Two-Loop Effective Potential of Supersymmetric Quantum Electrodynamics

by

Raymond Nadeau

Physics Department
McGill University
Montréal, May 1988

A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfillment of the requirements
for the degree of Doctor of Phylosophy

Two-Loop Effective Potential of Supersymmetric Quantum Electrodynamics

ABSTRACT

The formalism of effective potential method is first studied for usual field theory and extended to supersymmetric field theory. The specific case of supersymmetric quantum electrodynamics is then introduced. The superfields are shifted as required by Weinberg's method for the evaluation of effective potentials and superpropagators are derived with the method developed by Helayël-Neto for cases where supersymmetry is explicitly broken. Then, the one and two loop corrections to the effective potential may be calculated. These corrections are seen to be complex everywhere but at the minimum of the potential. The theory is then renormalized in a modified minimal substraction scheme and a finite expression is finally obtained for the effective potential. Thereon, the renormalized coupling constant and the β-function are calculated.

RESUME

La méthode du potentiel effectif est d'abord étudiée dans le cadre de la théorie du champ usuelle et ensuite appliquée à la théorie du champ supersymétrique. Le cas plus spécifique de l'électrodynamique quantique est présenté. On fait subir une translation aux superchamps tel que l'exige la méthode de Weinberg pour évaluer un potentiel effectif et les superpropagateurs sont obtenus à l'aide de la méthode qu'Helayël-Neto a développée pour les cas où la supersymétrie est explicitement brisée. Les corrections du premier et second ordre de l'expansion en boucles au potentiel effectif sont alors calculées. Ces corrections s'avèrent être complexes partout sauf au minimum du potentiel. La théorie est ensuite renormalisée dans le cadre modifié de soustractions minimales et une expression finie est finalement obtenue pour le potentiel effectif. De plus, la constante de couplage renormalisée ainsi que la fonction β sont calculées.

REMERCIEMENTS

Je voudrais d'abord remercier le professeur R.T. Sharp pour m'avoir permis de travailler sur un sujet qui me tenait à coeur ainsi que pour les précieuses remarques qu'il m'a faites au cours de mes calculs. Je le remercie également, ainsi que le département de physique, pour le support financier qui m'a été accordé .

Je voudrais aussi souligner l'aide que m'ont apportée les personnes suivantes: Alain Legault, Jean-François Malouin, Mark Walton, Réjean Girard et Cliff Burgess qui ont répondu avec patience à plusieurs de mes questions ainsi que Juan Gallego, Alain Legault et Nicola Richards, du centre d'informatique de l'université McGill, pour le support qu'ils m'ont donné au point de vue informatique. Je ne peux que souligner l'apport exceptionnel que Pierre Valin m'a apporté pour rendre la version finale de cette thèse aussi exempte d'erreurs que possible et je l'en remercie beaucoup. Je me dois de remercier aussi tous les étudiants du département que j'ai connus au fil des ans pour leur support moral.

Dans le même ordre d'idée, je remercie tous les membres de ma famille pour leur intérêt et leurs encouragements, et plus particulièrement Andrée Perreault pour l'aide qu'elle m'a apportée lors de la préparation de ce manuscrit ainsi que pour sa grande patience à mon endroit et finalement pour le courage qu'elle a su m'insuffler.

CONTENTS

FIGURES

TABLES

Chapter I

INTRODUCTION

1.1 Introductory Remarks

The method of effective potential has an important role in today's quantum field theory. The symmetry properties of the vacua may be determined with the help of this method [1][2][3], which makes it a powerful instrument for the study of spontaneously broken theories.

Three methods for evaluating the effective potential were put forward in the early seventies. First, there was the method of Coleman and Weinberg [1]. It has the drawback of having to sum an infinite number of graphs for each loop order (except tree-level). Obviously, calculations at two-loop order or higher become very difficult with such an approach. The second method is the one devised by Jackiw [2] where the effective potential is evaluated functionally. The potential is given by a perturbation series of vacuum bubble graphs. With the third method, due to Weinberg [3], the effective potential is calculated by summing the scalar tadpole graphs of the translated theory[1]. This method has the advantage of being easy to use and to understand as the underlying theory is simple [4]. This last method is quite powerful for supersymmetric theories because of the reduced number of graphs to be evaluated as first remarked by Miller [5] who gave to this extension of Weinberg's method the name of "Auxiliary

---

[1] By this, it is meant that the scalar fields of the theory have been shifted by a constant.

Field Tadpole Method" (AFTM). Moreover, Weinberg's method may be simplified so that it corresponds to the Vacuum Bubble Method (VBM) of Jackiw for all orders but the first. For all above methods but AFTM, one can work with constrained or unconstrained theories, whichever is simpler. With AFTM, the auxiliary fields, as the name of the method implies, are explicitly needed. The potential can therefore be put on-shell only at the very end of the calculations.

All three methods have been used in the evaluation of supersymmetric (SUSY) effective potentials both with component field and superfield formalisms. The Coleman-Weinberg method has been applied by O'Raifeartaigh and Parravicini [6] with a component approach and by Grisaru et al. [7] with supergraphs. Jackiw's method was used by Huq [8] for the evaluation with component fields of the one-loop effective potential of the Wess-Zumino model and extended later on to superspace by the same author [9]. Weinberg's method has been applied by many people to the case of the Wess-Zumino model; at one-loop order by Miller [5] with components and by Srivastava [10] and Miller [11] with superfields. The two-loop contribution of the Wess-Zumino model was also calculated with this same method by Miller [12] and Fogleman and Viswanathan [13], both with component fields. Coleman-Weinberg's method has also been used for the superspace evaluation of the effective potential of supersymmetric gauge theories [7] as well as Weinberg's [4] and Jackiw's [14] with a component approach.

The main reason of working with superfields is the reduced number of graphs to evaluate. As higher loop contributions are considered, the benefit of using superfields becomes overwhelming. However, as the superfields have to be shifted to evaluate the effective potential with VBM or AFTM, supersymmetry is explicitly broken and that brings about a lot of problems [11]. Fortunately, a method has recently been developed by Helayël-Neto et al. [15] for calculating superpropagators in such cases. With this method, the superpropagators are expressed in terms of a series of projection operators which form a basis. Thus, the advantage of having fewer graphs to evaluate is somewhat counterbalanced by the increased complexity of the superpropagators.

Although this method is a major improvement for the evaluation of superpropagators for explicitly broken supersymmetric theories, it is still not as good as one could wish. Indeed, even for supersymmetric quantum electrodynamics (SQED), one of the simplest SUSY gauge theories, Helayël-Neto's method yields unmanageable results when working with a SUSY gauge fixing condition. Hence, much progress will have to be made before it is possible to calculate effective potentials in superspace of more interesting (or realistic) models.

A combination of two of the methods presented for calculating effective potentials, VBM and AFTM, will be used herein for the case of the supersymmetric extension of quantum electrodynamics in superspace. As mentioned before, superpropagators cannot be calculated with a SUSY gauge fixing condition. Thus, they will be evaluated within a Wess-Zumino gauge scheme supplemented by Lorentz condition (or equivalently with a Landau gauge). Within this framework, the one-loop results should coincide with the ones Miller [4] obtained with a component field formalism. The two-loop effective potential has never been calculated before and therefore is an original result of this thesis as well as the quantities derived from it, the renormalized coupling constant and the β-function.

1.2 Work Outline

This work is divided in five main parts. The second chapter is devoted to the study of the method of effective potential in usual[2] field theory; the Φ^4 model is used as an

[2] As opposed to supersymmetric

example. The basics of supersymmetry are given in the first part of Chapter III. At the same time, some of the notation is set. In the second part of this chapter, the effective potential formalism is extended to supersymmetry and the case of the Wess-Zumino model is worked out in detail. Supersymmetric quantum electrodynamics is introduced in Chapter IV. The tree-level potential is given and the superpropagators are derived to be used in the next chapter to calculate the one and two loop contributions to the effective potential. The fact that these contributions develop an imaginary part is analyzed in the fifth chapter. In the sixth one, the theory is renormalized and the β-function as well as the running coupling constant are derived. Finally, in Chapter VII, some last comments are made and conclusions are drawn.

1.3 References

[1] S.Coleman, E.Weinberg, Phys.Rev.D7(1973)1888

[2] R.Jackiw, Phys.Rev.D9(1974)1686

[3] S.Weinberg, Phys.Rev.D7(1973)2887

[4] R.D.C.Miller, Nucl.Phys.229B(1983)189

[5] R.D.C.Miller, Phys.Lett.124B(1983)59

[6] L.O'Raifeartaigh, G.Parravicini, Nucl.Phys.11B(1976)516

[7] M.T.Grisaru, F.Riva, D.Zanon, Nucl.Phys.214B(1983)465

[8] M.Huq, Phys.Rev.D14(1976)3548

[9] M.Huq, Phys.Rev.D16(1976)1733

[10] P.P.Srivastava, Phys.Lett.132B(1983)80

[11] R.D.C.Miller, Nucl.Phys.228B(1983)316

[12] R.D.C.Miller, Nucl.Phys.241B(1984)535

[13] G.Fogleman, K.S.Viswanathan,Phys.Rev.D30(1984)1364

G.Fogleman, K.S.Viswanathan,Phys.Lett.133B(1983)393

[14] G.Woo, Phys.Rev.D12(1975)975

[15] J.A.Helayël-Neto, Phys.Lett.135B(1984)78

F.Feruglio, J.A.Helayël-Neto, F.Legovini, Nucl.Phys.249B(1985)533

Chapter II

EFFECTIVE POTENTIAL IN FIELD THEORY

In order to study the behaviour of vacuum states of field theories at higher order in the perturbation expansion, one should resort to the method of effective potential which allows the survey of all the minima of the theory. This method is well suited for studying the phenomenon of spontaneous symmetry breakdown and for obtaining the renormalized parameters (masses, coupling constants) of the theory considered.

2.1 Method of Effective Potential

We shall develop the technique by looking at the case of a self-coupled scalar field, as done in most references. The method is really relevant when the scalar field is coupled to other fields, such as a gauge field. However, the point here being to study how the method works, the simpler the model the better.

Consider the Lagrangian

$$L = \frac{1}{2}\ \partial^{\mu}\Phi\, \partial_{\mu}\Phi \ - \ V(\Phi) \qquad (1)$$

with

$$V = \frac{a}{4!}\ \Phi^4 \ - \ \frac{m^2}{4}\ \Phi^2 \qquad (2)$$

The vacuum expectation value for the field Φ is defined as

$$<\Phi> = \lim_{J\to 0} \frac{<\Phi^+|\,\Phi_{op}\,|\Phi^->_J}{<\Phi^+|\Phi^->} = \lim_{J\to 0} \Phi_c \tag{3}$$

with the notation

$|\Phi^->$: *vacuum state at a time* $t = -\infty$

$|\Phi^+>$: *vacuum state at a time* $t = +\infty$

Φ_{op} : *Heisenberg operator*

J : *current*

$\Phi_c(x)$: *vacuum expectation value (VEV) of the field* Φ

in presence of an external source $J(x)$

and is not necessarily the same as the classical minimum of the potential. The field Φ may be expressed in terms of the generating functional $W(J)$, that is

$$\Phi = \frac{\delta\, W(J)}{\delta J(x)} \tag{4}$$

with the usual definition for $W(J)$:

$$\exp[\,(\frac{i}{h})\; W(J)] = N \int D\Phi \;\exp\,[(\frac{i}{h}\,) \int d^4x\; L(x) + (J,\Phi)] \tag{5}$$

where

$$(J,\Phi) = \int d^4x \;\; J(x)\; \Phi(x) \tag{6}$$

and N is the normalization constant. Now let's define an effective action by doing a Legendre transformation of the generating functional

$$\Gamma(\Phi_c) = W(J) - (J,\Phi_c) \tag{7}$$

The current J in equation (7) can be eliminated by solving equation (4). This permits the use of Φ_c as an independent variable. Differentiating equation (7) with respect to Φ_c yields after a straightforward calculation

$$\frac{\delta\,\Gamma(\Phi_c)}{\delta\,\Phi_c(x)} = -J(x) \tag{8}$$

Then, putting J equal to 0, the VEV $<\Phi_c>$ becomes the root of

$$\frac{d\Gamma(\Phi_c)}{d\Phi_c}\Big|_{\Phi_c=<\Phi>} = 0 \tag{9}$$

It can be shown [1] that $\Gamma(\Phi_c)$ is the generating functional of the one-particle irreducible (1-PI) Green's functions.

The effective action can be expanded in terms of Φ_c and its derivatives as follows

$$\Gamma(\Phi_c) = \int d^4x\,[\,-U(\Phi_c(x)) + \frac{1}{2}\,\partial^\mu\,\Phi_c\,\partial_\mu\,\Phi_c\,F(\Phi_c) + \ldots] \tag{10}$$

where U is what is called the effective potential. Equating $\Phi_c(x)$ to a constant g , one gets

$$\Gamma(g) = -\int d^4x\,U(g) = -\,\Omega\,U(g) \tag{11}$$

where Ω is the total volume of spacetime. The effective action could also be expanded in the following way.

$$\Gamma(\Phi_c) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 .... \int d^4x_n \; \Gamma_n(x_1,...,x_n) \; \Phi_c(x_1)...\Phi_c(x_n) \tag{12}$$

which gives, once the right-hand side has been Fourier-transformed

$$\Gamma(\Phi_c) = \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d^4k_1}{(2\pi)^4} .... \int \frac{d^4k_n}{(2\pi)^4} [\; \delta^4(k^1,...,k^n) \; \tilde{\Gamma}_n(k_1,...,k_n) \; \tilde{\Phi}_c(k_1)... \; \tilde{\Phi}_c(k_n)] \tag{13}$$

where

$$\tilde{\Phi}_c(k) = \int d^4x \; \exp^{ik\cdot x} \; \Phi_c(x)$$

$$\tilde{\Gamma}(k_1,...,k_n) \;\; \delta^4(k_1,...,k_n) = \int d^4x_1 \; .... \int d^4x_1 \; \Gamma_n(x_1,...,x_n) \;\; \exp(ik_1 \cdot x_1 + ... + ik_n \cdot x_n) \tag{14}$$

Putting once again $\Phi_c(x)$ equal to g , one obtains for the Fourier transform of Φ_c ,

$$\tilde{\Phi}_c(k) = \Omega \, g \tag{15}$$

and for the effective action,

$$\Gamma(g) = \Omega \sum_{n=0}^{\infty} (\frac{g^n}{n!}) \;\; \tilde{\Gamma}_n(0) \tag{16}$$

with the notation

$$\tilde{\Gamma}_n(0) = \tilde{\Gamma}_n(0,0,...,0) \tag{17}$$

Comparing (11) to (12) , one finds that the effective potential may be written as

$$U(g) = -\sum_{n=0}^{\infty} \left(\frac{g^n}{n!}\right)\tilde{\Gamma}_n(0) \quad . \tag{18}$$

Thus, the effective potential is given by the infinite sum of the one-particle irreducible graphs with n external legs. In such a form, there is little hope of being able to calculate the effective potential of any but the simplest field theories. However, differentiating U n times near zero yields the following expression.

$$\frac{d^n U(g)}{dg^n}\Big|_{g=0} = -\tilde{\Gamma}_n(0) \tag{19}$$

From there, one may develop a suitable method to evaluate the effective potential [2]. We must first translate the theory

$$\Phi_c = \Phi'_c + c \tag{20}$$

and expand it around $\Phi'_c = 0$. This gives

$$\frac{d^n U(\Phi_c)}{d\Phi'^n_c}\Big|_{\Phi_c'=0} = -\tilde{\Gamma}_n'(0) \tag{21}$$

where $\tilde{\Gamma}_n'$ is the 1-PI amplitude for the translated theory. The last equation can be written as

$$\frac{d^n U(\Phi_c)}{d\Phi^n_c}\Big|_{\Phi_c=c} = -\tilde{\Gamma}_n'(0) \tag{22}$$

As $U(\Phi_c)$ is the potential expressed in terms of the untranslated theory, it has no c dependence and therefore, equation (22) can be written under the form

$$\frac{d^n U(c)}{dc^n} = -\tilde{\Gamma}_n'(0) \quad . \tag{23}$$

As said previously, n refers to the number of external legs. Thus , if one uses $n = 0$,

$$U(c) = -\tilde{\Gamma}_0'(0) \tag{24}$$

i.e. that U is given by the summation of the vacuum bubbles in the theory as shown in Figure 1. As mentioned in the introduction, this corresponds to Jackiw's method [3] for evaluating effective potentials.

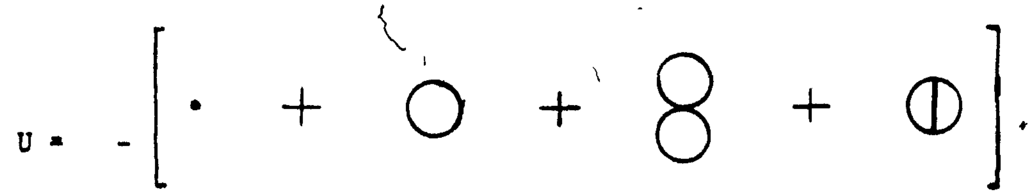


Figure 1: Vacuum Bubble graphs for translated Self-Coupled Scalar Field

The point in Figure 1 represents the constant terms in the translated Lagrangian. However, there is a problem at one-loop order in the perturbation expansion. Indeed, there is no propagator nor vertex for this graph. For this reason, one has to resort to the tadpole method [4] at this order. This method results from taking n equal to 1 in equation (23).

$$\frac{dU(c)}{dc} = -\tilde{\Gamma}_1'(0) \tag{25}$$

The contributing graphs are drawn in Figure 2.

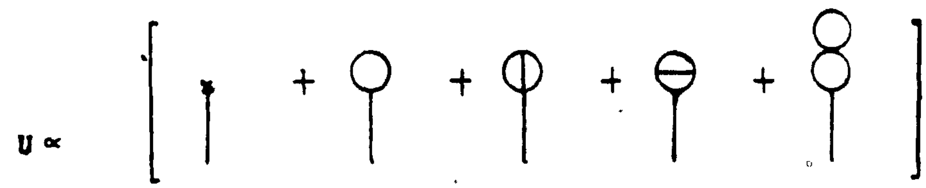


Figure 2: Tadpole graphs for translated Self-Coupled Scalar Field

The major drawback of the tadpole method is the extra integration that has to be performed once the graphs have been evaluated. Hence, the easiest way to calculate the effective potential, order by order in the loop expansion, is to use the vacuum bubble method except for the one-loop case. At this order, the extra integration should not usually be a major problem.

2.2 Application to the case of a Self-Coupled Scalar Field

We will now work out the simple example of the self-coupled scalar field whose Lagrangian is given in equation (1). Shifting the field by a constant c yields

$$L = \frac{1}{2}\,\partial^{\mu}\Phi\partial_{\mu}\Phi + \frac{m^2}{4}\,(\Phi^2 + 2c\Phi + c^2) - \frac{a}{4!}\,(\Phi^4 + 4c\Phi^3 + 6c^2\Phi^2 + 4c^3\Phi + c^4) \qquad (26)$$

At zero-loop order, the effective potential is simply given by the constant terms in the translated Lagrangian.

$$U_0(c) = -\frac{1}{4}m^2 c^2 + \frac{a}{4!}\,c^4 \qquad (27)$$

At one-loop order, the Φ^3 tadpole has to be evaluated. Simple calculations lead to the following finite part for U_1:

$$U_1(c) = -\left(\frac{a}{32\pi^2}\right)\int dc\,[\,c\,\{-K + K\ln(\frac{K}{M})\}\,] \qquad (28)$$

$$K = \frac{(m^2 - ac^2)}{2} \qquad (29)$$

$$M = 4\pi\mu^2 e^{-\gamma_e} \qquad (30)$$

The evaluation of divergent integrals is carried out in Appendix A. One can then perform the c-integration to get

$$U_1(c) = \left(\frac{1}{32\pi^2} \right) \left\{ \frac{K^2}{2} \ln\left(\frac{K}{M}\right) - \frac{K^2}{4} \right\} \tag{31}$$

which is the standard result [5].

Having calculated the effective potential, one can derive from it some physically useful quantities such as the renormalized masses and coupling constants. For the special case of Φ^4-theory, we have

$$\frac{d^2U}{dc^2}\Big|_{c=c_{\min}} = m^2 \qquad (\textit{renormalized mass}) \tag{32}$$

$$\frac{d^4U}{dc^4}\Big|_{c=c_{\min}} = a \qquad (\textit{renormalized coupling constant}) \tag{33}$$

$$\frac{dU}{dc}\Big|_{c=c_{\min}} = 0 \qquad (\textit{condition on } c_{\min}) \tag{34}$$

With $m^2 \geq 0$, equation (32) and equation (34) just state the fact that $c_{\min}$ must be the minimum of the potential. This last set of equations shows how useful is the effective potential method for deriving the renormalized quantities appearing in the Lagrangian.

Before closing this chapter, it would be good to recall that the effective potential is not itself a physical quantity. Indeed, it has been common knowledge that it is a gauge-dependent quantity since the very beginning of its development [3]. Nonetheless, very interesting physical results were obtained such as the scalar to vector mass ratio for spontaneously broken electrodynamics of massless scalar mesons [6]. Since then, it has been shown [7] that the minimum and the quantities derived at this point are gauge-independent as it has to be for physical quantities if the theory is to make any sense.

2.3 References

[1] K. Huang, "Quarks, Leptons and Gauge Fields", World Scient.Publ., Singapore, 1984, p.202

[2] R.D.C. Miller, Physics Letters 124B(1983)59

[3] R.Jackiw, Phys.Rev. D9(1974)1686

[4] S.Weinberg, Phys.Rev. D7(1973)2887

[5] K. Huang, ibid., p.208

[6] S.Coleman, E.Weinberg, Phys.Rev. D7(1973)1888

[7] G. Thompson, H-L. Yu, Physical Review D31(1985)2141

Chapter III

EFFECTIVE POTENTIAL IN SUPERSYMMETRIC FIELD THEORIES

In this chapter, the method of effective potential developed in the last chapter for usual field theories will be extended to the supersymmetric ones. First, a quick review of supersymmetry will be done, allowing us to set the notation and conventions to be used thereafter. In the second section, working out the effective potential of the Wess-Zumino model will show how to use the method for supersymmetric theories as well as how to derive superpropagators when supersymmetry is explicitly broken.

3.1 Supersymmetry, A Quick Review

Symmetries have played an important role in the development and the understanding of particle physics. They lead to the unification of theories describing the interactions of particles. One of these symmetries, supersymmetry, was put forward in the early seventies [1]. It unifies bosons with fermions as well as spacetime symmetries with internal ones. Moreover, the local version of supersymmetry (supergravity) unifies gravity with matter.

In addition to being aesthetically appealing, this symmetry has also led to technical improvements such as the cancellation of the quadratic divergences which plague 'usual' field theories. Some models have even been shown to be finite at all orders in perturbation expansion. Even though there is no experimental evidence yet that this symmetry exists in nature, there is hope it will. If not, it is believed that, at least, it will be a step in the good direction, towards the "right theory".

The goal of this section is to give to the reader a quick overview of supersymmetry and, at the same time, to set the notation and conventions.[3] What follows is based mainly on the book written by Wess and Bagger [2].

