index used to identify a specific bus, or node on a simplex, or an indexing parameter as defined in the text, or a bus number.

I_l^2 Square of line currents, where l is the line number defining a line between nodes i and j .

I_l^M Maximum current limit of line, in pu amperes. 7

j A bus number.

k A constant parameter defined in the text, or a bus number, the context being clarified in the text.

k_i A positive constant associated with node v_i .

K An iteration or a sequence count.

l An indexing parameter, defined in the text.

$l(S^j)$ Largest edge of simplex S^j .

L Set of nodes with specified disturbances, or the lower limit of a device.

$LD = (NB - NG)$, Number of load buses.

$LF = (2NB - NG - 1)$, Number of load-flow equations in standard form.

$\mathcal{L}(\cdot)$ The Lagrangian Function.

$\mathcal{L}_A(\cdot)$ The Augmented Lagrangian Function.

M An iteration count.

n The dimension of vector as defined in the text.

- N Variable network branches associated with controllers, or an iteration count.
- NB Number of system buses.
- ND Number of TCUL transformers.
- NF Number of functional variables.
- NG Number of generation buses, including those connected to synchronous condensers.
- NL Number of transmission lines.
- NP_i Number of units per plant i .
- NOG Total number of generation units.
- NSH Number of SVC devices.
- p Length measured along a path H^{-1} .
- p^k A specific point on the homotopy path.
- p^k Step length vector, defined by $p^k = x - x^k$.
- P_b, P_s Vectors, representing the step increments applied to the vectors x_b and x_s .
- P_v, P_u Sub-vectors of vector p_k , partitioned as $p^k = (P_v^t, P_{ut})^t$.
- P_{di} Real demand at bus ' i '.
- P_D Total system real demand.
- P_{Di} % of P_D carried at the bus.
- $P_g = (P_{gi}; i \in G)$, The vector of generation at PV buses.
- P_g^E Value of P_g at the easy optimum.
- P_{gi} Real generation at bus i , or the generation of unit i , the context is clarified in the text.
- P_{gi}^m Lower limit for real generation of unit i .
- P_{gi}^M Upper limit for real generation of unit i .
- P_{gs} Real generation at the slack bus.
- P_{gs}^L Limiting value of the slack generation, i.e., P_{gs}^m or P_{gs}^M .
- P_{ij} Real power flow on line $i-j$.
- P_{ij}^E Value of line flow on line $i-j$ at the easy optimum.
- P_{ij}^L Limiting value of line flow, i.e., either P_{ij}^m or P_{ij}^M .
- P_{ij}^m Lower real power limit on line $i-j$.
- P_{ij}^M Upper real power limit on line $i-j$.
- $P_L(\xi, x)$ Total system real losses.

ΔP Maximum absolute mismatch among all line flows P_{ij} , defined by the following expression $\Delta P = \text{Max} |(P_{ij}^E - P_{ij}^L)|$.

Q_{di} Reactive demand at bus 'i'.

Q_{gi} Reactive generation at bus i.

Q_{gi}^L Limiting value of Q_{gi} , i.e., Q_{gi}^m or Q_{gi}^M .

Q_{gi}^m Lower limit for reactive generation of unit i.

Q_{gi}^M Upper limit for reactive generation of unit i.

Q_D Total system reactive demand.

Q_{Di} % of Q_D carried at the bus.

r The dimension of a vector as defined in the text.

r_k A descent direction.

$r(S^j)$ Volume of the simplex S^j .

R Longitudinal line resistance, in pu, on a 100 MVA base.

R^n The n-dimensional Euclidean Space.

s Subscript index referencing the slack bus.

s Vector of positive components, providing bounds on d.

S, S^j Notation used for simplices contained in R^{n+1} .

S_{gi} Maximum pu MVA rating of synchronous machine i.

T Set of all transmission line numbers.

t Turns ratio of a tap changing transformer, or used as a symbol to represent such a device the context being clarified in the text.

τ A vector whose elements constitute of tap-changing transformers.

U Set of decision (control) variables, or is a sub-matrix of $g'(x_k)$ partitioned as $g'(x^k) = (V, U)$, or used to represent the upper limit of a device, the context being clarified in the text.

$u = (\xi, P_g)$, Vector representation of U.

u_1, u_2 Partitioned u, explained in text.

U^M, U^m Upper and lower bounds on U.

v_i The ith node of a simplex.

- V Belongs to the partitioned matrix defined by $g'(x^k) = (V, U)$.
 $V_G = (V_i : i \in G)$, a vector of voltages at PV-buses.
 V_i Complex voltage at bus i , i.e., $V_i = V_i \angle \theta_i$.
 V_i Magnitude of voltage at bus i .
 V_i^M, V_i^m Upper and Lower limits for bus voltage V_i .
 V^N Initial voltage setting at the bus.
- w A vector representing a point in space, which is used to approximate another point y which lies on a homotopy path.
- $w^j \in B^j$ A point on facet B^j .
- x Used under different contexts, i.e., either as a vector representation of X , or as a general $(n+1)$ -vector of variables used to explain path following, or as a general n -vector to explain an optimization methodology.
- x^* An arbitrary point on path H^{-1} .
- x^* The terminal point of the sequence $\{x_k\}$.
- x^0 Value of x at the beginning of path H^{-1} .
- x^1 Value of x at the end of path H^{-1} .
- x_b Basic variables among x .
- x^E Value of state vector x at the easy optimum.
- x_i The i th element of x .
- $\{x_k\}$ An iterative sequence.
- x_k The k th element of the iterative sequence.
- x^k A feasible point used to demonstrate a feasible direction algorithm, or the value of x at the k -th iteration.
- x^m Lower limits on the n -dim vector x .
- x^M Upper limits on the n -dim vector x .
- x_n Non-basic variables among x .
- x_s Super-basic variables among x .
- x_v, x_u Sub-vectors of x partitioned as $x = (x_v^t, x_u^t)^t$.
- x_b^L Limits, i.e., upper or lower, of variables x_b .
- X Set of dependent (state) variables, also called simple functional variables.
- X_L Longitudinal line reactance, in pu, on a 100 MVA base.

$y(p) = (x(p), \theta(p))$, by definition.

$y^k = (x^k, \theta^k)$, a general point on H^{-1} .

Y_{ij} Complex admittance of line $i-j$, i.e., $Y_{ij} = Y_{ij} \angle -\theta_{ij} = G_{ij} - jB_{ij}$.

Y_{ij} Magnitude of Y_{ij} .

Y_C Total capacitive susceptance of the line, in pu, on a 100 MVA base.

z_i i th component of the vector of injections. This is defined such that it can be any one of the following, the context being clarified in the text, i.e.,

- $P_{gi} - P_{di}$, if bus ' i ' is a PV or PQ bus,
- $Q_{gi} - Q_{di}$, if bus ' i ' is a PQ bus,
- $Q_{gi}^L - Q_{di}$, if bus ' i ' is a PV bus, and only if Q_{gi} is specified at a limit.
- I_{ij}^2 , only if the line current $i-j$ is specified at a limit. In this case z_i is defined either at location i or at location j , but not at both i and j .

z Vector of injections.

z_F Dependent variables (functionals) specified as injections.

z_I Inactive vector of injections.

z_{Fi} The i th element of z_F .

z_F^L Limiting values (maximum or minimum) of the functional variables.

z^k Basis vectors of dimension $(n-r)$ orthogonal to the rows of matrix $g'(x)$.

α Vector of Lagrange Multipliers used un-partitioned, or partitioned as (α_P, α_X) ; the usage being clarified in the text.

α_P A vector component of α corresponding to active P_g 's.

α_s A scalar Lagrange multiplier corresponding to active P_{gs} .

α_X A vector component of α corresponding to active ξ 's.

α_{Xi} An element of α_X .

α_{Xi}^M The largest of $|\alpha_{Xi}|$, taken over all free ξ -variables.

β A vector whose components constitute of Lagrange Multipliers defined in the text.

γ Scalar step-length parameter, or the asymptotic error constant; the context being clarified in the text.

γ A vector whose components are Lagrange Multipliers.

- γ_i The i th element of vector γ .
 δ A given 'small' value.
 ϵ A pre-defined precision value.
 ϵ_d Displacement error, i.e., an indicator which establishes the closeness of a point in space to the optimal point being sought.
 θ_i Bus voltage angle at bus i .
 θ_{ij}^M Maximum allowable spread in bus voltage angles between buses i and j .
 θ^k A permissible value of θ .
 θ The Continuation Parameter, limited by $0 \leq \theta \leq 1$.
 θ^0 The value of θ at the beginning of path H^{-1} , i.e., $\theta^0 = 0$.
 θ^1 The value of θ at the end of path H^{-1} , $\theta^1 = 1$.
 λ Scalar Lagrange Multiplier corresponding to the power balance equation.
 λ A vector whose components constitute of Lagrange Multipliers.
 λ_i The i -th component of λ .
 μ^k Vector of Lagrange Multipliers corresponding to equality constraints.
 ν^k Vector of Lagrange Multipliers corresponding to active inequality constraints.
 $\xi = (V_g, V_g, t, \varphi, B_{gh})$, a vector.
 ξ_I Vector, comprising inactive elements of ξ .
 ξ_1^E Value of ξ_1 at the easy optimum.
 ξ_1^M The i th element of ξ^M .
 ξ_1^N Homotopically deformed bound ξ_1^M , i.e., $\xi_1^N = \xi_1^E + (\xi_1^M - \xi_1^E)\theta$.
 ξ_1^m The i th element of ξ^m .
 ξ_1^n Homotopically deformed bound ξ_1^m , i.e., $\xi_1^n = \xi_1^E + (\xi_1^m - \xi_1^E)\theta$.
 ξ^m Lower bound on elements of vector ξ .
 ξ^M Upper bound on elements of vector ξ .
 ρ A scalar penalty parameter, or expressed as $\rho(\theta)$ becomes a homotopy.
 ρ_i Penalty parameter corresponding to the i -th functional.
 ρ_1 A physically unattainable value of system demand.
 ρ_0 A constant parameter, or a threshold value of ρ , or initial value of the homotopy $\rho(\theta)$.
 $\rho(\theta)$ A linear homotopy, defined by $\rho(\theta) = \rho_0 + \theta \rho_1$.

ϕ Phase shifter angle, or used as a symbol to represent such a device the context being clarified in the text.

$\vec{\phi}$ A vector whose elements constitute of phase-shifting transformers.

$\rightarrow$ Used to describe a mapping from left to right.

(.) Used to define a set.

ACRONYMS

AGC	Automatic Generation Control.
BIC	Bus Incremental Cost.
CM	Continuation Method.
CP	Continuation Parameter.
CPU	Central Processing Unit.
ED	Economic Dispatch.
EDF	Electricité de France
EMS	Energy Management System.
GRG	Generalized Reduced Gradient.
HDE	Homotopy Differential Equation.
IC	Incremental Cost.
ITL	Incremental Transmission Losses.
K-T	Kuhn Tucker.
LF	Load Flow.
LM	Lagrange Multiplier.
LP	Linear Programming.
MINOS	Modular In-core Nonlinear Optimization System.
MIPS	Million Instructions Per Second.
NP	Nonlinear Programming.
OPF	Optimal Power Flow.
PF	Penalty factor.
PL	Piece-wise Linear.
QP	Quadratic Programming.
RHCM	Restart Homotopy Continuation Method.
RHS	Right Hand Side.
SIL	System Incremental Cost.
SLP	Successive LP.
SMSM	Safety Margin Security Metric.
SQP	Successive QP.
SVC	Static VAR Control Device.
TCUL	Tap Changing Under Load.
VAR	Volt-Ampere Reactive.

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CHAPTER I

INTRODUCTION TO THE PRESENT TREND IN OPTIMAL POWER FLOW DEVELOPMENTS

1.1. General Discussion Relating to an OPF Study

The task of operating a power system economically, and securely, i.e., without violating any engineering design limits on its components, is commonly referred to as the optimal power flow (OPF) problem.

An optimal power flow solution gives the optimal active and reactive power dispatch for a static power system loading condition. Computationally this is a very demanding non-linear programming problem due to the large number of variables, and in particular to the much larger number and types of limit constraints¹ which define the boundaries of technical feasibility². The constraints can be either equalities or inequalities. The equality constraints are the standard set of nodal load-flow equations, while the inequalities are due to the plant and transmission system operating limits. Interest in OPF has been growing since the early sixties, and is perhaps at its peak at present, with highly competitive algorithms and solution methodologies, [Carpentier 1987, '85], [Happ et al., 1986], [Happ 1977, '75], [Stott et al., 1980, '79a], [Sasson et al., 1974]. The mathematical formulation of the problem and a review of some of the past successful OPF techniques are presented in Chapter II.

The purpose underlying an optimal power flow is to find a power flow solution, for real and reactive power dispatch, in which a given objective is minimized while respecting all engineering limits on power system components. The most used and least controversial objective in an OPF study is the production cost, [Stott et al., 1980]. Many other objectives are also possible, a discussion of which is deferred until Chapter II.

The real time solution of the OPF is increasingly coming closer to reality due to the recent advances in solution methodologies, and the advent of fast

¹ The limit constraints are due to engineering design.

² Feasibility implies the non violation of engineering limits of power system components.

and reliable computers. At present more than 18 utilities in North America have an active OPF program on VAX, PRIME, and IBM computers, [Happ 1986]. Judging by some recent technical publications, this number appears to be growing internationally, [De La Fuente et al., 1987]. However, some of the recent publications such as [Tinney et al., 1987], [El-Kady et al., 1986], express the concern that some of the underlying assumptions embodied in an OPF may limit the scope of the resulting solution. The reasons for concern expressed in these publications are due to the following, i.e.,

1. The external equivalents used in an OPF study,
2. Provision for handling discrete variables,
3. Application of selective control actions.

These points will be briefly elaborated.

1.1.1 Equivalencing External Systems for OPF Studies

The problem of equivalencing is best illustrated in figure 1.1. It is clear from this figure that, the accuracy of the OPF solution can be influenced by the model used to represent the external system. The following publications address and compare candidate models for equivalencing, [Alvarado et al., 1977], [Deckman et al., 1980a,b], and [Tinney et al., 1986,'87].

Illustrating Equivalencing in an Interconnected Power System

Figure 1.1

An accurate alternative to having an equivalent model is to have no model at all, i.e., consider the external system in its entirety. This no doubt

increases the dimensionality and computational burden relating to the problem, thereby demanding excessive computing power.

1.1.2 Discrete Variables in OPF Studies

Variables such as shunt capacitors and reactors of the non solid state variety, and tap changing under load (TCUL) transformers can be adjusted only in discrete steps. At present, all existing OPF algorithms ignore this fact in the initial solution process, but incorporate them later in an approximate manner. Tinney and colleagues [1987] contend that such a treatment can lead to higher sub-optimal costs, and could even lead to infeasibilities if the tap settings are situated far apart.

Circuit breaker positions in a network can also constitute discrete variables. An inclusion of such variables would no doubt enhance the usefulness of the OPF in areas of operations and system expansion planning, [ibid]. Limited experimentation by Tinney and colleagues has revealed that, rounding discrete controls to the nearest step is marginally acceptable for controls such as transformers having small uniform steps. However, for controls with large nonuniform steps, such as line switching, and some forms of controllable shunts, the resulting errors can be quite large, [ibid].

1.1.3 Selective Control Actions in OPF Studies

In order to implement an off-line OPF simulation for differing loading conditions, all inactive control variables, i.e., variables which are not constrained to a limit as a result of the solution, must be dispatched, i.e., updated as dictated by the OPF solution. This would be unrealistic from a practical view point due to the large number of control variables requiring an update. Further many of such controls would prove trivial, i.e., the changes necessary in order to implement the exact optimum would prove to be very small. Unfortunately the present OPF technology is unable to isolate such controls a-priori, so that the overall dimensionality of the problem can be reduced by treating the trivial controls as parameters in solving the OPF. A partial solution to this dilemma would be to pre-select

dispatchable controls according to prevailing operating practices, and treat the remainder as parameters [ibid]:

Tinney and colleagues [1987] indicate that with closed loop automatic controls any number of control actions can be executed. However, OPF has reached only a limited level of advancement in this vein as demonstrated by the New York Power Pool energy management system (EMS) [Elacqua et al., 1982], and the efforts of Tohoku Electric Power Company in Japan, [Isoda et al., 1985]. Further, it is very likely that even with automatic closed loop controls the number of variables to be adjusted per adjustment cycle will be limited, and methods applicable to discrete variables will decrease the number of control actions by holding trivial controls at their initial settings.

The control actions resulting in an OPF cannot be ranked. Further, the importance of an action is not necessarily related to its magnitude, i.e., large changes in a control need not necessarily imply an elevated position for the control variable in a hypothetical priority list of controls participating to minimize the objective and enforce the constraints. Assuming that it is possible to select the most important set of controls from an OPF output, the correct magnitude for the selected actions would be unknown because they would have to be readjusted to compensate for the omitted actions [ibid].

Tinney and colleagues [1987] provide figure 1.2 as a hypothetical rendition of the variation in cost as a function of the number of controls. Each point on the curve represents the minimum cost OPF solution for the indicated number of control actions used, i.e., the set of control actions corresponding to each point is the best set of that size. Point A in figure 1.2 corresponds to the smallest set of actions that can produce a feasible solution. The optimal cost decreases towards point D, which is also the solution currently provided by existing OPF algorithms. An OPF with curtailed control actions will be able to obtain a solution anywhere between points A and D. Figure 1.2 does not explicitly identify the control elements per control actions involved with each point. This is due to the possibility that, as the number of control actions change the composition of the sets

change as well. Elaborating, although the set at B is smaller than the set at C, elements of set B are not necessarily a subset of the elements of C, i.e., set C can include elements not in set B.

Optimal cost as a function of the
control actions used to solve the OPF problem

Figure 1.2

If a fast method for solving the selective control action problem were available, the curve of figure 4.2 can be approximately established by solving the OPF for a several different preassigned control actions. For relative measures, the optimum cost at point D would be necessary. At this point the experience of the operator becomes indispensable in order to select the number of control actions to solve a given problem. At present this seems to be a viable, but exhaustive alternative to the curtailed control action problem.

An alternative to the above scheme would be to assign a meaningful cost to initiating each kind of control action whose number is to be limited. The solution to such a scheme would produce the correct number of actions. Although such costs exist, they are difficult to quantify. [Tinney et al., 1987].

The discussion next focuses on the aspect of security in OPF.

1.2 Introduction to the Security Constrained OPF

The security constrained OPF³ is an expanded version of the general OPF in which a set of postulated (pre-selected) contingencies are incorporated as additional constraints into the overall OPF formulation. This aids to maintain security before and after a contingency. Pioneering work in this vein can be traced to that of Alsac and colleagues [1974].

Security can be defined under different contexts in the OPF. The following sections will briefly address all pertinent items relating to this issue.

1.2.1 Security of the Intact System

This is perhaps the simplest and the most commonly considered aspect of security among OPF formulations. Contingencies are not considered in the formulation. The topics underlying this thesis fall under this category. The details of the formulation are presented in Chapter II.

The solution to the above problem is gradually being dismissed by operators on grounds that it is too optimistic. Elaborating, should the system experience a contingency there is no guarantee that the system would reach a steady state secure solution subsequently. Even feasibility is not guaranteed.

The essence of current proposals in order to incorporate security into the OPF formulation will be considered next.

³ The term security is used to refer to the steady state operation of a power system in which no engineering limits on power system components are violated prior to, or after a contingency. A contingency is an unforeseen loss in power system components, eg., loss of generation, load, lines etc.

1.2.2 Security Under Contingencies

In order to maintain the security of a power system following a contingency, it is necessary to restrict the feasible (permissible) operating region of the power system through additional problem constraints. Mathematically this is achieved by appending to the OPF formulation sets of nodal load flow equations, plant and transmission system operating limit constraints corresponding to each contingency considered. This will cause the optimal solution (operating point) to move to a location which is secure before and after a contingency. Elaborating, the effect of additional constraints are to restrict the feasible region of operation such that, the system can reach a steady state secure solution following a previously postulated contingency, purely due to the actions of automatic controls in the system, eg., automatic generation controls. Stated alternately, the post-contingency secure operating point becomes a function of the pre-contingency secure operating point. The penalty one pays for this security is the additional pre-contingency operating expenditure incurred by the power system, i.e., the secure OPF solution is always more expensive than the OPF (generally insecure under contingency) solution. Attempts to solve the OPF along these lines with limited success can be traced to, [Alsaç et al., 1974], [Carpentier 1975], [Elfstrom et al., 1983], [Merritt et al., 1987].

An alternative to the above scheme would be to relax some of the additional constraints subject to the limitation that, in the event of a contingency it is possible to implement corrective controls within a permissible time period in order to restore security. The problem magnitude increases due to the post contingency controls now becoming unknowns. However, the relaxed constraints would result in a smaller pre-contingency operational expenditure, [Carpentier 1987].

At present, as applied to the OPF problem, the state-of-the-art with respect to 'security under contingency' is still in its infancy. The alternative of switching transmission lines as a corrective measure in OPF studies has received only limited attention in the past. However, a recent paper by Glavitsch [1987] appears to change the course of present OPF research towards integrating corrective switching into such studies.

Corrective switching is an inexpensive alternative for solving many security related problems. However, the algorithms at present are not fast enough to implement such measures in real time. The practical difficulty being due to the nature of the problem, i.e., the multitude of switching combinations to be considered. Hence, designing and integrating such algorithms into the OPF remains a challenge to power engineers, [Carpentier 1987].

Past solution attempts in this vein with little emphasis to integration with the OPF can be traced to [Koglin et al., 1985], [Rossier et al., 1983], and [Bacher 1986].

The following sections will study the possibilities of applying techniques on artificial intelligence to OPF studies.

1.3 Artificial Intelligence in OPF Studies

At present, artificial intelligence is in its absolute infancy as applied to the OPF problem. The rationale for such studies is to answer operational questions of the form,

'Among all changes suggested via a simulation, what minimal changes to the system are needed to alleviate all resulting violations, due to the occurrence of a single or multiple events from a specified set of contingencies?'

This is a very difficult question to answer due to the discrete nature of the variables involved, the multitude of possible solutions, and perhaps the non-availability of a unique solution. Furthermore, this is an area where the knowledge of an operator familiar with the idiosyncracies of a particular system becomes indispensable, in selecting one particular alternative among the many numerically suggested options.

The inception of a project of limited scope to take some account of the above factors can be traced to Electricité de France, [Carpentier 1987]. In it, the operators, in anticipation of future contingencies provide the

computer continuously with solution proposals at regular intervals ranging from a few minutes to a few hours. Under this scheme, the operator proposed solution will be accepted by the OPF should any other solution resulting from built in algorithms fail to provide a better alternative.

A foreseeable extension to the above project, would be to build a knowledge base to encompass frequently encountered topological structures, past successful operator proposals, and the corresponding off line OPF solutions which has provision to incorporate line switching as an alternative to, or in conjunction with, control variable adjustments to solve a specific OPF problem. Such a data base along with advanced techniques in artificial intelligence will aid to determine a rapid solution to a given OPF problem, given the actual network topology and its state. Developments along these lines are currently underway, [ibid].

1.4 The Use of OPF in Daily Scheduling Tasks

The previous discussion has considered the OPF as a dispatching task. Such consideration greatly limits the scope of the OPF solution even when the forecasts prove correct. In order to integrate the OPF into the daily scheduling tasks one has to take dynamic constraints, eg., ramping constraints which limit the rate at which the generation of a unit can be increased or decreased, and the generally fixed amount of water to be used in hydro plants into consideration. The consideration of ramping constraints is complicated by the fact that they apply differently to thermal fossil units, and nuclear units.

Carpentier [1984a, 1987] derives the theoretical property of 'Dynamic Dispatch Separability', in which the quadratic operating costs of thermal units are appropriately incremented by a set of anticipated over-costs, under certain restrictive assumptions, in order to adjust the optimum of the dispatching problem to that of the scheduling problem, [Carpentier 1984a, '87].

In implementing this property, only the operating costs of the OPF change. Carpentier claims that the computation of anticipated over-costs are equiva-

lent to at most two power flow iterations, and the equivalent costs for hydro plants can also be computed automatically given the solution to a scheduling problem at an instant of the scheduling cycle. The OPF solution meets all security and dynamic constraints under limited scheduling conditions, [ibid]. However, at present, this theory is still being evaluated.

The present trend in the development of OPF is partly due to the ever increasing user expectations, and indicates it to remain an evolving problem with one line of development advancing towards complete generality in formulation, and the other towards special applications with the resulting solutions however being limited in scope. Although OPF is currently considered as a static optimization problem, the present advancement indicates that it will eventually be merged with optimization applications extending over a period of time. Mathematically optimal solutions to realistic OPF problems may be difficult to achieve in real time. However as stated by Tinney and colleagues [1987], it is better to have good near optimal solutions to realistic problems rather than optimal solutions to unrealistic ones.

The presentation to follow next outlines the contents of the present thesis.

1.5 The Present Thesis

The following sections briefly overviews the motivations and contents of the present thesis.

1.5.1 General Comments

The present thesis addresses the OPF problem in the classical sense where all power system controllable variables are considered in the optimization relating to a given load and network parameter specification, i.e., a static loading condition of the intact system. It is of course possible to carry out the optimization with a limited number of controls, with the others fixed (parameterized), provided such controls can be made available a priori, usually via intelligent operator judgement.

All control variables are assumed continuously adjustable. This is generally true for most generation units, but need not necessarily be true for tap changing under load (TCUL) transformers, and shunt controllers. However this study makes the reasonable assumption that the step changes within these controls are reasonably close, and uniform, to be considered continuous. Although switching transmission lines in and out of the system can be considered as a discrete control action, this thesis does not consider this possibility, i.e., the system is assumed to be intact throughout the duration of the optimization.

This thesis considers the aspect of security from the point of view of the intact system. The solution so obtained may not prove secure in the event of a contingency. However, the thesis proposes a scheme whereby a secure feasible solution can be reached following a contingency. In this all controls participate in achieving a secure optimal solution, unless some controls are prevented from participating by the operator. The scheme in itself does not possess the intelligence to preselect the best set of controls in order to reach a secure optimal solution following a contingency, but rather allows all controls to vary. This scheme is considered to be better than adding constraints to enforce security, for a postulated set of contingencies, for the following reasons.

1. The optimal operating costs would be lower
2. A secure optimal solution is always possible to be obtained provided it is feasible, within a reasonable time frame.
3. The proposed scheme is very flexible, since the solution obtained due to the addition of many security constraints can be very restrictive in the sense that, the solution so provided will be able to protect the system only in the event of the postulated contingencies, i.e., should the system experience a contingency other than the ones postulated, there is no guarantee that the system would reach a secure optimal operating point following the contingency.

This thesis considers the system external to the power system being optimised in its entirety, i.e., equivalent models are not used due to their limited ability to represent the external system accurately. Since the

external system generators are non-dispatchable from within the power system being optimised, their controls, namely generations and other control variable settings would be normally fixed during the optimization process.

The developments in this thesis can be used for scheduling by repeatedly executing the optimization over predefined intervals of a few minutes (10 to 15). This process can be implemented very efficiently provided the system structure does not change, by utilising the solution to the last optimization as the starting solution to the next optimization. However, should the system undergo changes to its structure, the previous solution may not prove to be a good starting point.

The generation costs of all dispatchable, committed⁴, thermal units are assumed to be pure quadratic functions. This aids to maintain the convexity assumptions needed for the success of most nonlinear programming algorithms. Hydro units, if present, are assumed to be non-dispatchable, with a known generation, during the period of the optimization. The total generation cost of all thermal units in the system is chosen to be the objective function to be minimized

The effect of ramping constraints are ignored on all dispatchable units, since the optimal power flow problem is generally envisaged as a static optimization problem.

The complete formulation of the OPF problem is presented in Chapter II.

The principle objective underlying this thesis has been to investigate the potential of the continuation method⁵ (CM) to solve the OPF problem, under the assumptions outlined above. This has led to the development of the restart homotopy continuation method in which, the continuation method is applied with restarts (repeatedly) to solve the OPF problem.

⁴ Committed units are those generation units which are in operation synchronized to the network.

⁵ Principles underlying the continuation method are presented in Chapter II

Concisely, the continuation method has the potential to track the optimal solution to a deformed problem efficiently and reliably when the problem deformations are gradually removed. The tracked solution remains optimal and feasible with respect to each of the deformed problems. Once the problem deformation is successfully removed, the current solution becomes the solution to the original problem, since the original problem is initially so deformed so that once the deformations are removed the resulting problem becomes the original problem. In removing the problem deformation a series of intermediate problems are optimised

This feature of the CM sharply contrasts with other nonlinear programming techniques which address one problem, namely the original problem from the beginning to the end of the optimization. By deforming the problem to ones own advantage it is possible to optimise the deformed problem rapidly. Now if the deformation is slightly relaxed, there is reasonable grounds to believe that the solution to the resulting problem (which will better resemble the original problem than its predecessor) will lie very close to the previous problem, thereby enabling a quick solution to be obtained. A series of such solutions, resulting due to the gradual removal of the deformation would eventually lead to the optimal solution of the original problem. The relatively small computational effort per deformed problem, occurring within the series, gives reasonable grounds to believe that the overall process will be faster than a conventional nonlinear programming technique. This thesis has been able to positively confirm this.

1.5.2 Motivation Underlying the Present Research

The following enumerates the motivations underlying the present thesis.

1. To develop an algorithm, capable of exploiting the special properties of a power system, in solving the optimal power flow (OPF) problem with a specific objective function, namely the operational expenditure, with speed, reliability and accuracy
2. Making the solution process independent of an initial feasible starting point.

3: To assess the potential of the continuation method (CM), previously not applied to the nonlinear OPF problem, in accomplishing the following, i.e.,

- a) Meeting the requirements in (1) above with the least possible computational effort.
- b) Systematically identifying the optimal active set, by tracking a trivially found initial starting solution to each sub-problem.
- c) Identifying infeasibilities in problem specifications.
- d) Achieving convergence, where methods based on techniques other than the continuation method would either have convergence difficulties, or fail to reach an optimal solution.
- e) To investigate the stability of the continuation process in solving the optimal power flow problem described in (1) above.

4. To minimise the core required to solve a given OPF problem, such that problems of reasonably large dimensions can be solved on microcomputers with limited memory.

5. To compare the performance characteristics of such a special purpose algorithm, to a general purpose commercial code such as MINOS, under identical starting conditions, and computing environments.

An outline of this thesis will be presented next.

1.5.3 Outline of the Thesis

The following sections will outline briefly the contents of the discussions contained in the remainder of this thesis.

1.5.3.1 Chapter II

All definitions and assumptions, currently accepted for the OPF are reviewed. A comparison is made on general OPF modelling practices, in which the two most popular models, namely the sparse and compact models are compared.

The principles underlying past OPF solution methodologies are presented, followed by a discussion on algorithmic details pertaining to certain key methods, which have made an impact in the power industry.

The continuation method, and its underlying principles are considered next, followed by a set of definitions relating to different homotopies. Conditions relating to the existence and uniqueness, and the orientation of the homotopy path are also presented. This discussion concludes with a consideration of different path following algorithms, their merits, and principles underlying their relevant mechanisms.

The features desirable in a modern OPF solution technique are considered subsequently, followed by a section on the chronological developments in OPF methodologies, and their capacities to meet the stringent requirements demanded by a modern control centre. The scope of the discussion is limited to techniques which address the simultaneous dispatch of both real and reactive powers, with the exception of techniques which in spite of their limited scope have exhibited excellent numerical performances.

1.5.3.2 Chapter III

Chapter III develops the theory underlying the restart homotopy continuation method. The discussion begins by partitioning all variables into control, dependent, and injection variables. This is followed by the derivation of the first order Kuhn-Tucker conditions for the relaxed OPF problem. In determining the trivial initial solution to the relaxed problem, Algorithms Lambda and Coordination are developed. It is shown that the invocation of Algorithm Coordination provides the trivial initial optimal solution, also called the easy optimum, which will enable the inception of a continuation process to solve the relaxed problem.

At this juncture past successful homotopies in power engineering are discussed, and one which is hybrid in nature, and not previously tried is chosen as the candidate to track the Easy-Optimum.

Thenceforth Algorithm Corrector is developed, which is designed to correct for any drift in tracking the homotopy path, followed by Algorithm Phase I which completely solves the relaxed OPF problem.

Attention subsequently focuses on the full fledged OPF problem. In an attempt to enhance the overall performance of the algorithm, all functional violations at the termination of Algorithm Phase I are further divided into Types I and II functions. A special load flow specification is proposed, as a means of getting rapid convergence onto the optimal active set of functional variables. In this, Type I functions are placed among controllable variables, and Type II functions are placed among injections. It is shown that this specification provides the bus and system incremental costs as by-product information.

Algorithm Update is developed to implement the special load flow specifications, followed by Algorithm Phase II, designed to solve the complete OPF problem in conjunction with Algorithm Phase I.

A discussion on degeneracies and infeasibilities are included in order to understand certain numerical problems. Under this topic, a discussion on numerical oscillations, and singularities are presented. Techniques are also proposed to circumvent such problems. Special cases of degeneracy, in which the problem has no degrees of freedom are also considered.

As a means to reduce the computational burden certain heuristics are also developed which have proved to be numerically successful on many test cases.

Finally a discussion on the special features embodied in a computer code, designed to assess the theory developed herein, is provided.

1.5.3.3 Chapter IV

Simulations results, utilising the algorithms developed in Chapter III are presented here. Number of systems varying in dimension up to 118 buses, 179 transmission lines, 68 controllable variables, and 62 generation units are studied in detail. The performance of the proposed algorithm is compared to that of an augmented Lagrangian via MINOS Ver. 4.0.

The convergence characteristics of the proposed algorithm is also discussed.

1.5.3.4 Chapter V

This chapter contains the concluding remarks, and recommendations for extending the scope of the present study.

1.5.4 Original Contributions

As presented in this thesis, the following is a list of items considered to be original contributions to the area power systems.

- A. The formulation, and implementation of a predictor-corrector path following (continuation method) algorithm to solve the minimum cost optimal power flow (OPF) problem. In this, the noteworthy contributions are:
 - a) Demonstrating that, given the solution to the set of modified coordination equations, a homotopic deformation can be applied to an optimal power flow (OPF) sub-problem, such that this solution falls on a solution trajectory of the sub-problem, thereby enabling the inception of the continuation process which has the potential to follow the solution trajectory and reach the optimal solution of the sub-problem.
 - b) The manner in which the sub-problems are constructed, so that, should the optimization fail within a sub-problem due to a degeneracy or infeasibility, the solution to the previous sub-

problem will be able to identify the constraint(s) and variable(s) responsible for this failure.

- c) A new way of defining the homotopic deformations on a select set of network controllable variables, such as tap-changing under load (TCUL) devices, shunt controllers, voltages (at PV and sometimes at PQ buses), by shifting the variable bounds according to the algebraic signs of their respective Lagrange Multipliers at the beginning of each continuation process.
- d) Formulation of linear homotopies ~~on~~ a sub-set of controllable variables (called the set of ξ -variables), rather than the whole set of controllable and dependent variables, in order to guide the starting solution to each sub-problem towards its optimal value. This feature greatly reduces the computational burden associated with otherwise, explicitly updating and monitoring all the individual controls and dependent variables.
- e) The development and use of an elevator predictor, Newton corrector algorithm to track the solution to the homotopically deformed optimal power flow problem.
- f) The development and use of a restart feature, as a means to improve each sub-optimal solution resulting at the termination of a continuation process.
- g) The formulation of a set of intelligent rules, derived from experimental evidence to bring functional violations within bounds.
- h) A formulation in which all bus incremental costs for both real and reactive demand result as by-products, i.e., require no additional computation.
- i) A formulation which does not require an initial feasible operating point to begin the optimization.

- j) The implementation of a variable structure Jacobian, with nonconventional injections and states.

B. Development and demonstration of a working code, based on the above theory, on micro (IBM PC Family), mini (Vax 11/750, Microvax), and mainframe (Amdahl 5860) computers. In this the noteworthy contributions are:

- a) The modular structure of the code, with each module performing one specific task in the overall optimization.
- b) The use of sparsity oriented programming, with a minimum number of overhead arrays.
- c) Provision of a totally simplified dimensioning task; i.e., the user needs to dimension only three arrays, namely one double precision array, an integer array, and one half precision integer array, to execute the entire optimization. Formulae are provided to estimate the minimum dimensions needed for these arrays. After each successful completion of an optimization, information on the exact array dimensions⁶ are provided, which the user may opt to use for future simulations on the same system.
- d) Provision of a simplified output file, with facilities to expand the output as a means of viewing intermediate results, and progress of iterations.
- e) The proposed structure for a data file, which can be trivially created by the user, based on an information format generally provided for all IEEE test systems.

C. Comparison of the performance characteristics of the proposed method, to one based on an Augmented Lagrangian, through the use of the popular commercial code MINOS (Ver. 4.0). The noteworthy contributions are;

⁶ These arrays have a dimension less than or equal to their respective estimates.

- a) Numerous test systems considered, ranging in dimension up to 118 buses, 179 lines, and 68 controllable variables, all of which are evaluated under identical computing environments.
- b) The verification of the numerical results, as a means to evaluate the accuracy of the proposed method, and its performance, in comparison to a general purpose commercial code.
- c) A numerical study on the convergence characteristics of Algorithm Phase I, i.e., the solution process involved with each sub-problem.
- d) Documentation of all pertinent results.

CHAPTER II

REVIEW OF SOME OPTIMAL POWER FLOW TECHNIQUES

This chapter will review some of the better known optimal power flow techniques which have made an impact in the power industry. The following sections will initially clarify the definitions, assumptions, and notations used in the review, and in the development of the theory pertaining to the present work.

2.1 An Overview of Definitions and Assumptions

The sections to follow will define the terminology used in the OPF literature, which relate to the objectives, constraints, dependent, independent and disturbance variables.

2.1.1 Definitions and Assumptions Relating to Objectives and Constraints

One of the important concerns of the power system operator is, or will be to determine a control strategy which would securely operate the system such that an a-priori defined objective function is optimised (minimized), for quasi static operating conditions, without violating any of the engineering limits imposed on the system components. In the jargon of power system engineers, this problem is referenced as the optimal power flow (OPF) problem.

Depending on the characteristics of the system and company norms, the objective functions take different forms. As classified by Stott et al., [1980], Talukdar et al., [1981], Carpentier [1985] and Hap et al., [1986], the commonly used objective functions are;

1. The operating fuel cost of the power system,
2. The real or reactive losses in the system,
3. The departure of selected, or all, control variables from a pre-set points; i.e., minimum control action, or stated alternately, the departure from a given solution,

4. The amount of total environmental polluting emissions, referred to as emission dispatch,
5. Minimum load shedding.

As envisaged by most authors, the OPF is a static optimization problem, solved many times over a certain period. The period involved depends on the types of generation sources in the system, varying anywhere from a fraction of a day, to one year.

Seasonal storage hydro plants generally give rise to one year periods, while if run-of-river and pumped storage plants are involved the period will not exceed one week. Such hydro plants which cannot continuously maintain full output are classified in the IEEE Committee Report [1971] as limited energy sources. The formulation of the OPF must be such that, maximum use can be made of limited energy sources, to supplant fuel at plants having an unlimited fuel supply, notably thermal plants.

Over the horizon of optimization, taking the minimization of the fuel cost as a candidate for an objective, one needs to know precisely, the instantaneous variations in the cost components involved with each piece of committed generating equipment. The principal elements of the cost components are the fuel and plant maintenance costs.

The fuel cost can be further broken into cost components such as those needed in the storage and handling of fuel. Further, due credit must be applied towards any revenue generated by the sale of any by-products such as fly ash and sulfur.

A number of utilities translate the maintenance costs into an incremental maintenance cost, for ease of adaptability into the incremental fuel cost. This issue is discussed by Light [1957], Steinberg [1958], and Zelenka et al., [1959]. The AIEE Committee Report [1958] has a publication on established practices followed by the utilities in determining incremental maintenance costs. No significant changes in these practices have been published since.

In order to enhance the accuracy of the OPF solution, all these cost components must be incorporated into the input-output characteristics of thermal plants. At the plant end, the incremental operating costs of combustion turbines are determined quite accurately through in-plant performance testing. However, there are limitations to the accuracy embodying such tests. The AIEE Working group report on 'The Application of Incremental Heat Rates', [AIEE Publication S-104] discusses these limitations

The nuclear plant, while similar in many respects to a thermal plant, poses many new problems; since the rate of fuel consumption does not yield to direct measurements. In a conventional thermal plant fossil fuel energy is released almost instantaneously, while in a nuclear plant this energy is released over an extended period. Further, the extent of utilization of the nuclear fuel may not yield to precise determination.

Hydro plants do not have an explicit fuel cost. However, the usage of water can be translated in to an equivalent fuel cost, thus enabling these units to be considered as fictitious thermal units. The principles underlying an equivalent cost are based on realising that, in instances where the available quantity of the basic energy resource to a particular plant is a limiting factor in the operation of the plant, the per unit price of the energy delivered, bears no relation to the per unit price of the basic energy resource. In such instances, the aim is to utilize the available limited energy to the best possible advantage, over the period in which it is available. Such operation will bring about the greatest possible value in terms of fuel savings at plants where the fuel supply is not a limiting factor. The IEEE Committee Report [1971] exemplifies the relationship between the water value and the equivalent cost. Ringlee [1964] and Kennedy et al., [1965] describe methods for adjusting the water value to account for deviation of load, and availability of equipment from forecasted values. Anstine [1965] describes the applicability of the concept of incremental water value to run-of-river, storage and pumped storage plants. Kennedy et al., [1965] further describes alternate approaches applicable for systems

having little or no hydro, except for pumped¹ storage plants, to determine the pumping cycles.

In the general OPF formulation, it is customary to assume that incremental cost information for all committed units is known. If the system contains hydro plants, their real generations over the optimising horizon are assumed known.

The basic components of a power system are generating plants of the fossil type (coal, oil and gas), nuclear and hydro, transmission lines, on-line controllers, i.e., tap and phase-shifting transformers, and switchable shunts and reactors. These components have built-in engineering design limitations, which makes the OPF all the more complex, as the operator has to respect these limits on maintaining the quality of service. In translating these physical limitations into mathematical expressions, referred to as modelling in the IEEE Working Group Report [1981], one is faced with a multitude of inequality constraints, even for systems of moderate size. These constraints define a feasible secure region of operation for the intact system.

As applied to the OPF problem, and used in this thesis, the term security has a special meaning, which requires elaboration. Security, by definition, refers to a set of next contingencies. Dy Liacco [1967] has provided the framework to examine security. The generality of this makes it difficult to be included into the OPF study. However, Talukdar [1981], among other notions, specifies the simplest of all security notions as a binary (0 or 1) variable. He calls it the Safety Margin Security Metric (SMSM), and says that if equipment loading, after a postulated contingency, stays within specified bounds, then the system is secure; i.e., SMSM = 1; else the system is insecure SMSM = 0. This static notion of security, suits the OPF problem best compared to other proposed notions (both static and dynamic), and will be described next.

¹ Pumping units are generally operated at full capacity.

Talukdar says that, if certain safety margins, brought about by proper engineering judgement, can be maintained in a power system, this would make it secure in an SMSM sense. Common types of margins given are;

1. Spinning reserve; i.e., the amount of unused but committed generation capacity,
2. Ready reserve; i.e., the amount of generation capacity that can be committed at short notice,
3. Bus angle spread; i.e., a measure of both, the lines unused capacity and the systems likelihood of becoming unstable should a disturbance occur,
4. Equipment margins; i.e., the unused capacity in other pieces of equipment.

The engineering limitation of a piece of equipment can be suitably constrained to accommodate 1, 3 & 4 above, while 2, being an uncommitted generation does not fit in this manner, and becomes an exogenous variable with respect to the OPF problem. In this study 'security' will be applied in the SMSM sense.

The nodal voltage magnitudes also play a key role in the operation of a power system. Happ et al., [1978, '86], Fernandes et al., [1983], Palmer et al., [1983], El-Kady et al., [1986], regard the system voltage magnitudes as an equipment engineering limit and a system security limit. The reasons enumerated below, will serve to justify this claim.

The OPF imposes limits on voltages to respect equipment voltage ratings and to maintain the quality of service. As envisaged by authors Happ et al., [1978], such limits serve to preclude an excessive strain on reactive sources, such as generators, synchronous condensers, and static VAR control devices (SVC's), and to improve the real power transfer capability of the system. Stated alternately, the limited thermal capacity of the transmission system, restraints both inter and intra area real power transfer, if excessive amounts of reactive power are allowed to flow.

Palmer et al., [1983] and El-Kady et al., [1986] have pointed out that it is necessary to maintain voltage levels within limits; i.e., in an SMSM sense, to avoid voltage stability crisis, and the risk of a voltage collapse, following a normal, or an anticipated post contingency condition.

Nodal analysis, for a balanced three-phase power system of NB buses, in which the network parameters are represented by their positive-phase-sequence components, gives rise to the familiar $2 \times NB$ power flow equations. Stott [1974], surveys different methods that have been applied in the past to solve the above set of equations. In the context of OPF methods which must use a load flow (LF) as part of the solution process, the Polar-Version of the Newton-Raphson method takes precedence over any other method due to its ability to solve the power flow rapidly, reliably, and very accurately. Sasson et al., [1974] and Dommel et al., [1968], show how such a choice would enhance the optimization process by providing information within the LF Jacobian which can be used in subsequent iterations to enforce constraints, if necessary. Further, since voltage magnitudes are iterated upon directly, one can identify without any additional computation when they violate their bounds. For these reasons the present study will also formulate the problem in polar form. The load-flow equations will amount to $2 \times NB$ equality constraints, which must be satisfied at all times in the OPF problem.

This section has provided a concise account of the objective function, and constraints generally involved in an OPF study. The next section will classify the variables available for dispatch by the operator, and other dependent, i.e., non dispatchable variables, referred to as functional variables, which simply react to the directly dispatchable variables.

2.1.2 Definitions and Classifications of Decision and Dependent Variables

The following classification of power system variables, can be considered as conventional, but not necessarily mandatory for the successful implementation of an OPF solution strategy. Often their roles can be interchanged, as will be evidenced subsequently. The directly controllable independent or decision variables are;

1. The real power output $P_{g,i}$ of each plant i , except one plant s henceforth referred to as the slack generator. The slack generation $P_{g,s}$ is a mathematical entity used in the solution of the LF equations, having the physical capacity to bring about a power balance in the network. Subscript index i can assume integer values in the range 1 to NG, NG being the number of buses with generation capabilities. This also includes load buses with synchronous condensers, in which case $P_{g,i}$ would be zero²
2. The voltage magnitudes V_i at all generation buses,
3. The complex voltage phase angle θ_s at the slack³ bus,
4. Parameters of tap changing under load (TCUL) transformers, namely the angle ϕ of phase shifters, and tap position t of tap-changers,
5. Equivalent shunt susceptance of static VAR control (SVC) devices

This study assumes that, all these variables can be adjusted either continuously, or in small enough steps such that the variation can be considered continuous.

Happ et al., [1978] gives a model to represent the SVC variable in 5 above. Unlike its older counterpart; the conventional switchable shunt reactor or capacitor, the SVC is a solid state device, controlled by a thyristor. A practical configuration capable of providing both leading and lagging reactive power consists of, a series combination of a thyristor and inductor in parallel with a capacitor bank. The SVC has a very fast response time, (typically half a cycle), compared to its competitors, the synchronous condenser, and the switched reactor or capacitor. Further description of this device can be found in the work by Concordia [1973], Working Group 31-01 [1974], Moran [1975], Moran et al., and Trump [1976].

* An alternate choice would have been to consider such buses to be of the PQ (load) variety. However, the present choice is favoured as it simplifies the coding task, by giving rise to only one mismatch equation.

³ θ_s can also be considered as an arbitrarily specified reference angle. Since limits are not generally imposed on complex bus voltage angles, $\theta_s = 0$, is a popular choice in OPF studies

Variables which produce large network changes, incompatible with the underlying concepts on dispatching, such as the breaker configuration in the network and the number of committed units, are left out of this study, on the presumption that they are invariant under the optimization period under study.

The dependent, or functional variables are;

1. The slack generation, P_{gs} ,
2. Reactive generation Q_{gi} of all generating plants, including those of synchronous condensers,
3. Voltage magnitudes at all load buses,
4. Complex voltage angles⁴ at all buses except the slack bus,
5. Line current flows in all the transmission lines in the network.

Given the above classification of variables, the following sets can be defined. These sets serve to simplify the notational complexity in the theory to follow. However, during a solution process it is possible for elements of sets to switch positions among sets.

2.1.3 Sets Defining Decision, Dependent and Disturbance Variables

- If U denotes the set of decision, i.e., the set of variables yielding to direct control action, X the set of dependent (simple functional, also called the set of state variables), H the set of functional (set of variables dependent on both U and X), and D^5 the set of exogenously specified

⁴ Generally in the OPF formulation, bus angles do not have any limits imposed on them. However for stability reasons limits can be imposed on the angular difference, by taking the angles defining the line flow. This is seldom done in the OPF, as the general practice is to assume that the lines reach their thermal limits (current limit) before the steady state stability limit. This has been an accepted practice as seen from the work by Stadlin [1971], Peschon et al., [1971], Cory et al., [1972], Carpentier [1972, '73, '85], Glavitsh [1973], Podmore [1974], and Alsac et al., [1974]. In the pioneering work by Carpentier [1962], limits were imposed on angular differences.

⁵ Set D may become a subset of set U when minimum load shedding is enforced as an objective.

disturbance variables, i.e., real & reactive demands, including any tie-line flows to neighboring areas, the defining equations can be written as follows; i.e.,

$$U = (V_s, \theta_s, V_i, P_{gi}, t_j, \varphi_j, B_j : i \in G, j \in N) \quad \dots (2.1.a)$$

$$X = (\theta_i, V_k, \theta_k : i \in G, k \in L) \quad \dots (2.1.b)$$

$$H = (P_{gs}, Q_{gs}, Q_{gi}, I_l^2 : i \in G, l \in T) \quad \dots (2.1.c)$$

$$D = (P_{dk}, Q_{dk} : k \in L) \quad \dots (2.1.d)$$

Sets G, L, N and T are defined below;

1. G is the set of nodes connected to buses with generation capabilities, except the slack bus 's',
2. L is the set of nodes connected to load centers. Sets G and L may have a common intersection,
3. N is the set of network branch impedances affected by the control actions of TCUL transformers and SVC devices.
4. T is the set of all transmission line numbers.

Note: In the analysis to follow, the vectors u , x , h will be used to represent sets U, X, and H.

The OPF methodology solves for X and U for a given D, such that a predefined objective is minimized, while respecting system engineering limitations. Since set H is dependent on both X and U, it becomes known once X and U are known. The elements of the disturbance set D are not used, as a control measure in the optimization, but are treated as parameters, i.e., fixed quantities, during the optimization process. Hence, elements of D will not explicitly appear in the problem formulation.

In practice the operator can exert the greatest control action by dispatching system generation units. Auxiliary control is normally provided through controllers belonging to the set N. Of these, the phase shifting transformers and DC lines (if present) influence the real power flow primarily, while tap changing transformers and SVC devices have their strongest influence on the reactive power flow. In most power systems, the number of phase shifters

and DC lines are low, which implies that the dispatcher has relatively little control over the paths between energy sources and sinks.

2.1.4 Sets of Equations Defining the OPF

Using the general concise mathematical form for the OPF, as defined by Dommel et al., [1968] results in the following formulation; i.e.,

Minimize a scalar objective function,

$$f(u, x) \quad \dots (2.2.a)$$

Subject to the constraints,

$$g(u, x) = 0 \quad \dots (2.2.b)$$

$$h(u, x) \leq 0 \quad \dots (2.2.c)$$

$$u^m \leq u \leq u^M \quad \dots (2.2.d)$$

System security under steady state operating conditions is maintained by satisfying constraints 2.2c&d. Equation 2.2.b defines the set of $2 \times NB$ load flow equations, while equations 2.2.c&d, which can outnumber 2.2.b, is formed by respecting the engineering limitations on power system equipment and voltages as described in section 2.1.1. As defined previously in section 2.1.3, elements of both vectors x , and h can be generally classified as dependent OPF variables.

As observed by Stott et al., [1980], in a well designed power system, apart from controls u , only a few of the functional inequalities 2.2.c will be active at the optimum, over the entire working range of system demand.

Equations 2.2.b&c are nonlinear and algebraic. The nonlinearities among the load flow equations are relatively mild, so that a linearization about a given operating point approximates this set of nonlinear equations fairly well. Furthermore the corresponding LF Jacobian is very sparse, and the objective function can be normally expressed in an algebraic differentiable

form, thereby permitting the overall OPF formulation in 2.2 to be placed under classical nonlinear programming category.

The mathematical properties of the OPF are affected by the following, namely;

1. The engineering scope (accuracy of solution),
2. Approximations made in system modelling,
3. The chosen objective function.

Thus, in general, solution uniqueness cannot be assured, [Stott et al., 1980].

2.2 General Modelling Practices

Two distinct modelling practices, namely, sparse and compact (non-sparse) are used in the literature. Sparse modelling results when an optimization technique is applied directly to the problem defined by the set of equations 2.2.a-d. The major drawback here arises from the high dimensionality of the problem. However, by expressing problem 2.2 in terms of the control variables u , the dimensionality can be reduced from $\dim(u) + \dim(x)$ to $\dim(u)$. Added computing expenditure is the penalty one has to pay, for the decrease in dimensionality, and hence the reduced storage requirements for the solution process.

The compact version of problem 2.2 becomes possible through a solution of the LF equations 2.2.b which relates elements of vector x to those of vector u . The dimensionality of the compact problem, i.e., $\dim(u)$, defines the maximum possible degrees of freedom available. Stated alternately, the number of active constraints cannot exceed $\dim(u)$. Expressed mathematically, the compact problem becomes:

Minimize a scalar objective function,

$$f[u, x(u)]$$

.....(2.3.a)

Subject to the constraints,

$$g[u, x(u)] = 0 \quad \dots (2.3.b)$$

$$h[u, x(u)] \leq 0 \quad \dots (2.3.c)$$

$$u^m \leq u \leq u^M \quad \dots (2.3.d)$$

Equation 2.3.b is a scalar representation of the nonlinear load-flow manifold in the space of u , originally defined by 2.2.b.

In the compact formulation 2.3, it is necessary to solve the set of LF equations in an outer loop. The relatively mild nonlinearity among some of the LF equations, along with the efficient NR algorithm in polar form, greatly simplifies this task as outlined by Stett [1974], and later confirmed by Carpentier [1985].

Nonlinear programming techniques proposed to solve the OPF in full, depend either explicitly or implicitly on the Kuhn-Tucker first order necessary conditions, [Kuhn et al., 1951]. Here, the objective function and constraints are assumed to be continuous, with continuous first derivatives. This is true for the OPF. Sufficiency conditions, or optimality conditions, [Gill et al., 1981], require the objective and constraints to have continuous second derivatives. However, authors Bazaraa and Shetty [1979] have proved that, if the objective is convex, and the constraints form a non empty convex set, the first order K-T conditions would guarantee a global optimum, i.e., the necessary conditions would be sufficient. Authors who address the OPF problem generally assume the validity of the convexity assumptions to hold true for their study.

The literature on OPF carries a multitude of proposals for solving problems of the type 2.2 (or 2.3). Most proposals are based on a set of certain basic methods, and their variations. In order to limit the scope of the present study, the principles underlying the basic methods are considered in the next section, and the methods are reviewed subsequently.

2.3 Candidate Solution Techniques for the OPF Problem

Solution techniques proposed to date to solve the full fledged OPF can be classified under the following general categories, i.e.,

1. Nonlinear Programming (NP),
2. Linear Programming (LP),
3. Successive LP (SLP),
4. Successive QP (SQP),
5. Continuation Method (CM)⁶.

The principles underlying these techniques will be reviewed next.

2.3.1 Principles of Nonlinear Programming Techniques

NP techniques can be applied either to the sparse or compact version of the OPF formulation. In both versions, active functional variables from equation 2.2.c (or 2.3.c) get appended to the equalities 2.2.b (or 2.3.b) which represents the set of LF equations, or the LF manifold. The number of degrees of freedom is equal to the $\dim(u)$.

Once the reduced set of controls⁷ are identified, the NP technique seeks to find a direction of descent d , and a scalar step length parameter γ , to update the controls u by the addition of the term $\gamma*d$. This process is repeated until no further improvements are possible, at which point, the K-T conditions are considered satisfied to a given precision ϵ .

⁶ Although the continuation method has not been applied to solve the OPF in the past, it is included here due to its success in solving problems of secure economic dispatch. This can be seen in the work by Fahmideh-Vojdani [1982], Galiana et al., [1983], and Juman [1983]. A summary of these, along with possible ideas for extending the method to cover generation and line parameter variations are given by Huneault and colleagues [1984].

⁷ Each active functional constraint, removes one degree of freedom. This causes a change in the status of control and dependent variables. Further, in order to proceed with the optimization, it is necessary that the constraint gradients at the present expansion point be linearly independent. In NP terminology these conditions are termed constraint qualifications, Gill et al., [1981].

2.3.1.1 On the Direction of Descent

The methods for computing directions of descent are based on either of the following principles; i.e.,

1. Gradient information of the objective,
2. Newton direction,
3. Quasi-Newton (Variable Metric) direction.

In an unconstrained optimization of the objective function $f(u)$, the negative gradient ∇f_u gives the direction of descent. However, close to the optimum this gradient approaches zero, thereby retarding the progress of the optimization. Stated alternately, infinitely many iterations are needed to reach the optimum.

The Newton direction of descent for an unconstrained NP does not have this drawback. The Newton direction of descent arises from a quadratic approximation to the unconstrained objective, [Gill et al., 1981], and is given by the row vector, $-\nabla f_u H^{-t}$, where H^{-t} is the transpose of the inverse of the Hessian. Here, it is necessary that the Hessian H be non-singular and non-negative-definite. In an optimization problem where one has to search over a wide range of feasible points, it is often difficult to maintain the above conditions. Further, the inverse of the Hessian is difficult to evaluate. The quasi-Newton method overcomes these difficulties by using an approximation to the Hessian, with the desirable properties. Gill et al., [1981], give algorithms for forming the approximate Hessian. In practical implementations the Hessian is initially approximated by an identity matrix, and updated by the Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula, [ibid].

2.3.1.2 On the Selection of Step Length

Step sizes are selected to minimize either the objective function, or the Lagrangian along a direction of descent, respecting feasibility. In order to assure convergence, the step length γ must produce a sufficient decrease. This sufficient decrease requirement can be satisfied by several alternative

sets of conditions on γ . For example the Goldstein-Armijo Principle, [Gill et al., 1981], is one such condition.

The value of γ is dynamic, i.e., changes with iteration count in an NP. The success of a step length depends to a great extent on the initial value chosen for γ . The experience of Gill and colleagues is that an initial value of $\gamma=1$, performs well for Newton based algorithms.

There is a tradeoff between the effort expended in determining a good γ , and the resulting benefits. A theoretically rigorous γ may not always provide the best decrease in the objective. Many OPF algorithms fix γ at a value which minimizes objective f along direction d at each iteration. This involves a univariate minimization, often referred to as a line search or linear search. Gill et al., [1981], give the following practical criteria for choosing γ at a given expansion point u^k , i.e.;

$$|f'(u^k + \gamma d) d| \leq -k_1 f'(u^k) d \quad \dots (2.4.a)$$

$$f(u^k) - f(u^k + \gamma d) \geq -k_2 \gamma f'(u^k) d \quad \dots (2.4.b)$$

Where k_1, k_2 are constants such that, $0 \leq k_1 \leq 1$, and $0 \leq k_2 \leq 0.5$. Techniques are available, which determine γ based on equations 2.4.a & b.

Further coverage of step sizes in nonlinear programs can be found in the following references; [Kirchmayer 1958], [Dommel et al., 1968], [Peschon et al., 1971], [Velghe et al., 1972], [Dy Liacco 1973, '74, '79], [Sasson et al., 1973, '74], [Avriel 1976], [Barcelo et al., 1977], [Powell 1977], [Wu et al., 1979], [Giras et al., 1981], and [Talukdar et al., 1981].

