Impact Evaluation of Noise Uncertainty in Spectrum Sensing under Middleton Class A Noise

Enwei Xu
School of Electrical and Information Engineering
Harbin Institute of Technology, Harbin, China 150001
Email: xewmickey@gmail.com

Fabrice Labeau
Department of Electrical and Computer Engineering
McGill University, Montreal, Canada H3A 0G4
Email: fabrice.labeau@mcgill.ca

***Abstract*—Reliable spectrum sensing is required to enable the effective use of Cognitive Radio (CR) networks. In practice, CRs not only have to cope with impulsive noises but also need to consider noise uncertainties. Small uncertainties of noise parameters are inevitable in any practical detector and the impact of these must be considered. This paper considers the energy detector based spectrum sensing under Middleton Class A noise with parameter uncertainties. Firstly, we show how analytical expressions of the probabilities of detection and false alarm can be derived. Secondly, we show that the mismatch of impulsive parameters A and Γ have little impact on performance and can thus generally be ignored but the uncertainty of noise power σ_Z^2 can induce SNR Walls to the detector, which means that, no matter how many observations are obtained, the detector cannot robustly detect the primary signal. These results have implications for designing an enhanced energy detector for robust spectrum sensing.**

***Index Terms*—Cognitive Radio, Spectrum Sensing, impulsive noise, noise uncertainties.**

I. Introduction

In wireless communication systems, the need for higher data rates has increased due to multimedia applications. As a result, the limitation of the natural frequency spectrum has become more obvious. Cognitive Radio (CR) as a tempting solution to exploit the available spectrum has emerged. In order to make opportunistic usage of the frequency bands that are not heavily occupied by Primary Users (PU), the ability to measure or sense the absent of PU signal is necessary [1]. The task of obtaining awareness about the spectrum usage and the absence of primary users is called spectrum sensing. A number of different methods are proposed for identifying the presence of signal transmissions, such as energy-detector-based sensing [2], matched filtering based sensing [3], waveform-based sensing [4], cyclostationarity-based sensing [5] and radio identification based sensing [6] etc. Among these methods, energy-based approaches are the most common because of their low computational and implementation complexities. In addition, energy-based detectors do not need any knowledge about the PU's signal.

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Most of the existing literature on spectrum sensing considers impairment by additive white Gaussian noise (AWGN) only. However, this assumption fails to model the behavior of certain noise types in practice, such as impulsive noise. The Middleton Class A noise model is one of the widely investigated statistical distributions that is used to model the man-made interference and the impulsive noise in different systems [7]. In energy-based sensing, the signal is detected by comparing the output of the energy detector with a threshold which depends on the noise floor. The threshold can be selected for finding an optimum balance between the probability of false alarm P_{f_a} and the probability of detection P_d. In practice, the threshold is chosen to obtain a certain false alarm rate [8]. In AWGN scenario, the selection of the threshold is dependent, as we will show, on the noise variance; a small noise power estimation error can cause significant performance loss [9] [10]. Similarly, the selection of the threshold under Middleton Class A noise is dependent on the parameters of the noise model. The impact of the parameter mismatch on the sensing performance must be considered. However, none paper is dealing with this issue.

We consider the energy-detector based spectrum sensing under impulsive noise with noise uncertainty, providing the following contributions.

- We derive the analytical expressions of the probability of false alarm P_{f_a} and the probability of detection P_d when the PU signal is contaminated by Middleton Class A noise. In particular, we show the relationship between the observation samples required and the parameters of noise and the SNR for achieving the target P_{f_a} and P_d.
- We analyze the impact of the noise parameters uncertainty on sensing performance. We conclude that the uncertainty of impulsive parameters A and Γ have little impact on sensing performance and can thus generally be ignored, but the SNR Wall phenomenon will arise with the uncertainty of noise power σ_Z^2.

The remainder of this paper is organized as follows. The signal model is defined in Section II. In Section III we give the derivation of the analytical expressions of the probability of false alarm and detection. We analyze the impact of the noise parameters uncertainty on sensing performance in Section IV and the conclusion is explained in Section V.