Superfields are a function of superspace variables: $F(x, \theta, \bar{\theta})$. The θ and $\bar{\theta}$ are Grassmann variables which obey the rules given below and x represents the position in spacetime.

$$\begin{aligned} &\{ \theta^\alpha , \theta^\beta \} = \theta^\alpha \theta^\beta + \theta^\beta \theta^\alpha = 0 \\ &[P_m , \theta^\alpha] = P_m \theta^\alpha - \theta^\alpha P_m = 0 \end{aligned} \tag{35}$$

$$\begin{aligned} &\int d\theta = 0 \\ &\int \theta \, d\theta = 1 \end{aligned} \tag{36}$$

$$\begin{aligned} &\theta^2 = \theta^\alpha \theta_\alpha \\ &\bar{\theta}^2 = \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \\ &\theta^4 = \theta^2 \bar{\theta}^2 \end{aligned} \tag{37}$$

The Latin indices refer to spacetime and run from 0 to 3. The Greek indices run from 1 to 2. They denote the two-component Weyl spinors, the undotted indices being the representation (1/2,0) of SL(2,C) and the dotted ones their complex conjugate repre-

---

[3] For more details, the reader should consult the books by Wess and Bagger or the one by Gates et al. listed in the bibliography.

sentation (0,1/2)[4]. These indices can be moved up and down with the help of $\epsilon^{\alpha\beta}$ ($\epsilon^{\dot\alpha\dot\beta}$) and $\epsilon_{\alpha\beta}$ ($\epsilon_{\dot\alpha\dot\beta}$) where

$$\epsilon_{21} = \epsilon^{12} = 1 \qquad (38)$$

$$\epsilon_{12} = \epsilon^{21} = -1$$

$$\epsilon_{11} = \epsilon^{11} = \epsilon_{22} = \epsilon^{22} = 0$$

With this, the most general form of $F(x, \theta, \bar\theta)$ can be written down

$$F(x,\theta,\bar\theta) = f + \theta\phi + \bar\theta\bar\chi + \theta^2 m + \bar\theta^2 n + \theta\sigma^m\bar\theta v_m + \theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\psi + \theta^2\bar\theta^2 d \qquad (39)$$

where f, ϕ, m, n, v_m, χ, λ, ψ and d are all functions of x, and σ^m is the Pauli matrix with one dotted and one undotted index: $\sigma^m_{\alpha\dot\alpha}$. Its complex conjugate takes the form $\bar\sigma^{m\dot\alpha\alpha}$. Both are related by

$$\bar\sigma^{m\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\beta}\,\epsilon^{\alpha\beta}\,\sigma^m_{\beta\dot\beta} \qquad (40)$$

To go any further, differential operators must be defined in superspace. They are

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\,\sigma^m_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_m$$

$$\bar D_{\dot\alpha} = \frac{-\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\,\sigma^m_{\alpha\dot\alpha}\,\partial_m \qquad (41)$$

To define D^α *and* $\bar D^{\dot\alpha}$, one uses $\epsilon^{\alpha\beta}$ *and* $\epsilon^{\dot\alpha\dot\beta}$ along with

---

[4] In Minkowski space

$$\epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^{\beta}} = - \frac{\partial}{\partial\theta_{\alpha}} \tag{42}$$

To be really useful, the superfield F has to be restrained in some way with appropriate conditions. The first class of superfields that can be defined is the scalar superfield Φ whose definition is

$$D^{\alpha}\Phi = 0 \tag{43}$$

In terms of usual fields, Φ may be decomposed as

$$\Phi = A(y) + \sqrt{2}\theta\ \psi(y) + \theta^{2}F(y) \tag{44}$$

where

$$y^{m} = x^{m} + i\theta\sigma^{m}\bar{\theta} \tag{45}$$

or

$$\Phi = A(x) + i\,\theta\,\sigma^{m}\,\bar{\theta}\,\partial_{m}A(x) + 1/4\ \theta^{4}\Box\,A(x) + \sqrt{2}\,\theta\,\psi(x) - i\,\theta^{2}\,\partial_{m}\,\psi(x)\sigma^{m}\,\bar{\theta} + \theta^{2}F(x) \tag{46}$$

with

$$\Box = \partial^{m}\,\partial_{m} \quad . \tag{47}$$

A(x) is a scalar field and gives its name to the whole multiplet, $\psi(x)$ is a spinor and F(x) the auxiliary field. The most general renormalizable Lagrangian in terms of scalar superfields only is

$$L = \Phi_i^+ \Phi_i + [(\frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{g_{ijk}}{3} \Phi_i \Phi_j \Phi_k + \lambda_i \Phi_i)|_{\theta^2} + h.c.] \quad (48)$$

The second type of superfield most commonly encountered is the vector superfield. It must obey the relation

$$V = V^+ \quad . \quad (49)$$

Its expansion reads

$$V(x,\theta,\bar{\theta}) = C(x) + i\,\theta\,\chi(x) - i\,\bar{\theta}\,\bar{\chi}(x) + \frac{i}{2}\theta^2 [\, M(x) + iN(x)] - \frac{i}{2}\bar{\theta}^2 [M(x) - iN(x)] - \theta\sigma^m \bar{\theta}\, v_m(x) + i\,\theta^2\bar{\theta}[\, \bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^m \partial_m \bar{\chi}(x)] - i\,\bar{\theta}^2\,\theta[\, \lambda(x) + \frac{i}{2}\sigma^m \partial_m \chi(x)\,] + \frac{1}{2}\theta^4 [\, D + \frac{1}{2}\Box C(x)] \quad (50)$$

where C, M, N, D and v_m must all be real in order to satisfy equation (49). C, M, N are scalar fields, χ *and* λ are spinors and D is the auxiliary field. With a vector field, one may define a field strength.

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$$
$$\bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 \bar{D}_{\dot{\alpha}} V \quad (51)$$

It is possible to obtain a supersymmetric extension of quantum electrodynamics, usually called SUSY QED, using both vector and scalar superfields.

$$L_{QED} = \frac{1}{4}(WW|_{\theta^2} + \overline{WW}|_{\bar{\theta}^2}) + \Phi_+^+ e^{eV} \Phi_+ |_{\theta^2\bar{\theta}^2} + \Phi_-^+ e^{-eV} \Phi_- |_{\theta^2\bar{\theta}^2}$$
$$+ m(\Phi_+ \Phi_- |_{\theta^2} + \Phi_+^+ \Phi_-^+ |_{\bar{\theta}^2}) \qquad (52)$$

Even though this Lagrangian may look nonrenormalizable because of the infinite series of terms coming from e^{eV}, it may still be evaluated in the Wess-Zumino gauge where the third power of V vanishes. In this gauge, the vector field has the following θ-expansion

$$V = -\theta \sigma^m \bar{\theta} v_m + i\, \theta^2 \bar{\theta}\bar{\lambda}(x) - i\, \bar{\theta}^2 \theta\lambda(x) + \frac{1}{2}\theta^4 D(x) \quad . \qquad (53)$$

Such a gauge choice explicitly breaks supersymmetry. This means that the superfields are no longer a valid representation of the supersymmetric algebra. However, as will be seen later, the potential of SQED is still supersymmetric. Usual gauge transformations for v_m are still possible.

To calculate superpropagators, one must introduce a set of projection operators.

$$P_1 = \frac{D^2\bar{D}^2}{(16\,\Box)}$$

$$P_+ = \frac{D^2}{(16\,\Box)^{\frac{1}{2}}}$$

$$P_2 = \frac{\bar{D}^2 D^2}{(16\,\Box)}$$

$$P_- = \frac{\bar{D}^2}{(16\,\Box)^{\frac{1}{2}}}$$

$$P_T = -\frac{D\bar{D}^2 D}{(8\,\Box)} \qquad (54)$$

They have the following multiplication table.

Table 1: Multiplication table of the projection operators.

	P_1	P_2	P_+	P_-	P_T
P_1	P_1	0	P_+	0	0
P_2	0	P_2	0	P_-	0
P_+	0	P_+	0	P_1	0
P_-	P_-	0	P_2	0	0
P_T	0	0	0	0	P_T

When applied on scalar fields, one gets the following results.

$$P_1\,\Phi = 0$$

$$P_1\,\overline{\Phi} = \overline{\Phi}$$

$$P_2\,\Phi = \Phi$$

$$P_2\,\overline{\Phi} = 0$$

$$P_1 + P_2 + P_T = 1 \tag{55}$$

With this set of projection operators, it is possible to calculate superpropagators for unbroken supersymmetric theories (in SUSY gauges). It will be seen later on that a larger basis of projection operators can be defined with the above operators and the θ's. They will be used for deriving superpropagators for theories with explicitly broken supersymmetry.

This will close this short review of supersymmetry. It should be noticed that we work in an Euclidian space with a metric $\delta^{\mu\nu} \sim (1,1,1,1)$.

3.2 Wess-Zumino Model

This section will be divided in two parts. First, the effective potential for the Wess-Zumino model will be derived with the component field formalism which will prove very similar to the case studied for usual field theories. In the next part, superpropagators will be calculated in order to work out the effective potential with a superfield approach.

3.2.1 Component Field Formalism

The Wess-Zumino model is one of the simplest example of a supersymmetric field theory. Its Lagrangian reads

$$L = i\,\partial_m \bar{\psi} \bar{\sigma}^m \psi + \bar{A} \Box A + FF - [\, m\,(AF - \frac{1}{2}\psi\psi) + \frac{1}{2}\lambda\,(AAF - \psi\psi A) + h.c.\,] \tag{56}$$

The auxiliary fields, F and F , are kept in order to be able to use AFTM.

Following the last chapter recipe, the Bose fields are translated by a constant value.

$$A = A' + a \tag{57}$$

$$F = F' + f \tag{58}$$

The Fermi fields are not translated because a vacuum expectation value <ψ> different from zero would violate Lorentz invariance.

This leads to a new, shifted Lagrangian

$$\begin{aligned} L' = i\, \partial_m \bar{\psi} \sigma^m \psi + \bar{A}' \Box A' + \bar{F}' F' - [\, \nu\, (A'F' - 1/2\, \psi \psi\,) \\ + 1/2\, \lambda\, (A'A'F' - \psi \psi A') + 1/2\, \lambda f A'A' + F' (ma + 1/2\, \lambda aa - \bar{f}) \\ + \nu f A' + h.c.\,] + \bar{f} f + [\, m\, af + 1/2\, \lambda\, aaf + h.c.\,] \end{aligned} \tag{59}$$

with

$$\nu = m + \lambda\, a \tag{60}$$

The tree-level effective potential is simply given by the constant terms in the Lagrangian times minus one (-1).

$$U_0 = -\, \{\, \bar{f} f + [\, m\ af + \frac{1}{2}\, \lambda\, aaf + h.c.]\} \tag{61}$$

Using equations of motion to go on-shell, the effective potential may be written under its usual form [3].

$$U_0 = \bar{f} f \tag{62}$$

This result will be rederived with the help of the tadpole method in order to show one peculiarity which simplifies calculations in the case of supersymmetric theories when using this method. Once again, at zero-loop order, one can read directly from the Lagrangian the linear terms:

$$\Gamma_{F,0}(0) = -\left(ma + \frac{1}{2}\lambda aaf \right) \tag{63}$$

which yields, after an integration over f,

$$U_0 = -\bar{f}f + \left(ma + \frac{1}{2}\lambda aa \right) f + H(\bar{f},a,\bar{a}) \tag{64}$$

where H is the integration constant. Now, by simple symmetry between f and $\bar{f}$, it is easy to see that

$$H(\bar{f},a,\bar{a}) = \left(m\bar{a} + \frac{1}{2}\lambda aa \right) f + I(a,\bar{a}) \ . \tag{65}$$

Hence, the effective potential is

$$U_0 = -\bar{f}f + \left[\left(ma + \frac{1}{2}\lambda aa \right) f + h.c. \right] + I(a,\bar{a}) \tag{66}$$

At this point, one would normally have to evaluate the contribution coming from $\Gamma_{A,0}(0)$ to obtain $I(a,\bar{a})$. However, it is well known that the potential for a supersymmetric theory must vanish at its minimum. This means that

$$U_0(f=\bar{f}=0) = 0 \tag{67}$$

Using this condition, it becomes obvious that $I(a,\bar{a})$ must be zero and (61) is retrieved. This is one great simplification that occurs in SUSY theories. Only the auxiliary field tadpoles have to be evaluated, the contribution from other fields being automatically generated by the imposition of SUSY boundary condition [4]. An additional advantage to this method is that the auxiliary fields couple to fewer fields due to their higher dimensionality and so there are less graphs to consider. Equations of motion can be used at the very end of the calculation of the effective potential.

To calculate the one-loop correction, only the bosonic operators are required. Writing the Bose action under the compact form:

$$S_0 = \int d^4x [\, \frac{1}{2} \Phi^T A \, \Phi + \Phi^T B] \tag{68}$$

where

$$\Phi^T = (A, \bar{A}, F, \bar{F}) \quad , \tag{69}$$

$$B^T = (J_A, J_{\bar{A}}, J_F, J_{\bar{F}}) \quad , \tag{70}$$

the source term, and

$$A = \begin{pmatrix} -\lambda f & \Box & -\nu & 0 \\ \Box & -\bar{\lambda}\bar{f} & 0 & -\bar{\nu} \\ -\nu & 0 & Q & 1 \\ 0 & -\bar{\nu} & 1 & 0 \end{pmatrix} \tag{71}$$

Then, the generating functional can be written as

$$\ln(Z_0) = \frac{-1}{2} \int d^4x \, B^T A^{-1} B \tag{72}$$

and the propagators can be found by looking at $\frac{\delta^2 \ln(Z_0)}{\delta J_1 \delta J_2}$. The results are condensed in Figure 3.

Figure 3: Bosonic propagators for the Wess-Zumino model.

A——A	$-\frac{\lambda f}{Q}$
F——F	$-\frac{\lambda f \bar{\nu}^2}{Q}$
A——$\bar{A}$	$\frac{(p^2 + \nu\bar{\nu})}{Q}$
A——F	$\frac{\nu(p^2 + \nu\bar{\nu})}{Q}$
A——$\bar{F}$	$-\frac{\lambda\bar{f}\nu}{Q}$
F——$\bar{F}$	$\frac{\lambda^2\bar{f}f - p^2(p^2 + \nu\bar{\nu})}{Q}$

$$Q = (p^2 + \nu\bar{\nu})^2 - \lambda^2\bar{f}f$$

Applying the Auxiliary Field Tadpole Method [5], it is realized that the only tadpole to be evaluated is the FAA one. The Feynman rules for the effective action yields :

$$\Gamma = \int d^4\bar{k}(2\pi)^2\delta^4(k) \int d^4\bar{p} \, (-\frac{\lambda}{2})\Delta_{AA} \, F_{cl}(k) + \dots \tag{73}$$

where

$$d^4\bar{k} = \frac{d^4k}{(2\pi)^4} \tag{74}$$

One can read off from the previous equation

$$\Gamma_{F,1}(0) = \frac{1}{2} \int d^4\bar{p} \, \frac{\lambda^2 \bar{f}}{Q} \tag{75}$$

The integration over f leads to

$$U_1 = \frac{1}{2} \int d^4\bar{p} \, \ln Q + H(a,\bar{a}) \tag{76}$$

The requirement that SUSY is to remain unbroken by radiative corrections [3] implies that $H(a,\bar{a})$ cancels the first term of the previous equation when $f = \bar{f} = 0$ so that U_1 can be written as

$$U_1 = \frac{1}{2} \int d^4\bar{p} \, \ln[\, 1 - \frac{\lambda^2 \bar{f} f}{(p^2 + v\bar{v})^2}] \ . \tag{77}$$

To go beyond one-loop, one would have to calculate the fermion propagators and use the vacuum bubble method. This calculation , to two-loop order, has been performed by Miller [4] and also by Fogleman and Viswanathan [6]. It should be remarked that, even though the Wess-Zumino model is one of the simplest SUSY models, it shows some peculiarities not encountered in usual field theory models. One of these is the choice of a renormalization scheme which can be tricky because of the presence of only one renormalization constant; Amati and Chou [7] have shown that a minimal subtraction scheme leads to a kinetic term with the wrong sign and to an effective potential with a "pathological behaviour" [6]. Thus, one should be careful when applying usual methods of field theory to supersymmetric models.

3.2.2 Superfield Formalism

The calculations of the previous sub-section will be done all over again with a superfield approach in order to introduce the method used in the forthcoming chapters.

The Lagrangian of the Wess-Zumino model in superfield notation takes the form :

$$L = \int d^4\theta \, \{\Phi^+ \Phi + \Box^{-\frac{1}{2}} [(\frac{1}{2} m \Phi P_+ \Phi + (\frac{\lambda}{3!}) \Phi^2 P_+ \Phi) + h.c.]\} \tag{78}$$

where the field Φ is given by equation (46).

The superfield is then shifted by a constant superfield Ξ.

$$\Xi = a + \theta^2 f \tag{79}$$

The shifted Lagrangian is

$$L' = \int d^4\theta \, \{ \Phi^+\Phi + \Xi^+\Xi + \Box^{-\frac{1}{2}} [\frac{1}{2} (m + \lambda\Xi) \Phi P_+ \Phi + \frac{\lambda}{3!} \Phi^2 P_+ \Phi + (\Xi^+ + m\Xi P_+ + \frac{\lambda}{2} \Xi^2 P_+) \Phi + \frac{1}{2} m \, \Xi P_+ \Xi + \frac{\lambda}{3} \Xi^2 P_+ \Xi + h.c.] \} \tag{80}$$

The zero-loop effective potential can be read directly from the shifted Lagrangian.

$$U_0 = -\int d^4\theta \, \{ \Xi^+ \Xi + \Box^{-1/2} [\frac{m}{2} \Xi P_+ \Xi + \frac{\lambda}{3!} \Xi^2 P_+ \Xi + h.c.] \} \tag{81}$$

Performing the θ-integration is a trivial matter and it brings back equation (61) as it should.

As before, to evaluate the one-loop effective potential given by the Φ^3 tadpole, the superpropagators have to be derived. This may be done with the method developed by Helayël-Neto et al. [8] for the cases where supersymmetry has been explicitly broken.

First, the action is written in a compact form.

$$S_0 = \int d^8z \, \{ \frac{1}{2} \Phi^T A \Phi + \Phi^T P B \} \tag{82}$$

where

$$\Phi^T = (\Phi, \Phi^+) \tag{83}$$

$$B^T = (J, J^+) \tag{84}$$

$$P = \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} \tag{85}$$

and, finally,

$$A = \begin{pmatrix} (m+\lambda\Xi)\,\Box^{-\frac{1}{2}}P_+ & 1 \\ 1 & (m+\overline{\lambda\Xi})\,\Box^{-\frac{1}{2}}P_- \end{pmatrix} \tag{86}$$

From equation (82) , one obtains for the generating functional Z_0,

$$\ln Z_0 = -\frac{1}{2\Box} \int d^8z \ \{ B^T P A^{-1} P B\} \tag{87}$$

from which the superpropagators may be derived.

$$\Delta = -\begin{pmatrix} P_2 & 0 \\ 0 & P_1 \end{pmatrix} A^{-1} \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \delta^4_{12}. \tag{88}$$

This may be written under the form[5]

$$\Delta = -\begin{pmatrix} aP_+ + bP_1\theta^2 P_+ & P_1 \\ P_2 & aP_- + bP_2\bar{\theta}^2 P_- \end{pmatrix}^{-1} \delta^4_{12} \tag{89}$$

where we used the properties of the projection operators and where a and b were defined as

$$a = \Box^{-\frac{1}{2}} \nu \qquad \bar{a} = \Box^{-\frac{1}{2}} \bar{\nu} \tag{90}$$

$$b = \Box^{-1} f \qquad \bar{b} = \Box^{-1} \bar{f} \tag{91}$$

---

[5] The symbol δ^4_{12} stands for $\delta^2(\theta_1 - \theta_2)\ \delta^2(\bar{\theta}_1 - \bar{\theta}_2)$.

As only $\Delta_{\Phi\Phi}$ is needed for the evaluation of U_1 , only the (1,1) term of equation (89) will be calculated. The other terms may be derived in the same fashion.

For a matrix of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the (1,1) element of the inverse matrix is $c^{-1}d\,[ac^{-1}d - b]^{-1}$. Therefore, the superpropagator reads

$$\Delta_{\Phi\Phi} = (a + bA_2)P_- \,[\, P_1 - (aP_+ + bP_+A_3)\, p^2\, (a + bA_2)P_-]^{-1}\delta^4_{12} \tag{92}$$

where the A's are projection operators defined in reference [8]. The list of those needed herein are tabulated in Table 2 on page 35, with their multiplication table, at the end of the chapter. These seven projection operators form a basis which can be used to invert expressions like equation (92). There are antichiral partners to these projection operators which are denoted by P_1 , $\bar{A}_2$, $\bar{A}_3$, $\bar{A}_4$, $\bar{A}_7$, $\bar{A}_9$ and $\bar{A}_{10}$, and which also form a basis. The above superpropagator may be expressed in terms coming from both sets.

$$\Delta_{\Phi\Phi} = (a + bA_2)P_- \;[\, (1\text{-}\bar{a}a)P_1 - \bar{a}b\bar{A}_2 - a\bar{b}\,\bar{A}_3 - b\bar{b}\,\bar{A}_4\,]^{-1}\; \delta^4_{12} \tag{93}$$

Tedious calculations yield

$$\begin{aligned}\bar{\Delta}_{\Phi\Phi} = \frac{(\bar{a} + \bar{b}A_2)P_-}{W}\,[\, P_1 &+ \frac{\bar{a}b}{(W - \bar{b}b)}\bar{A}_2 + \frac{a\bar{b}W}{(W^2 - \bar{b}b)}\bar{A}_3 \\ &+ \frac{\bar{b}b}{(W - \bar{b}b)}\bar{A}_4 + \frac{\bar{a}a\bar{b}b}{(W - \bar{b}b)\,(W^2 - \bar{b}b)}\bar{A}_7 + \frac{\bar{a}a\bar{b}b}{(W^2 - \bar{b}b)}\bar{A}_9 \\ &+ \frac{\bar{a}a\bar{b}b\bar{a}b}{(W - \bar{b}b)\,(W^2 - \bar{b}b)}\bar{A}_{10}]\end{aligned} \tag{94}$$

with

$$W = 1 - \bar{a}a \tag{95}$$

From there, using the algebra of the P's and θ's, it can be shown that

$$\Delta_{\phi\phi} = \frac{1}{64} \{ 16 \frac{\nu}{(p^2 + \bar{\nu}\nu)} \bar{D}^2 + \frac{1}{[(p^2 + \bar{\nu}\nu)^2 - \lambda^2 \bar{f} f]} [f \bar{D}^2 D^2 \theta^2 \bar{D}^2 - \frac{\lambda \nu^2 f}{p^2} \bar{D}^2 D^2 \theta^2 \bar{D}^2 - \frac{\lambda^2 \nu \bar{f} f}{(p^2 + \bar{\nu}\nu)} (\bar{D}^2 \theta^4 D^2 \bar{D}^2 + \bar{D}^2 D^2 \theta^4 \bar{D}^2)] \} \delta_{12}^4 \tag{96}$$

Superpropagators for a theory where supersymmetry has been explicitly broken have a more complex structure than those for an unbroken theory. Obviously, it makes little sense to go through such calculations and obtain such an expression to do one-loop calculations. This work gets its reward at higher order in perturbation expansion because of the lower number of graphs to be evaluated.

Returning to the evaluation of the one-loop effective potential, we have the effective action:

$$\Gamma = \int d^4k \, (2\pi)^2 \delta^4(k) \int d^8 z \, \{ \Delta_{\Phi\Phi} \}_{\theta_1 = \theta_2 = \theta} \, \Phi(k, \theta, \bar{\theta}) + \ldots \tag{97}$$

Using equation (96) for $\Delta_{\phi\phi}$ and the following relations

$$\delta_{12}^4 |_{\theta_1 = \theta_2} = 0 \tag{98}$$

$$\theta_1^2 D_1^2 \bar{D}_1^2 \delta_{12}^4 |_{\theta_1 = \theta_2} = 16 \, \theta_1^2 \tag{99}$$

$$D_1^2 \theta_1^2 \bar{D}_1^2 \delta_{12}^4 |_{\theta_1 = \theta_2} = 16 \, \theta_1^2 \tag{100}$$

$$\theta_1^4 D_1^2 \bar{D}_1^2 \delta_{12}^4 |_{\theta_1=\theta_2} = 16\,\theta_1^4 \tag{101}$$

$$D_1^2 \theta_1^4 \bar{D}_1^2 \delta_{12}^4 |_{\theta_1=\theta_2} = 16\,\theta_1^4 \tag{102}$$

$$\int d^4\theta \;\; \theta^2\, \Phi = 0 \quad , \tag{103}$$

the effective action takes the form

$$\bar{\Gamma} = (\frac{\lambda}{2}) \int d^4\bar{p} \;\; \frac{\lambda \bar{f}}{[\,(p^2 + \bar{v}v)^2 - \lambda^2 \bar{f} f\,]} \;\; \int d^4\theta \,[\, 1 - \frac{2\lambda v f}{(p^2 + \bar{v}v)} \,\theta^2]\, \bar{\theta}^2 \Phi(0,\theta,\bar{\theta}) + \ldots \tag{104}$$

The auxiliary field of the superfield Φ is the one in θ^2. Hence, only the first term in the θ-integration is needed and the effective potential is found to be:

$$U_1 = (-\frac{\lambda}{2}) \int df \int d^4\bar{p} \, \frac{\lambda \bar{f}}{[\,(p^2 + \bar{v}v)^2 - \lambda^2 \bar{f} f\,]} \tag{105}$$

This result matches perfectly the one obtained by the component formalism. It would be good to recall that this superfield approach will prove itself to be really useful beyond one-loop order. For example, the vacuum bubble method applied to the calculation of the two-loop effective potential for the Wess-Zumino model in superspace requires only the evaluation of two supergraphs as opposed to the six graphs needed in a component approach. The method developed in this section will now be applied to SUSY QED.

