THE FLOW INDUCED BY TWO-DIMENSIONAL AND AXISYMMETRIC TURBULENT JETS ISSUING NORMALLY TO AN INFINITE PLANE SURFACE

by

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1. INTRODUCTION

When a jet emerges into a region of stagnant fluid, the mass flow in the jet increases in the downstream direction while its momentum is very nearly conserved.[1] This inflow of air exists in both laminar and turbulent jets and is associated with small pressure gradients both inside and outside the jet. Existing analyses of jets emerging into quiescent fluid assume that a constant pressure exists throughout the jet and its surroundings, and the theoretical velocity distribution derived on this basis agrees well with experiment.[1,2] Tollmien[3] computed the pressure variations within, but not outside, a turbulent jet and found that they did not cause any appreciable modification in velocities when resubstituted into the boundary layer equation of motion. G.I. Taylor[4] established the streamline pattern for various forced and thermal turbulent jets by replacing the jet with a continuous sink. A similar approach has been made in the extension of jet-flap theory to account for entrainment effects.[5] The following analysis expanded Taylor's approach for isothermal jets in order to predict the detailed pressure distribution on the flat surface from which the jet is ejected. This pressure distribution is compared with new experimental data for two-dimensional and axisymmetric incompressible isothermal jets.

The reduced pressure in the vicinity of a blowing slot due to jet entrainment leads to the "jet drag"[6] on a jet flapped aerofoil.

2. ANALYSIS

A jet emerging from an orifice into unbounded and undisturbed fluid separates at the lips of the orifice. Provided the orifice follows a smooth contraction of large contraction ratio, the jet velocity at the exit will be practically uniform; hence the jet may be considered separated from its surroundings by a vortex sheet. This vortex sheet diffuses rapidly due to the action of viscosity and becomes turbulent[7] when $R_e = \frac{U_J x}{\nu} = 7 \times 10^4$ (x is the distance along the jet axis measured from the from the orifice and U_J is the jet velocity at the orifice). The thickness of the mixing layers increase until the layers meet internally and the jet eventually becomes fully-developed (figure 1).

2.1 The Two-Dimensional Jet

In the two-dimensional jet the mixing layers meet about 6 slot widths downstream of the nozzle. Values close to 6 may be obtained from Tollmien's analysis supported by experiment, or by matching[8] the fully-developed jet velocity profile to velocity profiles for the mixing region measured by Liepmann and Laufer.

The inflow velocity at the outside edge of the mixing region is:[3]

$$Q_J = 0.032\ U_J \qquad \ldots(1)$$

hence the increase of volume flow per unit length of the jet in this region is:

$$Q_m = 0.064\ U_J \qquad \ldots(2)$$

The axial velocity in the fully developed incompressible jet is well represented by Gortler[2] in the equation

$$u = \left(\frac{3J\sigma}{4\rho x}\right)^{1/2} \operatorname{sech}^2 \frac{\sigma y}{x} \qquad \ldots(3)$$

where J is the jet momentum per unit span = $\rho {U_J}^2 b$

σ is determined experimentally to be 7.7[2]

Consequently the rate of increase of volume flow per unit length in <u>this</u> region is;

$$Q_J = \left(\frac{3J}{4\rho\sigma x}\right)^{1/2} \qquad \ldots(4)$$

The flow induced by the jet, in otherwise quiescent fluid, may be obtained by representing the effect of the jet by a series of sinks of strength equivalent to the rate of increase of volume flow in the jet. By assuming that the jet is thin in comparison with the distance from the orifice, the sinks may be located on the jet centre line (i.e. the x-axis). To account for the presence of the wall, which coincides with the y axis, image sinks are located on the negative x axis. Hence the velocity at any point (0, y) along the wall is:

$$-v_y = \sum_1^N \frac{Q_n}{\pi} \frac{y}{y^2 + x_n^2} \qquad \ldots(5)$$

where the subscript n denotes the nth sink.