2.3.1.3 On the Handling of Inequality Constraints

Functional inequality constraints, (constraints represented by equations 2.2.c or 2.3.c) pose a great computational burden on the OPF. The literature discusses many methods for handling such inequalities. Among these, the principles of some methods with a historical significance will be enumerated next. The implementation details and references are presented subsequently in section 2.5.

Penalty Functions:

Here, at every iteration, the objective function is augmented by a scalar multiple of the binding (violated) functional constraints. The scalar multiplier is the penalty factor (PF). Often it becomes necessary to increase the PF to obtain better satisfaction of the active functional constraints. This retards convergence due to ill conditioning to the Hessian introduced by the large penalty factors. Further the PF's are problem dependent, and require tuning on the part of the user. Functional constraints are satisfied only in a soft sense, leading to inaccuracies. The only advantage is the simplicity in implementing this procedure.

Projections:

Here, the original descent direction is projected onto the sub-space defined by the set of active constraints. Optimality conditions now refer to the projected gradient and projected Hessian and are derived in Gill et al., [1981]. Implementation is difficult when functional constraints, (constraints represented by equations 2.2.c) or 2.3.c) are present.

Switching Variables:

Here, whenever a functional variable (an element from set X, or H) violates its bound in the midst of an iterative process, an exchange is made in which it is replaced by a control variable from set U. Any element of set U qualifies for such an exchange provided it is not at a limit, and does not represent a previously violated functional constraint. However, there is always the possibility that the control variable so exchanged can violate its limit in the subsequent iteration(s), requiring a further swap. An important aspect sometimes overlooked here is the possibility that, a previously functional constraint, presently in set U, can subsequently come off its limit in the ensuing iterations.

Linearization:

Here, the constraints are linearized about an expansion point. A suitable NP (or LP) technique, specifically a quadratic program, or a linear program is applied to this now simplified problem, depending on the type of the approximations used for the objective. If the linearization is performed only once, the accuracy of the final optimum becomes questionable. However, if applied to a sequence of sub-problems, by updating the expansion point, the method will converge to the constrained nonlinear optimum. The linearization resembles the N-R method for solving the LF equations, and is one of the better ways known to handle nonlinear constraints. The relevant references are cited in the following discussion on linear, successive linear, and successive quadratic programming techniques.

2.3.2 Linear Programming Techniques

It is known that the solution of an LP lies at one of the vertices of the constraint set. The Simplex method searches for the optimum by moving systematically from one vertex to the next, [Luenberger 1984]. The significance of this search as interpreted by Stott et al., [1979], is that at each iteration, one of the binding constraints gets released from its bound, and a previously free constraint becomes binding. This feature is called constraint exchange. The handling of constraint exchanges are unique to the particular implementation of the LP algorithm. One such implementation, i.e., the Revised Simplex method has proved its worthiness in the literature.

In setting up the LP problem, the objective function is assumed convex and separable. All nonlinear constraints are linearized about a feasible expansion point, and converted to equalities by the addition of slack, and, where needed, artificial variables. The array coefficients of these equality constraints forms the LP tableau. In the Revised Simplex method a basis matrix is formed within this tableau. In the ensuing iterations, due to the constraint exchange, this basis needs to be updated regularly. Considering

the number of possible exchanges permissible, one has to be overly concerned about the efficiency of the basis-inversion process. This requires efficient techniques to exploit sparsity, and serves as the core of the LP process.

2.3.2.1 On the Dual Method

It is possible to translate any LP problem into its dual counterpart. This can then be solved by a primal LP. Stott and colleagues [1979] are of the opinion that the Dual Method is better suited to the real power dispatch sub-problem of the OPF.

The dual algorithm starts with an optimum feasible solution to a subset of the problem constraints, and proceeds by introducing a new constraint at each iteration, maintaining both optimality and feasibility until all violations are removed. This will then correspond to the optimal feasible solution.

The salient steps defining the dual method are enumerated below.

- a) Obtain an optimum basic feasible solution of the dual by ignoring line constraints, and form the basis matrix.
- b) Determine candidate c_1 among the free line-flows and injections, having the greatest violation, at the current expansion point.
- c) If c_1 does not exist, the optimum is reached, hence stop.
- d) Determine candidate c_2 among the active set of line-flows and injections such that it would become free, with the least added cost, should c_1 become binding instead.
- e) Swap c_1 and c_2
- f) Update the basis, and recycle from (a).

The dual solution is initialized by ignoring network constraints, and dispatching, to minimize the objective. Depending on the objective, such a starting point is easily obtained. On the other hand, the primal phase I is computationally more demanding. Stott and colleagues [1979] claim that, the above procedure for selecting a starting point, makes the dual starting

point closer to the true optimum, than the primal starting point. This means that the dual would execute faster.

2.3.3 Successive Linear Programming

Successive Linear Programming (SLP), also called the Method of Approximation Programming, solves a Nonlinear Program through a sequence of Linear Programs. It is necessary that the objective function and constraints have continuous first derivatives over the feasibility region. Separability of the objective and gradients is not a requirement. The starting point of each successor LP is the optimum of its predecessor LP. The first LP in this chain, should be started with a feasible solution. Palacios-Gomez and colleagues [1982], provide the theory for SLP, and conclude by stating that, SLP techniques are best suited to very large-scale problems with few degrees of freedom.

SLP techniques were proposed by Merlin in 1972 as a possible solution technique to solve the P-dispatch problem. The literature to date has not revealed a successful SLP application to the full fledged OPF problem. The potentials, and limitations, of SLP techniques have been assessed by Stott and colleagues [1978, '79 a,b, '83]. However the existing contention on the suitability of SLP techniques, to solve NP problems, has to be resolved by the many concerned authors. Current information on SLP is inadequate, and inconclusive, to recommend SLP techniques to solve the OPF problem. Further, since the solution to the LP sub-problem, lies on the vertex of the linearized constraint set, it is generally unlikely that this would coincide with the true nonlinear optimum. In order to enforce convergence, it will be necessary to make the constraints of the linearized sub-problem, reflect the true nonlinearity, by the addition of further bounds on variables, [Gill et al, 1981].

2.3.4 Successive Quadratic Programming

Successive Quadratic Programming (SQP), solves a Nonlinear Program through a sequence of Quadratic Programs, or linearly constrained nonlinear programs. Wilson [1963], Beale [1967], Gill et al., [1981] describe the former

strategy, while Robinson [1972] describes the latter. Convergence will be quadratic, provided the sequence is started close enough to the Kuhn-Tucker (K-T) solution point of the original nonlinear program. A K-T expansion point, for a general NP of the form 2.3 can be characterized by the triple u^k, μ^k, ν^k . Here, u^k is the value of vector u at the end of iteration k , and vectors μ^k, ν^k are the Lagrange Multipliers corresponding to the equality and active inequality constraints from equations 2.3.b&c respectively. The starting point of each successor QP is the optimum of its predecessor QP. The first QP in this chain, should be started with a K-T feasible point⁸ u^0, μ^0, ν^0 , such that LM estimate $\nu^0 \geq 0$ for the initial active set of functional variables. The expansion point u^0 , need not be feasible for the original NP. However, for the technique to work, it is necessary that each sub-problem be convex. This will be true by construction, i.e., by having $\nu^k \geq 0$ for active functional variables, provided, the original NP is convex.

The solution of the SQP depends heavily on the solver chosen for the QP sub-problem. Feasible search techniques are popular candidates for such a solver, [ibid].

2.3.5 The Continuation Method

To date, the Continuation Method (CM) has not been applied to solve the OPF problem. Hence, the presentation to follow may not quite fit the main title of this chapter. Nevertheless, the reason for its inclusion here is its past success in solving Economic Dispatch (ED), i.e., a sub class of OPF, where only the real power secure economic dispatch is considered. This is evidenced in the work by Vojdani et al., [1983], Galiana et al., [1983], and Huneault et al., [1984]. Further, this success served to motivate the present research, in which the method does prove a successful candidate to solve the OPF problem.

Application of CM's can be traced in different areas of mathematics and economics, where alternate names such as capacity; [Houthakker 1960],

⁸ Requirement of a feasible point is considered a drawback here, as it involves much computational effort, especially for OPF problems of large magnitude.

parameter variation; [Deist et al., 1967], imbedding; [Meyer 1968], homotopy; [Eaves 1972], and parametric programming; [Garcia et al., 1981], are used instead of CM.

The CM sets itself apart from conventional NP techniques, as it does not derive any necessary and sufficient conditions. In its most fundamental form the CM is a technique for solving systems of nonlinear equations as demonstrated by Garcia et al., [1981]. Houthakker [1960] and Zlobec et al., [1982] have used the CM, through transforming the original problem into one which requires the solution of equations, to study the effects of parameter variation on the optimal solution to mathematical programming problems.

The principles underlying the workings of the CM method are presented next, along with the necessary definitions and terminologies. For expository convenience, the notation x is used for a general set of n variables. It should not be confused with the state variables, and could very well represent either the state, the control, or both, depending on the formulation of the OPF. The context under which x is used is clarified in the text.

2.3.5.1 The Principle of Continuation

This can be best explained starting from the rudimentary problem in mathematics, i.e., solving a system of equations. In applying the principle of continuation, one will have to start off by deforming the original problem to a simple system, whose solution is almost obvious. This is by no means trivial in a general sense, and often requires some ingenuity. Most practical engineering problems can be exploited in one way or another to accomplish this as evidenced in the work of Vojdani et al., [1983], Galiana et al., [1983], Huneault et al., [1984], Calderon et al., [1986], and Richter et al., [1983, '84]. The reader exposed to the CM for the first time is referred to the work of Garcia et al., [1981], for simple examples illustrating the deformation process. By starting from this simple system, it is possible to track the solution trajectory, when the introduced deformation is continuously removed. At zero deformation the solution at hand will be the solution sought. The solution trajectory so formed may or may not provide usable information for intermediate problem deformations. This will

be explained subsequently. The continuation concept or the homotopy concept, [Garcia et al., 1981], is indeed very powerful, considering the different classes of problems that can be solved using this concept.

The deformation referred to above is accomplished through the introduction of a continuation parameter or a homotopy parameter, [Garcia et al., 1981]. The deformed problem is referred to as a homotopy, [ibid]. Three different homotopies, namely the Newton, Fixed point, and Linear can be traced in the literature. The subsequent sections will clarify the definitions and features relating to the continuation parameter, and homotopies.

2.3.5.2 The Continuation Parameter

The concept of the continuation parameter is best explained through the following general example. In this, the solution to a set of general n equations $F(x)=0$ is sought. For this purpose, the vector function $F(x)$ is deformed via the homotopy $H(x,\theta)=0$, such that when the continuation parameter⁹ $\theta=0$, results in $H(x,0)=E(x)=0$, i.e., the easy-to-solve problem whose solution is trivially obtained. However, when $\theta=1$, results in $H(x,1)=F(x)=0$. Thus if this deformation were to work, one needs to monitor only the solution trajectory or path, i.e., $x(\theta)=H^{-1}$, to the homotopy $H(x,\theta)=0$, in order to obtain the desired solution.

A formal definition of the functions are given below;

$$F(x): \mathbb{R}^n \rightarrow \mathbb{R}^n \quad \dots (2.5.a)$$

$$E(x): \mathbb{R}^n \rightarrow \mathbb{R}^n \quad \dots (2.5.b)$$

$$H(x,\theta): \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n \quad \dots (2.5.c)$$

$$H^{-1} = \{(x,\theta): H(x,\theta)=0\} \quad \dots (2.5.d)$$

For the continuation process to work, the solution $x(\theta)$ has to draw a continuous path, starting from x^0 , the solution to $E(x)=0$, to x^1 , the desired solution. Stated alternately, the solution to the successor problem must be started from the solution to the predecessor. If the successor

⁹ The continuation parameter is limited by $0 \leq \theta \leq 1$.

problem and the predecessor problem are not connected, the method would fail. The mathematical form of the continuous path is given by equation 2.5.d. The continuation process will work even for multiple solutions, provided the easy-to-solve problem $E(x)=0$ is formed to have the same multiplicity of solution. In this, x^0 has multiple values, each defining the beginning of a unique continuous solution path. The analysis above assumed the existence and uniqueness of the solution path(s) H^{-1} . The path exists only under certain, not too restrictive, assumptions. The sections to follow will examine these assumptions after presenting the three commonly used homotopies.

2.3.5.3 Types of Homotopies

The path H^{-1} , depends on the form of the homotopy chosen to represent H . The final solution is of course independent of the homotopy chosen. Numerous forms of homotopies can be traced in the general literature [Garcia et al., 1981]. However, most of these can be either classified as falling into the three categories defined below, or form a hybrid combination among these three. The three fundamental homotopy forms will now be defined to conform with the definitions of the previous section.

Definition of the Newton Homotopy

$$H(x, \theta) = F(x) - (1-\theta)F(x^0) \quad \dots\dots(2.6.a)$$

This form of homotopy can be started quite easily, i.e., by assigning $E(x)=F(x)-F(x^0)$, where x^0 can in theory be arbitrary, but is seldom chosen this way. An intelligent choice would take advantage of specific properties of the problem. This will be further elaborated subsequently. Note that the solution of $E(x)=0$ is x^0 by construction, thereby enabling an easy start.

Definition of the Fixed-Point Homotopy

$$H(x, \theta) = (1-\theta)(x-x^0) + \theta F(x) \quad \dots\dots(2.6.b)$$

It is clear that, as before, the solution to the easy-to-solve problem $E(x) = x - x^0$, is $x = x^0$ by construction, which provides an easy starting point.

Definition of the Linear Homotopy

$$\begin{aligned} H(x, \theta) &= \theta F(x) + (1-\theta)E(x) \\ &= E(x) + \theta[F(x) - E(x)] \end{aligned} \quad \dots (2.6.c)$$

This can be considered as a general definition encompassing the previous two. The Newton homotopy, or the fixed point homotopy results when $E(x)$ in equation 2.6.c is replaced by either $F(x) - F(x^0)$, or $x - x^0$.

The work of Vojdani et al., [1983], Galiana et al., [1983], Huneault et al., [1984], is based on the linear homotopy. The present research is also based on a linear homotopy. This will be explained subsequently.

2.3.5.4 Existence and Uniqueness of the Homotopy Path

In sections 2.3.5.1&2, the existence and uniqueness of the path was assumed. This section will specify the necessary and sufficient conditions for this to be true, and for the path to be well behaved. These conditions in a nutshell are given by the Implicit Function Theorem, which will be stated and proved.

Theorem 2.1: The Implicit Function Theorem

Let $H: \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ have continuous first derivatives, $(x^*, \theta^*) \in H^{-1}$, and Jacobian $H'(x^*, \theta^*)$ have full rank. Then in an ϵ neighborhood of (x^*, θ^*) all points (x, θ) that satisfy $H(x, \theta) = 0$ are on a single continuously differentiable path through (x^*, θ^*) .

Proof

A linear approximation of H about the point (x^*, θ^*) gives:

$$H(x, \theta) = H(x^*, \theta^*) + H'(x^*)(x - x^*) + \frac{\partial H}{\partial \theta} (\theta - \theta^*) \quad \dots (2.7.a)$$

Note that in the above equation, the partial derivative of H has been separated into two components, one with respect to the general problem variables x , and the other with respect to θ . The notation $H'(x^*)$ is used here to imply the partial derivative of $H(x, \theta)$ with respect to vector x , evaluated at $x = x^*$.

Since (x^*, θ^*) lies on path H^{-1} , $H(x^*, \theta^*) = 0$. Further if (x, θ) lies in an ϵ neighborhood of (x^*, θ^*) , $H(x, \theta) = 0$. Thus equation 2.7.a yields:

$$x - x^* = -[H'(x^*)]^{-1} \frac{\partial H}{\partial \theta} (\theta - \theta^*) \quad \dots (2.7.b)$$

Equation 2.9.b has n equations in $(n+1)$ variables. Hence the set of all solutions about the expansion point (x^*, θ^*) must lie on a unique straight line through this point, i.e., given (x^*, θ^*) , (x, θ) is uniquely defined and lies on the continuously differentiable path. This is true for all expansion points on path H^{-1} , provided $H'(x^*)$ is invertible.

QED

It can be seen that had H been linear, $H(x, \theta) = 0$ would have been exact, and the path would have been linear through (x^*, θ^*) . For nonlinear H , even a slight movement along the linearization would mean leaving the path H^{-1} , which means that the homotopy $H(x, \theta) = 0$ is satisfied only approximately. This exemplifies the arduous task underlying the tracking of a nonlinear path. Hence care must be exercised that the step sizes are very small; i.e. $(\theta - \theta^*)$ should be incremental in moving from x^0 to x^1 . Theorem 2.1 also guarantees that under the stated conditions the path H^{-1} is well behaved, and unique. Figure 2.1 shows such a path, path 3. Notice here, that paths 1, 2, 4 & 5 exhibit certain unacceptable behaviour. Additional conditions are needed to exclude these possibilities. These conditions are given in the

next section. In NP terminology the condition on Jacobian $H'(x^*)$, is referred to as constraint qualifications, [Gill et al., 1981]. This forms a very fundamental concept determining the success of the CM to the solution of the OPF problem.

Recapitulating, theorem 2.1 implies that under the given hypothesis (x, θ) is unique, and lies on the continuously differentiable path through (x^*, θ^*) . This feature excludes fears that the path may split, bifurcate, cross itself, or get into an infinite spiral. Ortega et al., [1970] give a second proof for theorem 2.1, based on the very same ideas, to which the interested reader is referred.

Examples of H^{-1} Paths Defined by Theorem 2.1

Figure 2.1

The consequence of the Implicit Function Theorem, is the Path Theorem, which will now be stated and proved.

Theorem 2.2 The Path Theorem

Let $H: \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ be continuously differentiable, $(x, \theta) \in H^{-1}$, and the Jacobian $H'(x, \theta)$ be of full rank for all $0 \leq \theta \leq 1$. Then H^{-1} consists of only continuously differentiable paths.

Proof

If $(x^*, \theta^*) \in H^{-1}$, by hypothesis, some or all the $(n+1)$, n square sub-matrices, which can be formed from $H'(x^*, \theta^*)$ are invertible. Theorem 2.1 ensures that all (x, θ) , in an ϵ neighborhood of an arbitrary point $(x^*, \theta^*) \in H^{-1}$ lie on a continuously differentiable path through this point. As this holds for any point on H^{-1} , it holds for all points. Hence H^{-1} consists of continuously differentiable paths. Note that the existence and uniqueness of the path is guaranteed by the Implicit Function Theorem.

QED

Theorem 2.2 provides the hypothesis ensuring that a path exists. The important point to note here is that in order to follow a path, one has to prove that a path exists. Theorem 2.2 does just that.

Theorems 2.1 and 2.2 give the necessary and sufficient conditions, to identify the uniqueness and existence of continuously differentiable paths, when solving systems of nonlinear equations. It is necessary to follow these paths to reach the final solution. The sections to follow will present, and compare, two methods available that path-follow, after presenting the following discussion on the orientation of a path.

2.3.5.5 Additional Conditions to Orient a Path

Although examples 1,2,4&5 in figure 2.1 satisfy the definition of paths, they outline two distinct behaviour patterns which calls for concern. The first is seen in examples 1,2&5, where the path reverses itself, and the second is seen in example 4, where the path diverges to infinity. These may cause the CM to fail 'en route'. Hence it is necessary to impose additional conditions on the homotopy H , which would exclude such behaviour, or serve to identify such behaviour at an early stage for possible corrective measures. This will ensure that the path itself reaches the proper destination. The Index Theorem, takes care of the first concern by giving conditions necessary for a path to maintain a proper orientation. For presenting the index theorem, the following function is defined; i.e.,

$$\begin{aligned} \text{sgn } (a) &= +1 \text{ if } a > 0 \\ &= 0 \text{ if } a = 0 \\ &= -1 \text{ if } a < 0 \end{aligned} \quad \dots (2.8.a)$$

Theorem 2.3 The Index Theorem

Let homotopy $H: \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$, be differentiable and regular (full rank) at both $\theta=0$, and $\theta=1$. Let the beginning and end of path H^{-1} be x^a and x^b . If both $(x^a, 0)$ and $(x^b, 0) \in H^{-1}$, then;

$$\text{sgn det } H'(x^a) = -\text{sgn det } H'(x^b) \quad \dots (2.8.b)$$

Else if $(x^a, 0) \in H^{-1}(0)$, and $(x^b, 0) \in H^{-1}(1)$, then;

$$\text{sgn det } H'(x^a) = \text{sgn det } H'(x^b)_0 \quad \dots (2.8.c)$$

Preliminaries to the proof of Theorem 2.3:

The path parameter p , is defined as the distance moved along a path H^{-1} , starting from any given reference point on the homotopy path H^{-1} . The reference point is usually taken to coincide with $\theta=0$. This would enable both the variable x and the continuation parameter θ to be expressed as functions of the path parameter p .

$$x = x(p) \quad \dots (2.8.d)$$

$$\theta = \theta(p) \quad \dots (2.8.e)$$

also; the following definitions are introduced for expository convenience; i.e.,

$$y = (x, \theta) = y(p) \quad \dots (2.8.f)$$

$$H'_{-i}(y) = H'(y), \text{ with the } i\text{th column removed.} \quad \dots (2.8.g)$$

Proof

The following equation holds if $(y) \in H^{-1}$, i.e.,

$$H(y) = 0 \quad \dots (2.8.h)$$

A Taylor expansion of equation 2.8.h about y gives;

$$H'(y) \frac{dy}{dp} = 0 \quad \dots (2.8.i)$$

The theorem on Basic Differential Equations (BDE), [Garcia et al., 1981], gives the solution to equation 2.8.i as;

$$\frac{dy_i}{dp} = (-1)^i \det H'_{-i}(y) \quad \dots (2.8.j)$$

If $i=n+1$, i.e., for $y_1 = \theta$, equation 2.8.j becomes;

$$\frac{d\theta}{dp} = (-1)^{n+1} \det H'(x) \quad \dots (2.8.k)$$

Hence for two terminal points, $(x^a, 0), (x^b, 0) \in H^{-1}$, the following results, assuming the path length to be measured from $p^1=0$; i.e.,

$$\text{for } p = p^1; \frac{d\theta}{dp} = (-1)^{n+1} \det H'(x^a) \quad \dots (2.8.l)$$

$$\text{for } p = p^2; \frac{d\theta}{dp} = (-1)^{n+1} \det H'(x^b) \quad \dots (2.8.m)$$

The homotopy path will reverse itself only if a positive increment in the path parameter causes a negative increment in the continuation parameter θ . This is clearly demonstrated in curve 5 of figure 2.1. Hence, the homotopy path to reverse itself, equations 2.8.l&m must have opposite signs, which is possible only if $\text{sgn } \det H'(x^a)$, equals, $-\text{sgn } \det H'(x^b)$. A similar reasoning will prove equation 2.8.c, under the specified conditions.

QED

The second concern, i.e., the case where the path H^{-1} diverges to infinity will be considered next.

In this case, it is clear that as $\|x\| \rightarrow \infty$, it is not possible to maintain $H(x, \theta) = 0$, as the terms in H are likely to explode. Under these conditions H is said to be norm-coercive. In short, if the homotopy chosen to path-follow is such that it is norm coercive, the path H^{-1} will not reach its final destination. The only solution to overcome a norm-coercive H is to choose a different homotopy at the onset. This exemplifies that choosing the proper homotopy is by no means trivial, and requires some prior experimentation. This can often lead to a hybrid homotopy, which encompasses some or all of the three types described previously in section 2.3.5.3. A norm-coercive H may also imply that a solution does not exist to the original problem.

Given the proper homotopy H , and the assurance that following path H^{-1} does indeed lead to a solution, the next item to influence the success of the CM, is the path-following algorithm. The section to follow will review such algorithms.

2.3.5.6 Algorithms that Path-Follow

Two classes of methods exist that path-follow. The first and the oldest¹⁰ is based on simplicial algorithms, and the second is based on solutions to differential equations. In both instances the algorithm will generate its own path, which approximates the path to follow, i.e., H^{-1} . This is an important concept to be accepted from the onset, as no method exists which will exactly path-follow, unless the path itself is linear, or piece-wise linear.

The sections to follow will review the essence of the methods proposed to path-follow, as found in the literature pertaining to mathematics. It must be noted that, not all these methods have been tried in the past for solving

¹⁰ Path-following was initially accomplished through simplicial algorithms. Subsequently path-following techniques were developed, based on solutions to differential equations. The latter served to overcome certain limitations of the former [Garcia et al., 1981].

the OPF. The method chosen for the present work is hybrid in nature, and is described in full in Chapter III. A knowledge of some of the the descriptions to follow will be assumed in Chapter III.

2.3.5.6.1 Essence of Simplicial Algorithms

Simplicial algorithms operate by building linear functions on simplices. If the simplices are constructed in a manner such that a linear function within the simplex can be made to approximate the path H^{-1} , a procedure is developed which can path-follow

By definition, a $(n+1)$ -simplex consists of $(n+2)$ points. Hence if simplex $S \subset \mathbb{R}^{n+1}$ is a $(n+1)$ -simplex, any point $w \in \mathbb{R}^{n+1}$ within the simplex can be expressed as the linear combination of the $(n+2)$ vertices $(v_i, i=0, \dots, n+1)$; i.e.,

$$w = \sum_{i=0}^{n+1} k_i v_i \quad \dots\dots(2.9.a)$$

$$\text{such that, } \sum_{i=0}^{n+1} k_i = 1; \text{ for } k_i \geq 0 \quad \dots\dots(2.9.b)$$

The set $S = \{v_i: i=0, \dots, n+1\}$ represents the $(n+2)$ vertices defining the simplex S . Further, if point w lies on a facet¹¹ $\{v_i: i=0, \dots, i-1, i+1, \dots, n+1\}$, then constant $k_i = 0$. If $k_i = 1$, w coincides with the vertex v_i . Further, $k_i \geq 0$ implies that point w is interior to the simplex S . Thus all points within S can be uniquely defined by the constants k_i .

Given the above definitions, a function $G(w): S \rightarrow \mathbb{R}^n$ is now defined which will approximate the homotopy function $H(y): \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ on simplex S . Notationally w approximates y .

¹¹ A facet is the largest face of the simplex, [Garcia et al., 1981].

$$\text{given, } w = \sum_{i=0}^{n+1} k_i v_i \quad \dots\dots(2.10.a)$$

$$\text{such that, } \sum_{i=0}^{n+1} k_i = 1; \text{ for } k_i \geq 0 \quad \dots\dots(2.10.b)$$

$$\text{define, } G(w) = \sum_{i=0}^{n+1} k_i H(v_i) = 0 \quad \dots\dots(2.10.c)$$

Observe that, $G(w)$ is linear, and serves as the desired linear approximation to H by drawing a linear path from one facet of the simplex to the other. This can be explained as follows.

From equations 2.10.b&c one has $(n+1)$ equations with $(n+2)$ unknowns. Under regularity¹² on G , i.e., given that G has full row rank, the solution describes a linear path in $\mathbb{R}^{n+1}$. This is the approximation to the homotopy path H , which will be interior to simplex S between two facets. Figure 2.2-3 illustrates this approximation. The simplicial algorithm will now be enumerated.

1. Given an initial point $y^0 \in H^{-1}$, choose a point w^0 close to y^0 , such that both y^0 , and w^0 will lie on facet B^0 of simplex S^0 .
2. At the j th iteration, solve equations 2.10.b&c to generate the linear approximation to the homotopy path $H(y)=0$ from $w^{j-1} \in B^{j-1}$, to $w^j \in B^j$, interior to simplex S^{j-1} .
3. Generate a new vertex, v_j on the other side of S^{j-1} , such that the new simplex becomes $S^j = B^j \cup v_j$. Note that $B^j = S^{j-1} \cap S^j$.
4. Recycle from 2, until the final point w^k is reached which approximates $y^1 \in H^{-1}$.

¹² Regularity almost always holds [Garcia et al., 1981]. If under a rare circumstance regularity fails, it can be easily enforced by changing one or more of the vertices v_i by an arbitrary small amount, [ibid].

Illustrating the approximation of Homotopy H on simplex S^j by function G .

Figure 2.2

All simplicial algorithms follow the basic steps 1-4 above. Step 2 of the above algorithm can be implemented through a simplex-like pivoting strategy proposed by Garcia et al., [1981]. The accuracy of the piece-wise linear (PL) approximation relies on the size of the simplex, i.e., smaller the simplex, the better the accuracy, and lesser the fear of divergence. Variations among these algorithms focus their attention on how the new vertex v_j in step 3 is generated. Two simplicial algorithms referenced in the literature are the flex simplicial, and triangulation algorithms. A well documented literature survey on these methods can be found in the text by Garcia et al., [1981].

Simplicial algorithms are slow in nature. In order to follow the homotopy path closely it is necessary to make the simplices small and robust¹³. It is possible to accelerate the process by choosing large simplices close to $\theta=0$, and shrinking them as θ approaches unity. However this is not advisable as the process can result in a path way off the true homotopy path H , and could cause divergence or infeasibility midway. The limitations on the size of the simplices forces one to inch along, making the process numerically

¹³ A robust simplex is one which satisfies the relation $[r(S^j)/(l(S^j)^{n+1})] > a$, where $r(S^j)$ is the volume, $l(S^j)$ the largest edge of the simplex S^j , and $a > 0$ is a given fraction. It is assumed that $S^j \subset \mathbb{R}^{n+1}$.

expensive, as can be evidenced from equations 2.10.a-c. Hybrid methods have been proposed to accelerate the process, but the results are inconclusive with respect to their effectiveness. The interested reader is referred to Garcia et al. [1981], for a related bibliography and discussion.

An illustration of how $G(w)$ may approximate $H(y)$.

Figure 2.3

The sections to follow will review the modern concept in path-following, i.e., through the direct solution of differential equations. The three methods reviewed are the Euler, Newton, and the predictor-corrector. Although the Euler and Newton methods are conceptually identical, they will be treated separately to highlight certain differences.

2.3.5.6.2 Euler's Method

This is by far the simplest among the three methods. However it has limited scope to path-follow. Nevertheless since this provides the basic ideas for better methods, the principles underlying the method will be reviewed.

Recalling the discussion of section 2.3.5.2; the homotopy path is defined as;

$$H(x, \theta) = 0$$

.....(2.11.a)

The total derivative of equation 2.11.a gives;

$$\frac{dx}{d\theta} = -[H'(x)]^{-1} \frac{\partial H}{\partial \theta} \quad \dots\dots(2.11.b)$$

Equation 2.11.b is also known as the homotopy differential equation (HDE), Garcia et al., [1981]. This is the most fundamental equation defining path-following techniques embodying differential equations. Written in difference form, it becomes the Euler formula, i.e.,

$$x^{k+1} = x^k - (\theta^{k+1} - \theta^k) [H'(x^k)]^{-1} \frac{\partial H}{\partial \theta} \quad \dots\dots(2.11.c)$$

The step length d^k is defined as follows, i.e.,

$$d^k = \theta^{k+1} - \theta^k \quad \dots\dots(2.11.d)$$

At the k th iteration the Euler process takes a linear approximation to the path H^{-1} at x^k and moves a distance $d^k [H'(x)]^{-1} H'(\theta)$ to a new point x^{k+1} . The process is repeated at x^{k+1} . It is necessary that step lengths d^k must be small for the piece-wise linear (PL) Euler path to track the homotopy path accurately. Figure 2.4 is a hypothetical rendition of the appearance of the PL Euler path and the homotopy path.

A major problem with the Euler is the resulting drift, also shown in figure 2.4. The repeated linear approximations tend to go further and further from the homotopy path H^{-1} . Much of the drifting is overcome by using correction formulae to the basic Euler step, i.e., equation 2.11.c, by utilizing information on earlier linear approximations. One such method is the Adams Bashforth method, described in the text by Hamming [1962].

The attention next focuses on where the Euler might fail. An examination of equation 2.11.b will reveal that, the HDE fails if the n -square matrix $H'(x)$ becomes singular.

The piece-wise linear Euler path and the homotopy path
for a hypothetical problem.

Figure 2.4

Setting $i=n+1$ in equation 2.8.j results in the following, i.e.,

$$\frac{dy_{n+1}}{dp} = (-1)^{n+1} \det H'_{-(n+1)}(y) \quad \dots (2.12.a)$$

Observing that $y_{n+1}=\theta$, and $H'_{-(n+1)}(y)=H'(x)$, it is clear that the Euler step would fail, when the homotopy path bends on itself. This feature is illustrated in figure 2.5.

While traversing the homotopy path, should $H'(x)$ become singular, the traverse can be continued, provided, $H'(y)$ is regular, i.e., has full row rank. This is justified by the implicit function theorem of section 2.3.5.4. Elaborating, the implicit function theorem requires that $H'_{-i}(y)$, for $i=n+1$, i.e., $H'(x)$, be invertible for a unique path. The theorem is valid provided $H'_{-i}(y)$ for any $i: i=1, \dots, n+1$ is invertible, and guarantees one and only one path. This result rescues the path following algorithm from breaking down, should $H'(x)$ become singular. The procedure to be adopted is explained next.

An inadmissible homotopy path.

Figure 2.5

Points a-d in figure 2.5 characterise the failure of the normal path following algorithm, i.e., with θ as the continuation parameter. These points are characterized by the $\det H'(x^i) = 0$, for $i = a, \dots, d$. At each of these points, assuming regularity on $H(y)$, θ must be replaced by an x_i in order to continue the traverse. The traverse may now be continued utilizing the modified HDE, until the singularity on $H'(x)$ is removed. At this juncture, a switch is made back to the conventional parameterization in which θ is the continuation parameter. This procedure is continued until $\theta = 1$, or the regularity on $H(y)$ becomes invalid, in which case the problem at hand is concluded to be infeasible.

The drift of the Euler path away from the homotopy path can be corrected through a restart feature. This feature serves as a key idea in the success of the CM, as utilized in this research. It is numerically wasteful to exactly path-follow, as the solution sought, in most cases, is the terminal point of the path. Exceptions do exist, an example being the formulation of the OPF such that the continuation parameter is the system demand P_D . In this case, since the path gives information on the much sought optimal solution trajectory, it is imperative to exactly path-follow, and the

restart feature becomes superfluous. However, since the present study seeks the terminal point of the path, i.e., $(x^1, 1) \in H^{-1}$, the restart feature will be presented under this context. The principles underlying the restart feature will now be considered.

The restart feature assumes that, any intermediate point (x^k, θ^k) not on H^{-1} , but close to H^{-1} , can be connected to the terminal point $(x^1, 1) \in H^{-1}$, through the restart feature. The new path $\underline{H}^{-1}$ generated will coincide with H^{-1} only at the terminal point x^1 . These features are illustrated in figure 2.6.

A Hypothetical Rendition of a Restart Path.

Figure 2.6

In the present work, and in most general cases the terminal point $\theta^k=1$. Hence the new homotopy $\underline{H}^{-1}$ must be designed such that θ^k is reset to zero at the onset of the new path. For expository convenience one such homotopy is given below, where for the sake of consistency in presentation it is assumed that the system to be solved is $H(x, \theta)=0$.

$$\underline{H} = H(y) - (1-\theta)H(x^k, 0) \quad \dots (2.13.a)$$

$$\text{at } \theta=0, y=y^k, \text{ giving } \underline{H} = H(x^k, 0) - H(x^k, 0) = 0 \quad \dots (2.13.b)$$

$$\text{at } \theta=1, y=y^1, \text{ giving } \underline{H} = H(x^1, 1) \quad \dots (2.13.c)$$

Thus restarting from (x^k, θ^k) , by choosing the Newton homotopy of equation 2.13.a, equations 2.13.b&c illustrate that the proper terminal point can be reached. In a typical practical situation, many such restarts may be needed before the original problem is solved to a given precision.

Recapitulating, the problem of interest is viewed as terminal value problem, in which the solution sought is x^1 . Both differential information, and a restart homotopy are employed to reach the terminal point. The restart homotopy serves to correct the drift from the desired path. The restart feature becomes superfluous in applications which require the homotopy path to be followed exactly.

Next, the Newton's method for path following will be described.

2.3.5.6.3 Newton's Method

The Newton method is very similar to the Euler, and operates by taking linear approximations to the path. However the Newton method takes larger steps, uses the Newton homotopy, and is based on solving nonlinear equations rather than differential equations. It can be viewed as a large-step, path-following approach with restarts.

Specifying the Newton homotopy to solve $F(x)=0$, gives;

$$H(x, \theta) = F(x) - (1-\theta)F(x^0) \quad \text{.....(2.14.a)}$$

On differentiating equation 2.15.a, and assuming $F'(x)$ to be invertible gives;

$$\frac{dx}{d\theta} = - [F'(x)]^{-1} F(x^0) \quad \text{.....(2.14.b)}$$

Equation 2.14.b written in difference form, assuming a unit step size, i.e., $d^1=1.0$ gives the following equation underlying the beginning of the Newton path; i.e.,

$$x^1 = x^0 - [F'(x^0)]^{-1} F(x^0) \quad \text{.....(2.14.c)}$$

It is seen here that equations 2.11.c & 2.14.c would be identical at (x^0, θ^0) , if the Euler utilized the Newton homotopy, and the unit step length. Further, since the Newton attempts to leap to the terminal point y^1 , it almost always misses this point, unless, the starting point for the Newton path is very close to the terminal point. This calls for an application of the restart homotopy, and the one to be used is that of equation 2.13.a.

Equation 2.13.a, for the Newton homotopy of equation 2.14.a becomes;

$$H = F(x) - (1-\theta)F(x^1) \quad \dots\dots(2.14.d)$$

The Newton step for the restart homotopy H becomes;

$$x^2 = x^1 - [F'(x^1)]^{-1}F(x^1) \quad \dots\dots(2.14.e)$$

Thus the general formula for the Newton path at iteration k becomes;

$$x^{k+1} = x^k - [F'(x^k)]^{-1}F(x^k) \quad \dots\dots(2.14.f)$$

The conceptual similarity among both the Euler and Newton methods should be evident. The Euler takes small steps, while the Newton takes large steps and tries to reach the solution fast. The nonlinearity of the homotopy path always makes it necessary for both processes to be restarted until, $F(x)=0$ has been solved to the required precision. Garcia and colleagues [1981] prove that the Newton method has very rapid convergence close to the solution, but may very well diverge or oscillate if started further away from the solution.

The similarity of the Newton process to that of solving a set of nonlinear equations by the classical Newton method should be evident. Variations to the Euler and Newton methods aim at improving their performances and are reported in Ortega et al., [1970].

Recapitulating, the Euler and the Newton technique for path-following embody a linear approximation to the path, reach the terminal point y^1 , and restart, if the termination criteria are not met. The speed of the Euler is controllable through proper choice of a step length d^k , while the Newton operates with a fixed step length of unity.

The third and the last among methods proposed in the literature to path-follow are the predictor-corrector method. These embody two stages, namely the predictor and corrector stages. The function of the predictor is to take a step in the correct direction as dictated by the orientation of the homotopy path, while the corrector, corrects for drift away from the true path. Applied repeatedly the homotopy path can be followed to any desired degree of accuracy. The details will be presented next.

2.3.5.6.4 Predictor-Corrector Methods

In applying the predictor-corrector method, one starts at a point y^k on the homotopy path and tries to predict the next point y^{k+1} along the path. Due to drift caused by the predictor, a corrector is needed to get back close to the path. Never is one interested in getting on to the path per-se, as this would be numerically wasteful.

Among the many techniques which can function as the predictor, the Euler is a popular candidate. For expository convenience, the elevator predictor and horizontal corrector are employed in the discussions to follow. The new terminology cited is introduced in the discussion to follow.

Given $y^k = (x^k, \theta^k)$, and that $H'(x^k)$ is non-singular the elevator step is defined as;

$$y^{k+1} = (x^k, \theta^k + d^k) \quad \dots\dots(2.15.a)$$

Observe that the movement is along the θ axis. Recalling the discussion of section 2.3.5.6.2, the elevator step would be meaningful only if the path does not bend on itself at y^k , i.e., $H'(x^k)$ is non-singular.

Clearly y^{k+1} does not lie on the homotopy path H^{-1} , which calls for a corrector step, designed to get very close to the path. One of the most favoured corrector in the literature is the Newton corrector. As always, there are many ways in which to implement the corrector. One such scheme will now be discussed.

Recall that the current point is y^{k+1} . For a vector constant $b \in \mathbb{R}^{n+1}$, a hyperplane $b^t(y - y^{k+1}) = 0$ is constructed to intersect the homotopy path at a unique point $y(p^k)$. Now a new function $F(y) = 0$ is constructed as shown below:

$$F = [H(y), b^t(y - y^{k+1})]^t = 0 \quad \dots (2.15.b)$$

Note that the function $F=0$, by construction has a solution $y=y(p^k)$, since $y(p^k)$ lies on the intersection of the homotopy path and the hyperplane. Hence, by starting from y^{k+1} , and applying Newton iterations to function F , one can get very close to the homotopy path. If points y^{k+1} and y^k are close to each other, convergence of the corrector step would be rapid. Figure 2.7 illustrates some of the concepts explained herein.

In practice the condition that y^k should be on the path can be relaxed to, close to the path. The corrector iterations are applied until $\|H(y)\| \leq \epsilon$. At this juncture another elevator step is applied, and the process repeated until the terminal point is reached.

The Newton corrector can be simplified by the proper choice of b . By choosing b parallel to the θ axis, i.e., $b = (0, \dots, 0, 1)$, one is able to obtain a corrector which traverses horizontally, henceforth called the horizontal corrector. The advantage of the horizontal corrector is its simplicity. An example of the elevator predictor and the horizontal corrector are shown in figure 2.8. To ensure convergence, as in any Newton based method, the starting point should be close to the solution. This can be ensured by taking small elevator steps. Path following by the elevator predictor and horizontal corrector is summarised in the following theorem.

One iteration of the elevator predictor and
Newton corrector.

Figure 2.7

Theorem 2.4

Given that matrix $H'(x)$ is non-singular at a point $y^k = (x^k, \theta^k) \in H^{-1}$, and that the homotopy H is twice continuously differentiable. If the initial starting point be $y^0 = (x^k, 0) \in H^{-1}$, and the path H^{-1} be of finite length, the application of an elevator predictor and horizontal corrector will follow the entire path to any desired degree of accuracy, provided the predictor step length is sufficiently small.

Proof

Since $H'(x)$ at $y^k = (x^k, \theta^k)$ is non-singular, by the previous discussion given in section 2.3.5.6.2, the path H^{-1} does not change orientation. Hence the step length d^k would ensure that the path is traversed, i.e., $\theta^k < \theta^{k+1}$.

Since step length d^k is small, y^{k+1} is sufficiently close to the path H^{-1} such that the Newton horizontal corrector can be made to converge to a point arbitrarily close to H^{-1} . This along with the previous argument will ensure that the new point is further along the path than y^k .

Since the homotopy path is of finite length, it is compact. Hence the method is capable of following the entire path length to any desired degree of accuracy.

QED

An illustration of the elevator predictor
and horizontal corrector.

Figure 2.8

Should $H'(x)$ become singular at y^k , the continuation parameter θ must be swapped for another variable x_i in order to continue the traverse. This feature has been discussed before in section 2.3.5.6.2, i.e., assuming regularity on $H(y)$, one attempts to find variable i , such that $1 \leq i \leq n$, for which $H'_{-i}(y)$ is non-singular, in order to exchange x_i with θ . Under this situation the vector $b = (0, \dots, 1, \dots, 0)$, where 1 is in position i . The predictor step should be such that the incremental step d^k is applied only to x_i in vector y^k . Theorem 2.4 remains valid under this variable switching. The essence of the argument supporting this is given below; i.e.,

The path H^{-1} is subdivided into a finite number of sub-paths, such that $H'_{-i}(y)$, for $1 \leq i \leq n$ is non-singular within each sub-path. With x_i as the

independent variable, theorem 2.4 can be utilized to traverse each sub-path, and eventually the whole path.

One way to accelerate the elevator predictor horizontal corrector algorithm is to have large initial step sizes tapering gradually, and becoming infinitesimally small close to $\theta=1$. However, with large step sizes, it is possible that the algorithm may latch onto a different path, oscillate, or cycle endlessly [Garcia et al., 1981]. The only cure for such difficulties is to backup and restart the tracking process with smaller step sizes.

The predictor-corrector method is a globalization of the method of Davidenko [1953]. Convergence proofs are given by [Menzel et al., 1978], and [Garcia et al., 1979]. Variations of the basic method are given in [Schmidt 1979].

Recapitulating, sections 2.3.5.6.1-4 presented two techniques fundamental for following a path to any degree of accuracy. The flex simplicial will path-follow as long as the simplices are small and robust. The predictor-corrector will path-follow with small predictor steps. The predictor corrector is faster than the flex simplicial, however, the risk of drift is greater, and unless proper precautions are taken, it may not be possible to return to the homotopy path. Alternatively, since the flex simplicial approximates the homotopy path by enclosing sections of it within simplices, problems due to drift do not occur.

2.4 Desirable Features in an OPF Algorithm

The following identifies the desirable features in an algorithm, picked to solve the OPF problem, i.e.,

1. Accuracy, i.e., the accuracy of the solution should be no worse than the accuracy of provided data.
2. Resolution, i.e., computing time should not be wasted on variables which can provide only marginal improvement to the objective.
3. Reliability, i.e., the method employed should be capable of advising the operator on constraints which act as bottlenecks in preventing an optimum solution from being reached easily.

4. Speed, i.e., the method should be fast enough for off-line applications on moderately fast computers.
5. Robustness, i.e., the method should be independent of the power system under scrutiny.

A review of some survey papers, notably Sasson [1974], Happ [1975, 1977], Stott et al., [1980, 1979] and Carpentier [1985] express the concern that no single method is able to meet all five requirements above. However, each of the major methods reported, has its own distinct advantage.

The following section discusses the key methodologies in as near a chronological order per category as possible. The scope of the discussion will be limited only to the OPF problem in which both active and reactive power dispatch take place, unless a method applied to the decoupled P or Q dispatch discloses a remarkable feature of interest.

2.5 Chronological Development in OPF Methodologies

The work of Squires in 1961, probably marked the inception of OPF studies. Squires formulated the problem in cartesian form, without utilizing the B-coefficients. However, the real components of voltages at generation buses were fixed at specified values, mainly to simplify the overall formulation. Line-flow limits, Q-limits on generators, on-line controllers, and SVC devices were not considered. The optimization was carried out over the remaining variables by solving the first order necessary conditions utilizing the Newton Raphson method. Although the formulation does not encompass the full OPF, this method produced better results over varying demands, than the B-coefficient methods available at that time.

The complete formulation of the OPF is due to Carpentier, in his classical introduction to the Method of Injections in 1962. The basic formulation, i.e., the Method of Injection was refined in 1968 into the method of Differential Injections. The historical importance of this work warrants the following review.

2.5.1 The Method of Injection due to Carpentier

Carpentier's major contribution was to place the OPF problem on a sound mathematical footing, and to distinguish its salient features clearly from classical economic dispatch. The original paper, in French, written in 1962 used Kuhn-Tucker conditions, [Kuhn et al., 1951], for the first time in power system engineering in order to derive the first order necessary conditions for optimality. Subsequently improvements to the basic methodology were proposed by Carpentier and colleagues in their 1968 version of Differential Injections. Means of extending the theory of Differential Injections to encompass unit-commitment and security were also proposed in 1968 by Carpentier and colleagues. The success of both methods were reviewed by Carpentier in 1972. In 1973, the final versions of the Differential and Total injections methodologies were published. Improvements and modifications continued to appear in the literature in the years 1975 and 1979. Experimentation of the Differential Injection Formulation on problems such as P-Q optimization, P-dispatch or Q-dispatch sub-problems of the OPF and decomposed P-Q dispatch have been reported by Carpentier and colleagues in 1975.

In the Method of Injections, i.e., [Carpentier 1962], the overall problem is cast into the sparse form of equations 2.2.a-d. Here, the 2*NB, LF equations, 2.2.b are written as follows:

$$z_{pi} = P_{gi} - P_{di} = P_i(u, x) \quad \dots\dots(2.16.a)$$

$$z_{qi} = Q_{gi} - Q_{di} = Q_i(u, x) \quad \dots\dots(2.16.b)$$

z_{pi} , z_{qi} are referred to as the injected real and reactive powers. Hence perhaps the title, the Method of Injections.

This formulation translated equipment engineering limitations, i.e., inequality constraints of types 2.2.c&d into 2.2.c. Expressed explicitly they become:

$$P_{gi}^2 + Q_{gi}^2 - S_{gi}^2 \leq 0 \quad \dots(2.16.c)$$

$$P_{gi}^m - P_{gi} \leq 0 \quad \dots(2.16.d)$$

$$Q_{gi} - Q_{gi}^m \leq 0 \quad \dots(2.16.e)$$

$$Q_{gi}^m - Q_{gi} \leq 0, \text{ for } i = 1, \dots, NG \quad \dots(2.16.f)$$

$$V_i - V_i^m \leq 0 \quad \dots(2.16.g)$$

$$V_i^m - V_i \leq 0, \text{ for } i = 1, \dots, NB \quad \dots(2.16.h)$$

$$\theta_i - \theta_j - \theta_{ij}^m \leq 0, \text{ for the NL nodal pair } (i-j) \quad \dots(2.16.i)$$

On line controllers and SVC devices have not been considered. The transmission flow limits are expressed in terms of the maximum spread θ_{ij}^m in bus voltage angles. The Lagrangian embodying equations 2.16.a-i, along with the objective $f(u,x)$ which represents the variable operating expenditure of the power system becomes.

$$L = f(u,x) + \alpha^t g(u,x) + \beta^t h(u,x) \quad \dots (2.16 j)$$

Where, α and β are vectors of Lagrange Multipliers.

To conform with the K-T conditions, elements of α are unrestricted in sign, while all elements of β are positive. The first order necessary conditions stipulate that the partial derivative of equation 2.16 j with respect to decision variables u , dependent variables x , and LM's α , and β must be zero. Further, the condition on complementary slackness stipulates that, should any of the inequalities $h(u,x) \leq 0$ become active, its LM becomes an unknown. Alternatively, an inactive inequality has zero LM.

Carpentier utilized the Gauss-Seidel iterative process to solve the aforementioned first order necessary conditions for the dependent variables x . However, the convergence characteristics were slow and unpredictable. This led to a revision of the basic formulation. The modified formulation became known as the method of Differential Injections. The Generalized Reduced Gradient (GRG)¹⁴ [Carpentier et al., 1966], [Abadie et al., 1969] was employed as a solver

¹⁴ The GRG method is described in Appendix E

2.5.2 Gradient Based OPF

All gradient techniques applied to the OPF are derived either partially or completely from the Wolfe's reduced gradient method, [Wolfe 1963]. In it the gradient of the objective is expressed in terms of a given set of controls. The discussion to follow will present gradient based techniques, which have been applied successfully to solve the OPF problem.

2.5.2.1 The Method of Differential Injections

In the formulation of the method of Differential Injections, Carpentier adopted the compact OPF formulation discussed in section 2.2. Efficiency in coding was implemented by adopting sparse matrix, and ordered elimination techniques along the lines recommended by, Carpentier [1963a,b], Sato et al., [1963], Baumann [1965], Tinney et al., [1967a,b], and Peschon [1968]. The generalized reduced gradient (GRG) method was applied to solve the OPF. A discussion on the general form of the GRG is presented in Appendix E.

In the formulation of Differential Injections, the OPF problem is translated into the form of equations E.1. The objective function E.1.a reflects the variable operating expenditure of the power system. To conform with the GRG code, this cost is convex, and assumed piece-wise linear. The active set, i.e., equation E.1.b comprises the first order linearization of the violated, or near violated functional constraints, and the power balance equation expanded to second order terms, about a given expansion point. It is almost impossible to have a functional constraint exactly satisfied at the given expansion point, unless this happens by chance. The version of the GRG code utilized to solve the problem is described in [Carpentier et al., 1968]. This description is different from that given for a general nonlinear problem as given in [Carpentier et al., 1966], and [Abadie et al., 1969]. The salient features of the solution technique are enumerated below.

- a) Determine the initial expansion point by executing a Lambda-Iteration with all voltages fixed at their upper bounds, and on-line controllers at their nominal settings.

- b) Run a classical load-flow to filter out the violated and near violated¹⁵ functional constraints. At this stage positive slack variables are added to convert these constraints to equalities of type E.1.b.
- c) Linearize the functional constraints about the expansion point found in step 1 up to first order terms, and add to this set the power balance equation expanded to second order terms about the same expansion point. This linearization must be carried out with respect to the reduced set of free control variables
- d) If iteration count is 1, this step is skipped. Else the constraints of (c) above are checked for variations in magnitude from their previous linearizations, and their present magnitudes, in nonlinear form computed in (b). If these variations are acceptably small the iterations are terminated.
- e) A two-phase method is now applied to find the optimum of the linearized problem. Phase I uses a method, undisclosed in the paper by Carpentier [1973a] to obtain feasibility, while Phase II utilises the GRG described in Appendix E, adopting Carpentier's proposals.
- f) Recycle from step (b) taking the optimum from step (e) as the new expansion point.

The original paper utilises the Beale Correction to update the dependent variables due to the control actions. A description of this can be found in [Carpentier et al., 1968], and [Carpentier 1985]. This process is essentially approximate, and is aimed at saving execution time by adopting an approximate Jacobian. This form of economy is not always very popular, [Carpentier 1985]. This also forms the basic difference between the general GRG reported by Carpentier and colleagues, [1966], Abadie and colleagues, [1969], and the GRG specially tailored for power system studies reported by Carpentier and colleagues [1968]. In the Total Injections Method, Carpentier [1973b] extends the Differential Injections formulation to cover the unit-commitment problem.

¹⁵ A near violated functional constraint is one which is essentially free but has reached or exceeded 85% of its nominal rating.

Peschon et al., [1971], Landqvist, et al., [1984], and Gribik, et al., [1983] were also able to apply the GRG successfully to the OPF problem. Peschon et al., [1971] have demonstrated that the EDF general purpose GRG code is suited for small scale OPF studies with security. Landqvist et al., [1984] used the GRG in a limited sense, i.e., for the Q-dispatch only with success. The GRG on a large scale was successfully applied by authors Gribik et al., limiting their study by ignoring the hard to handle functional line-flows. In this latter case a two-stage technique involving GRG and Augmented Lagrangian techniques were applied with success to systems with as many as 1500 buses.

As reported by Carpentier [1985], the current version of the GRG code at EDF accommodates a secure exact OPF, and is presently in use at major energy management system (EMS) centres. The execution times remain competitive with other methods published in the power system literature.

2.5.2.2 Dommel-Tinney Reduced Gradient Based OPF

The sparse formulation and a reduced gradient based solution methodology as envisioned by Dommel and Tinney in 1968 has, until recently, been accepted as the most suitable method for the OPF problem, [Happ 1977], [Burchett et al., 1982a]. The solution algorithm does not utilize the complete Wolfe Reduced Gradient algorithm. Instead the reduced gradient is computed in terms of the control variables u . The control set remains unchanged during the progress of the iterations. Violated functional constraints are appended to the objective function through the adaptation of simple exterior penalty modelling described by Gill and colleagues [1981].

The formulation based on the notation of problem 2.2 becomes:

Minimize a scalar objective,

$$f(x,u) = f(x,u) + \sum_{i \in A_F} \rho_i h_i^2(x,u) \quad \dots (2.17.a)$$

Subject to,

$$g(x, u) = 0 \quad \dots\dots(2.17.b)$$

$$u^m \leq u \leq u^M \quad \dots\dots(2.17.c)$$

The term augmented to the objective corresponds to the violated set of dependent variables, i.e., state and functional variables, in the course of the optimization. This quadratic penalty term serves to close the equicost contours in the vicinity of the violated functional constraint(s), such that the optimum lies in an unconstrained space with respect to these functional variables. Hence the omission of explicit inclusion of functional variables among the constraint set of problem 2.17. The reduced gradient corresponding to problem 2.17 is given by,

$$\nabla f_u = f'(u) - f'(x) g'(x)^{-1} g'(u) \quad \dots\dots(2.17.d)$$

Equation 2.17.d above dictates the progress of the iterations. Iterates advance step-wise in the direction of the negative reduced gradient. In order to compute the step length Dommel and Tinney proposed to approximate the augmented objective function 2.17.e given below, along the steepest descent direction, into a mathematical function and compute the approximate minimum.

$$F(x, u, \lambda) = f(x, u) + \sum_{j \in LE} \lambda_j g_j(x, u) + \sum_{i \in A_F} \rho_i h_i^2(x, u) \quad \dots\dots(2.17.e)$$

Here Lagrange Multiplier λ_j corresponds to the j-th load flow equation g_j , and LE amounts to the total number of LF equation in standard form.

Terms $f'(u)$ and $f'(x)$ change as functional constraints move in and out of the active set of functional variables, A_F . Whenever a control u_i reaches a limit, it is fixed at this limit, and the components of the reduced gradient ∇f_u corresponding to this are set to zero. This amounts to projecting the reduced gradient onto active control(s). This is the difference between the Dommel-Tinney method and the Wolfe Reduced Gradient method.

This method has had good success with systems ranging in sizes up to 500 buses. The penalties ρ_i 's introduce a major drawback to the overall performance of the algorithm. The dependent constraints are satisfied only in a soft sense. The step size to be estimated is also considered a disadvantage, as a small step will retard convergence, while a large step can cause oscillations, as pointed out by Happ, [1977], and Carpentier, [1985]. These reasons make the overall method rather unattractive for on-line applications, while the method can still be successfully used for off-line studies and system planning.

2.5.2.3 Conjugate Gradient Based OPF

Burchett and colleagues [1982a], experimented in improving the basic ideas of the Dommel-Tinney approach in order to overcome some of the difficulties reported therein. Sparse OPF modelling, as defined in problem 2.2 was employed. The principal problem of the Dommel-Tinney method as seen by Burchett and colleagues was the use of the negative reduced gradient, i.e., $-\nabla f_u$, or the direction of steepest descent, as the direction of optimization. It has been demonstrated by Burchett and colleagues [1982a] and Gill and colleagues [1981], that this direction can be greatly improved by taking into consideration the gradient at adjacent points, as follows:

$$r_k = -\nabla f_u + \beta_k r_{k-1} \quad \dots(2.18.a)$$

$$\beta_0 = 0 \quad \dots(2.18.b)$$

Where k is the iteration count, and r_k the new descent direction. The constant β_k is defined depending on the conjugate direction technique used. The most popular techniques being the Fletcher-Reeves and the Polak-Ribiere. Description of these techniques are given in [Fletcher et al., 1964], [Polak et al., 1969], and [Gill et al., 1981].

For quadratic forms of the objective function the conjugate gradient method is quadratically convergent. Burchett and colleagues [1982a] report that for the power system optimization, i.e., minimum loss or minimum cost problems, the technique is super-linearly convergent. Further it is reported that the convergent characteristics can be improved by periodically switching to the

negative reduced gradient technique, although no reason for this improvement was given.

The overall performance of this scheme is claimed to be better than the linear convergence characteristic of the Dommel-Tinney reduced gradient technique.

Functional constraints were handled much the same manner as done by Dommel and Tinney, except that the following heuristic formula has been suggested to increase the penalty parameters, i.e.;

$$\rho_1 = \rho_0 k^l \quad \dots (2.18.c)$$

Where ρ_0 and k are constants, and l is the number of times, functional constraint h_1 violated its bounds.

As a means of improving the step length procedure, Burchett and colleagues proposed the use of Zangwill's method, [Zangwill 1967]. In it a one-dimensional function is minimized without the use of derivatives. The objective of equation 2.17.e is approximated as a quadratic along the step length in three consecutive points, and its minimum obtained in closed form. This minimum replaces one of the three points. The process is repeated until no further improvement is possible. This step corresponds to a set of minor iteration within a larger loop, and amounts to about three to six power flow iterations to locate the minimum along the negative reduced gradient.

It is generally known that some control variables u_1 are insensitive to the optimization while others u_2 are sensitive. This causes the overall optimization to slow down as variables u_1 take a long time to settle. Dommel and colleagues [1968], and Wu and colleagues [1979] have proposed scaling the negative reduced gradient as a solution to this problem. However Burchett and colleagues [1982a] have proposed the decomposition of the overall OPF into a P and Q optimization problem as a solution, and claim this as an alternative form of scaling. Control variables u_1 , and u_2 are claimed to settle quickly to their optimum settings. The method has been tested successfully on systems up to 600 buses.

