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ASYMMETRIC BUILDING STRUCTURES

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Abstract

A simple method of forming a planar model for the two-dimensional computer analysis of a three-dimensional asymmetric structure that translates and twists under horizontal load is presented. The structure must consist of parallel bents which may be of any type and whose properties may vary with height. The bents and member properties are represented directly in the model, without transformation, while the results for displacements and forces from the analysis may be read directly, also without transformation. The use of a planar model makes it possible to simulate a three-dimensional analysis using a planar analysis program. The simplification allows a reduction in both the engineer's and the computer's time, as well as allowing the analysis of large structures on smaller computers.

Introduction

Until now, two-dimensional computer models for analyzing building structures have mostly been confined to representing systems that are symmetric about the axis of loading, such as the one shown in Fig. 1. These systems translate without twisting. The authors have presented, in a previous paper (Ref. 1), various techniques for the planar modelling of non-twisting behaviour. Eccentric lateral loads, on the other hand, produce both translation and twisting responses in buildings. In symmetrical structures, these modes of behaviour can easily be separated, enabling other simple two-dimensional models to be developed (Ref. 2). The bending and twisting that occurs in asymmetrical buildings, however, is much more difficult to describe and is almost universally thought of as requiring a complete three-dimensional model for analysis. The purpose of this paper is to introduce a planar modelling technique that is capable of fully and accurately duplicating the behaviour of asymmetrical structural systems.

Two other two-dimensional techniques for analyzing such structures have preceded the method to be described herein. In Ref. 3, a technique was proposed for a certain class of symmetrical structures, i.e. ones which consist of only "two types of framing systems, each comprising several vertical planar assemblages having similar stiffness properties and a common variation thereof along the height". The method is based on a mathematical technique of co-ordinate transformation, yielding uncoupled systems of equations. It involves transforming the stiffnesses of the framing systems, analyzing the resulting analogous systems two-dimensionally, reconverting the forces and displacements that result, and linearly recombining them to obtain the real structure's response. The technique involves much manual manipulation or specially written computer software for handling the numerous computations that both precede and succeed the actual two-dimensional analysis. In Ref. 4, a planar method of analysis was proposed for structures that are asymmetric about an axis parallel to the axis of horizontal loading but symmetric about the orthogonal axis. The method involves converting the original bents to analogous columns, which represent the shear and torsion properties, and assembling them into a planar model for analysis. The method is restricted to bents that behave as 'shear' members and, because of the 'lumping' procedure, it is not usually possible to recover the individual column forces.

The planar technique proposed in this paper, on the other hand, avoids having to transform member properties or resulting forces and displacements. As a result, the model's response to loading has actual physical significance and can lead to a better understanding of the coupled/twisting behaviour of an asymmetrical structural system. The method is unrestricted in the number of types of framing systems it may include, and in the possible variations of these systems with height. The model is restricted, however, to structures comprising a system of parallel bents aligned with the direction of horizontal loading, Fig. 2. It's formulation assumes the floor slabs to be infinitely rigid in, and infinitely flexible out of, their planes. The first assumption is generally accepted as correct and is inherent to the technique. The second assumption, that of an out-of-the-plane flexible floor between bents, can truly only approximate reality when the bents are widely spaced and the floor slab between them is thin. It should be noted that this is a relatively common situation in typical apartment building construction. However, work is currently proceeding on finding a way of incorporating into the planar model the effect of perpendicular-to-the-load framing.

Structural Behaviour

The feature of structural behaviour that has long delayed the development of the planar model for an asymmetrical structural system is the difficulty in uncoupling its simultaneous translational and twisting responses to an arbitrary lateral loading distribution. In the symmetrical case, the amount of lateral load causing translation only, and of torque producing twisting load only, can easily be deduced from the eccentricity of the applied load (since the centre of resistance and the centre of twist are known at every floor level to be at the geometrical centre of the structural system). It is possible to then construct and analyze two planar models, one for translation, the other for torsion, and to combine the results of the two appropriately to obtain the complete structural behaviour (Ref. 2).

When dealing with an asymmetrical structure, however, the behaviour is much more complex. The structure's centres of resistance and twist are obviously off-centre; further, these centres have been found to be distinct from one another and to vary in location from one floor level to the next. Establishing the positions of these centres is possible, but only after having previously executed matrix analyses that incorporate the complete structural system (Refs. 5, 6). Evidently, if the planar technique being developed for analyzing asymmetrical structures is to be of practical use, a different concept from that used for symmetrical structures must be found. Ironically, it turns out that the key to the problem lies in taking an apparent step backward by no longer trying to separate translational from twisting responses, but instead by developing a model that can incorporate both behaviours simultaneously.

The way to begin the development of the model is to consider one floor level of a structure and its displacement under loading. A floor slab of a very simple asymmetrical building is shown in Fig. 3. As a result of lateral loading applied to the structure, this floor displaces to some equilibrium position such as indicated. To displace in this manner, the floor must undergo both lateral translation and rotation. It is usual from the treatment of symmetrical structures to describe those motions as occurring about the centre of twist. The displacement in the direction of loading of a randomly selected point B on the floor slab can then be seen from Fig. 3 to be given by

$$\delta_{B,CT} = \{\delta[\text{due to transl. of ctr. of twist}]\} + \{\delta[\text{due to rotation about ctr. of twist}]\}$$

$$= \{\Delta\} + \{V\sin(\theta+\alpha) - V\sin\alpha\} \qquad (1)$$

where $\Delta_A, V, W, \alpha, \theta$ and ϕ are defined in the figure. It bears repeating, however, that the location of the centre of the twist is not easily found in an asymmetrical structure. As a result, another method of describing the displacement of the floor slab is required.