II. Signal Model

In spectrum sensing problem, the PU signal to be sensed is considered as a random process (called Bayesian model) in some works, and it is also considered as an unknown deterministic signal (called classical model) in others [11]. Here we choose Bayesian model and consider a source x with a zero-mean Gaussian PDF

$$f_X(x) = G(x;\sigma_X^2) = \frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{x^2}{2\sigma_X^2}} \tag{1}$$

transmitted over a channel impaired by a Middleton Class A noise z, whose PDF is

$$f_Z(z) = \sum_{m=0}^{\infty} \beta_m G(z;\sigma_m^2) = \sum_{m=0}^{\infty} \frac{\beta_m}{\sqrt{2\pi}\sigma_m} e^{-\frac{z^2}{2\sigma_m^2}} \tag{2}$$

where $\beta_m = \frac{e^{-A}A^m}{m!}$ indicates that m noise sources contribute to the impulsive event simultaneously, and $A = E\{m\} = \sum_{m=0}^{\infty} m\beta_m$ is the corresponding overlap index denoting the average number of impulse noise sources active at any given time. Larger values of A make the characteristic of the noise closer to Gaussian noise. Moreover, $\sigma_Z^2 = E\{z^2\} = \sigma_G^2 + \sigma_I^2$ is the noise power, where σ_G^2 is the Gaussian power, σ_I^2 is the impulsive power. $\Gamma = \frac{\sigma_G^2}{\sigma_I^2}$ is the power-ratio of the Gaussian component to the impulsive component, and $\sigma_m^2 = \frac{\frac{m}{A}+\Gamma}{1+\Gamma}\sigma_Z^2 = \sigma_G^2 + \sigma_I^2\frac{m}{A}$. Thus, the Middleton Class A noise is totally characterized by the parameters A, Γ and σ_Z^2 [12].

III. Energy detector under Middleton Class A noise

The problem of detecting the presence of primary users can be considered as the following binary hypothesis testing problem [13]:

$$\begin{aligned} H_0 &: y(n) = z(n) \\ H_1 &: y(n) = x(n) + z(n) \end{aligned} \tag{3}$$

where $n = 1, 2, 3, ..., N$, N is the number of observed samples; $y(n)$ is the signal observed by sensing receiver with $x(n)$ and $z(n)$ denoting the PU signal and the additive impulsive noise respectively. It is obvious that under hypothesis H_0 , the PU signal is absent and $y(n)$ consists only of noise $z(n)$. On the contrary, under hypothesis H_1 the PU signal is present along with noise $z(n)$.

Assume the noise $z(n)$ and the signal $x(n)$ are independent of each other, we can obtain that

$$\begin{aligned} f_Y(y(n))|_{H_0} &= f_Z(y(n)) = \sum_{m=0}^{\infty} \frac{\beta_m}{\sqrt{2\pi}\sigma_m} e^{-\frac{y(n)^2}{2\sigma_m^2}} \\ f_Y(y(n))|_{H_1} &= f_X(y(n)) * f_Z(y(n)) \\ &= \sum_{m=0}^{\infty} \frac{\beta_m}{\sqrt{2\pi(\sigma_m^2+\sigma_X^2)}} e^{-\frac{y(n)^2}{2(\sigma_m^2+\sigma_X^2)}} \end{aligned} \tag{4}$$