The decomposition casts considerable doubt on the accuracy of the overall optimization. As pointed out by Carpentier, [1985], it is quite possible for a P-Q decomposition based optimization to give an objective, nearly equal in value to the exact optimum, but with the controls quite distant from the optimum settings. This may mean that the optimum is insensitive to the solution.

2.5.2.4 Wolfe's Reduced Gradient Based OPF

The Stanford University MINOS code, [Murtagh et al., 1980, '81] which is based upon the Wolfe's reduced gradient algorithm was used by power system engineers Burchett and colleagues [1982b] for solving the OPF problem. In order to use the MINOS code, the OPF formulation must be sparse, i.e., in the form of problem 2.2, and the constraints must be explicitly modelled. The salient features of the method will now be reviewed.

In order to simplify the presentation no distinction will be made among the variables u and x , i.e., both u and x will be considered to be incorporated into x . Hence the OPF problem adapted to be solved by MINOS can be expressed concisely as,

Minimize a scalar function,

$$f(x) \quad \text{.....(2.19.a)}$$

Subject to the constraints,

$$h(x) \leq 0 \quad \text{.....(2.19.b)}$$

$$x^m \leq x \leq x^M \quad \text{.....(2.19.c)}$$

In order to understand the mechanisms underlying the MINOS algorithm, it will be assumed that the active set of h , i.e., h_A is known, along with the active variables among x . Hence problem 2.19 reduces to the linearly constrained sub-problem:

Minimize a scalar function,

$$f(x) \quad \dots (2.20.a)$$

Subject to the constraints,

$$h_{AL}(x) = h_A(x_k) + h_A'(x_k)(x - x_k) = 0 \quad \dots (2.20.b)$$

$$x_n = x_n^L \quad \dots (2.20.c)$$

$$x_b^m \leq x_b \leq x_b^M \quad \dots (2.20.d)$$

$$x_s^m \leq x_s \leq x_s^M \quad \dots (2.20.e)$$

Where x_n , x_b , x_s are the non-basic, basic and super-basic variables; and x_k is the value of x at the k -th iteration. The non-basic variables are at a bound while the basic and super-basic variables are within bounds.

In order to ensure that the projected Hessian of the Lagrangian of problem 2.20 is non-negative-definite, an Augmented Lagrangian function is constructed for problem 2.20 as explained by Gill and colleagues [1981]. Based on this the augmented Lagrangian takes the form;

$$\mathcal{L}_A = f(x) - \lambda^t (h_A - h_{AL}) + \frac{1}{2} \rho (h_A - h_{AL})^t (h_A - h_{AL}) \quad \dots (2.21.a)$$

Here, both h_{AL} and h_A are functions of x , where h_{AL} is the linearization of the active set of h , i.e., h_A about a given expansion point, ρ is a scalar penalty parameter, and λ a vector of Lagrange multipliers. The augmented Lagrangian 2.21.a does not replace the objective $f(x)$ in equation 2.20.a. The value of ρ is such that if it is set above a threshold value ρ_0 , the optimum of the augmented Lagrangian will converge to the optimum of the corresponding Lagrangian. As the optimum ρ is set to 0.

The basic features of the algorithm are enumerated below

- a) Form the linearization h_A ,
- b) Test for convergence, stop if converged.
- c) Optimise the linearly constrained sub-problem 2.20.
- d) Recycle from (a)

The key point in the MINOS algorithm is the solution of the sub-problem 2.20, which will be reviewed next.

Equation 2.20 b can be written in the form;

$$h_A' x = b \quad \text{.....(2.22.a)}$$

Where;

$$b = h_A'(x_k) x_k - h_A(x_k) \quad \text{.....(2.22.b)}$$

In order to explicitly express the non-basic variables, equation 2.22.a can be split into basic, super-basic, and non-basic as follows;

$$\begin{vmatrix} B & S & N \\ 0 & 0 & I \end{vmatrix} \begin{vmatrix} x_b \\ x_s \\ x_n \end{vmatrix} = \begin{vmatrix} b_b \\ b_s \\ b_n \end{vmatrix} \quad \text{.....(2.22.c)}$$

In order to stay on the active set defined by 2.22.c, the following conditions must be satisfied, [Gill et al., 1981];

$$\begin{vmatrix} B & S & N \\ 0 & 0 & I \end{vmatrix} \begin{vmatrix} p_b \\ p_s \\ 0 \end{vmatrix} = 0 \quad \text{.....(2.22.d)}$$

Where p_b and p_s are the increments applied to the corresponding (basic and super-basic) values of x .

It follows from equation 2.22.d that,

$$Bp_b = -Sp_s \quad \text{.....(2.22.e)}$$

This clearly demonstrates that the super-basic variables form the driving force in the optimization. Hence the optimization is carried out in a much reduced space than the whole space x , (space of u and x).

In the course of the iteration if a super-basic variable reaches a limit, it is moved into the non-basic set. Should a basic variable reach a limit, it is moved into the non-basic set and is replaced by an element from the superbasic set. Hence unlike in the Dommel-Tinney reduced gradient algorithm, the B,S,N partitions of 2.25 can change in the course of the iterations. Highly efficient coding is employed in updating the matrix factorizations in the event of an update. Further a module which is fundamental to the success of MINOS is the dlinsearch algorithm of Gill and Murray, [Murtagh et al., 1978]. The FORTRAN implementation can be found in [Gill et al., 1976].

MINOS has many options to solve sub-problem 2.20. The options are based on either the conjugate gradient method or the quasi-Newton method. The conjugate gradient technique available are;

1. Fletcher Reeves, [Fletcher et al., 1964],
2. Polak and Ribiere, [Polak et al., 1969],
3. Perry, [Perry 1976],
4. Memoryless Davidon-Fletcher-Powell (DFB),
5. Memoryless Complementary DFB.

The quasi-Newton technique implemented is due to the theoretical developments reported in publications [Davidon 1959], and [Fletcher et al., 1963]. The quasi-Newton method is more attractive when the number of superbasic variables are of the order of a few hundred.

Burchett and colleagues [1982b] used MINOS to solve OPF problems up to 600 buses. As expected the quasi-Newton performance is superior to the two conjugate methods tried, namely the Polak-Ribiere, and the Fletcher-Reeves. Convergence difficulties are reported for the 600 bus system when solved using the conjugate direction algorithm. If this is to be true generally, it would amount to a drawback on the overall performance of MINOS for large scale OPF, as it is impossible to use the quasi-Newton for large scale NP problems due to the large memory requirement.

The reason for the failure may be partly due to the small convergence tolerance of 10^{-4} being used. For the 118 bus minimum loss example the performance statistics are indeed good. The minimum cost problem has not been tried by these people. Since the number of variables involved in this latter instant are large, it would amount to a certain degree of retardation in overall the performance. The potential for on-line application is indeed quite promising.

2.5.3 Sasson's Hessian Based OPF

Here, Sasson and colleagues [1971] considered the general OPF problem 2.2 to be in the following form, i.e.,

Minimize the scalar objective,

$$f(x) \quad \text{.....(2.23.a)}$$

Subject to,

$$h(x) \geq 0 \quad \text{.....(2.23.b)}$$

Equation 2.26.b effectively incorporates the equivalent equations 2.2.b-d. Further no distinction is made among the state variables x and the control variables u .

Sasson translated problem 2.23 into an unconstrained sparse problem of the form;

Minimize the scalar objective,

$$F(x) = f(x) + \sum_{i \in A_F} \rho_i h_i^2(x) \quad \text{.....(2.24.a)}$$

The Fletcher-Powell quasi-Newton technique, [Fletcher et al., 1963] was used directly to solve problem 2.24. The Hessian is dense, and hence the storage requirements can be substantial. Further, in theory the penalty parameter ρ_i tends to ∞ as the optimum is reached. This causes the Hessian of problem

2.24 to become ill-conditioned and to retard the overall convergence. Further, as in the case of Dommel-Tinney method, the constraints are satisfied only in a soft sense, making the optimum often infeasible. These reasons make the method suitable only for planning purposes of moderate size systems, i.e., systems less than 100 buses in magnitude, [Carpentier 1985].

Barcelo and colleagues [1977] used a simplified Hessian and solved a 1200 bus system by limiting the number of active generations at 22, and the maximum number of binding constraints at only 5

2.5.4 Linear Programming Based OPF

In order to apply LP techniques the OPF problem must be linearized. The solution accuracy can be improved through successive linearization. Merlin in his 1972 publication proposed successive linearization and the repeated application of the Dual Simplex Method as a solution strategy for solving the P-dispatch sub-problem of the OPF. In this, a scheme was adopted to successively add system losses until a convergence tolerance was met.

The speed and efficiency of the Dual Simplex based successive LP for the P-only sub-problem is demonstrated by Dodu et al., [1975], Stott et al., [1978, '79a,b]. The paper by Stott et al., [1979a], proposes numerous improvements to the original proposals by Merlin. The notable ones being a specially tailored Dual Simplex Algorithm with adaptive constraint relaxation, the use of compact modelling, and the incorporation of the system losses into the overall model. The reported solution accuracy is better than that obtainable from a classical DC load flow used by Stott and colleagues [1978].

Successive LP techniques based on the Dual Simplex method have been tried by Stott and colleagues [1983], in a very limited sense by iterating on a decomposed P and Q only sub-problem. The algorithm is reported to perform poorly on the Q sub-problem. A much improved version of the LP based OPF package is now available with PCA Corporation, Banakar [1983]. However, the details of which are unavailable at present.

The general LP based algorithms present two major deficiencies. The first being the inaccuracies caused by the linearizations, and the second being the invalidity of the linearization for the Q-sub-problem, specifically with the system loss as the objective, [Carpentier 1985]. The LP seems well suited to the P-dispatch sub-problem. As observed by Broussolle and colleagues [1984], and Elacqua and colleagues [1982], the LP can be seriously considered as a solution alternative for the OPF only as a means of alleviating limited computing power, or when speed with which a solution can be obtained is the prime concern.

The LP cannot be considered as a viable alternative for on line OPF applications, [Carpentier 1985]

2.5.5 Successive Quadratic Programming Based OPF

The techniques underlying Successive Quadratic Programming are outlined in section 2.3.4. Giras and colleagues [1981] applied the Han-Powell quasi-Newton method; [Han 1976], [Powell 1977], to solve the compact formulation of the OPF problem given by equations 2.3 a-d. The success was demonstrated on a small system. Subsequently, feasibility studies on this method was carried out by [Nadira 1983]. Franchi and colleagues [1983] applied this technique successfully to solve the Q-dispatch problem on systems of up to 355 buses. These people used Beale's method, [Beale 1968], [Land et al., 1973], to solve the quadratic program sub-problem, until the active set is identified. Thenceforth, the first order K-T conditions are solved for directly through a Newton iterative technique. As in all quasi-Newton methods the overall performance slows down for large dimensions of the Hessian.

Burchett and colleagues [1984] applied successive quadratic program (SQP) to the sparse OPF formulation given by problem 2.2. Here, problem 2.2 was translated into a quadratic program via appropriate expansions of the mathematical functions. The publication claims to achieve quadratic convergence characteristics on a wide variety of objective functions such as, production cost, active losses, reactive loss, and VAR addition. The details of the mechanisms underlying the exact algorithm are sketchy. The reported

timings are indeed impressive, and can very well be adopted for on-line applications. Since exact second derivatives are used, the scheme can also fit into a Newton process, with the QP sub-problem solved through the Han-Powell, [Han 1976], [Powell 1977], quasi-Newton technique. The SQP method appears to be faster than an equivalent augmented Lagrangian technique by a factor of 4. Carpentier [1985] claims this technique to be equivalent in performance to the current version of the Differential Injection technique. However the latter has been tested only on systems up to 600 buses and 230 controls, while the former has been tested on systems up to 1900 buses and 186 controls.

2.5.6 Augmented Lagrangian Based OPF

All OPF trials based on the augmented Lagrangian method centers around the Stanford University MINOS/Augmented code, [Murtagh et al., 1980, '81]. The Q-dispatch problem, using MINOS is solved by Rouhani and colleagues [1983] and Balu and colleagues [1984] up to systems of 1100 buses. As discussed previously in section 2.5.2.4 the same code has been tested by Burchett and colleagues [1982] on systems up to 600 buses to solve the minimum loss problem. No evidence in the literature is found on the application of this code to the minimum cost problem.

The augmented Lagrangian method is very accurate, reliable and reasonably fast to be used off line for planning purposes.

2.5.7 Newton Based OPF

Sun and colleagues [1984] proposed an algorithm which uses a genuine Newton approach to solve the OPF. The problem was modelled in the sparse format of 2,2, with violated functional constraints incorporated through the addition of quadratic penalty terms to the Lagrangian. The Lagrangian is always designed to contain the power-flow equations, and has an appearance very similar to equation 2.17.e. The Newton method attempts to solve the first order K-T conditions derived for this Lagrangian through a Newton-Raphson process.

Two solution schemes are proposed, one based on the full formulation, and the other based on the decoupled formulation. Efficiency in coding is stressed to a great extent, as defining the success of the overall methodology. The method is general enough to handle all types of objective functions such as production cost, real and reactive losses, departure from a preset control variable values are typical.

The use of quadratic penalties do not produce negative effects as in gradient based techniques. Instead, they aid in clamping a variable at its target value

The method has been tested on systems up to 900 buses and 296 controls, with computing times of the order of 18 Newton power flow iterations. The challenging issue still prevalent is to efficiently identify the binding set of constraints. Once this hurdle is bypassed, this approach certainly appears to be the breakthrough in OPF computations power engineers have been waiting for.

2.6 Conclusion

Judging by the literature the history on OPF development spans a period of two decades. In order to limit the scope of the present study, section 2.5 has limited its scope only to the basic methods that have been introduced to date to solve the full fledged OPF. The goals to be met, remain unchanged and remain as enumerated in section 2.4.

LP based techniques are reliable and fast, but their limited accuracy make them unattractive as a serious OPF solution strategy either for on-line or off-line applications. If the intention is P-dispatch, the LP technique would serve ideally. Further, P-Q decomposition based techniques lack the accuracy and reliability often needed in OPF solution strategies and cannot be recommended.

Among the other basic techniques reviewed, the following methods compete for recognition. i.e.,

1. The GRG technique with compact and explicit (non penalised inclusion of functional variables), [Carpentier 1973],
2. Quadratic quasi-Newton techniques with compact explicit modelling, [Franchi et al., 1983],
3. Quadratic quasi-Newton techniques with sparse explicit modelling, [Burchett et al., 1984],
4. Quadratic Newton approach with sparse penalty modelling as described in the award winning paper by Sun and colleagues in 1984. This has not been tried on a production scale. However, an independently developed production grade OPF package based on the Newton method is currently available with PCA corporation which has superior performance characteristics on OPF problems, Banakar [1986].

Most recent techniques tend to use quadratic approximations, and quasi-Newton methods to solve the full fledged OPF. The minimum cost problem is not strongly nonlinear as compared to the minimum loss problem. This would make gradient techniques competitive with quasi-Newton for the minimum cost OPF. The observation is that gradient methods perform fast at the onset but tend to slow down as the optimum is reached. Further gradient techniques with explicit modelling require an initial feasible starting point, which is not always available. Hence much ingenuity in coding is needed to get gradient techniques to converge onto the optimum. If the current pace in algorithmic development continues, it is not hard to envisage the realization of an OPF solution strategy fast and accurate enough for all applications within the next decade.

CHAPTER III

SOLUTION OF THE OPF VIA THE RESTART HOMOTOPY CONTINUATION METHOD

This chapter develops the theory and algorithms underlying the work reported in this thesis. The following paragraphs will introduce the overall restart homotopy continuation method (RHCM) algorithm, and the variable classification embodied in this work

The RHCM algorithm embodies in essence two phases, namely Phase I and II.

The pertinent steps of the first phase are as follows:

1. Create a sub-problem of the complete optimal power flow (OPF) problem by relaxing boundary limits on all functional variables, namely voltages at load buses, line flows and reactive generations.
2. Parameterize a subset of the whole set of controls which comprise initially of tap-changers, phase-shifters, shunt controllers, and the voltages at generation buses. Note that variables so parameterized will be referred to as the set of ξ -variables, and will form members of vector ξ .
3. Optimize the resulting problem.

The solution in step (3) is used as an initial starting point in a continuation process, designed to track this solution to the optimal solution of the sub-problem defined in step (1). The tracking is accomplished via a predictor-corrector path following algorithm embodying certain special features, such that the solution accuracy can be improved to any desired degree through a flexible restart feature. Within the tracking process only the ξ -variables need to be explicitly monitored for break-points¹. Termination of the first phase marks an operating point in which all controls are strictly feasible and optimal with respect to the relaxed sub-problem

¹ Break-points are values of the continuation parameter at which elements of vector ξ are deactivated.

If following the termination of the first phase functional variables previously ignored prove to be within bounds, the solution to the relaxed problem becomes the solution to the complete OPF problem. However, should functional variables violate their bounds, Phase II of the algorithm is invoked, which in essence changes the roles of the control and violated functional variables, such that the newly modified sub-problem maintains the same basic structure as its predecessor. Under this modification elements of vector ξ can change. Phase I is invoked at this juncture to solve the new sub-problem. This process is repeated in cycles until the Kuhn-Tucker optimality conditions are satisfied. Simulations suggest that convergence is usually achieved within a few Phase I/II cycles.

The advantage of this formulation is that, should the restart homotopy continuation algorithm fail to converge following an update, the solution to the previous sub-problem can be used to identify the functional constraints causing the convergence problems. This feature will aid greatly in system planning to identify areas of the power system which may require expanded facilities.

The overall flow-chart of the restart homotopy continuation method algorithm is summarised in figure 3.1. All key algorithms are identified in figure 3.1. The following discussion will outline briefly the functions of these algorithms. A detailed discussion of these algorithms are deferred until later sections.

The caption Algorithm Lambda is used to describe the algorithm which solves the modified coordination equations for the real generations, given the losses and incremental transmission losses (ITL's). This step is solved by the classical Lambda technique often used to solve the classical or modified coordination equations in the past, [Sasson, A.M., 1976], [Vojdani et al., 1979].

Flow diagram for the restart homotopy continuation algorithm

Figure 3.1

Algorithm Lambda is used in an inner loop (within Algorithm Coordination), while Algorithm Coordination iteratively updates the losses and ITL's in an outer loop, for given values of the ξ -variables, i.e., the ξ -variables are not controllable during iterations involving Algorithm Coordination, instead they act as parameters. Algorithm Coordination provides an operating point which can be used as a starting point to proceed with the continuation process. The operating point provided by Algorithm Coordination at the beginning of a continuation process (at $\theta=0$) marks the easy optimum, i.e., the solution to be tracked by the succeeding continuation process

Algorithm Phase I encompasses both Algorithms Lambda and Coordination to track the easy optimum by incrementing the continuation parameter θ . During this process the ξ -variables no longer act as parameters, instead they are made controllable via suitable homotopic deformations. In incrementing θ , situations may arise in which it becomes necessary to update the active set of the ξ -variables. Values of θ at which such updates are needed are called break points, which reveal themselves via producing a conflicting situation in which the computed LM's do not correspond with the homotopically deformed problem (this situation will be clarified later, when the Algorithms are discussed in detail). Should Algorithm Phase I detect a Lagrange Multiplier conflict, Algorithm Corrector will be invoked to remove the conflict. At CP $\theta=1$, the relaxed OPF problem is considered to be solved provided, the absolute values of the computed LM's of all free ξ -variables are below a precision ϵ . If this were not so, a new (restart) homotopy is constructed, such that the whole tracking process can be restarted by making the current operating point the optimum of a second homotopically deformed problem. This second problem is now solved exactly like the first. If the convergence tolerances are not met, a third problem is constructed, and so on until the convergence tolerances are met, at which point the relaxed OPF problem is solved.

After Algorithm Phase I converges, all functional variables are checked for possible boundary violations. If no violations prevail, the whole OPF is solved. However, should violations prevail, Algorithm Phase II is invoked, which alters roles of the dependent and controllable variables, in order to remove these violations, by in turn invoking Algorithm Update, such that the

resulting sub-problem formulation maintains the same structure as its predecessor sub-problem, which can then be solved via Algorithm Phase I.

The details of each algorithm will be presented in section 3.2.

3.1 Classification of the Control, Dependent, and Injection Variables.

The following discussion on control, dependent, and injection variables are presented in order to clarify their roles in the overall optimization process.

The general OPF problem definition remains as given by equations 2.2.a-d in sparse form, or 2.3.a-d in compact form. The intention here is to reformulate the general problem, into one specific problem, such that a path following algorithm can easily be adapted as a solver. For this reason the following problem definition is proposed and used throughout the remainder of this thesis, i.e.,

Minimize a scalar objective,

$$f(z) \quad \text{.....(3.1.a)}$$

Subject to the constraints,

$$z = g(u, x) \quad \text{.....(3.1.b)}$$

$$u^m \leq u \leq u^M \quad \text{.....(3.1.c)}$$

$$x^m \leq x \leq x^M \quad \text{.....(3.1.d)}$$

Where the elements of the vector of injections z can have one of the following definitions,

- $z_f = P_{gi} - P_{di}$, if bus i is a PV or PQ bus²,(3.2.a)
 $- Q_{gi} - Q_{di}$, if bus i is a PQ bus,(3.2.b)
 $- Q_{gi}^L - Q_{di}$, if bus i is a PV bus, and appears if Q_{gi} is specified at a limit.(3.2.c)
 $- I_{ij}^2$, appears at location i , if the current magnitude on line $i-j$ is specified at a limit. .. (3.2.d)

Equations 3.2.a&b constitute the set of standard load flow (LF) equations as given by Elgerd [1982]. These are also equivalent to equation 2.2.b. One departs from this standard when equations 3.2.c&d are added to the former set as dictated by the optimization process

Vectors u and x represent the independent controls, and the set of dependent variables, i.e., includes both x and h variables. The dependent variables can further be classified into simple dependent, and functional dependent variables. As discussed before in section 2.1.3, the simple dependent variables are the voltages at PQ buses, while the functional dependent variables are the transmission line currents, the reactive generation at PV buses, and the slack generation P_{gs} .

For expository convenience, among the control variables u , the angle at the slack bus, θ_s , is set to zero. This brings about a unity reduction in the dimension of vector u . The remaining elements of u are further classified into those of the generation variety, and those not of this variety, i.e.,

$$u = (P_g, \xi) \quad \dots\dots(3.2.e)$$

$$\xi = (V_s, V_g, t, \phi, B_{sh}) \quad \dots\dots(3.2.f)$$

Further, conforming to equation 2.1.a, the elements of vectors V_s and P_g are defined as follows, i.e.,

² The real generation, and the bus voltage magnitude are specified at a PV bus. Both the real and reactive generations, are specified at zero, at a PQ bus. The (customer) demand at both buses are assumed known. These specifications are used in solving the system of load flow equations 2.2.b.

$$V_g = (V_i : i \in G) \quad \dots (3.2.g)$$

$$P_g = (P_{gi} : i \in G) \quad \dots (3.2.h)$$

Based on this variable partition, problem 3.1 becomes;

Minimize a scalar objective,

$$f(z) \quad \dots (3.3.a)$$

Subject to the constraints,

$$z(P_g) = z(\xi, x) \quad \dots (3.3.b)$$

$$\xi^m \leq \xi \leq \xi^M \quad \dots (3.3.c)$$

$$P_g^m \leq P_g \leq P_g^M \quad \dots (3.3.d)$$

$$x^m \leq x \leq x^M \quad \dots (3.3.e)$$

Formulation 3.3 represents the complete OPF problem. In order to simplify this formulation, the dependent variables x and h , previously defined by the set of equations 2.1.b&c, are incorporated into equation 3.3.e as x .

In the section to follow, problem 3.3 above will be relaxed, to give a sub-problem to the OPF problem, such that at a value of the continuation parameter (CP) $\theta=0$, the solution to the relaxed problem is trivially found. This marks the beginning of a tracking process, which when successful, leads to the solution of the relaxed OPF problem (sub-problem).

3.2 The Relaxed OPF Problem

The relaxed OPF problem is identical to problem 3.3 except for constraint 3.3.e. Among all elements of 3.3.e, only the following is considered, i.e.,

$$P_{gs}^m \leq P_{gs} \leq P_{gs}^M \quad \dots (3.3.f)$$

As will be seen in the discussions to follow, this will constitute a problem whose first order Kuhn-Tucker conditions yield well to a solution via a path following algorithm. Since this solution is dependent upon the objective

function considered, the constituents of the objective, i.e., equation 3.3.a, will be established next.

Among all candidate objective functions classified in section 2.1.1, the commonly used, and ~~least~~ ^{least} disputed objective is the variable operating costs of the committed units, [Stott et al., 1980], [Carpentier 1985]. Hence this thesis ⁷ adopts the operating cost as the objective to demonstrate the performance of a path-following algorithm, as a candidate solver of the OPF problem

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As stated in section 2.1.1, the operating cost of individual generating units comprises of many components. The dominating one is the input output characteristics of the unit. As addressed by the IEEE Committee Report in 1971, these characteristics are seldom accurately known. Further at valve points the curves become non-convex. This causes the problem addressed to become non-convex.

The literature proposes many solutions to circumvent this problem. The most relevant and easily adaptable of these, approximates the input-output characteristics by a polynomial, [Stott et al., 1980], [Carpentier 1985], [Happ et al., 1986], or piece-wise linear functions, [Happ et al., 1986], [Stott 1979, 1980], or a mix of the two above, [Happ et al., 1986]. The first approximation is popular among people utilizing general NP solution techniques, while the second among those utilising LP solution techniques. The third objective, although proposed as a possible candidate by Happ and colleagues, has not gained popularity in the literature. Further, in all three instances the objective is convex in the all P-Space, and must increase monotonically. These assumptions will make the overall OPF problem at least locally convex, [Happ et al., 1986], which implies that the solution to the first order K-T conditions will yield a locally optimal solution, [Bazaraa et al., 1979].

This thesis assumes an objective of the first type, in which the cost curve is approximated by a quadratic function as defined below, [Sasson et al., 1971], [Stott et al., 1979, '80].

$$C_i(P_{gi}) = c_i + a_i P_{gi} + \frac{1}{2} b_i P_{gi}^2 \text{ \$/hr} \quad \dots\dots(3.4.a)$$

Where, C_i is the variable operating cost of the i th unit, while c_i , a_i and b_i are the cost coefficients. Evidence of a third order polynomial can be found in the work by Alsac and colleague in 1974. However, for reasons unknown, the numerical example cited in this reference uses only a quadratic cost curve. The widespread use of quadratic cost curves, has motivated its use in this thesis. Furthermore, this choice would make comparison of published results, to those reported in this thesis more meaningful.

3.2.1 Derivation of the First Order Necessary Conditions for the Relaxed OPF Problem

Section 2.2 outlined the two general modelling practices, namely sparse and compact, adopted to address the OPF problem in the power system literature. However, for reasons enumerated below, this thesis adopts the compact formulation to problem 3.3 and its corresponding relaxed counterpart.

The advantages of the compact formulation over the sparse formulation are,

1. Reduction in the overall number of equations to be handled by the optimising algorithm, i.e., the set of LF equations 3.3.b gets replaced by a single power balance equation, [Carpentier 1985, '87]
2. Since the problem is expressed in terms of the control variables u , the overall memory requirements for solving the problem are reduced.
3. Increased overall solution speed, [ibid].
4. Ease of adaptability to security constraints, [ibid].
5. The nonlinearities prevalent among functional variables are relatively mild, and the losses take on near quadratic forms in the reduced space of u , [ibid]. Hence, Any linearization performed is valid over a larger region in the neighborhood of a given expansion point.
6. Ease of adaptability to Parametric Programming methods, [ibid].

Features 1-6 above are certainly very desirable in any OPF algorithm, and serves to justify the compact OPF formulation used in this thesis. The

discussion to follow will derive the first order K-T conditions for the relaxed OPF problem, in the space of controls u .

The Lagrangian function corresponding to the relaxed OPF problem becomes,

$$\mathcal{L} = \sum_{i=1}^{\text{NOG}} C_i(P_{gi}) - \lambda(\sum_{i=1}^{\text{NOG}} P_{gi} - P_D - P_L(u)) - \alpha^t(u - u^L) - \alpha_s(P_{gs} - P_{gs}^L) \quad (3.5.a)$$

Where NOG is the number of generation units, and P_{gs}^L is the limiting value of the slack generation, i.e., either P_{gs}^m , or P_{gs}^M . Here it is assumed that the slack generation is operating at a limiting value. Should the slack generation be within bounds, the last term in equation 3.5.a will not appear, as $\alpha_s = 0$.

The vector of LM's α , partitioned as (α_p, α_x) , is such that component vectors α_p and α_x correspond respectively to the binding P_g and ξ components of u .

The first order K-T conditions for equation 3.5.a become,

$$\frac{\partial \mathcal{L}}{\partial P_{gi}} = \frac{dC_i}{dP_{gi}} - \lambda(1 - \frac{\partial P_L}{\partial P_{gi}}) - \alpha_{pi} = 0 \quad (3.5.b)$$

$$\frac{\partial \mathcal{L}}{\partial P_{gs}} = \frac{dC_s}{dP_{gs}} - \lambda - \alpha_s = 0 \quad (3.5.c)$$

$$\frac{\partial \mathcal{L}}{\partial \xi_i} = \lambda \frac{\partial P_L}{\partial \xi_i} - \alpha_{xi} = 0 \quad (3.5.d)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{\text{NOG}} P_{gi} - P_D - P_L(u) = 0 \quad (3.5.e)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = u - u^L = 0 \quad (3.5.f)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_s} = P_{gs} - P_{gs}^L = 0 \quad (3.5.g)$$

By complementary slackness, [Kuhn-Tucker 1951], [Bazaraa et al., 1979], [Avriel, 1976], the Lagrange multiplier corresponding to unit i is known if the unit is not operating at a limit, i.e., $\alpha_{pi} = 0$, and its generation P_{gi} becomes unknown. Alternately, if the unit is at a limit, the generation becomes known and the Lagrange multiplier becomes unknown. Similar reasoning

holds true for the ξ -variables and their Lagrange multipliers. Hence among the set of equations 3.5.b-g, the number of equations will always equal the number of unknowns, and if solved should give the optimum of the relaxed OPF problem.

A situation of interest arises when the variable is at a limit, and its LM becomes zero at a solution point. This will correspond to the situation where that variable has either just been activated or deactivated, the status being easily clarified from the context of the solution process. Elaborating, if the value of the variable solves to be at limit then one concludes that the variable has just been activated. However, if the solved LM is zero, one concludes that the variable has just been deactivated. Such a situation almost never occurs in nonlinear programs of the class addressed in this thesis, because the finite precision real arithmetic operations involved in the computation of the variable, or its Lagrange multiplier, seldom works out to an exact zero.

The first order K-T conditions 3.5 b-g are trivially solved, if the optimal active set of controls is known a-priori. However this remains an unknown until the problem is completely solved. Further, it is always desirable in engineering applications, to have a means of systematically converging onto the active set, within a reasonable time frame.

The continuation method has been chosen as the solution candidate due to its past success in systematically, rapidly, and unequivocally, identifying the active set in problems of economic dispatch (ED) origin, documented in the work by Fahmideh-Vojdani et al., [1983], Galiana et al., [1983], and Huneault et al., [1984]. It is worthy of mention that the fundamental motivation underlying the work reported herein stems from the above references, specially the work by Galiana et al., [1983].

Following the discussion of section 2.3.5, the continuation method can be applied to solve the first order K-T conditions 3.5.b-f provided, an initial solution point can be found trivially, to be referred subsequently as the easy optimum. This point will then mark the starting point for a homotopy path which will eventually lead to the final solution of these equations.

The following section will address two key algorithms, one which identifies the easy optimum, and the other which tracks this optimum to the terminal point, which is also the solution of the relaxed OPF problem.

3.2.1.1 The Trivial Initial Solution

Among the set of equations 3.3 b-g, by ignoring temporarily the ξ -variables, i.e., equation 3.5.d one arrives at the well known modified coordination equations, [Shipley et al., 1956], [Kirchmayer 1958]. The analytical studies performed by Galiana and colleagues, and later published in 1978 and '79, show that the modified coordination equations are indeed physically and mathematically well posed. Galiana and colleagues [1979] prove that the solution to the modified coordination equations are independent of the choice of the slack bus, and converge to the true optimum irrespective of the nature of the quadratic generation cost coefficients. Numerical examples, to strengthen this theory were later published by Vojdani and colleagues [1979]. However, the numerical examples cited in these references assumed fixed losses and incremental transmission losses. These rather restrictive assumptions are relaxed in this thesis to variable losses, and variable incremental transmission losses to comply with a general OPF.

The overall solution methodology to solve the relaxed OPF closely follows the recommendations of Dy-Liacco [1977], along with a Binary Search technique or the Method of Bisection [Gill et al., 1981], which is used to solve the modified coordination equations. The salient features of the binary search algorithm will be considered next.

The binary search method is based on systematically reducing the interval of uncertainty, within which the solution is supposed to lie, by comparing the value of a scalar function at the two edges of this interval. As applied to the problem at hand, the function becomes the mismatch in the power balance equation, i.e., equation 3.5.e, and the interval is the interval within which the system incremental cost λ lies. If the initial closed interval is $[a, b]$, the method is guaranteed to converge in $\log_2((b-a)/\delta)$ iterations, for a given small δ , [Gill et al., 1981]. For practical systems tested, this

amounts to less than about 30 iterations, for convergence to a precision of 10^{-6} (pu MW), and appears to be independent of the number of generation units involved. In the cases tested, the initial interval boundaries were the lowest and highest values of the incremental costs (IC's) taken over all available units. Since the work per iteration is small, the overall performance is competitive with those proposed by Vojdani [1979].

The method of bisection³ can be shown to be the optimal algorithm for the class of functions that change sign in the closed interval $[a,b]$, i.e., it yields the smallest interval of uncertainty for a specified number of function evaluations, [Oll et al., 1981]. The interval of uncertainty converges to zero at a rate³ of $\frac{1}{2}$, [ibid].

Dy-Liacco [1977] proposes that the penalty factors be computed from the transposed Jacobian of the LF problem. This blends well with the compact formulation adopted for the OPF problem in this thesis. Further, for a given loss and incremental transmission loss coefficients, the solution of the modified coordination equation can be obtained rapidly and reliably through the binary search technique. Furthermore, this technique is also capable of adapting well to discontinuities in the IC segments of the equivalent unit per plant, although these IC segments are neither explicitly constructed, nor are they needed. This technique has been tested on the examples and counter-examples suggested by Vojdani et al., [1979], and is found to converge consistently to the optimum solutions provided in the above reference, thereby verifying its effectiveness and reliability.

3.2.1.1.1 Description of Algorithm Lambda

Algorithm Lambda solves the modified coordination equations, for the real generations, given the losses and incremental transmission losses (ITL's). Algorithm Lambda needs to be invoked many times in tracking the homotopy path. This will be evidenced in the subsequent description of Algorithm Phase I. Hence, there is a need for Algorithm Lambda to converge reliably, and rapidly. Despite its simplicity, simulation results reported in Chapter

³ Appendix D has a discussion on the rates of convergence of iterative sequences.

IV suggests that Algorithm Lambda meets these requirements. Its pertinent steps are enumerated below.

- a) When invoked for the first time, define the boundaries of the closed interval $[a, b]$, by setting $b = \lambda^M$ and $a = \lambda^m$, where λ^M and λ^m are the maximum and minimum IC's taken over all committed units. Define $\lambda = \frac{1}{2}(\lambda^M + \lambda^m)$.
- b) For iteration count greater than unity, update the losses and ITL's. Else these are initialized to zero
- c) Check if the demand and losses, i.e., $P_D + P_L$ can be furnished with the available generation. If not stop by stating 'Insufficient Generation'.
- d) At $SIC = \lambda$, update initial boundaries of the closed interval $[a, b]$, i.e., set $b = \lambda^M$ & $a = \lambda$, if there is surplus demand ($P_D + P_L$), or set $a = \lambda^m$ & $b = \lambda$, if there is surplus generation.
- e) Update the estimate of the system incremental cost (SIC), i.e., $\lambda = \frac{1}{2}(a + b)$.
- f) Update the generation of units as follows; If the system incremental cost λ violates the limiting values of the specified IC's of units, set the corresponding generations at the violated bound; else update their generation according to equations 3.5.b or c.
- g) Recycle from (d) until the changes in λ are below ϵ .

The following section will describe Algorithm Coordination, the solution of which marks the beginning of the homotopy path leading to the solution of the relaxed OPF problem.

3.2.1.1.2 Description of Algorithm Coordination

Algorithm Coordination solves the set of coordination equations by updating the incremental transmission losses, and the real generations, after fixing the ξ -variables at certain specified values within their bounds, until the change in losses from the previous iteration fall below a certain precision ϵ . The pertinent steps are enumerated below.

- a) Invoke Algorithm Lambda, to find the corresponding real generations.
- b) Solve the set of LF equations 3.3.b to update the state vector x , the losses $P_L(\xi, x)$ and incremental transmission losses.
- c) Recycle from (a) until the change in losses are below ϵ .

At this point in the optimization, the solution obtained satisfies equations 3.5.b-g, except 3.5.d. Hence in the frame of controls P_g and state x the solution obtained will be an optimum. One could call this an easy optimum provided one is able to homotopically deform the relaxed OPF such that even when ξ -variables are added to the existing controls P_g , the optimum remains an optimum, but of the homotopically deformed problem. As suggested by the discussion of section 2.3.5, the overall solution task is greatly simplified through an intelligent choice of the homotopy. The suggestion of Galiana et al., [1983] is adopted in a modified form to define the relevant homotopy. The topic of the following section is designed to give the relevant details.

3.2.1.2 The Homotopically Deformed Relaxed OPF Problem

Among the set 3.5.b-g, only equation 3.5.d remains unsatisfied at the easy optimum. This is seen by computing the partial $\lambda(\partial P_L / \partial \xi_1)$, for variables ξ_1 , specified within bounds, which under real arithmetic will be always non-zero, hence violating the condition on complementary slackness for these variables. This feature is exploited in the homotopic deformation of the relaxed OPF as explained below.

Since the SIC λ has a positive sign, the algebraic sign of the LM's α_{x_1} is determined by the algebraic sign of the partial $\partial P_L / \partial \xi_1$, which computes as a byproduct of the previous optimization, i.e., the solution of the coordination equations. By making the upper bound or lower bound of control ξ_1 coincide via a homotopic deformation with its current specification ξ_1^E , based on whether the LM α_{x_1} is negative or positive, one is able to make the easy optimum coincide with the optimum of the homotopically deformed relaxed OPF problem. The justification for this proposal will now be discussed.

Equation 3.5.a suggests that the optimum of the objective coincides with the optimum of the Lagrangian. Variations to this optimum for positive increments in limits to controls u is given below, i.e.,

$$\frac{\partial \mathcal{L}}{\partial u} = \alpha \quad \dots (3.6.a)$$

Positive increments to upper limits u^M amounts to relaxing the problem, while positive increments for lower limits u^m further constraints the problem. The optimal cost of a relaxed problem is lower, while that of a constrained problem higher. Hence this discussion serves to justify the actions proposed in the previous paragraph. The homotopic deformation claimed therein can be achieved via the following linear homotopies defined next; see equation 2.6.c, i.e.,

- a) For homotopic deformations to the upper bound, i.e., for $(\partial P_L / \partial \xi_1) < 0$, define the homotopy:

$$\xi_1^N = \xi_1^E + (\xi_1^M - \xi_1^E)\theta \quad \dots (3.7.a)$$

- b) For homotopic deformations to the lower bound, i.e., for $(\partial P_L / \partial \xi_1) > 0$, define the homotopy:

$$\xi_1^N = \xi_1^E + (\xi_1^m - \xi_1^E)\theta \quad \dots (3.7.b)$$

Where ξ_1^E is the value of ξ_1 at the easy optimum, i.e., provided by the successful completion of Algorithm Co-ordination. Variables ξ_1^N and ξ_1^m represent the homotopically deformed bounds ξ_1^M and ξ_1^m . It must be noted that for each ξ_1 either equation 3.7.a or 3.7.b applies, but not both at the same time.

Under these specifications, at $\theta=0$, the conditions on complementary slackness are exactly satisfied. This makes the easy optimum coincide with the optimum of the homotopically deformed relaxed OPF, and marks the beginning of the path leading to the solution of the relaxed OPF. Before a description of the path following algorithm is presented, it is necessary to distinguish clearly the differences among the homotopies 3.7 and the ones successfully

demonstrated in ED applications by Vojdani et al., [1983], Galiana et al., [1983], and Huneault et al., [1984].

3.2.1.3 A Comparison of Different Homotopic Deformations.

The scope of the discussion is limited to the application of CM to the dispatch problem. Two distinct homotopic deformations can be traced in the power system literature. These will now be discussed.

3.2.1.3.1 Homotopic Deformation via Relaxed Demand

The first successful application of the CM to economic dispatch is due to Vojdani and colleagues [1983]. Here the easy optimum is made to coincide with the point of minimum generation of all committed thermal units, by relaxing the demand to meet the generation. The feasible region under this relaxation amounts to a single point. This is an exact optimum provided no functional constraints (line flows in ED) are violated, and marks the beginning of the homotopy path. In comparing this to the proposals of section 3.2.1.2, one observes that in order to meet the K-T conditions the demand is deformed in one case, while in this thesis the bounds of a subset of controls are subjected to deformation, in which the deformation imposed are dependent on the current specification of a subset of controls u , specifically the ξ -variables.

By assuming the bus loads to be linearly correlated to the system demand, the linear nature of the ED problem is such that the first order K-T conditions at the above optimum take on analytical forms in which all key variables, i.e., generations of inactive units, LM's of active units, LM's of active functional variables, and the values of inactive functional variables yield to deformations via linear homotopies (see equation 2.6.c) of the form,

$$\rho(\theta) = \rho_0 + \theta \rho_1 \quad \dots (3.8.a)$$

Variable $\rho(\theta)$ satisfies the K-T conditions at the onset. Scalar ρ_1 is a

physically unattainable value of system demand, and ρ_0 marks the beginning of the homotopy path.

As shown by Vojdani et al., [1983], the above homotopy can be used to systematically update the active set, until an increase in the continuation parameter θ causes a violation in the first order K-T conditions. This can mark either a resolvable or ir-resolvable degenerate case. The latter marks the loadability limit, while the former, once resolved can be continued until the loadability limit is reached.

The homotopy path for all variables provide the optimal trajectory for varying demand. Further, the nature of the ED problem is such that the path is piece-wise linear, and in terms of tracking provides the least computational effort.

The next successful application of the CM to the dispatch problem is due to Galiana et al., [1983], which will be considered next.

3.2.1.3.2 Homotopic Deformation via Relaxed Functional Limits

The second suggested application of the CM to the economic dispatch problem is credited to the efforts of Galiana and colleagues [1983]. Here, a classical economic dispatch as specified by Kirchmayer [1958] is carried out, ignoring the functional constraints, i.e., transmission line flows in ED. If no functional violations are present at this point, this will mark the optimum. If functional violations are detected this will mark the easy optimum. The homotopies for tracking the easy optimum are generated according to the following criteria, i.e.,

$$P_{ij}^M = P_{ij}^M + (1-\theta) \Delta P, \text{ for upper limit violations} \quad \dots (3.9 a)$$

$$P_{ij}^M = P_{ij}^M - (1-\theta) \Delta P, \text{ for lower limit violations} \quad \dots (3.9 b)$$

Where ΔP is defined as follows,

$$\Delta P = \text{Max } |(P_{ij}^U - P_{ij}^L)|, \text{ taken over all lines } i-j \quad \dots (3.9 c)$$

Once again the nature of the ED problem is such that analytical expressions can be derived for key variables in terms of the continuation parameter θ . This is possible via the application of the K-T necessary conditions at the easy optimum. The active set is updated in a manner very similar to that proposed by Vojdani and colleagues [1983]. In this application, an irresolvable degeneracy amounts to an infeasible optimum. Unlike the previous case, the optimal trajectories generated do not constitute usable information to the dispatcher.

If the optimum sought corresponds to a single base case, this method is preferred, while for generating an optimal trajectory the former is preferred.

In comparing the homotopy 3.9 to 3.7, one observes that in 3.9 the initial deformation is uniformly applied to all violated functional variables, (line flows in ED) while in 3.7 the initial deformations are proportional to the specifications of the controls ξ at the easy optimum. Further as mentioned before, the nature of the deformations are such that the K-T conditions are satisfied at the easy optimum, which also marks the beginning of the homotopy path.

Huneault and colleagues [1984] extended the theory of sections 3.2.1.3.1&2 to recompute the optimum should the following events occur, or are necessary for planning purposes, i.e.,

1. Loss of generation,
2. Loss of transmission facilities,
3. Addition of generation,
4. Addition of transmission facilities.

It is also pointed out that general changes in generation sets and network parameters result in nonlinear homotopies. These are very difficult to handle, and the tracking process becomes numerically prohibitive even for economic dispatch.

Although the homotopies addressed in sections 3.2.1.2&3 are linear by definition, as discussed they achieve the homotopic deformation in a unique manner. Further the analytic nature of the tracking step, seen in section 3.2.1.3 does not apply to the OPF problem due to its nonlinear characteristics. This necessitates the development of a tracking algorithm capable of following the homotopy path. This will be the topic of the subsequent discussion.

3.2.1.4 Tracking the Easy Optimum

This thesis assumes that the solution of interest is the terminal point of the solution trajectory, similar to the study discussed in section 3.2.1.3.2. Hence no attempt is made to follow the solution trajectory closely. The point of concern here being to reach the terminal point within a given precision.

This thesis chooses the predictor corrector method of section 2.3.5 6 4 as the candidate to path-follow. The flow diagram of the overall algorithm is given in figure 3.1. The reasons for choosing the predictor corrector are given below, i.e.;

1. Problems due to drift, inherent in the Euler method, are eliminated by the Newton corrector step,
2. The controlled step-lengths of the predictor eliminates problems of divergence often present in the Newton's method, especially when the easy optimum is outside the region of convergence with respect to the terminal point of the trajectory,
3. Can be implemented with lesser effort than a flex simplicial algorithm, with competitive accuracy, obtained via controlled predictor steps,
4. The homotopy 3.7 is such that it adapts trivially to the predictor corrector path following algorithm.

Once the homotopies 3.7 are formed, an increment in θ amounts to the predictor step. If $\theta=1$, the predictor steps become the Newton steps, with the associated problems discussed before. Hence it is necessary to exercise

caution in assigning step sizes for θ . In the computer implementation, θ is incremented in multiples of 10, in which the first incremented value from zero is 10^{-4} . The reasons for this will be justified subsequently in Chapter IV, via numerical simulations.

Algorithm Phase I is designed to track the easy optimum along the solution trajectory. As discussed before, since the point of interest is the terminal point of this trajectory, no attempt is made to stay on the path at all times. The predictor corrector method employed is such that at each predictor step one moves off the path in a direction of increasing path length, i.e., along proper orientation of the path, and the corrector step is designed to bring this point back onto the path. The mathematics underlying this are discussed in section 2.3.5 6.4

The nature of this process is such that at $\theta=1$, the displacement error, i.e., a measure of the deviation from the true solution falls outside an acceptable precision ϵ . This necessitates a restart feature, discussed in section 2.3.5.6 2, whereby one is able to close in onto the true solution within a specified precision ϵ .

The predictor steps applied via the homotopy 3.7 are such that changes in specifications apply only to all the inactive⁴ ξ -variables. This amounts to a step taken parallel to the θ axis, henceforth referred to an elevator predictor; see section 2.3.5.6.4. The corrector step must be such that equations 3.5.b-g are satisfied at the new specification for ξ . This is accomplished via Algorithm Corrector, which will be discussed next.

3.2.1.4.1 Algorithm Corrector

The implementation of the corrector phase is not as trivial as that of the predictor. Recalling the discussion of section 3.2.1.2, all ξ -variables are included into the active set at the initial point of the solution trajectory. This does not mean that they will remain in the active set at the terminal point. Hence the corrector step must be designed to systematically

⁴ An inactive ξ variable is one whose specification lies within its natural bounds at $\theta=0$.

update the active set, as the solution trajectory is traversed. Since the path itself is nonlinear, the break points, i.e., points on the path where the active set requires updating, cannot be analytically identified, cf [Vojdani et al., 1983] and [Galiana et al., 1983], where the break points are analytically identified. This thesis uses a linear interpolation to identify the break points (and hence the variable settings) initially, and uses the restart feature subsequently to update the variable settings if necessary.

As evidenced in the work by Vojdani and colleagues [1983], and Galiana and colleagues [1983], the homotopy path changes course beyond a break point. This is due to the active set defining the path being different on either side of the break point. As explained below, in Algorithm Corrector, given the specifications of the ξ -variables, updates to the active set involving variables P_i are trivially made via Algorithm Coordination. However, updates to the active set involving the ξ -variables are more difficult to estimate. Algorithm Corrector solves this problem via the following steps:

- a) Invoke Algorithm Coordination to obtain the optimum corresponding to a given specification in the ξ -variables
- b) Compute the Lagrange Multipliers of all ξ -variables in the active set via equation 3.5 d, and check for any conflicts, i.e., a conflict is detected for variable ξ_i if its LM α_{ξ_i} has changed its algebraic sign from the previous iteration. In the event of a conflict, update the active set by dropping ξ_i from the active set, after specifying it at a value estimated via a linear interpolation⁵.
- c) If the continuation parameter $\theta=1$, repeat steps (a) & (b) until there are no conflicts, and exit.

⁵ The smaller the increments in θ , the better the accuracy of the linear interpolation. However, too small a step length will retard progress. The logarithmic increments chosen for θ strikes a compromise between speed and accuracy. As demonstrated in Chapter IV, in the first tracking step, changes in the active set are identified in the interval of θ ranging from 0.1 to 1.0. This makes the estimated positions of the free ξ variables inaccurate. However, as one closes in on the optimum, via the restart feature, this interval narrows down, thereby giving better accuracy to the estimates.

d) If the continuation parameter $\theta < 1$ exit.

Note: For arbitrary indices i and j , releasing ξ_i in step (b) can cause ξ_j to leave the active set immediately or in a later iteration, although, ξ_j did not exhibit a conflict before in step (b). If this update were immediate, the variable will be released only in the interval corresponding to the next higher value of θ . Such an updating scheme may cause a variable to be released at a value beyond its optimal setting. However this causes no problem as the subsequent restart iteration(s) will correct this situation. If this update were to occur in a subsequent iteration, it will be correctly identified, and specified subject only to the normal errors involved in the linear interpolation. Further since $\theta = 1$ (step c) corresponds to the final interval increment in θ , it is necessary to repeat steps (a) and (b) until all conflicts among the variables in vector ξ and their LM's are removed.

In order to detect immediate updates to the active ξ -set, step (c) could have been repeated following each update detected per interval increment in θ , for $\theta < 1$. However, this is found to be unnecessary, as it tends to slow down the tracking process.

At the completion of Algorithm Coordination, the current operating point will lie exactly on the path, should the active set require no updates, otherwise it will be close to the path (optimal trajectory), the closeness being determined by the accuracy of the estimates in step (b). In invoking Algorithm Coordination, the accompanying Newton LF iterates play an equivalent role to that proposed in section 2.3.5.6.4, which makes the corrector a Newton corrector. Hence by theorem 2.4 repeated applications of the elevator predictor and Newton corrector will follow the entire path.

The error in the estimates computed in step (b) causes the iterates to deviate from the true homotopy path. This brings about a displacement error at the terminal point, quantitatively estimated as the absolute value of the largest LM computed among all free control variables u . The nature of Algorithm Corrector is such that this error always occurs among the ξ -variables

This is also partly due to the precise computations in Algorithm Coordination, and the relatively mild coupling which exists among the generations and the ξ -variables, which preclude large errors in computed LM's from occurring among the free generations.

The restart feature, i.e., step (e) in Algorithm Phase I, to be considered next, is designed to make the displacement error fall below a certain precision ϵ . The pertinent steps of Algorithm Phase I are enumerated below.

3.2.1.4.2 Algorithm Phase I

Algorithm Phase I, (see the loops for Phase I highlighted in figure 3.1) solves the relaxed OPF problem to completion. If no functional violations are detected, this will mark the optimum of the full fledged OPF. The numerical simulations presented in Chapter IV indicate that a few functional violations do prevail in most cases. Algorithm Phase II, to be presented subsequently is designed to remove these functional violations.

The steps in Algorithm Phase I are as follows, i.e.,

- a) Set $\theta = \theta * 10$, and update the specifications of variables ξ in active set via homotopy 3.7
- b) Invoke Algorithm Corrector
- c) If θ is less than unity, go to step (a)
- d) Compute $\alpha_{x_1}^M$, i.e., the absolute value of the largest LM of all ξ variables not in the active set.
- e) If $\alpha_{x_1}^M < \epsilon$ STOP. Else incorporate all ξ -variables into the active set via homotopy 3.7, and recycle from (a).

In the computer implementation a value of about 10^{-4} is found to be satis

⁶ The symbol $\theta = \theta * 10$ implies, set θ to θ times 10

⁷ This step marks the restart feature discussed before in section 2.3.5.6.2. Further, it is necessary to incorporate all the ξ -variables into the active set, for the displacement error may be such that the previous active set may be incorrect.

factory for ϵ . A lower value will result in greater number of iterations, for a marginal improvement in the objective.

The section to follow will summarize the overall concepts underlying Phase I iterations.

3.2.1.5 Summary of Phase I Iterations

The elevator predictor Newton corrector steps in Algorithm Phase I are illustrated in figure 3.2 below. For clarity, only the projected path in the space of θ - ξ , and θ -(x, P_g) are shown.

In figure 3.2 the broken lines indicate the piece-wise linear approximations of the projected homotopy path on the appropriate sub-spaces. The full lines on the space θ -(x, P_g) indicate the progress of the elevator predictor, Newton corrector, steps. Since the corrector steps move parallel to the (x, P_g)-axis, the term horizontal corrector can be used synonymously with the Newton corrector, cf. section 2.3 5.6.4.

Hypothetical trajectory of a homotopy path in two sub-spaces

Figure 3.2

Recapitulating Algorithm Phase I, the corrector iterates computed are exact for a given set of injections, provided the active set of ξ -variables remain static from the initial to the terminal point. In such a situation, the exact optimum is reached at $\theta=1$, and no restarts are needed. However, the simulations of Chapter IV indicate that such a situation almost never exists in practice.

The difficulty arises in estimating the position of the free ξ -variables due to the sensitive coupling prevalent among these variables. The restart feature progressively filters errors, and moves the estimates towards their optimal settings, i.e., it functions as an active set filter. The displacement error is a function of the estimation error, as each approximate estimate defines an approximate trajectory. Figure 3.3 gives an illustration involving a single break point.

Displacement error due to incorrect estimation of free ξ -variables

Figure 3.3

The restart feature is designed to reduce the displacement error ϵ_d below a precision ϵ . The nature of the homotopy 3.7 is such that, when reinvoked as specified in Algorithm Phase I, step (e), it gives a restart point on a path closer to the true trajectory. This is due to the fact that most of the

members of the active ξ -variables being correctly identified after the first pass of the continuation process, i.e., with restart count zero. As observed in the numerical simulations presented in Chapter IV it is possible for ϵ_d to diverge marginally with higher restart iteration counts. This is due to the highly nonlinear nature of the problem, and the sensitive coupling which exists among the free ξ -variables. However, as the optimum is approached, the convergence becomes monotonic.

The progress of the restart steps are shown in figure 3.4. Convergence to the true trajectory takes a few restart iterations. Every restart iteration takes the operating point closer to the true optimum. For clarity, figure 3.4 assumes that convergence to the actual path is possible, for a restart count of unity.

Progress of restart iterations

Figure 3.4

This discussion concludes the solution of the relaxed OPF. If no violated functional variables exist, as pointed out before, this will mark the end of the overall optimization. In most instances, functional violations do prevail at the end of the Phase I iterations. The sections to follow will discuss different schemes whereby functional violations can be alleviated, and will present a method tried successfully in this thesis.

3.3 The OPF Problem with Functional Violations

Once a successful solution to the relaxed OPF is obtained, the solution to the constrained OPF will be 'quite near'. This is evidenced from the numerical simulations of Chapter IV. Improving the relaxed optimum in order to respect functional violations is a non-trivial numerical task. The complexity is increased by the coupling influence among the violated functional variables. Stated alternately, relieving the violations on a subset among all violated functional variables, could result in the violations being removed among the remainder. Choosing this subset requires some conjecturing. It is also quite possible that this subset is the whole set. This thesis utilises logic based on the LM's in this endeavour

3.3.1 Effect of Violated Functional Variables on the Relaxed OPF Formulation

Among all functional variables x , the relaxed OPF formulation of section 3.2, incorporates only the slack generation. This leads to a solution in which the limits on P_{gs} are respected. However, the non inclusion of other functional variables may cause boundary violations among some of these.

At the constrained solution, among all variables x , some may reach their violated bounds while the rest may come within their bounds. It is also possible for more functional variables, not previously violated, to enter the active set of functional variables. Hence the problem at hand now becomes combinatorial in nature, and reduces to identifying the optimal active set of functional variables at the constrained optimum.

In order to commence the identification process, perhaps the best logic would be to begin the next update iteration by incorporating all violated functional variables into the active set, specified at their violated bounds. Thenceforth a scheme has to be designed to systematically converge onto the optimal active set of functional variables, by dropping those functional variables which do not belong to this set, at which point the complete OPF problem is solved. During this process the active set of

controls must also be monitored, and updated. Monitoring the active set of controls can be achieved trivially via Algorithm Coordination.

In sections 2.1.3 and 3.1, two types of functional variables were identified. To simplify the discussions to follow, Type I will refer to simple functional variables, (those identified in equation 2.1.b), while Type II will refer to the functional dependent variables, (those identified in equation 2.1.c) excluding the slack generation.

In order to preserve the structure of formulation 3.5, functional violations of Types I&II are handled such that, Type II violations result in a change in the dimension of z , while Type I violations do not. This form of structuring is necessary, so that Algorithm Phase I can be reapplied to reach the optimum to conform with the new specification of u , and x of the OPF problem 3.1. The procedure will be explained shortly after a discussion on Type I violations is presented.

3.3.1.1 Type I Violations

Type I functional variables, i.e., voltages at PQ buses, if violated at the easy optimum, are initially specified at the violated limit in formulation 3.5, and are swapped⁸ for an equal number of the ξ -variables. Hence, such variables become parameters for subsequent iterations, and are no longer dependent variables. This specification preserves the structure of formulation 3.5. Hence the new optimum, resulting due to a updated specification among the controls u , and the dependent variables x , can be reached via Algorithm Phase I. Recalling the steps of Algorithm Phase I, it is possible for an exchanged Type I functional, now among the ξ -variables, to come off its bound during the succeeding Phase I iterations.

Unlike their counterpart, i.e., Type II functional variables, violations among simple dependent variables do not constitute a change in the dimension of z .

⁸ The swapping strategy is discussed subsequently.

Next, a discussion on Type II functional variables will be presented.

3.3.1.2 Type II Violations

Functional violations of Type II, specified at their violated bounds, modify the Lagrangian 3.5.a as follows, i.e.,

$$\mathcal{L} = \sum_{i=1}^{\text{NOG}} (P_{g,i}) - \lambda (\sum_{i=1}^{\text{NOG}} P_{g,i} - P_D - P_L(z, \xi)) - \alpha^t (u - u^L) - \alpha_s (P_{g,s} - P_{g,s}^L) - \gamma^t (z_f - z_f^L) \quad (3.10.a)$$

Vector γ represents a set of LM's, and vector z_f is the set of violated Type II functional variables, now incorporated as additional injections. These specifications are carried out according to definitions 3.2.c or d, and result in the following additions to the first order K-T conditions of formulation 3.5, i.e.,

$$\frac{\partial \mathcal{L}}{\partial z_{f,i}} - \lambda \frac{\partial P_L}{\partial z_{f,i}} - \gamma_i = 0 \quad (3.10.b)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} - z_f - z_f^L = 0 \quad (3.10.c)$$

Equation 3.10.c, is incorporated into the set of LF equations 3.1.b. Equation 3.10.b, gives the LM of the active set of functional variables. The discussion on the algebraic sign of LM's given in section 3.2.1.2 applies here as well, and is used to update this set at the termination of each Phase I iteration. During Phase I iterations, any new Type I violation incorporated among the ξ -variables get updated along the lines discussed before for updating the active set of ξ -variables. The complete procedure involved will be discussed shortly after the following discussion on Bus Incremental Costs (BIC's) is presented.