Table 2: **Extended Table of Projection operators**

$$A_2 = \Box^{1/2} P_2 \theta^2$$

$$A_3 = \Box^{1/2} \theta^2 P_2$$

$$A_4 = \Box P_2 \theta^4 P_2$$

$$A_7 = \Box \theta^4 P_2$$

$$A_9 = \Box P_- \theta^4 P_+$$

$$A_{10} = \Box \theta^4 P_+$$

	P_2	A_2	A_3	A_4	A_7	A_9	A_{10}
P_2	P_2	A_2	A_3	A_4	A_4	A_9	$-A_2$
A_2	A_2	0	A_7	0	0	A_2	0
A_3	A_3	A_9	0	A_3	A_3	0	$-A_9$
A_4	A_4	A_2	0	A_4	A_4	0	$-A_2$
A_7	A_7	A_{10}	0	A_7	A_7	0	0
A_9	A_9	0	A_3	0	0	A_9	0
A_{10}	A_{10}	0	$-A_7$	0	0	A_{10}	0

3.3 References

[1] A.Neveu, J.H.Schwarz, Nucl.Phys.31B(1971)86

P.Ramond, Phys.Rev.D3(1971)2415

J.L.Gervais, B.Sakita, Nucl.Phys.34B(1971)632

Y.A.Gel'Fand, E.P.Likhtman, JETP Lett. 13(1971)323

D.V.Volkov, V.P.Akulov, Phys.Lett.46B(1973)109

J.Wess, B.Zumino, Nucl.Phys.70B(1974)39

J.Wess, B.Zumino, Phys.Lett.49B(1974)52

[2] J.Wess, J.Bagger, "Supersymmetry and Supergravity", Princeton Univ. Press, Princeton, 1983, 180p.

[3] R.D.C.Miller,Phys.Lett.124B(1983)59

[4] R.D.C.Miller, Nucl.Phys.241B(1984)535

[5] R.D.C.Miller, Nucl.Phys.229B(1983)189

[6] G.Fogleman, K.S.Viswanathan, Phys.Rev.30D(1984)1364

G.Fogleman, K.S.Viswanathan, Phys.Lett.133B(1983)393

[7] A.Amati, K.Chou, Phys.Lett.114B(1982)129

[8] J.A.Helayël-Neto, Phys.Lett.135B(1984)78

F.Feruglio, J.A.Helayël-Neto, F.Legovini, Nucl.Phys.249B(1985)533

Chapter IV

SUPERSYMMETRIC QUANTUM ELECTRODYNAMICS

This chapter will be devoted to the study of the supersymmetric extension of quantum electrodynamics. The Lagrangian is composed of a vector superfield and two scalar superfields along with their hermitian conjugates. It includes all the usual QED terms as well as the ones from scalar electrodynamics in addition to some other terms. SQED was the first step in the development of supersymmetric gauge theories [1].

Because of its relative simplicity, SQED is a good model to use for developing new techniques of calculation which, hopefully, could be extended to other field theories. Indeed, it can be noticed in the work done by Miller [2] on the evaluation of the effective potential at one-loop order for gauge theories that the results for SQCD (Supersymmetric Quantum Chromodynamics) are a mere generalization of the ones obtained for SQED.

In the first section, the shifted Lagrangian will be derived and the zero-loop effective potential obtained. Then, in the next section, superpropagators for the shifted theory will be calculated and tabulated at the very end of the chapter for later use.

4.1 Tree-Level Effective Potential

The supersymmetric extension of Quantum electrodynamics is a U(1) gauge theory with one vector field and two scalar fields R and S whose Lagrangian is

$$L = \int d^8z\,[\,\bar{S}\,e^{eV}S + \bar{R}\,e^{-eV}R\,] + \{\,[\,\int d^4x d^2\theta\,(\tfrac{1}{4})WW + m\,SR\,] + h.c.\,\} \quad (106)$$

To use the method developed in the previous chapters, one must shift each superfield by a constant superfield.

$$V \rightarrow V' + u \quad (107)$$

$$S \rightarrow S' + s \quad (108)$$

$$R \rightarrow R' + r \quad (109)$$

The constant superfields u, s, r have the following θ-expansion:

$$u = c + \frac{i}{2}\,\theta^2\,(m+in) - \frac{i}{2}\,\bar{\theta}^2\,(m-in) + \frac{1}{2}\,\theta^4\,(D + \frac{\Box c}{2}) \quad (110)$$

$$s = a_s + \theta^2 f_s \quad (111)$$

$$r = a_r + \theta^2 f_r \quad (112)$$

These constant superfields are functions of only spin-0 fields because of Lorentz invariance. Dropping the primes, one obtains for the shifted Lagrangian:

$$L = \int d^8z\,\{\,[\,(\bar{S}+\bar{s})\,e^{e(V+u)}\,(S+s) + (\bar{R}+\bar{r})\,e^{-e(V+u)}\,(R+r)\,] +$$
$$(\,[\,(V+u)\,(-\Box P_T)\,(V+u) - m\,(S+s)(\Box^{-1/2}P_+)(R+r)\,] + h.c.)\,\} \quad (113)$$

One can read directly from equation (113) the tree-level effective potential, U_0.

$$U_0 = -\int d^4\theta\,\{\,\bar{s}e^{eu}\,s + \bar{r}\,e^{-eu}\,r + u\,(-\Box P_T)\,u - m\,sr\,\delta^2(\bar{\theta}) - m\,\bar{s}\bar{r}\,\delta^2(\theta)\,\} \quad (114)$$

This last expression may be simplified with the help of equations of motion. The potential then becomes:

$$U_0 = \int d^4\theta \; \{ \; u \, (\Box P_T) \, u + \bar{s} \, e^{eu} \, s + \bar{r} \, e^{-eu} \, r \; \} \tag{115}$$

The usual expression for U_0 is obtained by using the Wess-Zumino gauge where

$$u = \frac{1}{2} \, \theta^4 \, d \tag{116}$$

In this case, U_0 takes the form

$$U_0 = \int d^4\theta \; \{ \; -u \Box P_T \, u + \bar{s}s + \bar{r}r \; \} \tag{117}$$

or, in terms of the component fields,

$$U_0 = \frac{1}{2} \; d^2 + \bar{f}_s f_s + \bar{f}_r f_r \quad . \tag{118}$$

Before going any further, it would be good to define properly the notation to be used hereafter for specifying the order of perturbation. The subscript will indicate the number of loops of the graph considered and the superscript will correspond to the order in the $\hbar$-expansion. Both will be needed once the renormalization constants are introduced. So, we have

$$U = U_0 + U_1 + U_2 + \ldots \tag{119}$$

with

$$U_0 = U_0^{(0)} + \hbar U_0^{(1)} + \hbar^2 U_0^{(2)} + \ldots$$

$$U_1 = \hbar U_1^{(1)} + \hbar^2 U_1^{(2)} + \ldots$$

$$U_2 = \hbar^2 U_2^{(2)} + \ldots \tag{120}$$

Then, the effective potential has the following $\hbar$. expansion:

$$U = U_0^{(0)} + \hbar\,(U_0^{(1)} + U_1^{(1)}) + \hbar^2\,(U_0^{(2)} + U_1^{(2)} + U_2^{(2)}) + \ldots \ . \tag{121}$$

It is rather easy to see from the previous expression that the cancellation of the infinite terms arising from divergent integrals, if any, will have to take place between terms having identical superscripts.

Now that this point is settled, we may pursue with the derivation of the superpropagators for the shifted theory.

4.2 Superpropagators for Broken SUSY QED

As stated earlier, a very general method for the calculation of superpropagators in the case of explicitly broken supersymmetric theories has been developed by Helayël-Neto et al. [3]. This method may be applied here. However, this will require much care because, in the Wess-Zumino gauge, the superfields V, S and R are no longer superfields as their θ-expansion has been in some way cut off.

Before doing the actual calculation, it should be explained why the supersymmetric gauges were discarded. The method of projection operators devised by Helayël-Neto requires that the breaking terms be decomposed in a θ-expansion. In the case of SQED, u has four terms and r, $\bar{r}$, s and $\bar{s}$ two each. With so many terms to start with, the calculation of the superpropagators quickly becomes hardly manageable, not to say anything about further calculations with expressions so derived.

Another method has been tried by Grisaru et al. [4] with a supersymmetric gauge choice for SQED at one-loop order. They were not able to obtain an answer in a closed form. These two points lead us to choose the Wess-Zumino gauge.

Once this choice is made, one could work out the propagators of the component fields and place them in the appropriate θ-expansion in order to obtain the superpropagators. Alternatively , one can try to stick to the superfield notation all along and use Helayël-Neto's method. This last method will be the one used herein.

This way of doing things requires much care as 'truncated'[6] superfields are considered. As calculations progress, the reader will notice what kind of problems may arise from this situation. The rest of this work will be restricted to the massless case to simplify matters. The quadratic part of the SQED action, equation (113) , in the Wess-Zumino gauge is

$$S_0 = (\frac{1}{2}) \int d^4\theta \, \{ \, V_{WZ} \, (c - 2\Box P_{\tilde{T}}) \, V_{WZ} + 2\, \bar{S}S + 2\, \bar{R}R + 2eu\, \bar{S}S - 2eu\, \bar{R}R + 2e\, (\bar{s}S' + sS^{\prime\prime} - \bar{r}R' - rR^{\prime\prime}) \, V_{WZ} \, \} \tag{122}$$

The constant superfield u is given by equation (116) , the scalar fields, R and S, are of the type given in equation (46) and V_{WZ} is defined in equation (53). The new terms are:

$$c = e^2 \, (\bar{r}r + \bar{s}s)$$
$$S' = A_S + \sqrt{2}\, \theta\psi_S + i\, \theta\sigma\bar{\theta}\cdot\partial A_S \tag{123}$$

---

[6] Here, 'truncated' means that some terms in the θ-expansion are absent.

$$R' = A_R + \sqrt{2}\,\theta\psi_R + i\,\theta\sigma\bar{\theta}\cdot\partial A_R \tag{124}$$

with

$$\begin{aligned} s &= a_s \\ r &= a_r \end{aligned} \tag{125}$$

The disappearance of terms in the θ-expansion of these fields comes from the fact that only the ones multiplied by θ^4 will survive the θ-integration. Then, why should one bother about these if they don't survive after integration ? After all, this is what is usually done in such cases. The reason is that the projection operators used for deriving superpropagators will not have the same effect if applied on a 'full' superfield or on a truncated one as will be seen in the coming set of equations (141) to (144). It should be noticed that the number of breaking terms has been seriously reduced by using this gauge. Only a_s , $\bar{a}_{\bar{s}}$, a_r , $\bar{a}_{\bar{r}}$ and $(\frac{1}{2})\,\theta^4\,d$ are left.

The next step will be to make the gauge choice for v^m which, in the Wess-Zumino gauge, is still free as opposed to the case of supersymmetric gauge fixing where everything is set at once. The Lorentz gauge is the best choice for simplifying calculations as v^m disconnects completely from the rest of the quadratic action. The Lorentz condition may be stated as

$$\partial_m V|_{\theta\bar{\theta}} = 0. \tag{126}$$

The action becomes

$$S_0 = (\frac{1}{2})\int d^4\theta\,\{\,V_{WZL}\,(2\Box P_L)\,V_{WZL} + V_L\,(c - 2\,\Box\,)\,V_L + 2\,\bar{S}S + 2\,\bar{R}R +$$

$$2eu\,\bar{S}S - 2eu\,\bar{R}R + 2e\,(\bar{s}S_A + s\bar{S}_A - \bar{r}R_A - r\bar{R}_A)\,V_{WZL}\} \tag{127}$$

with

$$S_A = S' - i\,\theta\sigma\bar{\theta}\cdot\partial A_S \tag{128}$$

$$V_L = -\,\theta\sigma^m\bar{\theta}\,v_m \tag{129}$$

$$V_{WZL} = V_{WZ} - V_L \tag{130}$$

$$P_L = P_1 + P_2 \tag{131}$$

Thereon, one may calculate the superpropagators with the method developed in chapter III. To do so, equation (127) will be written in a matrix form. This yields an action of the form

$$S_0 = \frac{1}{2}\int d^4\theta$$

$$\left[(\bar{S}_A, S_A, \bar{R}_A, R_A, V_{WZL}) \begin{pmatrix} 1+u & 0 & 0 & 0 & es \\ 0 & 1+u & 0 & 0 & e\bar{s} \\ 0 & 0 & 1-u & 0 & -er \\ 0 & 0 & 0 & 1-u & -e\bar{r} \\ e\bar{s} & es & -e\bar{r} & -er & 2\,\Box P_L \end{pmatrix} \begin{pmatrix} S_A \\ \bar{S}_A \\ R_A \\ \bar{R}_A \\ V_{WZL} \end{pmatrix} \right.$$

$$\left. + (\bar{S}_T, S_T, \bar{R}_T, R_T) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_T \\ \bar{S}_T \\ R_T \\ \bar{R}_T \end{pmatrix} + V_L\,(c-2\Box)\,V_L \right] \tag{132}$$

where

$$S_T = S - S_A \tag{133}$$

The superpropagators to be derived from the last two terms are quite easy to obtain. Indeed, the last one corresponds to a massive gauge field whose propagator in the Landau gauge may be found in most textbooks on Field Theory.

$$\Delta_{V_L V_L} = -\frac{1}{2}\,\theta_1\theta_2\bar{\theta}_1\bar{\theta}_2\,\frac{\left(\delta^{mn} - \frac{p^m p^n}{p^2}\right)}{\left(p^2 \mp \frac{c}{2}\right)} \tag{134}$$

The second term corresponds to a purely scalar action and the answer can be found in Wess and Bagger [5], with a small correction.

$$\Delta_{S_T \bar{S}_T} = (\, P_2 \delta^4_{12} \,)_T \tag{135}$$

The correction lies in the truncation of the θ-expansion of $(P_2\delta^4_{12})$ where the removed terms are the ones included in the superpropagator of $S_A\bar{S}_A$. The θ-expansion of equation (135) reads

$$\begin{aligned}\Delta_{S_T\bar{S}_T} &= -\frac{1}{p^2}\,\{\, \exp[\,(\theta_1\sigma\bar{\theta}_1 + \theta_2\sigma\bar{\theta}_2 - 2\,\theta_1\sigma\bar{\theta}_2\,)\cdot p\,]\ \}_T \\ &= \frac{1}{p^2}\,\{\, -\theta_1\sigma\theta_1\cdot p - \theta_2\sigma\bar{\theta}_2\cdot p + 1/4\,\theta_1^4 - \theta_1^2\bar{\theta}_1\bar{\theta}_2\,p^2 + 1/2\,\theta_1\theta_2\bar{\theta}_1\bar{\theta}_2\,p^2 + \\ &\qquad \theta_1^2\bar{\theta}_2^2\,p^2 - \theta_2^2\bar{\theta}_2\bar{\theta}_1\,p^2 + 1/4\,\theta_2^4 + 1/4\,p^2\,\theta_1^4\,\theta_2\sigma\bar{\theta}_2\cdot p - \\ &\qquad p^2\,\theta_1^2\bar{\theta}_2^2\theta_2\sigma\bar{\theta}_1\cdot p + 1/4\,p^2\,\theta_2^4\theta_1\sigma\bar{\theta}_1\cdot p - 1/16\,\theta_1^4\theta_2^4\,p^4\,\}\end{aligned} \tag{136}$$

The expression for $\Delta_{R_T\bar{R}_T}$ is identical. Only the first part of equation (132) is now left to evaluate. The best way to tackle the problem is to perform independently the calculation for the purely scalar part of the action, then the purely vector sector and finally for the scalar-vector part. Using equations of motion, one may write the scalar sector of the action as:

$$S_0^{sc} = \frac{1}{2} \int d^4\theta \, \Phi_A^T \, (\Box I - AP_L + \theta^4 B) \, \Phi_A \tag{137}$$

with

$$\Phi_A^T = (\bar{S}_A, S_A, \bar{R}_A, R_A) \quad , \tag{138}$$

$$B = \frac{ed}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad . \tag{139}$$

and

$$A = \frac{e^2}{\Box} \begin{pmatrix} -s\bar{s} & s^2 & \bar{r}s & -rs \\ \bar{s}^2 & -s\bar{s} & \bar{s}\bar{r} & -r\bar{s} \\ -r\bar{s} & -rs & -r\bar{r} & r^2 \\ \bar{r}\bar{s} & -s\bar{r} & \bar{r}^2 & -r\bar{r} \end{pmatrix} \tag{140}$$

and I, which is the 4 by 4 identity matrix.

At this point, it is of the utmost importance to distinguish the double-primed superfields from the "full" scalar superfields. The reason lies in the next set of equations.

$$\int d^4\theta \, [\, \bar{S} P_L S \,] = \int d^4\theta \, [\, \bar{S} S \,] \tag{141}$$

$$\int d^4\theta \, [\, \bar{S}_A \, P_L S_A \,] = \int d^4\theta \, [\, \bar{S}_A \, (\tfrac{1}{2} \, \theta^4 \, \Box) \, S_A + \bar{S}_A|_{\bar{\theta}} \, (\theta\sigma\bar{\theta}\cdot p) \, S_A|_{\theta} \,]$$

$$= \int d^4\theta \, [\, \bar{S} \, (\tfrac{1}{2}\, \theta^4 \, \Box) \, S + S|_{\bar{\theta}} \, (\theta\sigma\bar{\theta}\cdot p) S|_{\theta} \,] \tag{142}$$

$$\int d^4\theta \, [\, S \, P_L \bar{S} \,] = \int d^4\theta \, [\, \bar{S} \, (\tfrac{1}{2}\, \theta^4 \Box) \, S + \bar{S}|_{\bar{\theta}} \, (-\theta\sigma\bar{\theta}\cdot p) \, S|_{\theta} \,] \tag{143}$$

$$\int d^4\theta \, [\, S_A P_L S_A \,] = \int d^4\theta \, [\, S \, (\tfrac{1}{2}\, \theta^4 \, \Box) \, S \,] \tag{144}$$

Thus, with the above relations, the action (137) may be written as

$$S_0 = \frac{1}{2} \int d^4\theta \, \{\, \bar{\Phi}^T \, [\, I + \theta^4 (B - \frac{A}{2}) \,] \, \Phi - \bar{\Phi}^T|_{\bar{\theta},\theta} \, [\, (\theta\sigma\bar{\theta}\cdot p) \, E \,] \, \Phi|_{\theta,\bar{\theta}} \,\} \tag{145}$$

where the following notation has been introduced :

$$\bar{\Phi}^T|_{\bar{\theta},\theta} = (\, \bar{S}|_{\bar{\theta}}, S|_{\theta}, \bar{R}|_{\bar{\theta}}, R|_{\theta} \,) \tag{146}$$

and

$$E = \frac{e^2}{4} \begin{pmatrix} -\bar{s}s & 0 & -\bar{r}s & 0 \\ 0 & \bar{s}s & 0 & \bar{r}s \\ -r\bar{s} & 0 & -\bar{r}r & 0 \\ 0 & r\bar{s} & 0 & \bar{r}r \end{pmatrix} \tag{147}$$

When multiplied by θ^4 , the Φ_A term may be replaced by Φ as it does not make any difference. It should also be noticed that Φ has replaced Φ_A in the first term of equation (145). By doing this, the contribution of the second term of equation (132) has been reincluded in the chiral action. Thus, superpropagators calculated with the action (145) will include the part given by equation (135). One should then be careful not to take this contribution twice into account.

As the $\theta\,(\bar{\theta})$-component singles out in equation (145) , it will be calculated on its own . So, one defines an "amputated" Φ ,

$$\Phi_A = \Phi - \Phi|_{\theta,\bar{\theta}} \tag{148}$$

and gets the following action

$$S_0 = \frac{1}{2} \int d^4\theta \, \{ \bar{\Phi}_A^T \, [\, I + \theta^4 \, (B - \frac{A}{2}\,) \,] \, \Phi_A + \bar{\Phi}^T|_{\bar{\theta},\theta} \, [\, (\theta\sigma\bar{\theta}\cdot p) \, (L\text{-}E) \,] \, \Phi|_{\theta,\bar{\theta}} \, \} \tag{149}$$

where

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{150}$$

The $\Phi|_{\theta,\bar{\theta}}$ being made out of only one component each , the calculation of their superpropagators is similar to the component case ; one has simply to add the proper number of θ's to the propagators found by usual methods of calculation. The result is :

$$\Delta_{\bar{\Phi}|_{\bar{\theta},\theta}\,\Phi|_{\theta,\bar{\theta}}} = \frac{-1}{(L\text{-}E)} \begin{pmatrix} f_1(\theta) + g_1(\theta) & 0 & 0 & 0 \\ 0 & f_2(\theta) + g_2(\theta) & 0 & 0 \\ 0 & 0 & f_1(\theta) + g_1(\theta) & 0 \\ 0 & 0 & 0 & f_2(\theta) + g_2(\theta) \end{pmatrix} \tag{151}$$

$$\equiv - \frac{1}{2} \; C(p,\theta,\bar{\theta}) \; (L\text{-}E)^{-1.}$$

and the different θ-functions are

$$f_1(\theta) = 2\,\theta_2 \sigma \bar{\theta}_1 \cdot p \tag{152}$$

$$g_1(\theta) = -\,\theta_1^2 \theta_2^2 \theta_1 \sigma \bar{\theta}_2 \cdot p$$

$$f_2(\theta) = -2\,\theta_1 \sigma \bar{\theta}_2 \cdot p$$

$$g_2(\theta) = \theta_1^2 \theta_2^2 \theta_2 \sigma \bar{\theta}_1 \cdot p$$

For the calculation of $\Delta_{\bar{\Phi}_A \Phi_A}$, the procedure will be the one described in chapter III along with the truncation of the θ-expansion at the end as done before for $\Delta_{\bar{\Phi}_T \Phi_T}$. First, let's define the matrix M as being

$$M = B - \frac{A}{2} \tag{153}$$

Then, one adds a term involving the currents to equation (149) and drops the subscript "A" for the time being.

$$S_0^{sc} = \frac{1}{2} \int d^4\theta \, \{ \, \bar{\Phi}^T (I + \theta^4 M) \, \Phi + \bar{\Phi}^T P_{\mp} J_{\Phi} \, \} \tag{154}$$

The symbol $P_{\mp}$ represents the matrix composed of the projection operators P_- and P_+.

$$P_{\mp} = \frac{1}{\Box^{1/2}} \begin{pmatrix} P_- & 0 & 0 & 0 \\ 0 & P_+ & 0 & 0 \\ 0 & 0 & P_- & 0 \\ 0 & 0 & 0 & P_+ \end{pmatrix} \tag{155}$$

$$P_{\pm} = \bar{P}_{\mp} \tag{156}$$

The current J_{Φ} is a vector built from the four amputated scalar currents :

$$J_\Phi = \begin{pmatrix} J_{\bar{S}} \\ J_S \\ J_{\bar{R}} \\ J_R \end{pmatrix} \tag{157}$$

With the above definitions, the generating functional for the amputated scalar sector reads

$$\begin{aligned} \ln Z_0 &= \frac{-1}{2} \int d^4\theta \, \{ (P_\mp J_\Phi)^T (I + \theta^4 M)^{-1} P_\pm J_\Phi \} \\ &= -\frac{1}{2} \int d^4\theta \, \{ J_\Phi^T P_\mp (I + \theta^4 M)^{-1} P_\pm J_\Phi \} \end{aligned} \tag{158}$$

The second derivative of $\ln Z_0$ with respect to J_{Φ_1}, J_{Φ_2} yields the superpropagator:

$$\Delta_{\bar{\Phi}_1 \Phi_2} = \frac{-1}{2} X_{12} \, \{ P (I + \theta^4 M)^{-1} P \} \, \delta^4_{12} \tag{159}$$

where

$$P = P_\pm P_\mp = \begin{pmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_1 & 0 \\ 0 & 0 & 0 & P_2 \end{pmatrix} \tag{160}$$

and

$$X_{12} = \begin{cases} 2 & \text{if } \Phi_1 = \Phi_2 \\ 1 & \text{if } \Phi_1 \neq \Phi_2 \end{cases} \tag{161}$$