Replacing the individual sink by a continuous line sink whose strength is:

$$Q_n = Q_m \quad \text{for} \quad 0 \leqslant x \leqslant 6t$$

$$Q_u = Q_J \quad \text{for} \quad 6t \leqslant x \leqslant \infty$$

the velocity v_y is given by:

$$-v_y = 0.0204\ U_J \int_0^{6t} \frac{y\,dx_n}{y^2 + x_n^2} + \frac{1}{\pi}\left(\frac{3J}{4\sigma\rho}\right)^{\frac{1}{2}} \int_{6t}^{\infty} \frac{y\,dx_n}{(x_n)^{1/2}\,(y^2+x_n^2)} \quad \ldots(6)$$

Integrating equation (6) and substituting $J = \rho U_J^2 t$ and $\sigma = 7.7$:

$$-\frac{v_y}{U_J} = 0.0204\ \tan^{-1}\frac{6}{(\frac{y}{t})} + 0.0704\left(\frac{y}{t}\right)^{-\frac{1}{2}}\left\{\pi - (0.5)\ln\left[\frac{6+(\frac{y}{t})+(12\frac{y}{t})^{\frac{1}{2}}}{6+(\frac{y}{t})-(12\frac{y}{t})^{1/2}}\right] - \tan^{-1}\left[1 + \left(\frac{12}{y/t}\right)^{\frac{1}{2}}\right] + \tan^{-1}\left[1 - \left(\frac{12}{y/t}\right)^{\frac{1}{2}}\right]\right\} \quad \ldots(7)$$

and

$$C_p = \frac{p_s - p_\infty}{\frac{1}{2}\rho U_J^2} = -\left(\frac{v_y}{U_J}\right)^2 \quad \ldots(8)$$

where p_s is the local static pressure and p_∞ is the static pressure at infinity.

2.2 The Axisymmetric Jet

In an axisymmetric jet the cylindrical mixing layer behaves in much the same way as in the two-dimensional case.(9) Hence the increase of volume flow per unit length of the jet is given approximately by:

$$Q_m = 2\pi a(0.032\ U_J) = 0.201\ a\ U_J \quad \ldots(9)$$

where a is the radius of the nozzle at the exit.

The equation may underestimate slightly the actual Q_m since the spread of the mixing layer increases the average radius of the jet to a value somewhat greater than that of the nozzle. The mixing layer is again assumed to terminate 6 diameters downstream of the nozzle where the jet becomes fully developed (i.e. at the point $x = 12a$). The rate of increase of volume flow in the fully developed jet per unit length is given by:[2]

$$Q_J = 0.404 \left(\frac{J}{\rho}\right)^{\frac{1}{2}} \qquad \ldots(10)$$

and is independent of x. J is once again the jet momentum.

Representing the entrainment of the jet in the same way as before the radial inflow velocity along the wall is:

$$-v_r = \int_{\frac{\pi}{2}}^{\tan^{-1}\left(\frac{r}{12a}\right)} \frac{0.201}{2\pi}\left(\frac{a}{r}\right)\sin\theta\, d\theta + \int_{\tan^{-1}\left(\frac{r}{12a}\right)}^{0} \frac{0.404}{2\pi}\left(\frac{J}{\rho}\right)^{\frac{1}{2}}\sin\theta\, d\theta \qquad \ldots(11)$$

where r is the radial distance measured from the centre line (figure 1) and $\theta = \cot^{-1}\left(\frac{x}{r}\right)$.

Substituting $J = \rho\pi a^2 U_J{}^2$ and integrating

$$-\frac{v_r}{U_J} = \frac{a}{2\pi r}\left\{0.715 - \frac{0.514}{\left[1 + \left(\frac{r}{12a}\right)^2\right]}\right\} \qquad \ldots(12)$$

and $C_p = -\left(\frac{v_r}{U_J}\right)^2 \qquad \ldots(13)$

DISCUSSION

Equations (8) and (13) are plotted in figures (2) and (3) and compared with new experimental data. In these measurements the slot boundary layer was thin, and $J = 0.97\ \rho U_J^2 t$. The agreement in both cases is good. In addition the following observations may be made:

(i) In the two-dimensional case the reduction of pressure associated with jet entrainment is significant <u>several hundred</u> slot widths away from the blowing slot, while in the axisymmetric case this reduction of pressure is smaller and becomes very small within a few nozzle diameters.