3.3.2 The Bus Incremental Costs

Bus Incremental Costs are a means of assessing the incremental production costs, due to increments in various bus loads. Author Squires [1961], was perhaps the first person to introduce and interpret this concept for a

problem which closely approximated the OPF. The role of BIC's in economic dispatch has been studied at depth by Vojdani [1982], Ponrajah [1984], and Ponrajah and colleagues [1985]. The following discussion will present the derivations underlying BIC's for the full fledged OPF problem.

In order to compute BIC's, at the termination of each Phase I iteration, the bus loads are considered to be variables rather than parameters. Under this assumption, the Lagrangian 3.5.a can be expressed in terms of the inactive injections z_i , and inactive controls of the ξ -variety, i.e., ξ_i as,

$$L = L(z_i, \xi_i, D) \quad \dots (3.10.d)$$

A Taylor series expansion of 3.10.d gives,

$$dL = \frac{\partial L}{\partial z_i} dz_i + \frac{\partial L}{\partial \xi_i} d\xi_i + \frac{\partial L}{\partial D} dD \quad \dots (3.10.e)$$

At the optimum, by the K-T optimality conditions, (cf. equation 3.5), the first two terms of equation 3.10.e are zero. Hence, equation 3.10.e reduces to,

$$\frac{dL}{dD} = \frac{\partial L}{\partial D} \quad \dots (3.10.f)$$

Equation 3.10.f will serve as the fundamental equation to be used in the computation of BIC's. Two cases will be discussed next. Case I corresponds to the i th element of D , i.e., D_i being a real demand P_{di} , and Case II corresponds to this being a reactive demand Q_{di} .

Case I $D_i = P_{di}$

After some algebraic manipulations on Equation 3.10.f, the BIC for increments in real demand at bus i becomes,

$$\frac{dL}{dP_{di}} = \lambda (1 - \frac{\partial P_L}{\partial z_i}) \quad \dots (3.10.g)$$

Where, $z_i = P_{gi} - P_{di}$.

A situation of special interest arises when bus 1 becomes the slack bus. Recall that in the compact OPF formulation, the slack generation is a dependent LF variable, and is not a member of the injections z . This makes the incremental transmission loss term zero at the slack bus. Hence, equation 3.10.g reduces to,

$$\frac{dr}{dP_{ds}} = \lambda \quad (3.10.h)$$

The variable λ , now represents the BIC of the slack bus, and not the system incremental cost as in economic dispatch. In ED applications, since the system losses P_L are fixed, λ becomes the system incremental cost in the absence of active functional variables (real power flow on transmission lines), and will be equal to the IC of the slack unit provided it is within bounds; else, the difference amounts to the LM computed for the corresponding unit, [Ponrajah et al., 1985].

Case II. $D_1 = Q_{d1}$

Similar algebraic manipulations on Equation 3.10.f, gives the BIC for increments in reactive demand at bus 1 as follows, i.e.,

$$\frac{dr}{dQ_{d1}} = -\lambda \frac{\partial P_L}{\partial z_1} \quad \dots (3.10.i)$$

Where, $z_1 = Q_{s1}^L - Q_{d1}$.

When Q_{d1} coincides with z_{r1} , it can be shown from equation 3.10.b, that equation 3.10.i becomes equal to $-\gamma_1$. This is a very useful relationship, to be used in updating the active set of functional variables. Further, through a discussion similar to the one presented in section 3.2.1.2, γ_1 becomes equal to the LM corresponding to the active functional z_{r1} .

These discussions indicate that the BIC's for both real, and reactive loads, can be computed as byproduct information resulting from the successful completion of Algorithm Phase I.

Many power system operators consider the system incremental cost, (SIC), as an indicator for tariff settings in economy interchanges, i.e., transactions involving the utility and its customers, [Coughan et al., 1983]. Hence, the following section will derive the SIC's resulting in optimal power flow applications.

3.3.2.1 The System Incremental Cost

The system incremental cost, for increments in real demand P_D is given by the following equation, i.e.,

$$\frac{d\mathcal{L}}{dP_D} = \lambda \left(1 - \frac{\partial P_L}{\partial z^t} * \frac{\partial z}{\partial P_D} \right) \quad \dots (3.10.j)$$

Given the correlation among the bus loads and system demand, the system incremental cost is trivially computed from equation 3.10.j.

The system incremental cost provides global information, while the bus incremental costs provide local information. This local information is more meaningful considering that economy interchanges always take place locally.

In the work of Ponrajah and colleagues [1985], it has been shown that, in economic dispatch applications, all BIC's equal the SIC, provided no active functional variables, i.e., real power flow on transmission lines are present. Variations begin to appear, once functional variables enter the active set. The observations reported in Chapter IV for the full fledged OPF is such that, never, are the incremental costs at two different buses equal, irrespective of the presence or absence of functional variables (Types I&II) among active constraints. This marks a sharp contrast to the economic dispatch problem, and is understandable, considering that losses, do prevent equality among incremental costs.

The special nature of the specifications in section 3.3.1, proposed for violated functional variables, calls for suitable modifications to be made to the conventional LF formulation. These modifications will be considered next.

3.3.3 Special LF Specifications

In order to solve the system of LF equations 3.1, it is necessary to have unspecified variables, equal in number to the set of specified injections. Hence, for every specification of violated functional variables, an equal number of unspecified variables must appear in the optimization, or an equal number of injections must leave z , i.e., be unspecified.

In the former case, the additional variables come from among the ξ -variables, in which any member qualifies as a candidate. In the latter case, injections involving inactive real generation are the only eligible candidates to be dropped from z , i.e., becomes dependent LF variables, as the other members in z have fixed specifications and cannot be made dependent. However, by the following theorem, the second case involving inactive real generations is excluded as a mathematically viable alternative.

Theorem 3.1

In the compact formulation of the optimal power flow problem, taken over all dependent variables, no more than one dependent real generation may exist in the load flow formulation.

Proof:

By assuming more than one dependent generation, eg., $s1$ and $s2$, to prevail in the compact OPF formulation, one obtains the following equations for the BIC's, i.e.,

$$\frac{dr}{dP_{ds1}} = \lambda \quad \dots\dots(3.10.j)$$

$$\frac{dr}{dP_{ds2}} = \lambda \quad \dots\dots(3.10.k)$$

Equations 3.10.j&k are simultaneously possible, only if buses $s1$ and $s2$ are electrically identical, i.e., connected by a loss less link. Since such a situation can never arise in a lossy network, the theorem is proved by contradiction.

QED

The previous discussion has made it clear that, at the termination of Algorithm Phase I, it may be necessary to take any or all of the following corrective measures, i.e.,

1. Drop certain Type II functional variables from the active set,
2. Incorporate Type II functional variables into the active set.

It must be noted that the nature of the homotopy 3.7 is such that, no violations occur among the ξ -variables. Hence, it is necessary to devise a systematic scheme, which will unequivocally determine the functional variables to leave the active set. This will be the topic of the following discussion.

3.3.4 Algorithm Update

Algorithm Update is invoked after each successful completion of Algorithm Phase I (see figure 3.1). At this juncture, should a functional update become necessary, this algorithm systematically updates the active set of functional variables, and re-specifies the control and dependent variables as discussed before, preparing the formulation for a subsequent re-invocation of Algorithm Phase I. The steps are enumerated below, i.e.,

- a) Go to step (e) if Algorithm Phase I is invoked for the first time.
- b) Compute all Lagrange multipliers γ , via equation 3.10.1.
- c) Delete from the active set, the i th element of Type II functional z_{p_i} , if the algebraic sign of its LM γ_i conflicts with the specification of z_{p_i} (see discussion in section 3.2.1.2), and correspondingly increment the number of controls comprising the ξ -variables by adding to it one of the variables now in x , which can function as a control, eg., V_{pv} , tap, phase-shifter, or a shunt controller. Note that this amounts to a reduction in the dimensionality of the standard load flow dependent variables.
- d) Set a flag, if an update of x becomes necessary in step (c) above.

- e) Identify all violated Types I&II functional⁹ variables.
- f) In the event of no violations in step (e), and if the flag in step (d) is not set STOP, the optimum is reached.
- g) For all functional violations in step (e), specify each violated Type I functional at its violated bound and place it among the ξ -variables, and correspondingly release a variable¹⁰ from among the ξ -variables to become a dependent LF variable. For each Type II violation, append to the set of injections z the violated Type II functional variables specified at their violated bound, and correspondingly increase the dimension of the LF dependent variables by releasing a variable (see footnote) from among the ξ -variables.

Algorithm Phase II, to be presented next, solves the full fledged OPF problem. Once successfully executed, this will give a locally optimal solution. Under the assumptions on convexity, discussed in section 2.2 this will also mark the global optimum.

The steps involved in Algorithm Phase II will be enumerated next

3.3.5 Algorithm Phase II

Algorithm Phase II invokes Algorithms Phase I, and Update, until the first order K-T conditions are satisfied in the overall OPF problem. The pertinent steps are,

- a) Invoke Algorithm Phase I.
- b) Invoke Algorithm Update.
- c) Recycle from (a).

Note that, Algorithm Phase II will terminate successfully within the module, Algorithm Update. However, there are instances in which unsuccessful

⁹ Only Type II functional variables need to be computed here. The values of Type I functional variables are available as a result of the previous LF solution.

¹⁰ Candidate variables for an exchange, from among the ξ -variables are discussed subsequently

termination could occur. Such abnormal terminations are due to degeneracies and infeasibilities. The sections to follow will address all such cases experienced in the present work, and will propose remedial measures to circumvent the degeneracies, and in the event of infeasibilities, provide the best possible information in the partial solution which would assist in its alleviation

3.4 Degeneracies and Infeasibilities

Degeneracies and infeasibilities, if present, manifest themselves in the form of oscillations or singularities, at some stage of the optimization. Once these are detected, remedial measures are needed to progress with the optimization. Very often, premature termination becomes the only remedial action. The sections to follow, will commence this discussion, by considering first, numerical oscillations which may occur in Algorithm Coordination

3.4.1 Numerical Oscillations in Algorithm Coordination

Numerical oscillations arise, due to the solution to Algorithm Coordination being situated at a point of inflection. Figure 3.5 depicts graphically a situation of this nature. One immediate solution which comes to mind, is to use a smaller step length. Generally, such an action would merely increase the frequency of oscillation, and will not solve the problem effectively, Bach [1982].

In order to reach a solution of the above nature, one has to resort to direct search methods, i.e., methods based on function comparison, [Gill 1981], [Bach 1982]. These methods are primitive, and their simplicity does not necessarily guarantee any convergence.

All direct search methods, eg., The Polytope Algorithm, [Gill 1981], The Bracketing Algorithm, [Bach 1982], rely on function comparisons, to systematically narrow down interval in which the solution lies. With problems of higher dimensions, like the OPF, the 'curse of dimensionality' soon takes control.

Solution at a point of inflection

Figure 3 5

For all practical purposes, it is better to terminate the iterations, should such oscillations occur. The numerical experience gained from simulations reported in Chapter IV, suggest that, such oscillations are rare, and can often be alleviated with a slight change in the system demand, or the unit commitment schedule.

Singularities can also occur in Algorithm Coordination. If a singularity prevails it will reveal itself when a solution is attempted to the set of LF equations in step (b) of this algorithm. The next section will address this issue.

3.4.2 Singularities in Algorithm Coordination

The discussion here will commence with the following remark taken from [Watson 1986], i.e., 'Despite considerable success on practical problems and a large amount of supporting theory, all these homotopy methods suffer from one fatal flaw. A Jacobian matrix somewhere could become singular, and the computer implementation would either experience great difficulty or the method would fail completely (the more common occurrence)'. The truth of this manifests itself via a singular LF Jacobian in Algorithm Coordination.

The experience gained via the numerical simulations reported in Chapter IV suggest that, most often singularities occur either

- 1) Immediately following an exchange of variables specifications in Algorithm Update, or
- 2) After a few predictor corrector steps in Algorithm Phase I.

Cases 1 and 2 above, will be discussed separately under Type I and Type II singularities.

3.4.2.1 Type 1 Singularity

Type I singularities occur due to the nonconventional, but mathematically valid LF specifications within Algorithm Update. If present, such singularities manifests itself immediately following the invocation of Algorithm Phase I. Such singularities are difficult to resolve, due to the generally multiple nature of the specifications. Had the multiplicity been unity, the last exchange would be the most likely candidate causing this singularity, i.e., the nullity of the LF Jacobian would be unity.

Although single specification updates, i.e., considering one functional violation at a time, usually the most violated, possess this advantage over multiple specification updates in which all functional violations are considered at the same time, the former would result in many more Phase I iterations. This will make the overall solution process numerically more expensive. Multiple specification updates converge faster, as the working set converges to the optimal active set quickly. However, the price one pays for multiple specification updates is the risk of facing perhaps, a difficult to resolve degeneracy.

The numerical experience gained in the preparation of this thesis suggests that, such degeneracies depend on the characteristics of the power system. Further, large systems at high load levels tend to run into degeneracies faster than smaller systems. Judging by the computer times reported in Chapter IV, the algorithm proposed here does not seem to be at fault, as the bench mark algorithm used in this thesis, i.e., MINOS/AUGMENTED, Ver. 4.0,

[Murtagh et al., 1977, 1978a&b, 1980 a&b, 1981], [Preckel 1980], [Saunders 1977], also runs into convergence difficulties under the same conditions.

In order to resolve a degeneracy, one has to know the nullity of the LF Jacobian. This would involve a singular value decomposition, [Gofub et al., 1983]. Given the nullity n of the LF Jacobian, one has to investigate the various combinations among the ξ -variables, taken n at a time, which when incorporated into the LF jacobian is capable of removing this singularity: Such an action is justified by the fact that, just before the new variable specifications, the LF Jacobian was non-singular. If this process fails, one concludes that the problem is degenerate beyond the current operating point.

The procedure involved is better understood via Algorithm Resolve Singularity, which will be proposed next

3.4.2.1.1 Algorithm Resolve Singularity

- a) Identify the nullity n of the LF Jacobian via a singular value decomposition.
- b) Enumerate the different combinations that would arise, if the ξ -variables, to become dependent at the beginning of the current Phase I iteration were permuted n at a time.
- c) Try each combination in b, one at a time in the LF specification until the singularity is removed.
- e) If step (c) fails STOP, as the problem is infeasible.

Note that variations to the above algorithm are also possible

Next, attention focuses on Type II singularity

3.4.2.2 Type II Singularity

Type II singularities manifests itself within algorithm Phase I. Such a singularity could either mean that the homotopy path is bending back on itself (see The Implicit Function Theorem, section 2.3.5.4), or that the homotopy path has reached a boundary of the LF feasibility region. The latter conclusion is made via the theorem proposed by Jarjis [1980].

The former reason for the singularity is eliminated on grounds that, generally, the conditions for the stability of the continuation process, specified by Garcia and colleagues [1981], holds true for the OPF problem. This has been observed to be true in past work of a related nature, reported by Huneault and colleagues [1984].

Hence, the singularity of the LF Jacobian can be implicated only with the boundary of the LF feasibility region (or the steady state stability region). Figure 3.6 illustrates such a situation.

Further numerical tests are needed at this juncture, before any conclusive remarks can be made. This is necessary for, by changing the space of injections and states, i.e., $z-x$, in the LF problem, one can in most cases circumvent this singularity. It must be noted that the work of Jarjis, i.e., Jarjis [1980], is carried out in the conventional fixed LF space, by assuming on-line controllers capable of producing asymmetric elements in the LF Jacobian to be absent. The presence of on-line controllers in the present work can be exploited in situations of this nature, by changing the space of states x , by swapping elements with the ξ -variables, to remove this singularity. Numerical experience has shown this to work. If the swapping action does not remove the singularity, one has to conclude that the problem is degenerate, and that the optimum is infeasible.

Algorithm Resolve Singularity, proposed before in section 3.4.2.1.1, needs to be invoked for corrective measures in a situation of this nature. For success, one needs to assume that, the newly added ξ -variables at the beginning of the current Phase I iteration, are within bounds at the point of singularity. If this assumption were not valid, the resolution of the singularity becomes tedious, as one has to return to the beginning of the current Phase I iteration, to restart with a new set of exchanged ξ -variables. Unfortunately, the multiple possibilities prevalent under such circumstances, makes such a procedure almost impossible to complete within a given time frame.

Singularity due to the intersection of the homotopy path and the boundary of steady state stability

Figure 3.6

3.4.3 Loss of Degrees of Freedom in Algorithm Update

Recall that, in Algorithm Update the dimension of the ξ -variables control the degrees of freedom for subsequent Phase I iterations. A loss in the degrees of freedom can occur, when the number of violated Type II functional variables, exceeds the number of ξ -variables. At the onset of the optimization, the dimension of the ξ -variables is given by;

$$\text{Dim. of vector } \xi = NG + ND + NSH - 1 \quad \dots(3.11.a)$$

Where the following definitions apply, i.e.,

NG: Number of generation buses,

ND: Number of on-line controllers, i.e., TCUL transformers,

NSH: SVC devices,

For each specified Type II functional, the dimension of vector ξ reduces by unity. As mentioned before, each member of the ξ -variables is a candidate, to be swapped into the state vector x . Hence, it is clear that, when the

number of Type II functional variables exceeds the dimension given by equation 3.11.a, a degenerate situation will arise, and the Algorithm Phase II has to be terminated prematurely.

In a well designed power system, seldom are many Type II functional variables active at a normal optimum. Hence one can make the reasonable assumption that such an occurrence is generally rare. This is further justified by the fact that, results of commercial algorithms reported by Burchett et al., [1982a, b, 1984], El-Kady et al., [1986] do not concern themselves with line-flow limit violations.

Section 3.4 has presented all candidate degeneracies to be experienced in implementing Algorithm Phase II. Often heuristics can be incorporated to accelerate the convergence process. As in all heuristics, one takes the risk of misclassification, i.e., a potentially feasible solution can be classified as being infeasible. The sections to follow will present the heuristics tried successfully in this thesis. All failures in Algorithm Phase II are verified using MINOS Ver. 4.0, with satisfactory results to augment the confidence on the applied heuristics.

The following sections will present these heuristics, after the basic ideas underlying the rules are presented.

3.5 Heuristics to Overcome Degeneracies

Heuristics are often designed to circumvent tedious numerical computations. If successful, the overall solution process can be accelerated many times. The nature of the heuristics are such that, one sacrifices consistency for speed, i.e., the accelerated convergence may not apply equitably among all problem categories. However, this becomes a minor set back considering the magnitude of problems that can be solved with a given set of rules. Further, the more the user gets used to the underlying rules, via experience gained in simulation trials, the better are the user's judgments to overcome certain commonly encountered constraint violations. This would aid in the selection of correct system parameters to be altered, such that in the best

possible attempt, the complete OPF solution can be obtained in just one Phase I iteration.

The basis for the heuristic rules are the general solution characteristics of the full fledged OPF problem. These characteristics will be presented next, and the rules will be developed subsequently.

3.5.1 General Observations on OPF Solution Characteristics

Test simulations of the full OPF, performed and verified on systems of varying dimensions reveal the following general observations about the vector of ξ -variables, i.e.,

1. If capacitive SVC devices are within bounds at the termination of the first Phase I iteration, they will almost always reach their upper bounds at the constrained optimum. However, should they be at an upper bound, they will tend to remain here at the constrained optimum.
2. If voltages at generation buses (V_{pv} 's) are within bounds at the termination of the first Phase-I iteration, then they tend to remain free at the constrained optimum. However, if such voltages reach their upper bound at the termination of the Phase I iteration, then they tend to remain there at the constrained optimum.
3. No conclusive remarks of the above nature are possible with respect to TCUL transformers.

These observations certainly narrow down the choice of candidate ξ -elements, qualified for an exchange in Algorithm Update. This is due to reasons that, not much gain in computing is possible, immediately following Algorithm Phase I, if variables such as V_{pv} at a limit, or an SVC devices at a limit are chosen as exchange candidates, due to their likelihood of violating their operating limit if they are made dependent LF variables. It is always better to leave such variables within vector ξ , as they can be maintained within limits (or at a limit) as dictated by the optimization process via Algorithm Phase I.

The heuristic rules, to replace the exchange mechanism specified in step (g) of Algorithm Update will be presented.

3.5.2 Heuristics for the Exchange Mechanism in Algorithm Update

The heuristic rules, for swapping ξ -variables for x variables, is best given by the following priority list. This list is consulted at the termination of each Phase I iteration, and is such that variables higher up have a greater priority to be swapped back into x . The reasons underlying the hierarchy of this list will be given subsequently.

The Priority List

1. Voltages at load buses (V_{PQ} 's).
2. Voltages at generation buses (V_{PV} 's) within bounds.
3. TCUL transformers within bounds.
4. SVC devices within bounds.

Numerical experience suggests that, the LF Jacobian has the best condition¹¹ number whenever all voltages at PQ buses are incorporated into the state vector x . Hence, the greatest priority given to these variables. Further for reactive power violations at PV buses, the best condition number is possible by incorporating the voltage at this bus into x . However, as explained before, this is possible only if the corresponding voltage is within bounds. If this voltage is at a bound, the nearest voltage, in a geographic sense, at a PV bus must be chosen.

Different combinations of TCUL devices produce marginal difference to the condition number of the LF Jacobian. Hence in the present implementation, the device higher up on the priority list, within bounds, is chosen to enter the state vector x .

A similar observation is made with respect to capacitive SVC devices. However, since these devices tend to reach their limiting values at the

¹¹ The LSQR algorithm, [Paige 1982a,b], is used in estimating the condition number of the LF Jacobian.

constrained optimum, they do not make good exchange candidates. Hence, lowest priority is given to such variables.

The simulations of Chapter IV are based on these rules. The wide variety of cases reported here, suggest that the rules are adequately justified. Cases suggested as infeasible by the proposed algorithm, remain infeasible when a solution is attempted via MINOS Ver.4.0.

The following sections present, the pertinent features of a FORTRAN-77 code, developed to evaluate the performance of the herein proposed algorithm, versus the general purpose Stanford University MINOS Ver 4 0

3.6 The Computer Code

All computations are implemented in Double Precision. This is essential considering the convergence tolerances used in the study. Different phases of the algorithm employ different tolerances. The magnitudes of these are chosen carefully, after performing many simulations on systems of widely varying dimensions. The emphasis throughout is to reach the best possible solution for a given data.

This project being the first attempt of a path following scheme to solve the full fledged OPF, approximations to enhance speed are not investigated. Hence enhancements based on approximations may be possible, and are left for future interested persons.

For a meaningful comparison, the convergence tolerances are comparable with those employed by MINOS. These are as follows, i.e.,

1. 10^{-6} pu MW for ϵ in Algorithm Lambda
2. 10^{-6} pu MW on MVR or A^2 for the LF iterations in step (b) of Algorithm Coordination.
3. 10^{-4} pu MW for ϵ in Algorithm Coordination.
4. 5×10^{-4} for ϵ in Algorithm Phase I.

These tolerances may be too tight for industry, which is principally interested, for practical reasons, in two to three significant decimal digits of accuracy, [Robichaud 1987]. Exceptions being persons from GE, where tolerances of 10^{-9} have been used in their study on a quadratically convergent OPF, [Burchett et al., 1984].

Tighter tolerances are considered essential to evaluate the performance, and convergence characteristics, of new algorithms on a theoretical framework. As a final remark, the nonlinear nature of the problem being addressed, and the limited precision for floating point operations, do indeed justify the need for reasonably tight tolerances, especially in work being developed for the first time.

The overall code was developed in modular form. The different modules represent the steps outlined for Algorithms Lambda, Coordination, Corrector, Phase I, Update, and Phase II to perform specific tasks and interact appropriately to solve the whole problem. In developing the computer code, careful planning and prior thought was given to the following, in order to make it as competitive as possible. The items given special consideration are embedded among the different modules, and are identified below, i.e.,

1. The data file,
2. The structure of the LF Jacobian,
3. Elements of the LF Jacobian, and auxiliary vectors,
4. The solver for the set of LF equations.

The sections to follow will outline the relevant details of the four items above.

3.6.1 The Data File

The user provides a data file, for each case to be studied. Appendix A gives the general structure for a sample data file, where the data for Example 1.6, i.e., one particular case of a six bus system simulated in Chapter IV, is used as an example. All elements herein are clearly identified, and are

in free format. This file is read once through completely, and all elements are stored in linked-lists for maximum flexibility and economy, [George et al., 1981], [Enns 1975]. Additional indexing overhead arrays are kept to a minimum. Certain important points to note in the preparation of this file will now be discussed.

The buses in the system can be numbered arbitrarily in ascending order. However, one bus with generation capabilities, i.e., real and reactive, or reactive only, is given No. 1, and becomes the slack (and reference) bus. Transmission lines are identified by the buses defining the line, and are arranged in ascending order of line numbers. Line numbers themselves are assigned by drawing a cut-set arc about each bus taken consecutively in ascending order. A similar strategy is employed in numbering the on-line controllers. SVC devices are numbered sequentially in ascending order of bus numbers.

In providing data for the generation units, each unit is considered separately, rather than combining all units per plant to form an equivalent machine. This format is considered more flexible in areas of expansion planning studies (or experimentation), and requires a minimum amount of computation to be performed by the user. Since most generation data is provided in the form of heat rate information, eg., [IEEE Reliability Test System 1979], [El-Hawary et al., 1979], the only computation needed by the user would be to convert this information to the relevant c-a-b cost coefficients given in equation 3.4.a.

The pertinent features of the general LF Jacobian, constructed in this thesis, will be presented next.

3.6.2 The Structure of the LF Jacobian

The specifications in steps (c&g) of Algorithm Update, makes it necessary to have a nonconventional, and flexible format for the LF Jacobian.

For purposes of efficiency in coding, the fixed sub structure of the Jacobian must be separated from its variable counterpart. This will involve

some sacrifices to be made with respect to the structural symmetry, and non-trivial diagonals, present in the conventional Jacobian. The Jacobian structure, to best suit the present needs is given below.

	Rows	Columns
1	P at PQ Buses	θ_{PQ}
2	P at PV Buses	θ_{PV}
3	Q at PQ Buses	V_{PQ} (if unspecified)
4	Specified Q	V_s, V_{PV} (if unspecified)
5	Specified I_L^2	t, ϕ, B_{sh} (if unspecified)

Here, the rows identify elements forming the injections z , and the columns correspond to the dependent LF variables. The elements forming the fixed substructure of the Jacobian is identified in bold, with the exception of t, ϕ, B_{sh} in row 5. The property of structural symmetry, and non-trivial diagonals, is preserved only in the block identified by the row-column combination in 1&2 above. The remainder of the Jacobian can assume any arbitrary structure, and variable dimensions. However, this is of little consequence, as what really matters is the non-singularity of the LF Jacobian.

The discussion next focuses on the elements of the LF Jacobian.

3.6.3 Elements of the LF Jacobian and Auxiliary Vectors

For reasons given in section 2.1.1, the elements of the LF Jacobian are computed in polar coordinates. The trigonometric terms involved are such that they can easily be partitioned into terms which occur repeatedly, both within the Jacobian and outside, and others which do not. In the computer code, care has been exercised in identifying these common terms, so that repetitions in computations are totally eliminated. Further matrix-vector products are carried out only when needed, and all trivial multiplications are indexed and eliminated.

Appendix B gives analytical expressions for all terms appearing in the

Jacobian, and various auxiliary vectors. Further, the models for TCUL devices used in this thesis are also presented in this appendix.

Finally, the discussion focuses on solving the set of LF equations 3.3 b, in Algorithm Coordination

3.6.4 The Solver for The Set of LF Equations

The greatest computational burden in the overall algorithm proposed herein, lies within the solver for the set of LF equations 3.3 b, needed in Algorithm Coordination. Since large scale system simulation, plays a mandatory role in modern software of the nature addressed in this thesis, the use of special coding techniques taking sparsity and ordering to minimize fill-in become inevitable, if not mandatory, [Stott et al, 1986], [George et al, 1981]. Such techniques are well known for their computational efficiencies, [ibid].

An overview of Algorithm Phase I will reveal that, Algorithm Coordination needs to be invoked many times in the overall solution process. This brings on a further desirable feature, namely rapid solution to the set of LF equations, which makes it almost mandatory to use a direct method, rather than an iterative method. The solutions due to such methods are exact, disregarding rounding errors. Most direct methods are based on the principles outlined by [Tinney et al, 1967].

The nature of power system problems are such that, arithmetic precision is preserved almost always, in computations involving 32-bits or more, [ibid]. Hence, one need not be overly concerned about techniques used principally to preserve numerical precision. Further since the problem of preserving precision increases with 2by2, or 4by4 matrix structures, due to higher factorization effort, there is some advantage to be gained by using the 1by1 structure, i.e., the full LF Jacobian, [Stott et al, 1986].

Within Algorithm Phase I, the structure of the LF Jacobian, and the RHS vector of mismatches remain constant. However, their elements change as the iterations progress. Hence, much computational gain is possible with a code

which factorizes the LF Jacobian only once per Phase I iteration, and stores the non-trivial factors competitively, i.e., with minimum overhead. Further the code must easily adapt to solving the transposed Jacobian equations needed to compute the ITL's in step (b) of Algorithm Coordination.

The routine MA-28, from the Harwell Library of Routines, [Duff 1977], [Harwell User Guide 1977], meets almost all of the desirable features specified above. Hence it is used as the solver in this study. The method used here is a sparse variant of Gaussian elimination, in which the LF Jacobian is decomposed into factors using a pivotal strategy. This is designed to compromise between maintaining sparsity, and controlling loss of accuracy through round-off, [Duff 1977]. The algorithm initially tries to permute the given LF Jacobian into a lower triangular form. If successful, the solution becomes trivial. If this attempt fails, the sparse variant of Gaussian elimination is invoked.

There are built-in safeguards, to inform the user when the factorizations within a Phase I step require updating, and to highlight any potential numerical ill-conditioning.

CHAPTER IV

DOCUMENTATION AND ANALYSIS OF SIMULATION RESULTS

This chapter documents, and analyses, simulation results on a number of test systems varying in size up to 118 buses, 179 transmission lines, 68 controllable variables, and 62 generation units. Recommendations in the EPRI report [EPRI 1977] are utilized, wherever necessary, to complete any missing data specifications on transmission line current flow limits, and generation cost parameters. All data files used for simulations, and schematic diagrams for the 6, 10, 30 and 118 bus systems are given in Appendix C.

4.1 Simulations on a 6-Bus System

Simulation results on a 6-bus system will be presented next. The system dimensions, in brief, are as follows, i e.,

No. of Buses	- 6
No. of Transmission Lines	- 7
No. of On-Line Controllers ¹	- 3
No. of Shunt Controllers	- 3
No. of Generation (PV) Buses	- 2
No. of Load (PQ) Buses	- 3
No. of Generation Units	- 5

The discussion to follow, will commence with a study of results, resulting from simulations based on specification C.1.1 (example 1).

4.1.1 Simulations Based on Specification C.1.1

Table 4.1.a summarises the solution process, and table 4.1.b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 1 (specification C.1.1).

¹ Tap-changing transformers and phase-shifters.

Table 4.1.a
Summary of the solution process - Example 1

	Phase I (No.1)	Phase I (No.2)
Number of Restarts	1	0
Functional Violations	Q(1)	NONE
Variables Leaving x	NONE	NONE
Variables Entering x	V(1)	NONE

Table 4.1.b
Performance statistics of the RHCM and MINOS Ver. 4.0 - Example 1

	RHCM	MINOS
Number of Jacobian Evaluations	47	71
Exec Speed (CPU sec on Amdahl 5860)	0.380	1.150
Core Req'd. (M-bytes on Amdahl 5860)	0.312	0.616

The highlights of table 4.1 a will now be discussed.

In the solution process, two Phase I iterations are involved. In the first, one restart is required for convergence, i.e., to bring down the absolute value of the largest LM of the free controllable ξ -variables below a specified precision ϵ . In the second invocation of Algorithm Phase I no restarts are required.

The termination of the first Phase I iteration identifies one violation among the Type II functions, and no violations among the Type I functional variables. The Type II violation is due to the reactive generation at bus(1), i.e., Q(1) being above its upper limit. In order to remove this violation, Algorithm Phase II appends Q(1) to the set of injections, after specifying it at its upper limit, and changes of the role of the voltage at bus(1), i.e., V(1), from that of a controllable variable, to that of a dependent LF variable, i.e., a Type I function. The subsequent invocation of Algorithm Phase I removes all violations, and provides the optimal solution.

At the optimum, Q(1) remains at its upper limit, and V(1) within bounds.

For purposes of comparison², the final solutions due to the RHCM and MINOS are presented next.

4.1.1.1 Comparison of Results

The results summarized below in table 4.1.c give the values of all key variables at the final solution, obtained from the RHCM and MINOS.

Table 4.1.c
Solution summary - Example 1

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Lwr.Lmt	02.400	02.400
Unit(2)	Lwr.Lmt	68.950	68.950
Unit(3)	Lwr.Lmt	54.250	54.250
Unit(5)	Free	81.445	81.443
React. Generations (MVAR):			
Q(1)	Upr.Lmt	06.000	06.000
Q(2)	Free	10.027	10.351
Q(4)	Free	36.564	34.898
Bus Voltages (pu V) & Ang:			
V(1)	Free	1.051/0.000	1.051/0.000
V(2)	Upr.Lmt	1.100/0.195	1.100/0.194
V(3)	Free	1.028/0.108	1.025/0.108
V(4)	Upr.Lmt	1.100/0.118	1.100/0.118
V(5)	Free	0.991/-0.033	0.991/-0.033
V(6)	Free	1.025/0.006	1.025/0.006

² In obtaining these results, identical computing environments are used, i.e., job class, priority, and compiler used are the same, and the jobs have been executed during a time in which the computer is lightly loaded.

Table 4.1.c cont.

Tap & Phi (rad) Position :			
$\phi(1)$	Free	-.032	-.030
$t(2)$	Free	0.938	0.933
$t(3)$	Free	0.999	0.998
Sht. Controllers (pu mho):			
$B_{sh}(1)$		0.037(FR) ³	0.050(UL) ⁴
$B_{sh}(2)$	Upr.Lmt	0.050	0.050
$B_{sh}(3)$	Upr.Lmt	0.050	0.050
Optimal Cost in (\$/hr):			
		2773.62	2773.60

Note: Even though MINOS places shunt controller $B_{sh}(1)$ at its upper limit, its LM is very small, implying that this device is on the verge of being released. The difference in setting brings about only a marginal change to the objective. Further since the line currents on all lines fall within their limiting values they are not reported in table 4.1.c.

Experimental evidence shows that, the number of restart iterations resulting during the initial invocation of Algorithm Phase I, are generally influenced by the initial settings of voltages, on-line and shunt controllers. The computer code allows only the initial voltage specifications to be set by the user, as it has been found that off nominal initial specifications for on-line controller and shunt controller positions tend to cause convergence difficulties when the LF module is invoked for the first time. In this example, since the voltages at generation buses 2, and 4, reach their upper bounds at the terminal optimum, the number of restart iterations in the first invocation of Algorithm Phase I can be minimized by initializing these voltages at their upper limits, and maximized by initializing them at their lower limits. It must be noted that the optimal solution to the relaxed OPF

³ FR. Used as an acronym for 'Free'

⁴ UL: Used as an acronym for 'Upper Limit'

problem, obtained at the termination of Algorithm Phase I invoked for the first time, is independent of the initial voltage (variable) specifications.

The following discussion considers two cases. Case I corresponds to the situation in which all voltages have an initial specification as given in section C.1.1, while Case II corresponds to the situation in which voltages $V(2)$ and $V(4)$ are initially specified at their lower limit. Note that in cases I and II, the initial specification for other voltages at PQ buses, on-line and shunt controllers remain as given in C.1.1.

Case II provides an opportunity to study the convergence characteristics of Algorithm Phase I, as it involves a larger number of restart iterations than Case I, caused by the starting solution being further from the true optimal solution. The section to follow will discuss the convergence characteristics of Algorithm Phase I.

4.1.1.2 Convergence Characteristics of Algorithm Phase I

The convergence of Algorithm Phase I is determined by the sequence $\{(\alpha_1^M)_K\}$, where α_1^M is the absolute value of the largest LM (the displacement error ϵ_d) computed over all free ξ -variables at $\theta=1$, i.e., the terminal point of a continuation process (restart iteration) within Algorithm Phase I. As discussed in section 3.2.1.4.2, Algorithm Phase I is terminated when $\alpha_1^M \leq \epsilon$, for some value of K , where K is the restart iteration count.

As discussed before, the value of K is influenced by the initial voltage (variable) specifications. For Case I $K=1$, and for Case II $K=5$. This observation indicates that, further the initial starting point from the optimum of the relaxed OPF being solved, the greater the number of restart iterations.

The sequence $\{(\alpha_1^M)_K\}$, corresponding to Case II is presented in table 4.1.d. This will be analyzed through Algorithm Rate-of-Convergence which is described in Appendix D, in order to estimate the rate of convergence of Algorithm Phase I. The terminal point of the sequence, i.e., $(\alpha_1^M)^*$ is assumed to be 5×10^{-3} , as this is the value used for ϵ in this study.

Table 4.1.d
Elements of the Sequence $\{(\alpha_1^M)_K\}$ - Example 1, Case II

K	$(\alpha_1^M)_K$
0	$0.7767 \cdot 10^{-1}$
1	$0.2331 \cdot 10^{-1}$
2	$0.4201 \cdot 10^{-1}$
3	$0.4482 \cdot 10^{-1}$
4	$0.1907 \cdot 10^{-1}$
5	$0.4024 \cdot 10^{-2}$

Figure⁵ 4.1, is a plot of the variations in the asymptotic error constant⁶ γ , for differing values of the asymptotic rate of convergence r . The salient features of this figure will be discussed next.

Observations:

The characteristics of figure 4.1 suggests an asymptotic rate of convergence r in the vicinity of 1.5, suggesting super linear convergence, [Gill et al., 1981]. This can be seen as follows, i.e., observing the family of curves towards $K=5$, i.e., the terminal point of the sequence $\{(\alpha_1^M)_K\}$, one sees that for values of the asymptotic rate r greater than 1.5, the tendency is for the asymptotic error constant γ to violate the upper bound of inequality D.1.a (Appendix D), by indicating a tendency to attain a large value for $K>5$. In order to observe this explicitly, one needs to have a value for ϵ , smaller than $5 \cdot 10^{-3}$. However, values smaller than $5 \cdot 10^{-3}$ for ϵ are not recommended, as they tend to be numerically wasteful, i.e., the higher restart iteration counts bring about improvements which for all practical purposes become insignificant.

It has been observed that the redefinition of the controls u , and the states x following each invocation of Algorithm Phase I, influences the number of

⁵ Note that all figures appearing within this chapter are placed at the end of this chapter.

⁶ The asymptotic error constant γ , and the asymptotic rates of convergence r are defined in Appendix D.

restart iterations resulting in the subsequent invocation of Algorithm Phase I. In the example under consideration, Algorithm Phase II, and hence Algorithm Update needs to be invoked only once to reach the optimal solution, which indicates that $Q(1)$ has been correctly identified to be active at the optimal solution. In the subsequent Phase I iterations, no restarts are required, i.e., $K=0$. This implies that the optimal solution to the overall problem has been very close to that of the relaxed problem. This is further confirmed by the optimal costs of the relaxed and complete problems, i.e., 2773.13 \$/hr., and 2773.62 \$/hr.

The discussion next analyses the break points which result for Cases I and II, during the progress of the continuation processes within each invocation of Algorithm Phase I.

4.1.1.3 Study of Break Points

Algorithm Phase I, once invoked, is capable of systematically identifying the active set of all variables. Recall that Algorithm Coordination determines the active set of vector P_g , (generations), while Algorithm Corrector identifies the active set of controls in vector ξ , (ξ -variables) through the restart homotopy continuation algorithm. At the termination of Phase I, the active set of functions, i.e., Types I and II are updated, if necessary, via invoking Algorithm Phase II. Within Algorithm Phase I, one needs to monitor explicitly only the break points corresponding to the active set of ξ -variables, as the active set of the remaining controls will be correctly identified.

The reduced number of variables to be monitored is considered to be a distinct advantage here, as the monitoring of all power system variables simultaneously would be computationally prohibitive, with increased system dimensions.

At inception of each continuation process, within each invocation of Algorithm Phase I, all elements of ξ -variables are placed in the active ξ -set, via appropriate homotopic deformations applied to the boundaries of these variables. Subsequently, some of them are deactivated. The values of the

continuation parameter θ , at which these variables are deactivated mark the break points, and are explicitly monitored within Algorithm Phase I. Each break point of variable ξ_1 , influences the optimal trajectory of all other dependent, and independent variables. Such trajectories are implicitly updated by this algorithm, eg., the controls P_s are updated via Algorithm Coordination, and the dependent LF variables are updated within the load flow module. Recall that Algorithm Coordination, and the LF module are invoked within Algorithm Phase I, following each predictor step.

In example C.1.1, Case I, (where the initial starting point is closer to the optimal solution of the relaxed OPF being solved) requires one restart iteration within the first invocation of Algorithm Phase I, while Case II (where the initial starting point is further away from the optimal solution than in Case I) requires 5. In both cases Algorithm Phase I needs to be invoked twice prior to reaching the optimal solution of the complete OPF problem. The resulting break points are given in table 4.1.e.

Table 4.1.e

Summary of break points resulting in the first invocation of Algorithm Phase I - Example 1, Case I

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.03776	$t(3)$ is deactivated at setting 1.0048
	0.14873	$\phi(1)$ is deactivated at setting -0.0436
	0.16543	$V(1)$ is deactivated at setting 1.0717
	0.23419	$V(4)$ is deactivated at setting 1.1000
	0.58707	$B_{sh}(1)$ is deactivated at setting 0.0000
	0.63205	$t(2)$ is deactivated at setting 0.9328
	1.00000	Termination of the continuation process
1	0.00000	Beginning of the continuation process
	0.03342	$t(2)$ is deactivated at setting 0.9401
	0.05556	$t(3)$ is deactivated at setting 0.9979
	0.06349	$\phi(1)$ is deactivated at setting -0.0271
	0.06246	$B_{sh}(1)$ is deactivated at setting 0.0036
	0.32479	$V(1)$ is deactivated at setting 1.0830
	1.00000	Termination of the continuation process

At this juncture functional variable $Q(1)$ is above its upper limit. In order to contain this violation a re-specification of injection and dependent variables become necessary. This is achieved via invoking Algorithm Phase II, followed by Algorithm Phase I to solve the new problem under the modified variable specifications. The resulting break-points are given in table 4.1.f.

Table 4.1.f

Summary of break points resulting in the second invocation of Algorithm Phase I - Example 1, Case I

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.00616	$t(3)$ is deactivated at setting 0.9987
	0.01877	$\varphi(1)$ is deactivated at setting -0.0316
	0.03673	$t(2)$ is deactivated at setting 0.9382
	0.68556	$B_{sh}(1)$ is deactivated at setting 0.0370
	1.00000	Termination of the continuation process

Table 4.1.g

Summary of break points resulting in the first invocation of Algorithm Phase I - Example 1, Case II

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.13378	$V(1)$ is deactivated at setting 0.9900
	0.21526	$t(3)$ is deactivated at setting 1.0302
	0.22930	$\varphi(1)$ is deactivated at setting -0.0565
	0.32792	$V(4)$ is deactivated at setting 1.0342
	0.59020	$B_{sh}(1)$ is deactivated at setting 0.0000
	0.64002	$t(2)$ is deactivated at setting 0.9320
	1.00000	Termination of the continuation process

Table 4.1.g cont.

1	0.00000 0.02599 0.26315 0.26530 0.26701 0.40081 0.45850 1.00000	Beginning of the continuation process $\varphi(1)$ is deactivated at setting -0.0478 $t(3)$ is deactivated at setting 1.0542 $B_{sh}(1)$ is deactivated at setting 0.0174 $t(2)$ is deactivated at setting 0.9904 $V(4)$ is deactivated at setting 1.0649 $V(1)$ is deactivated at setting 1.0470 Termination of the continuation process
2	0.00000 0.03804 0.07229 0.14238 0.26689 0.32755 0.65335 1.00000	Beginning of the continuation process $V(1)$ is deactivated at setting 1.0442 $\varphi(1)$ is deactivated at setting -0.0290 $t(3)$ is deactivated at setting 1.0177 $B_{sh}(1)$ is deactivated at setting 0.0113 $t(2)$ is deactivated at setting 0.9541 $V(4)$ is deactivated at setting 1.0892 Termination of the continuation process
3	0.00000 0.00080 0.00401 0.00472 0.01457 0.09919 0.42062 1.00000	Beginning of the continuation process $\varphi(1)$ is deactivated at setting -0.0292 $t(3)$ is deactivated at setting 1.0171 $V(4)$ is deactivated at setting 1.0886 $B_{sh}(1)$ is deactivated at setting 0.0110 $t(2)$ is deactivated at setting 0.9482 $V(1)$ is deactivated at setting 1.0712 Termination of the continuation process
4	0.00000 0.00317 0.00513 0.02687 0.06960 0.25545 1.00000	Beginning of the continuation process $t(2)$ is deactivated at setting 0.9480 $\varphi(1)$ is deactivated at setting -0.0278 $V(1)$ is deactivated at setting 1.0682 $t(3)$ is deactivated at setting 1.0077 $B_{sh}(1)$ is deactivated at setting 0.0242 Termination of the continuation process
5	0.00000 0.00701 0.06457 0.14202 0.21603 0.38649 1.00000	Beginning of the continuation process $\varphi(1)$ is deactivated at setting -0.0262 $t(3)$ is deactivated at setting 0.9997 $B_{sh}(1)$ is deactivated at setting 0.0185 $t(2)$ is deactivated at setting 0.9335 $V(1)$ is deactivated at setting 1.0826 Termination of the continuation process

The situation at this point is theoretically identical to that which prevails at the end of table 4.1.e, since the relaxed OPF sub-problem is identical in both instances.

Table 4.1.h

Summary of break points resulting in the second invocation of Algorithm
Phase I - Example 1, Case II

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.00842	t(3) is deactivated at setting 0.9987
	0.01923	t(2) is deactivated at setting 0.9385
	0.02444	$\varphi(1)$ is deactivated at setting -0.0316
	0.03915	B _{sh} (1) is deactivated at setting 0.0200
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the first time, the variables contained in vector ξ are,

Voltages : $V_S(1)$, $V_{PV}(2)$ & $V_{PV}(4)$

On-line Controllers: $\varphi(1)$, t(2) & t(3)

Shunt Controllers: B_{sh}(1), B_{sh}(2) & B_{sh}(3)

It must be noted that each time the continuation process de-activates a variable, it no longer belongs to vector ξ , i.e., generally within each restart iteration count, the dimension of vector ξ decreases as θ approaches unity as more and more variables get deactivated. Thus the computational burden (within Algorithm Corrector), for computing and checking for conflicts among the ξ -variables and their LM's, decreases as θ approaches unity.

Recall that in Case I, voltages at generation buses 2 and 4, i.e., V(2), and V(4), have an initial specification equal to their optimal settings, i.e., a value equal to their upper bounds. Table 4.1.e which summarises the break points corresponding to Case I indicates that, for restart iteration count K=0, and the first invocation of Algorithm Phase I, V(2) remains active at its initial (optimal) setting, while V(4) gets released from its optimal setting at $\theta=0.23419$. However during the next restart iteration K=1, V(4) remains active. Note that if a ξ -variable remains in the active set in which it is placed just prior to invoking the continuation process ($K \geq 0$), the

nature of the homotopy corresponding to this variable is such that it is bound to take it to an appropriate limiting value at $\theta=1$. Recall that, should none of the controls from vector ξ be deactivated, the optimum of the relaxed OPF problem will be reached once the continuation parameter θ reaches unity, for any restart iteration count $K=0$. Further, the homotopy path generated for all controls in vector ξ will be exact. However this almost never happens as such a situation would result in a degeneracy should functional violations prevail at $\theta=1$. Such cases have been discussed before in Chapter III.

In Case II, voltages at generation buses 2 and 4, i.e., $V(2)$ and $V(4)$, are specified initially at their lower bound of 0.99 pu. Table 4.1.g which summarises the break points corresponding to Case II indicates that, for restart iteration count $K=0$, and the first invocation of Algorithm Phase I, $V(2)$ remains in the active set. This enables $V(2)$ to move to its optimal setting of 1.1 pu, at the continuation parameter θ reaches unity. However, under these conditions voltage $V(4)$ is released from the active set after increasing its value up to 1.0342 pu, at $\theta=0.32792$. In the subsequent restart iterations, i.e., $K=1, 2$ and 3 , $V(4)$ is released from the active set at values of 1.0649, 1.0892 and 1.0886 pu. Note that $V(2)$ remains active at 1.1 pu (optimal setting) for $K=1, 2$ and 3 . During the iteration corresponding to $K=4$, $V(4)$ reaches its upper bound of 1.1 pu (optimal setting), and remains in this position for the subsequent restart iteration. Since both $V(2)$ and $V(4)$ are at their optimal settings, and the other variables close to their optimal settings, at the beginning of restart iteration $K=5$, convergence is achieved as the continuation parameter reaches unity.

Shunt controllers $B_{sh}(2)$ and $B_{sh}(3)$ reach their optimal settings (upper bounds of 0.05 pu), during $K=0$, for both Cases I and II. Further, it must be noted that during the second invocation of Algorithm Phase I (tables 4.1.f and 4.1.h), $V(1)$ becomes a dependent LF variable, due to the addition of $Q(1)$ among the set of injections. Hence neither $V(1)$ nor its LM are explicitly monitored, since all dependent variables are ignored when Algorithm Phase I is invoked.

Theoretically, the terminal point following the first invocation of Phase I must be independent of the initial specifications. As evidenced in tables 4.1.e, and 4.1.g, this is found to be true except for a small error corresponding to the setting of $B_{sh}(1)$. This error is subsequently reflected in tables 4.1.f and 4.1.h, which are otherwise identical.

Experience shows that by increasing the precision ϵ , it is possible to bridge the error. However, this increases the computational burden, as it involves more restart iterations, but with little to no improvement of practical significance to the optimal cost.

The discussion next considers the scatter pattern observed for the break points, i.e., number of break points occurring per interval increment in the continuation parameter, during the progress of restart iterations within Algorithm Phase I. Recall that the continuation parameter is incremented in steps of 10 (section 3.2.1.4) and is such that the first incremented value is 10^{-4} . The analysis to follow will also serve to justify this choice.

4.1.1.3.1 Scatter Pattern of Break Points

Figures 4.2 and 4.3 give the scatter pattern of break points for Cases I and II respectively, i.e., the plot of the interval θ , and the corresponding number of ξ -variables that become inactive during this interval for different invocation counts of Algorithm Phase I.

Recall that each time Algorithm Phase I is invoked, there will be varying number of restart iterations necessary, before convergence actually takes place with respect to the relaxed OPF problem being solved. Further, each restart iteration uses the solution of the previous restart iteration as the solution to be tracked, and increments the continuation parameter θ from 0 to 1 of the homotopically deformed relaxed OPF problem, which remains the same until Phase I is completed. Each time θ is incremented, Algorithm Corrector must be reinvoked to move the operating point onto the load flow manifold, and resolve any conflicting situations (update the active ξ -set) which may arise at the new operating point among the ξ -variables and their Lagrange Multipliers. The only features which change per restart iteration

are the homotopies which define the problem deformations, as the homotopies are dependent on the variable settings following each completed restart iteration, i.e., upon reaching $\theta=1$

Following each completion of Algorithm Phase I, if a change in variable specification becomes necessary, i.e., should the roles of the control and dependent variables change in order to maintain a dependent variable within bounds, the needed changes are implemented via invoking Algorithm Phase II, and a new starting solution for the subsequent Phase I iteration is established by invoking Algorithm Coordination. This is the only instance where the solution at the termination of a previous continuation process, does not qualify as a starting solution for the subsequent continuation process. Note that in obtaining the new starting solution to conform with the updated variable specifications, θ remains at unity, and the control variables in vector ξ do not change their settings. Only the dependent (Type I) variables change. Further, the updated solution may correspond to one of lower or higher losses, and correspondingly provide a lower or higher operating cost than at the previous operating point. Once the new operating point is obtained successfully, Algorithm Phase I is invoked to solve a new relaxed OPF problem, under the new variable specifications.

Tables 4.1.e to 4.1.h give the break-points occurring during the optimization process for cases I and II. In both figures 4.2 and 4.3 the total number of ξ -variables amount to 9 during the first invocation of Phase I, and reduce to 8 in the second, since $V(1)$ becomes a dependent LF variable. Further, among the 9 ξ -variables carried in the first invocation of Phase I, 5 become inactive, and among the 8 ξ -variables carried in the second invocation of Phase I, 4 become inactive at the termination of the respective Phase I iterations.

A common pattern can be observed in both figures 4.2 and 4.3. During restart iteration, count 0, the majority of the break points occur in the last interval of θ , i.e., 0.1 to 1. However, for higher restart iteration counts, the break points occur within smaller interval increments of θ , giving improved estimates for these variables, i.e., smaller the interval size, greater the accuracy of estimates. This is reflected numerically within the

sequence $((\alpha_1^M)_K)$, (see table 4.1.d which corresponds to Case II), where $(\alpha_1^M)_K$ is the absolute value of the largest LM computed among all free ξ -variables. This is a very desirable result, and serves to justify the choice of a logarithmic interval step size for θ . Had θ been incremented linearly, the errors introduced by the linear interpolation would have slowed down the overall convergence.

The value for ϵ used in this study is 5×10^{-3} . A smaller value for ϵ would shift the distribution further to the left with very marginal improvement to the objective, but with increasingly better values for the LM's of free ξ -variables. This observation serves to demonstrate that the objective is almost insensitive to about the 4th and higher significant figures of the ξ -variables. Hence, attempting to improve the setting of the free ξ -variables beyond the 4th significant digit would be numerically wasteful, and is not recommended.

Viewed alternately, figures 4.2 and 4.3 illustrate the function of the restart iterations in shifting the ξ -variables towards their optimal settings. This is prominently featured in the latter figure, where it is seen that with increased restart iteration counts better estimates are provided for the free ξ -variables, due to the increasingly smaller intervals in which they are estimated.

Ideally the scatter pattern for the second invocation of Phase I, shown by the hatched lines in figures 4.2 and 4.3, must be identical for both Cases I and II, since the problem being solved is the same, namely the relaxed OPF under the same set of control and dependent variable specifications. The small difference arises due to variable $B_{sh}(1)$, shunt controller at bus(1), becoming inactive during the 5th interval in Case I, and 4th interval in Cases II. This is not considered a serious error, as the effect of this is almost not felt among the objective and other dependent and independent variables, (see Comparison of Results in section 4.1.1.1).

In Cases I and II, the largest LM computed among all inactive ξ -variables is due to $t(2)$. The break point of this variable is estimated in the 4th interval. The observation is that, not all variables whose break points are

estimated in higher order intervals have high LM's. Figures 4.2 and 4.3 suggest that it is only necessary to accurately identify the optimal settings of a few ξ -variables in order to get the remainder to converge to their optimal settings.

The study next considers the variation to the optimal cost from the point where the co-ordination equations converge for the first time, until the optimal solution is reached.

4.1.1.4 Trajectories of the Optimal Cost, Generation and Incremental Transmission Losses

Generally as the optimal solution is tracked via Algorithm Phase I, the computed optimal cost draws a path of its own. This will constitute the trajectory of the optimal cost. In tracking the optimal solution, Algorithm Corrector needs to be invoked in order to keep the operating point on the load flow manifold, and to resolve the conflicting situations which may arise among the ξ -variables and their LM's, i.e., update the active set corresponding to the ξ -variables. Figure 4.4 gives the plot of the computed optimal costs, corresponding to specification C.1.1 (example 1), each time control exits from Algorithm Corrector, i.e., each time the operating point is moved onto the LF manifold, or a conflict is resolved successfully (i.e., active ξ -set is updated). Additional scales are provided on top of figure 4.4, to identify the beginning ($\theta=0$) of each restart iteration, corresponding to each invocation of Phase I. Recall that in order for Phase I to converge, generally few restart iterations are needed, and within each restart iteration θ is incremented from 0 to 1, marking a continuation process.

Figure 4.5 gives the trajectory drawn by the optimal generations, and figure 4.6 the trajectories of incremental transmission losses corresponding to the various injections whose principal components are identified in the figure. The salient features of figures 4.4, 4.5, and 4.6 will be considered next.

In figure 4.4, the operating cost decreases from 2778.9 \$/hr (at $\theta=0$) to 2778.1 \$/hr (at $\theta=10^{-4}$) during the first increment in the corrector iteration.

tion count N . This is found to be generally true, since the starting point usually constitutes a higher cost, often due to non optimal variable settings. No change in the active set of ξ -variables occurs, but the ξ -variables do undergo a certain change to their initial setting as dictated by their respective homotopies, i.e., variables $V(4)$, $V(2)$, $t(3)$, $B_{sh}(3)$, and $B_{sh}(2)$ move towards their upper bounds, while variables $V(1)$, $t(2)$, $\phi(1)$ and $B_{sh}(1)$ move towards their lower bounds. It must be noted that if a ξ -variable were at its upper (lower) limit, and the optimization required it to move towards its upper (lower) limit, the nature of the homotopy is such that the variable will physically undergo no change to its setting. Due to this, variables $V(2)$, $V(4)$ and $B_{sh}(1)$ do not change their settings, i.e., the former two remain at their upper bound while the latter remains at the lower bound during this iteration. However, the subsequent movement of the remainder of the ξ -variables towards their optimal settings, causes the initial reduction in the operating cost, thereby confirming the correctness of the updates. However, it is possible to have instances in which the optimal cost can rise as the optimization progresses, due to different reasons. Such situations will be individually discussed, if and when they arise.

For subsequent corrector iterations, the change in the optimal cost is marginal, as the active set of ξ -variables remains almost static, although the variables do undergo changes to their settings, until $N=3$ ($\theta=10^{-2}$). For $N=4$ ($\theta=0.1$), the optimal cost decreases from 2778.1 \$/hr to 2778.0 \$/hr. Within this interval increment in θ , $t(3)$ is released from the active set, its new setting being obtained via a linear interpolation. Recalling the steps of Algorithm Corrector, the cost 2780.0 \$/hr corresponds to the specification in $t(3)$ being beyond its optimal setting. The collective influence of other variables in the active set causes the cost to decrease instead of rise. If the cost were updated to correspond to the optimal setting of $t(3)$, this cost should be even lower. However such updates are considered unnecessary, as the cost will be appropriately corrected in the next interval increment in θ . If such an occurrence were to happen in the last interval increment in θ then Algorithm Corrector would have been reinvoked to provide an updated solution. Corrections to the optimal cost

following each update is unnecessary as it tends to slow down the tracking process.

During the next corrector iteration ($N=5$, and the last interval increment in θ) the cost increases sharply from 2780.0\$/hr to 2784.5 \$/hr due to variables $V(1)$, $V(2)$, $\varphi(1)$, and $B_{sh}(1)$ not being released from the active ξ -set. The higher cost is due to these variables being moved to a value beyond their optimal settings, prior to the optimization process realising (via a conflict among these variables and their computed LM's) that they must be released from the active set. On releasing these variables at settings estimated via linear interpolations, and updating the cost (since the change to the active set occurred during the last interval increment in θ) the updated cost corresponding to the new settings ($N=6$, $\theta=1$) amounts to 2773.6 \$/hr, as expected lower than the cost of 2784.5 \$/hr which amounts to the cost which would have resulted had these variables been maintained in the active set. The decrease in costs act as a means of confirming that the active set of ξ -variables has been correctly updated. Note that the continuation parameter $\theta=1$ at this point, and marks the termination of the iterations corresponding to restart iteration count $K=0$.