The two extra $P_\pm$ matrices come from the variational derivatives with respect to chiral superfields. The rule for these derivatives may be found in Wess and Bagger's book [6].

$$\frac{\delta}{\delta J(x,\theta,\bar\theta)} \int d^4\theta \int d^4x J(x',\theta',\bar\theta') F(x',\theta',\bar\theta') = -\frac{\bar D^2}{4} F(x,\theta,\bar\theta)$$

$$= \Box^{1/2} P_- F(x,\theta,\bar\theta) \qquad (162)$$

Equation (159) is inverted as follows :

$$\Delta_{\bar\Phi_1\Phi_2} = \frac{-1}{2} X_{12} [\, IP + P\,\theta^4 M P\,]^{-1} \delta^4_{12}$$

$$= \frac{-1}{2} X_{12} [\, IP - P\,\theta^4 M P + P\,\theta^4 M P P\,\theta^4 M P - \ldots\,] \delta^4_{12}$$

$$= \frac{-1}{2} X_{12} [\, IP - \frac{\Box M}{(\Box + M)} P\,\theta^4 P\,] \delta^4_{12} \qquad (163)$$

This yields the set of superpropagators for the amputated scalar sector which takes the form :

$$\Delta_{\bar\Phi_A\Phi_A} = \frac{-1}{2} X_{12} \left[\begin{pmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_1 & 0 \\ 0 & 0 & 0 & P_2 \end{pmatrix} - \frac{M}{(\Box I + M)} \begin{pmatrix} \bar A_4 & 0 & 0 & 0 \\ 0 & A_4 & 0 & 0 \\ 0 & 0 & \bar A_4 & 0 \\ 0 & 0 & 0 & A_4 \end{pmatrix} \right]_A \delta^4_{12}$$

$$= \frac{-1}{2} X_{12} [\, P - \frac{M}{(\Box + M)} A\,]_A \delta^4_{12} \qquad (164)$$

where the subscript "A" has been reincluded to recall that the θ-expansion has to be truncated. The matrix A is formed of the projection operators A_4 and $\bar A_4$ which have been defined in the previous chapter, more precisely in Table 2 on page 35. The term $(P_2\delta^4_{12})_A$ is given by a simple modification to equation (136).

$$(P_2\delta^4_{12})_A = \frac{-1}{p^2} + (P_2\delta^4_{12})_{T.} \qquad (165)$$

and $(A_4\delta^4_{12})|_A$ is found to be equal to its untruncated version.

$$(A_4\delta^4_{12})_A = \frac{1}{p^2}[-1 - \theta_1\sigma\bar{\theta}_1{\cdot}p - \theta_2\sigma\bar{\theta}_2{\cdot}p + 1/4\,\theta^4_1\,p^2 + 1/2\theta_1\theta_2\bar{\theta}_1\bar{\theta}_2\,p^2$$

$$+ 1/4\,\theta^4_2\,p^2 + 1/4\,p^2\,\theta^4_1\,\theta_2\sigma\bar{\theta}_2{\cdot}p + 1/4\,p^2\,\theta^4_2\,\theta_1\sigma\bar{\theta}_1{\cdot}p$$

$$- \frac{1}{16}\,\theta^4_1\theta^4_2\,]$$

$$= A_4\delta^4_{12} = \Box.P_2\theta^4P_2\,\delta^4_{12} \qquad (166)$$

Finally, collecting all the constituents of the scalar superpropagators, one gets

$$\Delta_{\bar{\phi}\phi} = \frac{-1}{2}\,X_{12}\,\{\,[\,P - \frac{M}{(\Box I + M)}\,(\Box P\theta^4P)\,]_A\,\delta^4_{12} - \frac{C(p,\theta,\bar{\theta})}{4(L\text{-}E)}\,\} \qquad (167)$$

where all the terms have been previously defined. The decomposition of $\Delta_{\bar{\phi}\phi}$ into each of its constituting components $(\Delta_{SS}, \Delta_{RR}, \ldots)$ is straightforward, the only technical difficulty being the inversion of the matrices $(\Box I + M)$ and $(L - E)$. To ease further references, the result of this calculation is given at the very end of this chapter in Table 3 on page 56.

This completes the most intricate part of this chapter as the remaining superpropagators will be derived quite straightforwardly because we will not have to worry about the θ composition of the superfields. This simplifies the calculations as well as the understanding of the method.

Using the equations of motion in equation (132) , one obtains a quadratic action for the vector sector:

$$S_0 = \frac{1}{2} \int d^4\theta \, V_{WZL} \, \{ \, 2\Box \, P_L - e^2 \bar{s}s \, [\, P_L - \frac{ed}{2\,(\Box + \frac{ed}{2})} A_L \,]$$

$$- e^2 \bar{r}r \, [\, P_L + \frac{ed}{2\,(\Box - \frac{ed}{2})} A_L \,] \, \} \, V_{WZL} \qquad (168)$$

The projection operator A_L is formed in the same manner as P_L , i.e.

$$A_L = A_4 + \bar{A}_4 \qquad (169)$$

Both P_L and A_L will have to be truncated at the end of the calculation in a way similar to the scalar case. Adding the current term to equation (168) , one may easily obtain the logarithm of the generating functional Z_0 from which one finds

$$\Delta_{V_{WZL} V_{WZL}} = - \{ \, (2\Box - c) \, P_L + \frac{ed}{2} \left[\frac{e^2 \bar{s}s}{(\Box + \frac{ed}{2})} - \frac{e^2 \bar{r}r}{(\Box - \frac{ed}{2})} \right] A_L \, \}^{-1} \delta^4_{12} \qquad (170)$$

The technique for inverting in the curly bracket is the same as the one used for equation (163) and the result so derived is

$$\Delta_{V_{WZL} V_{WZL}} = \frac{1}{2(p^2 + \frac{c}{2})} \{ \, P_L -$$

$$\frac{[\, e^2 \bar{s}s \, x(p^2 + x) - e^2 \bar{r}r \, x(p^2 - x) \,]}{[\, 2p^2 (p^4 - x^2) + e^2 \bar{s}s \, p^2 (p^2 + x) + e^2 \bar{r}r \, p^2 (p^2 - x) \,]} A_L \, \}_T \delta^4_{12} \qquad (171)$$

where

$$x = \frac{ed}{2} \tag{172}$$

and the truncated projection operators are

$$(P_2\delta^4_{12})_T = \frac{-1}{8}\,\theta^4_1\theta^4_2\,p^2 - (\theta^2_1\,\bar\theta_1\bar\theta_2\theta_2\sigma\bar\theta_2 + \theta^2_2\,\bar\theta_1\bar\theta_2\theta_1\sigma\bar\theta_1)\cdot p \tag{173}$$

$$(A_L\delta^4_{12})_T = \frac{-1}{8}\,\theta^4_1\theta^4_2\,p^2 \tag{174}$$

The evaluation of the scalar-vector sector is most simply done by starting with the action for the vector part, equation (168) , and using equations of motion to replace one of the V_{WZL}'s. This yields

$$S_0 = -\frac{1}{2}\int d^4\theta\, V_{WZL}\,\{\;[\,2\Box\, P_L - e^2\,\bar{s}s\,(P_L - \frac{x}{(\Box + x)}\,A_L) - e^2\,\bar{r}r\,(P_L + \frac{x}{(\Box - x)}\,A_L)\,]\;e^{-1}\;(s^{-1},s^{-1},-r^{-1},-r^{-1})\,(I + B)\,\}\,\Phi_A \tag{175}$$

where B and Φ_A are defined in equations (138) and (139) The same technique is used over again to find the superpropagators:

$$\Delta_W = \frac{e}{2}\left\{\begin{pmatrix} P_1 + x\bar{A}_7 & 0 & 0 & 0 \\ 0 & P_2 + xA_7 & 0 & 0 \\ 0 & 0 & P_1 - x\bar{A}_7 & 0 \\ 0 & 0 & 0 & P_2 - xA_7 \end{pmatrix}\begin{pmatrix} \bar{s} \\ s \\ -\bar{r} \\ -r \end{pmatrix}\frac{1}{(p^2 + \frac{c}{2})}\right.$$

$$(P_L - \frac{[e^2 \bar{s}s\, x\,(p^2 + x) - e^2 \bar{r}r\, x\,(p^2 - x)]}{2p^2}$$

$$[(p^4 - x^2) + 1/2\, e^2 \bar{s}s(p^2+x) + 1/2\, e^2 \bar{r}r\,(p^2-x)\,]^{-1} A_L)\,\}_T\, \delta^4_{12} \qquad (176)$$

The inverse of the terms in the first parenthesis is easy to obtain with the help of the multiplication table for the projection operators, Table 2 on page 35.

$$[P_2 + xA_7]^{-1} = [P_2 + xA_7 P_2]^{-1}$$

$$= P_2 [1 + xA_7]^{-1}$$

$$= P_2 [1 - \frac{x}{(\Box + x)} A_7]$$

$$= P_2 - \frac{x}{(\Box + x)} A_4 \qquad (177)$$

Once again, the subscript T in equation (176) means that the θ-expansion has to be cut off. For this case, they read

$$(P_2 \delta^4_{12})_T = \frac{1}{4} (\theta_1^4 - 4\, \theta_1^2\, \bar{\theta}_1 \bar{\theta}_2) \qquad (178)$$

$$(A_4 \delta^4_{12})_T = \frac{1}{4}\, \theta_1^4 \qquad (179)$$

All the results derived in this section are collected in Table 3 on page 56 at the end of this section. One may check that these results, for the bosonic sector, are in complete agreement with the ones derived with component fields [2]. In this same table, each of the truncated θ-expansions found earlier is given a different name in order to avoid confusion in further calculations.

To conclude this chapter, we would like to underline the fact that the technique for deriving superpropagators worked quite well, except for the scalar sector, even though the fields considered were not superfields. Indeed, only at the end of the calculation, one had to worry about this fact by truncating the θ-expansion in an appropriate way. The case of the scalar sector was more involved because of the appearance of purely chiral terms, SS and RR , or antichiral, $\overline{SS}$ and $\overline{RR}$, whose action is defined only over $d^2\theta$ or $d^2\bar{\theta}$ and not over $d^4\theta$. If these terms were absent, this case would be as simple as the others.

Obviously, all these calculations would be simplified if the Wess-Zumino gauge could be implemented in such a manner that the superfield technique would not be too much affected. It seems that such a method has been recently devised [7]. With this method, Kreuzberger et al. have been able to derive superpropagators for SQED in a Wess-Zumino like gauge with the help of an extended algebra of projection operators (2 projection operators in addition to the original five found in Table 1 on page 22) .

However, it is not evident whether or not this technique would have been a major improvement here. As the theory was explicitly broken, an extended basis of projection operators, the A's, had to be defined from P_1 , P_2 , P_+ and P_- . If this new set had been applied herein, it would have required an extended basis of projection operators of the A-type which would have been much larger and intermediate steps in the derivation of the superpropagators could have been much more involved.

Nonetheless, it should be interesting to try this method to rederive the results obtained in this chapter. If the method proved to be manageable, it would be a major improvement over the method used herein (especially for the purely scalar sector), first from an aesthetical point of view and second, for the possibilities it would open for the calculation of effective potentials in superspace of more complex theories.

Table 3: Superpropagators

Superpropagators for SUSY QED in a Wess-Zumino gauge supplemented with the Lorentz condition.

1-Scalar Sector

$$\Delta_{\bar{S}S} = \frac{-1}{(p^2-x)} \left[1 - \frac{a}{2}(p^2 + x)\ Z \right] A_p + B_p + \frac{(p^2 + b)}{2\ (p^2 + \frac{c}{2})}\ C_p$$

$$\Delta_{\bar{R}R} = \frac{-1}{(p^2+x)} \left[1 - \frac{b}{2}(p^2 - x)\ Z \right] A_p + B_p + \frac{(p^2 + a)}{2\ (p^2 + \frac{c}{2})}\ C_p$$

$$\Delta_{SS} = (\frac{1}{4})\ s^2\ \frac{(p^2+x)}{(p^2-x)}\ ZA_p$$

$$\Delta_{\bar{S}\bar{S}} = (\frac{1}{4})\ \bar{s}^2\ \frac{(p^2+x)}{(p^2-x)}\ ZA_p$$

$$\Delta_{RR} = (\frac{1}{4})\ r^2\ \frac{(p^2-x)}{(p^2+x)}\ ZA_p$$

$$\Delta_{\bar{R}\bar{R}} = (\frac{1}{4})\ \bar{r}^2\ \frac{(p^2-x)}{(p^2+x)}\ ZA_p$$

$$\Delta_{RS} = (\frac{-1}{4})\ rs\ ZA_p$$

$$\Delta_{\bar{R}\bar{S}} = (\frac{-1}{4})\ \overline{rs}\ ZA_p$$

$$\Delta_{\bar{R}S} = (\frac{-1}{4})\ \bar{r}s\ ZA_p + B_p + \frac{\bar{r}s}{2\ (p^2+\frac{c}{2})} C_p$$

$$\Delta_{RS} = (\frac{-1}{4})\, r\bar{s}\, Z A_p + B'_p + \frac{r\bar{s}}{2\,(p^2 + \frac{c}{2})}\, C'_p$$

The following variables have been introduced:

$$a = \frac{e^2}{2}\,\bar{s}s$$

$$b = \frac{e^2}{2}\,\bar{r}r$$

$$Z = [\,(p^4 - x^2) + a(p^2 + x) + b(p^2 - x)\,]^{-1}$$

$$= [\,(p^2 + w_+)\,(p^2 + w_-)\,]^{-1}$$

where

$$w_+ = \frac{c}{4} + \gamma \qquad w_- = \frac{c}{4} - \gamma$$

and

$$\gamma = \sqrt{\frac{c^2}{16} - (a-b-x)x}\ .$$

$$\det(\,p^2 I - M) = (p^4 - x^2)\,(p^2 + w_+)\,(p^2 + w_-)$$

$$A_p = p^{-2}\,[\,-\theta_1\sigma\bar{\theta}_2{\cdot}p - \theta_2\sigma\bar{\theta}_2{\cdot}p + \frac{1}{4}\,\theta_1^4\,p^2 - \theta_1^2\bar{\theta}_1\bar{\theta}_2\,p^2 + \frac{1}{2}\,\theta_1\theta_2\theta_1\theta_2\,p^2 - \theta_2^2\bar{\theta}_2\bar{\theta}_1\,p^2$$

$$+ \frac{1}{4}\,\theta_2^4\,p^2 + \frac{1}{4}\,p^2\,\theta_1^4\theta_2\sigma\bar{\theta}_2{\cdot}p + \frac{1}{4}\,p^2\,\theta_2^4\theta_1\sigma\bar{\theta}_1{\cdot}p - \frac{1}{16}\,\theta_1^4\theta_2^4\,p^4\,]$$

$$B_p = \bar{\theta}_1^2\theta_2^2$$

$$B'_p = \theta_1^2\bar{\theta}_2^2$$

$$C_p = -2\theta_2\sigma\bar{\theta}_1{\cdot}p + \bar{\theta}_1^2\theta_2^2\theta_1\sigma\bar{\theta}_2{\cdot}p$$

$$C'_p = 2\theta_1\sigma\theta_2{\cdot}p - \theta_1^2\bar{\theta}_2^2\theta_2\sigma\theta_1{\cdot}p$$

<u>**2-Vector Sector**</u>

$$\Delta_{VV} = \frac{1}{2\,(p^2 + \frac{c}{2})}\,\{\, D_p - \frac{1}{p^2}\,[\, ax(p^2+x) - bx(p^2-x)\,]\; Z\, E_p + (\delta^{mn} - \frac{p^m p^n}{p^2}\,)\, F_p \,\}$$

The following variables have been introduced:

$$D_p = (\frac{-1}{8})\,\theta_1^4\theta_2^4\,p^2 - \theta_1^2\bar{\theta}_1\bar{\theta}_2\theta_2\sigma\bar{\theta}_2{\cdot}p - \theta_2^2\bar{\theta}_1\bar{\theta}_2\theta_1\sigma\theta_1{\cdot}p$$

$$E_p = (\frac{-1}{8})\,\theta_1^4\theta_2^4\,p^2$$

$$F_p = -\,\theta_1\theta_2\bar{\theta}_1\bar{\theta}_2$$

<u>**3-Scalar-Vector Sector**</u>

$$\Delta_{VS} = \frac{s}{2\,(p^2+\frac{c}{2})}\,\{\, G_p - \frac{[\,(ax(p^2+x) - bx(p^2-x))\, Z - x\,]}{(p^2 - x)}\, K_p \,\}$$

$$\Delta_{VS} = \frac{\bar{s}}{2\,(p^2+\frac{c}{2})}\,\{\, H_p - \frac{[\,(ax(p^2+x) - bx(p^2-x))\, Z - 2x\,]}{(p^2 - x)}\, K_p \,\}$$

$$\Delta_{VR} = \frac{r}{2\,(p^2+\frac{c}{2})}\,\{\, G_p - \frac{[\,(ax(p^2+x) - bx(p^2-x))\, Z + 2x\,]}{(p^2 + x)}\, K_p \,\}$$

$$\Delta_{VR} = \frac{\bar{r}}{2\,(p^2+\frac{c}{2})} \left\{ H_p - \frac{\left[\, (ax(p^2+x) - bx(p^2-x))\, Z + 2x \,\right]}{(p^2+x)} K_p \right\}$$

The following variables have been introduced:

$$G_p = (\frac{1}{4})\,(\theta_1^4 - 4\theta_1^2\bar{\theta}_1\bar{\theta}_2)$$

$$H_p = (\frac{1}{4})\,(\theta_1^4 - 4\theta_1\bar{\theta}_1^2\theta_2)$$

$$K_p = (\frac{1}{4})\,\theta_1^4$$

4.3 References

[1] J.Wess, B.Zumino, Nucl.Phys.B78(1974)1

[2] R.D.C. Miller, Nucl. Phys. B229(1983)189

[3] J.A.Helayël-Neto, Phys.Lett. 135B(1984)78

F.Feruglio, J.A.Helayël-Neto, F.Legovini, Nucl.Phys.B249(1985)533.

J.A.Helayël-Neto, F.A.B.Rabelo de Carvalho, A.W.Smith, Nucl.Phys. B271(1986)175

[4] M.T.Grisaru, F.Riva, D.Zanon, Nucl.Phys.B214(1983)465

[5] J Wess, J.Bagger, "Supersymmetry and Supergravity", Princeton University Press, Princeton, 1983, p.72

[6] J.Wess, J.Bagger, ibid., p.82

[7] T.Kreuzberger, W.Kummer, O.Piguet, A.Rebhan, M.Schweda, Phys.Lett. 167B(1986)393

Chapter V

HIGHER ORDER CORRECTIONS TO THE EFFECTIVE POTENTIAL

In this chapter and the next one are concentrated the main results of this thesis. First, in this chapter, the one and two loop order contributions to the effective potential (U_1 , U_2) will be calculated with the help of the superpropagators derived in the previous chapter.

The results of the first section, i.e. the one-loop effective potential, have already been obtained by Miller [1] by means of a calculation with components. Hence, this will serve as a test for the superfield method applied to SUSY gauge theories. With this point verified, the method will be applied to two-loop order. This task will be carried out in the second part of this chapter.

This chapter will be divided into four sections. As mentioned above, the first and second will be devoted to evaluating the one and two loop order effective potential. These expressions will be in terms of four-momentum integrals which will be calculated in the third section. In the last section, the different parts of the effective potential will be gathered in their analytical form for further use in chapter VI.

5.1 One-Loop Effective Potential

The calculation of the one-loop effective potential for SUSY QED resembles closely the one already done for the Wess-Zumino model in Chapter III. The contributing graphs are given in Figure 4. There are six of them to start with but only two of them need to be evaluated because only the auxiliary field tadpole contribution is needed when using AFTM, along with the supersymmetric boundary condition, and because there are no F-tadpoles:

$$\int d^4\theta\, S|_{\theta^2}\, \Delta_{VV} = 0 \tag{180}$$

and similarly for $\bar{S}$, R, $\bar{R}$.

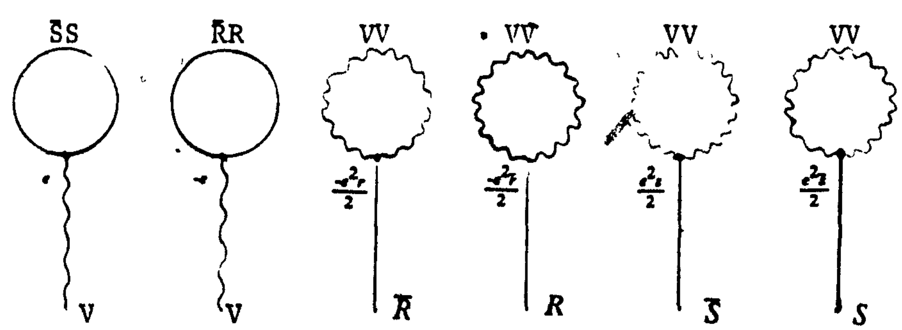


Figure 4: One-Loop Tadpole Graphs in SUSY QED

This leaves us with the two last graphs whose contributions to the effective action are

$$\Gamma_1 = e\int d^4\bar{p}\,(2\pi)^4\delta^4(p_{ext})\int d^4\bar{k}\int d^4\theta\,[\,V(p_{ext},\theta,\bar{\theta})\,\{\,\Delta^k_{\bar{S}S} - \Delta^k_{\bar{R}R}\,\}_{\theta_1=\theta_2=\theta}\,] \tag{181}$$

As mentioned above, with AFTM, one needs only to know the term corresponding to the auxiliary field of V, i.e. D . Knowing this, one may write down the one-particle irreducible D tadpole .

$$\Gamma_{D,1}'(p_{ext}=0) = \frac{e}{2} \int d^4k \, [\, \Delta_{SS}^k - \Delta_{RR}^k \,]_{\phi=0} \tag{182}$$

With the help of the expressions for Δ_{SS} and Δ_{RR} found in Table 3 on page 56, equation (182) becomes

$$\Gamma_{D,1}'(0) = \frac{e}{2} \int d^4k \, \{ \frac{-[\,1 - \frac{a}{2}(k^2+x)Z^k\,]}{(k^2-x)} + \frac{[\,1 - \frac{b}{2}(k^2-x)Z^k\,]}{(k^2+x)} \} \tag{183}$$

It would be possible to start from this expression, to integrate over d and apply SUSY boundary condition in order to obtain the one-loop effective potential. However, there is a simpler way to obtain the same result. First, one uses matrix notation for the superpropagators instead of their explicit version given in Table 3 on page 56. So, retaining the notation of equation (163), one may write equation (183) as

$$\Gamma_{D,1}'(0) = (\frac{-e}{2}) \int d^4k \, \{ \, [\,(\Box I + M)^{-1}\,]_{11} - [\,(\Box I + M)^{-1}\,]_{33} \} \tag{184}$$

With the clever trick used by Miller [1],

$$\frac{\partial \ln [\, det(\Box I + M)\,]}{\partial d} = e \, \{ \, [\,(\Box I + M)^{-1}\,]_{11} - [\,(\Box I + M)^{-1}\,]_{33} \} \; , \tag{185}$$

equation (184) can be written in a very simple fashion:

$$\Gamma_{D,1}'(0) = (\frac{-1}{2})\int d^4k \frac{\delta \ln [\, det(\Box I + M)\,]}{\delta d} \tag{186}$$

The above quantity is equal to minus the derivative of the one-loop effective potential with respect to d. Thus, the integration over d is trivial and the only thing left to do is to determine the integration constant with help of the SUSY boundary condition. The result is

$$U_1 = (1/2)\int d^4k \ln \left\{ \frac{[\,(k^4 - x^2) + a\,(k^2 + x) + b\,(k^2 - x)\,]\,[\,k^4 - x^2\,]}{[\,k^2 + \frac{c}{2}\,]\,k^6} \right\} \tag{187}$$

This leaves us with the task of performing the integration over the internal four-momentum k. The regularization technique of 't Hooft and Veltman [2] will be used to work out these integrals. As mentioned in the introductory remarks at the beginning of this chapter, this will be done in the third section and the final result given in the next one.