Consider relating a floor's translation and rotation to any arbitrarily chosen point in the plane of the slab (point A in Fig. 3, for instance) instead of to its centre of twist. It is postulated that the slab displaces about this point as though an imaginary rigid arm connects the slab to A. The lateral movement of point B, in terms of the displacement and rotation occurring at A, can then be defined as

$$\delta_{B.A} \quad \{\delta[\text{due to transl. of } A]\} + \{\delta[\text{due to rotation about } A]\}$$

$$= \{\Delta_A\} - \{[W \sin\phi + V \sin\alpha] - [V \sin(\theta + \alpha) - W \sin(\theta - \phi)]\} \quad (2)$$

where are as shown in Fig. 3.

From geometry, however the following relationship can be established

$$\Delta_A = \Delta + W \sin\phi + W \sin(\theta - \phi) \quad (3)$$

Back substituting this expression for into Eqn. 2 leads to

$$\delta_{B.A} = \Delta + V \sin(\theta + \alpha) - V \sin\alpha \quad (4)$$

which is identical to the expression for the lateral displacement at B as initially derived with respect to the centre of twist in Eqn. 1.

Thus it has been shown that the equilibrium position of a laterally loaded floor slab can be described equally well with respect to an arbitrarily selected reference centre A as to the centre of twist.

So far, only the response to loading of an individual floor has been considered. A complete structural system, however, comprises many such floors, each of which will most likely translate and rotate by a different amount. As a corollary of the fact that each floor's displacement can be referred to any point in its plane, it follows that all floors, no matter what their response, can be related to a commonly located reference point in each of the floor level plans; i.e., the reference points for all of the floor levels can be chosen in such a way that they lie on a single vertical line, Fig.4. By this choice, the previously mentioned obstacle to developing a two-dimensional model for asymmetrical structures - the difficulty in determining each floor's actual centres of resistance and twist - is sidestepped, and a simple means is achieved for describing the complete response of the structures. It is shown in the following section how this new concept can be utilized to develop a planar model for the analysis of asymmetrical structural systems.

In order to simplify the construction of the two-dimensional model it is recommended that the vertical axis of reference centres be located outside the building envelope, as opposed to within it. By following this suggestion, it is ensured that all structural elements' lateral displacements caused by the rotation of floors about the reference centres occur in the same direction. If the reference centre axis was situated within the building itself, such lateral displacements would take place in opposite directions on either side of the axis. While this latter behaviour is only slightly more difficult to incorporate into a planar model, its inclusion is a needless complication and, might as well be avoided.

Two-Dimensional Model

The process of constructing the two-dimensional model begins by standing all the load-resisting structural bents side by side in a single plane. This is demonstrated in Fig. 5 for an example of an asymmetrical structure. The bents need not be positioned in any particular order, can be arbitrarily spaced, and are located relative to some convenient co-ordinate origin typically situated at the bottom left-hand corner of the model. The only rule that must be observed in assembling these bents is that they are all as though being viewed in the real structure looking in the same direction. For example, the bents in Fig. 5b have been drawn from the perspective of an observer who is looking at the structural system shown in Fig. 5a from left to right.

Having represented all the vertical structural elements, only the constraint of the horizontal rigid floor diaphragms on these elements' relative displacements, and on the distribution of forces between them remains to be included. For this, sets of additional nodes with each bent of the structure are defined, as are members that connect these nodes to corresponding floor levels, Fig. 6. The special characteristics of these extra nodes and members are described in the following paragraphs.

The nodes added to the model, termed governing nodes, replace the function of the reference centres in the three-dimensional system; i.e., the structural elements displace laterally with respect to these points. Each bent has associated with it governing nodes equal in number to its floor levels. The location of an individual governing node is established as follows. Horizontally, its coordinate is set the same as that of the bent on its corresponding floor level. In the example shown in Fig. 6, each governing node is thus horizontally located, although to aid the clarity of the drawing these nodes are shown offset further left. Vertically, a governing node is positioned above its associated floor level a distance equal to the distance in plan between the bent and the chosen reference centre axis. Fig. 6 demonstrates this for the example structural system and reference centre axis shown in Fig. 5a. The governing nodes are constrained against vertical displacement; however, their remaining two degrees of freedom, rotation and horizontal movement, are not unconditionally free. It is necessary, using the internodal degree of freedom constraint option, available on the more comprehensive general purpose structural analysis programs, to constrain the horizontal displacement of all of the governing nodes associated with a particular floor level of a structure to be identical. For example, the nodes in Fig. 6 indicated by an asterisk, which govern the behaviour of the structure's fifth floor, are so constrained. By the same technique, the rotations of these floor level sets of governing nodes are also constrained to match one another. In this manner, the governing nodes defined for each floor level in the planar model take the place of the assigned single reference centre about which the different levels of the actual three-dimensional structure are taken to rotate.

The function of the members that join the governing nodes to the structural bents in the model is to duplicate the effect in the real structure of a) the in-plane rigidity of the floor diaphragms and b) the rigid arms imagined to connect these diaphragms to their reference centres. In order to carry out this function, it is necessary that the connecting members are assigned to be very stiff in both flexure and shear. Also, so that only shear force is transferred from these members into the structural bents - just as the actual structure shear force alone is considered to be passed from the floor slabs into vertical structural elements - moment and axial force releases must be specified at the ends of the connecting members attached to the bents. As a guide to the term "very stiff", it is suggested that the inertias of the rigid arms should be assigned values so that, if a rigid arm were considered as a vertical cantilever, fixed at the governing node and free at its lower end, its horizontal stiffness at the lower end would be of an order 10^3 to 10^5 greater than the estimated horizontal stiffness of the bent at the level to which the rigid arm connects.

This completes the description of the physical elements that compose the planar model of an asymmetrical structural system. It must now be established how the model is to be loaded.