At this point ($N=6$, $\theta=1$), ξ -variables $V(2)$, $B_{sh}(3)$, and $B_{sh}(2)$ are active at their natural (zero homotopic deformation) upper bounds, and variables $V(1)$, $V(4)$, $\varphi(1)$, $t(2)$, $t(3)$ and $B_{sh}(1)$ are inactive and within their natural bounds.

However, since the absolute value of the largest LM, taken over all free variables, i.e., $\alpha_{x_i}^M$, where index i corresponds to $V(4)$, exceeds the convergence tolerance ϵ , due to errors in the settings among the free ξ -variables, all the ξ -variables are placed once again in the active set, via homotopic deformations, in which the respective variable bounds are deformed to coincide with the variable settings, such that the current operating point becomes an optimum of a homotopically deformed relaxed OPF problem. This sets the stage for the beginning of the first restart iteration, i.e., $K=1$ ($\theta=0$), principally designed to bring $\alpha_{x_i}^M \leq \epsilon$.

The operating point at $N=6$, corresponds to $\theta=0$, for the restart iteration $K=1$, i.e., the optimum to be tracked in the successor iteration, i.e., $K=1$ becomes the optimum of the predecessor iteration, i.e., $K=0$. The corrections applied to the variable settings in the first interval increment in θ causes the optimum cost computed to decrease to 2773.0\$/hr. At $N=10$ ($\theta=10^{-1}$), variables $t(3)$, $t(2)$, $\varphi(1)$ and $B_{sh}(1)$ are released from the active set. However, this does not make a significant change in the cost, as the cost is now insensitive to the ξ -variables, due to most of them being in the vicinity of their optimal settings. At $N=11$ ($\theta=1$), $V(1)$ is released from the active set, and at $N=12$ the optimal cost is updated to correspond to the new setting of $V(1)$, obtained via a linear interpolation, in this last interval increment in θ . Recall that this step is necessary whenever the active set undergoes a change during the last interval increment in θ . At this point in the optimization, the absolute value of the largest LM among the free ξ -variables, i.e., $\alpha_{x_1}^M$ falls below ϵ , thereby marking the termination of Phase I iterations.

At this point ($N=12$, $\theta=1$), ξ -variables $V(4)$, $V(2)$, $B_{sh}(3)$, $B_{sh}(2)$ are active at their natural bounds (all upper bounds), and variables $V(1)$, $\varphi(1)$, $t(2)$, $t(3)$ and $B_{sh}(1)$ are within their natural bounds and satisfy the K-T conditions with respect to the relaxed OPF problem.

An analysis of the functional variables (Types I and II) reveals that the reactive generation at bus(1), i.e., $Q(1)$ a Type II function, exceeds its upper limit. In order to correct this violation, $Q(1)$ is specified at its violated bound and is appended among the set of injections, while $V(1)$, previously part of the controllable ξ -variables, is now made dependent (Type I functional) by the algorithm.

Recall that such exchanges are carried out according to the priority list specified in section 3.5.2. In this instance, voltage $V(1)$ is picked for the swap, as it had the highest priority.

At this juncture all the ξ -variables as before, except $V(1)$ are placed in the active set of ξ -variables, via appropriate homotopic deformations applied to their natural bounds, and Algorithm Coordination is invoked to

provide a new operating point. Recall that this step is necessary since the previous operating point does not qualify as a starting solution once the control and dependent variable specifications change. The new operating point corresponds to $N=13$ ($\theta=0$), and has a cost of 2772.8 \$/hr, which is lower than the cost at $N=12$, indicating lower system losses at the new operating point. In the tracking step immediately following this, i.e., $N=14$, the cost rises to 2773.6 \$/hr (an increase from the cost at $N=12$, the terminal point of the previous continuation process), reflecting the impact of the newly added constraint $Q(1)$.

At $N=16$, ($\theta=10^{-2}$), variable $t(3)$ is released from the active ξ -set, at $N=17$, ($\theta=10^{-1}$), variables $t(2)$ and $\varphi(1)$ are released from the active set, and at $N=18$, ($\theta=1$) variable $B_{sh}(1)$ is released from the active set. The cost at $N=18$ amounts to 2773.7 \$/hr. The updated optimal cost at $N=19$ ($\theta=1$), to correct for the last update in the active set due to variable $B_{sh}(1)$ is 2773.62 \$/hr, as expected lower than the previous cost, since the previous cost corresponds to a value of $B_{sh}(1)$ being moved beyond its optimal setting.

This marks the solution of the complete OPF problem, as the K-T conditions are satisfied within precision ϵ , and no functional violations prevail.

Figure 4.5 gives the variation in the total generation, and figure 4.6 the variations in the incremental transmission losses (ITL's), for corresponding values of the corrector iterations specified in figure 4.4. Figures 4.4 and 4.5 are similar, since the generation cost plotted in figure 4.4 bears a direct relationship to the real generation.

In figure 4.6, the ITL corresponding to $Q(1)$ appear only at $N=13$. This is due to $Q(1)$ being added to the set of injections only at $N=12$. Hence for values of $N < 12$, the ITL corresponding to $Q(1)$ is zero. The variations in this from $N=12$ to $N=13$ are not explicitly computed by the algorithm as it does not have any use in the optimization, i.e., all ITL's result as by-products in the optimization. Note that apart from the changes in ITL's due to large shifts in the operating point (usually occurring towards the termination of the continuation process), the variational pattern remains

almost static indicating that changes to the load-flow manifold, along the direction (path orientation) in which the solution is approached, are gradual. This conclusion can be drawn due to the ITL's being a direct function of the operating point on the LF manifold. The largest ITL prevails at bus(2), due to the large line resistances of lines 2-3 and 2-5 emanating from this node. Note that the ITL at bus(1) is zero since it is taken as the slack bus. A further interesting feature seen in figure 4.6 is that, all ITL's corresponding to both real and reactive injections are positive, implying that increased generation will amount to increased losses.

4.1.1.5 Trajectories of the ξ -Variables

The following control variables are contained in vector ξ at the beginning of the solution process, i.e., $M=1$, $K=0$ and $N=0$, where M , K , and N are the Phase I invocation, restart iteration, and corrector iteration counts respectively.

Voltages : $V_S(1)$, $V_{PV}(2)$ & $V_{PV}(4)$

On-line Controllers: $\phi(1)$, $t(2)$ & $t(3)$

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

The trajectories of the generation voltage, phase shifter, tap-changer and shunt controller settings, and their Lagrange Multipliers⁷ are shown in figures 4.7, 4.8, 4.9 and 4.10.

For restart iteration counts $N=0$ ($\theta=0$) to $N=3$ ($\theta=10^{-2}$), no changes occur within the active ξ -set. At $N=4$ ($\theta=0.1$) the LM of $t(3)$ (figure 4.9) changes its algebraic sign⁸, indicating that this variable must be released from the active ξ -set. The setting of $t(3)$, and the corresponding value of the continuation parameter, estimated via linear interpolation is given in table

⁷ The LM's plotted are α_{x_i}/λ , where index i corresponds to the i th elements of vector ξ .

⁸ Each time a LM changes its algebraic sign, i.e., crosses the zero mark, the corresponding variable is dropped from the active ξ -set, after its optimal setting is obtained via a linear interpolation, and its LM reset to zero.

4.1.e. At $N=5$ ($\theta=1$) the LM's of $V(4)$ and $V(1)$ (figure 4.7), $\phi(1)$ (figure 4.8), $t(2)$ (figure 4.9), and $B_{sh}(1)$ (figure 4.10) change their algebraic sign, indicating that these variables must be released from the active ξ -set. The estimated variable settings and the corresponding values of the continuation parameter are given in table 4.1.e. A further corrector iteration is necessary to update the operating point to correspond with the variable settings estimated in the last interval increment in θ , for the restart iteration count $K=0$. The updated solution corresponds to $N=6$.

At this juncture, as discussed before, since $\alpha_{x_1}^M$, i.e., the largest LM computed over all free ξ -variables (i corresponds to $V(4)$), exceeds the convergence tolerance ϵ , a restart iteration becomes necessary in order to respect the convergence tolerance. Corrector iteration count $N=6$ marks the beginning of this restart iteration, which also happens to be the first restart iteration ($K=1$) within the current Phase I iteration. Recall that all the 9 ξ -variables must now be placed within the active ξ -set, via appropriate homotopic deformations, and the continuation parameter θ reset to 0.

At $N=10$ ($\theta=0.1$), the LM's of variable $\phi(1)$ (figure 4.8), $t(2)$ and $t(3)$ (figure 4.9) and $B_{sh}(1)$ (figure 4.10) change their algebraic sign indicating that these variables must be released from the active ξ -set. Their estimated optimal settings and the corresponding LM's are given in table 4.1.e. At $N=11$ ($\theta=1$), the LM of $V(1)$ (figure 4.7) changes its algebraic sign, thereby causing this variable to be released from the active ξ -set at an estimated value given in table 4.1.e. A further corrector iteration is necessary to update the operating point to correspond with the variable settings estimated in the last interval increment in θ , for the restart iteration count $K=1$. The updated solution corresponds to $N=12$ ($\theta=1$).

At this juncture, since $\alpha_{x_1}^M$, i.e., the largest LM computed over all free ξ -variables (i corresponds to $V(1)$), is within the convergence tolerance ϵ , the relaxed OPF problem is solved. However, since variable $Q(1)$ violates its upper bound, Algorithm Phase II is invoked to update the control and dependent variable specifications. This causes $V(1)$ to become dependent, and no longer part of vector ξ . Hence neither the variations in $V(1)$, nor its LM

are explicitly tracked in the succeeding iterations. Recall that following each update to the control and dependent variable specifications, the previous solution does not qualify as a starting solution for subsequent Phase I iterations. Hence, Algorithm Corrector is reinvoked to provide an updated solution (at $N=13$), to correspond with the new variable specifications, and marks the solution to be tracked in the next invocation of Algorithm Phase I. The continuation parameter θ is reset to zero, and the 8 remaining ξ -variables (since $V(1)$ is no longer part of vector ξ) are placed in the active ξ -set via suitable homotopic deformations applied to their natural bounds.

In the ensuing iterations, at $N=16$ ($\theta=10^{-2}$), the LM of variable $t(3)$ (figure 4.9) changes its algebraic sign and causes this variable to be released from the active ξ -set, at an estimated setting given in table 4.1.f. At $N=17$ ($\theta=0.1$), the LM's of variables $\phi(1)$ (figure 4.8), and $t(2)$ (figure 4.9) change their algebraic sign, thereby causing these variables to be released from the active ξ -set, at estimated settings given in table 4.1.f. At $N=18$, the LM of variable $B_{sh}(1)$ (figure 4.10) changes its algebraic sign, and this variable is released from the active set at an estimated setting given in table 4.1.f. A further corrector iteration is necessary to update the operating point to correspond with the variable settings estimated in the last interval increment in θ , for the restart iteration count $K=0$. The updated solution corresponds to $N=19$.

At this juncture, since $\alpha_{x_1}^M$, i.e., the largest LM computed over all free ξ -variables (i corresponds to $t(2)$), is within the convergence tolerance ϵ . Hence the relaxed OPF problem is solved. Further since no functional violations prevail, the current solution is the solution to the complete OPF problem.

In figure 4.7 the LM's of $V(2)$ and $V(4)$ remain negative throughout the optimization process, except at $N=5$, where the LM of $V(4)$ changes sign. This maintains both $V(2)$ and $V(4)$ at their homotopically deformed upper bounds, during the progress of the iterations, i.e., during the removal of this deformation. Since these variables are initially set at their natural upper bound (1.1 pu V), no changes to their settings can be observed, since the

homotopically deformed bounds become insensitive to the continuation parameter θ . In the case of $V(1)$, since the initial value of its LM is positive, the variable follows its homotopically deformed lower bound. This is seen as a decrease in the variable setting. For $N=5$ to $N=11$, the negative LM causes this variable to now follow its homotopically deformed upper bound. At $N=12$, the LM takes on a small positive value, within the convergence precision ϵ , indicating that the optimal solution can be further improved by decreasing the setting of $V(1)$. In the case of $V(2)$ and $V(4)$, the negative LM's simply confirms the correctness of the solution, since these variables are already at their natural upper bounds.

In figure 4.8, the initially positive LM causes $\phi(1)$ to follow its homotopically deformed lower bound. At $N=5$, the LM changes its algebraic sign. For subsequent iterations up to $N=10$, the LM remains negative, thereby allowing the variable to follow its homotopically deformed upper bound. At $N=10$, the LM changes sign and the variable is released from the active set at a setting estimated via linear interpolation. This variable maintains its setting at this value until the continuation process corresponding to the current restart iteration ($K-1$) is completed. However, the changes in the settings of other variables which are updated during the remaining interval increment in θ , corresponding to $K-1$, causes the LM to assume a positive value at $N=12$ ($\theta=1$), i.e., the termination of the continuation process corresponding to $K-1$. In the subsequent iterations, the positive LM causes $\phi(1)$ to move along with its homotopically deformed lower bound until $N=17$, where the LM changes sign. At the termination of the continuation process ($N=19$), the LM has a small negative value, whose absolute value falls within convergence precision ϵ , indicating that the optimal solution can be further improved by increasing the setting of this variable.

In figure 4.9 the positive LM of $t(2)$ causes this variable to follow its homotopically deformed lower bound, while the negative LM of $t(3)$ causes it to follow its homotopically deformed upper bound. At $N=4$, the LM of $t(3)$ changes its algebraic sign, and at $N=5$ the LM of $t(2)$ changes its algebraic sign. From $N=4$ to $N=10$ the LM of $t(3)$ remains positive causing this variable to move towards its lower bound. The opposite happens to $t(2)$, where from $N=5$ to $N=10$ the negative LM causes variable $t(2)$ to move towards its upper

bound. At $N=12$, i.e., the termination of the first invocation of Algorithm Phase I the LM's of both variables remain positive. During the second invocation of Algorithm Phase I, up to $N=17$, the LM of $t(2)$ is positive while that of $t(3)$ remains marginally negative, causing these variables to follow their homotopically deformed lower and upper bounds respectively. At the termination of the continuation process, i.e., at $N=19$, corresponding to the second invocation of Algorithm Phase I, the LM's of $t(2)$ and $t(3)$ are marginally positive, but within the convergence precision of ϵ , indicating that the optimal solution can be further improved by decreasing their settings.

In figure 4.10, the LM's of variables $B_{sh}(2)$ and $B_{sh}(3)$ remain negative, thereby getting these variables to follow their upper limits. At $N=5$, these variables reach their natural upper bounds, and remain there at $N=6$, which marks the termination of the continuation process corresponding to $K=0$. For subsequent iterations, the variables are forced to follow their homotopically deformed upper bounds. However, since the variables have a setting equal to the natural upper bounds at the beginning of each succeeding continuation process, the homotopies become insensitive to the continuation parameter θ . Notice the similarity of this to the case involving variables $V(2)$ and $V(4)$, where these variables remain at their natural upper bound from the beginning to the end of the continuation process. The behaviour of $B_{sh}(1)$ is different. The initially positive LM of $B_{sh}(1)$ causes this variable to move with its homotopically deformed lower bound. However, since this variable is initially set at its lower bound, the homotopy defining the lower bound becomes insensitive to the continuation parameter θ , and no change to the variable setting occurs. At $N=5$, the LM changes its algebraic sign, causing $B_{sh}(1)$ to now follow its homotopically deformed upper bound. This continues until $N=10$, at which point the LM changes sign, and the variable is released from the active set. At $N=13$, the beginning of the second invocation of Algorithm Phase I, the LM of $B_{sh}(1)$ takes on a negative value causing this variable to follow its homotopically deformed upper bound, until $N=18$. At $N=19$, the terminal point of Phase I iterations corresponding to $K=0$, the LM of $B_{sh}(1)$ assumes a small positive value, within precision ϵ , indicating that a decrease in the variable setting is capable of improving the optimal solution.

In figures 4.7, 4.8, 4.9, and 4.10 the striking similarity is that the termination of the first continuation process, i.e., $K=0$, within each Phase I invocation, brings all ξ -variables very close to their optimal setting corresponding to the relaxed OPF problem being solved. Stated alternately, the active set of ξ -variables is almost correctly identified. Note that the plots depict the solutions of two different relaxed OPF problems, one terminating at $N=12$, and the other terminating at $N=19$. The restart iterations merely act to improve the LM's of the free ξ -variables, and bring very little change to the variable settings observed at $K=0$, per each invocation of Algorithm Phase I, provided $\alpha_{x_i}^M$, i.e., the largest LM computed over all free ξ -variables at $K=0$, is reasonably small at $\theta=1$, which is almost always true. The LM $\alpha_{x_i}^M$ is a good indicator of the convergence characteristics of the restart homotopy continuation algorithm, as it is capable of faithfully reflecting the separation of the current operating point from the optimal solution of the relaxed OPF problem being solved.

4.1.1.6 Bus Incremental Costs corresponding to Real and Reactive Demands

Figure 4.11 gives a plot of the bus incremental costs (BIC's) corresponding to both real (dL/dP_d in \$/pu MW) and reactive (dL/dQ_d in \$/pu MVAR) demands, each time Algorithm Phase I terminates successfully, i.e., at the solution of each relaxed OPF problem. The BIC λ (\$/pu MW) at the slack bus, corresponding to the real demand is shown by the broken line. The system incremental cost (equation 3.10.j, chapter 10), will be close to λ .

At the termination of the first Phase I iteration ($M=1$), the BIC's corresponding to the real demand at buses 1, 5 and 6 are almost equal to λ , while those at buses 2, 3, and 4 are lower than λ . The differences among the bus incremental costs are brought on by the losses in the system, and not due to functional constraints, since violations among functional constraints are ignored in Phase I iteration $M=1$. The BIC's corresponding to the reactive demand are very much smaller and are zero at all generation buses, and non-zero at the load buses 3, 5 and 6, although the one at bus(3) is insignificantly small.

Generally if the voltage at a PQ bus is within limits, and if any shunt controllers (of the capacitive variety) connected to this bus are also within limits, the BIC at this bus corresponding to the reactive demand becomes insignificantly small. This will be true even if the corresponding voltage is at an upper limit. This is so since any reactive demand can be generated locally, without incurring additional expenditure. However should the voltage become constrained at an lower limit, and should the shunt controller still remain inactive, the corresponding incremental cost with respect to the reactive demand will still be small although not insignificant. In the event that the voltage is constrained at a lower limit, and the shunt controller constrained at an upper limit, the incremental bus costs with respect to the reactive demands will begin to show a marked increase, since the reactive demand at this bus cannot be generated locally, and must be generated and transmitted from elsewhere in the system. The reverse will be true if the shunt controller is of the inductive type.

An exactly similar phenomenon can be observed at a generation bus, depending on the status of the reactive generation at this bus.

In this example, at the termination of Phase I iteration $M=1$, the inactive voltage at the load bus(3), and the inactive reactive source $B_{sh}(1)$ help in maintaining a low incremental bus cost (falling outside the resolution of the plot) with respect to the reactive demand at this bus. In the case of load buses 5 and 6 although their voltages are within limits, the corresponding shunt controllers 2 and 3 are at an upper limit pushing the incremental cost of reactive power at these busses. Since the reactive violation $Q(1)$ is ignored at this juncture, the BIC corresponding to the reactive demand at this bus remains zero.

At the termination of Phase I iteration $M=2$ $V(3)$, and $B_{sh}(1)$ are inactive thereby maintaining the BIC at this bus insignificantly small and negative. Note that a negative BIC implies that an increase in reactive demand will cause a reduction in the optimal cost. At buses 5 and 6 the voltages continue to remain within limits, but since the shunt controllers remain active the BIC with respect to the reactive demands remain significant, and slightly above the values at $M=1$, due to reactive power limitations in the

system brought on by $Q(1)$ being constrained at a limit. The voltage at the slack bus, i.e., bus(1) is within limits, however the active reactive generation $Q(1)$ causes the incremental cost of reactive power at this bus to rise from its previous value at $M-1$, although rather insignificant in magnitude, due to $V(1)$ being within limits, i.e., stated alternately any further reactive demand from this bus can still be supplied at minimum cost at the expense of $V(1)$.

The active reactive generation $Q(1)$, specified at its upper limit, during Phase I iteration $M-2$ does not bring about any noticeable changes among the BIC's corresponding to the real demand and λ , although λ increases marginally from 10.958 to 10.978 \$/MW-hr to reflect the new problem constraint. Note that the changes to the BIC's, with respect to the real demands at all buses maintain approximately the same relative positions with respect to λ from Phase I iteration $M-1$ to $M-2$.

This completes the discussion of results, resulting from simulation corresponding to specification C.1.1 (example 1). The following discussion will summarise the results, resulting from simulation corresponding to specification C.1.2 (example 2).

4.1.2 Simulations Based on Specification C.1.2

Specification C.1.2 is identical to C.1.1, except for the following changes, i.e.,

1. The current limit on transmission line 1₄ (i.e., line 2-5) is reduced from 1.5 to 0.32 pu A.
2. The maximum reactive generation capability of unit(1) is increased from 6 to 26 MVAR.
3. The upper limit on the voltage at bus(3) is reduced from 1.10 to 1.01 pu V.
4. The lower limit on the voltage at bus(5) is increased from 0.99 to 1.05 pu V.

The reason for these changes are to investigate the solution potential of the RHCM algorithm, in handling different types of functional violations following the termination of Phase I iterations M=1. Note that, within the first invocation of Algorithm Phase I the solution process is identical to the previous example.

Table 4.2.a summarises the solution process, and table 4.2.b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 2 (specification C.1.2).

Table 4.2.a
Summary of the solution process - Example 2

	Phase I (No.1)	Phase I (No.2)
Number of Restarts	1	2
Functional Violations	V(3), V(5), I ₄	NONE
Variables Leaving x	V(3), V(5)	NONE
Variables Entering x	$\varphi(1)$, t(2), V(1)	NONE

Table 4.2.b
Performance statistics of the RHCM and MINOS Ver. 4.0 - Example 2

	RHCM	MINOS
Number of Jacobian Evaluations	68	53
Exec Speed (CPU sec on Amdahl 5860)	0.540	0.970
Core Reqd. (M-bytes on Amdahl 5860)	0.312	0.616

The highlights of table 4.2.a will now be discussed.

At the termination of Phase I iterations M=1, Type I functions V(3) and V(5) exceed their upper and lower bounds respectively, and Type II functional I₄ (line current on line 2-5) exceeds its bound. Note that Q(1) is within bounds. Algorithm Phase II is invoked at this point, which places variables V(3) and V(5) among the set of ξ -variables, and appends I₄ among the set of

injections after specifying it at its violated bound. In response to these changes variables $V(1)$, $\phi(1)$ and $t(2)$ change their role from a controllable variable (ξ -variable) to a dependent variable. In the ensuing Phase I iterations ($M=2$), two restart iterations ($K=2$) are needed prior to reaching the solution of the second relaxed OPF problem. Note that in the previous example (i.e., example 1) no restart iteration were needed within the second invocation of Algorithm Phase I, since the solution to second relaxed OPF problem was very close to that of the first. At this juncture since no functional violations prevail, this marks the solution to the complete OPF problem. At the optimal solution, among the variables which changed roles, variable $V(3)$ is free, $V(5)$ at a lower bound, I_4 at an upper bound and variables $V(1)$, $\phi(1)$ and $t(2)$ within bounds.

For purposes of comparison, the final solutions due to the RHCM and MINOS are be presented in table 4.2.c.

Table 4.2.c
Solution summary - Example 2

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Lwr.Lmt	02.400	02.400
Unit(2)	Lwr.Lmt	68.950	68.950
Unit(3)	Lwr.Lmt	54.250	54.250
Unit(5)	Free	81.817	81.816
React. Generations (MVAR):			
Q(1)	Free	22.010	22.166
Q(2)	Free	01.982	01.977
Q(4)	Free	28.056	27.899
Bus Voltages (pu V) & Ang:			
V(1)	Free	1.082/0.000	1.082/0.000
V(2)	Upr.Lmt	1.100/0.195	1.100/0.162
V(3)	Free	0.996/0.108	0.996/0.122
V(4)	Upr.Lmt	1.100/0.118	1.100/0.127
V(5)	Lwr.Lmt	1.050/-0.033	1.050/-0.042
V(6)	Free	1.021/0.006	1.022/0.004

Table 4.2.c cont.

Tap & Φ (rad) Positions:			
$\phi(1)$	Free	-.135	-.135
$t(2)$	Free	0.909	0.909
$t(3)$	Free	1.074	1.074
Sht. Controllers (pu mho):			
$B_{sh}(1)$	Upr.Lmt	0.050	0.050
$B_{sh}(2)$	Upr.Lmt	0.050	0.050
$B_{sh}(3)$	Upr.Lmt	0.050	0.050
Line Current ² (pu A ²):			
$(I_A)^2$	Upr.Lmt	0.1024	0.1024
Optimal Cost in (\$/hr):			
		2777.48	2777.47

Note that since the line currents on the remaining lines fall within their limiting values they are not reported.

The following discussion will consider the break-points resulting within the second invocation of Algorithm Phase I.

Table 4.2.d

Summary of break points resulting in the second invocation of Algorithm Phase I - Example 2

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.71229	V(3) is deactivated at setting 0.9951
	0.78777	t(3) is deactivated at setting 1.0807
	0.82705	V(4) is deactivated at setting 1.1000
	0.83801	V(5) is deactivated at setting 1.0500
	0.84683	$B_{sh}(2)$ is deactivated at setting 0.0500
	0.93285	$B_{sh}(3)$ is deactivated at setting 0.0500
	1.00000	Termination of the continuation process

Table 4.2.d cont.

1	0.00000	Beginning of the continuation process
	0.03506	$t(3)$ is deactivated at setting 1.0724
	0.03637	$V(3)$ is deactivated at setting 0.9958
	1.00000	Termination of the continuation process
2	0.00000	Beginning of the continuation process
	0.01632	$V(3)$ is deactivated at setting 0.9956
	0.03605	$t(3)$ is deactivated at setting 1.0737
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the second time, the variables contained in vector ξ are,

Voltages : $V_{PV}(2)$, $V_{PQ}(3)$, $V_{PV}(4)$ & $V_{PQ}(5)$

On-line Controllers: $t(3)$

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

Table 4.2.d gives the value of the continuation parameter θ at which the above variables are released from the active set, and their corresponding settings. Variables not released reach their bounds (upper or lower) at the termination of the continuation process. Note that among all the inactive ξ -variables at the termination of restart iteration count $K=0$, variables $V(4)$, $B_{sh}(2)$ and $B_{sh}(3)$ are inactive at their upper limits, and $V(5)$ inactive at its lower limit. In the subsequent restart iterations these variables become active at their respective natural bounds. Variable $V(3)$, which represented a Type I violation at the solution to the previous relaxed OPF problem, and set to its violated bound just prior to invoking Algorithm Phase II for the second time, comes off its violated bound. This example shows that variables which constitute a Type I (or Type II) violation at the termination of a relaxed OPF problem need not remain at the violated bound at the solution to the next relaxed OPF (or full OPF) problem. Note that all the ξ -variables are very close (attain at least the first three significant digits) to their optimal setting corresponding to the relaxed OPF problem being solved, at the termination of the first continuation process, $K=0$. Further restart iterations aid to improve the LM of the free ξ -variables, but bring about only marginal improvement to the objective.

The variations in the optimal cost are shown in figure 4.12. Recall that restart iteration count $N=12$ marks the termination of Algorithm Phase I invoked for the first time. At $N=12$, Algorithm Phase II is invoked in order to update the control and dependent variable specifications, to remove the Type I functional violations on variables $V(5)$, $V(3)$, and Type II functional violation on variable I_4 . The cost at the updated operating point corresponding to the new variable specifications, computed by invoking Algorithm Coordination corresponds to $N=13$. The rise in the cost from 2773.13 (at $N=12$) to 2798.82 (at $N=13$) \$/hr, is due to increased losses at the new operating point. The termination of the subsequent continuation process, i.e., $K=0$, within Phase I iteration $M=2$, comes very close to the optimal solution of the current relaxed OPF problem, except for the optimal settings on variables $V(3)$ and $t(3)$, i.e., the ξ -variables inactive at the solution. The subsequent restart iterations, within Phase I iteration $M=2$, i.e., $K=1$ and 2 are needed to update the settings on $V(3)$ and $t(3)$, in order to satisfy the K-T conditions within the specified precision ϵ . Note that the termination of the first continuation process, within each invocation of Algorithm Phase I, is able to move the variable settings very close to their optimal values. In this example, at $N=19$, the terminal point of the first continuation process $K=0$, all variables whose optimal settings correspond to their natural bounds have reached these values. Further, all the free control variables in vector ξ have up to three correct significant figures of their optimal settings. These starting values, used in the subsequent continuation process aids to settle rapidly the free ξ -variables, by giving them the additional significant figures. Stated alternately, the additional restart iterations, merely update the remaining significant figures of the free ξ -variables, until the K-T conditions are satisfied to precision ϵ . Note that the final optimal cost at $N=29$, i.e., 2777.48 \$/hr, (the solution to the second relaxed OPF problem) is higher than the cost at $N=12$, i.e., 2773.13 \$/hr (the solution to the first relaxed OPF problem), demonstrating the impact of active functional constraints, i.e., the optimal cost rises with increased number of active problem constraints.

Figure 4.13 gives, the BIC's obtained at $N=12$, and $N=29$, i.e., the terminal points of Phase I iterations $M=1$ and 2, or at the solution of each relaxed OPF problem considered within the two Phase I iteration. As before λ

represents the BIC at the slack bus corresponding to the real demand. Since the second relaxed OPF problem is more constrained than the first, the value of λ is higher in the second problem than in the first.

The incremental costs of real demand, plotted at the termination of restart iteration M-1 does not take consideration of the current violation on transmission line I_4 (i.e., line 2-5). The subsequent restart iteration M-2 is designed to contain this violation, and the corresponding changes to the incremental cost of real demand show that at bus(2), the BIC corresponding to the real demand decreases in magnitude (and with respect to λ) from 8.823 to 8.119 \$/MW-hr, while the BIC corresponding to the real demand at bus(5) increases in magnitude from 11.028 to 11.121 \$/MW-hr. The computed power flow (both real and reactive) on line I_4 , at the termination of Phase I iterations M-1 and M-2, is from bus(2) to bus(5). The reason for this change is given below.

Generally when the power flow is from bus 2 to bus 5 on line 2-5 is limited by the optimization process, a scenario develops in which bus 5 experiences a scarcity of inexpensive (as determined by the previous optimization process in which the line current restriction on line 2-5 is neglected) electricity, and bus 2 experiences a surplus of the same. Hence by the fundamental principles of supply and demand, the cost at bus 2 decreases while that at bus 5 increases. The value of λ will generally rise to reflect the impact of containing the line current on line 2-5.

At the termination of Phase I iteration M-1, the incremental costs with respect to the reactive demands at load buses 5 and 6 take on significant values since the shunt controllers 2 and 3 at these buses are active at an upper limit. The voltage at bus(6) is within limits, while that at bus(5) exceeds its lower limit. Note that this lower limit violation has no effect on the BIC since such functional violations are ignored. The incremental cost of reactive demand at bus(3) remains insignificant since the shunt controller 1 at this bus is within limits, and the upper limit violation on $V(3)$ is ignored.

At the termination of Phase I iteration M=2, all three shunt controllers are at an upper limit, voltages at buses 3 and 6 inactive and the voltage at 5 constrained at a lower limit. The inactive voltages at buses 3 and 6 are able to limit the rise in the incremental cost of reactive demand at this buses, although marginal rises are inevitable. The incremental cost of reactive power at bus(3) continues to remain small and outside the resolution of the plot. The largest rise in the incremental cost of reactive power is recorded at bus(5) since $V(5)$ is constrained at a lower limit, the shunt controller 2 at this bus is at an upper limit, and due to limiting the reactive power flow from bus(2) to bus(5).

Note that an active line current flow generally brings about more pronounced changes among the incremental bus costs with respect to the real demand, and λ , in comparison to an active reactive generation, which primarily affects the incremental bus costs with respect to the reactive demands.

The following discussion will summarise the results, resulting from simulation corresponding to specification C.1.3 (example 3).

4.1.3 Simulations Based on Specification C.1.3

Specification C.1.3 is identical to C.1.2 except for the following changes, i.e.,

1. The current limit on transmission line I_4 (i.e., line 2-5) is increased from 0.32 to 1.32 pu A.
2. The current limit on transmission line I_6 (i.e., line 4-6) is reduced from 2.5 to 0.32 pu A.

As before, the changes are to investigate the solution potential of the RHCM algorithm, in handling different types of functional violations following the termination of Phase I iterations M=1. Note that, within the first invocation of Algorithm Phase I, the solution process is identical to the previous example.

Table 4.3.a summarises the solution process, and table 4.3.b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 3 (specification C.1.3).

Table 4.3.a
Summary of the solution process - Example 3

	Phase I (No.1)	Phase I (No.2)	Phase I (No.3)
Number of Restarts	1	2	0
Functional Violations	V(3), V(5), I_6	V(1)	NONE
Variables Leaving x	V(3), V(5)	V(1)	NONE
Variables Entering x	$\phi(1)$, $t(2)$, V(1)	$t(3)$	NONE

Table 4.3.b
Performance statistics of the RHCM and MINOS Ver. 4.0 - Example 3

	RHCM	MINOS
Number of Jacobian Evaluations	91	66
Exec Speed (CPU sec on Amdahl 5860)	0.700	1.140
Core Req'd. (M-bytes on Amdahl 5860)	0.312	0.616

The highlights of table 4.3.a will now be discussed.

At the termination of Phase I iterations M-1, Type I functions V(3) and V(5) exceed their upper and lower bounds respectively, and Type II function I_6 (line current on line 4-6) exceeds its bound. Note that I_4 (line current on line 4-6) is within bounds. Algorithm Phase II is invoked at this point, which places variables V(3) and V(5) among the set of controllable ξ -variables, and appends I_6 among the set of injections after specifying it at its violated bound. In response to these changes variables V(1), $\phi(1)$ and $t(2)$ change their role from a controllable variable (ξ -variable) to a dependent variable.

Note that these variables are identical to those which changed roles in the previous example following Phase I invocation count $M=1$. This is so since the relaxed OPF problem solved during the previous Phase I iteration (i.e., $M=1$) is the same in both examples, which in turn creates the same priority list of ξ -variables which are eligible to become dependent.

In the ensuing Phase I iterations ($M=2$), two restart iterations ($K=2$) are needed prior to reaching the solution of the second relaxed OPF problem. Note that in the previous example (i.e., example 2) also required two restart iterations within the second invocation of Algorithm Phase I.

At this juncture, among the variables which changed roles, it is found that the variable $V(3)$ is at its upper limit, $V(5)$ at its lower limit, and line current I_6 at a bound. Further $\phi(1)$, $t(2)$ are within limits, and $V(1)$ exceeds its upper limit. This is an indication that perhaps following $M=1$, $V(1)$ should have remained in vector ξ , and not made dependent. However there is no way in which such an occurrence(s) of event(s) can be seen at the beginning of a Phase I iteration, immediately following a Phase II update. The only solution would be to allow for the resolution of such situations, rather than try to prevent their occurrence. The formulation in this thesis is able to resolve such a situation quite trivially, i.e., as far as the problem is concerned, the violation in $V(1)$ is a Type I functional violation. In order to resolve this, Algorithm Phase II is reinvoked which places $V(1)$ among the set of controls in vector ξ , and makes $t(3)$, i.e., the next element in vector ξ with the highest priority for an exchange, a dependent variable. Now a third relaxed OPF problem is solved by invoking Algorithm Phase I. It is found that no restart iterations ($K=0$) are necessary within Phase I iteration $M=3$, in order to reach the solution.

At this juncture, among all variables which changed roles during the previous iterations, $V(3)$ is at an upper limit, $V(5)$ at a lower limit, I_6 at a limit, $\phi(1)$, $t(2)$ are within bounds, $V(1)$ at an upper limit, and $t(3)$ is within bounds. Variable $B_{sh}(1)$ is the only inactive control in vector ξ to be present at this time, indicating that the system is being operated under very restrictive conditions. Since no further functional violations prevail at this point, this marks the solution to the complete OPF problem.

For purposes of comparison, the final solutions due to the RHCM and MINOS are be presented in table 4.3.c.

Table 4.3.c
Solution summary - Example 3

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Lwr.Lmt	02.400	02.400
Unit(2)	Lwr.Lmt	68.950	68.950
Unit(3)	Lwr.Lmt	54.250	54.250
Unit(5)	Free	81.851	81.851
React. Generations (MVAR):			
Q(1)	Free	31.084	31.058
Q(2)	Free	03.647	03.647
Q(4)	Free	20.447	20.179
Bus Voltages (pu V) & Ang:			
V(1)	Upr.Lmt	1.100/0.000	1.100/0.000
V(2)	Upr.Lmt	1.100/0.252	1.100/0.252
V(3)	Upr.Lmt	1.010/0.107	1.010/0.107
V(4)	Upr.Lmt	1.100/0.122	1.100/0.122
V(5)	Lwr.Lmt	1.050/-0.007	1.050/-0.007
V(6)	Free	1.023/0.019	1.023/0.019
Tap & Phi rad) Positions:			
$\phi(1)$	Free	-.080	-.080
t(2)	Free	0.919	0.919
t(3)	Free	1.087	1.087
Sht. Controllers (pu mho):			
B _{sh} (1)	Free	0.024	0.026
B _{sh} (2)	Upr.Lmt	0.050	0.050
B _{sh} (3)	Upr.Lmt	0.050	0.050
Line Current ² (pu A ²):			
(I _g) ²	Upr.Lmt	0.1024	0.1024

Table 4.3.c cont.

Optimal Cost in (\$/hr):		
	2777.83	2777.82

Note that since the line currents on the remaining lines fall within their limiting values they are not reported above.

The following discussion will consider the break-points resulting within the second and third invocation of Algorithm Phase I.

Table 4.3.d

Summary of break points resulting in the second invocation of Algorithm Phase I - Example 3

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.09544	V(3) is deactivated at setting 1.0100
	0.67657	B _{sh} (1) is deactivated at setting 0.0367
	0.72635	t(3) is deactivated at setting 1.0751
	0.81604	V(4) is deactivated at setting 1.1000
	0.82355	V(5) is deactivated at setting 1.0500
	0.91314	B _{sh} (2) is deactivated at setting 0.0500
	1.00000	Termination of the continuation process
1	0.00000	Beginning of the continuation process
	0.01318	t(3) is deactivated at setting 1.0709
	0.78376	B _{sh} (1) is deactivated at setting 0.0474
	1.00000	Termination of the continuation process
2	0.00000	Beginning of the continuation process
	0.05279	t(3) is deactivated at setting 1.0728
	0.13227	B _{sh} (1) is deactivated at setting 0.0366
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the second time, the variables contained in vector ξ are,

Voltages : $V_{PV}(2)$, $V_{PQ}(3)$, $V_{PV}(4)$ & $V_{PQ}(5)$

On-line Controllers: $t(3)$

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

Table 4.3.d gives the value of the continuation parameter θ at which the above variables are released (if necessary) from the active ξ -set, and their corresponding settings. Variables not released reach their bounds (upper or lower) at the termination of the continuation process. Note that, among all the inactive ξ -variables at the termination of restart iteration count $K=0$, variables $V(4)$, $V(3)$ and $B_{sh}(2)$ are inactive at their upper limits, and $V(5)$ inactive at its lower limit. In the subsequent restart iterations these variables become active at their respective natural bounds. Unlike in example 2, $V(3)$ reaches an upper limit at the solution to the second relaxed OPF problem. Further as before all the ξ -variables come very close (attain at least the first three significant digits) to their optimal setting corresponding to the relaxed OPF problem being solved, at the termination of the first continuation process, $K=0$. Further restart iterations aid to improve the LM of the free ξ -variables and bring about only marginal improvements to the objective.

The functional violation (Type I) on $V(1)$, following the termination of Phase I iterations corresponding to $M=2$, makes it necessary to invoke Algorithm Phase I for the third time, after implementing a suitable variable update by invoking Algorithm Phase II. Within the second invocation of Algorithm Phase II, $V(1)$ becomes a controllable variable, and $t(3)$ becomes a dependent variable. The resulting break-points during Phase I iterations $M=3$ are given below in table 4.3.e.

Table 4.3.e

Summary of break points resulting in the third invocation of Algorithm
Phase I - Example 3

K	θ	Reason
0	0 00000	Beginning of the continuation process
	0 27360	$B_{sh}(1)$ is deactivated at setting 0.0236
	1 00000	Termination of the continuation process

In invoking Algorithm Phase I for the third time, the variables contained in vector ξ are,

Voltages $V_s(1)$, $V_{PV}(2)$, $V_{PQ}(3)$, $V_{PV}(4)$ & $V_{PQ}(5)$

On-line Controllers: NONE

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

Table 4.3.e indicates that except for variable $B_{sh}(1)$, all other variables reach their bounds, including $V(1)$, which became controllable within Phase I iteration $M=3$. This makes it possible for Phase I iterations to converge quickly, usually within the first continuation process ($K=0$), to a very good precision (0.237×10^{-4} in this example). Note that fewer the number of free controllable variables in vector ξ , the quicker the convergence of Phase I iterations, since the interactive influence among the free ξ -variables is a minimum

The variations in the optimal cost are shown in figure 4.14. Recall that restart iteration count $N=12$ marks the termination of Phase I iterations $M=1$. At $N=12$, Algorithm Phase II is invoked in order to update the control and dependent variable specifications, to remove the Type I functional violations on variables $V(5)$, $V(3)$, and Type II functional violation on variable I_6 . The cost at the updated operating point corresponding to the new variable specifications, computed by invoking Algorithm Coordination, corresponds to $N=13$. The rise in the cost from 2773.13 (at $N=12$) to 2800.09 (at $N=13$) \$/hr, is due to increased losses at the new operating point. The termination of the subsequent continuation process, i.e., $K=0$, within Phase

I iteration M=2 comes very close to the optimal solution of the current relaxed OPF problem, except for the optimal settings on variables $t(3)$ and $B_{sh}(1)$, i.e., the ξ -variables inactive at the solution. The subsequent restart iterations, within Phase I iteration M=2, i.e., K=1 and 2 are needed to update the settings on $t(3)$ and $B_{sh}(1)$, by giving them the additional significant figures, in order to satisfy the K-T conditions within the specified precision ϵ . Note that the termination of the first continuation process, within each invocation of Algorithm Phase I, is able to move all the ξ -variable settings very close to their optimal values. In this example, at N=19, the terminal point of the first continuation process K=0, all variables whose optimal settings correspond to their natural bounds have reached these values. Further, all the free control variables in vector ξ have up to three correct significant figures of their optimal settings. These starting values, used in the subsequent continuation process aids to settle rapidly the free ξ -variables to their optimal values. Stated alternately, the additional restart iterations, merely update the remaining significant figures of the free ξ -variables, until the K-T conditions are satisfied to precision ϵ . Note that the optimal cost at N=31 (2776.81 \$/hr), i.e., the termination of Phase I iterations M=2 (the solution to the second relaxed OPF problem) is higher than the cost at N=12, i.e., 2773.13 \$/hr, (the solution to the first relaxed OPF problem), demonstrating the impact of the newly active constraints, upon the optimal cost.

At this juncture Algorithm Phase II has to be reinvoked in order to remove functional violation on variable V(1). The updated cost at the new operating point resulting from the control and dependent variable update amounts to 2776.81 \$/hr at N=32, with no change from the previous cost at N=31. This implies that the operating points, before and after the invocation of Algorithm Phase II are either very close, or lie on a region of the LF manifold in which the losses are relatively insensitive to the operating point. No additional tests are carried out to identify which of these two situations apply to the problem at hand, since such identification is unnecessary to the success of the optimization. Note that this situation sharply contrasts the ones at N=12, and N=13, where the operating points are such that, the difference in losses become quite apparent. The solution to the third relaxed OPF problem, which is also the solution to the complete

OPF problem, is reached at the termination of the continuation process $K=0$, at corrector iteration count $N=38$, within Phase I iteration $M=3$. The optimal cost amounts to 2777.83 \$/hr, higher than the cost at $N=31$, (2776.8 \$/hr), the solution to the previous relaxed OPF problem. Since the successor relaxed OPF is more constrained than the predecessor relaxed OPF, the cost of the successor will be always higher than the cost of its predecessor.

Figure 4.15 gives the BIC's obtained at $N=12$, $N=31$ and $N=38$, i.e., the terminal points of Phase I iterations $M=1, 2$ and 3 , or at the solution of each relaxed OPF problem considered within each Phase I iteration. As before λ is the BIC at the slack bus corresponding to the real demand. Since the relaxed OPF problems are more constrained with higher values of Phase I iteration counts, the value of λ increases progressively.

The BIC's plotted at $M=2$, i.e., the solution of the second relaxed OPF problem, corresponds to one active transmission line, namely I_6 (line 4-6), in which the real and reactive powers flow from bus(4) to bus(6). The effect of this active line on the BIC's at buses 4 and 6 defining the line, is to cause the incremental cost of reactive power at bus(4) to decrease in magnitude from its previous value, and that at bus(6) to increase relative to λ . At the solution to Phase I iterations $M=3$, this difference is more pronounced due to $V(1)$ being constrained at an upper limit. The reasons for this differential costs are the same as those given in the previous example (i.e., example 2), and hence will not be repeated here.

At the termination of Phase I iteration $M=1$, as in the previous example, among the voltages at the load buses, $V(3)$ exceeds its upper limit, $V(5)$ exceeds its lower limit, and $V(6)$ is within bounds. Note that these functional violations are not taken into consideration in computing the BIC's. However, the incremental costs with respect to the reactive demand at buses 5 and 6 remain significant compared to the one at bus(3).

At the termination of Phase I iteration $M=2$, $V(3)$ is at an upper limit, $V(5)$ at a lower limit, and $V(6)$ within limits. Shunt controllers 2 and 3 at buses 5 and 6 remain active at an upper limit, while shunt controller 1 at bus 3 is inactive. The rise in the BIC at bus(6) is partly due to limiting the

Line current on line I_6 . However the inactive shunt controller 1 at bus(3), and $V(3)$ which is at an upper limit, tend to maintain the incremental cost of reactive power at bus(3) very small. A third Phase I iteration is needed at this juncture to contain the violation on $V(1)$.

At the termination of Phase I iteration $M=3$, the scenario of the previous paragraph remains unchanged, except for the violation on $V(1)$ which no longer exists. The impact of containing $V(1)$ at an upper limit is reflected further among the incremental cost of reactive demand at buses 5 and 6, while the cost at bus(3) continues to remain insignificant.

Note that as before an active line current flow brings about more pronounced changes among the incremental bus costs with respect to the real demand, and λ , in comparison to an active reactive generation, which primarily affects the incremental bus costs with respect to the reactive demands.

The following discussion will summarise the results, resulting from simulation corresponding to specification C.1.4 (example 4).

4.1.4 Simulations Based on Specification C.1.4

Specification C.1.4 is identical to C.1.3 except for the following changes, i.e.,

1. The current limit on transmission line I_6 (i.e., line 4-6) is increased from 0.32 to 2.50 pu A.
2. The upper limit on the reactive generation $Q(1)$ is decreased from 46 to 6 MVAR.

As before, the changes are to investigate the solution potential of the RHCM algorithm, in handling different types of functional violations following the termination of Phase I iterations $M=1$.

Table 4.4.a summarises the solution process, and table 4.4.b gives a comparative summary of the performance of the restart homotopy continuation

method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 4 (specification C.1.4).

Table 4.4.a
Summary of the solution process - Example 4

	Phase I (No.1)	Phase I (No.2)	Phase I (No.3)
Number of Restarts	2	0	0
Functional Violations	V(3), V(5), Q(1)	t(3)	NONE
Variables Leaving x	V(3), V(5)	t(3)	NONE
Variables Entering x	t(2), t(3), V(1)	$\varphi(1)$	NONE

Table 4.4.b
Performance statistics of the RHCM and MINOS Ver. 4.0 - Example 4

	RHCM	MINOS
Number of Jacobian Evaluations	70	48
Exec Speed (CPU sec on Amdahl 5860)	0.550	0.940
Core Reqd. (M-bytes on Amdahl 5860)	0.312	0.616

The highlights of table 4.4.a will now be discussed.

In this example the voltages are given an arbitrary initial specification (as given in C.1.4), different from the ones used in the previous examples. This makes the variations in the optimal cost, within Phase I iteration M-1 (figure 4.16) different to those observed before. However, the optimal solution at the termination of Phase I iterations M-1 is the same, within acceptable numerical precision. Note that due to the starting point used, this example requires two restart iterations (unlike the previous examples which required only one restart iteration) to converge within Phase I iteration M-1.

The starting point used in this example is designed to highlight a certain feature of the algorithm which manifests itself within Phase I iterations

M-1. This will be discussed next by considering the break-points occurring within Phase I iterations M-1, given in table 4.4.c.

Table 4.4.c

Summary of break points resulting in the first invocation of Algorithm
Phase I - Example 4

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.10636	V(1) is deactivated at setting 1.0443
	0.12397	t(3) is deactivated at setting 1.0220
	0.17779	$\phi(1)$ is deactivated at setting -0.0482
	0.59487	B _{sh} (1) is deactivated at setting 0.0000
	0.64394	t(2) is deactivated at setting 0.9317
	0.80135	V(4) is deactivated at setting 1.0912
	1.00000	Termination of the continuation process
1	0.00000	Beginning of the continuation process
	0.06240	t(3) is deactivated at setting 1.0131
	0.07415	$\phi(1)$ is deactivated at setting -0.2900
	0.07705	t(2) is deactivated at setting 0.9464
	0.15449	B _{sh} (1) is deactivated at setting 0.0124
	0.55284	V(1) is deactivated at setting 1.0778
	1.00000	Termination of the continuation process
2	0.00000	Beginning of the continuation process
	0.01720	$\phi(1)$ is deactivated at setting -0.2314
	0.06786	B _{sh} (1) is deactivated at setting 0.0114
	0.12312	t(3) is deactivated at setting 0.9883
	0.12396	V(1) is deactivated at setting 1.0827
	0.15749	t(2) is deactivated at setting 0.9348
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the first time (i.e., M-1), the variables contained in vector ξ are,

Voltages : $V_{PV}(1)$, $V_{PV}(2)$ & $V_{PV}(4)$

On-line Controllers: $\phi(1)$, t(2) & t(3)

Shunt Controllers: B_{sh}(1), B_{sh}(2) & B_{sh}(3)

Variables V(1), t(3), $\phi(1)$, B_{sh}(1) and t(2) are released from the active ξ -set, in that order, during the last interval increment in θ , during the

initial continuation process ($k=0$). Variable settings, and the corresponding continuation parameters are given in table 4.4.c. Recall that since the above five variables were inactivated during the last interval increment in θ , it becomes necessary to update the current solution to correspond with the new variables specifications. During this update iterations the variables remaining active in vector ξ must be monitored for any conflicts, i.e., the computed LM must not contradict the bound at which the variable is situated. Should conflicts occur, they must be resolved by repeating steps (a) and (b) of Algorithm Corrector. In the previous examples, i.e., example 1, 2 and 3 no such conflicts were observed during each update iteration at $\theta=1$. However in this example, i.e., example 4, immediately following the update iteration at $N=6$ (implemented by invoking Algorithm Coordination, within Algorithm Corrector), a conflict is detected for variable $V(4)$, thereby making it necessary to release this variable from the active set at a value estimated to be 1.0912 pu V. Note that the break point corresponding to $V(4)$, occurs beyond the break-points corresponding to the variables which have been previously released. A further update iteration ($N=7$, $\theta=1$) is needed, in order to update the current operating point to correspond with the new setting of $V(4)$. Following this iteration no conflicts exist among the variables, thereby marking the termination of the continuation process $K=0$, within Phase I iteration $M=1$. At this point, unlike in the previous examples, i.e., example 1 (Case I), example 2&3, not all the control and dependent variables have attained values close enough to their optimal settings. This happens since the initial starting point is further away from the optimal solution to the relaxed OPF problem being solved. Additional restart iterations are necessary in order to reach the optimal solution.

In order to satisfy the K-T conditions corresponding to the current relaxed OPF problem, two further restart iterations become necessary, namely $K=1$ and $K=2$. The break-points and the corresponding variable settings are given in table 4.4.c. During the last interval increment in θ , (i.e., 0.1 to 1), within restart iteration $K=2$, variables $t(3)$, $V(1)$ and $t(2)$ become inactive. On updating the operating point to correspond to the new setting of these variables it is found that the optimal cost rises from 2772.87 \$/hr (at $N=18$) to 2776.29 \$/hr (at $N=19$). One would have usually expected the cost to be less at $N=19$, than at $N=18$, since the controls in vector ξ at $N=18$ have

been moved beyond their optimal positions by the respective homotopies. However, in this instant, it appears that the real losses at $N=19$ are higher than those at $N=18$, thereby masking the reduction in cost brought about by the update to the active ξ -set.

At the termination of Phase I iterations $M=1$ (at $N=19$), Type I functions $V(3)$ and $V(5)$ exceed their upper and lower bounds respectively, and Type II function $Q(1)$ (reactive generation at bus 1) exceeds its upper bound. Algorithm Phase II is invoked at this point, which places variables $V(3)$ and $V(5)$ among the set of controllable ξ -variables, and appends $Q(1)$ among the set of injections after specifying these functional variables at their violated bounds. In response to these changes variables $V(1)$, $t(2)$ and $t(3)$ change their role from a controllable variable (ξ -variable) to a dependent variable.

Note that these variables are identical to those which changed roles in the previous example following Phase I iteration $M=1$, except for $t(3)$, which replaces $\phi(1)$. Recall that, when it comes to a variable update involving on-line controllers, all on-line controllers are given an equal priority to become dependent variables should the need arise. The RHCM algorithm stores the on-line controllers in a linked list array, according to the order in which they are released from the active ξ -set, during each continuation process, i.e., $K=0,1,2 \dots$ etc. This order is influenced by the initial variable setting. Hence depending on the starting solution, one could have different controllable ξ -variables becoming dependent to alleviate the same set of functional violations, with respect to a given problem. The success of the succeeding iterations often depend on the control variables which become dependent, and the characteristics of the power system involved. Selecting the correct set of controllable variables to become dependent at the onset of the succeeding Phase I iteration remains a challenge, due to the vastly varying nature of the sensitivities of these variables to the violated functional constraints, during the progress of the succeeding iterations.

In the ensuing Phase I iterations ($M=2$), no restart iteration ($K=0$) is needed prior to reaching the solution to the second relaxed OPF problem.

Note that in the previous example (i.e., example 3) required two restart iterations within Phase I iterations $M=2$.

At this juncture, among the variables which changed roles, it is found that the variable $V(3)$ is at its upper limit, $V(5)$ at its lower limit, and reactive generation $Q(1)$ remain at an upper bound. Further $t(2)$, $V(1)$ are within limits, and $t(3)$ exceeds its upper limit, thereby implying a Type I functional violation. This is an indication that perhaps following $M=1$, $t(3)$ should have remained in vector ξ , and not made dependent. However there is no way in which such an occurrence(s) of event(s) can be seen at the beginning of a Phase I iteration, immediately following a Phase II update. In order to resolve this, Algorithm Phase II is reinvoked, which places $t(3)$ among the set of controls in vector ξ after specifying it at its violated bound, and makes $\phi(1)$ from vector ξ a dependent variable. Now a third relaxed OPF problem is solved by invoking Algorithm Phase I. It is found that no restart iterations ($K=0$) are necessary within Phase I iteration $M=3$, in order to reach the solution.

At this juncture, among all variables which changed roles during the previous iterations, $V(3)$ is free, $V(5)$ at a lower limit, $Q(1)$ at an upper limit, $\phi(1)$, $t(2)$ and $V(1)$ are within bounds, and $t(3)$ is at an upper limit. Among the set of controls in vector ξ , $V(3)$ is the only inactive control indicating that the system is operating under very restrictive conditions. Since no further functional violations prevail at this point, this marks the solution to the complete OPF problem.

For purposes of comparison, the final solutions due to the RHCM and MINOS are be given in table 4.4.d.

Table 4.4.d
Solution summary - Example 4

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Lwr.Lmt	02.400	02.400
Unit(2)	Lwr.Lmt	68.950	68.950
Unit(3)	Lwr.Lmt	54.250	54.250
Unit(5)	Free	81.849	81.838
React. Generations (MVAR):			
Q(1)	Upr.Lmt	06.000	06.000
Q(2)	Free	01.733	01.597
Q(4)	Free	45.431	45.549
Bus Voltages (pu V) & Ang:			
V(1)	Free	1.040/0.000	1.040/0.000
V(2)	Upr.Lmt	1.100/0.176	1.100/0.176
V(3)	Free	1.003/0.115	1.005/0.115
V(4)	Upr.Lmt	1.100/0.121	1.100/0.121
V(5)	Lwr.Lmt	1.050/-0.038	1.050/-0.038
V(6)	Free	0.999/0.003	0.999/0.003
Tap & Phi (rad) Positions:			
$\phi(1)$	Free	-.094	-.095
t(2)	Free	0.915	0.917
t(3)	Upr.Lmt	1.100	1.100
Sht. Controllers (pu mho):			
B _{sh} (1)	Upr.Lmt	0.050	0.050
B _{sh} (2)	Upr.Lmt	0.050	0.050
B _{sh} (3)	Upr.Lmt	0.050	0.050
Optimal Cost in (\$/hr):			
		2777.81	2777.70

Note that since the line currents on all lines fall within their limiting values they are not reported above.

The following discussion will consider the break-points resulting within the second and third invocation of Algorithm Phase I.

Table 4.4.e

Summary of break points resulting in the second invocation of Algorithm
Phase I - Example 4

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.05866	$\varphi(1)$ is deactivated at setting $-.0340$
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the second time, the variables contained in vector ξ are,

Voltages : $V_{PV}(2)$, $V_{PQ}(3)$, $V_{PV}(4)$ & $V_{PQ}(5)$

On-line Controllers: $\varphi(1)$

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

Table 4.4.e gives the value of the continuation parameter θ at which the above variables are released (if necessary) from the active ξ -set, and their corresponding settings. Variables not released reach their bounds (upper or lower) at the termination of the continuation process. Note that, only one variable, i.e., $\varphi(1)$ is inactive at the final solution, and all the other variables remain active and reach an appropriate bound. No restart iterations are needed for convergence, i.e., at the termination of continuation process $K=0$, the K-T conditions relating to the second relaxed OPF problem are satisfied to within precision ϵ .

The functional violation (Type I) on $t(3)$, following the termination of Phase I iterations corresponding to $M=2$, makes it necessary to invoke Algorithm Phase I for the third time, after implementing a suitable variable update by invoking Algorithm Phase II. Within the second invocation of Algorithm Phase II, $t(3)$ becomes a controllable variable, and $\varphi(1)$ becomes a dependent variable. The resulting break-points during Phase I iterations $M=3$ are given in table 4.4.f.

Table 4.4.f

Summary of break points resulting in the third invocation of Algorithm
Phase I - Example 4

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.26198	V(3) is deactivated at setting 1.0031
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the third time, the variables contained in vector ξ are,

Voltages : $V_{PV}(2)$, $V_{PQ}(3)$, $V_{PV}(4)$ & $V_{PQ}(5)$

On-line Controllers: $t(3)$

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

Table 4.4.f indicates that except for variable $V(3)$, all other variables remain at their bounds, including $t(3)$, which became controllable within Phase I iteration $M=3$. As before (example 3) this makes it possible for Phase I iterations to converge quickly, usually within the first continuation process ($K=0$), to a very good precision (0.221×10^{-3} in this example).