5.2 Two-Loop Effective Potential

At two-loop order, the Vacuum Bubble Method may be used again. Calculations are more straightforward than for the one-loop case as no extra integrations are needed and as everything is done with superfield notation. At this order, there are many contributing graphs. They are all listed in Figure 8 on page 86.

The two-loop effective potential is given by minus the sum of all these graphs.

$$U_2 = -\,\Gamma_2'(0)$$

$$= -e^2 \int d^4\theta_1 \int d^4\theta_2 \int d^4\bar{p} \int d^4\bar{q} \int d^4\bar{l}\; \delta^4(p + q + l)$$

$$\{ \sum_{\substack{i=1 \\ j=1}}^{4} \Delta^p_{VV}\, \Delta^q_{\Phi_i \bar{\Phi}_j}\, \Delta^l_{\bar{\Phi}_i \Phi_j} + \frac{e}{2}\, \Delta^p_{VV} \sum_{\substack{i=1 \\ j=1}}^{4} t_j\, \Delta^q_{\Phi_i V}\, \Delta_{\bar{\Phi}_i \Phi_j}\, n_{ij} +$$

$$\frac{e^2}{2}\, \Delta^p_{VV} \sum_{\substack{i=1 \\ j=1}}^{4} t_i t_j\, \Delta^q_{V\Phi_i}\, \Delta^l_{\bar{\Phi}_j V} + \sum_{\substack{i=1 \\ j=0}}^{4} \Delta^p_{\Phi_i \bar{\Phi}_j}\, \Delta^q_{V\Phi_j}\, \Delta^l_{\bar{\Phi}_i V}\, n_{ij} +$$

$$\frac{e^2}{2}\, \Delta^p_{VV}\, \Delta^q_{VV} \sum_{\substack{i=1 \\ j=1}}^{4} t_i t_j\, \Delta^l_{\Phi_i \Phi_j} \}$$

$$-\frac{e^2}{2} \int d^4\theta \int d^4\bar{p} \int d^4\bar{q}\; \delta^4(p+q)\; \{ \Delta^p_{VV} [\, \Delta^q_{\bar{S}S} + \Delta^q_{\bar{R}R} \,]$$

$$+ [\, \Delta^p_{SV}\, \Delta^q_{VS} + \Delta^p_{RV}\, \Delta^q_{VR} \,] \} \qquad (188)$$

$$\Phi = (\bar{S}, S, \bar{R}, R) \qquad t = (\bar{s}, s, \bar{r}, r) \qquad (189)$$

$$n_{ij} = \begin{vmatrix} 1 & \text{if number of } \bar{R}R \text{ (or } \bar{r}R\text{ , } \bar{R}r \text{ or } \bar{r}r \text{) pairs is even} \\ -1 & \text{if number of } \bar{R}R \text{ (or } \bar{r}R\text{ , } \bar{R}r \text{ or } \bar{r}r \text{) pairs is odd} \end{vmatrix} \qquad (190)$$

The number of graphs to evaluate is quite large but it would be even worse if a component approach had been considered. Nonetheless, many of these terms will be discarded once the integration over the θ variables have been performed. To facilitate things, each type of graphs will be evaluated separately. Each contribution will be identified with the letter used in Figure 8 on page 86. This way, the two-loop effective potential is defined as

$$U_2 = - \left[U_a + U_b + U_c + U_d + U_e + U_f + U_g + U_h \right] \tag{191}$$

To evaluate U_a through U_h , one uses the superpropagators found in Table 3 on page 56 , then integrates over $d^4\theta_1\, d^4\theta_2$ ($d^4\theta$ for the last two) as well as over one of the four-momentum and finds :

$$U_a = e^2 \int d^4p \int d^4q \; \{ (\tfrac{1}{8}) (l^4 - x^2) Z^l \, [(p^2 - x)^{-1} (q^2 - x)^{-1} (1 - \tfrac{a}{2} (p^2 + x) Z^p -$$

$$\tfrac{a}{2} (q^2 + x) Z^q + \tfrac{a^2}{2} (p^2 + x) (q^2 + x) Z^p Z^q) + (p^2 + x)^{-1} (q^2 + x)^{-1}$$

$$(1 - \tfrac{b}{2} (p^2 - x) Z^p - \tfrac{b}{2} (q^2 - x) Z^q + \tfrac{b^2}{2} (p^2 - x) (q^2 - x) Z^p Z^q) - ab \, Z^p Z^q]$$

$$- \frac{1}{2} (p\text{-}q)_m \frac{(\delta^{mn} - \frac{l^m l^n}{l^2})}{(l^2 + \frac{c}{2})} (p\text{-}q)_n \, [\frac{1}{2} (p^2 + x)^{-1} (q^2 + x)^{-1}$$

$$+ \frac{1}{2} (p^2 - x)^{-1} (q^2 - x)^{-1} + \frac{1}{8\gamma^2} (aN\text{-}bO)^2 (p^2 + w_-)^{-1} (q^2 + w_-)^{-1}$$

$$+ \frac{1}{8\gamma^2} (aP\text{-}bQ)^2 (p^2 + w_+)^{-1} (q^2 + w^+)^{-1} + \frac{ab}{2\gamma^2} (p^2 + w_-)^{-1} (q^2 + w_+)^{-1}]$$

$$+ \frac{1}{2} \frac{p_m}{p^2} \frac{(\delta^{mn} - \frac{l^m l^n}{l^2})}{(l^2 + \frac{c}{2})} \frac{q_n}{q^2} \left[\frac{(p^2 + b)}{(p^2 + \frac{c}{2})} \frac{(q^2 + b)}{(q^2 + \frac{c}{2})} \right.$$

$$\left. + \frac{(p^2 + a)}{(p^2 + \frac{c}{2})} \frac{(q^2 + a)}{(q^2 + \frac{c}{2})} - \frac{2ab}{(p^2 + \frac{c}{2})(q^2 + \frac{c}{2})} \right] + \frac{p \cdot q}{p^2 (q^2 + \frac{c}{2})}$$

$$[(l^2 - x)^{-1} (1 - \frac{a}{(p^2 + \frac{c}{2})}) + (l^2 + x)^{-1} (1 - \frac{b}{(p^2 + \frac{c}{2})}) - \frac{a}{4\gamma} (\frac{N}{(l^2 + w_-)}$$

$$- \frac{P}{(l^2 + w_+)} + \frac{2\gamma}{a(l^2 - x)}) (1 - \frac{a}{(p^2 + \frac{c}{2})}) - \frac{b}{4\gamma} (\frac{O}{(l^2 + w_-)} - \frac{Q}{(l^2 + w_+)}$$

$$+ \frac{2\gamma}{b(l^2 + x)}) (1 - \frac{b}{(p^2 + \frac{c}{2})}) - \frac{ab}{2\gamma} (l^2 + w_-)^{-1} - (l^2 + w_+)^{-1} (p^2 + \frac{c}{2})] \} \quad (192)$$

where

$$N = \frac{w_- - x}{w_- + x}$$

$$O = \frac{w_- + x}{w_- - x}$$

$$P = \frac{w_+ - x}{w_+ + x}$$

$$Q = \frac{w_+ + x}{w_+ - x} \quad (193)$$

$$U_b = 0 \quad (194)$$

$$U_c = 0 \quad (195)$$

$$U_d = - e^2 \int d^4\tilde{p} \int d^4\tilde{q} \{ \{ \frac{1}{8} Z^p Z^q [a (p^2 + x) (q^2 + x) (l^2 - x)^{-1} (1 - \frac{a}{2} (l^2 + x) Z^l)$$

$$+ b (p^2 - x) (q^2 - x) (l^2 + x)^{-1} (1 - \frac{b}{2} (l^2 - x) Z^l) - \frac{a^2}{2}$$

$$(p^2 + x) (q^2 + x) (l^2 + x) (l^2 - x)^{-1} Z^l - \frac{b^2}{2} (p^2 - x) (q^2 - x) (l^2 - x) (l^2 + x)^{-1} Z^l$$

$$+ ab Z^l (p^2 - x) (q^2 + x) + (p^2 + x) (q^2 - x)]$$

$$+ \frac{1}{2} (p^2 + \frac{c}{2})^{-1} (q^2 + \frac{c}{2})^{-1} Z^l [a^2 \frac{(l^2 + x)}{(l^2 - x)} + b^2 \frac{(l^2 - x)}{(l^2 + x)} - 2ab] \} \quad (196)$$

$$U_e = \frac{-e^2}{4} \int d^4p \int d^4q \, \{ \frac{(\delta^{mn} - \frac{p^m p^n}{p^2})}{(p^2 + \frac{c}{2})} \frac{(\delta_{nm} - \frac{q_n q_m}{q^2})}{(q^2 + \frac{c}{2})}$$

$$[a(l^2 - x)^{-1} (1 - \frac{a}{2} (l^2 + x) Z^l) + b(l^2 + x)^{-1} (1 - \frac{b}{2} (l^2 - x) Z^l)$$

$$- \frac{a^2}{2} \frac{(l^2 + x)}{(l^2 - x)} Z^l - \frac{b^2}{2} \frac{(l^2 - x)}{(l^2 + x)} Z^l + 2abZ^l] \} \tag{197}$$

where one e^2 has been combined to the constant scalar superfields and written as the constants a and b defined in Table 3 on page 56,

$$U_f = \frac{3e^2}{4} \int d^4p \left\{ \frac{[1 - \frac{a}{2} (p^2 + x) Z^p]}{(p^2 - x)} + \frac{[1 - \frac{b}{2} (p^2 - x) Z^p]}{(p^2 + x)} \right\} \tag{198}$$

$$U_g = 0 \tag{199}$$

In the first five terms, U_a through U_e, " l " has been kept as a short writing.

$$l = p + q \tag{200}$$

All the different terms in equation (192) to equation (198) may be put in such a form that only two types of integrals will ultimately need to be evaluated. These typical integrals take the form

$$I(a,b,c) = \int d^4p \int d^4q \, \frac{1}{[(p^2 + a) (l^2 + b) (q^2 + c)]} \tag{201}$$

and

$$J(a,b) = \int d^4p \int d^4q \; \frac{1}{[\,(p^2 + a)\,(q^2 + b)\,]} \tag{202}$$

In terms of these two basic integrals, the different parts of the two-loop effective potential U become :

$$U_a = e^2 \{ -(1 + \frac{6x^2}{c^2})J(\frac{c}{2},\frac{c}{2}) - \frac{5}{16} J(x,x) - \frac{5}{16} J(-x,-x) - \frac{5x^2}{4\gamma^2} J(w_+,w_+) -$$

$$\frac{5x^2}{4\gamma^2} J(w_-,w_-) - \frac{5ab}{8\gamma^2} J(w_+,w_-) + [\, \frac{(aP + bQ)}{8\gamma} + \frac{(a^2P\text{-}2ab + b^2Q)}{8c\gamma} + 2ab\frac{w_+}{c\gamma^2} +$$

$$\frac{(aP\text{-}bQ)^2}{8\gamma^2}\,]\, J(w_+,\frac{c}{2}) + [\, \frac{-(aN + bO)}{8\gamma} - \frac{(a^2N\text{-}2ab + b^2O)}{8c\gamma} + \frac{2abw_-}{c\gamma^2} + \frac{(aN - bO)^2}{8\gamma^2}\,]$$

$$J(w_-,\frac{c}{2}) + (\frac{3}{4} + \frac{b}{2c})J(x,\frac{c}{2}) + (\frac{3}{4} + \frac{a}{2c})J(-x,\frac{c}{2}) + \frac{1}{8\gamma}(1 + \frac{2w_+}{c})(a^2N - 2ab + b^2O)$$

$$I(\frac{c}{2},w_-,\frac{c}{2}) - \frac{1}{8\gamma}(1 + \frac{2w_-}{c})(a^2P - 2ab + b^2Q)\, I(\frac{c}{2},w_+,\frac{c}{2}) - \frac{a}{2}(1 + \frac{x}{c})\, I(\frac{c}{2},-x,\frac{c}{2})$$

$$- \frac{b}{2}\, 1 - \frac{x}{c})\, I(\frac{c}{2},x,\frac{c}{2}) - \frac{abw_-}{4c\gamma}(P + Q + 2)\, I(\frac{c}{2},w_+,0) +$$

$$\frac{abw_+}{4c\gamma}(N + O + 2)\, I(\frac{c}{2},w_-,0) + \frac{(aN\text{-}bO)^2}{16\gamma^2}(\frac{c}{2} - 4w_-)\, I(w_-,\frac{c}{2},w_-)$$

$$+ \frac{(aP\text{-}bQ)^2}{16\gamma^2}(\frac{c}{2} - 4w_+)\, I(w+,\frac{c}{2},w_+) - (\frac{4abw_+w_-}{c\gamma^2})\, I(w_+,\frac{c}{2},w_-) -$$

$$\frac{w_-^2 - x^2}{32\gamma} I(-x,w_-,-x) + \frac{w_+^2 - x^2}{32\gamma} I(-x,w_+,-x) - \frac{w_-^2 - x^2}{32\gamma} I(x,w_-,x)$$

$$+ \frac{w_+\text{-}x^2}{32\gamma} I(x,w_+,x) + (x + \frac{c}{8})\, I(-x,c/2,-x) + (-x + \frac{c}{8})\, I(x,c/2,x) -$$

$$\frac{ab(\frac{c^2}{4} - 4w_-w_+)}{cG^2} I(w_-,0,w_+) + (\frac{c}{8} - \frac{3x^2}{c})\, I(0,c/2,0) + \frac{3x^2}{c} I(c/2,c/2,c/2) -$$

$$\frac{b}{4}\left(1+\frac{2x}{c}\right) I\left(\frac{c}{2},-x,0\right) - \frac{a}{4}\left(1-\frac{2x}{c}\right) I\left(\frac{c}{2},x,0\right) + \frac{(w_+^2-x^2)}{128\gamma^3}(aP\text{-}bQ)^2 I(w_+,w_+,w_+)$$

$$- \frac{(w_-^2-x^2)}{128\gamma^3}(aN\text{-}bO)^2 I(w_-,w_-,w_-) + \frac{[\,8ab(w_+^2-x^2) - (aP\text{-}bQ)^2 (w_-^2-x^2)\,]}{128\gamma^3}$$

$$I(w_+,w_-,w_+) - \frac{[\,8ab(w_-^2\text{-}x^2) - (aN\text{-}bO)^2(w_+^2\text{-}x^2)\,]}{128\gamma^3} I(w_-,w_+,w_-)\ \} \qquad (203)$$

$$U_d = e^2\ \{\ \frac{(a^2N - 2ab + b^2O)}{4\gamma} I\left(\frac{c}{2},w_-,\frac{c}{2}\right) - \frac{(a^2P - 2ab + b^2Q)}{4\gamma} I\left(\frac{c}{2},w_+,\frac{c}{2}\right) +$$

$$\frac{a}{2} I\left(\frac{c}{2},-x,\frac{c}{2}\right) + \frac{b}{2} I\left(\frac{c}{2},x,\frac{c}{2}\right) + \frac{(w_+^2\text{-}x^2)}{64\gamma^3}(aP\text{-}bQ)^2 I(w_+,w_+,w_+) -$$

$$\frac{(w_-^2-x^2)}{64\gamma^3}(aN\text{-}bO)^2 I(w_-,w_-,w_-) - [\ (aw_+-x)^2(w_--x)(w_+ + 2w_- + 3x) -$$

$$(bw_+ + x)^2(w_- + x)(w_+ + 2w_- -3x) - 4abx(w_+^2 + 2w_+w_- -3x^2)\ ](128x\gamma^3)^{-1} I(w_+,w_-,w_+) +$$

$$[\ (aw_- -x)^2(w_+-x)(w_- + 2w_+ + 3x) - (bw_- + x)^2(w_+ + x)(w_- + 2w_+\text{-}3x) -$$

$$4abx(w_-^2 + 2w_+w_- -3x^2)\ ](128x\gamma^3)^{-1} I(w_-,w_+,w_-)\ \} \qquad (204)$$

$$U_e = e^2\ \{\ [\ \frac{(a^2P + b^2Q - a^2N - b^2O)}{16c\gamma} + \left(\frac{w_-}{8c^2\gamma}\right)(a^2P - 2ab + b^2Q) - \left(\frac{w_+}{8c^2\gamma}\right)$$

$$(a^2N - 2ab + b^2O)\ ]\ J\left(\frac{c}{2},\frac{c}{2}\right) - \frac{(a^2P - 2ab + b^2Q)}{8c\gamma} J\left(w_+,\frac{c}{2}\right) + \frac{(a^2N - 2ab + b^2O)}{8c\gamma}$$

$$J\left(w_-,\frac{c}{2}\right) - \frac{(a^2N - 2ab + b^2O)}{32\gamma}\left[\ 8 + 4\frac{(w_+ + \frac{c}{2})^2}{c^2}\ \right] I\left(\frac{c}{2},w_-,\frac{c}{2}\right) +$$

$$\frac{(a^2P - 2ab + b^2Q)}{32\gamma}\left[\ 8 + 4\frac{(w_- + \frac{c}{2})^2}{c^2}\ \right] I\left(\frac{c}{2},w_+,\frac{c}{2}\right)$$

$$-(\frac{w_-^2}{4c^2\gamma})(a^2P - 2ab + b^2Q)\, I(\frac{c}{2},w_+,0) + (\frac{w_+^2}{4c^2\gamma})(a^2N - 2ab + b^2O)\, I(\frac{c}{2},w_-,0) +$$

$$(\frac{w_+^2}{8c^2\gamma})(a^2P - 2ab + b^2Q) I(0,w_+,0) - (\frac{w_-^2}{8c^2\gamma})(a^2N - 2ab + b^2O)\, I(0,w_-,0) \}$$

(205)

$$U_f = e^2 \{ \frac{-3}{16\gamma}(aP + bQ)\, J(w_+,\frac{c}{2}) + \frac{3}{16\gamma}(aN + bO)\, J(w_-,\frac{c}{2}) -$$

$$\frac{3}{8} J(-x,\frac{c}{2}) - \frac{3}{8} J(x,\frac{c}{2}) \} \qquad (206)$$

To obtain these results, the on-shell value of the auxiliary field has been used.

$$d = \frac{e(\bar{r}r - \bar{s}s)}{2} = b - a \qquad (207)$$

This yields

$$x = \frac{e(b-a)}{2} \qquad (208)$$

$$\gamma = \sqrt{\frac{c^2}{16} + 3x^2}$$

As both a and b are semi-positive definite, d may only exist between certain limits :

$$-\frac{c}{2} \leq d \leq \frac{c}{2} \qquad (209)$$

where

$$c = 2\,(a + b) \qquad (210)$$

The four constituent terms of the two-loop effective potential will be added together once the integrals I and J will have been evaluated. This will be done in the coming section.

5.3 Evaluation of the Divergent Integrals

The problem of having to solve divergent integrals is as old as the Feynman graphs themselves. Many methods have been used to circumvent this difficulty. All of them have the same goal, that is to isolate the infinite part in order to remove it from the original Lagrangian within an appropriate renormalization scheme. One of these methods is the dimensional regularization of 't Hooft and Veltman [2]. This method which is simple to use yields an infinite part factorized as a residue at some pole. Moreover, infrared divergencies are handled without additional problems. For these reasons, this technique will be used to evaluate the divergent integrals found in the previous sections.

5.3.1 Dimensional Regularization

Dimensional regularization is so broadly used nowadays that it can be found in most of the recent textbooks on field theory [3]. The idea behind this theory is that Feynman integrals converge if the size of spacetime dimension considered, n, is small enough. Hence, one may evaluate these integrals in n dimensions and at the end return to 4 dimensions in an appropriate way. There, infinities will show up as finite residues at poles in (n-4). To make things clearer, a typical integral in the evaluation of Feynman diagrams will be worked out.

$$A = \mu^{4-n} \int d^n q \frac{1}{(q^2 + m^2)^a}$$

$$= \mu^{4-n}(2\pi)^{-n} \int dq \frac{q^{n-1}}{(q^2 + m^2)^a} \int d\theta_1 \int d\theta_2 \sin\theta_2 \ldots \int d\theta_{n-1} \sin^{n-2}\theta_{n-1} \tag{211}$$

The arbitrary mass term μ is introduced to keep the overall dimension of A unchanged. The θ's are angles in the n-space and should not be confused with Grassmann variables defined earlier.

The integral A as defined in equation (211) is finite for a number of dimensions n smaller than two times a, the exponent of the parenthesis. The angular integration is easy to perform with the help of the relation

$$\int d\theta \sin^k\theta = \Gamma(\frac{1}{2}) \frac{\Gamma(\frac{1}{2} + \frac{k}{2})}{\Gamma(1 + \frac{k}{2})} \tag{212}$$

The properties of Euler Gamma functions may be found in Appendix A. The remaining integral over q can be done with the help of the standard result :

$$\int dx \frac{x^{2b-1}}{(x^2 + m^2)^a} = \frac{\Gamma(b)\ \Gamma(a-b)}{2\ \Gamma(a)} (m^2)^{a-b} \tag{213}$$

With these two expressions, one finds that the integral A is equal to :

$$A = \mu^{4-n}(4\pi)^{-n/2} \frac{\Gamma(a-n/2)}{\Gamma(a)\ (m^2)^{a-\frac{n}{2}}} \qquad a > \frac{n}{2} \tag{214}$$

For the case $a>\frac{n}{2}$, A is finite and one has only to replace n by 4 to obtain the final result. However, equation (214) is still a valid representation of A for $a<\frac{n}{2}$ as long as $(a-\frac{n}{2})$ is not equal to a nonpositive integer. This is the idea behind dimensional regularization. The technique is to take an analytic continuation of these integrals to complex values of n where it can be evaluated. The infinities reappear at the end as poles in the limit of n going to 4. For example, for the case $a = 1$, equation (214) becomes

$$A = (4\pi)^{-2}\Gamma(-1+\epsilon)\mu^{2\epsilon}m^{2(1-\epsilon)}$$

$$= \frac{m^2}{(4\pi)^2}\left[\frac{1}{\epsilon} + (1-\gamma_e) + (\frac{1}{2})(1 - 2\gamma_e + \gamma_e^2 + \frac{\pi^2}{6}) + \ldots\right]$$

$$\left[1 - \epsilon\ln(\frac{m^2}{4\pi\mu^2}) + \frac{\epsilon^2}{2}\ln^2(\frac{m^2}{4\pi\mu^2}) + \ldots\right]$$

$$= (\frac{m}{4\pi})^2\left[-\frac{1}{\epsilon} + (1 - \gamma_e + \ln(\frac{m^2}{4\pi\mu^2})) + O(\epsilon^2)\right] \tag{215}$$

where γ_e is Euler's constant and where n has been expressed as $(4-2\epsilon)$ close to 4. Results from Appendix A have been used to obtain equation (215) as well as the following relation:

$$a^\epsilon = e^{\epsilon\ln a} = 1 + \epsilon\ln a + \frac{\epsilon^2}{2}\ln^2 a + \ldots \tag{216}$$

The infinite part of the integral A has been factorized as the residue $(\frac{m}{4\pi})^2$ at the pole $\frac{1}{\epsilon}$ where the limit ϵ going to zero is understood. With an appropriate renormalization scheme, all these poles should cancel and the limit give a finite result.

The typical one-loop integrals have been calculated in the original paper of 't Hooft and Veltman, [2], and some are tabulated in the first appendix. They will serve as the building blocks in the evaluation of the forthcoming integrals.

5.3.2 One-Loop Integrals

The integrals found in the evaluation of the one-loop effective potential, equation (187), are all of the same type, i.e.

$$I_1 = \int d^4p \ \ln(p^2 + a) \tag{217}$$

where a may take the values x , $-x$, w_+ , w_- and 0. Equation (217) may be written as

$$I_1 = \int da \int d^4p \frac{1}{(p^2 + a)} \tag{218}$$

The four-momentum integral has now exactly the form found in Appendix A and its solution is

$$\begin{aligned} I_1 &= \int da \left[\frac{a}{(4\pi)^2} \Gamma(-1+\epsilon) \left(\frac{a}{4\pi\mu^2}\right)^{-\epsilon} \right] \\ &= \frac{a^2}{(4\pi)^2} \frac{1}{(2-\epsilon)} \Gamma(-1+\epsilon) \left(\frac{a}{4\pi\mu^2}\right)^{-\epsilon} \end{aligned} \tag{219}$$

Hence, for $a = 0$, the integral vanishes but for w_-, there is an imaginary part as well as for $a = x$ when x is negative and for $a = -x$ when x is positive. This is a genuine effect which will be discussed in subsection 5.3.4.