It has been shown that in three-dimensional space a building's response to loading can be decomposed into lateral translation and rotation relative to an arbitrarily located vertical reference centre axis. External horizontal loading, no matter where or how it is applied, can also be interpreted relative to the reference centres by statically equivalent sets of loads. In Fig. 7b this is shown for the concentrically applied lateral load acting on the floor of the example structure, Fig. 7a. The lateral load can be transformed to a similar load at the reference centre and a torque. Therefore, for the building as a whole, whatever the form of the actual lateral loading and whatever variations it exhibits with height, it is possible to transform it into a simple equivalent distribution of forces acting at the floor reference centres and torques.

In the planar model the governing nodes, which are analogous to the reference centres in the real structure, are used to apply the lateral loading. The torques and lateral forces, determined as acting at the structure's reference centres, are applied to one bent's governing nodes, Fig. 7c. The constraint between the displacements and rotations of the governing nodes causes the force and torque applied to one governing node to be effectively applied simultaneously to all the other governing nodes for the same floor level. The directions of the forces and torques applied to the model are decided so that the displacements they cause in the bents conform with those resulting from the actual loading on the real structure. For example, in the planar model shown in Fig. 7c, horizontal forces acting from left to right and counterclockwise moments are depicted applied to the governing nodes. Both the forces and the moments induce rightward deflections in the bents by means of the rigid connecting members.

Testing of the Model

For comparison purposes the results of the analysis of the two-dimensional model just described are presented in Tables 1 and 2, beside those from a conventional three-dimensional structural model. In both models, beam-type elements are used to represent columns and beams, and quadrilateral membrane-type elements are used to represent wall segments. Except for the joints at the base, which are fixed, every joint in the three-dimensional model has six degrees of freedom. The rigid-floor diaphragm effect is incorporated into the three-dimensional model by a set of perimeter beams that have relatively large axial areas and moments of inertia in the plane of the slab. Some structural analysis programs, specially written for the analysis of buildings, have a rigid-floor diaphragm option that would seemingly eliminate the need for this perimeter set of beams and simultaneously cut almost in half the degrees of freedom in the three-dimensional model. However, even when this option is available it is not always possible to use it since some of these programs restrict its use to models constructed of only beam-type elements, which is not the case in this example. Table 3 compares the size of the two and three-dimensional models.

All the results of the analysis using the two-dimensional model are within five percent of the corresponding values obtained from the three-dimensional representation. This is certainly an acceptable difference for building design, given the inevitable uncertainties and inaccuracies inherent in any numerical representation of a real structure. The discrepancies between the results of the two types of analyses were probably attributable to the lack of complete rigidity in the connecting arms of the two-dimensional model, and in the horizontal frames of the three-dimensional model. Further, as indicated in Table 3 by the number of equations to be solved for the analysis of each model, the two-dimensional technique is much more concise than the three-dimensional method. The saving in computer time is partly a function of the analysis program and the size of the structure. For this example problem, using the SAP 80 program, the proposed two-dimensional analysis ran in one-third of the time for the three-dimensional analysis.

Condensed Two-Dimensional Model

Having described the concept and demonstrated the validity of the two-dimensional model, a condensed version of this model can easily be introduced, Fig. 8. Instead of defining an separate set of governing nodes for each bent of the structure, only a single set of governing nodes need be defined. The bents must then be offset vertically from this set of nodes according to their in-plan distance from the chosen reference centre. All the bents are then joined at their floor levels to this single set of governing nodes. In all other respects, this condensed model is the same as the original two-dimensional one. Obviously, the reduced size of this model enables even greater reductions in computer storage requirements and running time.

Conclusion

Three-dimensional asymmetrical structural systems can be analyzed simply and accurately in two dimensions as a result of the techniques developed in this paper. An original concept for the division of translational and twisting responses to loading is developed to permit these behaviours to be incorporated simultaneously in a single planar model. This model significantly broadens the range of structures that can be investigated when restricted to a two-dimensional program. Also, it significantly reduces the time and storage capacity required of a computer for an analysis of a very large building structure, rendering an analysis by a micro-computer much more feasible.

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Planar Modelling Techniques for Asymmetric Building Structures

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Table 1: Lateral Deflection at the Top of the Building

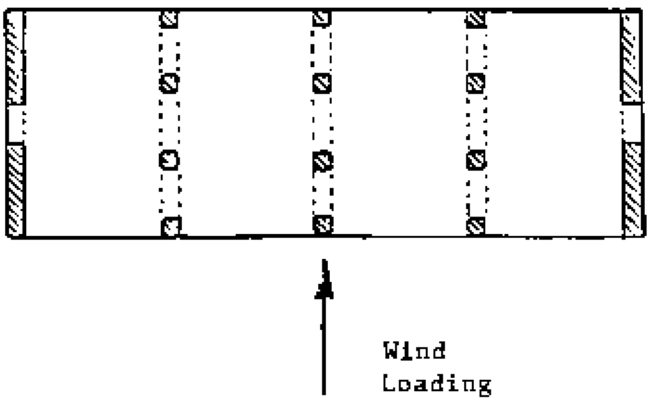
	2 - D Model (in.)	3 - D Model (in.)	% difference
Bent #1	0.510	0.527	3
Bent #2	0.392	0.406	3
Bent #3	0.274	0.285	3
Bent #4	0.156	0.163	4

Table 2: Distribution of Joint Displacements Over Height of the Building for Windward Side of Bent #1