As for the energy detector based spectrum sensing, the corresponding test statistic is expressed as:

$$T(y) = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^2 \tag{5}$$

According to the central limit theorem [14], when N is large, the metric $T(y)$ can be approximated as a Gaussian random variable and

$$\begin{aligned} T(y)|_{H_0} &\sim \mathcal{N}(\mu_0, \frac{\sigma_0^2}{N}) \\ T(y)|_{H_1} &\sim \mathcal{N}(\mu_1, \frac{\sigma_1^2}{N}) \end{aligned} \tag{6}$$

in which

$$\begin{aligned} \mu_0 &= \mathcal{E}\{\,|y(n)|^2\big|\,H_0\} = \sum_{m=0}^{\infty}\beta_m\sigma_m^2 = \sigma_Z^2 \\ \mu_1 &= \mathcal{E}\{\,|y(n)|^2\big|\,H_1\} \\ &= \sum_{m=0}^{\infty}\beta_m(\sigma_m^2+\sigma_X^2) = \sigma_Z^2 + \sigma_X^2 \\ \sigma_0^2 &= \mathcal{E}\{\,(|y(n)|^2-\mu_0)^2\big|\,H_0\} \\ &= \mathcal{E}\{\,|y(n)|^4\big|\,H_0\} - \mu_0^2 = \sum_{m=0}^{\infty} 3\beta_m\sigma_m^4 - \sigma_Z^4 \\ \sigma_1^2 &= \mathcal{E}\{\,(|y(n)|^2-\mu_1)^2\big|\,H_1\} \\ &= \mathcal{E}\{\,|y(n)|^4\big|\,H_1\} - \mu_1^2 \\ &= \sum_{m=0}^{\infty} 3\beta_m\sigma_m^4 - \sigma_Z^4 + 2\sigma_X^4 + 4\sigma_X^2\sigma_Z^2 \end{aligned} \tag{7}$$

Thus, the probability of false alarm P_{f_a} and probability of detection P_d can be given in terms of the Q function by

$$\begin{aligned} P_{f_a} &= P_r(T(y) > \gamma|\,H_0) = Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}}) \\ P_d &= P_r(T(y) > \gamma|\,H_1) = Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}}) \end{aligned} \tag{8}$$

where γ is the threshold for the energy-based spectrum sensing.

If the variance of the signal σ_X^2 can be obtained in addition to the noise parameters, we can calculate N by fixing the P_{f_a} and P_d as Eq.(9). Then the threshold γ can also be obtained from N.

$$N = [\frac{Q^{-1}(P_{f_a})\sigma_0 - Q^{-1}(P_d)\sigma_1}{\mu_1-\mu_0}]^2 \tag{9}$$

However, although the noise parameters can be estimated, the signal power is difficult to estimate since it depends on many varying factors such as transmission and propagation characteristics. In practice, the threshold is normally chosen to satisfy a certain P_{f_a} according to the Neyman-Pearson criterion, which only requires the noise power to be known.

As shown in Eq.(10):

$$\gamma = \sqrt{\frac{\sigma_0^2}{N}} Q^{-1}(P_{fa}) + \mu_0. \quad (10)$$

IV. Impact Evaluation of Uncertainty on Noise Characteristics

Let $SNR = \frac{\sigma_X^2}{\sigma_Z^2}$, substituting into Eq.(9), we have

$$N = [\frac{Q^{-1}(P_{f_a})K(A,\Gamma) - Q^{-1}(P_d)\sqrt{K(A,\Gamma)^2 + 4SNR + 2SNR^2}}{SNR}]^2 \quad (11)$$

where

$$K(A,\Gamma) = \sqrt{\sum_{m=0}^{\infty} 3\beta_m (\frac{m/A+\Gamma}{1+\Gamma})^2 - 1} \quad (12)$$

The Middleton Class A noise is totally characterized by the parameters A, Γ and σ_Z^2. As in Eq.(11),when the three parameters are all known, $K(A,\Gamma)$ is determined. It is clearly that no matter how low the SNR is, the target P_{f_a} and P_d can be achieved by increasing the number of observation samples. However, the accurate noise parameter estimation is not always possible for the receiver [15].There must be small errors on the estimation of each parameter during the detecting process. So the impact of each of them must be considered. For discussion convenience, we assume without loss of generality, that the actual parameters are $A = 0.01$, $\Gamma = 0.1$ and $\sigma_Z^2 = 1$.