5.3.3 Two-Loop Integrals

In the case of the two-loop effective potential, there are two types of integral to be evaluated. The first one,

$$J(a,b) = \int d^4p \int d^4q \frac{1}{(p^2 + a)(q^2 + b)} , \tag{220}$$

is simple enough as the answer is given by the multiplication of two one-loop integrals.

$$J(a,b) = \frac{\Gamma^2(-1+\epsilon)}{(4\pi)^4} \frac{ab}{(\frac{a}{4\pi\mu^2})^\epsilon (\frac{b}{4\pi\mu^2})^\epsilon} \tag{221}$$

The other type of integral, I(a,b,c), is far more difficult to evaluate. For this case, it will be necessary to use Feynman parameters for folding denominators into one. For the case of two denominators, Feynman's formula reads

$$\frac{1}{D_1^i D_2^j} = \frac{\Gamma(i+j)}{\Gamma(i)\,\Gamma(j)} \int_0^1 dx \frac{x^{i-1}(1-x)^{j-1}}{[\,xD_1 + (1-x)D_2\,]^{i+j}} \tag{222}$$

where i and j do not have to be integers. Thus, let us rewrite the expression for I(a,b,c).

$$I(a,b,c) = (\mu^2)^{4-n} \int d^n p \int d^n q \frac{1}{(p^2 + a)(l^2 + b)(q^2 + c)} \tag{223}$$

with l still defined as

$$l = p + q \quad . \tag{224}$$

With the judicious use of the expression

$$1 = \frac{1}{2n}\left(\frac{\partial p^m}{\partial p^m} + \frac{\partial q^m}{\partial q^m} \right) \quad . \tag{225}$$

in equation (223), one may prevent the ultraviolet divergences to find their way into the parametric integrals [4]. This yields

$$I(a,b,c) = -\frac{(\mu^2)^{4-n}}{n-3} \int d^n p \int d^n q \frac{\left[\frac{a}{(p^2+a)} + \frac{b}{(l^2+b)} + \frac{c}{(q^2+c)} \right]}{(p^2+a)\,(l^2+b)\,(q^2+c)} \tag{226}$$

The best way to evaluate this expression is to introduce Feynman parameters one at a time, starting with the most divergent four-momentum integral. Let us start with the first term of equation (226).

$$\int d^n p \int d^n q \frac{1}{(p^2+a)^2\,(l^2+b)(q^2+c)}$$

$$= \int dy \int d^n p \frac{1}{(p^2+a)^2} \int d^n q \frac{1}{[\, q^2 + y(p^2 + 2p{\cdot}q + b) + (1{-}y)c \,]^2}$$

$$= \frac{1}{(4\pi)^2}\,\Gamma(\epsilon)\int_0^1 dy\, y^{-\epsilon}\,(1\text{-}y)^{-\epsilon}\int d^n p \frac{1}{(p^2+a)^2}\,\frac{1}{(p^2+\frac{b}{(1\text{-}y)}+\frac{c}{y})^{\epsilon}}$$

$$= \frac{1}{(4\pi)^2}\Gamma(2\epsilon)\int_0^1 dy\int_0^1 dz\,\frac{y^{\epsilon}\,(1\text{-}y)^{\epsilon}\,z(1\text{-}z)^{\epsilon\text{-}1}}{[\,yz(1\text{-}y)a+(1\text{-}z)yb+(1\text{-}y)(1\text{-}z)c\,]^{2\epsilon}} \tag{227}$$

Obviously, the third term of equation (226) will give a similar result with a and c interchanged. For the second term, one simply has to make a change of variable ($p \rightarrow p\text{-}q,\ l \rightarrow p,\ q \rightarrow q'$) to retrieve the same answer as before with a and b interchanged.

This leaves us with the integration over the two Feynman parameters, y and z. Integrating first over z, one uses the fact that

$$(1-z)^{\epsilon\text{-}1} = \frac{-1}{\epsilon}\,\frac{d(\,1-z)^{\epsilon}}{dz} \tag{228}$$

to obtain with partial integration

$$\frac{1}{\epsilon}\int dy\, y^{\epsilon}(1\text{-}y)^{\epsilon}\,[\,1-\epsilon-2\epsilon\ln a-2\epsilon\ln y-2\epsilon\ln(1\text{-}y)+O(\epsilon^2)\,] \tag{229}$$

$$= \frac{1}{\epsilon}\,[\,1+\epsilon-2\,\epsilon\,\ln a+O(\epsilon^2)\,]$$

With this result, the integral I(a,b,c) may be written as

$$I(a,b,c) = -\frac{\mu^{4\epsilon}}{(1\text{-}2\epsilon)}\,\frac{1}{(4\pi)^{4-2\epsilon}}\,\frac{\Gamma(2\epsilon)}{\epsilon}\,\{\,(1+\epsilon)(a+b+c)-$$

$$2\epsilon[\,a\ln a+b\ln b+c\ln c\,]+O(\epsilon^2)\,\} \tag{230}$$

With the relations of Appendix A, this finally becomes

$$I(a,b,c) = \frac{-1}{(4\pi)^4} \{ [\frac{1}{2\epsilon^2} + \frac{3}{2\epsilon} + 3 + \frac{\pi^2}{6})] (a+b+c) -$$

$$(2 + \frac{1}{\epsilon}) [a\ln\frac{a}{M} + b\ln\frac{b}{M} + c\ln\frac{c}{M}] + (\frac{1}{2\epsilon})O(\epsilon) \} \quad (231)$$

where

$$M = 4\pi\mu^2 e^{-\gamma_e} \quad (232)$$

This last term implies that the integrals over Feynman parameters should have been carried at order ϵ^2 so that the finite part of I(a,b,c) be fully determined. The problem is that this calculation is quite involved and that the result cannot be put in closed form [5] [6] [7] . In the three references just quoted, three similar types of approximation were used for the finite part of I(a,b,c). Each of these reproduces the correct limiting cases, such as $a = b = c$, where the finite part may be found exactly. Choosing the method of Mahanthappa and Sher [7], one obtains for I(a,b,c) a form which the authors claim to be numerically good within a 10% margin of error.

$$I(a,b,c) = \frac{-1}{(4\pi)^4} \{ [(\frac{1}{2\epsilon^2}) + (\frac{3}{2\epsilon}) + 3 + \frac{\pi^2}{6})] (a+b+c) -$$

$$(3 + \frac{1}{\epsilon}) [a\ln\frac{a}{M} + b\ln\frac{b}{M} + c\ln\frac{c}{M}] + 3 [a\ln^2\frac{a}{M} +$$

$$b\ln^2\frac{b}{M} + c\ln^2\frac{c}{M}] + O(\epsilon) \} \quad (233)$$

If this calculation were to be pursued up to three-loop order, the remainder of equation (233) would have to be put down under one form or another in order to cancel the new infinities which would arise.

Like in the case of the one-loop integrals, a, b and c may become negative and give rise to an imaginary part. This problem will now be discussed.

5.3.4 Negative Mass Terms in Feynman Integrals

Mathematically, the fact of having a negative mass term in a Feynman integral presents very little difficulty. Let's consider the simplest case. In Minkowski space, we have

$$I = \int d^4p \frac{1}{(p^2 - m^2 - i\alpha)} \tag{234}$$

In the complex p^0-plane, there are two possible poles, one along each of the curves drawn in Figure 5 on page 80 and of value $(\pm(\sqrt{p^2 - m^2 - i\alpha}))$. If these poles are real, the contour of integration is the one given in Figure 6 on page 80 and one may replace

$$\int_{-\infty}^{\infty} dp^0 \rightarrow - \int_{-i\infty}^{i\infty} dp^0 \tag{235}$$

This corresponds to a Wick rotation and is equivalent to going from a Minkowski space to an Euclidian space. For the case where the poles are purely imaginary, it is still possible to make a Wick-type rotation in the complex plane but the contour is slightly different as shown in Figure 7 on page 81. The way the contour is drawn allows us to perform the rotation without crossing any singularities. However, the integration over the two extra half-circles will result in an imaginary part.

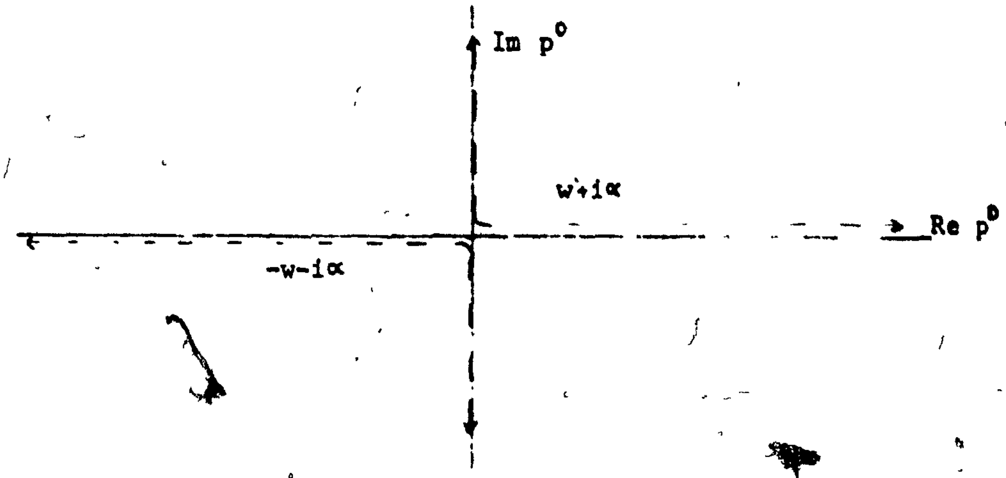


Figure 5: Poles in the complex p^0-plane

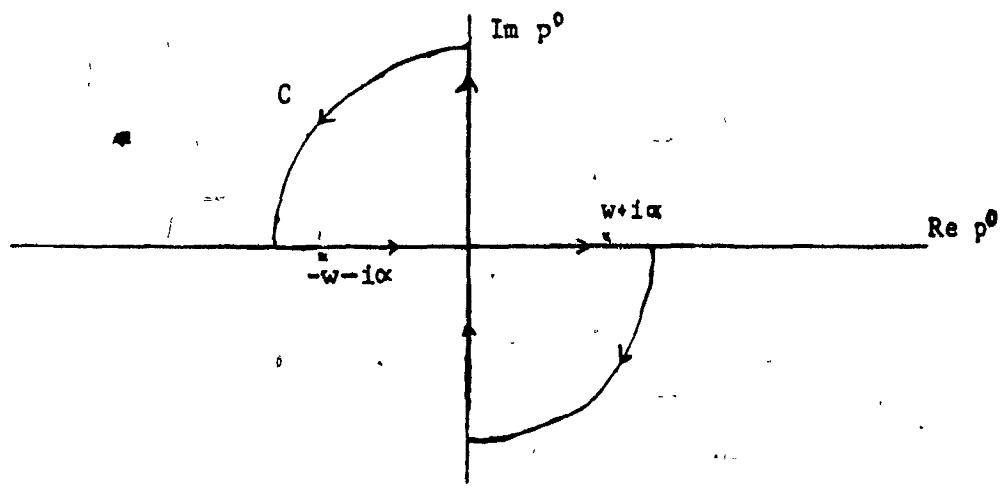


Figure 6: Contour of integration for the case where the poles are real.

Carrying out all the calculations in details, one arrives at the conclusion that the integrals tabulated in Appendix A´ are valid even with a negative argument, the real and imaginary part being automatically and correctly included.

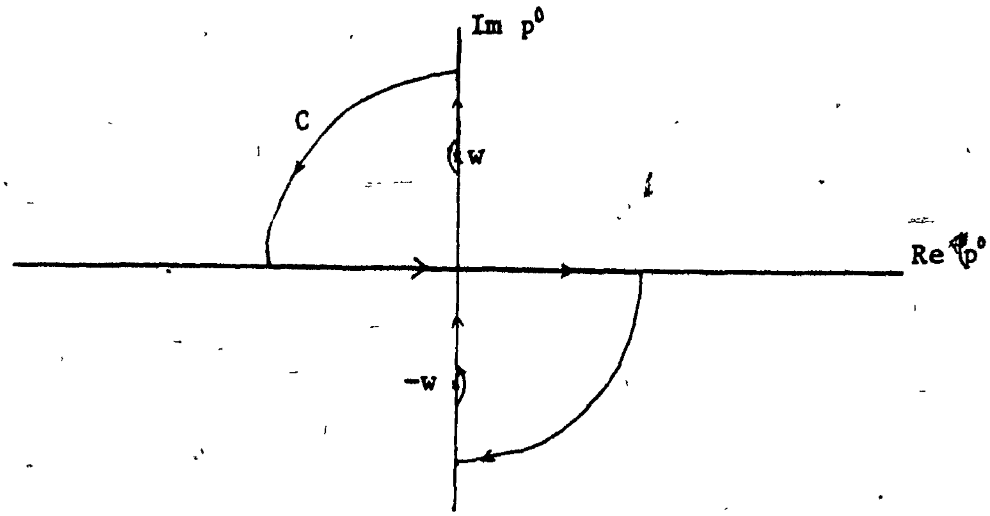


Figure 7: Contour of integration for the case where the poles are imaginary

The term $i\alpha$ ($\alpha << 1$) which is usually dropped once in Euclidian space should be kept in the case of a negative argument in order to determine the sign of the imaginary part as Log(-1) could be either plus or minus one: even though $i\alpha$ was not explicitly kept in expressions fot I(a,b,c) and J(a,b), it should be assumed that it is understood. This is very important from the physical point of view because the imaginary part of the effective potential is interpreted as being proportional to the decay probability per unit time per unit volume of the system [8]. Coleman and Weinberg [9] also studied this phenomenon in their work on effective potential and concluded that the vacuum becomes kinematically unstable because of the presence of a negative mass term. Thus, the system starts to decay.

In such a case, one must replace U by Re(U) in the relations which define physical quantities such as the set of equations (32) to (34) because the Lagrangian must be real. Moreover, if infinities arise in the imaginary part, they will have to cancel each other as the renormalization constants must also be real since they appear in the renormalized Lagrangian.

5.4 Analytical Form of the Effective Potential

This section will consist of only the one and two loop effective potential written down as an expansion in the parameter ϵ. The limit ϵ goes to zero is understood and left aside until the theory is renormalized.

So, using equation (219) along with the results of Appendix A, the one-loop effective potential given by equation (187) takes the form

$$U_1 = -\frac{1}{(64\pi^2)}\{\ [\ (\frac{1}{\epsilon})(8x^2) + 12x^2 - 2x^2\ln|\frac{x}{M}| + \frac{c^2}{4}\ln|\frac{c}{2M}| - w_+^2\ln|\frac{w_+}{M}|$$

$$- w_-^2\ln|\frac{w_-}{M}| + O(\epsilon)\ ] \ + i\pi\ [\ x^2 + w_-^2\ ]\ \} \tag{236}$$

where the dependence on the coupling constant e is hidden in the different parameters x, c, w_+ *and* w_- defined previously.

For the two-loop effective potential, one uses equation (221) and equation (231) in the expressions U_a, U_d, U_e and U_f given in equations (203) through (206) and obtains

$$U_2 = (\frac{e}{16\pi^2})^{2^2}\ \{\ [\ (\frac{1}{\epsilon^2})\ (\frac{33x^2}{8}) + (\frac{1}{\epsilon})\ (\frac{93x^2}{8}) - (\frac{1}{\epsilon})\ [\ (\frac{-3c^2}{16})\ \ln|\frac{c}{2M}|$$

$$+ (\frac{9x^2}{4})\ \ln|\frac{x}{M}| + (\frac{3c^2}{16} + 3x^2 + \frac{3c^3}{64\gamma} + \frac{21cx^2}{16\gamma})\ \ln|\frac{w_+}{M}| +$$

$$(\frac{3c^2}{16} + 3x^2 - \frac{3c^3}{64\gamma} - \frac{21cx^2}{16\gamma})\ \ln|\frac{w_-}{M}|\ ]\ - (\frac{11}{32} + \frac{3x^2}{2})\ \ln|\frac{c}{2M}| -$$

$$\frac{15x^2}{2}\ \ln|\frac{x}{M}| - (\frac{11c^2}{64} + \frac{51x^2}{4} + \frac{11c^3}{256\gamma} + \frac{51cx^2}{16\gamma})\ \ln|\frac{w_+}{M}| -$$

$$(\frac{11c^2}{64} + \frac{51x^2}{4} - \frac{11c^3}{256\gamma} - \frac{51cx^2}{16\gamma})\ \ln|\frac{w_-}{M}| - (\frac{15c^2}{64} - \frac{21x^2}{8})\ \ln^2|\frac{c}{2M}| +$$

$$8x^2 \ln^2|\frac{x}{M}| + (\frac{c^2}{128} + \frac{137x^2}{8} + \frac{c^3}{512\gamma} + \frac{11cx^2}{2\gamma} - \frac{5cx^2w_+}{16\gamma^2}) \ln^2|\frac{w_+}{M}| +$$

$$(\frac{c^2}{128} + \frac{137x^2}{8} - \frac{c^3}{512\gamma} - \frac{11cx^2}{2\gamma} - \frac{5cx^2w_-}{16\gamma^2}) \ln^2|\frac{w_-}{M}| + (\frac{7c^2}{64} -$$

$$\frac{5x^2}{8} + \frac{7c^3}{256\gamma} + \frac{cx^2}{2\gamma}) \ln|\frac{c}{2M}| \ln|\frac{w_+}{M}| - (\frac{7c^2}{64} - \frac{5x^2}{8} - \frac{7c^3}{256\gamma} - \frac{cx^2}{2\gamma}) \ln|\frac{c}{2M}| \ln|\frac{w_-}{M}|$$

$$+ x^2 \ln|\frac{x}{M}| \ln|\frac{c}{2}M| + \frac{15abx^2}{8\gamma^2} \ln|\frac{w_+}{M}| \ln|\frac{w_-}{M}|]$$

$$-i\pi [(\frac{1}{\epsilon})(\frac{-3c^2}{32} - \frac{33x^2}{8} + \frac{3c^3}{128\gamma} + \frac{21cx^2}{16\gamma}) + 8x^2 \ln|\frac{x}{M}| +$$

$$(\frac{7c^2}{64} + \frac{3x^2}{8} - \frac{7c^3}{256\gamma} - \frac{cx^2}{2\gamma}) \ln|\frac{c}{2M}| + \frac{15abx^2}{8\gamma^2} \ln|\frac{w_+}{M}|$$

$$+ (\frac{c^2}{64} + \frac{137x^2}{4} - \frac{c^3}{256\gamma} - \frac{11cx^2}{\gamma}) \ln|\frac{w_-}{M}| - (\frac{11c^2}{64} + \frac{33x^2}{8} -$$

$$\frac{11c^3}{256\gamma} - \frac{51cx^2}{16\gamma})]$$

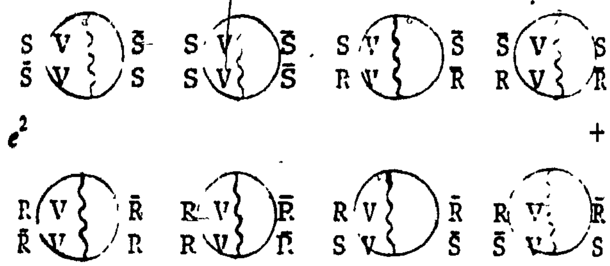
$$- \pi^2 [\frac{c^2}{128} + \frac{153x^2}{8} - \frac{c^3}{512\gamma} - \frac{11cx^2}{2\gamma} - \frac{5cx^2w_-}{16\gamma^2}] + O(\epsilon) \} \tag{237}$$

One may verify that the expression found for U_2 respects at each order in ϵ the SUSY boundary condition :

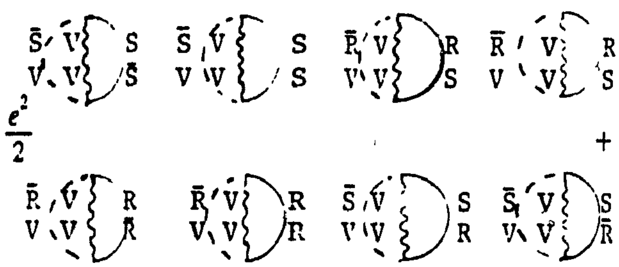
$$U_2(c,x=0) = 0 \tag{238}$$

The same is true for U_1 but this is no surprise as the boundary condition was used to derive it. It is recalled that the finite part of U_2 at order ϵ^0 is only an approximation as explained in section 5.3.3.

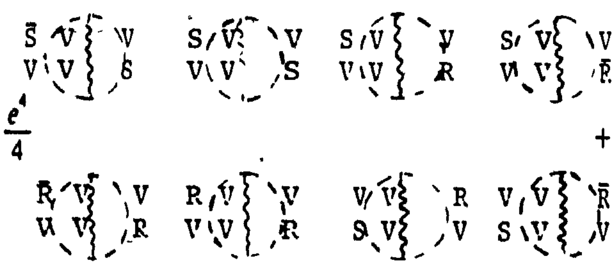
These expressions have now to be renormalized in order to obtain a finite (and final) form of the effective potential up to two-loop order. This will be done in the next chapter.



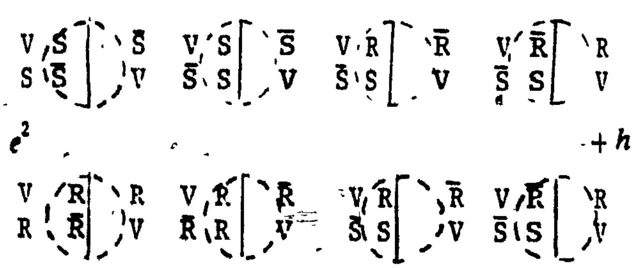
a) e^2 + h.c.



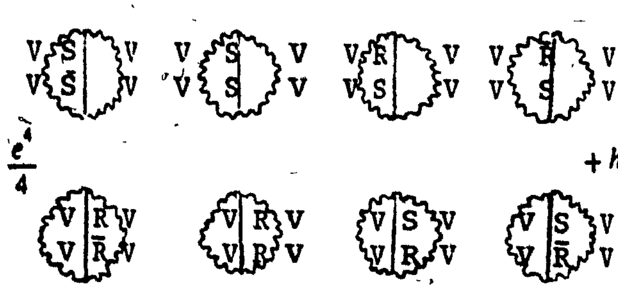
b) $\frac{e^2}{2}$ + h.c.



c) $\frac{e^4}{4}$ + h.c.



d) e^2 + h.c.



e) $\frac{e^4}{4}$ + h.c.

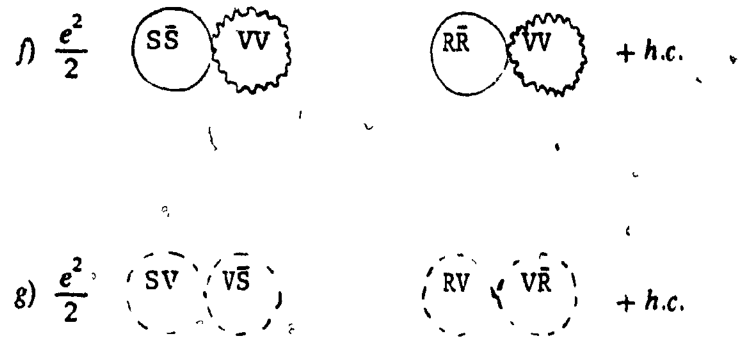


Figure 8; Two-Loop Vacuum Bubbles for SQED

5.5 References

[1] R.D.C.Miller,Nucl.Phys.B229(1983)189

[2] G.'t Hooft, M.Veltman, Nucl.Phys.B44(1972)189

[3] B.De Witt, J.Smith, "Field Theory in Particle Physics", North-Holland Personal Library, Netherland, 1986, p.305 P.Ramond, "Field Theory, a Modern Primer", Benjamin-Cummings Publ.Co., 1981, p.148

[4] P.Ramond, "FieldTheory, a Modern Primer", Benjamin-Cummings Publ. Co., 1981, p160

[5] G.Fogleman, K.Viswanathan, Phys.Rev.D301984)1364

[6] R.D.C.Miller, Nucl Phys.241B1984)535

[7] K.T.Mahanthappa, M.A.Sher, Phys.Rev.D22(1980)1711

[8] S.Coleman, "Laws of Hadronic Matter",ed. by Zichichi, Academic Press, 1973, p.169

[9] S.Coleman, E.Weinberg, Phys.Rev.D7(1973)1888

Chapter VI

RENORMALIZATION

The infinities arising in the expressions for the one and two loop order effective potential indicate the need of an appropriate renormalization scheme. The simplest way this may be done is to introduce renormalization constants in the original Lagrangian and reformulates Feynman rules in terms of the renormalized parameters. This way, counterterms are automatically included and divergences will be removed within a minimal subtraction scheme.