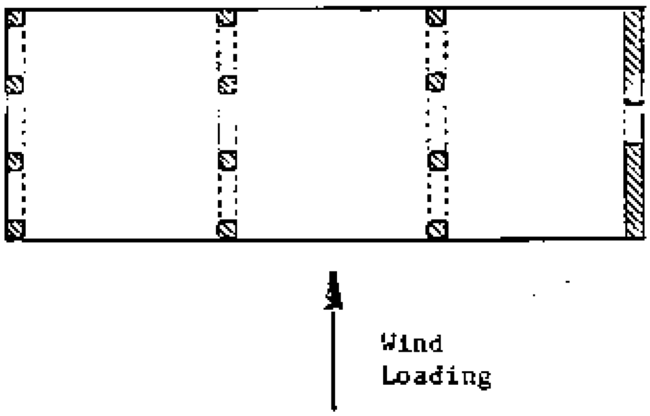
		10th Floor	6th Floor	3rd Floor
Horizontal Displacement (in.)	2 - D Model	0.510	0.270	0.077
	3 - D Model	0.527	0.279	0.080
	% difference	3	3	3
Verical Displacement (in.)	2 - D Model	0.0237	0.0192	0.0097
	3 - D Model	0.0235	0.0191	0.0096
	% difference	1	1	1
Rotation (rad.)	2 - D Model	0.000219	0.000353	0.000303
	3 - D Model	0.000216	0.000367	0.000314
	% difference	1	4	4

Table 3: Comparison of Size of 2 - D and 3 - D Models

	2 - D Model	3 - D Model
Number of Nodes	194	154
Number of Equations to be solved	340	810
Number of Beam-Type Elements	220	240
Number of Membrane-Type Elements	30	30