A. Uncertainty of A

With uncertainty, the estimated overlap index is assumed to be in an interval $\hat{A} \in [\frac{1}{\rho}A, \rho A]$, and $\rho > 1$ is a parameter that quantifies the size of the uncertainty. To achieve a target P_{f_a} and P_d robustly, the following equations need to hold:

$$\begin{aligned} P_{f_a} &= \max_{\hat{A}\in[\frac{1}{\rho}A,\rho A]} Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}}) \\ P_d &= \min_{\hat{A}\in[\frac{1}{\rho}A,\rho A]} Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}}) \end{aligned}. \quad (13)$$

Due to the fact that the Q function is a monotonically decreasing function and σ_0^2 and σ_1^2 are also monotonically decreasing functions of A, we obtain

$$\begin{aligned} P_{f_a} &= Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}})\Big|_{\hat{A}=\frac{1}{\rho}A} \\ P_d &= Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}})\Big|_{\hat{A}=\rho A} \end{aligned}. \quad (14)$$

Then we have

$$N = [\frac{Q^{-1}(P_{f_a})K(A/\rho,\Gamma) - Q^{-1}(P_d)\sqrt{K(A\rho,\Gamma)^2 + 4SNR + 2SNR^2}}{SNR}]^2 \quad (15)$$

Fig.1 shows the required number of samples N to achieve the target $P_{f_a} = 0.1$ and $P_d = 0.9$ with different uncertainty levels ρ on A. To achieve the target P_{f_a} and P_d, a larger quantity of samples is required when the uncertainty of A gets larger. Results for different SNR is shown in Fig.2.

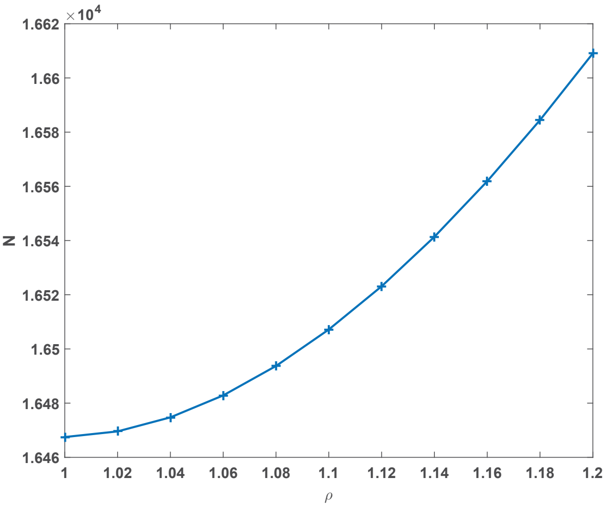

Fig. 1. Samples required vs. ρ, for different levels of uncertainty on A.(SNR=-5dB)

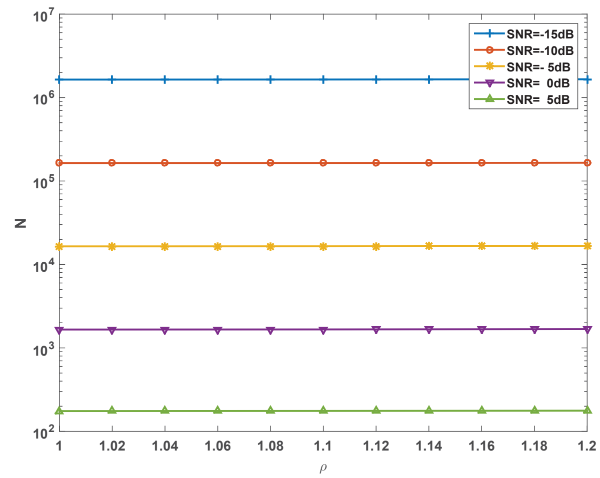

Fig. 2. Samples required vs. ρ , for different levels of uncertainty on A with different SNR.