Once a finite expression for the effective potential is found, it will be possible to derive the renormalized coupling constant and the β-function for SUSY QED.

6.1 Renormalization scheme

Looking back at the original SUSY QED Lagrangian, one realizes that three different renormalization constants need to be introduced : one for the vector field, one for the chiral fields and one for the coupling constant.

$$Z_v = 1 + \hbar\, Z_v^{(1)} + \hbar^2\, Z_v^{(2)} + \ldots \tag{239}$$

$$Z_c = 1 + \hbar\, Z_c^{(1)} + \ldots \tag{240}$$

$$Z_e = 1 + \hbar\, Z_e^{(1)} + \ldots \tag{241}$$

Because of the structure of the effective potential at tree-level, only $Z_v^{(2)}$ will be needed to make the theory finite at two-loop order. This will become evident later.

From the renormalized Lagrangian, one may calculate the new superpropagators, change the Feynman rules and calculate U all over again. But once the two first steps are done, one realizes that the expressions found up to now are valid if the following definitions are used instead of the old ones.

$$c = e^2 (\bar{s}s + \bar{r}r)\, Z_e\, Z_c = 4e^2\, Z_e\, Z_c\, \sigma \tag{242}$$

$$x = \frac{ed}{2} Z_e^{\frac{1}{2}} Z_v^{\frac{1}{2}} = \frac{e^2}{2} Z_e^{\frac{1}{2}} Z_v^{\frac{1}{2}} \eta \tag{243}$$

From these two equations, one may deduce that w_+ and w_- have now the following expansion in $\hbar$:

$$w_+ = e^2(\sigma + G) \{ 1 + \hbar Z_c^{(1)} \frac{[\, 2\sigma^2 + \eta^2 + \frac{2\sigma^3}{\gamma} + \frac{\sigma\eta^2}{\gamma}\,]}{(\sigma+\gamma)^2} + \hbar\, Z_v^{(1)} \frac{\eta^2(1+\frac{\sigma}{\gamma})}{(\sigma+\gamma)^2} +$$
$$\hbar\, Z_e^{(1)} \frac{2\sigma^2 + 2\eta^2 + \frac{2\sigma^3}{\gamma} + \frac{5\sigma\eta^2}{\gamma}}{(\sigma+\gamma)^2} \} \tag{244}$$

$$w_- = e^2(\sigma - \gamma) \{ 1 + \hbar\, Z_c^{(1)} \frac{[\, 2\sigma^2 + \eta^2 - \frac{2\sigma^3}{\gamma} - \frac{\sigma\eta^2}{\gamma}\,]}{(\sigma-\gamma)^2} + \hbar\, Z_v^{(1)} \frac{[\, \eta^2(1-\frac{\sigma}{\gamma})\,]}{(\sigma-\gamma)^2}$$
$$\hbar\, Z_e^{(1)} \frac{[\, 2\sigma^2 + 2\eta^2 - \frac{2\sigma^3}{\gamma} - \frac{5\sigma\eta^2}{\gamma}\,]}{(\sigma-\gamma)^2} \} \tag{245}$$

With these seven equations, one can now work out the $\hbar$-expansion of each loop order of the effective potential. The task is tedious but straightforward. The result is

$$U_0^{(0)} = \frac{d^2}{2} \tag{246}$$

$$U_0^{(1)} = Z_\nu^{(1)} \frac{d^2}{2} \tag{247}$$

$$U_0^{(2)} = Z_\nu^{(2)} \frac{d^2}{2} \tag{248}$$

$$U_2^{(2)} = U_2 \tag{249}$$

with U_2 given in equation (237) and

$$U_1^{(1)} = -\hbar\left(\frac{e^2}{16\pi}\right)\left\{ \left(\frac{1}{\epsilon}\right)(2\eta^2) + 3\eta^2 - \frac{\eta^2}{2}\ln\left|\frac{\eta}{2M}\right| + 4\sigma^2\ln\left|\frac{2\sigma}{M}\right| - (\sigma-\gamma)^2 \right.$$
$$\left. \ln\left|\frac{\sigma-\gamma}{M}\right| - (\sigma+\gamma)^2\ln\left|\frac{\sigma+\gamma}{M}\right| + i\pi\left[\frac{\eta^2}{4} + (\sigma-\gamma)^2\right]\right\} \tag{250}$$

$$U_1^{(2)} = -\hbar^2\left(\frac{e^2}{16\pi}\right)^2 \frac{1}{4}\left\{ \left(\frac{1}{\epsilon} + \frac{3}{2}\right)\left[\left(\frac{3\eta^2}{2}\right) Z_\nu^{(1)} + \eta^2 Z_c^{(1)} + \left(\frac{5\eta^2}{2}\right) Z_e^{(1)}\right] + \right.$$
$$\left(\frac{\pi^2}{6}\right)\left[\left(\frac{3\eta^2}{4}\right)Z_\nu^{(1)} + \left(\frac{\eta^2}{2}\right)Z_c^{(1)} + \left(\frac{5\eta^2}{4}\right)Z_e^{(1)}\right] - Z_\nu^{(1)}\left[\frac{\eta^2}{2}\ln\left|\frac{\eta}{2M}\right| + \frac{\eta^2}{2}\right.$$
$$\left(1+\frac{\sigma}{\gamma}\right)\ln\left|\frac{\sigma+\gamma}{M}\right| + \frac{\eta^2}{2}\left(1-\frac{\sigma}{\gamma}\right)\ln\left|\frac{\sigma-\gamma}{M}\right|\Big] - Z_c^{(1)}\Big[-8\sigma^2\ln\left|\frac{2\sigma}{M}\right| +$$
$$\left(4\sigma^2 + \frac{\eta^2}{2} - \frac{4\sigma^3}{\gamma} - \frac{2\sigma\eta^2}{\gamma}\right)\ln\left|\frac{\sigma-\gamma}{M}\right| + \left(4\sigma^2 + \frac{\eta^2}{2} + \right.$$
$$\left.\frac{4\sigma^3}{\gamma} + \frac{2\sigma\eta^2}{\gamma}\right)\ln\left|\frac{\sigma+\gamma}{M}\right|\Big] - Z_e^{(1)}\left[\left(\frac{\eta^2}{2}\right)\ln\left|\frac{\eta}{2M}\right| - 8\sigma^2\ln\left|\frac{2\sigma}{M}\right|\right.$$
$$+\left(4\sigma^2 + \eta^2 - \frac{4\sigma^3}{\gamma} - \frac{5\sigma\eta^2}{2\gamma}\right)\ln\left|\frac{\sigma-\gamma}{M}\right| + \left(4\sigma^2 + \eta^2 + \right.$$
$$\left.\frac{4\sigma^3}{\gamma} + \frac{5\sigma\eta^2}{2\gamma}\right)\ln\left|\frac{\sigma+\gamma}{M}\right|\Big] + \epsilon\, Z_\nu^{(1)}\left[\left(\frac{\eta^2}{4}\right)\ln^2\left|\frac{\eta}{2M}\right| - \left(\frac{\eta^2}{2}\right)\ln\left|\frac{\eta}{2M}\right| + \right.$$
$$\left(\frac{\eta^2}{4}\right)\left(1-\frac{\sigma}{\gamma}\right)\left(\ln^2\left|\frac{\sigma-\gamma}{M}\right| - 2\ln\left|\frac{\sigma-\gamma}{M}\right|\right) + \left(\frac{\eta^2}{4}\right)\left(1+\frac{\sigma}{\gamma}\right)$$

$$(\ln^2|\frac{\sigma+\gamma}{M}| - 2\ln|\frac{\sigma+\gamma}{M}|)] + \epsilon Z_c^{(1)} [-4\sigma^2 (\ln^2|\frac{2\sigma}{M}| - \ln|\frac{2\sigma}{M}|) +$$

$$(2\sigma^2 + \frac{\eta^2}{4} - \frac{2\sigma^3}{\gamma} - \frac{\sigma\eta^2}{\gamma}) (\ln^2|\frac{\sigma-\gamma}{M}| - 2\ln|\frac{\sigma-\gamma}{M}|) +$$

$$(2\sigma^2 + \frac{\eta^2}{4} + \frac{2\sigma^3}{\gamma} + \frac{\sigma\eta^2}{\gamma}) (\ln^2|\frac{\sigma+\gamma}{M}| - 2\ln|\frac{\sigma+\gamma}{M}|)]$$

$$+ \epsilon Z_e^{(1)} [(\frac{\eta^2}{4}) (\ln^2|\frac{\eta}{2M}| - 2\ln|\frac{\eta}{2M}|) - 4\sigma^2 (\ln^2|\frac{2\sigma}{M}| - 2\ln|\frac{2\sigma}{M}|) +$$

$$(2\sigma^2 + \frac{\eta^2}{2} - \frac{2\sigma^3}{\gamma} - \frac{5\sigma\eta^2}{4})(\ln^2|\frac{\sigma-\gamma}{M}| - 2\ln|\frac{\sigma-\gamma}{M}|) + (2\sigma^2 + \frac{\eta^2}{2} +$$

$$\frac{2\sigma^3}{\gamma} + \frac{5\sigma\eta^2}{4\gamma}) (\ln^2|\frac{\sigma+\gamma}{M}| - 2\ln|\frac{\sigma+\gamma}{M}|)]$$

$$- i\pi [- Z_\nu^{(1)} (\frac{3\eta^2}{4} - \frac{\sigma\eta^2}{2\gamma}) - Z_c^{(1)} (4\sigma^2 + \frac{\eta^2}{2} - \frac{4\sigma^3}{\gamma} - \frac{2\sigma\eta^2}{\gamma})$$

$$- Z_e^{(1)} (4\sigma^2 + \frac{5\eta^2}{4} - \frac{4\sigma^3}{\gamma} - \frac{5\sigma\eta^2}{4\gamma}) + \epsilon Z_\nu^{(1)} (\eta^2 \ln|\frac{\eta}{2M}| +$$

$$(1 - \frac{\sigma}{\gamma}) \frac{\eta^2}{2} \ln|\frac{\sigma-\gamma}{M}| - \frac{3\eta^2}{4} + \frac{\sigma\eta^2}{2\gamma}) + \epsilon Z_c^{(1)} ((4\sigma^2 + \frac{\eta^2}{2} -$$

$$\frac{4\sigma^3}{\gamma} - \frac{2\sigma\eta^2}{\gamma}) (\ln|\frac{\sigma-\gamma}{M}| - 1)) + \epsilon Z_e^{(1)} (\frac{\eta^2}{4}(\ln|\frac{\eta}{2M}| - 1) +$$

$$(4\sigma^2 + \eta^2 - \frac{4\sigma^3}{\gamma} - \frac{5\sigma\eta^2}{2\gamma}) (\ln|\frac{\sigma-\gamma}{M}| - 1))]$$

$$- \pi^2 [\epsilon Z_\nu^{(1)} (\frac{\eta^2}{16} - \frac{\sigma\eta^2}{4\gamma}) + \epsilon Z_c^{(1)} (2\sigma^2 + \frac{\eta^2}{4} - \frac{2\sigma^3}{\gamma} -$$

$$\frac{\sigma\eta^2}{\gamma}) + \epsilon Z_e^{(1)} (2\sigma^2 + \frac{\eta^2}{2} - \frac{2\sigma^3}{\gamma} - \frac{5\sigma\eta^2}{4\gamma})] \} \qquad (251)$$

The extra e^2 in the logarithmic terms have been absorbed in the μ^2 term understood in M.

The renormalization constants will serve to eliminate the divergent terms as well as real constant terms in a modified minimal subtraction scheme. This way, the higher order corrections to the potential are expressed as functions of logarithms.

Equating terms of same order in $\hbar$, one first gets

$$\frac{\eta^2}{2} Z_v^{(1)} - \left(\frac{e^2}{64\pi^2}\right)\left(2\frac{\eta^2}{\epsilon} + 3\eta^2\right) = 0 \tag{252}$$

which means that

$$Z_v^{(1)} = \left(\frac{e^2}{16\pi^2}\right)\left(\frac{1}{\epsilon} + \frac{3}{2}\right) \tag{253}$$

As pointed out at the beginning of this section, only $Z_v^{(1)}$ is determined at first order in $\hbar$. Both $Z_c^{(1)}$ and $Z_e^{(1)}$ are determined at second order in $\hbar$ along with $Z_v^{(2)}$. This is due to the structure of the tree-level potential which depends only on d but not on the chiral fields nor the coupling constant. This way $Z_v^{(n)}$ always appears at one order lower in $\hbar$ than the two other renormalization constants.

At next order in $\hbar$, the terms in $\frac{1}{\epsilon}\ln(\ldots)$ must first be solved to obtain $Z_c^{(1)}$ and $Z_e^{(1)}$. Then, these values for the first order renormalization constants will be used to determine $Z_v^{(2)}$.

Gathering all the terms in $\frac{1}{\epsilon}\ln(\ldots)$ from $U_1^{(2)}$ and $U_2^{(2)}$, one gets five different relations between the Z's of which only two are linearly independent,

$$\begin{aligned} 2Z_c^{(1)} + 2Z_e^{(1)} &= \left(\frac{e^2}{16\pi^2}\right)\frac{3}{\epsilon} \\ 2Z_v^{(1)} + 2Z_e^{(1)} &= \left(\frac{e^2}{16\pi^2}\right)\frac{9}{\epsilon} \end{aligned} \tag{254}$$

The values of $Z_c^{(1)}$ and $Z_e^{(1)}$ are readily obtained.

$$Z_c^{(1)} = \left(\frac{e^2}{16\pi^2}\right)\left(\frac{-2}{\epsilon}\right) \qquad (255)$$

$$Z_e^{(1)} = \left(\frac{e^2}{16\pi^2}\right)\left(\frac{7}{2}\epsilon\right) \qquad (256)$$

Now that all of the first order constants are determined, their value may be used to get $Z_v^{(2)}$ which appears in terms of order $\frac{1}{\epsilon^2}$, $\frac{1}{\epsilon}$ and ϵ^0. The calculation is rather simple and gives

$$Z_v^{(2)} = \left(\frac{e}{4\pi}\right)^4 \left[\frac{35}{16\epsilon^2} - \frac{21}{8\epsilon} - \frac{3}{8}\left(\frac{5\pi^2}{6} + \frac{87}{2}\right)\right] \qquad (257)$$

At second order in $\hbar$, there is only one equation to determine the constant parts of $Z_v^{(2)}$, $Z_c^{(1)}$ and $Z_e^{(1)}$. Hence, for the sake of simplicity, the whole constant part has been put into $Z_v^{(2)}$. This minimizes the number of changes to be done to the logarithmic part of the effective potential. To resume, let's write down the complete expressions for the different renormalization constants.

$$Z_c = 1 + \hbar\left(\frac{e}{4\pi}\right)^2\left(\frac{-2}{\epsilon}\right) \qquad (258)$$

$$Z_e = 1 + \hbar\left(\frac{e}{4\pi}\right)^2\left(\frac{7}{2\epsilon}\right) \qquad (259)$$

$$Z_v = 1 + \hbar\left(\frac{e}{4\pi}\right)^2\left(\frac{1}{\epsilon} + \frac{3}{2}\right) + \hbar^2\left(\frac{e}{4\pi}\right)^4\left[\frac{35}{16\epsilon^2} - \frac{21}{8\epsilon} - \frac{3}{8}\left(\frac{5\pi^2}{6} + \frac{87}{2}\right)\right] \qquad (260)$$

With these expressions, the effective potential is cured from all its divergences up to $\hbar^2$ order, including the ones in $\frac{1}{\epsilon}\,ln(...)$ as well as the imaginary ones :

$$(\frac{e^2}{16\pi})\, i\, \{ \frac{-3\sigma^2}{2} - \frac{33\eta^2}{32} + \frac{3\sigma^2}{2\gamma} + \frac{21\sigma\eta^2}{16\gamma} \} =$$

$$(\frac{i}{4})\, \{ \sigma^2 [\, 4Z_c^{(1)} + 4Z_e^{(1)}] + \eta^2 [\, \frac{3}{4} Z_v^{(1)} + \frac{1}{2} Z_c^{(1)} + \frac{5}{4} Z_e^{(1)} e\,] +$$

$$\sigma^3 [\, 4Z_c^{(1)} + 4Z_e^{(1)}] + \frac{\sigma\eta^2}{\gamma} [\, - \frac{1}{2} Z_v^{(1)} - 2Z_c^{(1)} - \frac{5}{4} Z_e^{(1)}] \} \tag{261}$$

One may verify that, with the values found for the different $Z^{(1)}$'s, the above equality holds true. At this point, the limit ϵ going to zero may safely be taken. The final expression for the renormalized effective potential up to two-loop order is

$$U = \frac{e^2\eta^2}{2} + \hbar\, e^2 (\frac{e}{4\pi})^2 \{ \frac{\eta^2}{8} \ln|\frac{\eta}{2M}| - \sigma^2 \ln|\frac{2\sigma}{M}| + \frac{1}{4} (\sigma + \gamma)^2 \ln|\frac{\sigma + \gamma}{M}|$$

$$+ \frac{1}{4} (\sigma - \gamma)^2 \ln|\frac{\sigma - \gamma}{M}| - i\, [\, \frac{1}{4} \eta^2 + (\sigma - \gamma)^2]\ \}$$

$$+ \hbar^2 e^2 (\frac{e}{4\pi})^4 \{ (\frac{5}{2} \sigma^2 + \frac{3}{8} \eta^2) \ln|\frac{2\sigma}{M}| - \frac{9}{8} \eta^2 \ln|\frac{\eta}{2M}| -$$

$$(\frac{5}{4} \sigma^2 + \frac{45}{16} \eta^2 + \frac{5}{4} \frac{\sigma^3}{\gamma} + \frac{9}{4} \frac{\sigma\eta^2}{\gamma}) \ln|\frac{\sigma + \gamma}{M}| - (\frac{5}{4} \sigma^2 + \frac{45}{16} \eta^2 -$$

$$\frac{5}{4} \frac{\sigma^3}{\gamma} - \frac{9}{4} \frac{\sigma\eta^2}{\gamma}) \ln|\frac{\sigma - \gamma}{M}| - (\frac{9}{4} \sigma^2 + \frac{21}{32} \eta^2) \ln^2|\frac{2\sigma}{M}| + \frac{55}{32} \eta^2 \ln^2|\frac{\eta}{2M}| -$$

$$[\, \frac{5}{8} \sigma^2 - \frac{125}{32} \eta^2 + \frac{\sigma}{\gamma} (\frac{5}{8} \sigma^2 - \frac{155}{32} \eta^2) + 5\sigma\eta^2 \frac{\sigma + \gamma}{16\gamma^2}] \ln^2|\frac{\sigma + \gamma}{M}| -$$

$$[\, \frac{5}{8} \sigma^2 - \frac{125}{32} \eta^2 - \frac{\sigma}{\gamma} (\frac{5}{8} \sigma^2 - \frac{155}{32} \eta^2) + 5\sigma\eta^2 \frac{\sigma - \gamma}{16\gamma^2}] \ln^2|\frac{\sigma - \gamma}{M}| +$$

$$[\, \frac{7}{4} \sigma^2 - \frac{5}{32} \eta^2 + \frac{\sigma}{\gamma} (\frac{7}{4} \sigma^2 + \frac{1}{2} \eta^2)]\ \ln|\frac{2\sigma}{M}| \ln|\frac{\sigma + \gamma}{M}| +$$

$$[\, \frac{7}{4} \sigma^2 - \frac{5}{32} \eta^2 - \frac{\sigma}{\gamma} (\frac{7}{4} \sigma^2 + \frac{1}{2} \eta^2)]\ \ln|\frac{2\sigma}{M}| \ln|\frac{\sigma - \gamma}{M}| +$$

$$\frac{1}{4} \eta^2 \ln|\frac{\eta}{2M}| \ln|\frac{2\sigma}{M}| + \frac{15}{32} (\sigma^2 - \frac{1}{4} \eta^2)(\frac{\eta}{\gamma})^2 \ln|\frac{\sigma + \gamma}{M}| \ln|\frac{\sigma - \gamma}{M}|$$

$$- i\pi \, [\, \frac{55}{32} \eta^2 \ln|\frac{\eta}{2M}| + \frac{15}{32} \frac{ab\eta^2}{\gamma^2} \ln|\frac{\sigma+\gamma}{M}| - (\frac{5}{8} \sigma^2 - \frac{125}{32} \eta^2 -$$

$$\frac{5}{8} \frac{\sigma^3}{\gamma} + \frac{145}{32} \frac{\sigma\eta^2}{\gamma}) \ln|\frac{\sigma-\gamma}{M}| + (\frac{7}{4} \sigma^2 + \frac{3}{32} \eta^2 - \frac{7}{4} \frac{\sigma^3}{\gamma} - \frac{1}{2} \frac{\sigma\eta^2}{\gamma}) \ln|\frac{2\sigma}{M}|$$

$$- \frac{5}{4} \sigma^2 - \frac{9}{4} \eta^2 + \frac{5}{4} \frac{\sigma^3}{\gamma} + \frac{9}{4} \frac{\sigma\eta^2}{\gamma}) \,]$$

$$- \pi^2 \, [\, - \frac{5\sigma^2}{8} + \frac{555\eta^2}{128} + \frac{5\sigma^3}{8\gamma} - \frac{145\sigma\eta^2}{32\gamma} + \frac{5\sigma^2\eta^2}{16\gamma^2} \,] \, \} \qquad . \qquad (262)$$

It should be noticed that the constant part of U coming from the square of the imaginary part has not been removed during renormalization as for the rest of the constants. The reason is that only terms in η^2 could be absorbed in the renormalization constants because of the structure of the terms involved. Moreover, this part of the potential originates from the $\ln^2$ terms which are just an approximation. Hence, it is preferable to keep it away from the expressions for the renormalization constants which are exact.

Graphs of the effective potential as a function of η for different σ are given in Appendix B. It may be seen that for small σ , all the contributions to the potential have the form of parabolas and that the sum of them all is positive definite as should be for a supersymmetric potential. For higher values of σ , the different contributions develop secondary extrema but the sum is still a positive definite parabola with its minimum at $\eta = 0$. This is due to the fact that the tree-level contribution dominates over the loop corrections by an appreciable factor.

The first contribution to the imaginary part of U comes from the term of order $\hbar$ in the perturbation expansion. It has much the form of a parabola. The maximum value of U_{im} is attained for $\eta = \sigma$ ($U_{im}^{(1)}(\sigma{=}\eta,\eta) \approx \frac{\hbar e^4}{16\pi^2} (6\eta^2)$). If the value of the finestructure constant is used for the coupling constant, it is easy to see that the imaginary part of the potential is quite small compared to the real part (about two orders of magnitude) ; this is quite normal as the imaginary part of U is due solely to radiative corrections. Thus, even if there is some instability away from the minimum, the system may still exist in that state for some time.

6.2 Running Coupling Constant

In this section, it will be shown that the supersymmetric extension of quantum electrodynamics displays the same long range behaviour as QED by determining its β-function. To do so, the renormalized coupling constant will be derived from the effective potential and from it, the β-function and the running coupling constant.