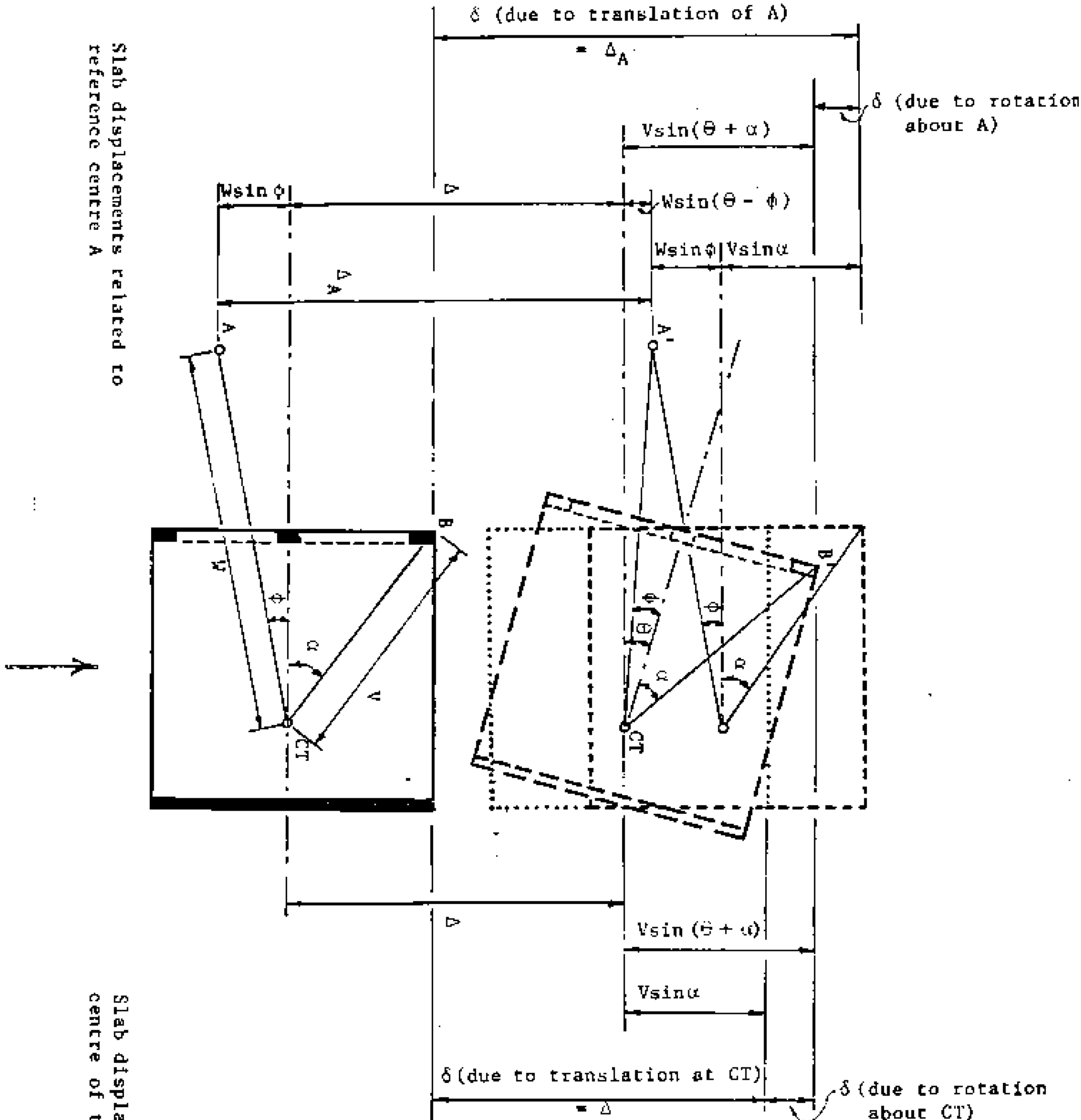
1

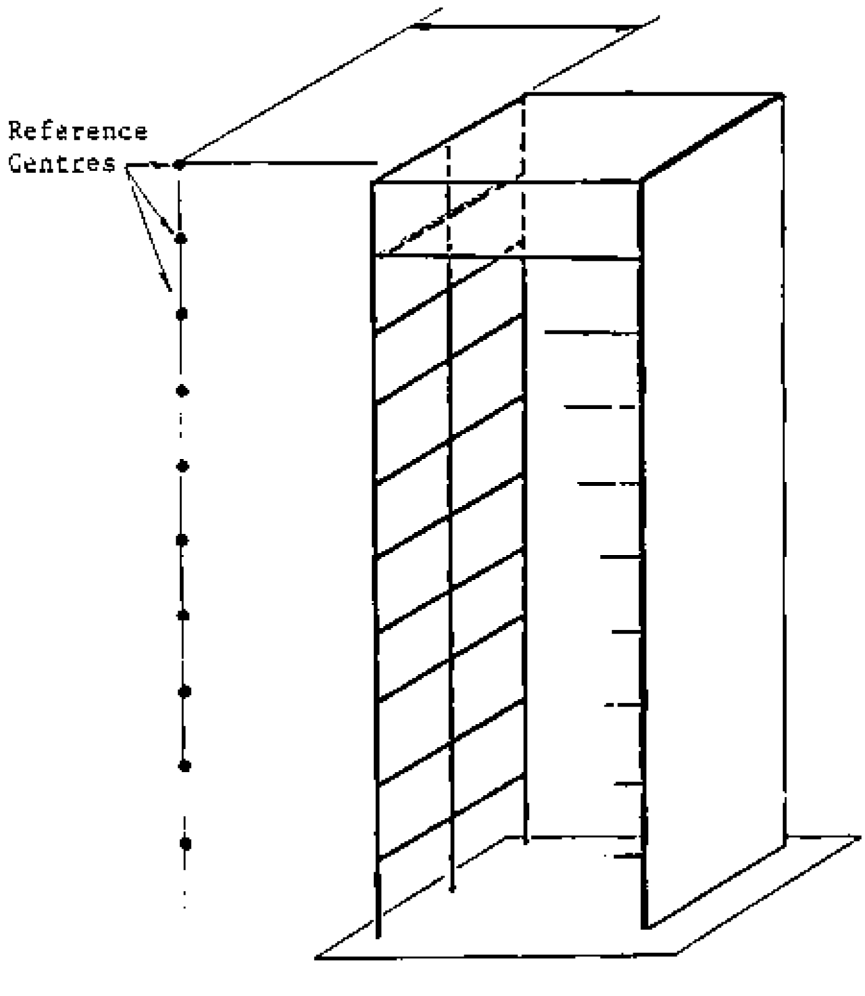


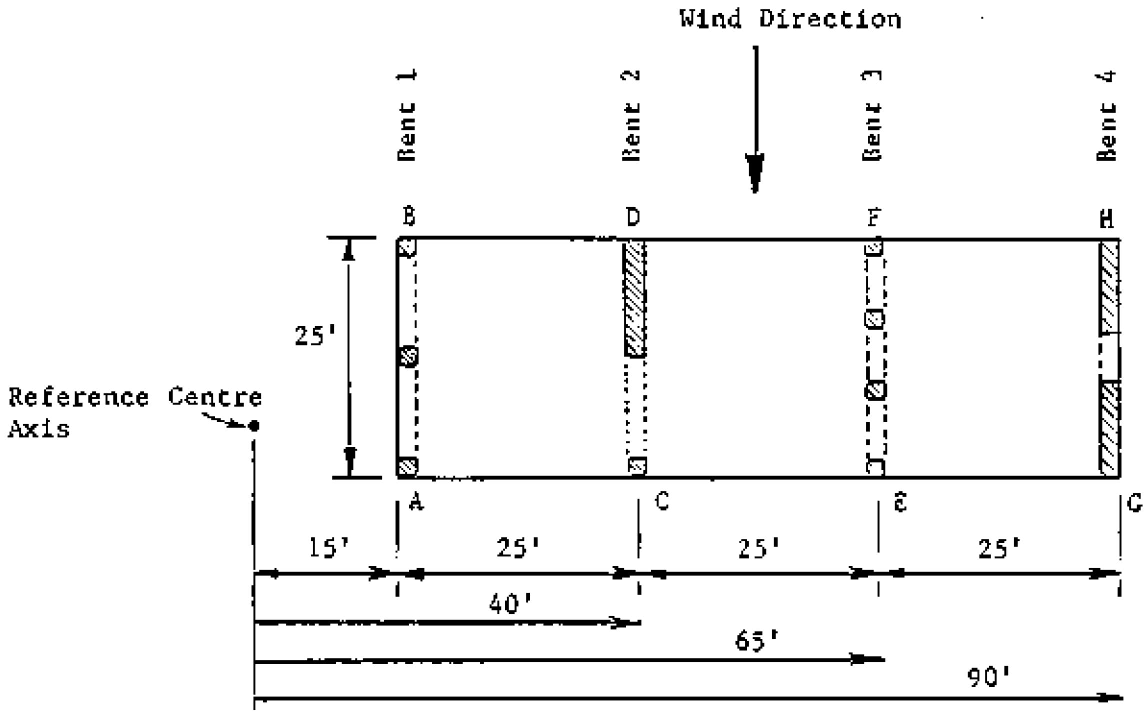
2

Slab displacements related to reference centre A

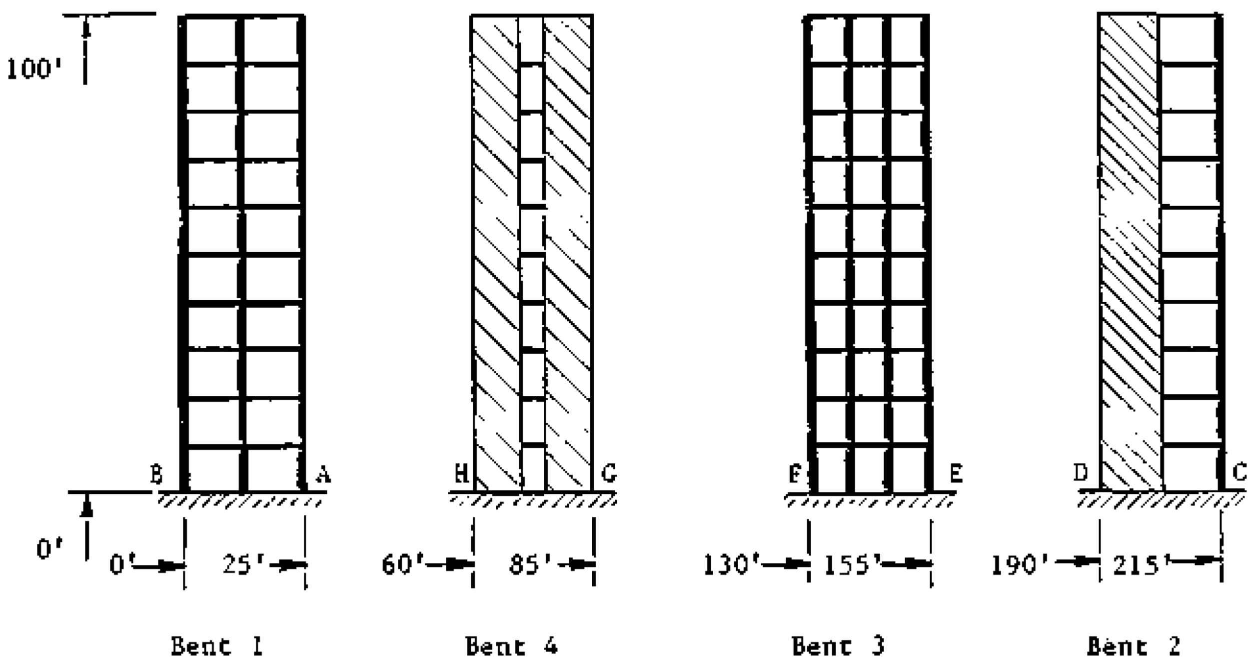