The two figures illustrate that we can slightly increase the number of samples N to achieve the target P_{f_a} and P_d,. The larger the uncertainty ρ is, the larger N is required. So the impact on the uncertainty of A can be easily eliminated by increasing N, and N is actually reasonably insensitive to ρ.

B. Uncertainty of Γ

Similarly, if the estimated power ratio is in an interval $\hat{\Gamma} \in [\frac{1}{\rho}\Gamma, \rho\Gamma]$, and $\rho > 1$ is a parameter that quantifies the size of the uncertainty, to achieve a target P_{f_a} and P_d robustly,the following equations need to hold:

$$\begin{aligned} P_{f_a} &= \max_{\hat{\Gamma}\in[\frac{1}{\rho}\Gamma,\rho\Gamma]} Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}}) \\ P_d &= \min_{\hat{\Gamma}\in[\frac{1}{\rho}\Gamma,\rho\Gamma]} Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}}). \end{aligned} \quad (16)$$

With similar argument as above, we obtain

$$P_{f_a} = Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}})\Big|_{\hat{\Gamma}=\frac{1}{\rho}\Gamma}. \\ P_d = Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}})\Big|_{\hat{\Gamma}=\rho\Gamma} \tag{17}$$

Then we have

$$N=[\frac{Q^{-1}(P_{f_a})K(A,\Gamma/\rho)-Q^{-1}(P_d)\sqrt{K(A,\Gamma\rho)^2+4SNR+2SNR^2}}{SNR}]^2 \tag{18}$$

Fig.3 shows the required number of samples N to achieve the target $P_{f_a}=0.1$ and $P_d=0.9$ with different uncertainty levels on Γ. To achieve the target P_{f_a} and P_d, a smaller quantity of samples is required when the uncertainty of Γ gets larger. Results for different SNR is shown in Fig.4.

The two figures illustrate that we just need a smaller quantity of samples N to achieve the target P_{f_a} and P_d. The larger the uncertainty ρ is, the smaller N is required, but the variation in N is very small. So the impact on the uncertainty of Γ can in most cases be ignored.

Comparing the four figures, we found that the impact of Γ is even lower than that of A, and the impact of these two parameters can be easily eliminated.

C. Uncertainty of noise power σ_Z^2

Finally, if the estimated noise power is assumed to be in an interval $\hat{\sigma}_Z^2 \in [\frac{1}{\rho}\sigma_Z^2, \rho\sigma_Z^2]$, and $\rho > 1$ is a parameter that quantifies the amount of the uncertainty. To achieve a target P_{f_a} and P_d robustly, the following equations need to hold:

$$P_{f_a} = \max_{\hat{\sigma}_z^2\in[\frac{1}{\rho}\sigma_z^2,\rho\sigma_z^2]} Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}}) \\ P_d = \min_{\hat{\sigma}_z^2\in[\frac{1}{\rho}\sigma_z^2,\rho\sigma_z^2]} Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}}). \tag{19}$$

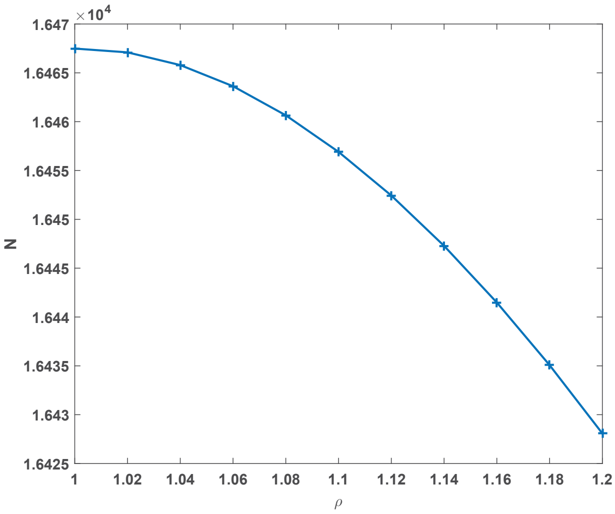

Fig. 3. Samples required vs. ρ, for different levels of uncertainty on Γ.(SNR=-5dB)