(ii) Since both the mixing region and the fully developed jet are analyzed individually a discontinuity in the inflow velocity occurs at the end of the mixing region. Although this is physically unacceptable it apparently does not lead to any significant error in the back wall pressure coefficient.

(iii) Far away from the nozzle the flow induced by the jet becomes independent of the slot width or nozzle diameter, i.e. $c_p(\frac{y}{t})$ or $c_p(\frac{r}{a})^2$ tend to become constant. This conclusion may be reached through dimensional analysis since for large y the jet momentum becomes the significant parameter governing the flow. In fact the assumption made by Taylor[4] that a jet of momentum J emerges from a slot of zero thickness (i.e. neglecting the existence of a mixing region) extrapolates the last observation right up to the slot. By making the same assumption in the present analysis, equations (7) and (12) become:

$$- \frac{v_y}{U_J} = 0.0704.\pi\left(\frac{y}{t}\right)^{-1/2} \qquad \ldots(14)$$

$$- \frac{v_r}{U_J} = \frac{0.715}{2\pi}\left(\frac{a}{r}\right) \qquad \ldots(15)$$

Using equations (14) and (15) $c_p(\frac{y}{t}) = -0.0488$ and $c_p(\frac{r}{a})^2 = -0.0130$, These are plotted in figures (2) and (3) for comparison.

(iv) There exist other approximate methods by which one may account for the existence of a finite slot. By assuming that the jet originates from some hypothetical origin inside the nozzle, the distance of this hypothetical origin from the nozzle lip may be determined by matching the mass flow from the nozzle to that of a fully developed turbulent jet.[10, 5] Thus in the two-dimensional jet the velocity is given now by $u = \left(\frac{3J\sigma}{4\rho(x+x_o)}\right)^{1/2} \operatorname{sech}^2 \frac{\sigma y}{x+x_o}$ where $x_o = \frac{\sigma t}{3}$.

The pressure distribution on the surface calculated using this method is also plotted in figure (2) (curve (3)). The pressure distribution may also be calculated by assuming that the usual fully developed jet velocity profile starts entraining 6 slots width downstream from the slot and that entrainment in the mixing region may be neglected. The pressure distribution calculated from the latter approach is also plotted in figure (2). It appears that the former method overestimates the local c_p approximately by the same amount as the latter method underestimates it.