Slab displacements related to centre of twist



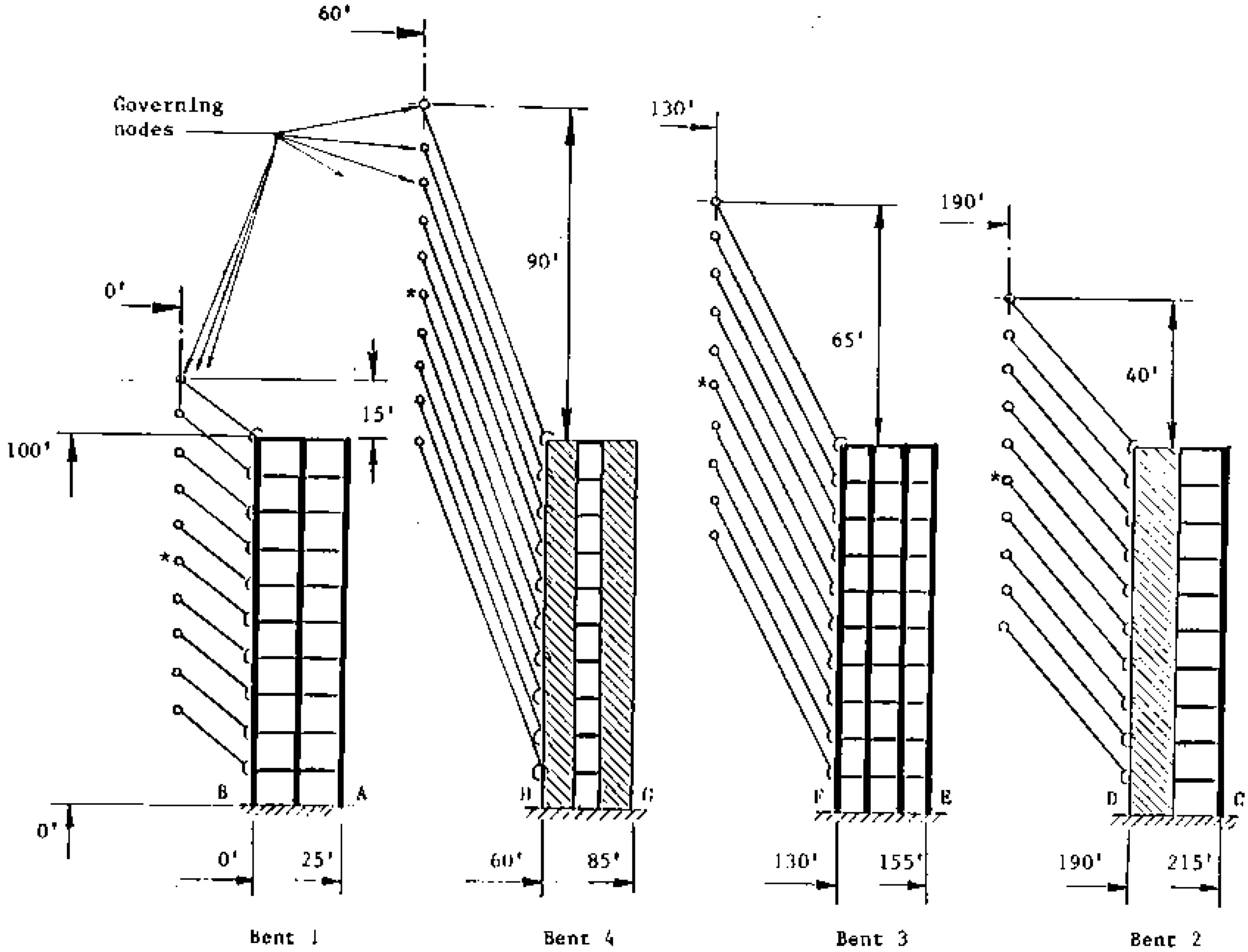

Reference
Centres


Wind Direction
Bent 1
Bent 2
Bent 3
Bent 4
B
D
F
H
25'
Reference Centre
Axis
A
C
E
G
15'
25'
25'
25'
40'
65'
90'