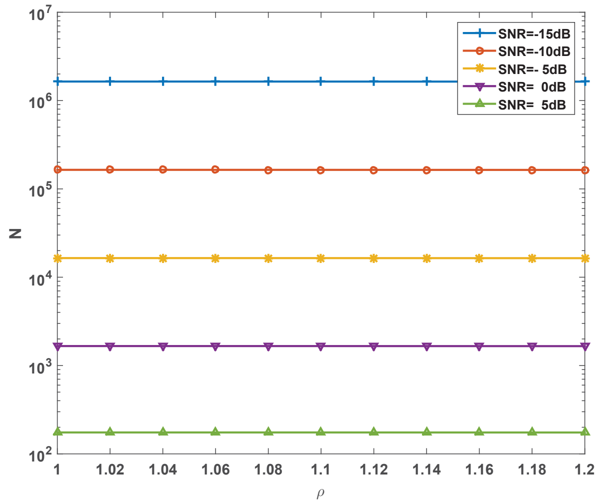

Fig. 4. Samples required vs. ρ, for different levels of uncertainty on Γ with different SNR.

Due to the Q function being a monotonically decreasing function, we obtain

$$P_{f_a} = Q(\frac{\gamma-\mu_0}{\sqrt{\sigma_0^2/N}})\Big|_{\hat{\sigma}_z^2=\rho\sigma_z^2} \\ P_d = Q(\frac{\gamma-\mu_1}{\sqrt{\sigma_1^2/N}})\Big|_{\hat{\sigma}_z^2=\frac{1}{\rho}\sigma_z^2} \tag{20}$$

Then we have

$$N=[\frac{Q^{-1}(P_{f_a})K(A,\Gamma)\rho-Q^{-1}(P_d)\sqrt{K(A,\Gamma)^2\frac{1}{\rho^2}+\frac{4}{\rho}SNR+2SNR^2}}{SNR-(\rho-\frac{1}{\rho})}]^2 \tag{21}$$

It is obvious that, when SNR approaches $(\rho-\frac{1}{\rho})$, $N\to\infty$. So an SNR Wall exists. It means that the detector cannot robustly detect the signal if the signal power is less than the uncertainty in the noise power. Fig.5 shows that the SNR wall will become higher with the uncertainty ρ becoming higher.(note that the vertical axis is a logarithmic scale)

V. Conclusion

We studied the problem of detecting the presence of a primary user in a cognitive radio setting with impulsive noise. Firstly, we give the expression of false alarm probability P_{f_a} and detection probability P_d derived under the specific condition of impulsive communication noise. Secondly, we found that the uncertainty on noise parameters A and Γ have little impact on the detection performance, and the impact can be eliminated by adapting the number of samples N. Although both are small, uncertainty of A has larger impact on performance than that of Γ. Finally, the uncertainty of noise power σ_Z^2 cannot be ignored since it can result in the emergence of SNR wall, in which case weak signals cannot be detected reliably no matter how many samples observed.

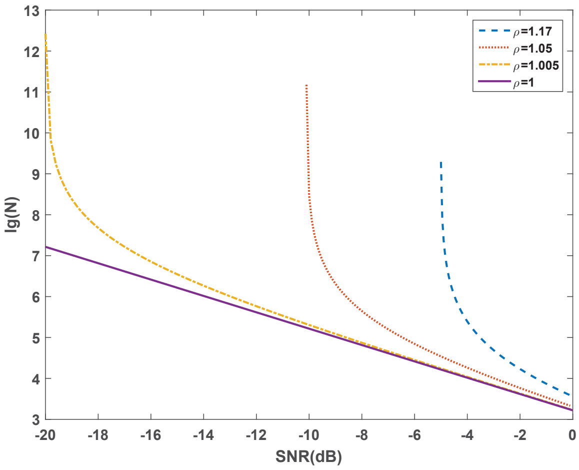

Fig. 5. SNR walls with different ρ.

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