The variations in the optimal cost are shown in figure 4.16. Coordination iteration count $N=19$ marks the termination of Phase I iterations $M=1$. At $N=19$, Algorithm Phase II is invoked in order to update the control and dependent variable specifications, to remove the Type I functional violations on variables $V(5)$, $V(3)$, and Type II functional violation on variable $Q(1)$. The cost at the updated operating point corresponding to the new variable specifications, computed by invoking Algorithm Coordination, corresponds to $N=20$. The reduction in the cost from 2776.29 (at $N=19$) to 2773.00 (at $N=20$) \$/hr, is due to decreased losses at the new operating point. Recall that in the previous example (example 3), in which different controls become dependent, and different injections prevail, the optimal cost following a new variable specification is higher than the cost at the termination of Phase I iterations $M=1$. This demonstrates the effect of the

control, dependent variables, and injections which are responsible for defining the starting solution to the subsequent continuation process. The termination of the subsequent continuation process, i.e., $K=0$, within Phase I iteration $M=2$ solves the current (second) relaxed OPF problem. Note that the optimal cost at $N=25$ (2776.97 \$/hr), i.e., the termination of Phase I iterations $M=2$ (the solution to the second relaxed OPF problem) is higher than the cost at $N=19$, i.e., 2776.29 \$/hr, (the solution to the first relaxed OPF problem), demonstrating the impact of the newly active constraints, upon the optimal cost. Note the fractional rise in cost associated with maintaining reactive generation $Q(1)$, voltages $V(3)$ and $V(5)$ at a limit. This is so, since a re-dispatch of reactive power affects the optimal cost only marginally. Had $Q(1)$ been replaced by a line current at a limit, the equivalent change in cost would have been noticeable, since a re-dispatch of real generation would have been necessary to maintain a line current at a limit, and yet meet the system demand at all system buses (cf example 2 and 3).

At this juncture Algorithm Phase II has to be reinvoked in order to remove functional (Type I) violation on variable $t(3)$. The updated cost at the new operating point resulting from the control and dependent variable updates amounts to 2776.95 \$/hr at $N=26$, with (almost) no change from the previous cost at $N=25$ (2776.97 \$/hr). This implies that the operating points, before and after the invocation of Algorithm Phase II are either very close, or lie on a region of the LF manifold in which the losses are relatively insensitive to the operating point. No additional tests are carried out to identify which of these two situations apply to the problem at hand, since such identification is unnecessary to the success of the optimization. Note that this situation sharply contrasts the ones at $N=18$, 19 and 20 where the operating points are such that, the difference in losses become quite apparent. The solution to the third relaxed OPF problem, which is also the solution to the complete OPF problem, is reached at the termination of the continuation process $K=0$, at corrector iteration count $N=32$, within Phase I iteration $M=3$. The optimal cost amounts to 2777.81 \$/hr, higher than the cost at $N=25$, (2776.97 \$/hr), the solution to the previous (second) relaxed OPF problem. Since the successor relaxed OPF is more constrained than the

predecessor relaxed OPF, the cost of the successor will be generally higher than the cost of the predecessor.

Figure 4.17 gives the BIC's obtained at $N=19$, $N=25$ and $N=32$, i.e., the terminal points of Phase I iterations $M=1, 2$ and 3 , or at the solution of each relaxed OPF problem considered within each Phase I iteration. As before λ represents the BIC at the slack bus. Since each successor relaxed OPF problem is more constrained than the predecessor, the value of λ becomes progressively higher.

The scenario at the termination of Phase I iteration $M=1$ remains the same as before in example 3, except that $Q(1)$ exceeds its upper limit. All line currents are within their bounds.

At the termination of Phase I iteration $M=2$, among the voltages at load buses, $V(3)$ is at an upper limit, $V(5)$ at a lower limit, and $V(6)$ within limits. All three shunt controllers at these buses are constrained at an upper bound. Since $V(5)$ is at a lower limit, the incremental cost of reactive power at this bus is above the other two. The BIC at bus(3) is smaller than its previous value, but remains insignificant. The voltage at generation bus(1) is within limits, but since the reactive generation at this bus is constrained at a limit, the incremental cost of reactive power increases from its previous value. A further Phase I iteration is needed at this point to contain the violation on $t(3)$.

At the termination of Phase I iteration $M=3$, the voltage and shunt controller scenarios remains as before except for $V(3)$ which is inactive. The optimization effort needed to contain variable $t(3)$ at an upper limit is reflected as higher incremental costs of reactive power at load buses 3, 5, 6 and generation bus 1, although the incremental cost at bus(3) continues to remain small and outside the resolution of the plot. Note that the BIC corresponding to the reactive demand at bus(5) has the highest magnitude since $V(5)$ is constrained at a lower limit, and the shunt controller 2 at this bus is at an upper limit.

From Phase I iteration M-1 to M-3, the BIC's with respect to the real demand increases marginally from their previous values, but maintain approximately the same relative position with respect to λ , which itself undergoes only a marginal increase. Note that λ and the BIC's with respect to the reactive demands at buses 1,3,5 and 6, increase monotonically from M-1 to M-3. This is due to the successor relaxed OPF being more constrained than the predecessor OPF

As observed previously, the active reactive generation affects the BIC's with respect to the reactive demand more than the BIC's with respect to the real demand. Stated alternately, the shortage in reactive generation causes the incremental costs in reactive demand to rise.

The following discussion will summarise the results, resulting from simulation corresponding to specification C 1.5 (example 5).

4.1.5 Simulations Based on Specification C.1.5

Specification C 1.5 is identical to C 1.4 except for the following changes, i.e.,

1. The current limit on transmission line I_3 (i.e., line 2-3) is increased from 1.00 to 1.11 pu A.
2. The current limit on transmission line I_4 (i.e., line 2-5) is increased from 1.32 to 3.32 pu A.
3. The current limit on transmission line I_6 (i.e., line 4-6) is increased from 2.50 to 3.32 pu A.
4. The upper limit on the reactive generation $Q(1)$ is increased from 6 to 16 MVAR, and the upper limits on generations $Q(4)$ and $Q(5)$ are decreased from 200 and 80 MVAR respectively, to 10 MVAR each.
5. The limits on voltages $V(3)$ and $V(5)$ are changed to match those at the remaining buses.

As before, the changes are to investigate the solution potential of the RHCM algorithm, in handling different types of functional violations following the termination of Phase I iterations M-1.

Table 4.5.a summarises the solution process, and table 4.5.b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 5 (specification C 1.5).

Table 4.5 a
Summary of the solution process - Example 5

	Phase I (No 1)	Phase I (No 2)
Number of Restarts	5	2
Functional Violations	Q(4), Q(1)	NONE
Variables Leaving x	NONE	NONE
Variables Entering x	V(1), t(2)	NONE

Table 4 5 b
Performance statistics of the RHCM and MINOS Ver 4 0 - Example 5

	RHCM	MINOS
Number of Jacobian Evaluations	135	82
Exec Speed (CPU sec on Amdahl 5860)	0 990	1 250
Core Reqd. (M-bytes on Amdahl 5860)	0 312	0 616

The highlights of table 4 5 a will now be discussed

In this example the voltages are given an initial flat specification (as given in C.1.5), different from the ones used in all the previous examples. This makes the variations in the optimal cost, within Phase I iteration M-1 (figure 4 18) different to those observed before. However, the optimal solution at the termination of Phase I iterations M-1 is the same, within acceptable numerical precision. Note that due to the starting point used, this example requires five restart iterations to convergence within Phase I iteration M-1, i.e., further the starting point from the optimal solution, higher the number of restart iterations to converge.

Table 4.5.c gives the break-points which result, within Phase I iterations M-1, when a flat voltage profile is used as a starting solution.

Table 4.5.c

Summary of break points resulting in the first invocation of Algorithm
Phase I — Example 5

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.09272	V(1) is deactivated at setting 0.9948
	0.21235	t(3) is deactivated at setting 1.0299
	0.22209	$\varphi(1)$ is deactivated at setting -0.0554
	0.29317	V(4) is deactivated at setting 1.0371
	0.59075	B _{sh} (1) is deactivated at setting 0.0000
	0.64035	t(2) is deactivated at setting 0.9320
	1.00000	Termination of the continuation process
1	0.00000	Beginning of the continuation process
	0.02715	$\varphi(1)$ is deactivated at setting -0.0464
	0.24730	t(3) is deactivated at setting 1.0530
	0.25882	B _{sh} (1) is deactivated at setting 0.0170
	0.25965	t(2) is deactivated at setting 0.9893
	0.39957	V(4) is deactivated at setting 1.0664
	0.43799	V(1) is deactivated at setting 1.0474
	1.00000	Termination of the continuation process
2	0.00000	Beginning of the continuation process
	0.02202	V(1) is deactivated at setting 1.0442
	0.06767	$\varphi(1)$ is deactivated at setting -0.0288
	0.12809	t(3) is deactivated at setting 1.0188
	0.26889	B _{sh} (1) is deactivated at setting 0.0111
	0.32976	t(2) is deactivated at setting 0.9533
	0.65649	V(4) is deactivated at setting 1.0897
	1.00000	Termination of the continuation process
3	0.00000	Beginning of the continuation process
	0.00026	$\varphi(1)$ is deactivated at setting -0.0288
	0.00552	V(4) is deactivated at setting 1.0889
	0.00738	B _{sh} (1) is deactivated at setting 0.0109
	0.01044	t(3) is deactivated at setting 1.0162
	0.08820	t(2) is deactivated at setting 0.9480
	0.44213	V(1) is deactivated at setting 1.0723
	1.00000	Termination of the continuation process

Table 4.5.c cont.

4	0.00000	Beginning of the continuation process
	0.00376	$\varphi(1)$ is deactivated at setting -0.0278
	0.00287	$t(2)$ is deactivated at setting 0.9479
	0.02417	$V(1)$ is deactivated at setting 1.0680
	0.06047	$t(3)$ is deactivated at setting 1.0080
	0.25775	$B_{sh}(1)$ is deactivated at setting 0.0242
	1.00000	Termination of the continuation process
5	0.00000	Beginning of the continuation process
	0.00698	$\varphi(1)$ is deactivated at setting -0.0262
	0.06609	$t(3)$ is deactivated at setting 0.9997
	0.13953	$B_{sh}(1)$ is deactivated at setting 0.0186
	0.21367	$t(2)$ is deactivated at setting 0.9335
	0.38869	$V(1)$ is deactivated at setting 1.0826
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the first time (i.e., $M=1$), the variables contained in vector ξ are,

Voltages : $V_S(1)$, $V_{PV}(2)$ & $V_{PV}(4)$

On-line Controllers: $\varphi(1)$, $t(2)$ & $t(3)$

Shunt Controllers: $B_{sh}(1)$, $B_{sh}(2)$ & $B_{sh}(3)$

Within Phase I iteration $M=1$, variables $V(1)$, $V(4)$, $\varphi(1)$, $t(2)$, $t(3)$ and $B_{sh}(1)$ are released from the active ξ -set, during restart iterations $K=0, 1$, and 3, at the values indicated in table 4.5.c. During restart iteration $K=2$, a phenomenon similar to the one observed in the previous example during restart iteration $K=0$ occurs, i.e., variable $V(4)$ becomes inactive only after the release of variables $t(3)$, $B_{sh}(1)$ and $t(2)$ during the last interval increment in θ . Note that the CP corresponding to $V(4)$, i.e., 0.65649, lies beyond those of the other three variables which were inactivated during this last interval. This is an indication that the release of $V(4)$ should follow the release of other variables, and the RHCM algorithm is able to correctly identify this

During restart iteration $K=4$, $V(4)$ reaches its upper limit and remains there during the succeeding restart iteration $K=5$. Since all variables active at the optimal solution to the initial relaxed OPF are correctly identified at the commencement of iteration $K=5$, the terminal point of this iteration

satisfies the K-T conditions within precision ϵ . This aids to confirm the previous observations, i.e., once the correct active set of ξ -variables are identified, convergence is very rapid.

At this juncture Type II functional constraints Q(1) and Q(4) exceed their upper limits. In invoking Algorithm Phase II to correct these violations, both Q(1) and Q(4) are appended to the set of injections after being specified at their upper bounds. In response to the added injections, control ξ -variables V(1) and t(2) become dependent.

In the ensuing Phase I iterations (M=2), two restart iterations are needed prior to reaching the optimal solution. At this solution, among the variables which changed roles, both Q(1) and Q(4) remain at an upper limit, while V(1) and t(2) are within limits. Controllable ξ -variables ϕ (1) and t(2) are also within limits. Since no functional violations prevail at this point, this marks the solution to the complete OPF problem.

For purposes of comparison, the final solutions due to the RHCM and MINOS are be given in table 4.5.d.

Table 4.5.d
Solution summary - Example 5

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Lwr.Lmt	02.400	02.400
Unit(2)	Lwr.Lmt	68.950	68.950
Unit(3)	Lwr.Lmt	54.250	54.250
Unit(5)	Free	81.433	81.423
React. Generations (MVAR):			
Q(1)	Upr.Lmt	16.000	16.000
Q(2)	Free	01.470	01.469
Q(4)	Upr.Lmt	20.000	20.000

Table 4.5.d cont.

Bus Voltages (pu V) & Ang:			
V(1)	Free	1.079/0.000	1.079/0.000
V(2)	Upr. Lmt	1.100/0.193	1.100/0.192
V(3)	Free	1.008/0.110	1.008/0.110
V(4)	Upr. Lmt	1.100/0.121	1.100/0.121
V(5)	Free	0.974/-0.032	0.974/-0.033
V(6)	Free	1.044/0.007	1.044/0.007
Tap & Phi (rad) Positions:			
$\phi(1)$	Free	-0.021	-0.024
t(2)	Free	0.917	0.917
t(3)	Free	0.959	0.958
Sht. Controllers (pu mho):			
B _{sh} (1)	Upr. Lmt	0.050	0.050
B _{sh} (2)	Upr. Lmt	0.050	0.050
B _{sh} (3)	Upr. Lmt	0.050	0.050
Optimal Cost in (\$/hr):			
		2773.49	2773.39

Note that since the line currents on all lines fall within their limiting values they are not reported above.

The following discussion will consider the break-points resulting within the second invocation of Algorithm Phase I.

Table 4.5.e

Summary of break points resulting in the second invocation of Algorithm
Phase I - Example 5

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.09480	V(4) is deactivated at setting 1.0843
	0.16894	$\varphi(1)$ is deactivated at setting 0.0275
	0.42086	t(3) is deactivated at setting 0.9514
	0.48653	B _{sh} (1) is deactivated at setting 0.0356
	0.68502	B _{sh} (3) is deactivated at setting 0.0500
	0.71397	V(2) is deactivated at setting 1.1000
	0.90793	B _{sh} (2) is deactivated at setting 0.0500
	1.00000	Termination of the continuation process
1	0.00000	Beginning of the continuation process
	0.06578	t(3) is deactivated at setting 0.9627
	0.19205	$\varphi(1)$ is deactivated at setting -0.0308
	0.97334	V(4) is deactivated at setting 1.0996
	1.00000	Termination of the continuation process
2	0.00000	Beginning of the continuation process
	0.05483	t(3) is deactivated at setting 0.9586
	0.03651	$\varphi(1)$ is deactivated at setting -0.0208
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the second time, the variables contained in vector ξ are,

Voltages : V_{PV}(2) & V_{PV}(4)
 On-line Controllers: $\varphi(1)$, t(3)
 Shunt Controllers: B_{sh}(1), B_{sh}(2) & B_{sh}(3)

Table 4.5.e gives the value of the continuation parameter θ at which the above variables are released (if necessary) from the active ξ -set, and their corresponding settings. Variables not released reach their bounds (upper or lower) at the termination of the continuation process. Among all the variables in vector ξ , only variables $\varphi(1)$ and t(3) are inactive at the final solution. Note that, at the termination of restart iteration K=0, variables V(2), B_{sh}(2) and B_{sh}(3) are inactive at their upper bounds. However during the subsequent restart iteration K=1, these variables remain

at their upper bound, thereby forcing $B_{sh}(1)$ to reach its upper limit at the termination of this iteration. At the termination of restart iteration $K=2$, $V(4)$ also reaches its upper limit. Since no functional violations prevail at this point, this marks the solution to the complete OPF problem.

The variations in the optimal cost are shown in figure 4.17. Within Phase I iterations $M=1$, coordination iteration count $N=5$ (cost 2786.38 \$/hr) marks the termination of restart iteration $K=0$. However, since variables $t(3)$, $\varphi(1)$, $V(4)$, $B_{sh}(1)$ and $t(2)$ indicate a conflict they are released from the active ξ -set, at settings estimated via linear interpolations, and Algorithm Coordination is reinvoked to update the operating point to correspond with the new specification of these variables. The termination of this iteration is marked by $N=6$ (cost 2779.4 \$/hr), with a reduction in optimal cost from the previous operating point. During the next restart iterations $K=1$, the cost rises sharply towards the last interval increment in θ , i.e., $N=11$ (cost 2790.77 \$/hr) since the controls $t(3)$, $B_{sh}(1)$, $t(2)$, $V(4)$ and $V(1)$ are moved beyond their optimal settings. On releasing these variables from the active ξ -set and updating the operating point the cost reduces at $N=12$ to 2775.89 \$/hr. A similar pattern of events can be observed for restart iterations $K=2, 3$ and 4. However, the magnitude of the variation in cost gradually diminishes as the solution is approached. The final solution is reached at $N=37$ cost 2773.89 \$/hr. Observe that in this last restart iteration $K=5$, the cost increases slightly following the update iteration due to the masking effect of system losses discussed in the previous example (example 4).

At this juncture, Algorithm Phase II is invoked to change the variable specifications in order to handle the functional violations prevalent among $Q(1)$ and $Q(4)$ at $N=37$. The new starting solution for Phase I iterations $M=2$ corresponds to $N=38$ (cost 2779.75), higher than the previous cost at $N=37$. A similar pattern of events, discussed in the previous paragraph can be observed during restart iteration counts $K=0$ and 1, and the last restart iteration $K=2$. Note that the optimal solution is reached at $N=55$ has a cost of 2773.50 \$/hr, almost equal to the cost at the solution to the previous relaxed OPF problem, i.e., at $N=37$ (cost 2773.89 \$/hr). Generally whenever active reactive sources are present, the change brought about to the optimal

cost to the succeeding relaxed OPF problem is marginal, in comparison to the cost of the previous sub-problem in which violations are permitted among the respective reactive sources.

Figure 4.19 gives the BIC's obtained at $N=37$ and $N=55$, i.e., the terminal points of Phase I iterations $M=1$ and 2, or at the solution of each relaxed OPF problem considered within each Phase I iteration. As before λ represents the BIC corresponding to the real demand at the slack bus. Since each successor relaxed OPF problem is more constrained than the predecessor, the value of λ is higher in the second problem than in the first.

The scenario at the termination of Phase I iteration $M=1$ remains as before in example 4, except that functional violations now prevail among the reactive generations at buses 1 and 4. Voltages for $V(4)$ and $V(2)$ are at their upper limits, and the remaining voltages are within limits. Further all line current flows are also within limits.

Phase I iteration $M=2$ is able to contain the reactive generations $Q(1)$ and $Q(4)$ within limits. At this juncture the same voltage scenario as above prevails. The effect of containing the reactive generation is a general increase in the cost of incremental reactive power at the load buses 3,5,6 and the generation buses 2 and 4.

As observed before, the insufficient reactive generation in the system causes the incremental cost of reactive power to rise. The effect of this is felt only marginally among the incremental cost of real demand, which maintains a near constant profile with respect to λ .

The following discussion will summarise the results, resulting from simulation corresponding to specification C.1.6 (example 6).

4.2 Simulations on a 10-Bus System

Simulation results on a 10-bus system will be presented next. The system dimensions, in brief, are as follows, i.e.,

No. of Buses	- 10
No. of Transmission Lines	- 13
No. of On-Line Controllers	- 05
No. of Shunt Controllers	- 00
No. of Generation (PV) Buses	- 06
No. of Load (PQ) Buses	- 03
No. of Generation Units	- 14

The discussion to follow, will commence with a study of results, resulting from simulations based on specification C.1 6 (example 6)

4.2.1 Simulations Based on Specification C.2.1

Table 4.6.a summarises the solution process, and table 4.6 b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 6 (specification C.1.6).

Table 4 6.a
Summary of the solution process - Example 6

	Phase I (No.1)	Phase I (No 2)	Phase I (No 3)
Number of Restarts	4	17	12
Functional Violations	Q(4)	V(6), Q(1)	NONE
Variables Leaving x	NONE	V(6)	NONE
Variables Entering x	V(6)	V(4), V(8)	NONE

Table 4.6.b
Performance statistics of the RHCM and MINOS Ver. 4 0 - Example 6

	RHCM	MINOS
Number of Jacobian Evaluations	367	830
Exec Speed (CPU sec on Amdahl 5860)	4 850	14 06
Core Req'd. (M-bytes on Amdahl 5860)	0 324	0.608

The highlights of table 4.6.a will now be discussed.

In the solution process, three Phase I iterations are involved. In the first, 4 restart iterations are required for convergence i.e., to bring down the absolute value of the largest LM of the free controllable ξ -variables below a specified precision ϵ . In the second and third invocation of Algorithm Phase I, 17 and 12 restart iterations are needed.

The termination of the first Phase I iteration (M-1) identifies one violation among the Type II functions, and no violations among the Type I functional variables. The Type II violation is due to the reactive generation at bus(4), i.e., $Q(4)$ being above its upper limit. In order to remove this violation, Algorithm Phase II appends $Q(4)$ to the set of injections, after specifying it at its upper limit, and changes of the role of the voltage at bus(6), i.e., $V(6)$, from that of a controllable variable, to that of a dependent LF variable, i.e., a Type I functional. At the termination of Phase I iteration M-2, $Q(4)$ is at an upper limit, but $V(6)$, now a Type I functional is above its maximum permissible value. Further, the reactive generation at bus(1), i.e., $Q(1)$ is also above its maximum permissible value thereby constituting a Type II violation. In order to remove these violations, Algorithm Phase II appends $Q(1)$ to the set of injections after specifying it at an upper limit, and places $V(6)$ among the set of controllable ξ -variables after specifying it at the violated bound. In response to these changes variables $V(8)$ and $V(4)$, from vector ξ , are now made dependent. At the termination of the subsequent Phase I iteration M-3, all functional violations are removed, and the optimum of the relaxed OPF becomes the optimum of the complete OPF problem. At the optimum, among the variables which changed roles following the previous Phase I iterations, variables $Q(4)$, $Q(1)$, $V(6)$ are at an upper limit, and $V(4)$ and $V(8)$ are within limits.

For purposes of comparison, the final solutions due to the RHCM and MINOS are given in table 4.6.c.

Table 4.6.c
Solution summary - Example 6

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Free	204.52	204.53
Unit(2)	Free	193.40	193.39
Unit(3)	Free	192.93	192.93
Unit(4)	Free	206.30	206.30
Unit(5)	Free	206.91	206.91
Unit(6)	Free	078.18	078.18
Unit(7)	Free	077.51	077.51
Unit(8)	Free	074.97	074.97
Unit(9)	Free	073.54	073.54
Unit(10)	Upr. Lmt	217.00	217.00
Unit(11)	Free	067.47	067.47
Unit(12)	Free	062.00	062.00
Unit(13)	Free	102.29	102.29
Unit(14)	Free	214.65	214.65
React. Generations (MVAR):			
Q(1)	Upr. Lmt	120.00	120.00
Q(2)	Free	006.96	006.62
Q(4)	Upr. Lmt	390.00	390.00
Q(6)	Free	109.28	109.40
Q(8)	Free	153.07	153.41
Q(9)	Free	054.66	054.53
Q(10)	Free	090.31	090.33
Bus Voltages (pu V) & Ang:			
V(1)	Upr. Lmt	1.050/-0.000	1.050/-0.000
V(2)	Upr. Lmt	1.050/-0.059	1.050/-0.059
V(3)	Free	0.978/-0.109	0.978/-0.109
V(4)	Free	1.023/-0.053	1.023/-0.053
V(5)	Free	1.019/-0.121	1.019/-0.121
V(6)	Upr. Lmt	1.050/-0.075	1.050/-0.075
V(7)	Free	1.018/-0.075	1.018/-0.075
V(8)	Free	1.046/-0.039	1.046/-0.039
V(9)	Upr. Lmt	1.050/-0.079	1.050/-0.079
V(10)	Upr. Lmt	1.050/-0.017	1.050/-0.017
Tap & Phi (rad) Positions:			
t(1)	Free	0.991	0.990
t(2)	Free	0.998	0.998
t(3)	Free	0.988	0.988
t(4)	Free	0.994	0.994

Table 4.6.c cont.

Optimal Cost in (\$/hr):		
	2786.28	2786.27

Note that since the line currents on all lines fall within their limiting values they are not reported above.

The following discussion will consider the break-points resulting within the first, second and third invocation of Algorithm Phase I. In all three instances, only the break-points resulting within the restart iteration count $K=0$, and those occurring within the final restart iteration are provided. The general characteristics observed and discussed previously in section 4.1.1.3, are found to hold true among the break-points occurring within the intermediate restart iterations.

Table 4.6.d

Summary of break points resulting in the first invocation of Algorithm Phase I - Example 6

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.00478	t(5) is deactivated at setting 0.9994
	0.01942	t(4) is deactivated at setting 0.9970
	0.01967	V(6) is deactivated at setting 1.0469
	0.03884	t(2) is deactivated at setting 0.9951
	0.04388	V(9) is deactivated at setting 1.0500
	0.05619	V(4) is deactivated at setting 1.0433
	0.07177	V(8) is deactivated at setting 1.0418
	0.07662	t(3) is deactivated at setting 0.9913
	0.08053	V(10) is deactivated at setting 1.0500
	0.09657	t(1) is deactivated at setting 0.9893
	1.00000	Termination of the continuation process

Table 4 6.d cont.

4	0.00000	Beginning of the continuation process
	0.00377	t(5) is deactivated at setting 0 9959
	0.00416	V(8) is deactivated at setting 1.0421
	0.00421	t(1) is deactivated at setting 0 9850
	0.00478	t(2) is deactivated at setting 0 9960
	0.00486	t(3) is deactivated at setting 0.9878
	0.00590	t(4) is deactivated at setting 0 9970
	0 06915	V(4) is deactivated at setting 1 0458
	0 36880	V(6) is deactivated at setting 1 0483
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the first time, the variables contained in vector ξ are,

Voltages $V_S(1)$, $V_{PV}(2)$, $V_{PV}(4)$, $V_{PV}(6)$, $V_{PV}(8)$, $V_{PV}(9)$ & $V_{PV}(10)$

On-line Controllers: $t(1)$, $t(2)$, $t(3)$, $t(4)$ & $t(5)$

Table 4.6 d gives the value of the continuation parameter θ at which the above variables are released (if necessary) from the active ξ -set, and their corresponding settings. Variables not released reach their bounds (upper or lower) at the termination of the continuation process. Among the voltages at generation buses, $V(1)$ and $V(2)$ reach their upper limits at the termination of iteration $K=0$. Note that at this point $V(9)$ and $V(10)$ are inactive at their upper limits. However, as seen in table 4 6 d, at the termination of Phase I iteration $M=1$, voltages $V(9)$ and $V(10)$ reach their upper limits, but voltages $V(4)$, $V(6)$, $V(8)$ and all TCUL devices remain inactive within their respective bounds.

Figure 4 20 gives a plot of the optimal costs computed at the termination of each restart iteration. The cost (2783 98 \$/hr) at corrector iteration count $N=0$, amounts to the value at which the coordination equations converge for the first time, with respect to the initial settings of the controls in vector ξ . This is the cost corresponding to the solution defining the initial ($K=0$) continuation process within Phase I iteration $M=1$. At the termination of Phase I iterations $M=1$ ($N=28$), this cost decreases to 2783.90 \$/hr. The interesting feature here is the amount of numerical computation

(28 corrector iterations) needed to bring about this marginal reduction in cost.

At this juncture Type II functional $Q(4)$ exceeds its upper limit. Algorithm Phase II is invoked in order to implement an appropriate variable update to contain this violation, along the lines discussed previously. Algorithm Phase I is invoked for the second time ($M=2$) in order to solve a second relaxed OPF problem to conform with the new variable specifications. At the termination of the ensuing continuation process $K=0$, the optimal cost rises to 2791.42 at $N=35$. This perhaps demonstrates the initial impact of the newly added injection specification, i.e., $Q(4)$, and the change in the variable specifications

The break-points resulting during Phase I iteration $M=2$ are given in table 4.6.e. As before, only the break-points resulting during restart iteration $K=0$, and those occurring within the final restart iterations are tabulated.

Table 4.6.e

Summary of break points resulting in the second invocation of Algorithm Phase I - Example 6

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.12307	V(8) is deactivated at setting 1.0438
	0.12582	t(5) is deactivated at setting 0.9746
	0.13654	t(1) is deactivated at setting 0.9653
	0.14437	V(4) is deactivated at setting 1.0230
	0.14541	V(9) is deactivated at setting 1.0500
	0.14622	t(3) is deactivated at setting 0.9668
	0.15578	t(2) is deactivated at setting 0.9722
	0.15678	V(10) is deactivated at setting 1.0500
	0.15880	t(4) is deactivated at setting 0.9726
	0.15929	V(1) is deactivated at setting 1.0500
	0.17099	V(2) is deactivated at setting 1.0500
	1.00000	Termination of the continuation process

Table 4.6.e cont

17	0.00000	Beginning of the continuation process
	0 00083	V(4) is deactivated at setting 1 0258
	0 00014	t(2) is deactivated at setting 0 9887
	0.00236	V(8) is deactivated at setting 1.0417
	0 00255	t(3) is deactivated at setting 0 9869
	0.00343	t(1) is deactivated at setting 0 9833
	0 00425	t(4) is deactivated at setting 0 9910
	0 00727	t(5) is deactivated at setting 0 9985
	1 00000	Termination of the continuation process

In invoking Algorithm Phase I for the second time (M=2), the variables contained in vector ξ are,

Voltages $V_S(1), V_{PV}(2), V_{PV}(4), V_{PV}(8), V_{PV}(9) \text{ \& } V_{PV}(10)$
 On-line Controllers $t(1), t(2), t(3), t(4) \text{ \& } t(5)$

At the termination of restart iteration K=0, all of the above controllable ξ -variables are released from the active set. Observe that variables V(1), V(2), V(9) and V(10) although at their upper limits, are not contained in the active set, i.e., these variables are inactive at their upper limits. During the subsequent restart iterations voltages at buses 1, 2, 9 and 10 become active at their upper bounds. At the termination of Phase I iteration M=2 at K=17, all TCUL devices, and the voltages at buses 4 and 8 are within limits. The cost at this point amounts to 2786.23 \$/hr, at N=125. Note that this is a rise from the cost at the termination at Phase I iterations M=1 (2783.90 \$/hr at N=28), indicating the impact of the newly added injection Q(4), but this cost is lower than the one at the termination of restart iteration K=0, within Phase I iteration M=2 (2791.4 \$/hr at N=35), substantiating the effort of the optimization.

At this juncture, functional violations prevail among variables Q(1) and V(6) making it necessary to invoke Algorithm Phase II for a second time. The change in the variable specifications remain as discussed before. Algorithm Phase I is invoked for the third time (M=3) in order to solve a third relaxed OPF problem to conform with the new variable specifications. At the

termination of the ensuing continuation process $K=0$, the optimal cost rises marginally to 2786.49 at $N=131$. As before this perhaps demonstrates the initial impact of the newly added injection specification, i.e., $Q(1)$, and the change in the variable specifications.

The break-points resulting during Phase I iteration $M=3$ are given in table 4.6.f. As before, only the break-points resulting during restart iteration $K=0$, and those occurring within the final restart iterations are tabulated.

Table 4.6.f
Summary of break points resulting in the third invocation of Algorithm
Phase I - Example 6

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.00986	t(4) is deactivated at setting 0.9901
	0.01681	t(3) is deactivated at setting 0.9845
	0.01897	V(1) is deactivated at setting 1.0470
	0.02150	t(5) is deactivated at setting 1.0018
	0.02213	t(1) is deactivated at setting 0.9872
	0.02266	t(2) is deactivated at setting 0.9924
	0.02354	V(9) is deactivated at setting 1.0500
	0.03149	V(6) is deactivated at setting 1.0500
	0.03318	V(2) is deactivated at setting 1.0500
	0.03747	V(10) is deactivated at setting 1.0500
	1.00000	Termination of the continuation process
12	0.00000	Beginning of the continuation process
	0.00133	t(4) is deactivated at setting 0.9941
	0.00139	t(3) is deactivated at setting 0.9878
	0.00165	t(2) is deactivated at setting 0.9981
	0.00188	t(1) is deactivated at setting 0.9905
	0.00211	t(5) is deactivated at setting 1.0040
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the third time ($M=3$), the variables contained in vector ξ are,

Voltages : $V_s(1)$, $V_{PV}(2)$, $V_{PV}(6)$, $V_{PV}(9)$ & $V_{PV}(10)$

On-line Controllers: t(1), t(2), t(3), t(4) & t(5)

At the termination of restart iteration $K=0$, all of the above controllable ξ -variables are released from the active set. Observe that variables $V(2)$, $V(6)$, $V(9)$ and $V(10)$ although at their upper limits, are not contained in the active set, i.e., these variables are inactive at their upper limits. During the subsequent restart iterations voltages at buses 1,2,6,9 and 10 become active at their upper bounds. At the termination of Phase I iteration $M=3$ at $K=12$, all TCUL devices are within limits, and all voltages in vector ξ are at their upper bounds. The cost at this point amounts to 2786 28 \$/hr., at $N=195$. Note that this is a marginal rise from the cost at the termination at Phase I iterations $M=2$ (2786 23 \$/hr at $N=125$), indicating the impact of the newly added injection $Q(1)$, and the change in the variable specifications. However, this cost is lower than the one at the termination of restart iteration $K=0$, within Phase I iteration $M=3$ (2786 49 \$/hr at $N=131$), substantiating the effort of the optimization.

Since no functional violations prevail at the termination of Phase I iterations $M=3$, this marks the optimal solution

Figure 4.21 gives the variations in the BIC's at the termination of Phase I iterations $M=1,2$ and 3. The incremental cost of real demand at the slack bus is represented by λ .

At the termination of Phase I iteration $M=3$, voltages at generation buses 1,2,9 and 10 are at an upper limit, and those at 4,8 and 6 are within limits. Among the load buses 3,5 and 7 all voltages are within limits. Note that this example does not carry any shunt controllers. Among all the load buses, bus(7) has the largest voltage magnitude, and the lowest incremental cost of reactive demand, which falls outside the resolution of the plot, and bus(3) has the smallest voltage magnitude and the largest incremental cost of reactive demand. The functional violation on $Q(4)$ is not taken into consideration in computing these incremental costs.

At the termination of Phase I iteration $M=2$, reactive generation $Q(4)$ is contained at an upper limit, but functional violations exist among the

voltage at the generation bus(6), and reactive generation $Q(1)$. The voltages at the load buses 3,5 and 7 remain within bounds. The effect of containing $Q(4)$ at an upper limit corresponds to a rise in the incremental costs of reactive demands at all the load buses, and a marginal rise in the incremental cost of reactive power at bus(4). Among all load buses, bus(3) continues to have the lowest voltage magnitude and the highest incremental cost of reactive demand.

At the termination of Phase I iteration $M=3$, reactive generations $Q(1)$ and $Q(4)$ are contained at an upper limit. The voltages at the load buses 3,5 and 7, remain within bounds, and the voltage at bus(1) is an upper bound. The effect of the active generation $Q(1)$ is marginally sensed throughout the system, since $V(1)$, and $Q(1)$ are at upper limits. Such a scenario at bus(1) implies that although the reactive generation is constrained at an upper limit, any further reactive demand from this bus can be supplied at the expense of $V(1)$ or $Q(1)$. As observed in the previous examples, active $Q(1)$ and $Q(4)$ have only marginal effect on the profile of BIC's corresponding to real demands. A general increase (although marginal) can be observed for λ , reflecting the progressively constrained solutions to the OPF sub-problems.

The following discussion will summarise the results, resulting from simulation corresponding to specification C.1.7 (example 7).

4.3 Simulations on a 30-Bus System

Simulation results on a 30-bus system will be presented next. The system dimensions, in brief, are as follows, i.e.,

No. of Buses	- 30
No. of Transmission Lines	- 41
No. of On-Line Controllers	- 04
No. of Shunt Controllers	- 09
No. of Generation (PV) Buses	- 05
No. of Load (PQ) Buses	- 24
No. of Generation Units	- 06

The discussion to follow, will commence with a study of results, resulting from simulations based on specification C.1.6 (example 6).

4.3.1 Simulations Based on Specification C.3.1

Table 4.7.a summarises the solution process, and table 4.7.b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver 4.0, when used to solve example 7 (specification C.1.7).

Table 4.7 a
Summary of the solution process - Example 7

	Phase I (No.1)
Number of Restarts	9
Functional Violations	NONE
Variables Leaving x	NONE
Variables Entering x	NONE

Table 4.7.b
Performance statistics of the RHCM and MINOS Ver. 4.0 - Example 7

	RHCM	MINOS
Number of Jacobian Evaluations	131	2158
Exec Speed (CPU sec on Amdahl 5860)	5.940	99.68
Core Req'd. (M-bytes on Amdahl 5860)	0.352	0.616

The highlights of table 4.7.a will now be discussed.

The solution process requires only one Phase I iteration. In this 9 restart iterations are required for convergence, i.e., to bring down the absolute value of the largest LM of the free controllable ξ -variables below a specified precision ϵ . Since the terminal solution of Phase I iteration M-1 does not have any functional violations, this corresponds to the complete solution to the OPF problem.

For purposes of comparison, the final solutions due to the RHCM and MINOS are given in table 4.7.c.

Table 4.7.c
Solution summary - Example 7

Variable	Status	Value(RHCM)	Value(MINOS)
Real Generations in (MW):			
Unit(1)	Free	176.05	176.11
Unit(2)	Free	048.84	048.84
Unit(3)	Free	021.52	021.52
Unit(4)	Free	022.16	022.20
Unit(5)	Free	012.25	012.26
Unit(6)	Lwr.Lmt	012.00	012.00
React. Generations (MVAR):			
Q(1)	Free	-13.94	-14.68
Q(2)	Free	030.32	029.92
Q(5)	Free	029.89	030.15
Q(8)	Free	036.48	036.66
Q(11)	Free	014.43	014.74
Q(13)	Free	008.44	008.97

Table 4.7.c cont.

Bus Voltages (pu V) & Ang;			
V(1)	Upr.Lmt	1.050/-0.000	1.050/-0.000
V(2)	Free	1.038/-0.064	1.038/-0.064
V(3)	Free	1.027/-0.100	1.027/-0.100
V(4)	Free	1.021/-0.120	1.022/-0.120
V(5)	Free	1.010/-0.183	1.011/-0.183
V(6)	Free	1.017/-0.140	1.018/-0.140
V(7)	Free	1.006/-0.167	1.007/-0.167
V(8)	Free	1.019/-0.145	1.019/-0.145
V(9)	Free	1.022/-0.177	1.021/-0.178
V(10)	Free	1.034/-0.209	1.033/-0.209
V(11)	Upr.Lmt	1.050/-0.154	1.050/-0.154
V(12)	Free	1.039/-0.195	1.038/-0.195
V(13)	Upr.Lmt	1.050/-0.179	1.050/-0.179
V(14)	Free	1.029/-0.212	1.029/-0.212
V(15)	Free	1.030/-0.217	1.029/-0.217
V(16)	Free	1.031/-0.206	1.031/-0.207
V(17)	Free	1.031/-0.213	1.030/-0.213
V(18)	Free	1.023/-0.228	1.022/-0.228
V(19)	Free	1.022/-0.231	1.021/-0.231
V(20)	Free	1.027/-0.228	1.026/-0.228
V(21)	Free	1.026/-0.219	1.025/-0.219
V(22)	Free	1.026/-0.218	1.026/-0.219
V(23)	Free	1.027/-0.226	1.026/-0.227
V(24)	Free	1.021/-0.228	1.020/-0.229
V(25)	Free	1.025/-0.224	1.024/-0.224
V(26)	Free	1.008/-0.231	1.007/-0.231
V(27)	Free	1.037/-0.217	1.036/-0.217
V(28)	Free	1.014/-0.149	1.015/-0.149
V(29)	Free	1.026/-0.242	1.025/-0.242
V(30)	Free	1.011/-0.255	1.010/-0.255
Tap & Phi (rad) Positions:			
t(1)	Free	1.005	1.002
t(2)	Free	0.956	0.954
t(3)	Upr.Lmt	1.100	1.100
t(4)	Free	1.037	1.035

Table 4.7.c cont.

Sht. Controllers (pu mho):			
B _{sh} (1)	Upr.Lmt	0.050	0.050
B _{sh} (2)	Upr.Lmt	0.050	0.050
B _{sh} (3)	Upr.Lmt	0.050	0.050
B _{sh} (4)	Upr.Lmt	0.050	0.050
B _{sh} (5)	Upr.Lmt	0.050	0.050
B _{sh} (6)	Upr.Lmt	0.050	0.050
B _{sh} (7)	Free	0.037	0.037
B _{sh} (8)	Upr.Lmt	0.050	0.050
B _{sh} (9)	Free	0.029	0.029
Optimal Cost in (\$/hr):			
		802.217	802.599

Note that the optimal cost provided by MINOS is marginally higher than that provided by the RHCM. Further, since the line currents on all lines fall within their limiting values they are not reported above.

The following discussion will consider the break-points resulting within the first invocation of Algorithm Phase I. The break-points occurring within the restart iteration count $K=0$, and those occurring within the final restart iteration are provided in table 4.7.d. The general characteristics observed and discussed previously in section 4.1.1.3, are found to hold true among the break-points occurring within the intermediate restart iterations.

Table 4.7.d

Summary of break points resulting in the first invocation of Algorithm
Phase I - Example 7

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.02053	V(2) is deactivated at setting 1.0016
	0.02507	t(2) is deactivated at setting 0.9964
	0.19041	V(8) is deactivated at setting 0.9860
	0.24631	t(4) is deactivated at setting 1.0329
	0.41003	t(1) is deactivated at setting 0.9516
	0.45997	B _{sh} (9) is deactivated at setting 0.0260
	0.54986	V(1) is deactivated at setting 1.0300
	0.57355	V(5) is deactivated at setting 0.9690
	0.77096	B _{sh} (1) is deactivated at setting 0.0398
	0.77424	V(11) is deactivated at setting 1.0400
	0.80645	B _{sh} (7) is deactivated at setting 0.0414
	0.83847	B _{sh} (5) is deactivated at setting 0.0428
	0.86716	t(3) is deactivated at setting 1.0882
	0.92388	B _{sh} (4) is deactivated at setting 0.0466
	0.94815	B _{sh} (3) is deactivated at setting 0.0477
9	0.96672	B _{sh} (8) is deactivated at setting 0.0485
	1.00000	Termination of the continuation process
	0.00000	Beginning of the continuation process
	0.00458	V(8) is deactivated at setting 1.0186
	0.00535	V(5) is deactivated at setting 1.0103
9	0.00634	t(1) is deactivated at setting 1.0051
	0.00681	B _{sh} (7) is deactivated at setting 0.0374
	0.00980	t(4) is deactivated at setting 1.0370
	0.01076	B _{sh} (9) is deactivated at setting 0.0288
	0.03170	V(2) is deactivated at setting 1.0379
	0.05636	t(2) is deactivated at setting 0.9559
	1.00000	Termination of the continuation process

In invoking Algorithm Phase I for the first time (M=1), the variables contained in vector ξ are,

Voltages : V_s(1), V_{PV}(2), V_{PV}(5), V_{PV}(8), V_{PV}(11) & V_{PV}(13)

On-line Controllers: t(1), t(2), t(3) & t(4)

Shunt Controllers: B_{sh}(1), B_{sh}(2), B_{sh}(3), B_{sh}(4), B_{sh}(5), B_{sh}(6),
B_{sh}(7), B_{sh}(8), B_{sh}(9)

At the termination of restart iteration $K=0$, i.e., the first continuation process variables $V(13)$, $B_{sh}(2)$ and $B_{sh}(6)$ reach their optimal settings, i.e., their respective upper bounds. Among the 16 inactive ξ -variables, most of those which have a break-point closer to unity, i.e., $B_{sh}(8)$, $B_{sh}(3)$, $B_{sh}(4)$, $t(3)$ and $B_{sh}(5)$ reach a bound at the final optimum. The exception being $B_{sh}(7)$ and $V(1)$, where although $B_{sh}(7)$ has a break-point closer to unity it remains inactive, while $V(1)$ which has a breakpoint further from unity reaches an upper bound at the optimum. An interesting feature is that, as observed in previous examples, most of the variables contained in vector ξ come very close to their optimal settings at the termination of the first continuation process, i.e., $K=0$.

At $K=0$, there are 16 inactive ξ -variables, which reduce gradually to 8 at the optimal solution at $K=9$. The influence of inactive ξ -variables among each other is such that generally, greater their number, higher will be the restart iteration count at the corresponding optimal solution. Having a large number of inactive ξ -variables is certainly advantages, since they assist to contain functional violations in subsequent Phase I iterations.

Two cases of this example have been investigated. The data pertaining to both examples are identical, except for the fact that in Case II the upper limits on all bus voltages have been relaxed to 1.10 pu. The starting solution embodied remains the same for both cases, i.e., flat voltage profile at all buses, nominal settings for on-line controllers, i.e., centre tap position for all TCUL devices, and zero susceptance for all shunt controllers.

Figures 4.22 and 4.24 give the variation in the optimal costs, for Cases I and II, computed at the termination of each continuation process. Observe that the starting cost is identical in both figures. The final optimal cost corresponding to Case I amounts to 802.217 \$/hr reached at restart iteration count $K=9$, while that for Case II is 799.15 \$/hr reached at the termination of restart iteration $K=15$. The relaxed constraints make the cost of Case II lower than that of Case I.

In Case II there are 10 inactive ξ -variables, i.e., $t(3)$ and $B_{sh}(5)$ in addition to the 8 in Case I. This perhaps explains the higher number of restart iterations. The solution times are 10.25 and 99.40 CPU sec, using the RHCM and MINOS on the Amdahl computer. Note that the solution time with respect to the RHCM has almost doubled, while that with respect to MINOS remains almost unchanged. The solution and the number of Jacobian evaluations needed by the RHCM* (799 14 \$/hr and 229) continues to be lower than that obtained from MINOS (799 50 \$/hr and 2105). The optimal active set of both controllable and dependent variables remain identical.

The variations in the BIC's for Cases I and II are plotted in figures 4.23 and 4.25. Note that in spite of the relative magnitudes of BIC's being lower in figure 4.25 than in figure 4.23, the general variation in the profile with respect to the BIC at the slack bus λ remains the same.

The incremental cost of reactive power at all generation buses, i.e., buses 1, 2, 5, 8, 11 and 13 are zero since the reactive generation at these locations are within their operating limits. Note that the incremental cost of reactive power at all load buses remain insignificant (almost zero), except at bus(26). Although the voltage at this bus is within limits, there are no shunt controllers directly connected to this bus, and the only neighbor to bus(26), i.e., bus(25) does not carry one either (see figure C.3 in Appendix C), which perhaps explains the significant (with respect to the same at other buses) cost of incremental reactive demand.

The following, and final discussion will summarise the results, resulting from simulation corresponding to specification C.1.8 (example 8).

4.4 Simulations on a 118-Bus System

Simulation results on a 118-bus system will be presented next. The system dimensions, in brief, are as follows, i.e.,

No. of Buses	- 118
No. of Transmission Lines	- 179
No. of On-Line Controllers	- 010
No. of Shunt Controllers	- 009
No. of Generation (PV) Buses	- 048
No. of Load (PQ) Buses	- 069
No. of Generation Units	- 062

The discussion to follow, will commence with a study of results, resulting from simulations based on specification C.1.8 (example 8).

4.4.1 Simulations Based on Specification C.4.1

Table 4.8.a summarises the solution process, and table 4.8.b gives a comparative summary of the performance of the restart homotopy continuation method (RHCM) algorithm, and MINOS Ver. 4.0, when used to solve example 8 (specification C.1.8).

Table 4.8.a
Summary of the solution process - Example 8

	Phase I (No.1)
Number of Restarts	20
Functional Violations	NONE
Variables Leaving x	NONE
Variables Entering x	NONE

Table 4.8.b
Performance statistics of the RHCM and MINOS Ver. 4.0 - Example 8

	RHCM	MINOS ⁹
Number of Jacobian Evaluations	315	NA
Exec Speed (CPU sec on Amdahl 5860)	48 590	> 900 0
Core Req'd (M-bytes on Amdahl 5860)	0 452	1 764

The highlights of table 4.8 a will now be discussed.

The solution process requires only one Phase I iteration. In this 20 restart iterations are required for convergence, i.e., to bring down the absolute value of the largest LM of the free controllable ξ -variables below a specified precision ϵ . Since the terminal solution of Phase I iteration M=1 does not have any functional violations, this corresponds to the complete solution to the OPF problem

The complete solution to the 118-bus system is given in Appendix F

The following discussion will consider the break-points resulting within the first invocation of Algorithm Phase I, given in table 4.8 c. In this only the break-points resulting within the restart iteration count K=0, and those occurring within the final restart iteration are provided. The general characteristics observed and discussed previously in section 4.1.1.3, are found to hold true among the break-points occurring within the intermediate restart iterations.

⁹ MINOS did not converge for this example.

Table 4.8.c

Summary of break points resulting in the first invocation of Algorithm
Phase I - Example 8

K	θ	Reason
0	0.00000	Beginning of the continuation process
	0.00125	t(10) is deactivated at setting 1.0005
	0 00297	V(55) is deactivated at setting 1.0500
	0 00629	V(70) is deactivated at setting 1.0493
	0.00678	V(90) is deactivated at setting 1.0492
	0 00846	V(104) is deactivated at setting 1.0490
	0 00868	t(5) is deactivated at setting 0.9981
	0.01038	V(92) is deactivated at setting 1.0479
	0 01302	V(19) is deactivated at setting 1.0476
	0.01330	V(62) is deactivated at setting 1.0476
	0.01487	V(15) is deactivated at setting 1.0474
	0.01898	V(80) is deactivated at setting 1.0500
	0 02004	V(18) is deactivated at setting 1.0500
	0 02064	V(6) is deactivated at setting 1.0500
	0.02244	V(107) is deactivated at setting 1.0500
	0 03004	V(105) is deactivated at setting 1.0459
	0.03036	V(56) is deactivated at setting 1.0459
	0 03078	V(46) is deactivated at setting 1.0458
	0.03321	V(1) is deactivated at setting 1.0500
	0.03845	V(61) is deactivated at setting 1.0500
	0.05103	V(12) is deactivated at setting 1.0438
	0 05571	V(91) is deactivated at setting 1.0500
	0.05958	V(32) is deactivated at setting 1.0430
	0.07100	V(49) is deactivated at setting 1.0500
	0.07227	t(2) is deactivated at setting 1.0165
	0.07677	V(74) is deactivated at setting 1.0413
	0.07937	V(89) is deactivated at setting 1.0500
	0.08050	V(54) is deactivated at setting 1.0409
	0.08377	t(4) is deactivated at setting 1.0188
	0.08526	V(77) is deactivated at setting 1.0405
	0.08773	t(6) is deactivated at setting 1.0196
	0.09242	V(27) is deactivated at setting 1.0398
	0.10266	V(69) is deactivated at setting 1.0500
	0.10669	V(31) is deactivated at setting 1.0295
	0.12365	$\phi(7)$ is deactivated at setting 0.1152
	0.13384	t(9) is deactivated at setting 0.9542
	0.14393	$\phi(3)$ is deactivated at setting -1.247
	0.14394	V(40) is deactivated at setting 1.0262
	0.14472	V(87) is deactivated at setting 1.0500
	0.15003	t(1) is deactivated at setting 0.9513
	0.15073	V(76) is deactivated at setting 1.0256
	0.24274	V(34) is deactivated at setting 1.0500
	0.34424	V(42) is deactivated at setting 1.0084
	0.37609	V(36) is deactivated at setting 1.0500
	0.40909	V(65) is deactivated at setting 1.0500
	1.00000	Termination of the continuation process

Table 4.8.c cont.

20	0.0000000	Beginning of the continuation process
	0.0000005	t(1) is deactivated at setting 0.9645
	0.0000005	V(87) is deactivated at setting 1.0440
	0.0000066	V(104) is deactivated at setting 1.0460
	0.0000145	V(105) is deactivated at setting 1.0448
	0.0000476	t(10) is deactivated at setting 0.9977
	0.0000477	$\varphi(7)$ is deactivated at setting 0.0683
	0.0000535	V(56) is deactivated at setting 1.0268
	0.0000812	V(90) is deactivated at setting 1.0492
	0.0000886	V(54) is deactivated at setting 1.0267
	0.0001284	t(5) is deactivated at setting 0.9956
	0.0002328	t(9) is deactivated at setting 0.9695
	0.0002893	V(107) is deactivated at setting 1.0475
	0.0003554	$\varphi(3)$ is deactivated at setting -0.0835
	0.0006914	V(55) is deactivated at setting 1.0278
	0.0007615	t(6) is deactivated at setting 1.0123
	0.0008297	V(18) is deactivated at setting 1.0419
	0.0010374	V(12) is deactivated at setting 1.0416
	0.0012827	t(2) is deactivated at setting 0.9987
	0.0013044	V(62) is deactivated at setting 1.0483
	0.0014859	V(74) is deactivated at setting 1.0250
	0.0016266	V(69) is deactivated at setting 1.0362
	0.0017528	V(42) is deactivated at setting 1.0252
	0.0021129	V(80) is deactivated at setting 1.0331
	0.0021560	V(40) is deactivated at setting 1.0301
	0.0023012	V(32) is deactivated at setting 1.0388
	0.0023926	V(70) is deactivated at setting 1.0335
	0.0026677	V(24) is deactivated at setting 1.0488
	0.0030502	V(77) is deactivated at setting 1.0255
	0.0030614	V(49) is deactivated at setting 1.0419
	0.0035477	V(92) is deactivated at setting 1.0476
	0.0053283	V(15) is deactivated at setting 1.0403
	0.0059441	V(76) is deactivated at setting 1.0200
	0.0067571	V(72) is deactivated at setting 1.0476
	0.0074722	V(99) is deactivated at setting 1.0470
	0.0077194	V(73) is deactivated at setting 1.0396
	0.0087988	V(46) is deactivated at setting 1.0379
	0.0109260	V(27) is deactivated at setting 1.0389
	0.0110270	V(31) is deactivated at setting 1.0376
	0.0111980	V(19) is deactivated at setting 1.0412
	0.0136230	V(59) is deactivated at setting 1.0427
	0.0140360	V(6) is deactivated at setting 1.0464
	0.0176270	V(1) is deactivated at setting 1.0465
	1.0000000	Termination of the continuation process

Note that ξ -variables V(61), V(91), t(4), V(36) and V(65) which were inactive at the termination of the initial continuation process (K=0),

become active at the optimum, i.e., at the termination of restart iteration K=20. Further, variables V(24), V(72), V(99), V(73) and V(59) which were active at the termination of the first continuation process, become inactive at the optimum.

At the beginning of the continuation process K=0, a total of 68 controllable ξ -variables are present, which comprise of 49 voltages at generation buses (including the voltage at the slack bus), 10 TCUL devices, and 9 shunt controllers. At the optimal solution 26 of these variables reach a limit, while 42 remain inactive.

Figure 4.26 gives the variation in the optimal cost, at the termination of each continuation process, from inception to termination of the optimization process. The incremental costs of real and reactive demands at all 118 buses are given in Appendix F.

Case I - Variations in γ with r for the sequence $((\alpha_1^H)_K)$

Figure 4.1

Scatter Pattern of Break Points for ξ -Variables - Case I

Figure 4.2

Scatter Pattern of Break Points for ξ -Variables - Case II

Figure 4.3

Variation in the optimal cost from inception to termination
of the optimization process - Example 1

Figure 4.4

Variation in the optimal generation from inception to termination
of the optimization process - Example 1

Figure 4.5

Variation in the incremental transmission losses from inception to termination of the optimization process - Example 1

Figure 4.6

Variation in the generation voltage settings, and their LM's, from inception to termination of the optimization process - Example 1

Figure 4.7

Variation in the phase-shifter setting, and its LM, from inception to termination of the optimization process - Example 1

Figure 4.8

Variation in the tap changer settings, and their LM's, from inception to termination of the optimization process - Example 1

Figure 4.9

Variation in the shunt controller settings, and their LM's, from inception
 to termination of the optimization process - Example 1

Figure 4.10

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , computed at the termination of each Phase I iteration - Example 1

Figure 4.11

Variation in the optimal cost from inception to termination
of the optimization process - Example 2

Figure 4.12

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , computed at the termination of each Phase I iteration - Example 2

Figure 4.13

Variation in the optimal cost from inception to termination
of the optimization process - Example 3

Figure 4.14

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , computed at the termination of each Phase I iteration - Example 3

Figure 4.15

Variation in the optimal cost from inception to termination
of the optimization process - Example 4

Figure 4.16

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , computed at the termination of each Phase I iteration — Example 4

Figure 4.17

Variation in the optimal cost from inception to termination
of the optimization process - Example 5

Figure 4.18

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , computed at the termination of each Phase

I iteration - Example 5

Figure 4.19

Variation in the optimal cost, at the termination of each continuation process, from inception to termination of the optimization process - Example 6

Figure 4.20

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , computed at the termination of each Phase I iteration - Example 6

Figure 4.21

Variation in the optimal cost, at the termination of each continuation process, from inception to termination of the optimization process - Example 7 (Case I)

Figure 4.22

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , Example 7 (Case I)

Figure 4.23

Variation in the optimal cost, at the termination of each continuation process, from inception to termination of the optimization process - Example 7 (Case II)

Figure 4.24

The BIC's corresponding to both real and reactive demands, and the BIC at the slack bus λ , Example 7 (Case II)

Figure 4.25

Variation in the optimal cost, at the termination of each continuation process, from inception to termination of the optimization process - Example 8

Figure 4.26

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

5.1 Conclusions

The principal objective underlying this thesis has been to investigate the potential of the continuation method in solving the complete non-linear minimum cost optimal power flow problem.

As part of this investigation the restart homotopy continuation method (RHCM) algorithm is developed and tested on numerous IEEE, and other test systems ranging in size up to 118 buses, 179 transmission lines, 68 controllable variables, and 62 generation units. The performance statistics are attractive, especially with higher system dimensions. The table 5.1 compares the performance statistics of the RHCM algorithm, to that of the general purpose commercial code MINOS Ver. 4.0.

The consistently low dynamic core required to execute the program via the RHCM is primarily due to the compact formulation of the optimal power flow (OPF) problem, and due to sparse storage techniques employed in developing the code.

The execution speeds reflect the consistently good performance of the restart homotopy continuation method algorithm. This is primarily due to the efficient solution possible to each sub-problem via the continuation method, which monitors explicitly only a few of the controllable variables, rather than the whole set of controls and dependent (functional) variables. Further reduction in computing has been possible via the non-conventional load flow specification employed in this study, which is able to rapidly identify the functional variables which become active at the optimum.

Table 5 1.a
Performance statistics of the RHCM algorithm and MINOS¹

	NSB	NAV	NAF	ES [RHCM & MINOS]		DC [RHCM & MINOS]	
Ex.1	6	8	1	0.380	1.150	0.312	0.616
Ex.2	6	10	2	0.540	0.970	0.312	0.616
Ex.3	6	11	3	0.700	1.140	0.312	0.616
Ex.4	6	11	2	0.550	0.940	0.312	0.616
Ex.5	6	10	2	0.990	1.250	0.312	0.616
Ex.6	10	8	2	4.850	14.06	0.324	0.608
Ex.7	30	12	0	5.940	99.68	0.352	0.616
Ex.8	118	43	0	48.59	> 900.0	0.452	1.764
Ex.9 ²	118	45	1	53.62	NA	0.448	NA

The RHCM algorithm is found to be stable, i.e., the subset of the controllable variables explicitly monitored by the continuation process reach their optimal settings decisively without any form of instability (oscillations). The success of the overall algorithm is however pivoted on the successful solution to the set of load-flow equations.

In most of the test cases it is found that the initial sub-problem solves to completion without experiencing any numerical difficulties. Numerical difficulties, if present, manifest themselves within the load-flow module, by giving rise to a singular Jacobian, or a load flow which does not converge. Under such circumstances the functional violations at the termination of the previous sub-problem can be identified as being responsible for the numerical ill-conditioning. Hence the decomposition of the overall

¹ The following acronyms are used in table 5.1.

NSB: Number of System Buses

NAV: Number of Active Variables (this encompasses all the individual system variables at the optimum)

NAF: Number of Active Functions (types I and II only at the optimum)

ES: Execution Speed (CPU sec. on the Amdahl 5860)

DC: Dynamic Core required for executing the program (M-bytes on the Amdahl 5860)

Note: MINOS did not converge for examples 8 and 9.