5a

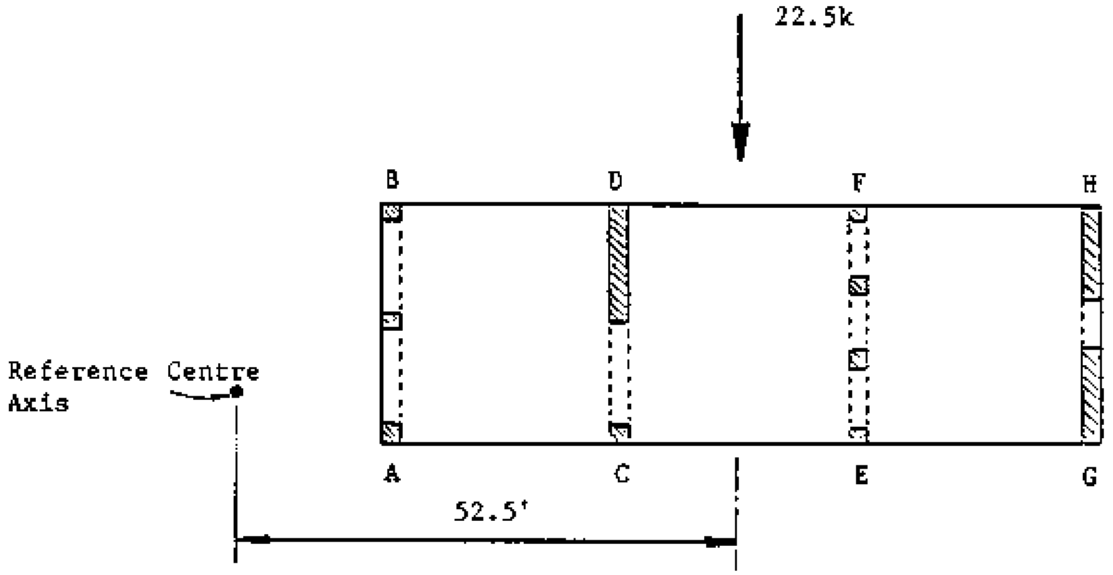

100'
0'
B
A
H
G
F
E
D
C
0'
25'
60'
85'
130'
155'
190'
215'
Bent 1
Bent 4
Bent 3
Bent 2

5b

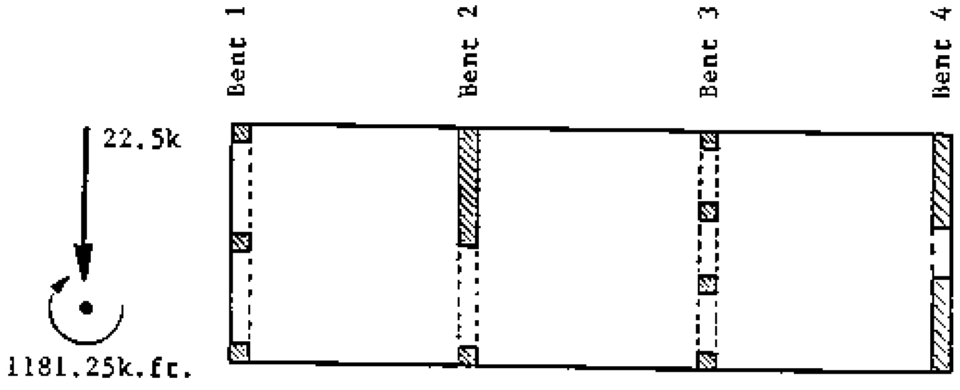


Governing nodes
60'
0'
130'
190'
90'
65'
40'
15'
100'
0'
B
A
H
G
F
E
D
C
0'
25'
60'
85'
130'
155'
190'
215'
Bent 1
Bent 4
Bent 3
Bent 2