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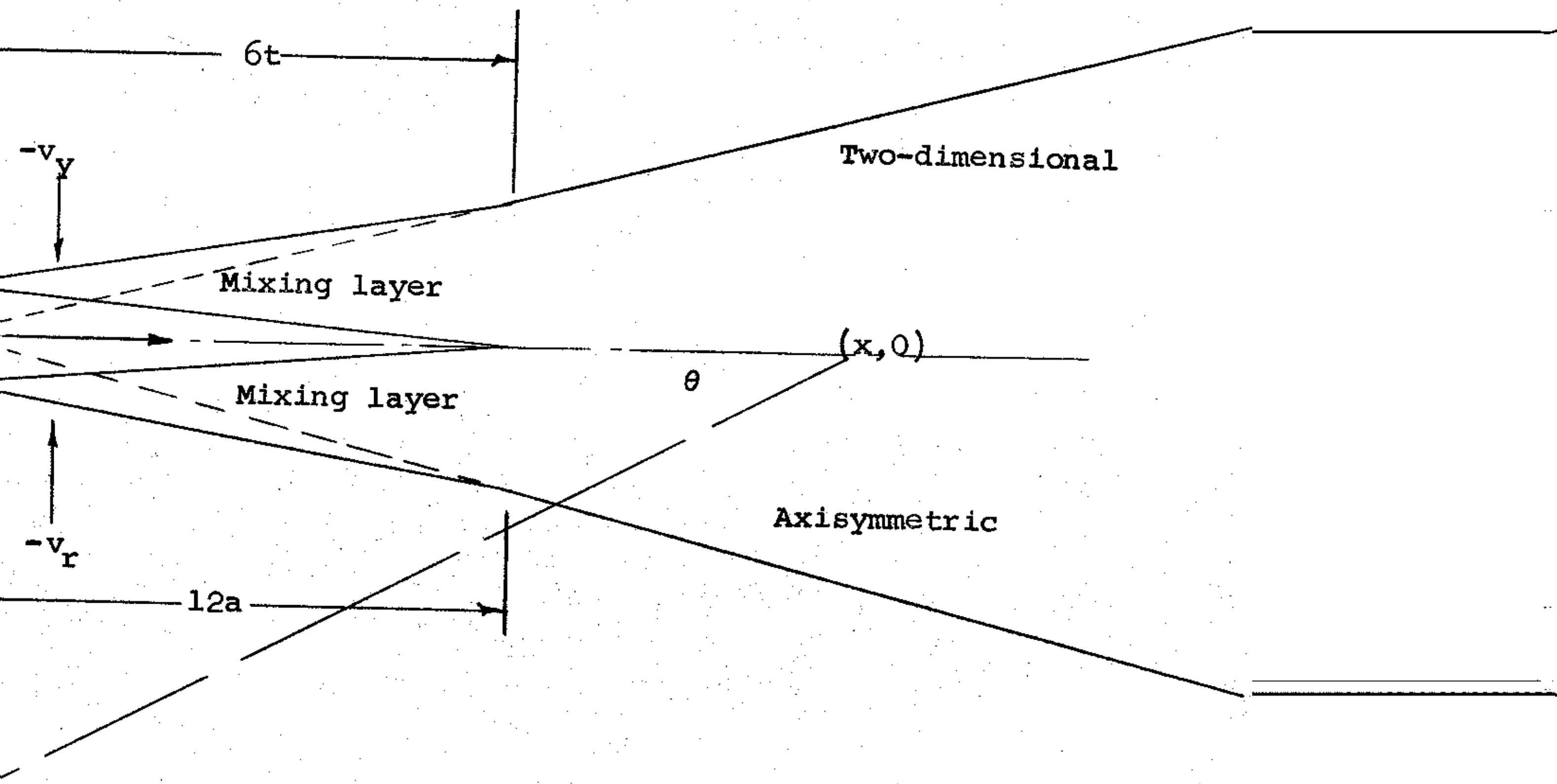
Figure 1