² Due to space limitations this example is not discussed in this thesis.

problem into sub-problems helps to pinpoint problem infeasibilities efficiently.

An important property of the RHCM algorithm is that it does not require an initial feasible starting point. This makes the RHCM algorithm simple to apply with minimal effort for testing systems of different sizes. In testing new systems an overall flat voltage profile of 1 pu V on all system buses, nominal settings for all tap changing under load (TCUL) transformers and shunt controllers is found to work well.

The incremental costs of both real and reactive demands at all the system buses, commonly called the bus incremental costs, furnished at the optimal solution to each sub-problem, are able to clearly identify the effects of active functional constraints upon these costs, so that, if desired, rates for economy interchanges (i.e., transactions between the utility and its customers) can be optimally computed. It is found that active reactive sources primarily affect the incremental costs of reactive demand, while active transmission lines primarily affect the incremental cost of real demand at all system buses. The system incremental cost is found to generally rise with added active functional constraints. Further, incremental cost of reactive demand is found to be much smaller than the incremental cost of real demand. Generally the bus incremental costs are different at all system buses, irrespective of the presence or absence of active functional constraints.

It is found that the restart homotopy continuation method is efficient, stable and robust when used to solve the minimum cost optimal power flow problem.

5.2 Recommendations for Future Research

The following is a set of recommendations to extend the scope of the present study, and to enhance the convergence characteristics of the overall algorithm.

5.2.1 Newton Steps in the Predictor

In the implementation of Algorithm Phase I, the predictor steps are applied logarithmically. This is necessary at the inception of the tracking process, to ensure that corrector iterations can come back onto the homotopy path. However, after a few restart iterations the operating point will be sufficiently close to the relaxed optimum. This makes the logarithmic steps superfluous.

Efficiency in computing can be enhanced by relaxing the predictor steps to Newton steps, i.e., incrementing the continuation parameter θ directly to unity, by observing a certain index sensitive to the error between the current sub-optimum and the optimum of the problem being solved. The absolute value of the largest LM among the set of free ξ -variables, i.e., $|\alpha_{x1}^M|$, will serve as this indicator. It is felt that this feature, if successfully implemented, may provide better performance statistics than those reported here.

5.2.2 Fixed Jacobian Factors in the LF

As the optimum is approached, the general tendency is for $|\alpha_{x1}^M|$ to gradually decrease in magnitude. Under these circumstances the Jacobian elements tend to vary marginally. Hence, having constant Jacobian factors may expedite the execution. Additional simulation experience is needed to determine the periodicity for Jacobian updates, which will expedite the convergence without sacrificing either the convergence stability, or the accuracy of the resulting solution.

5.2.3 A New Basis for x - ξ Exchanges

Algorithm Update uses a set of intelligent rules for updating the state vector x . The heuristic nature of these rules, casts some doubt on their general success. Although theoretically justified, the numerical experience is such that the success of an x - ξ exchanges relies heavily upon the condition number of the load-flow Jacobian immediately following such an exchange.

Stated alternatively, for a given injection specification, if each element from among the set of ξ -variables were substituted one at a time for any given element of the state vector x currently contained in the load-flow specification, the condition number of the load-flow Jacobian can assume widely varying numbers. In order to enhance the convergence of the load-flow it is imperative that the condition number of the load-flow Jacobian must be kept to a minimum possible. The conventional load-flow specification has the best possible condition number. However, the formulation adopted in this thesis is such that this conventional specification cannot be maintained if functional violations (both Type I & II) prevail following the solution to a relaxed OPF sub-problem (or following the termination of a Phase I iteration). Hence a scheme needs to be developed to update the load-flow specification in such a manner so as to keep the condition number of the Jacobian as small as possible under the new set of injection specifications. This scheme would then serve to systematise the x - ξ exchange mechanism.

5.2.4 Simulations in the Load Domain

The simulations reported in this thesis are carried out in the domain of a single continuation parameter. This is adequate for optimum solutions to a single base case. However, there are instances in which an optimal trajectory with respect to a varying demand is desirable. The utility of such information would serve ideally the requirements for on-line demand tracking applications.

There are many ways to develop such a trajectory. One method would be to solve several base cases in order to generate this trajectory. This is not considered elegant, although the starting point of a successor optimization can always be made the terminal point of the predecessor optimization. This will save computing time provided the successor optimization does not require an update to the optimal active set of the predecessor optimization.

An elegant approach would be to reformulate the entire problem, with the system demand as the continuation parameter. By varying this demand over the interval of interest, the required trajectory can be drawn. An investigation

to evaluate the possibility of such a scheme will provide a valuable contribution to the power industry.

5.2.5 Switching to an Active Set Method

The efficiencies of active set methods are well known. The principal problems underlying such methods are the identification of the optimal active set. Among all elements of the active set, the functional variables bring about the greatest computational requirement. Hence if incorrectly identified, this can result in much wasted computing.

Judging by the final optimal costs, it is clear that the objective optimum is quite close to the relaxed³ optimum. Hence, there is good grounds to believe the possibility that all violated Type II functions should reach their violated bounds at the objective optimum. The simulations presented in Chapter IV appear to confirm this. Hence, overall convergence can be expedited by switching to an active set method, immediately following the first invocation of Algorithm Update. Provision must be made to allow for any erroneously identified functional constraints to be dropped from this set. Details of such a scheme remains to be developed.

5.2.6 Incorporation of Dynamic Constraints

Dynamic constraints pose additional problems to the solution endeavour. For instance, ramp constraints influence the optimal solution. Their inclusion in a study of this magnitude would no doubt widen the scope of its application. It may not be possible to implement the generally nonlinear ramping characteristics of a unit. Linearization about an operating point, should be considered as a possible alternative in future attempts to widen the scope of the present study.

³ The optimum, resulting from the execution of Algorithm Phase I for the first time.

5.2.7 Other Constraints and Objectives

The pollutants discharged into the atmosphere by thermal generating units, have been drawing greater attention in recent times. This calls for future OPF studies to incorporate environmental constraints and consider objectives not addressed in this thesis. This calls for experimentation with flexible and general objective functions in the OPF formulation. Conventional optimization methods, and homotopy methods, may have to be intertwined in order to exploit their various efficiencies, and for handling such complex constraints and objectives. The algorithmic and other details, remain to be developed.

5.2.8 Achieving Load-Flow Feasibility Following a Change in its Specification

A change in the load-flow specification becomes necessary if functional violations prevail following the solution to a relaxed optimal power flow sub-problem. In the present implementation if the violations are Type II, they are specified at the violated bound and appended to the set of injections, while if the violations are Type I they are specified at their violated bound and placed among the set of ξ -variables. As discussed previously the state vector is appropriately adjusted following such an update in the load-flow specification. The previous solution will serve as the starting solution to the subsequent load-flow iteration. However, this starting solution can sometimes fail to solve the load-flow. The failure may be due to infeasibilities, or due to the starting solution being inappropriate. The following proposal is aimed at resolving this dilemma.

For the sake of argument, let the violated functional constraint $h_1(u)$ be limited by the following inequality;

$$h_1(u) \leq 0 \quad \dots\dots(5.1.a)$$

Let the value of this functional constraint immediately following a Phase I iteration be $h^* \geq 0$. Hence the violation amounts to h^* .

In the subsequent iteration let this functional be specified by;

$$h_1(u) = (1-\theta_1)h^* \quad \dots\dots(5.1.b)$$

where θ_1 is a continuation parameter limited by $0 \leq \theta_1 \leq 1$. The advantage of this specification is that at $\theta_1=0$, the solution to the system of load-flow equations under the new and old specifications coincide with each other, i.e., the solution to the new load-flow specification is known at $\theta_1=0$. Assuming feasibility the desired solution can be reached by incrementing θ_1 to unity. Subsequently the normal Phase I iterations can be commenced to solve the new relaxed OPF problem.

5.2.9 Re-dispatch following Single or Multiple Contingencies

In practice situations may arise in which the steady-state optimal solution following a single or multiple contingency is desired. Starting the solution to such a problem from 'cold' would be rather undesirable since it may prove to be time consuming, numerically expensive, and would involve discarding the numerical effort which led to the steady state optimal solution prior to the contingency. The scheme to follow can use an existing restart homotopy continuation algorithm to reach the desired solution. The underlying principles of this scheme are very similar to those seen previously in section 5.2.8.

For the sake of argument, let the contingency be single and involve line (i-j), or the failure of a TCUL device connected to this line. Let the pre-contingency admittance of line i-j be Y_{ij} . Since the post contingency value of Y_{ij} must be zero, a single contingency involving line i-j can be simulated by the following homotopy, i.e.,

$$y_{ij} = (1-\theta_2)Y_{ij} \quad \dots\dots(5.2.a)$$

Where y_{ij} is the homotopically deformed value of Y_{ij} , and the continuation parameter θ_2 is limited by $0 \leq \theta_2 \leq 1$. Note that the solution to the OPF problem at $\theta_2=0$ is known since it coincides with the pre-contingency solution. The post-contingency solution, if feasible, is found by incrementing θ_2 to

unity. Note that for each incremented value of θ_2 , a Phase I of the restart homotopy continuation algorithm must be solved, and if new functional violations are encountered at the termination of each Phase I iteration, they must be accommodated into the subsequent formulation exactly as discussed before in the theory.

Multiple line contingencies can be handled in exactly the same manner provided such contingencies do not tear the network apart.

Contingencies involving generation units (or shunt controllers) must be handled differently such that the limits of the generation (or shunt controller) units involved in the contingency are homotopically collapsed to zero in order to simulate a contingency, [Huneault et al., 1984]. This is accomplished by the following homotopy, i.e.,

$$(1-\theta_2)P_{gi}^m \leq P_{gi} \leq (1-\theta_2)P_{gi}^M \quad \dots\dots(5.2.b)$$

Where index i represents the unit which has just been subjected to a contingency.

As before, for each incremented value of θ_2 a Phase I iteration must be solved. Further if functional violations prevail following the solution of a Phase I, these functional variables must be appropriately incorporated into the formulation exactly as discussed in the theory of the restart homotopy continuation algorithm. Further improvement to this scheme is possible by replacing P_{gi}^M with P_{gi}^* , where P_{gi}^* is the generation of unit i just prior to the contingency, a similar operation being valid for shunt controllers as well.

A reverse procedure to those discussed can be adopted to simulate the addition of a transmission line, TCUL device, generation unit or shunt controller in planning studies.

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APPENDIX A

USER SUPPLIED DATA FILES

A.1 General Structure for User Supplied Data Files

The following is the general format to be used, in all data files to be supplied by the user. The data for Example 6.1 is chosen for purposes of demonstration. The character strings need not be entered. Their presence here is only to clarify the type and nature of each element.

General Network Data

NB: No. of buses
 NL: No. of transmission lines
 ND: No. of devices, i.e., taps and phase shifters
 NSH: No. of controllable shunts
 NG: No. of generation buses

Table A.1.a

Proposed structure for general network data

NB	NL	ND	NSH	NG
6	7	3	3	3

System Demand

P_D : Real Demand
 Q_D : Reactive Demand

Table A.1.b

Proposed structure for system demand

P_D	Q_D
200.000	60.000

Data Pertaining to Transmission Lines

i: From bus No

j: To bus No

R: Longitudinal line resistance in pu (100 MVA base)

X_L : Longitudinal line reactance in pu (100 MVA base)

Y_C : Total capacitive susceptance of the line in pu (100 MVA base)

I^M : Maximum current limit of line, in pu amperes

Table A.1.c

Proposed structure for transmission line data

i	j	R	X_L	Y_C	I^M
1	4	0.080D0	0.370D0	0.030D0	3.00D0
1	6	0.123D0	0.518D0	0.042D0	2.00D0
2	3	0.723D0	1.050D0	0.010D0	1.00D0
2	5	0.282D0	0.640D0	0.005D0	1.50D0
3	4	0.005D0	0.133D0	0.005D0	5.00D0
4	6	0.097D0	0.407D0	0.030D0	2.50D0
5	6	0.005D0	0.300D0	0.005D0	3.00D0

TCUL Device Data¹

Type²: Type of TCUL device, identified as either 'TAP' or 'PHI'

k: Bus at which device is connected

i: One end of line carrying the device

j: Other end of the line carrying the device

U: Upper limit of the TCUL device

L: Lower limit of the TCUL device

¹ See also section B.5 (Appendix B).

² The phase-shifter limits are in radians.

Table A.1.d
Proposed structure for TCUL device data

Type	k	i	j	U	L
'PHI'	3	3	2	0.18D0	-0.18D0
'TAP'	4	4	3	1.10D0	0.90D0
'TAP'	6	6	5	1.10D0	0.90D0

Controllable Shunt Data

- k: Bus carrying the controllable shunt element
 U: Upper limit of the shunt susceptance in pu (100 MVA base)
 L: Lower limit of the shunt susceptance in pu (100 MVA base)

Table A.1.e
Proposed structure for controllable shunt data

k	U	L
3	0.05D0	0.00D0
5	0.05D0	0.00D0
6	0.05D0	0.00D0

Type and % Load Carried by the Buses

- B_T : Bus Type
1. Slack Bus (V, δ)
 2. Load Bus (P, Q)
 3. Generation Buses, including any bus with synchronous condensers; (PV)

P_{D1} : % of P_D carried at the bus

Q_{D1} : % of Q_D carried at the bus

V^N : Initial (nominal) voltage setting at the bus

V^M : Upper limit for bus voltage variations

V^m : Lower limit for bus voltage variations

Note: Data is arranged in an ascending order of bus numbers:

Table A.1.f

Proposed structure for type and % load carried by the various buses

B_T	P_{D1}	Q_{D1}	V^N	V^M	V^m
1	0.200D0	0.140D0	1.10D0	1.10D0	0.99D0
3	0.350D0	0.100D0	1.10D0	1.10D0	0.99D0
2	0.100D0	0.100D0	1.10D0	1.10D0	0.99D0
3	0.000D0	0.250D0	1.10D0	1.10D0	0.99D0
2	0.250D0	0.200D0	1.10D0	1.10D0	0.99D0
2	0.100D0	0.210D0	1.10D0	1.10D0	0.99D0

Generation Data

k: Bus number at which the plant is situated

 NP_i : Number of units per plant i P_{gi}^M : Maximum real power rating of the unit 'i' P_{gi}^m : Minimum real power rating of the unit 'i' Q_{gi}^M : Maximum reactive power rating of the unit 'i' Q_{gi}^m : Minimum reactive power rating of the unit 'i'

a_i, b_i : Cost coefficients of units, whose quadratic cost curve takes the form $c_i + a_i P_i + b_i P_i^2$

Table A.1.g

Proposed structure for generation data

i	k	NP_i	P_{gi}^M	P_{gi}^m	Q_{gi}^M	Q_{gi}^m	a_i	b_i
(1)	1	1	12.00D0	2.400D0	6.000D0	0.00D0	23.2780D0	0.2746D0
(1)	2	2	197.0D0	68.95D0	100.0D0	-40.0D0	20.0227D0	0.0060D0
(2)			155.0D0	54.25D0	80.00D0	-50.0D0	9.27060D0	0.0134D0
(1)	4	2	0.000D0	0.000D0	200.0D0	-50.0D0	0.00000D0	0.0000D0
(2)			155.0D0	54.25D0	80.00D0	-50.0D0	9.27060D0	0.0134D0

Note: The plant number i is implied, i.e., need not be explicitly entered by the user. Hence, it is shown within parenthesis:

APPENDIX B

**ANALYTICAL EXPRESSIONS FOR INJECTIONS AND PARTIAL DERIVATIVES
APPEARING IN THE FORMULATION OF THE RHCM ALGORITHM**

B.1 Analytical Expressions for Injection Specifications

The derivations for the real and reactive injections are given in Elgerd [1982].

$$P_i = P_{gi} - P_{di} = \sum_{k=1}^{NB} |V_i V_k Y_{ik}| \cos(\theta_i - \theta_k + \theta_{ik}) \quad \dots\dots(B.1.a)$$

$$Q_i = Q_{gi} - Q_{di} = \sum_{k=1}^{NB} |V_i V_k Y_{ik}| \sin(\theta_i - \theta_k + \theta_{ik}) \quad \dots\dots(B.1.b)$$

$$I_i^2 = (I_i^M)^2 = [V_i^2 + V_j^2 - 2*V_i*V_j*\cos(\theta_i - \theta_j)]*Y_{ij}^2 \quad \dots\dots(B.1.c)$$

TCUL transformers (see section B.5) will modify equation B.1.c according to table B.1.a (section B.2)

The changes to the variables identified on the top of table B.1.a, due to the presence of controls indicate on the left, are given within table B.1.a.

In the above derivation the following definitions are assumed, i.e.,

$$V_i = V_i \angle \theta_i \quad \dots\dots(B.1.d)$$

$$V_j = V_j \angle \theta_j \quad \dots\dots(B.1.e)$$

$$Y_{ij} = Y_{ij} \angle -\theta_{ij} = G_{ij} - jB_{ij} \quad \dots\dots(B.1.f)$$

B.2 Elements of The LF Jacobian

The analytical expression for the diagonal and off diagonal elements of the LF Jacobian are given below, i.e.,

$$\frac{\partial P_i}{\partial \theta_i} = \sum_{j=1; \text{excluding } j=i}^{NB} -V_i V_j Y_{ij} \sin(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.2.a)$$

$$\frac{\partial P_i}{\partial \theta_j} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.2.b)$$

$$\frac{\partial P_i}{\partial V_i} = \sum_{j=1; \text{excluding } j=i}^{NB} V_j Y_{ij} \cos(\theta_i - \theta_j + \theta_{ij}) + 2 * V_i Y_{ii} \cos(\theta_{ii}) \quad \dots (B.2.c)$$

$$\frac{\partial P_i}{\partial V_j} = V_i Y_{ij} \cos(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.2.d)$$

$$\frac{\partial Q_i}{\partial \theta_i} = \sum_{j=1; \text{excluding } j=i}^{NB} V_i V_j Y_{ij} \cos(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.2.e)$$

$$\frac{\partial Q_i}{\partial \theta_j} = -V_i V_j Y_{ij} \cos(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.2.f)$$

$$\frac{\partial Q_i}{\partial V_i} = \sum_{j=1; \text{excluding } j=i}^{NB} V_j Y_{ij} \sin(\theta_i - \theta_j + \theta_{ij}) + 2 * V_i Y_{ii} \sin(\theta_{ii}) \quad \dots (B.2.g)$$

$$\frac{\partial Q_i}{\partial V_j} = V_i Y_{ij} \sin(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.2.h)$$

$$\frac{\partial I_i^2}{\partial \theta_i} = 2 * V_i * V_j * \sin(\theta_i - \theta_j) * Y_{ij}^2 \quad \dots (B.2.i)$$

$$\frac{\partial I_i^2}{\partial \theta_j} = 2 * V_i * V_j * \sin(\theta_j - \theta_i) * Y_{ij}^2 \quad \dots (B.2.j)$$

$$\frac{\partial I_i^2}{\partial V_i} = 2 * a_1 * [V_i - V_j \cos(\theta_i - \theta_j)] * Y_{ij}^2 \quad \dots (B.2.k)$$

$$\frac{\partial I_i^2}{\partial V_j} = 2 * a_2 * [V_j - V_i \cos(\theta_i - \theta_j)] * Y_{ij}^2 \quad \dots (B.2.l)$$

I. equations B.2.i-1 above, the variables a_1 - a_2 =1 whenever no TCUL devices are present. However, the presence of such devices bring about the following modifications to the variables as indicated below, i.e.,

Table B.1.a

Modification to voltages, angles, and variables a_1 , a_2 due to TCUL devices

	V_i	V_j	θ_i	θ_j	a_1	a_2
t at node i	tV_i	—	—	—	t	1
t at node j	—	tV_j	—	—	1	t
φ at node i	—	—	$\theta_i + \varphi$	—	1	1
φ at node j	—	—	—	$\theta_j + \varphi$	1	1

The changes to the variables identified on the top of the table B.1.a due to the presence of controls indicate on the left, are given within the table B.1.a.

B.3 Elements of Vector $\frac{\partial P_i}{\partial x}|_{\xi}$

The elements of the above vector are needed in the computation of the ITL coefficients.

The system losses are given by the following equation, i.e.,

$$P_L = P_L(x, \xi) \quad \text{.....(B.3.a)}$$

The compact modelling, described in section 2.2, is used to develop all the theory outlining this study. The compact model is derived from the sparse model.

The set of LF equations according to the sparse model 2.2 is given by the following equation, i.e.,

$$z = z(x, \xi) \quad \text{.....(B.3.b)}$$

From equations B.3.a&b, one arrives at the following transposed Jacobian equation, giving the set of ITL coefficients, i.e.,

$$J^t \frac{dP_i}{dz} \Big|_{\xi} = \frac{\partial P_i}{\partial x} \Big|_{\xi} \quad \dots (B.3.c)$$

Where J is the LF Jacobian, given by the following equation, i.e.,

$$J = \frac{\partial z}{\partial x} \quad \dots (B.3.d)$$

The elements of the partial derivative on the RHS of equation B.3.c are given below, i.e.,

$$\frac{\partial P_i}{\partial \theta_1} = V_1 V_i Y_{1i} \sin(\theta_1 - \theta_i + \theta_{1i}) + \sum_{k=2}^{NB} \frac{\partial P_k}{\partial \theta_1} \quad \dots (B.3.e)$$

$$\frac{\partial P_i}{\partial V_1} = V_i Y_{1i} \cos(\theta_1 - \theta_i + \theta_{1i}) + \sum_{k=2}^{NB} \frac{\partial P_k}{\partial V_1} \quad \dots (B.3.f)$$

B.4 Lagrange Multipliers of the ξ -Variables

Equation 3.5.d specifies the LM's of the ξ -variables. These are computed via the following matrix operations, derived from equations B.3.a&b, i.e.,

$$\frac{dP_i}{d\xi} \Big|_z = \frac{\partial P_i}{\partial \xi} \Big|_x - \frac{\partial z^t}{\partial \xi} \Big|_x \star \frac{dP_i}{dz} \Big|_{\xi} \quad \dots (B.4.a)$$

Observe that analytical expressions are not possible for these LM's, due to the presence of the vector of ITL's. However, analytical expressions can be derived for matrix $\partial z^t / \partial \xi \Big|_x$, and vector $\partial P_i / \partial \xi \Big|_x$. The following presentation will derive these expressions.

B.4.1 Elements of Matrix $\frac{\partial z^t}{\partial \xi} \Big|_x$

For purposes of derivation, if ξ_k represents a tap changer, at position 't', situated at bus 'i' on line i-j, the corresponding entries in the above matrix become;

$$\frac{\partial P_i}{\partial t} = 2*t*V_i^2 * G_{ij} + V_i V_j Y_{ij} \cos(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.4.b)$$

$$\frac{\partial P_j}{\partial t} = V_i V_j Y_{ij} \cos(\theta_j - \theta_i + \theta_{ij}) \quad \dots (B.4.c)$$

$$\frac{\partial Q_i}{\partial t} = 2*t*V_i^2 (B_{ij} - Y_C) + V_i V_j Y_{ij} \sin(\theta_i - \theta_j + \theta_{ij}) \quad \dots (B.4.d)$$

$$\frac{\partial Q_j}{\partial t} = V_i V_j Y_{ij} \sin(\theta_j - \theta_i + \theta_{ij}) \quad \dots (B.4.e)$$

$$\frac{\partial I_i^2}{\partial t} = 2*V_i * [t*V_i - V_j \cos(\theta_i - \theta_j)] * Y_{ij}^2, \text{ for } t \text{ at bus } i \quad \dots (B.4.f)$$

Further, for purposes of derivation, if ξ_k represents a phase shifter, at position ' φ ', situated at bus ' i ' on line $i-j$, the corresponding entries in the above matrix become;

$$\frac{\partial P_i}{\partial \varphi} = -V_i V_j Y_{ij} \sin(\theta_i - \theta_j + \theta_{ij} + \varphi) \quad \dots (B.4.g)$$

$$\frac{\partial P_j}{\partial \varphi} = V_i V_j Y_{ij} \sin(\theta_j - \theta_i + \theta_{ij} - \varphi) \quad \dots (B.4.h)$$

$$\frac{\partial Q_i}{\partial \varphi} = V_i V_j Y_{ij} \cos(\theta_i - \theta_j + \theta_{ij} + \varphi) \quad \dots (B.4.i)$$

$$\frac{\partial Q_j}{\partial \varphi} = -V_i V_j Y_{ij} \cos(\theta_j - \theta_i + \theta_{ij} - \varphi) \quad \dots (B.4.j)$$

$$\frac{\partial I_i^2}{\partial \varphi} = 2*V_i * V_j \sin(\theta_i + \varphi - \theta_j) * Y_{ij}^2, \text{ for } \varphi \text{ at bus } i \quad \dots (B.4.k)$$

Furthermore, for purposes of derivation, if ξ_k represents a SVC device at position ' B_{sh} ', situated at bus ' i ', the corresponding entries in the above matrix become;

$$\frac{\partial P_i}{\partial B_{sh}} = 0 \quad \dots (B.4.l)$$

$$\frac{\partial Q_i}{\partial B_{sh}} = -V_i^2 \quad \dots (B.4.m)$$

$$\frac{\partial I_1^2}{\partial B_{sh}} = 0 \quad \dots (B.4.n)$$

Analytical expressions for elements of the vector $\partial P_L / \partial \xi |_x$ in equation B.4.a will be derived next.

B.4.2 Elements of Vector $\frac{\partial P_L}{\partial \xi} |_x$

These elements are trivially obtained via the following equation, i.e.,

$$P_L = P_L(x, \xi) = \sum_{i=1}^{NB} P_i \quad \dots (B.4.o)$$

Thus the elements of the above vector become,

$$\frac{\partial P_L}{\partial t} |_x = \frac{\partial P_L}{\partial t} |_x + \frac{\partial P_L}{\partial t} |_x \quad \dots (B.4.p)$$

$$\frac{\partial P_L}{\partial \varphi} |_x = \frac{\partial P_L}{\partial \varphi} |_x + \frac{\partial P_L}{\partial \varphi} |_x \quad \dots (B.4.q)$$

$$\frac{\partial P_L}{\partial B_{sh}} |_x = 0 \quad \dots (B.4.r)$$

The following section will describe the models used in this study to represent TCUL devices.

B.5 Models for TCUL Devices

TCUL devices can be either tap-changers, or phase-shifters. Since these devices are fundamentally transformers, they are lossy. Much analytical simplification is possible if their losses are neglected, Elgerd [1982], Stevenson [1975]. Hence, the following analysis will assume such devices to be lossless.

B.5.1 Elements of the Y-Bus Due to a Tap-changer

A Tap-changer Connected to a Transmission Line

Figure B.1

The tap-changer, modifies the elements of the bus admittance matrix at locations i&j as shown below, i.e.,

$$\begin{matrix} & i & j \\ \begin{matrix} i \\ j \end{matrix} & \begin{bmatrix} t^2(Y_{ij} + kY_c) & -tY_{ij} \\ -tY_{ij} & (Y_{ij} + kY_c) \end{bmatrix} \end{matrix}$$

B.5.2 Elements of the Y-Bus Due to a Phase-shifter

A Phase-shifter Connected to a Transmission Line

Figure B.2

The phase-shifter, modifies the elements of the bus admittance matrix at locations i&j as shown below, i.e.,

$$\begin{matrix} & i & j \\ \begin{matrix} i \\ j \end{matrix} & \begin{bmatrix} (Y_{ij} + kY_c) & -Y_{ij} * 1/\varphi \\ -Y_{ij} * 1/\varphi & (Y_{ij} + kY_c) \end{bmatrix} \end{matrix}$$

The off diagonal asymmetry must be noted.

APPENDIX C

DATA SETS FOR TEST SYSTEMS

The following sections present the data sets used for simulations reported in Chapter IV. These specifications follow the recommendations given in Appendix A. The cases considered are designed such that, each, corresponds to a different combination of variables within the optimal active set. The schematic diagram of the 6-bus system is given below.

Schematic diagram of the 6-bus test system

Figure C.1

C.1 6-Bus Test System

This test system is taken from the text by Dhar [1982] and modified according to the specifications C.1.1 to C.1.5 given below.

C.1.1 Example 1

```

6 7 3 3 3
200.000 60.000
1 4 0.08000 0.37000 0.03000 3.0000
1 6 0.12300 0.51800 0.04200 2.0000
2 3 0.72300 1.05000 0.01000 1.0000
2 5 0.28200 0.64000 0.00500 1.5000
3 4 0.00500 0.13300 0.00500 5.0000

```



```

4 6 0.097D0 0.407D0 0.030D0 2.50D0
5 6 0.005D0 0.300D0 0.005D0 3.00D0
'PHI' 3 3 2 0.18D0 -0.18D0
'TAP' 4 4 3 1.10D0 0.90D0
'TAP' 6 6 5 1.10D0 0.90D0
3 0.05D0 0.00D0
5 0.05D0 0.00D0
6 0.05D0 0.00D0
1 0.200D0 0.140D0 1.10D0 1.10D0 0.99D0
3 0.350D0 0.100D0 1.10D0 1.10D0 0.99D0
2 0.100D0 0.100D0 1.10D0 1.10D0 0.99D0
3 0.000D0 0.250D0 1.10D0 1.10D0 0.99D0
2 0.250D0 0.200D0 1.10D0 1.10D0 0.99D0
2 0.100D0 0.210D0 1.10D0 1.10D0 0.99D0
1 1
12.00D0 02.40D0 006.0D0 00.0D0 23.2780D0 0.2746D0
2 2
197.0D0 68.95D0 100.0D0 -40.0D0 20.0227D0 0.0060D0
155.0D0 54.25D0 080.0D0 -50.0D0 09.2706D0 0.0134D0
4 2
0.000D0 0.000D0 200.0D0 -50.0D0 00.0000D0 0.0000D0
155.0D0 54.25D0 080.0D0 -50.0D0 09.2706D0 0.0134D0

```

G.1.2 Example 2

```

6 7 3 3-3
200.0D0 60.0D0
1 4 0.080D0 0.370D0 0.030D0 3.00D0
1 6 0.123D0 0.518D0 0.042D0 2.00D0
2 3 0.723D0 1.050D0 0.010D0 1.00D0
2 5 0.282D0 0.640D0 0.005D0 0.32D0
3 4 0.005D0 0.133D0 0.005D0 5.00D0
4 6 0.097D0 0.407D0 0.030D0 2.50D0
5 6 0.005D0 0.300D0 0.005D0 3.00D0
'PHI' 3 3 2 0.18D0 -0.18D0
'TAP' 4 4 3 1.10D0 0.90D0
'TAP' 6 6 5 1.10D0 0.90D0
3 0.05D0 0.00D0
5 0.05D0 0.00D0
6 0.05D0 0.00D0
1 0.200D0 0.140D0 1.10D0 1.10D0 0.99D0
3 0.350D0 0.100D0 1.10D0 1.10D0 0.99D0
2 0.100D0 0.100D0 1.01D0 1.01D0 0.99D0
3 0.000D0 0.250D0 1.10D0 1.10D0 0.99D0
2 0.250D0 0.200D0 1.15D0 1.15D0 1.05D0
2 0.100D0 0.210D0 1.10D0 1.10D0 0.99D0
1 1
12.00D0 2.40D0 26.00D0 0.0D0 23.278D0 0.2746D0
2 2
197.0D0 68.95D0 100.0D0 -40.0D0 20.0227D0 0.0060D0
155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0

```

4 2

0.000D0 0.000D0 200.0D0 -50.0D0 0.0000D0 0.0000D0
 155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0

C.1.3 Example 3

6 7 3 3 3

200.0D0 60.0D0

1 4 0.080D0 0.370D0 0.030D0 3.00D0

1 6 0.123D0 0.518D0 0.042D0 2.00D0

2 3 0.723D0 1.050D0 0.010D0 1.00D0

2 5 0.282D0 0.640D0 0.005D0 1.32D0

3 4 0.005D0 0.133D0 0.005D0 5.00D0

4 6 0.097D0 0.407D0 0.030D0 0.32D0

5 6 0.005D0 0.300D0 0.005D0 3.00D0

'PHI' 3 3 2 0.18D0 -0.18D0

'TAP' 4 4 3 1.10D0 0.90D0

'TAP' 6 6 5 1.10D0 0.90D0

3 0.05D0 0.00D0

5 0.05D0 0.00D0

6 0.05D0 0.00D0

1 0.200D0 0.140D0 1.10D0 1.10D0 0.99D0

3 0.350D0 0.100D0 1.10D0 1.10D0 0.99D0

2 0.100D0 0.100D0 1.01D0 1.01D0 0.99D0

3 0.000D0 0.250D0 1.10D0 1.10D0 0.99D0

2 0.250D0 0.200D0 1.15D0 1.15D0 1.05D0

2 0.100D0 0.210D0 1.10D0 1.10D0 0.99D0

1 1

12.00D0 2.40D0 46.00D0 0.0D0 23.278D0 0.2746D0

2 2

197.0D0 68.95D0 100.0D0 -40.0D0 20.0227D0 0.0060D0

155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0

4 2

0.000D0 0.000D0 200.0D0 -50.0D0 0.0000D0 0.0000D0

155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0

C.1.4 Example 4

6 7 3 3 3

200.0D0 60.0D0

1 4 0.080D0 0.370D0 0.030D0 3.00D0

1 6 0.123D0 0.518D0 0.042D0 2.00D0

2 3 0.723D0 1.050D0 0.010D0 1.00D0

2 5 0.282D0 0.640D0 0.005D0 1.32D0

3 4 0.005D0 0.133D0 0.005D0 5.00D0

4 6 0.097D0 0.407D0 0.030D0 0.32D0

5 6 0.005D0 0.300D0 0.005D0 3.00D0

'PHI' 3 3 2 0.18D0 -0.18D0

'TAP' 4 4 3 1.10D0 0.90D0

'TAP' 6 6 5 1.10D0 0.90D0

```

3 0.05D0 0.00D0
5 0.05D0 0.00D0
6 0.05D0 0.00D0
1 0.200D0 0.140D0 1.03D0 1.10D0 0.99D0
3 0.350D0 0.100D0 1.05D0 1.10D0 0.99D0
2 0.100D0 0.100D0 1.00D0 1.01D0 0.99D0
3 0.000D0 0.250D0 1.05D0 1.10D0 0.99D0
2 0.250D0 0.200D0 1.00D0 1.15D0 1.05D0
2 0.100D0 0.210D0 1.00D0 1.10D0 0.99D0
1 1
  12.00D0 2.40D0 6.00D0 0.0D0 23.278D0 0.2746D0
2 2
  197.0D0 68.95D0 100.0D0 -40.0D0 20.0227D0 0.0060D0
  155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0
4 2
  0.000D0 0.000D0 200.0D0 -50.0D0 0.0000D0 0.0000D0
  155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0

```

C.1.5 Example 5

```

6 7 3 3 3
200.0D0 60.0D0
1 4 0.080D0 0.370D0 0.030D0 3.00D0
1 6 0.123D0 0.518D0 0.042D0 2.00D0
2 3 0.723D0 1.050D0 0.010D0 1.11D0
2 5 0.282D0 0.640D0 0.005D0 3.32D0
3 4 0.005D0 0.133D0 0.005D0 5.00D0
4 6 0.097D0 0.407D0 0.030D0 3.32D0
5 6 0.005D0 0.300D0 0.005D0 3.00D0
'PHI' 3 3 2 0.18D0 -0.18D0
'TAP' 4 4 3 1.10D0 0.90D0
'TAP' 6 6 5 1.10D0 0.90D0
3 0.05D0 0.00D0
5 0.05D0 0.00D0
6 0.05D0 0.00D0
1 0.200D0 0.140D0 1.00D0 1.10D0 0.95D0
3 0.350D0 0.100D0 1.00D0 1.10D0 0.95D0
2 0.100D0 0.100D0 1.00D0 1.10D0 0.95D0
3 0.000D0 0.250D0 1.00D0 1.10D0 0.95D0
2 0.250D0 0.200D0 1.00D0 1.10D0 0.95D0
2 0.100D0 0.210D0 1.00D0 1.10D0 0.95D0
1 1
  12.00D0 2.40D0 16.00D0 0.0D0 23.278D0 0.2746D0
2 2
  197.0D0 68.95D0 100.0D0 -40.0D0 20.0227D0 0.0060D0
  155.0D0 54.25D0 80.0D0 -50.0D0 9.2706D0 0.0134D0
4 2
  0.000D0 0.000D0 10.0D0 -50.0D0 0.0000D0 0.0000D0
  155.0D0 54.25D0 10.0D0 -50.0D0 9.2706D0 0.0134D0

```

C.2 10-Bus Test System

The basic network and generation data for this test system is taken from [Adielson 1972]. The schematic diagram of the 10-bus system is given below.

Schematic diagram of the 10-bus test system

Figure C.2

C.2.1 Example 6

```

10 13 5 0 7
1950.000 1120.000
1 3 0.0113600 0.0553100 0.2025000 3.57100
1 4 0.0118500 0.0780200 0.3037500 2.53200
1 2 0.0098800 0.0484000 0.2025000 4.08200
1 8 0.0450400 0.1236500 0.2025000 1.59700
2 4 0.0098800 0.0484000 0.1012500 4.08200
3 5 0.0487900 0.1916000 0.2025000 1.03100
3 4 0.0487900 0.1916000 0.2025000 1.03100
4 6 0.0074100 0.0488900 0.1012500 4.04000
4 7 0.0163000 0.0651900 0.3037500 3.03000
4 10 0.0039500 0.0197500 0.2025000 10.0000
5 6 0.0187700 0.0628100 0.2025000 3.14500
5 9 0.0118500 0.0780200 0.3037500 2.53000

```

```

7 8 0.01630D0 0.06380D0 0.30375D0 3.090D0
'TAP' 3 3 1 1.10D0 0.90D0
'TAP' 4 4 1 1.10D0 0.90D0
'TAP' 4 4 2 1.10D0 0.90D0
'TAP' 5 5 9 1.10D0 0.90D0
'TAP' 7 7 8 1.10D0 0.90D0
1 0.00000D0 0.00000D0 1.05D0 1.05D0 0.95D0
3 0.00000D0 0.00000D0 1.05D0 1.05D0 0.95D0
2 0.12821D0 0.13393D0 1.05D0 1.05D0 0.95D0
3 0.51282D0 0.56250D0 1.05D0 1.05D0 0.95D0
2 0.07692D0 0.06696D0 1.05D0 1.05D0 0.95D0
3 0.05128D0 0.03125D0 1.05D0 1.05D0 0.95D0
2 0.05128D0 0.04464D0 1.05D0 1.05D0 0.95D0
3 0.12821D0 0.13393D0 1.05D0 1.05D0 0.95D0
3 0.05128D0 0.02679D0 1.05D0 1.05D0 0.95D0
3 0.00000D0 0.00000D0 1.05D0 1.05D0 0.95D0
1 1
217.0D0 80.0D0 120.0D0 -24.0D0 0.773D0 0.00680D0
2 2
217.0D0 80.0D0 120.0D0 -24.0D0 0.395D0 0.00886D0
217.0D0 80.0D0 120.0D0 -24.0D0 0.538D0 0.00814D0
4 4
217.0D0 80.0D0 120.0D0 -24.0D0 0.768D0 0.00700D0
217.0D0 80.0D0 120.0D0 -24.0D0 0.623D0 0.00768D0
108.0D0 40.0D0 75.0D0 -15.0D0 0.636D0 0.02016D0
108.0D0 40.0D0 75.0D0 -15.0D0 0.696D0 0.01956D0
6 2
108.0D0 40.0D0 75.0D0 -15.0D0 0.675D0 0.02066D0
108.0D0 40.0D0 75.0D0 -15.0D0 0.803D0 0.01932D0
8 2
217.0D0 80.0D0 120.0D0 -24.0D0 0.394D0 0.00784D0
108.0D0 40.0D0 75.0D0 -15.0D0 1.367D0 0.01246D0
9 2
72.0D0 30.0D0 40.0D0 -8.0D0 1.513D0 0.01204D0
108.0D0 40.0D0 75.0D0 -15.0D0 0.678D0 0.01546D0
10 1
217.0D0 80.0D0 120.0D0 -40.0D0 0.489D0 0.00786D0

```

C.3, 30-Bus Test System

The basic network and generation data for this test system (AEP 30-bus) is taken from [Alsaç et al., 1974]. The schematic diagram is given at the end of this appendix.

C.3.1 Example 7

30 41 4 9 6

283.40D0 126.20D0

1	2	0.0192D0	0.0575D0	0.0264D0	1.30D0
1	3	0.0452D0	0.1852D0	0.0204D0	1.30D0
2	4	0.0570D0	0.1737D0	0.0184D0	0.65D0
2	5	0.0472D0	0.1983D0	0.0209D0	1.30D0
2	6	0.0581D0	0.1763D0	0.0187D0	0.65D0
3	4	0.0132D0	0.0379D0	0.0042D0	1.30D0
4	6	0.0119D0	0.0414D0	0.0045D0	0.90D0
4	12	0.0000D0	0.2560B0	0.0000D0	0.65D0
5	7	0.0460D0	0.1160D0	0.0102D0	0.70D0
6	7	0.0267D0	0.0820D0	0.0085D0	1.30D0
6	8	0.0120D0	0.0420D0	0.0045D0	0.32D0
6	9	0.0000D0	0.2080D0	0.0000D0	0.65D0
6	10	0.0000D0	0.5560D0	0.0000D0	0.32D0
6	28	0.0169D0	0.0599D0	0.0065D0	0.32D0
8	28	0.0636D0	0.2000D0	0.0214D0	0.32D0
9	10	0.0000D0	0.1100D0	0.0000D0	0.65D0
9	11	0.0000D0	0.2080D0	0.0000D0	0.65D0
10	17	0.0324D0	0.0845D0	0.0000D0	0.32D0
10	20	0.0936D0	0.2090D0	0.0000D0	0.32D0
10	21	0.0348D0	0.0749D0	0.0000D0	0.32D0
10	22	0.0727D0	0.1499D0	0.0000D0	0.32D0
12	13	0.0000D0	0.1400D0	0.0000D0	0.65D0
12	14	0.1231D0	0.2559D0	0.0000D0	0.32D0
12	15	0.0662D0	0.1304D0	0.0000D0	0.32D0
12	16	0.0945D0	0.1987D0	0.0000D0	0.32D0
14	15	0.2210D0	0.1997D0	0.0000D0	0.16D0
15	18	0.1070D0	0.2185D0	0.0000D0	0.16D0
15	23	0.1000D0	0.2020D0	0.0000D0	0.16D0
16	17	0.0824D0	0.1932D0	0.0000D0	0.16D0
18	19	0.0639D0	0.1292D0	0.0000D0	0.16D0
19	20	0.0340D0	0.0680D0	0.0000D0	0.32D0
21	22	0.0116D0	0.0236D0	0.0000D0	0.32D0
22	24	0.1150D0	0.1790D0	0.0000D0	0.16D0
23	24	0.1320D0	0.2700D0	0.0000D0	0.16D0
24	25	0.1885D0	0.3292D0	0.0000D0	0.16D0
25	26	0.2544D0	0.3800D0	0.0000D0	0.16D0
25	27	0.1093D0	0.2087D0	0.0000D0	0.16D0
27	29	0.2198D0	0.4153D0	0.0000D0	0.16D0
27	30	0.3202D0	0.6027D0	0.0000D0	0.16D0
28	27	0.0000D0	0.3960D0	0.0000D0	0.65D0
29	30	0.2399D0	0.4533D0	0.0000D0	0.16D0

'TAP' 4 4 12 1.10D0 0.90D0

'TAP' 6 6 9 1.10D0 0.90D0

'TAP' 6 6 10 1.10D0 0.90D0

'TAP' 28 28 27 1.10D0 0.90D0

10 0.05D0 0.0D0

12 0.05D0 0.0D0

15 0.05D0 0.0D0

17 0.05D0 0.0D0

```

20 0.05D0 0.0D0
21 0.05D0 0.0D0
23 0.05D0 0.0D0
24 0.05D0 0.0D0
29 0.05D0 0.0D0
1 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
3 0.0766D0 0.1006D0 1.0000D0 1.0500D0 0.9500D0
2 0.0085D0 0.0095D0 1.0000D0 1.0500D0 0.9500D0
2 0.0268D0 0.0127D0 1.0000D0 1.0500D0 0.9500D0
3 0.3324D0 0.1506D0 1.0000D0 1.0500D0 0.9500D0
2 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
2 0.0805D0 0.0864D0 1.0000D0 1.0500D0 0.9500D0
3 0.1059D0 0.2377D0 1.0000D0 1.0500D0 0.9500D0
2 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
2 0.0205D0 0.0158D0 1.0000D0 1.0500D0 0.9500D0
3 0.0000D0 0.0000D0 1.1000D0 1.0500D0 0.9500D0
2 0.0395D0 0.0594D0 1.0000D0 1.0500D0 0.9500D0
3 0.0000D0 0.0000D0 1.1000D0 1.0500D0 0.9500D0
2 0.0219D0 0.0127D0 1.0000D0 1.0500D0 0.9500D0
2 0.0289D0 0.0198D0 1.0000D0 1.0500D0 0.9500D0
2 0.0124D0 0.0143D0 1.0000D0 1.0500D0 0.9500D0
2 0.0318D0 0.0460D0 1.0000D0 1.0500D0 0.9500D0
2 0.0113D0 0.0071D0 1.0000D0 1.0500D0 0.9500D0
2 0.0335D0 0.0269D0 1.0000D0 1.0500D0 0.9500D0
2 0.0078D0 0.0055D0 1.0000D0 1.0500D0 0.9500D0
2 0.0618D0 0.0887D0 1.0000D0 1.0500D0 0.9500D0
2 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
2 0.0113D0 0.0127D0 1.0000D0 1.0500D0 0.9500D0
2 0.0307D0 0.0531D0 1.0000D0 1.0500D0 0.9500D0
2 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
2 0.0124D0 0.0182D0 1.0000D0 1.0500D0 0.9500D0
2 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
2 0.0000D0 0.0000D0 1.0000D0 1.0500D0 0.9500D0
2 0.0085D0 0.0071D0 1.0000D0 1.0500D0 0.9500D0
2 0.0374D0 0.0151D0 1.0000D0 1.0500D0 0.9500D0
1 1
200.0D0 50.0D0 150.0D0 -20.0D0 2.000D0 0.00750D0
2 1
080.0D0 20.0D0 060.0D0 -20.0D0 1.750D0 0.03500D0
5 1
050.0D0 15.0D0 062.5D0 -15.0D0 1.000D0 0.12500D0
8 1
035.0D0 10.0D0 048.7D0 -15.0D0 3.250D0 0.01668D0
11 1
030.0D0 10.0D0 040.0D0 -10.0D0 3.000D0 0.05000D0
13 1
040.0D0 12.0D0 044.7D0 -15.0D0 3.000D0 0.05000D0

```

C.4 118-Bus Test System

The basic network data for this test system (AEP 118-bus) is taken from [Podmore et al, 1977]. The schematic diagram is given at the end of this

appendix. The generation data is constructed from the EPRI report [EPRI EM-285], and other published sources [IEEE Reliability Test System, 1979] and [Alsaç et al., 1974].

C.4.1 Example 8

```

118 179 10 9 29
4216.00D0 1338.80D0
001 002 0.03030D0 0.09990D0 0.02540D0 1.370D0
001 003 0.01290D0 0.04240D0 0.01082D0 1.370D0
002 012 0.01870D0 0.06160D0 0.01572D0 1.370D0
003 005 0.02410D0 0.10800D0 0.02840D0 1.370D0
003 012 0.04840D0 0.16000D0 0.04060D0 1.370D0
004 005 0.00176D0 0.00798D0 0.00210D0 1.370D0
004 011 0.02090D0 0.06880D0 0.01748D0 1.370D0
005 006 0.01190D0 0.05400D0 0.01426D0 1.370D0
005 008 0.00000D0 0.02670D0 0.00000D0 2.055D0
005 011 0.02030D0 0.06820D0 0.01738D0 1.370D0
006 007 0.00459D0 0.02080D0 0.00550D0 1.370D0
007 012 0.00862D0 0.03400D0 0.00874D0 1.370D0
008 009 0.00244D0 0.03050D0 1.16200D0 6.225D0
008 030 0.00431D0 0.05040D0 0.51400D0 6.225D0
009 010 0.00258D0 0.03220D0 1.23000D0 6.225D0
011 012 0.00595D0 0.01960D0 0.00502D0 1.370D0
011 013 0.02225D0 0.07310D0 0.01876D0 1.370D0
012 014 0.02150D0 0.07070D0 0.01816D0 1.370D0
012 016 0.02120D0 0.08340D0 0.02140D0 1.370D0
012 117 0.03290D0 0.14000D0 0.03580D0 1.370D0
013 015 0.07440D0 0.24440D0 0.06268D0 1.370D0
014 015 0.05950D0 0.19500D0 0.05020D0 1.370D0
015 017 0.01320D0 0.04370D0 0.04440D0 1.370D0
015 019 0.01200D0 0.03940D0 0.01010D0 1.370D0
015 033 0.03800D0 0.12440D0 0.03194D0 1.370D0
016 017 0.04540D0 0.18010D0 0.04660D0 1.370D0
017 018 0.01230D0 0.05050D0 0.01298D0 1.370D0
017 030 0.00000D0 0.03880D0 0.96000D0 2.055D0
017 031 0.04740D0 0.15630D0 0.03990D0 1.370D0
017 113 0.00913D0 0.03010D0 0.00768D0 1.370D0
018 019 0.01119D0 0.04930D0 0.01142D0 1.370D0
019 020 0.02520D0 0.11700D0 0.02980D0 1.370D0
019 034 0.07520D0 0.24700D0 0.06320D0 1.370D0
020 021 0.01830D0 0.08490D0 0.02160D0 1.370D0
021 022 0.02090D0 0.09700D0 0.02460D0 1.370D0
022 023 0.03420D0 0.15900D0 0.04040D0 1.370D0
023 024 0.01350D0 0.04920D0 0.04980D0 1.370D0
023 025 0.01560D0 0.08000D0 0.08640D0 2.055D0
023 032 0.03170D0 0.11530D0 0.11730D0 1.370D0
024 070 0.10221D0 0.41150D0 0.10198D0 1.370D0
024 072 0.04880D0 0.19600D0 0.04880D0 1.370D0
025 026 0.00000D0 0.03820D0 0.00000D0 8.000D0
025 027 0.03180D0 0.16300D0 0.17640D0 1.370D0

```


026 030 0.00799D0 0.08600D0 0.90800D0 6.225D0
027 028 0.01913D0 0.08550D0 0.02160D0 1.370D0
027 032 0.02290D0 0.07550D0 0.01926D0 1.370D0
027 115 0.01640D0 0.07410D0 0.01972D0 1.370D0
028 029 0.02370D0 0.09430D0 0.02380D0 1.370D0
029 031 0.01080D0 0.03310D0 0.00830D0 1.370D0
030 038 0.00464D0 0.05400D0 0.42200D0 6.225D0
031 032 0.02980D0 0.09850D0 0.02510D0 1.370D0
032 113 0.06150D0 0.20300D0 0.05180D0 1.370D0
032 114 0.01350D0 0.06120D0 0.01628D0 1.370D0
033 037 0.04150D0 0.14200D0 0.03660D0 1.370D0
034 036 0.00871D0 0.02680D0 0.00568D0 1.370D0
034 037 0.00256D0 0.00940D0 0.00984D0 1.370D0
034 043 0.04130D0 0.16810D0 0.04226D0 1.370D0
035 036 0.00224D0 0.01020D0 0.00268D0 1.370D0
035 037 0.01100D0 0.04970D0 0.01318D0 1.370D0
037 038 0.00000D0 0.03750D0 0.00000D0 2.055D0
037 039 0.03210D0 0.10600D0 0.02700D0 1.370D0
037 040 0.05930D0 0.16800D0 0.04200D0 1.370D0
038 065 0.00901D0 0.09860D0 1.04600D0 4.150D0
039 040 0.01840D0 0.06050D0 0.01552D0 1.370D0
040 041 0.01450D0 0.04870D0 0.01222D0 1.370D0
040 042 0.05550D0 0.18300D0 0.04660D0 1.370D0
041 042 0.04100D0 0.13500D0 0.03440D0 1.370D0
042 049 0.03575D0 0.16150D0 0.17200D0 1.370D0
043 044 0.06080D0 0.24540D0 0.06068D0 1.370D0
044 045 0.02240D0 0.09010D0 0.02240D0 1.370D0
045 046 0.04000D0 0.13560D0 0.03320D0 1.370D0
045 049 0.06840D0 0.18600D0 0.04440D0 1.370D0
046 047 0.03800D0 0.12700D0 0.03160D0 1.370D0
046 048 0.06010D0 0.18900D0 0.04720D0 1.370D0
047 049 0.01910D0 0.06250D0 0.01604D0 1.370D0
047 069 0.08440D0 0.27780D0 0.07092D0 1.370D0
048 049 0.01790D0 0.05050D0 0.01258D0 1.370D0
049 050 0.02670D0 0.07520D0 0.01874D0 1.370D0
049 051 0.04860D0 0.13700D0 0.03420D0 1.370D0
049 054 0.03993D0 0.14507D0 0.14680D0 1.370D0
049 066 0.00900D0 0.04595D0 0.04960D0 2.055D0
049 069 0.09850D0 0.32400D0 0.08280D0 1.370D0
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2	0.01660D0	0.01718D0	1.05000D0	1.05000D0	0.95000D0
3	0.01115D0	0.00747D0	1.05000D0	1.05000D0	0.95000D0
2	0.00806D0	0.01195D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00403D0	0.00523D0	1.05000D0	1.05000D0	0.95000D0
2	0.00569D0	0.00299D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00640D0	0.00822D0	1.05000D0	1.05000D0	0.95000D0
3	0.01565D0	0.01718D0	1.05000D0	1.05000D0	0.95000D0
2	0.00878D0	0.00747D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00474D0	0.00822D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00403D0	0.00299D0	1.05000D0	1.05000D0	0.95000D0
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3	0.01494D0	0.01643D0	1.05000D0	1.05000D0	0.95000D0
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3	0.00000D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
3	0.00925D0	0.01344D0	1.05000D0	1.05000D0	0.95000D0
2	0.00664D0	0.00523D0	1.05000D0	1.05000D0	0.95000D0
2	0.00000D0	0.00000D0	1.05000D0	1.10000D0	0.95000D0
3	0.00000D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
3	0.01565D0	0.01494D0	1.05000D0	1.05000D0	0.95000D0
2	0.00000D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
3	0.00285D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
3	0.00142D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
3	0.01613D0	0.02017D0	1.05000D0	1.05000D0	0.95000D0
2	0.01115D0	0.00822D0	1.05000D0	1.05000D0	0.95000D0
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2	0.01139D0	0.00747D0	1.05000D0	1.05000D0	0.95000D0
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3	0.03866D0	0.03137D0	1.05000D0	1.05000D0	0.95000D0
3	0.00237D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00901D0	0.01120D0	1.05000D0	1.05000D0	0.95000D0
2	0.00356D0	0.00672D0	1.05000D0	1.05000D0	0.95000D0
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3	0.00996D0	0.00000D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00522D0	0.01120D0	1.05000D0	1.05000D0	0.95000D0
2	0.00119D0	0.00224D0	1.05000D0	1.05000D0	0.95000D0
3	0.00546D0	0.01195D0	1.05000D0	1.05000D0	0.95000D0
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2	0.00522D0	0.00523D0	1.05000D0	1.05000D0	0.95000D0