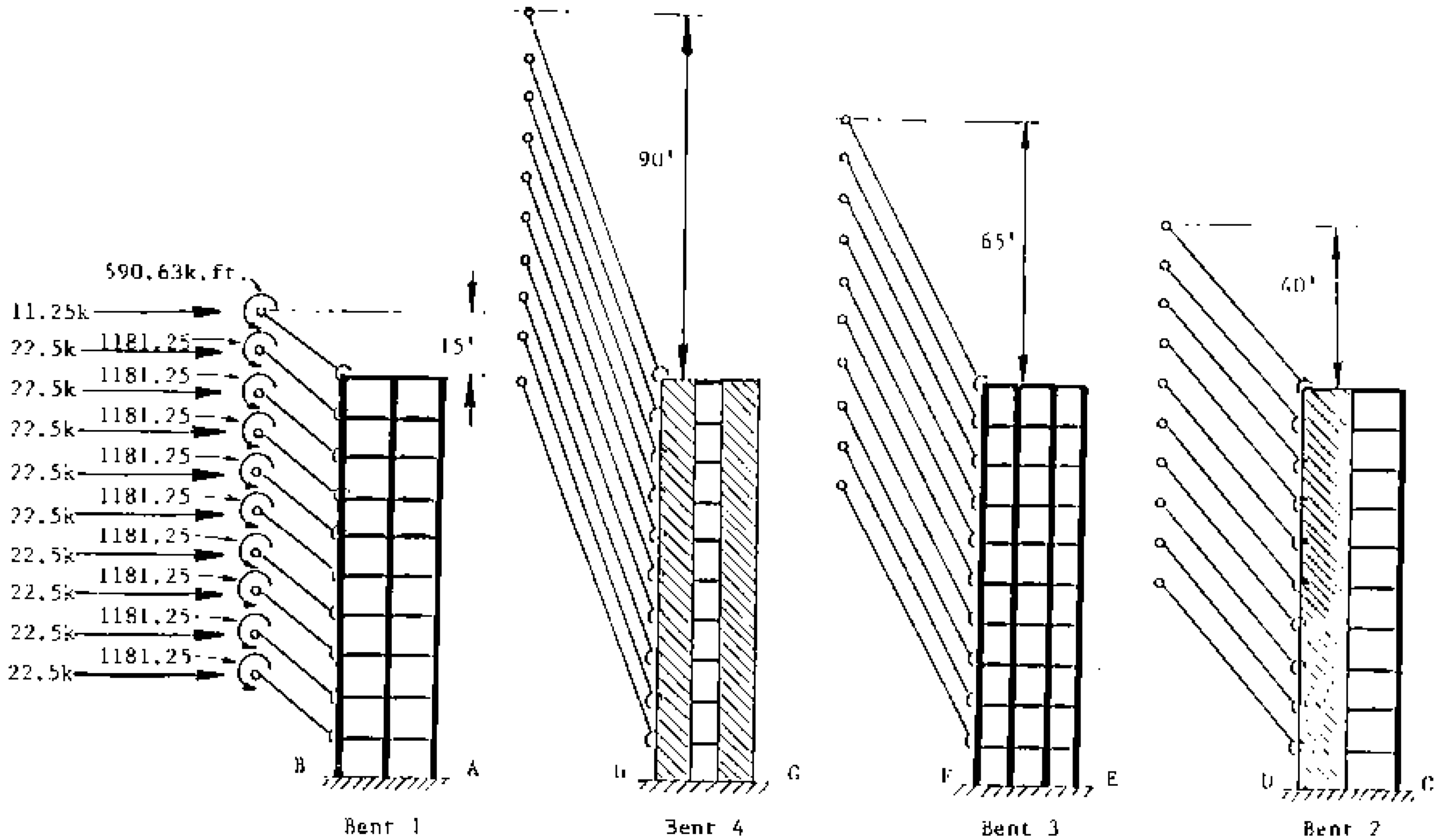
6


22.5k
B
D
F
H
Reference Centre Axis
A
C
E
G
52.5'

7a

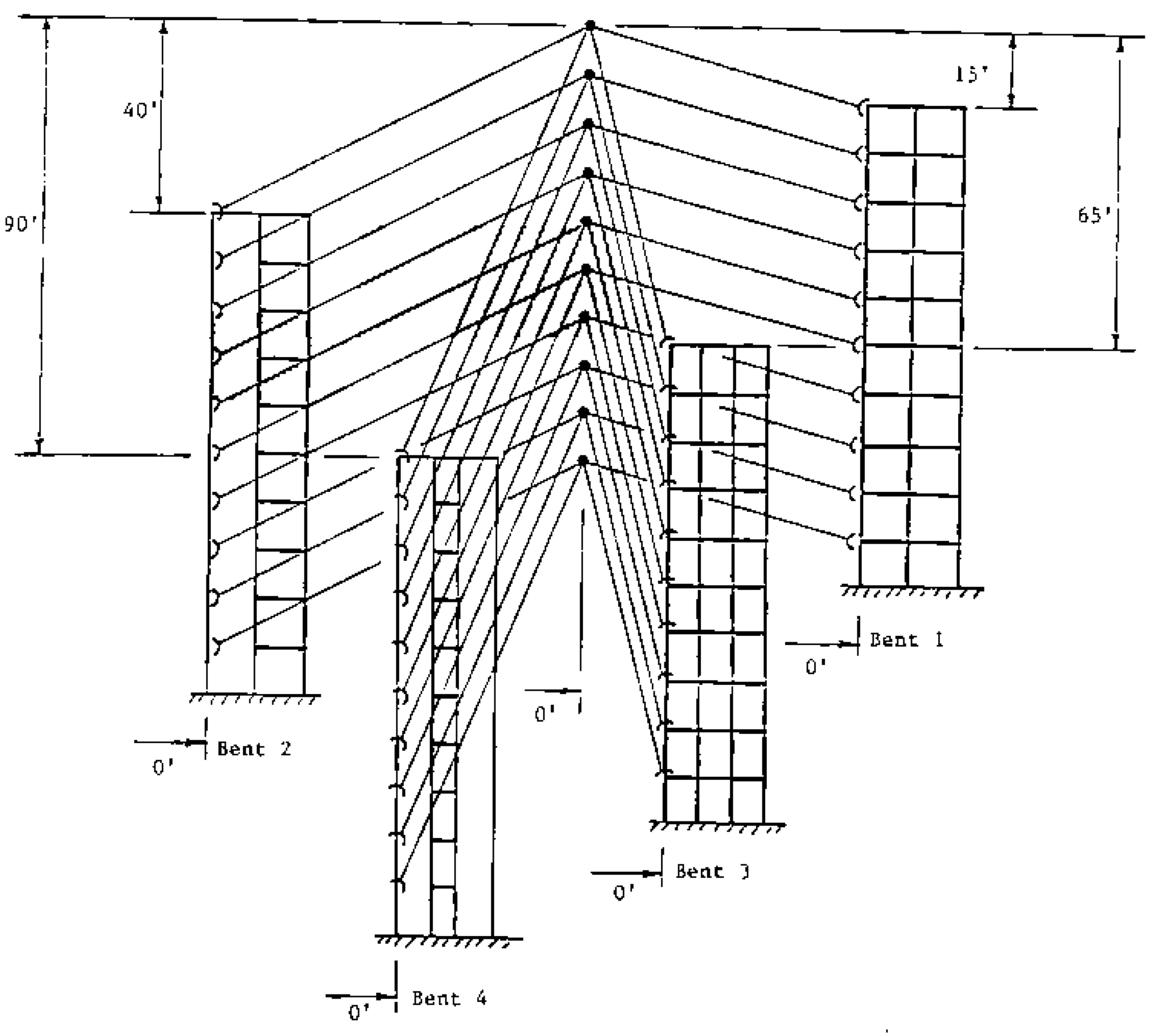

Bent 1
Bent 2
Bent 3
Bent 4
22.5k
1181.25k.ft.

7b



590.63k.ft
11.25k
22.5k
1181.25
15'
90'
65'
40'
A
B
G
H
E
F
C
D
Bent 1
Bent 4
Bent 3
Bent 2

7c



40'
90'
15'
65'
0'
Bent 1
0'
0'
Bent 2
0'
Bent 3
0'
Bent 4