A schematic representation of the jet



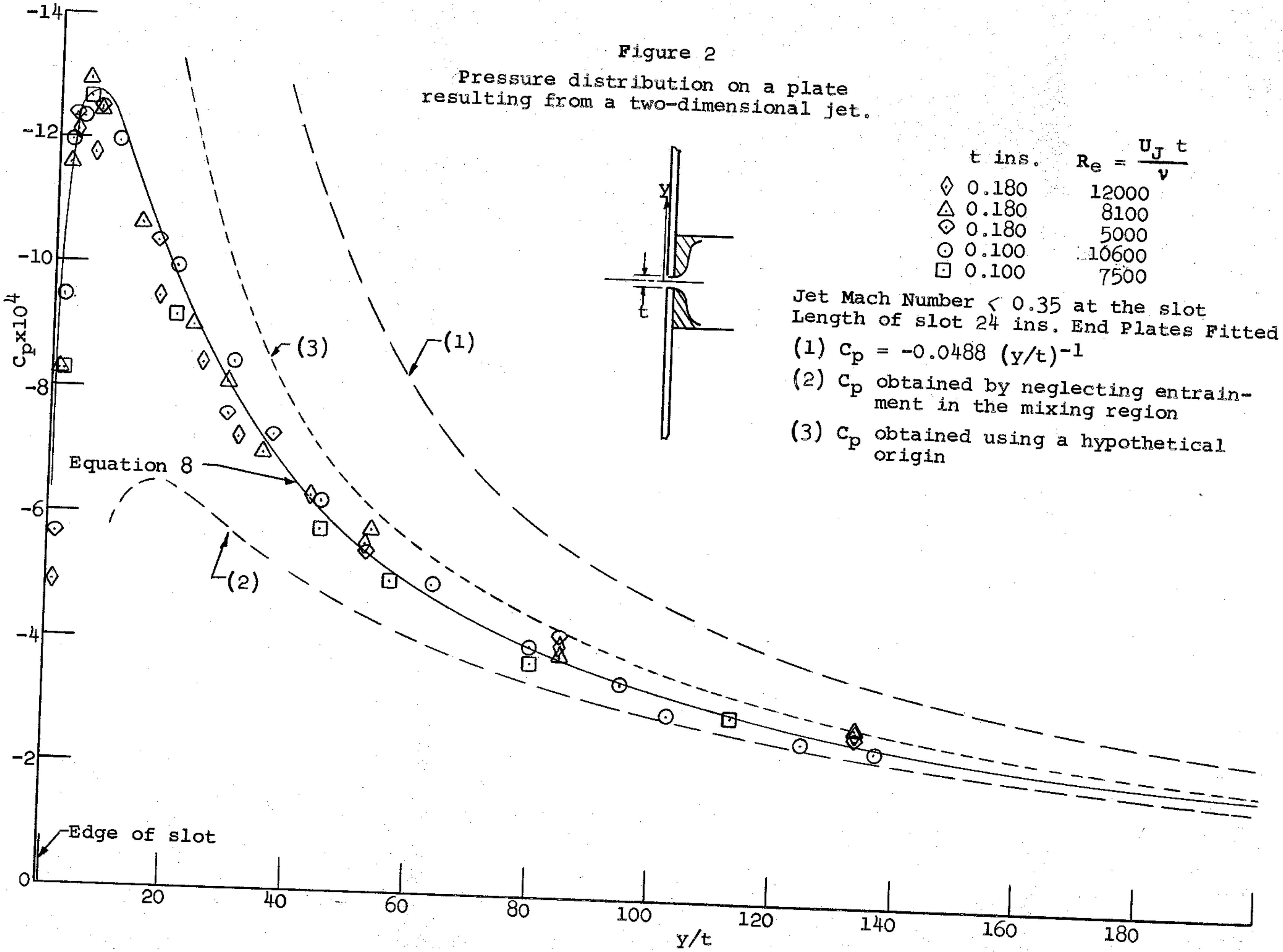


Figure 2
Pressure distribution on a plate resulting from a two-dimensional jet.
t ins.
Re = UJ t / ν
0.180 12000
0.180 8100
0.180 5000
0.100 10600
0.100 7500
Jet Mach Number < 0.35 at the slot
Length of slot 24 ins. End Plates Fitted
(1) Cp = -0.0488 (y/t)^-1
(2) Cp obtained by neglecting entrainment in the mixing region
(3) Cp obtained using a hypothetical origin
Equation 8
(1)
(2)
(3)
Edge of slot
y
t
Cpx10^4
-14
-12
-10
-8
-6
-4
-2
0
20
40
60
80
100
120
140
160
180
y/t