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006 1
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108.0D0 40.0D0 075.0D0 -200.0D0 0.696D0 0.01956D0
027 1
072.0D0 30.0D0 040.0D0 -08.0D0 1.513D0 0.01204D0
031 1
072.0D0 30.0D0 040.0D0 -08.0D0 1.513D0 0.01204D0
032 1
072.0D0 30.0D0 040.0D0 -08.0D0 1.513D0 0.01204D0
034 1
108.0D0 40.0D0 075.0D0 -15.0D0 1.367D0 0.01246D0
036 1
108.0D0 40.0D0 075.0D0 -15.0D0 0.675D0 0.02066D0
040 1
072.0D0 30.0D0 040.0D0 -08.0D0 1.513D0 0.01204D0
042 1
108.0D0 40.0D0 075.0D0 -15.0D0 1.367D0 0.01246D0
046 1
072.0D0 30.0D0 040.0D0 -08.0D0 1.513D0 0.01204D0
049 1
217.0D0 80.0D0 120.0D0 -24.0D0 0.773D0 0.00680D0
054 1
072.0D0 30.0D0 040.0D0 -08.0D0 1.513D0 0.01204D0
055 1
108.0D0 40.0D0 075.0D0 -15.0D0 0.803D0 0.01932D0
056 1

```

072.0D0	30.0D0	040.0D0	-08.0D0	1.513D0	0.01204D0
059 1					
108.0D0	40.0D0	200.0D0	-85.0D0	0.678D0	0.01546D0
061 1					
108.0D0	40.0D0	075.0D0	-15.0D0	0.678D0	0.01546D0
062 1					
108.0D0	40.0D0	075.0D0	-15.0D0	0.636D0	0.02016D0
065 2					
108.0D0	40.0D0	075.0D0	-400.0D0	0.678D0	0.01546D0
108.0D0	40.0D0	075.0D0	-400.0D0	1.367D0	0.01246D0
066 3					
108.0D0	40.0D0	200.0D0	-15.0D0	0.636D0	0.02016D0
108.0D0	40.0D0	200.0D0	-15.0D0	1.367D0	0.01246D0
108.0D0	40.0D0	200.0D0	-15.0D0	0.678D0	0.01546D0
069 2					
217.0D0	80.0D0	120.0D0	-75.0D0	0.623D0	0.00768D0
217.0D0	80.0D0	120.0D0	-75.0D0	0.773D0	0.00680D0
070 1					
072.0D0	30.0D0	040.0D0	-08.0D0	1.513D0	0.01204D0
072 1					
072.0D0	30.0D0	040.0D0	-08.0D0	1.513D0	0.01204D0
073 1					
072.0D0	30.0D0	040.0DQ	-08.0D0	1.513D0	0.01204D0
074 1					
108.0D0	40.0D0	075.0D0	-15.0D0	0.803D0	0.01932D0
076 1					
108.0D0	40.0D0	075.0D0	-15.0D0	0.803D0	0.01932D0
077 1					
072.0D0	30.0D0	200.0D0	-08.0D0	1.513D0	0.01204D0
080 2					
108.0D0	40.0D0	075.0D0	-15.0D0	0.636D0	0.02016D0
217.0D0	80.0D0	120.0D0	-24.0D0	0.623D0	0.00768D0
085 1					
108.0D0	40.0D0	075.0D0	-15.0D0	0.678D0	0.01546D0
087 1					
108.0D0	40.0D0	075.0D0	-15.0D0	0.803D0	0.01932D0
089 3					
108.0D0	40.0D0	075.0D0	-15.0D0	0.675D0	0.02066D0
108.0D0	40.0D0	075.0D0	-15.0D0	0.696D0	0.01956D0
108.0D0	40.0D0	075.0D0	-15.0D0	0.636D0	0.02016D0
090 1					
217.0D0	80.0D0	120.0D0	-40.0D0	0.489D0	0.00786D0
091 1					
072.0D0	30.0D0	040.0D0	-08.0D0	1.513D0	0.01204D0
092 1					
108.0D0	40.0D0	075.0D0	-15.0D0	1.367D0	0.01246D0
099 1					
072.0D0	30.0D0	040.0D0	-08.0D0	1.513D0	0.01204D0
100 2					
217.0D0	80.0D0	120.0D0	-24.0D0	0.394D0	0.00784D0
108.0D0	40.0D0	075.0D0	-15.0D0	0.803D0	0.01932D0
103 2					
217.0D0	80.0D0	120.0D0	-24.0D0	0.538D0	0.00814D0
217.0D0	80.0D0	120.0D0	-24.0D0	0.395D0	0.00886D0

104 1
108.0D0 40.0D0 075.0D0 -15.0D0 1.367D0 0.01246D0
105 1
108.0D0 40.0D0 075.0D0 -15.0D0 1.367D0 0.01246D0
107 1
108.0D0 40.0D0 075.0D0 -15.0D0 0.678D0 0.01546D0

(a) திருவள்ளூர் மாவட்டம்.

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Schematic diagram of the 30-bus test system

Figure C.3

Schematic diagram of the IEEE 118-bus test system - Part I

Figure C-4.a

Schematic diagram of the IEEE 118-bus test system - Part II
Figure C.4.b

Figure C.4.c

APPENDIX D

RATES OF CONVERGENCE OF ITERATIVE SEQUENCES

In an iterative optimization process, a converging sequence of numbers, i.e., $\{x_k\}$, is often generated. For practical reasons, this sequence must converge with some rapidity. Faster the rate of convergence, better the method. This section presents means of characterising the rate at which such sequences converge.

Assuming that a theoretical rate of convergence is given, seldom is one able to realize this, as the conditions under which this is possible can seldom be realized under practical circumstances. To elaborate, an infinite sequence is a meaningless entity in a computer, Gill et al., [1981]. Hence the presentation to follow will be numerical in nature.

D.1 Definitions and Assumptions

In the presentation to follow it is assumed that the sequence $\{x_k\}$ converges to x^* , such that the elements of the sequence are distinct, and never equal x^* . A meaningful measure for judging the rate of convergence is to compare the improvement at each step to the improvement at the previous step.

As defined by authors Gill et al., [1981], a sequence $\{x_k\}$ is said to converge with order r , when r is the largest number to satisfy the following inequality, i.e.,

$$0 \leq \lim_{K \rightarrow \infty} \frac{|x_{K+1} - x^*|}{|x_K - x^*|^r} < \infty \quad \dots (D.1.a)$$

Since the rate of convergence of interest occurs as $K \rightarrow \infty$, r is also called the asymptotic rate of convergence. Depending on whether $r=1$, or 2, the sequence is said to have linear, or quadratic convergence respectively. Any convergence rate greater than unity implies super-linear convergence. Super-linear convergence is also characterized by sequences for which $r \rightarrow 0$ in the

limit, when $r=1$, [ibid], where γ is the asymptotic error constant defined below.

The asymptotic error constant γ is defined as follows, i.e.,

$$\gamma = \lim_{K \rightarrow \infty} \frac{|x_{K+1} - x^*|}{|x_K - x^*|^r} \quad \dots (D.1.b)$$

It can be seen that for $r=1$, γ must be strictly less than unity for convergence. Further, for sequences with linear rates of convergence, the step decrease $|x_K - x^*|$ will vary greatly with γ .

For further information on rates of convergence, the reader is referred to Ortega et al., [1970].

The section to follow will present an algorithm to determine the convergence rate of a given numerical sequence.

D.2 Algorithm Rate-of-Convergence

Algorithm Rate-of-Convergence, dynamically increments the asymptotic rate of convergence, in given increments, until equation D.1.a is violated. This latter value of r gives an upper bound on r . The method is approximate, but has shown satisfactory performance on sequences known to have linear, superlinear, and quadratic convergence. The greater the number of elements in the sequence the better the estimate on r . The pertinent steps of this algorithm are enumerated below.

- a) Read bounds for equation D.1.a, increments on r , i.e., Δr , and the terminal point of the series, i.e., x^* .
- b) Initialize $r=1$.
- c) Increment r by Δr .
- d) Compute a sub-sequence in γ , i.e., (γ_K) , by scanning through the given sequence (x_K) in a sequentially ascending manner.
- e) Recycle from (c) until elements of (γ_K) violate equation D.1.a.

APPENDIX E

THE GENERAL FORM OF THE GRG METHOD

The motivation for the generalized reduced gradient (GRG) method stems from the active set method for linear constraints, documented in [Gill et al., 1981] and [Luenberger 1984]. The implication here is to stay on the active set, which is a subset of the nonlinear or linear constraint set, while the objective is minimized. This is not trivial, and to date no direct procedure is available to accomplish this. Hence one needs to resort to an iterative correction procedure to stay on a nonlinear active set. An ideal version of the GRG method would have the optimum active set satisfied at all iterations. One specific drawback in the use of the GRG is the need for an initial feasible starting point, which is not always available. This makes it necessary to embody a two-phase scheme where Phase I determines the feasible point to be used by the GRG in Phase II.

The Wolfe's reduced gradient method, [Wolfe 1963], generalised to handle nonlinear constraints, becomes the GRG.

The GRG in its generality solves problems of the form:

Minimize a scalar objective,

$$f(x) \quad \text{.....(E.1.a)}$$

Subject to the constraints,

$$g(x) = 0 \quad \text{.....(E.1.b)}$$

$$x^m \leq x \leq x^M \quad \text{.....(E.1.c)}$$

Where x is a general n -set of variables, and $g(x)$ a set of r ($r < n$) functional constraints. These functional constraints could be linear, nonlinear or a mixture of linear and nonlinear constraints. The optimization is carried out over a reduced space of dimension $(n-r)$. The optimization loses r -degrees of freedom due to the r -equations E.1.b.

There is no known method to stay on a general nonlinear active set (E.1.b). However, this problem can be circumvented by exploiting the very effective step length method designed to stay on a linear active set. For this purpose, equation E.1.b is linearized about a feasible expansion point x^k , giving;

$$g(x) = g(x^k) + g'(x^k)p^k \quad \dots (E.2.a)$$

$$p^k = x - x^k \quad \dots (E.2.b)$$

In order for point x to be feasible;

$$g'(x^k)p^k = 0 \quad \dots (E.2.c)$$

Matrix $g'(x^k)$ has full row rank, such that the requirements on constraint qualifications are not violated. Equation E.2.c suggests that step p^k must lie in the null space¹ of matrix $g'(x^k)$. Hence one is able to choose p^k to represent p^k , such that,

$$p^k = Z^k p^z \quad \dots (E.2.d)$$

The $(n-r)$ columns of Z^k spans the null space of $g'(x^k)$. The $(n-r)$ vector p^z serves as the driving force in the optimization. The optimization takes place in the reduced space of p^z , which is also the column space of Z^k . This is demonstrated by taking a quadratic approximation of the objective about x^k giving:

$$f(x) = f(x^k) + f'(x^k)Z^k p^z + \frac{1}{2}(Z^k p^z)^t f''(x^k) Z^k p^z \quad \dots (E.2.e)$$

The minimum of (E.2.e) occurs when,

$$(Z^k)^t f''(x^k) Z^k p^z = -(f'(x^k) Z^k)^t \quad \dots (E.2.f)$$

¹ The null space of $g'(x)$ is the same as the space orthogonal to the row space of $g'(x)$ since $n > r$.

Equation E.2.f determines the step p^k . In implementing the GRG, the Hessian $f''(x^k)$ is usually approximated by a suitable positive definite matrix, and Z^k is obtained through a variable reduction technique, as proposed in [Abadie et al., 1965,1969], and [Gill et al., 1981]. To implement the variable reduction technique the following partitions are made, i.e.,

$$x = (x_v^t, x_u^t)^t \quad \text{.....(E.2.g)}$$

In this the variables x are partitioned into dependent and independent sets. The dimension of x_v is r , and must be such that all elements are free. An exact analogy would be the basis variables in a linear program (LP). Elements of x_u can be either at a limit or free. The free elements of x_u define the remaining degrees of freedom in the optimization.

The following partitions are made to conform with the definition in (E.2.g), i.e.,

$$g'(x^k) = (V, U) \quad \text{.....(E.2.h)}$$

$$Z^k = (-V^{-1}U, I)^t \quad \text{.....(E.2.i)}$$

$$p^k = (p_v^t, p_u^t)^t \quad \text{.....(E.2.j)}$$

$$f'(x^k) = (f_v, f_u) \quad \text{.....(E.2.k)}$$

From equations E.2.d,i&j one obtains,

$$p_v = -V^{-1}Up^k \quad \text{.....(E.2.l)}$$

$$p_u = p^k \quad \text{.....(E.2.m)}$$

Equations E.2.l&m clarify the variational relationship among the dependent and independent variables. Further the projected gradient term in equation E.2.f, i.e., $(f'(x^k)Z^k)^t$ becomes $[(-U^tV^{-1}f_v^t)+f_u^t]$, to be known subsequently as the reduced gradient.

The key equations in the GRG process, i.e., equations E.2.f&l now become,

$$(Z^k)^t f''(x^k) Z^k p_u = U^t V^{-1} f_v^t - f_u^t \quad \text{.....(E.2.n)}$$

$$p_v = -V^{-1}Up_u \quad \text{.....(E.2.o)}$$

The differences among the reduced gradient type algorithms arise from the manner in which they are actually implemented. The interested reader is referred to the work by Abadie and colleagues [1965,'69], Gill and colleagues [1981], for a description of some variations in implementation. The scope of the subsequent review will be limited to the proposals by Carpentier and colleagues [1966], and Abadie and colleagues [1969].

Equation E.2.n gives the increments γp_u to be applied to the controllable variables, for some scalar step length parameter γ . This is possible only as long as none of the controllable x_u 's reach or violate a limit. Should an element of x_u violate a limit, the value of γ corresponding to the limit must be determined. For subsequent iterations the corresponding component of p_u must be set to zero. This feature of the GRG is illustrated in figure E.1.

Illustrating the progress of GRG iterations in the reduced x_u space when an element of x_u reaches a limit.

Figure E.1

The corrections to the dependent variables x_v can be obtained from equation E.2.o. However, this is valid only for infinitesimal variations about a

feasible point $\mathbf{x}^k$. A better update is obtained by actually solving equations E.1.b for $\mathbf{x}_v$.

To save on computational expenses, Carpentier proposed that one could resort to the latter updates only occasionally, while relying on the former updates for a greater part of the iterations. Gill and colleagues [1981] propose that equation E.2.n alone should not be used for control. Instead the step on control be adjusted adaptively by incorporating components from the range space of $(g'(\mathbf{x}^k))^t$ and null spaces of matrix $g'(\mathbf{x}^k)$.

The trajectory of the limiting value of a dependent variable x_{v1} ($=v_1$ say) in the reduced $\mathbf{x}_u$ space is depicted in figure E.2. Should the GRG iterations at step k cause x_{v1} to be violated, an exchange is carried out between x_{v1} and a free element of the control variables $\mathbf{x}_u$. Let x_{u1} ($=u_1$) be chosen for this exchange, as illustrated in figure E.3. This will cause a corresponding change to the dependent and independent roles played by these variables.

Illustrating the trajectory of a dependent x_{v1} in the reduced space of $\mathbf{x}_u$.

Figure E.2.

A numerically exhaustive situation arises when a functional variable limit is violated. In this instance, Gill et al., [1981] propose that one has to

iteratively reduce the step length γ until the functional just reaches its limit. For subsequent iterations this functional is added to the active set E.1.b. It is also possible that some constraints of the active set may be incorrectly included in the active set, and require deletion. Such updates can be based on the Lagrange multipliers of equations E.1.b.

Illustrating the reduced Space of x_u
after the roles of u_i and v_i are Interchanged

Figure E 3

In order to handle functional variables, Carpentier introduced positive slack variables to convert the violated and near violated functional constraints to equalities, which are then incorporated into the active set E.1.b. In the ensuing iterations the magnitude of the slack variables determine the constraints to be dropped from the active set, thus assisting in the update

For handling functional constraints, the method of Gill et al. [1981] is preferable as one can formulate the problem in such a manner so that the LM's result as a byproduct in the ensuing computations. If slack variables are used, they need be explicitly computed. The CRC process is terminated successfully when all elements of p_k are zero (or sufficiently small).

APPENDIX F

SOLUTION TO THE 118-BUS EXAMPLE

The complete solution to the 118-bus example is given below. Note that all TCUL devices are identified as DEV(N), where N is the device number. In order to identify whether a DEV(N) is a tap-changer or a phase-shifting transformer one has to refer to the specification file C.4.1 (example 8), e.g., DEV(3) and DEV(7) are phase-shifting transformers. The settings of other variables are self explanatory, and are not elaborated here. The solution file also contains information relating to the minimum dimension needed for certain arrays, which will be useful should a re-run of the same problem or a modified version of it becomes necessary. The order in which all remaining variables are reported is the order in which these variables are stored within the computer. This structure is found to be economical from a storage point of view.

#OF BUSES	-	118
#OF LINES	-	179
#OF CONTROL DEVICES	-	10
#OF SHUNT ELEMENTS	-	9
#OF GENERATION BUSES EXCLUDING THE SLACK	-	48
#OF LOAD BUSES (PQ)	-	69
#OF GENERATION UNITS	-	62

OPTIMUM SOLUTION FOUND

STATISTICS FOR REDIMENSIONING ARRAYS IRN & ICN
 MIN FOR LIRN: 1104
 MIN FOR LICN: 1449
 # OF NR ITRS: 315
 ESTIMATE OF ELEMENT GROWTH IN LU: -0.115392D-01
 MAX RESIDUAL IN UNSATISFIED EQS: 0.000000D+00
 # OF NONZEROS IN LF -JACOBIAN: 1104
 # OF NONZEROS IN MAS-JACOBIAN: 1474

VALUE OF LAMBDA : 0.17545D+01
 VALUE OF P-DEMAND : 0.42160D+04
 VALUE OF Q-DEMAND : 0.13388D+04
 SYSTEM LOSS (MW) : 0.67270D+02

TOTAL GENERATION COST: 0.55809735716D+04 \$/HOUR

BUS#	ANG(RAD)	dC/dP(\$/MWH)	dC/dQ(\$/MWH)
2	-0.191437D-01	0.177474D+01	0.273302D-02
3	-0.299498D-02	0.175879D+01	0.200673D-02
5	0.378592D-01	0.172912D+01	0.332510D-03
7	-0.623925D-02	0.176235D+01	0.305775D-03
9	0.146460D+00	0.171816D+01	-0.366794D-02
11	-0.122071D-01	0.177136D+01	0.475508D-02
13	-0.480419D-01	0.180321D+01	0.117662D-01
14	-0.483495D-01	0.179480D+01	-0.643383D-03
16	-0.510799D-01	0.179393D+01	0.156524D-02
17	-0.771709D-01	0.179638D+01	-0.585016D-02
20	-0.111854D+00	0.183334D+01	0.463011D-02
21	-0.103819D+00	0.182905D+01	0.774933D-02
22	-0.829595D-01	0.181385D+01	0.683340D-02
23	-0.351697D-01	0.177803D+01	0.128176D-02
28	-0.967768D-01	0.183011D+01	0.248324D-02
29	-0.100925D+00	0.183333D+01	0.144061D-02
30	-0.336346D-01	0.178680D+01	-0.569201D-02
33	-0.128736D+00	0.184347D+01	0.286829D-02
35	-0.140054D+00	0.183807D+01	0.198304D-03
37	-0.135667D+00	0.183351D+01	-0.184113D-02
38	-0.141137D+00	0.183378D+01	-0.242262D-02
39	-0.204695D+00	0.190926D+01	0.539122D-02
41	-0.253287D+00	0.195771D+01	0.343642D-02
43	-0.208053D+00	0.190096D+01	0.104062D-01
44	-0.270663D+00	0.195702D+01	0.122416D-01
45	-0.279654D+00	0.196399D+01	0.126462D-01
47	-0.250676D+00	0.192460D+01	-0.178830D-02
48	-0.256965D+00	0.192700D+01	0.129266D-02
50	-0.283182D+00	0.195143D+01	0.277459D-02
51	-0.325666D+00	0.199986D+01	0.108626D-01
52	-0.341124D+00	0.201788D+01	0.119925D-01
53	-0.356355D+00	0.202185D+01	0.968987D-02
57	-0.323489D+00	0.198783D+01	0.189320D-02
58	-0.337189D+00	0.200624D+01	0.670730D-02
60	-0.262601D+00	0.191303D+01	0.173439D-03
63	-0.263409D+00	0.192682D+01	0.110997D-02
64	-0.235432D+00	0.190644D+01	-0.314438D-04
67	-0.234998D+00	0.189899D+01	0.223115D-02
68	-0.159123D+00	0.186318D+01	-0.134101D-02
71	-0.136173D+00	0.185170D+01	-0.628711D-04
75	-0.185417D+00	0.190268D+01	0.174375D-02
78	-0.163977D+00	0.188508D+01	0.535894D-02
79	-0.160950D+00	0.188125D+01	0.806355D-02
81	-0.148820D+00	0.185595D+01	0.161756D-03
82	-0.117596D+00	0.184401D+01	0.158751D-01
83	-0.939063D-01	0.181785D+01	0.138706D-01
84	-0.487915D-01	0.176075D+01	0.928575D-02
86	0.530918D-02	0.169646D+01	-0.331385D-07
88	-0.158448D-01	0.172397D+01	0.237076D-02
93	-0.305921D-01	0.175444D+01	0.111798D-01
94	-0.457051D-01	0.177364D+01	0.163634D-01
95	-0.779258D-01	0.181029D+01	0.257212D-01

96	-0.101385D+00	0.183181D+01	0.169437D-01
97	-0.121451D+00	0.184493D+01	0.108865D-01
98	-0.940137D-01	0.181214D+01	0.743681D-02
101	-0.489473D-02	0.172542D+01	0.634956D-02
102	-0.677842D-02	0.172499D+01	0.226533D-02
106	0.109668D-01	0.171090D+01	0.298246D-02
108	-0.709462D-02	0.172897D+01	0.358134D-02
109	-0.192629D-01	0.174091D+01	0.465472D-02
110	-0.463797D-01	0.176549D+01	0.505441D-02
111	-0.465999D-01	0.176549D+01	0.427700D-02
112	-0.951235D-01	0.182324D+01	-0.186615D-01
113	-0.801948D-01	0.180073D+01	-0.764173D-02
114	-0.910625D-01	0.182491D+01	0.196388D-02
115	-0.915067D-01	0.182538D+01	0.213565D-02
116	-0.165933D+00	0.186523D+01	-0.199922D-02
117	-0.450802D-01	0.179732D+01	0.746904D-02
118	-0.194616D+00	0.191554D+01	0.492927D-02
1	0.000000D+00	0.175449D+01	0.000000D+00
4	0.405535D-01	0.172567D+01	0.000000D+00
6	0.617071D-02	0.175286D+01	0.000000D+00
8	0.737385D-01	0.173833D+01	0.000000D+00
10	0.226386D+00	0.169683D+01	0.000000D+00
12	-0.207968D-01	0.177499D+01	0.000000D+00
15	-0.977577D-01	0.182216D+01	0.000000D+00
18	-0.963769D-01	0.181624D+01	0.000000D+00
19	-0.103244D+00	0.182352D+01	0.000000D+00
24	-0.485313D-01	0.178694D+01	0.000000D+00
25	0.500933D-01	0.173166D+01	0.000000D+00
26	0.630967D-01	0.174047D+01	0.000000D+00
27	-0.803616D-01	0.181643D+01	0.000000D+00
31	-0.952559D-01	0.182596D+01	0.000000D+00
32	-0.841457D-01	0.181848D+01	0.000000D+00
34	-0.139001D+00	0.183796D+01	0.000000D+00
36	-0.138068D+00	0.183653D+01	0.000000D+00
40	-0.231120D+00	0.193573D+01	0.000000D+00
42	-0.270282D+00	0.196348D+01	0.000000D+00
46	-0.252146D+00	0.192578D+01	0.000000D+00
49	-0.249319D+00	0.191473D+01	0.000000D+00
54	-0.342250D+00	0.199941D+01	0.000000D+00
55	-0.340642D+00	0.199573D+01	0.000000D+00
56	-0.341678D+00	0.199869D+01	0.000000D+00
59	-0.317848D+00	0.194791D+01	0.000000D+00
61	-0.250886D+00	0.190460D+01	0.000000D+00
62	-0.250203D+00	0.190563D+01	0.000000D+00
65	-0.174884D+00	0.187158D+01	0.000000D+00
66	-0.196673D+00	0.187141D+01	0.000000D+00
69	-0.160647D+00	0.186330D+01	0.000000D+00
70	-0.155320D+00	0.186802D+01	0.000000D+00
72	-0.789424D-01	0.180670D+01	0.000000D+00
73	-0.126026D+00	0.184455D+01	0.000000D+00
74	-0.182053D+00	0.189834D+01	0.000000D+00
76	-0.189861D+00	0.190997D+01	0.000000D+00
77	-0.158093D+00	0.187767D+01	0.000000D+00
80	-0.129995D+00	0.184838D+01	0.000000D+00

85	-0.224049D-01	0.172344D+01	0.000000D+00
87	0.953811D-01	0.169646D+01	0.000000D+00
89	0.190156D-01	0.170325D+01	0.000000D+00
90	0.152773D-01	0.170529D+01	0.000000D+00
91	0.167321D-01	0.170226D+01	0.000000D+00
92	-0.521219D-02	0.172281D+01	0.000000D+00
99	-0.296803D-01	0.175104D+01	0.000000D+00
100	0.197607D-01	0.170640D+01	0.000000D+00
103	0.789348D-01	0.164823D+01	0.000000D+00
104	0.327936D-01	0.169133D+01	0.000000D+00
105	0.236246D-01	0.169797D+01	0.000000D+00
107	0.298572D-01	0.169161D+01	0.000000D+00

P—GENERATIONS

UNIT #	VALUE	LAG MULTI	LOWER LIMIT	UPPER LIMIT
61 LL	0.400000D+02	0.167430D+00	0.400000D+02	0.108000D+03
60 LL	0.400000D+02	0.174067D+00	0.400000D+02	0.108000D+03
55 LL	0.300000D+02	0.123155D+00	0.300000D+02	0.720000D+02
54 LL	0.400000D+02	0.142592D+00	0.400000D+02	0.108000D+03
53 LL	0.300000D+02	0.171937D+00	0.300000D+02	0.720000D+02
41 LL	0.300000D+02	0.296469D-01	0.300000D+02	0.720000D+02
40 LL	0.300000D+02	0.674962D-01	0.300000D+02	0.720000D+02
39 LL	0.300000D+02	0.617653D-02	0.300000D+02	0.720000D+02
20 LL	0.400000D+02	0.274378D-01	0.400000D+02	0.108000D+03
19 LL	0.300000D+02	0.557244D-01	0.300000D+02	0.720000D+02
18 LL	0.300000D+02	0.482395D-01	0.300000D+02	0.720000D+02
17 LL	0.300000D+02	0.577746D-01	0.300000D+02	0.720000D+02
12 LL	0.300000D+02	0.872580D-01	0.300000D+02	0.720000D+02
11 LL	0.400000D+02	0.418797D-01	0.400000D+02	0.108000D+03
10 LL	0.300000D+02	0.579610D-01	0.300000D+02	0.720000D+02
9 LL	0.300000D+02	0.520407D-01	0.300000D+02	0.720000D+02
2 LL	0.300000D+02	0.119709D+00	0.300000D+02	0.720000D+02
62 FF	0.655570D+02	-0.977623D-04	0.400000D+02	0.108000D+03
59 FF	0.141437D+03	-0.960746D-04	0.800000D+02	0.217000D+03
58 FF	0.136380D+03	-0.960746D-04	0.800000D+02	0.217000D+03
57 FF	0.467546D+02	-0.101574D-03	0.400000D+02	0.108000D+03
56 FF	0.167385D+03	-0.101574D-03	0.800000D+02	0.217000D+03
52 FF	0.154731D+03	-0.106026D-03	0.800000D+02	0.217000D+03
51 FF	0.529337D+02	-0.105619D-03	0.400000D+02	0.108000D+03
50 FF	0.514899D+02	-0.105619D-03	0.400000D+02	0.108000D+03
49 FF	0.497649D+02	-0.105619D-03	0.400000D+02	0.108000D+03
48 FF	0.462400D+02	-0.107250D-03	0.400000D+02	0.108000D+03
47 FF	0.676154D+02	-0.107832D-03	0.400000D+02	0.108000D+03
46 FF	0.159539D+03	-0.113160D-03	0.800000D+02	0.217000D+03
45 FF	0.601320D+02	-0.113160D-03	0.400000D+02	0.108000D+03
44 FF	0.302786D+02	-0.119399D-03	0.300000D+02	0.720000D+02
43 FF	0.572898D+02	-0.127092D-03	0.400000D+02	0.108000D+03
42 FF	0.566880D+02	-0.123301D-03	0.400000D+02	0.108000D+03
38 FF	0.160322D+03	-0.117173D-03	0.800000D+02	0.217000D+03
37 FF	0.161483D+03	-0.117173D-03	0.800000D+02	0.217000D+03
36 FF	0.771852D+02	-0.125357D-03	0.400000D+02	0.108000D+03
35 FF	0.404722D+02	-0.125357D-03	0.400000D+02	0.108000D+03

34 FF	0.612740D+02	-0.125357D-03	0.400000D+02	0.108000D+03
33 FF	0.404864D+02	-0.114793D-03	0.400000D+02	0.108000D+03
32 FF	0.771967D+02	-0.114793D-03	0.400000D+02	0.108000D+03
31 FF	0.629717D+02	-0.120736D-03	0.400000D+02	0.108000D+03
30 FF	0.793329D+02	-0.116589D-03	0.400000D+02	0.108000D+03
29 FF	0.821348D+02	-0.102505D-03	0.400000D+02	0.108000D+03
28 FF	0.403163D+02	-0.277162D-03	0.300000D+02	0.720000D+02
27 FF	0.617210D+02	-0.275948D-03	0.400000D+02	0.108000D+03
26 FF	0.403769D+02	-0.271176D-03	0.300000D+02	0.720000D+02
25 FF	0.167878D+03	-0.157739D-03	0.800000D+02	0.217000D+03
24 FF	0.342708D+02	-0.162826D-03	0.300000D+02	0.720000D+02
23 FF	0.478605D+02	-0.142836D-03	0.400000D+02	0.108000D+03
22 FF	0.351008D+02	-0.116838D-03	0.300000D+02	0.720000D+02
21 FF	0.562162D+02	-0.106852D-03	0.400000D+02	0.108000D+03
16 FF	0.533955D+02	-0.530357D-04	0.400000D+02	0.108000D+03
15 FF	0.515690D+02	-0.530357D-04	0.400000D+02	0.108000D+03
14 FF	0.485205D+02	-0.530357D-04	0.400000D+02	0.108000D+03
13 FF	0.170614D+03	-0.457877D-04	0.800000D+02	0.217000D+03
8 FF	0.503050D+02	-0.987714D-04	0.400000D+02	0.108000D+03
7 FF	0.132681D+03	-0.681605D-04	0.800000D+02	0.217000D+03
6 FF	0.139813D+03	-0.681605D-04	0.800000D+02	0.217000D+03
5 FF	0.141950D+03	-0.711735D-04	0.800000D+02	0.217000D+03
4 FF	0.540299D+02	-0.307940D-04	0.400000D+02	0.108000D+03
3 FF	0.157327D+03	-0.779912D-04	0.800000D+02	0.217000D+03
1 FF	0.522503D+02	0.934224D-06	0.400000D+02	0.108000D+03

Q-GENERATIONS

BUS #	VALUE	LAG MULTI	LOWER LIMIT	UPPER LIMIT
107 FF	0.887272D-01	0.000000D+00	-0.150000D+00	0.750000D+00
105 FF	0.335684D+00	0.000000D+00	-0.150000D+00	0.750000D+00
104 FF	0.196985D+00	0.000000D+00	-0.150000D+00	0.750000D+00
103 FF	-0.353183D+00	0.000000D+00	-0.480000D+00	0.240000D+01
100 FF	0.520046D+00	0.000000D+00	-0.390000D+00	0.195000D+01
99 FF	0.598343D-01	0.000000D+00	-0.800000D-01	0.400000D+00
92 FF	0.322414D+00	0.000000D+00	-0.150000D+00	0.750000D+00
91 FF	-0.588507D-01	0.000000D+00	-0.800000D-01	0.400000D+00
90 FF	0.320625D+00	0.000000D+00	-0.400000D+00	0.120000D+01
89 FF	-0.268252D+00	0.000000D+00	-0.450000D+00	0.225000D+01
87 FF	0.135726D+00	0.000000D+00	-0.150000D+00	0.750000D+00
85 FF	0.294116D+00	0.000000D+00	-0.150000D+00	0.750000D+00
80 FF	0.134886D+00	0.000000D+00	-0.390000D+00	0.195000D+01
77 FF	0.677969D+00	0.000000D+00	-0.800000D-01	0.200000D+01
76 FF	0.422974D+00	0.000000D+00	-0.150000D+00	0.750000D+00
74 FF	0.293262D+00	0.000000D+00	-0.150000D+00	0.750000D+00
73 FF	-0.117104D-01	0.000000D+00	-0.800000D-01	0.400000D+00
72 FF	-0.320682D-01	0.000000D+00	-0.800000D-01	0.400000D+00
70 FF	0.927317D-01	0.000000D+00	-0.800000D-01	0.400000D+00
69 FF	-0.808955D+00	0.000000D+00	-0.150000D+01	0.240000D+01
66 FF	0.598718D+01	0.000000D+00	-0.450000D+00	0.600000D+01
65 FF	-0.669412D+01	0.000000D+00	-0.800000D+01	0.150000D+01
62 FF	0.120254D+00	0.000000D+00	-0.150000D+00	0.750000D+00
61 FF	0.191202D+00	0.000000D+00	-0.150000D+00	0.750000D+00

59	FF	0.979128D+00	0.000000D+00	-0.850000D+00	0.200000D+01
56	FF	0.162873D+00	0.000000D+00	-0.800000D-01	0.400000D+00
55	FF	0.178764D+00	0.000000D+00	-0.150000D+00	0.750000D+00
54	FF	0.334432D+00	0.000000D+00	-0.800000D-01	0.400000D+00
49	FF	0.348374D+00	0.000000D+00	-0.240000D+00	0.120000D+01
46	FF	0.120071D+00	0.000000D+00	-0.800000D-01	0.400000D+00
42	FF	0.127798D+00	0.000000D+00	-0.150000D+00	0.750000D+00
40	FF	0.370812D+00	0.000000D+00	-0.800000D-01	0.400000D+00
36	FF	0.167818D+00	0.000000D+00	-0.150000D+00	0.750000D+00
34	FF	0.219955D-01	0.000000D+00	-0.150000D+00	0.750000D+00
32	FF	0.139090D+00	0.000000D+00	-0.800000D-01	0.400000D+00
31	FF	0.240948D+00	0.000000D+00	-0.800000D-01	0.400000D+00
27	FF	0.200420D+00	0.000000D+00	-0.800000D-01	0.400000D+00
26	FF	-0.555179D+01	0.000000D+00	-0.600000D+01	0.225000D+01
25	FF	0.545019D+01	0.000000D+00	-0.224000D+01	0.700000D+01
24	FF	-0.517434D-01	0.000000D+00	-0.800000D-01	0.400000D+00
19	FF	0.260273D+00	0.000000D+00	-0.150000D+00	0.750000D+00
18	FF	0.187039D+00	0.000000D+00	-0.800000D-01	0.400000D+00
15	FF	0.148454D+00	0.000000D+00	-0.800000D-01	0.400000D+00
12	FF	0.438293D+00	0.000000D+00	-0.150000D+00	0.750000D+00
10	FF	-0.133726D+01	0.000000D+00	-0.140000D+01	0.240000D+01
8	FF	-0.198282D+01	0.000000D+00	-0.350000D+01	0.220000D+01
6	FF	0.269454D+00	0.000000D+00	-0.150000D+00	0.750000D+00
4	FF	0.140023D-01	0.000000D+00	-0.400000D+00	0.120000D+01
1	FF	0.331987D+00	0.000000D+00	-0.230000D+00	0.115000D+01

BUS VOLTAGES

BUS #	VALUE	LAG MULTI	LOWER LIMIT	UPPER LIMIT
61 UU	0.105000D+01	-0.134995D+01	0.950000D+00	0.105000D+01
34 UU	0.105000D+01	-0.460805D+01	0.950000D+00	0.105000D+01
100 UU	0.105000D+01	-0.834051D+01	0.950000D+00	0.105000D+01
91 UU	0.105000D+01	-0.324125D+01	0.950000D+00	0.105000D+01
89 UU	0.105000D+01	-0.118637D+02	0.950000D+00	0.105000D+01
65 UU	0.105000D+01	-0.228513D+02	0.950000D+00	0.105000D+01
36 UU	0.105000D+01	-0.242497D+01	0.950000D+00	0.105000D+01
103 UU	0.105000D+01	-0.604862D+02	0.950000D+00	0.105000D+01
85 UU	0.105000D+01	-0.128969D+02	0.950000D+00	0.105000D+01
66 UU	0.105000D+01	-0.154808D+02	0.950000D+00	0.105000D+01
26 UU	0.105000D+01	-0.134614D+02	0.950000D+00	0.105000D+01
25 UU	0.105000D+01	-0.213643D+02	0.950000D+00	0.105000D+01
10 UU	0.105000D+01	-0.105994D+02	0.950000D+00	0.105000D+01
8 UU	0.105000D+01	-0.220933D+02	0.950000D+00	0.105000D+01
4 UU	0.105000D+01	-0.216505D+02	0.950000D+00	0.105000D+01
59 FF	0.104274D+01	0.239207D+00	0.950000D+00	0.105000D+01
19 FF	0.104120D+01	0.578141D+00	0.950000D+00	0.105000D+01
6 FF	0.104636D+01	0.216804D+00	0.950000D+00	0.105000D+01
1 FF	0.104655D+01	0.124793D+00	0.950000D+00	0.105000D+01
27 FF	0.103894D+01	0.431873D+00	0.950000D+00	0.105000D+01
31 FF	0.103758D+01	0.336371D+00	0.950000D+00	0.105000D+01
99 FF	0.104698D+01	-0.199159D+00	0.950000D+00	0.105000D+01
46 FF	0.103788D+01	-0.721114D+00	0.950000D+00	0.105000D+01
73 FF	0.103957D+01	-0.383760D+00	0.950000D+00	0.105000D+01

49 FF	0.104188D+01	-0.509342D+00	0.950000D+00	0.105000D+01
80 FF	0.103311D+01	-0.383990D+00	0.950000D+00	0.105000D+01
40 FF	0.103011D+01	-0.199484D+00	0.950000D+00	0.105000D+01
24 FF	0.104878D+01	0.215167D+00	0.950000D+00	0.105000D+01
69 FF	0.103623D+01	0.976473D-01	0.950000D+00	0.105000D+01
70 FF	0.103347D+01	-0.193641D+00	0.950000D+00	0.105000D+01
74 FF	0.102498D+01	0.142196D+00	0.950000D+00	0.105000D+01
77 FF	0.102548D+01	0.131898D-02	0.950000D+00	0.105000D+01
42 FF	0.102521D+01	0.286275D+00	0.950000D+00	0.105000D+01
15 FF	0.104031D+01	-0.406252D+00	0.950000D+00	0.105000D+01
32 FF	0.103878D+01	-0.696581D+00	0.950000D+00	0.105000D+01
72 FF	0.104765D+01	-0.126148D+00	0.950000D+00	0.105000D+01
62 FF	0.104833D+01	-0.130276D-01	0.950000D+00	0.105000D+01
76 FF	0.101995D+01	0.965341D-01	0.950000D+00	0.105000D+01
12 FF	0.104163D+01	-0.261997D+00	0.950000D+00	0.105000D+01
92 FF	0.104757D+01	-0.206668D-02	0.950000D+00	0.105000D+01
18 FF	0.104187D+01	-0.410408D+00	0.950000D+00	0.105000D+01
55 FF	0.102775D+01	-0.614979D-01	0.950000D+00	0.105000D+01
107 FF	0.104747D+01	0.682570D-03	0.950000D+00	0.105000D+01
56 FF	0.102684D+01	0.242073D-01	0.950000D+00	0.105000D+01
54 FF	0.102667D+01	0.253092D+00	0.950000D+00	0.105000D+01
104 FF	0.104604D+01	-0.208626D-03	0.950000D+00	0.105000D+01
90 FF	0.104923D+01	0.330708D-03	0.950000D+00	0.105000D+01
105 FF	0.104478D+01	-0.272849D-02	0.950000D+00	0.105000D+01
87 FF	0.104403D+01	0.163833D-04	0.950000D+00	0.105000D+01
118 FF	0.101508D+01	0.000000D+00	0.950000D+00	0.105000D+01
117 FF	0.102658D+01	0.000000D+00	0.950000D+00	0.105000D+01
116 FF	0.106111D+01	0.000000D+00	0.950000D+00	0.110000D+01
115 FF	0.103393D+01	0.000000D+00	0.950000D+00	0.105000D+01
114 FF	0.103431D+01	0.000000D+00	0.950000D+00	0.105000D+01
113 FF	0.105338D+01	0.000000D+00	0.950000D+00	0.110000D+01
112 FF	0.102792D+01	0.000000D+00	0.950000D+00	0.105000D+01
111 FF	0.102647D+01	0.000000D+00	0.950000D+00	0.105000D+01
110 FF	0.102570D+01	0.000000D+00	0.950000D+00	0.105000D+01
109 FF	0.103133D+01	0.000000D+00	0.950000D+00	0.105000D+01
108 FF	0.103491D+01	0.000000D+00	0.950000D+00	0.105000D+01
106 FF	0.103890D+01	0.000000D+00	0.950000D+00	0.105000D+01
102 FF	0.104342D+01	0.000000D+00	0.950000D+00	0.105000D+01
101 FF	0.103693D+01	0.000000D+00	0.950000D+00	0.105000D+01
98 FF	0.102893D+01	0.000000D+00	0.950000D+00	0.105000D+01
97 FF	0.101807D+01	0.000000D+00	0.950000D+00	0.105000D+01
96 FF	0.101220D+01	0.000000D+00	0.950000D+00	0.105000D+01
95 FF	0.100778D+01	0.000000D+00	0.950000D+00	0.105000D+01
94 FF	0.102318D+01	0.000000D+00	0.950000D+00	0.105000D+01
93 FF	0.103103D+01	0.000000D+00	0.950000D+00	0.105000D+01
88 FF	0.104285D+01	0.000000D+00	0.950000D+00	0.105000D+01
86 FF	0.105876D+01	0.000000D+00	0.950000D+00	0.110000D+01
84 FF	0.103546D+01	0.000000D+00	0.950000D+00	0.105000D+01
83 FF	0.102015D+01	0.000000D+00	0.950000D+00	0.105000D+01
82 FF	0.101357D+01	0.000000D+00	0.950000D+00	0.105000D+01
81 FF	0.105670D+01	0.000000D+00	0.950000D+00	0.110000D+01
79 FF	0.101613D+01	0.000000D+00	0.950000D+00	0.105000D+01
78 FF	0.101862D+01	0.000000D+00	0.950000D+00	0.105000D+01
75 FF	0.102201D+01	0.000000D+00	0.950000D+00	0.105000D+01

71 FF	0.103785D+01	0.000000D+00	0.950000D+00	0.105000D+01
68 FF	0.105943D+01	0.000000D+00	0.950000D+00	0.110000D+01
67 FF	0.104353D+01	0.000000D+00	0.950000D+00	0.105000D+01
64 FF	0.104934D+01	0.000000D+00	0.950000D+00	0.105000D+01
63 FF	0.104165D+01	0.000000D+00	0.950000D+00	0.105000D+01
60 FF	0.104764D+01	0.000000D+00	0.950000D+00	0.105000D+01
58 FF	0.101851D+01	0.000000D+00	0.950000D+00	0.105000D+01
57 FF	0.102634D+01	0.000000D+00	0.950000D+00	0.105000D+01
53 FF	0.101097D+01	0.000000D+00	0.950000D+00	0.105000D+01
52 FF	0.101055D+01	0.000000D+00	0.950000D+00	0.105000D+01
51 FF	0.101597D+01	0.000000D+00	0.950000D+00	0.105000D+01
50 FF	0.103220D+01	0.000000D+00	0.950000D+00	0.105000D+01
48 FF	0.103740D+01	0.000000D+00	0.950000D+00	0.105000D+01
47 FF	0.103806D+01	0.000000D+00	0.950000D+00	0.105000D+01
45 FF	0.101812D+01	0.000000D+00	0.950000D+00	0.105000D+01
44 FF	0.101897D+01	0.000000D+00	0.950000D+00	0.105000D+01
43 FF	0.102935D+01	0.000000D+00	0.950000D+00	0.105000D+01
41 FF	0.102210D+01	0.000000D+00	0.950000D+00	0.105000D+01
39 FF	0.103128D+01	0.000000D+00	0.950000D+00	0.105000D+01
38 FF	0.107062D+01	0.000000D+00	0.950000D+00	0.110000D+01
37 FF	0.105382D+01	0.000000D+00	0.950000D+00	0.110000D+01
35 FF	0.104941D+01	0.000000D+00	0.950000D+00	0.105000D+01
33 FF	0.103845D+01	0.000000D+00	0.950000D+00	0.105000D+01
30 FF	0.108132D+01	0.000000D+00	0.950000D+00	0.110000D+01
29 FF	0.103403D+01	0.000000D+00	0.950000D+00	0.105000D+01
28 FF	0.103294D+01	0.000000D+00	0.950000D+00	0.105000D+01
23 FF	0.104740D+01	0.000000D+00	0.950000D+00	0.105000D+01
22 FF	0.103284D+01	0.000000D+00	0.950000D+00	0.105000D+01
21 FF	0.102836D+01	0.000000D+00	0.950000D+00	0.105000D+01
20 FF	0.103175D+01	0.000000D+00	0.950000D+00	0.105000D+01
17 FF	0.105322D+01	0.000000D+00	0.950000D+00	0.110000D+01
16 FF	0.103787D+01	0.000000D+00	0.950000D+00	0.105000D+01
14 FF	0.103989D+01	0.000000D+00	0.950000D+00	0.105000D+01
13 FF	0.102489D+01	0.000000D+00	0.950000D+00	0.105000D+01
11 FF	0.103706D+01	0.000000D+00	0.950000D+00	0.105000D+01
9 FF	0.106693D+01	0.000000D+00	0.950000D+00	0.110000D+01
7 FF	0.104399D+01	0.000000D+00	0.950000D+00	0.105000D+01
5 FF	0.104989D+01	0.000000D+00	0.950000D+00	0.105000D+01
3 FF	0.104282D+01	0.000000D+00	0.950000D+00	0.105000D+01
2 FF	0.103872D+01	0.000000D+00	0.950000D+00	0.105000D+01

ONLINE DEVICES

DEV #	VALUE	LAG MULTI	LOWER LIMIT	UPPER LIMIT
4 LL	0.800000D+00	0.107889D-01	0.800000D+00	0.120000D+01
8 LL	0.800000D+00	0.330640D+00	0.800000D+00	0.120000D+01
2 FF	0.998698D+00	0.773264D-01	0.800000D+00	0.120000D+01
6 FF	0.101233D+01	-0.299036D-01	0.800000D+00	0.120000D+01
5 FF	0.995571D+00	-0.112428D-01	0.800000D+00	0.120000D+01
3 FF	-0.834716D-01	-0.461868D-01	-0.523600D+00	0.523600D+00
9 FF	0.969538D+00	-0.194605D-01	0.800000D+00	0.120000D+01
7 FF	0.682828D-01	0.704016D-02	-0.523600D+00	0.523600D+00
10 FF	0.997685D+00	+0.783866D-02	0.800000D+00	0.120000D+01
1 FF	0.964534D+00	-0.168176D-04	0.800000D+00	0.120000D+01

SHT CAPACITORS

SHT #	VALUE	LAG MULTI	LOWER LIMIT	UPPER LIMIT
9 UU	0.500000D-01	-0.531750D+00	0.000000D+00	0.500000D-01
8 UU	0.500000D-01	-0.144352D+01	0.000000D+00	0.500000D-01
7 UU	0.500000D-01	-0.163088D+01	0.000000D+00	0.500000D-01
6 UU	0.500000D-01	-0.832576D+00	0.000000D+00	0.500000D-01
5 UU	0.500000D-01	-0.139116D+00	0.000000D+00	0.500000D-01
4 UU	0.500000D-01	-0.131086D+01	0.000000D+00	0.500000D-01
3 UU	0.500000D-01	-0.127105D+01	0.000000D+00	0.500000D-01
2 LL	0.000000D+00	0.204463D+00	0.000000D+00	0.500000D-01
1 UU	0.500000D-01	-0.366515D-01	0.000000D+00	0.500000D-01

SQ.OF LNE.CUR.

LINE #	VALUE	LAG MULTI	LOWER LIMIT	UPPER LIMIT
179 FF	0.313033D-02	0.000000D+00	0.000000D+00	0.137000D+01
178 FF	0.533248D+00	0.000000D+00	0.000000D+00	0.137000D+01
177 FF	0.105364D-03	0.000000D+00	0.000000D+00	0.137000D+01
176 FF	0.123040D+00	0.000000D+00	0.000000D+00	0.137000D+01
175 FF	0.181855D+00	0.000000D+00	0.000000D+00	0.137000D+01
174 FF	0.127195D-01	0.000000D+00	0.000000D+00	0.137000D+01
173 FF	0.198742D+00	0.000000D+00	0.000000D+00	0.137000D+01
172 FF	0.137067D-02	0.000000D+00	0.000000D+00	0.137000D+01
171 FF	0.653745D-01	0.000000D+00	0.000000D+00	0.137000D+01
170 FF	0.611913D-01	0.000000D+00	0.000000D+00	0.137000D+01
169 FF	0.508242D+00	0.000000D+00	0.000000D+00	0.137000D+01
168 FF	0.115566D+00	0.000000D+00	0.000000D+00	0.137000D+01
167 FF	0.863334D-01	0.000000D+00	0.000000D+00	0.137000D+01
166 FF	0.349328D-02	0.000000D+00	0.000000D+00	0.137000D+01
165 FF	0.369879D-02	0.000000D+00	0.000000D+00	0.137000D+01
164 FF	0.463284D-02	0.000000D+00	0.000000D+00	0.137000D+01
163 FF	0.128122D+01	0.000000D+00	0.000000D+00	0.205500D+01
162 FF	0.498762D-01	0.000000D+00	0.000000D+00	0.137000D+01
161 FF	0.388796D+00	0.000000D+00	0.000000D+00	0.137000D+01
160 FF	0.428769D+00	0.000000D+00	0.000000D+00	0.137000D+01
159 FF	0.552604D-01	0.000000D+00	0.000000D+00	0.137000D+01

158	FF	0.176854D+00	0.000000D+00	0.000000D+00	0.137000D+01
157	FF	0.144589D+01	0.000000D+00	0.000000D+00	0.205500D+01
156	FF	0.402472D+00	0.000000D+00	0.000000D+00	0.137000D+01
155	FF	0.635383D+00	0.000000D+00	0.000000D+00	0.137000D+01
154	FF	0.516625D-01	0.000000D+00	0.000000D+00	0.137000D+01
153	FF	0.608565D-02	0.000000D+00	0.000000D+00	0.137000D+01
152	FF	0.758389D-02	0.000000D+00	0.000000D+00	0.137000D+01
151	FF	0.862368D-01	0.000000D+00	0.000000D+00	0.137000D+01
150	FF	0.123399D+00	0.000000D+00	0.000000D+00	0.137000D+01
149	FF	0.302956D-01	0.000000D+00	0.000000D+00	0.137000D+01
148	FF	0.383507D-03	0.000000D+00	0.000000D+00	0.137000D+01
147	FF	0.425834D+00	0.000000D+00	0.000000D+00	0.137000D+01
146	FF	0.354157D-02	0.000000D+00	0.000000D+00	0.137000D+01
145	FF	0.262536D+00	0.000000D+00	0.000000D+00	0.137000D+01
144	FF	0.213061D+00	0.000000D+00	0.000000D+00	0.137000D+01
143	FF	0.620073D-01	0.000000D+00	0.000000D+00	0.137000D+01
142	FF	0.908986D-02	0.000000D+00	0.000000D+00	0.137000D+01
141	FF	0.568946D-01	0.000000D+00	0.000000D+00	0.137000D+01
140	FF	0.192845D+00	0.000000D+00	0.000000D+00	0.137000D+01
139	FF	0.267972D+00	0.000000D+00	0.000000D+00	0.137000D+01
138	FF	0.117778D+00	0.000000D+00	0.000000D+00	0.137000D+01
137	FF	0.883842D-01	0.000000D+00	0.000000D+00	0.137000D+01
136	FF	0.424552D+00	0.000000D+00	0.000000D+00	0.137000D+01
135	FF	0.248731D+00	0.000000D+00	0.000000D+00	0.137000D+01
134	FF	0.113938D+00	0.000000D+00	0.000000D+00	0.137000D+01
133	FF	0.334474D-01	0.000000D+00	0.000000D+00	0.137000D+01
132	FF	0.375979D-01	0.000000D+00	0.000000D+00	0.137000D+01
131	FF	0.642826D+00	0.000000D+00	0.000000D+00	0.137000D+01
130	FF	0.248897D+00	0.000000D+00	0.000000D+00	0.137000D+01
129	FF	0.251088D-01	0.000000D+00	0.000000D+00	0.137000D+01
128	FF	0.226138D+00	0.000000D+00	0.000000D+00	0.137000D+01
127	FF	0.732515D+00	0.000000D+00	0.000000D+00	0.137000D+01
126	FF	0.495305D+00	0.000000D+00	0.000000D+00	0.137000D+01
125	FF	0.146215D-01	0.000000D+00	0.000000D+00	0.137000D+01
124	FF	0.454854D-01	0.000000D+00	0.000000D+00	0.137000D+01
123	FF	0.538474D-01	0.000000D+00	0.000000D+00	0.137000D+01
122	FF	0.182332D-01	0.000000D+00	0.000000D+00	0.137000D+01
121	FF	0.114737D-01	0.000000D+00	0.000000D+00	0.137000D+01
120	FF	0.533842D-01	0.000000D+00	0.000000D+00	0.137000D+01
119	FF	0.106322D+00	0.000000D+00	0.000000D+00	0.137000D+01
118	FF	0.501110D-01	0.000000D+00	0.000000D+00	0.137000D+01
117	FF	0.433849D-01	0.000000D+00	0.000000D+00	0.137000D+01
116	FF	0.308186D+00	0.000000D+00	0.000000D+00	0.137000D+01
115	FF	0.109920D-01	0.000000D+00	0.000000D+00	0.137000D+01
114	FF	0.515586D-01	0.000000D+00	0.000000D+00	0.137000D+01
113	FF	0.223173D-02	0.000000D+00	0.000000D+00	0.137000D+01
112	FF	0.332856D+01	0.000000D+00	0.000000D+00	0.622500D+01
111	FF	0.307143D+00	0.000000D+00	0.000000D+00	0.415000D+01
110	FF	0.577519D+01	0.000000D+00	0.000000D+00	0.622500D+01
109	FF	0.152815D+00	0.000000D+00	0.000000D+00	0.137000D+01
108	FF	0.141628D+01	0.000000D+00	0.000000D+00	0.622500D+01
107	FF	0.325192D+02	0.000000D+00	0.000000D+00	0.622500D+01
106	FF	0.439307D+01	0.000000D+00	0.000000D+00	0.622500D+01
105	FF	0.226991D+01	0.000000D+00	0.000000D+00	0.622500D+01

104	FF	0.192250D-01	0.000000D+00	0.000000D+00	0.137000D+01
103	FF	0.633166D-01	0.000000D+00	0.000000D+00	0.137000D+01
102	FF	0.403938D+00	0.000000D+00	0.000000D+00	0.137000D+01
101	FF	0.222986D-02	0.000000D+00	0.000000D+00	0.137000D+01
100	FF	0.513205D-01	0.000000D+00	0.000000D+00	0.137000D+01
99	FF	0.827268D+00	0.000000D+00	0.000000D+00	0.137000D+01
98	FF	0.227916D+01	0.000000D+00	0.000000D+00	0.205500D+01
97	FF	0.210394D+00	0.000000D+00	0.000000D+00	0.137000D+01
96	FF	0.152405D+00	0.000000D+00	0.000000D+00	0.137000D+01
95	FF	0.517197D-01	0.000000D+00	0.000000D+00	0.137000D+01
94	FF	0.860835D-02	0.000000D+00	0.000000D+00	0.137000D+01
93	FF	0.332042D-01	0.000000D+00	0.000000D+00	0.137000D+01
92	FF	0.160094D-01	0.000000D+00	0.000000D+00	0.137000D+01
91	FF	0.780011D-02	0.000000D+00	0.000000D+00	0.137000D+01
90	FF	0.162530D-01	0.000000D+00	0.000000D+00	0.137000D+01
89	FF	0.377109D-02	0.000000D+00	0.000000D+00	0.137000D+01
88	FF	0.736754D-03	0.000000D+00	0.000000D+00	0.137000D+01
87	FF	0.290881D-01	0.000000D+00	0.000000D+00	0.137000D+01
86	FF	0.835932D-02	0.000000D+00	0.000000D+00	0.137000D+01
85	FF	0.247157D-01	0.000000D+00	0.000000D+00	0.137000D+01
84	FF	0.709949D-01	0.000000D+00	0.000000D+00	0.137000D+01
83	FF	0.868865D-01	0.000000D+00	0.000000D+00	0.137000D+01
82	FF	0.742535D-01	0.000000D+00	0.000000D+00	0.137000D+01
81	FF	0.141273D+01	0.000000D+00	0.000000D+00	0.205500D+01
80	FF	0.417956D+00	0.000000D+00	0.000000D+00	0.137000D+01
79	FF	0.323613D+00	0.000000D+00	0.000000D+00	0.137000D+01
78	FF	0.208345D+00	0.000000D+00	0.000000D+00	0.137000D+01
77	FF	0.290014D-01	0.000000D+00	0.000000D+00	0.137000D+01
76	FF	0.103398D+00	0.000000D+00	0.000000D+00	0.137000D+01
75	FF	0.388372D-02	0.000000D+00	0.000000D+00	0.137000D+01
74	FF	0.641638D-03	0.000000D+00	0.000000D+00	0.137000D+01
73	FF	0.134322D-03	0.000000D+00	0.000000D+00	0.137000D+01
72	FF	0.392302D-01	0.000000D+00	0.000000D+00	0.137000D+01
71	FF	0.595504D-01	0.000000D+00	0.000000D+00	0.137000D+01
70	FF	0.981388D-02	0.000000D+00	0.000000D+00	0.137000D+01
69	FF	0.659933D-01	0.000000D+00	0.000000D+00	0.137000D+01
68	FF	0.273144D-01	0.000000D+00	0.000000D+00	0.137000D+01
67	FF	0.156895D-01	0.000000D+00	0.000000D+00	0.137000D+01
66	FF	0.449408D-01	0.000000D+00	0.000000D+00	0.137000D+01
65	FF	0.225208D+00	0.000000D+00	0.000000D+00	0.137000D+01
64	FF	0.185843D+00	0.000000D+00	0.000000D+00	0.137000D+01
63	FF	0.173973D+00	0.000000D+00	0.000000D+00	0.415000D+01
62	FF	0.329084D+00	0.000000D+00	0.000000D+00	0.137000D+01
61	FF	0.463377D+00	0.000000D+00	0.000000D+00	0.137000D+01
60	FF	0.336530D+01	0.000000D+00	0.000000D+00	0.205500D+01
59	FF	0.157245D-01	0.000000D+00	0.000000D+00	0.137000D+01
58	FF	0.430975D-01	0.000000D+00	0.000000D+00	0.137000D+01
57	FF	0.186149D+00	0.000000D+00	0.000000D+00	0.137000D+01
56	FF	0.283083D+00	0.000000D+00	0.000000D+00	0.137000D+01
55	FF	0.120966D-02	0.000000D+00	0.000000D+00	0.137000D+01
54	FF	0.131914D-01	0.000000D+00	0.000000D+00	0.137000D+01
53	FF	0.181855D-01	0.000000D+00	0.000000D+00	0.137000D+01
52	FF	0.511627D-02	0.000000D+00	0.000000D+00	0.137000D+01
51	FF	0.127004D-01	0.000000D+00	0.000000D+00	0.137000D+01

50 FF	0.458915D+01	0.000000D+00	0.000000D+00	0.622500D+01
49 FF	0.387962D-01	0.000000D+00	0.000000D+00	0.137000D+01
48 FF	0.207079D-02	0.000000D+00	0.000000D+00	0.137000D+01
47 FF	0.275196D-01	0.000000D+00	0.000000D+00	0.137000D+01
46 FF	0.248647D-02	0.000000D+00	0.000000D+00	0.137000D+01
45 FF	0.423585D-01	0.000000D+00	0.000000D+00	0.137000D+01
44 FF	0.155453D+01	0.000000D+00	0.000000D+00	0.622500D+01
43 FF	0.676617D+00	0.000000D+00	0.000000D+00	0.137000D+01
42 FF	0.303234D+02	0.000000D+00	0.000000D+00	0.800000D+01
41 FF	0.249366D-01	0.000000D+00	0.000000D+00	0.137000D+01
40 FF	0.699900D-01	0.000000D+00	0.000000D+00	0.137000D+01
39 FF	0.187671D+00	0.000000D+00	0.000000D+00	0.137000D+01
38 FF	0.120376D+01	0.000000D+00	0.000000D+00	0.205500D+01
37 FF	0.760703D-01	0.000000D+00	0.000000D+00	0.137000D+01
36 FF	0.101401D+00	0.000000D+00	0.000000D+00	0.137000D+01
35 FF	0.489789D-01	0.000000D+00	0.000000D+00	0.137000D+01
34 FF	0.106032D-01	0.000000D+00	0.000000D+00	0.137000D+01
33 FF	0.221279D-01	0.000000D+00	0.000000D+00	0.137000D+01
32 FF	0.117950D-01	0.000000D+00	0.000000D+00	0.137000D+01
31 FF	0.201937D-01	0.000000D+00	0.000000D+00	0.137000D+01
30 FF	0.102812D-01	0.000000D+00	0.000000D+00	0.137000D+01
29 FF	0.225683D-01	0.000000D+00	0.000000D+00	0.137000D+01
28 FF	0.140550D+01	0.000000D+00	0.000000D+00	0.205500D+01
27 FF	0.197455D+00	0.000000D+00	0.000000D+00	0.137000D+01
26 FF	0.283945D-01	0.000000D+00	0.000000D+00	0.137000D+01
25 FF	0.614739D-01	0.000000D+00	0.000000D+00	0.137000D+01
24 FF	0.196819D-01	0.000000D+00	0.000000D+00	0.137000D+01
23 FF	0.302719D+00	0.000000D+00	0.000000D+00	0.137000D+01
22 FF	0.635271D-01	0.000000D+00	0.000000D+00	0.137000D+01
21 FF	0.440154D-01	0.000000D+00	0.000000D+00	0.137000D+01
20 FF	0.414481D-01	0.000000D+00	0.000000D+00	0.137000D+01
19 FF	0.135784D+00	0.000000D+00	0.000000D+00	0.137000D+01
18 FF	0.151132D+00	0.000000D+00	0.000000D+00	0.137000D+01
17 FF	0.259118D+00	0.000000D+00	0.000000D+00	0.137000D+01
16 FF	0.239816D+00	0.000000D+00	0.000000D+00	0.137000D+01
15 FF	0.712913D+01	0.000000D+00	0.000000D+00	0.622500D+01
14 FF	0.549427D+01	0.000000D+00	0.000000D+00	0.622500D+01
13 FF	0.663146D+01	0.000000D+00	0.000000D+00	0.622500D+01
12 FF	0.191821D+00	0.000000D+00	0.000000D+00	0.137000D+01
11 FF	0.383171D+00	0.000000D+00	0.000000D+00	0.137000D+01
10 FF	0.571416D+00	0.000000D+00	0.000000D+00	0.137000D+01
9 FF	0.199340D+01	0.000000D+00	0.000000D+00	0.205500D+01
8 FF	0.364833D+00	0.000000D+00	0.000000D+00	0.137000D+01
7 FF	0.618527D+00	0.000000D+00	0.000000D+00	0.137000D+01
6 FF	0.120020D+00	0.000000D+00	0.000000D+00	0.137000D+01
5 FF	0.123696D-01	0.000000D+00	0.000000D+00	0.137000D+01
4 FF	0.153296D+00	0.000000D+00	0.000000D+00	0.137000D+01
3 FF	0.275687D-02	0.000000D+00	0.000000D+00	0.137000D+01
2 FF	0.120442D-01	0.000000D+00	0.000000D+00	0.137000D+01
1 FF	0.421704D-01	0.000000D+00	0.000000D+00	0.137000D+01