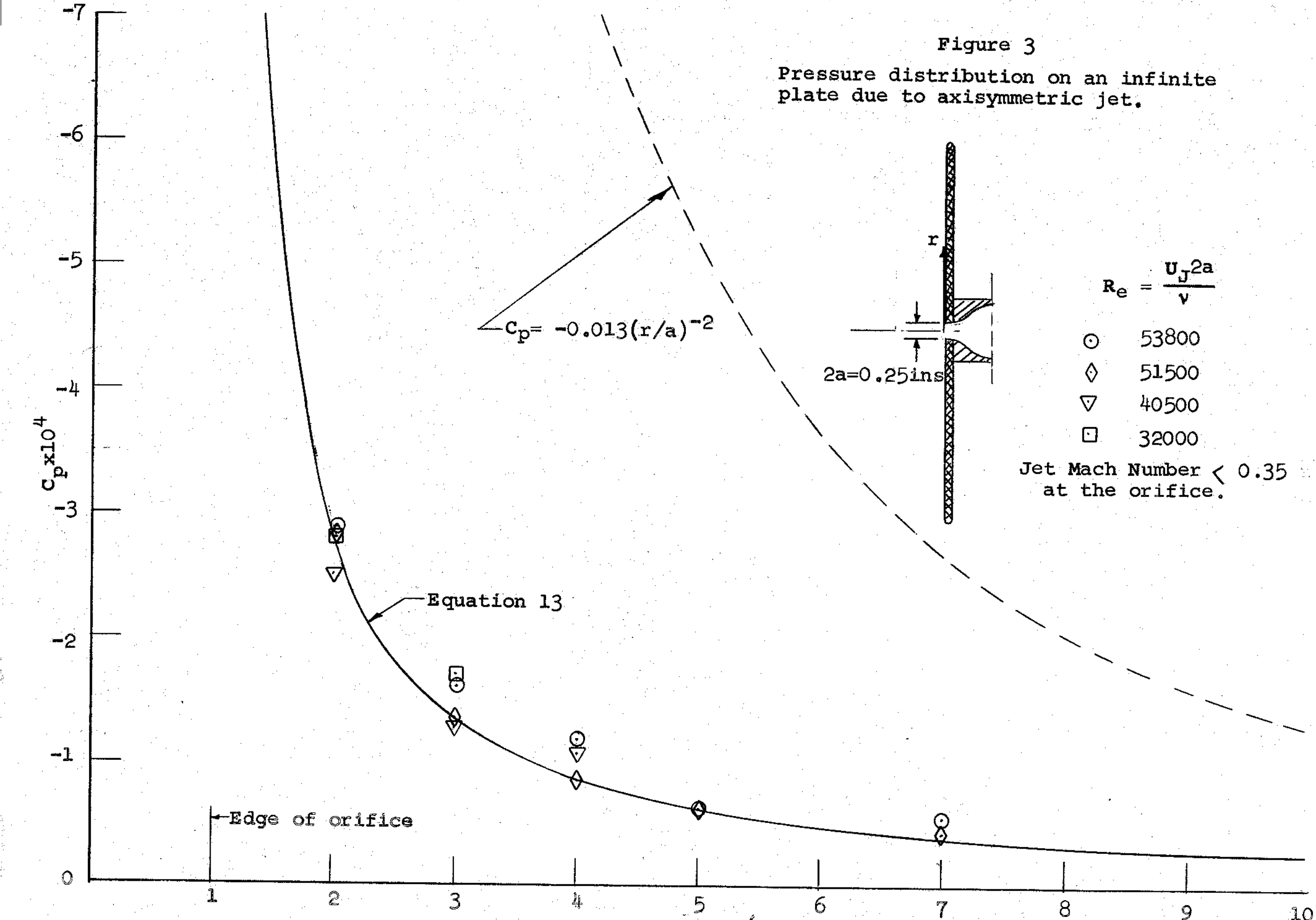


Figure 3
Pressure distribution on an infinite plate due to axisymmetric jet.
$C_p = -0.013(r/a)^{-2}$
Equation 13
Edge of orifice
r
2a=0.25ins
$R_e = \frac{U_J 2a}{\nu}$
53800
51500
40500
32000
Jet Mach Number < 0.35 at the orifice.
$C_p x10^4$
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10