The tree-level potential U_0 may be expressed in terms of η with the help of equation (243). It gives

$$U_0^{(0)} = \frac{e^2 \eta^2}{2} \tag{263}$$

Thus, as explained in Chapter II, one may define the renormalized coupling constant as the second derivative of the effective potential with respect to η at the minimum of the potential ($\eta = 0$). However, there will be logarithmic singularities at this point. Hence, the renormalized coupling constant must be defined away from it [1] at some point "m". Along the σ-axis, any point may be chosen as long as it is greater or equal to the one chosen for η so that equation (209) be respected. Thus, the definition for the renormalized coupling constant is

$$e_r^2 = \mathrm{Re}\left[\frac{\partial^2}{\partial\eta^2} U \right]_{\substack{\eta=m \\ \sigma=n\geq m}}$$

$$= e^2 \{ 1 + \hbar(\frac{e}{4\pi})^2 [\frac{3}{4} + \frac{9m^2}{16\Psi^2} + \frac{1}{4} \ln|\frac{m}{2M}| + \frac{3}{8} (1 - \frac{n^3}{\Psi^3}) \ln|\frac{n+\Psi}{M}| +$$

$$\frac{3}{8} (1 + \frac{n^3}{\Psi^3}) \ln|\frac{n-\Psi}{M}|] + \hbar^2(\frac{e}{4\pi})^4 [\frac{685}{16} - \frac{30n}{\Psi} - \frac{34n^2}{\Psi^2} - \frac{213m^2}{32\Psi^2} + \frac{51n^3}{4\Psi^3} +$$

$$\frac{39nm^2}{4\Psi^3} + \frac{51n^4}{8\Psi^4} + \frac{333n^2m^2}{64\Psi^4} - \frac{99m^4}{256\Psi^4} + \frac{93}{16} \ln|\frac{m}{2M}| + \frac{55}{16} \ln^2|\frac{m}{2M}| -$$

$$\frac{21}{16} \ln^2|\frac{2n}{M}| + (\frac{-3}{8} - \frac{85n^2}{16\Psi^2} + \frac{15m^2}{64\Psi^2} + \frac{63n^4}{8\Psi^4} + \frac{9n^2m^2}{4\Psi^4}) \ln|\frac{2n}{M}| +$$

$$(\frac{125}{16} - \frac{5n^2}{2\Psi^2} + \frac{75n^2m^2}{32\Psi^4} - \frac{45n^2m^4}{32\Psi^6}) (\ln^2|\frac{n+\Psi}{M}| + \ln^2|\frac{n-\Psi}{M}|) +$$

$$\frac{n}{\Psi} (\frac{145}{16} - \frac{1725m^2}{128\Psi^2} + \frac{15n^2}{32\Psi^2} - \frac{135n^2m^2}{128\Psi^4} + \frac{4185m^4}{512\Psi^4}) (\ln^2|\frac{n+\Psi}{M}| - \ln^2|\frac{n-\Psi}{M}|) +$$

$$(\frac{-9}{8} + \frac{93n^2}{16\Psi^2} + \frac{3075m^2}{128\Psi^2} - \frac{27n^4}{8\Psi^4} + \frac{2239n^2m^2}{256\Psi^4} + \frac{87m^4}{32\Psi^4})(\ln|\frac{n+\Psi}{M}| + \ln|\frac{n-\Psi}{M}|) +$$

$$\frac{n}{\Psi} (\frac{57n^2}{16\Psi^2} - \frac{737m^2}{256\Psi^2} + \frac{471n^4}{32\Psi^4} + \frac{177n^2m^2}{128\Psi^4} - \frac{207m^4}{64\Psi^4})(\ln|\frac{n+\Psi}{M}| - \ln|\frac{n-\Psi}{M}|) +$$

$$\frac{1}{2} \ln|\frac{2n}{M}| \ln|\frac{m}{2M}| + \frac{15}{32} (\frac{2n^2}{\Psi^2} - \frac{3m^2}{\Psi^2} + \frac{27m^4}{8\Psi^4} - \frac{15n^2m^2}{2\Psi^4} + \frac{9n^2m^4}{2\Psi^6} - \frac{9m^6}{8\Psi^6})$$

$$\ln|\frac{n+\Psi}{M}| \ln|\frac{n-\Psi}{M}| - \frac{5}{16} \ln|\frac{2n}{M}|(\ln|\frac{n+\Psi}{M}| + \ln|\frac{n-\Psi}{M}|) +$$

$$\frac{n}{\Psi} (1 - \frac{15m^2}{16\Psi^2} - \frac{21n^2}{16\Psi^2} + \frac{189n^2m^2}{64\Psi^4} + \frac{27m^4}{32\Psi^4}) \ln|\frac{2n}{M}| (\ln|\frac{n+\Psi}{M}| - \ln|\frac{n-\Psi}{M}|) -$$

$$\pi^2 (\frac{555}{64} - \frac{145n}{16\Psi} + \frac{5n^2}{8\Psi^2} + \frac{2175nm^2}{128\Psi^3} + \frac{15n^3}{32\Psi^3} - \frac{75n^2m^2}{32\Psi^4} -$$

$$\frac{135n^3m^2}{128\Psi^5} - \frac{3915nm^4}{512\Psi^5} + \frac{45n^2m^4}{32\Psi^6})] \} \qquad (264)$$

The constant Ψ corresponds to γ, as defined in Table 3 on page 56, evaluated at $(\sigma = n, \eta = m)$.

$$\Psi = \sqrt{n^2 + \frac{3m^2}{4}} \tag{265}$$

Equation (264) may be used to reexpress the effective potential in terms of the renormalized coupling constant.

$$U = \frac{1}{2} e_r^2 \eta^2 + (\frac{\hbar}{16\pi}) e_r^4 [\, U^{(1)} - \frac{1}{2} \hat{U}^{(1)} \eta^2 \,] +$$

$$(\frac{\hbar}{16\pi})^2 e_r^6 [\, U^{(2)} - 2\, U^{(1)} \hat{U}^{(1)} + (\hat{U}^{(1)})^2 \eta^2 - \frac{1}{2} \hat{U}^{(2)} \eta^2 \,] \tag{266}$$

The hat over U means taking the real part of the second derivative with respect to η evaluated at $\sigma = n$, $\eta = m$ with $n \geq m$. At one-loop order, this yields an expression for the effective potential which is free from the arbitrary constant M.

$$U = 1/2 e_r^2 \eta^2 + (\frac{\hbar}{16\pi}) e_r^4 \{ -3\frac{\eta^2}{8} + \frac{9m^2}{\Psi^2} - (\frac{\eta^2}{8}) \ln|\frac{\eta}{\Psi}| + \frac{\sigma^2}{2} (\ln|\frac{2\sigma}{n+\Psi}| + \ln|\frac{2\sigma}{n-\Psi}|) -$$

$$\frac{3\eta^2}{16} (\ln|\frac{\sigma+\gamma}{n+\Psi}| + \ln|\frac{\sigma-\gamma}{n-\Psi}|) - \frac{\sigma\Psi}{2} \ln|\frac{\sigma+\gamma}{\sigma-\gamma}| - \frac{3\sigma^3\eta^2}{16\gamma^3} \ln|\frac{n+\Psi}{n-\Psi}| \} \tag{267}$$

The arbitrary constant M has been replaced by the arbitrary point (n,m) in the (σ, η)-space. The same exercise could be carried out at second order in $\hbar^2$ but it is most probable that all dependency on M could not be removed because of the approximative nature ot this expression.

To see how the β-function is usually defined, the case of the Φ^4 model will be considered. The analysis will be carried along the lines of reference [2]. If the bare parameters of the theory are expressed in terms of the physical parameters and if the current J is properly renormalized, one may derive an equality for the one-particle-irreducible Green's functions :

$$\Gamma_0^{(n)}(p_1,\dots,p_n : \lambda_0, \mu_0, \epsilon) = Z_\Phi^{-n/2}\,\bar{\Gamma}^{(n)}(p_1,\dots,p_n : \lambda, m, \mu, \epsilon) \tag{268}$$

where the Γ's are finite in the limit ϵ goes to zero. The subscript "0" refers to the bare parameters. Using the fact that only one side of the equality contains μ , one may derive with respect to it and obtain the renormalization group equation :

$$\left[u\frac{\partial}{\partial u} + \mu\frac{\partial\lambda}{\partial\mu}\frac{\partial}{\partial\lambda} + \mu\frac{\partial m}{\partial\mu}\frac{\partial}{\partial m} - \frac{n}{2}u\frac{\partial \ln Z_\Phi}{\partial\mu}\right]\bar{\Gamma}^{(n)} = 0 \tag{269}$$

The coefficients of equation (269) are used to define the β-function and the γ functions.

$$\beta(\lambda, \frac{m}{\mu}, \epsilon) \equiv \mu\frac{\partial\lambda}{\partial\mu} \tag{270}$$

$$\gamma_d(\lambda, \frac{m}{\mu}, \epsilon) \equiv \frac{\mu}{2}\frac{\partial \ln Z_\Phi}{\partial\mu} \tag{271}$$

$$\gamma_m(\lambda, \frac{m}{\mu}, \epsilon) \equiv \frac{\mu}{2}\frac{\partial \ln m^2}{\partial\mu} \tag{272}$$

For the case of SQED, the renormalized quantities are function of two variables, n and m (not to counfond with the mass term of the Φ^4 model). It is easy to see that, for that case, the β-function may be defined as

$$\beta(e_r^2) = \left(m\frac{\partial}{\partial m} + n\frac{\partial}{\partial n}\right)e_r^2$$

$$= D_{op}\, e_r^2 \tag{273}$$

This combination of differential operators yields manageable results as it has the following properties.

$$\begin{aligned}
&D_{op}\, \Psi = \Psi && D_{op} \ln^2|n + \Psi| = 2\ln|n + \Psi| \\
&D_{op}\, n = n && D^{op} \ln^2|n - \Psi| = 2\ln|n - \Psi| \\
&D_{op}\, m = m && D_{op} \ln^2|2n| = 2\ln|2n| \\
&D_{op} \ln|n + \Psi| = 1 && D_{op} \ln^2|\frac{m}{2}| = 2\ln|\frac{m}{2}| \\
&D_{op} \ln|n - \Psi| = 1 && \\
&D_{op} \ln|2n| = 1 && D_{op} (\frac{n}{\Psi})^a = 0 \\
&D_{op} \ln|\frac{m}{2}| = 1 && D_{op} (\frac{m}{\Psi})^a = 0
\end{aligned} \tag{274}$$

With these relations, the β-function is found to be, with $\hbar = 1$,

$$\begin{aligned}
\beta(e_r^2) = \frac{e_r^4}{16\pi^2} + \frac{e_r^6}{(16\pi^2)^2} \{ &\frac{87}{8} + \frac{1463m^2}{64\Psi^2} - \frac{5859m^4}{512\Psi^4} + \\
&\frac{45}{4}(1 + \frac{3m^2}{2\Psi^2} + \frac{m^4}{16\Psi^4} - \frac{9m^6}{16\Psi^6})(\ln|\frac{n+\Psi}{M}| + \ln|\frac{n-\Psi}{M}|) + \\
&\frac{n}{H}(15 - \frac{7671m^2}{256\Psi^2} + \frac{351m^4}{64\Psi^4})(\ln|\frac{n+\Psi}{M}| - \ln|\frac{n-\Psi}{M}|) + \\
&\frac{59}{8}\ln|\frac{m}{2M}| - \frac{11}{4}\ln|\frac{2n}{M}| \}
\end{aligned} \tag{275}$$

The first order result is just as expected as it differs only by a constant factor from the result obtained for QED .[7] Both theories have the same long range behaviour, i.e.

---

[7] It should be noticed that the definition of the β-function differs from the usual one. To make the connection with the usual results, the radiative corrections to the auxiliary field d must be taken into account. This yields slightly different values for the renormalization constants and the β-function, as defined in ref.[3], becomes

that the charge appears weaker at larger distances. At second order, the β-function for SQED becomes more complicated as it cannot be expressed as a constant times e^6. Thus, it cannot be compared simply with the QED result.

The running coupling constant, e_R^2, may be found by solving the differential equation (273). It is found that, at first order,

$$e_R^2(n,m) = \frac{e_0^2}{\left(1 - \frac{e_0^2}{32\pi^2}\ln\left|\frac{m}{m_0}\right| - \frac{e_0^2}{32\pi^2}\ln\left|\frac{n}{n_0}\right|\right)} = \frac{e_0^2}{\left(1 - \frac{e_0^2}{32\pi^2}\ln\left|\frac{nm}{n_0 m_0}\right|\right)} \tag{276}$$

where

$$e_0^2 = e_R^2\,(n_0, m_0) \tag{277}$$

and n_0 , m_0 are arbitrary scales. One can see that the charge e_R^2 becomes weaker for small scales, which is equivalent to large distances.

$$\beta(e^2) = \frac{e^4}{4\pi^2} + \frac{9e^6}{128\pi^4} + \ldots$$

which corresponds to the result derived by other means [4][5] [6] .

The Landau point is the point at which the running coupling constant develops a singularity and becomes infinite. For SQED, this is rather a Landau line in (σ,η)-space whose equation is

$$nm = n_0 m_0 \exp[\, 32\pi^2\, e_0^{-2} \,] \tag{278}$$

This is a very large scale. So, before reaching such singularities, higher order corrections must be included. This contribution could be found by integrating the e_r^6-term of equation (275). However, merely by looking at the set of relations (274), one may deduce this second order contribution and obtain

$$\begin{aligned} e_R^2(n,m) = e_0^2 \{ 1 - \frac{e_0^2}{32\pi^2} \ln|\frac{nm}{n_0 m_0}| - \frac{e_0^4}{(512\pi^4} [(\frac{87}{8} + \frac{1463m^2}{\Psi^2} - \\ \frac{5859m^4}{512\Psi^4}) \ln|\frac{mn}{n_0 m_0}| + \frac{45}{4} (1 + \frac{3m^2}{2\Psi^2} + \frac{m^4}{16\Psi^4} - \frac{9m^6}{16\Psi^6}) \\ (\ln^2|\frac{n+\Psi}{M}| - \ln^2|\frac{n_0+\Psi_0}{M}| + \ln^2|\frac{n-\Psi}{M}| - \ln^2|\frac{n_0-\Psi_0}{M}|) + (\frac{n}{\Psi}) (15 - \\ \frac{7671m^2}{256\Psi^2} + \frac{351m^4}{64\Psi^4}) (\ln^2|n+\Psi| - \ln^2|\frac{n_0+\Psi_0}{M}| - \ln^2|\frac{n-\Psi}{M}| + \\ \ln^2|\frac{n_0-\Psi_0}{M}|) - \frac{1}{4} \ln^2|\frac{nm}{n_0 m_0}| + \frac{59}{8} (\ln^2|\frac{m}{2M}| - \ln^2|\frac{m_0}{2M}|) \\ - \frac{11}{4} (\ln^2|\frac{2n}{M}| - \ln^2|\frac{2n_0}{M}|)] \} \end{aligned} \tag{279}$$

where

$$\Psi_0 = \sqrt{ n_0^2 + \frac{3m_0^2}{4} } \quad . \tag{280}$$

It should be recalled that the e_0^6-term of equation (279) is an approximation as it is derived from the finite part of $U^{(2)}$. This term is given only to have an idea of the behaviour of the running coupling constant at this order of perturbation. One may see in Appendix C how the renormalized coupling constant, as given in equation (264), varies with m for different n's. The curves run rather smoothly on the logarithmic scale except for a very small bump near m equal to n when n is small. This is probably due to larger numerical errors or a greater sensibility to the approximation when one works with so minute numbers or still, to both reasons together. Thus, one should not pay too much attention to this feature and consider that the coupling varies quite monotonicly .

6.3 References

[1] S.Coleman, E.Weinberg, Phys.Rev.D7(1973)1888

[2] P.Ramond, "Field Theory, a Modern Primer", Benjamin-Cummings Publ.Co., Reading, 1981, p.174

[3] P.Ramond, ibid., p.183

[4] W.E.Caswell, Phys.Rev.Lett.33(1974)244

[5] D.R.T.Jones, N.P.75B(1974)531

[6] A.A.Belavin, A.A.Migdal, JETP Lett.19(1974)181

Chapter VII

CONCLUSION

7.1 Comments

A finite expression for the effective potential of supersymmetric quantum electrodynamics has been obtained at two-loop order. The different loop contributions are seen to converge quite rapidly except for small values of σ where the ratio of the first to second order correction is rather small (~ 10). This raises some questions whether or not there will be convergence at three-loop order.

It may also be remarked that the different loop contributions are not necessarily positive. This does not violate supersymmetry. Only the total effective potential has to be positive definite and this is indeed the case as the tree-level potential always dominates by a large factor over the radiative corrections. It is also seen that the minimum (d=0) is preserved and supersymmetry is unbroken.

A peculiarity about the effective potential for SQED is the appearence of an imaginary part. In fact, the potential is complex everywhere in the (c,d) plane except at the minimum (c,d=0). This was not altogether a new feature. The study of spontaneously massive dilatons [1] showed the same behaviour as well as a non-abelian SUSY gauge model with SU(2) internal symmetry [2]. The imaginary part was associated to the decay probability of the system which becomes unstable away from the minimum. It was also seen that this instability is small enough to allow the exixtence of such a state for a relatively long period.

The renormalized coupling constant was derived from the effective potential and was shown to have a rather smooth behaviour. The β-function was derived from it and proved that SQED running coupling constant had the expected long range behaviour.

It should be noticed that , in order to apply the effective potential method in superspace to more complex theories , an improved method for the calculation of superpropagators for broken supersymmetric theories would be required. For a case as "simple" as SQED, it was not possible to obtain superpropagators in a fully supersymmetric gauge ; a Wess-Zumino gauge was used and it was seen that it is a difficult gauge to implement in a supergraph approach. However, it was mentioned that there has been a breakthrough lately as how to deal with a Wess-Zumino gauge[7] and preserves at the same time supergraph techniques [3]. Hence, there could be hope that, with the appropriate modifications to Helayël-Neto's method, superpropagators for broken supersymmetric gauge theories be calculable in a more elegant and efficient fashion.

To conclude, it can be said that the effective potential of SQED remains supersymmetric up to two-loop order even if each term does not necessarily do so. The vacuum is unstable under small perturbations as the potential develops an imaginary part as soon as the minimum is left. Finally, it was shown that SQED running coupling constant shows the same long range behaviour as the one for QED.

---

[7] Or , more generally, with a non-SUSY gauge condition

7.2 References

[1] I.T.Drummond, Nucl.Phys.B72(1974)41

[2] G.Woo, Phys.REV. D12(1975)975

[3] T.Kreuzberger, W.Kummer, O.Piguet, A.Rebhan, M.Schweda, Phys.Lett. 167B(1986)393

Appendix A

DIMENSIONAL REGULARIZATION AND GAMMA FUNCTION

A.1 Glossary of Dimensional Regularization Formulae Used in this Work

$$\int d^n\bar{p}\,\frac{1}{(p^2+M^2+2k\cdot p)^a} = -\frac{\Gamma(a-\frac{n}{2})}{(4\pi)^{n/2}\Gamma(a)}\quad\frac{1}{(M^2-k^2)^{a-\frac{n}{2}}} \tag{281}$$

$$\int d^n\bar{p}\,\frac{p_i}{(p^2+M^2+2k\cdot p)^a} = -\frac{\Gamma(a-\frac{n}{2})}{(4\pi)^{n/2}\Gamma(a)}\quad\frac{k_i}{(M^2-k^2)^{a-\frac{n}{2}}} \tag{282}$$

$$\int d^n\bar{p}\,\frac{p_i p_j}{(p^2+M^2+2k\cdot p)^a} = -\frac{1}{(4\pi)^{\frac{n}{2}}\Gamma(a)}\left[\frac{k_i k_j\,\Gamma(a-\frac{n}{2})}{(M^2-k^2)^{a-\frac{n}{2}}}+\frac{\frac{1}{2}\,\delta_{ij}\,\Gamma(a-1-\frac{n}{2})}{(M^2-k^2)^{a-1-\frac{n}{2}}}\right] \tag{283}$$

A.2 Gamma Function

The definition of the Gamma function is, for n greater than zero,

$$\Gamma(n) = \int_0^\infty dt\, t^{n-1}\, e^{-t} \quad . \tag{284}$$

For n smaller or equal to zero, the Gamma function is defined with the help of the relation

$$\Gamma(n) = \frac{\Gamma(n+1)}{n} \tag{285}$$

From this definition, one may see that the Gamma function has poles for all nonpositive integers.

The expansion of the Gamma function as a function of ϵ is

$$\lim_{\epsilon \to 0} \Gamma(1+\epsilon) = 1 - \gamma_e \epsilon + \frac{\epsilon^2}{2}(\gamma_e^2 + \frac{\pi^2}{6}) - O(\epsilon^3) \tag{286}$$

The constant γ_e is the Euler constant ans its value is

$$\gamma_e = \lim_{n\to\infty} [\, 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln(n) \,] \tag{287}$$

$$= 0.5772156649....$$

With the relation

$$\gamma_e^{-\epsilon} = 1 - \epsilon\gamma_e + \frac{\epsilon^2}{2}\gamma_e^2 \quad , \tag{288}$$

equation (286) may be written as

$$\lim_{\epsilon\to 0} \Gamma(1+\epsilon) = \gamma_e^{-\epsilon}\,(1 + \epsilon^2 \frac{\pi^2}{12}) + O(\epsilon^3) \quad . \tag{289}$$

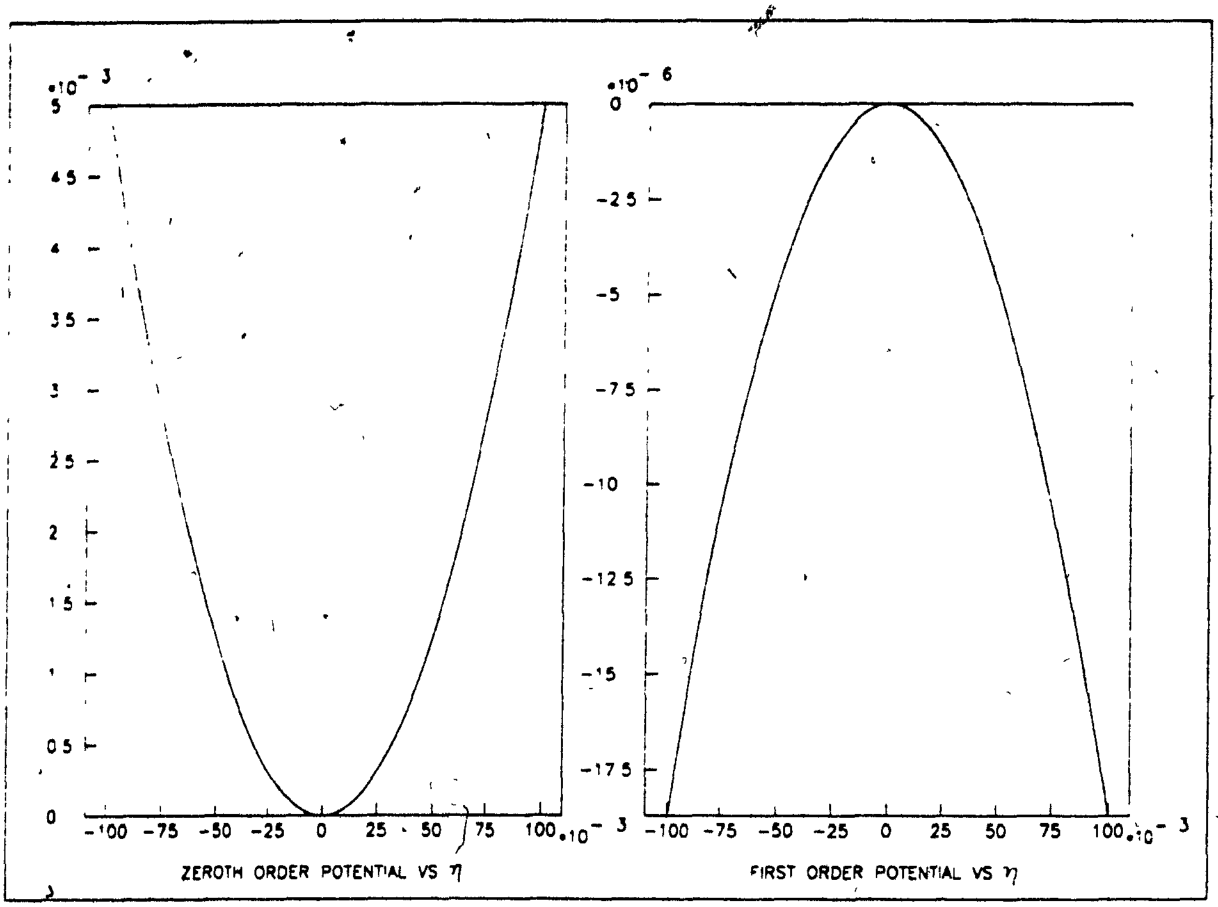
Appendix B

GRAPHS OF THE EFFECTIVE POTENTIAL FOR SQED

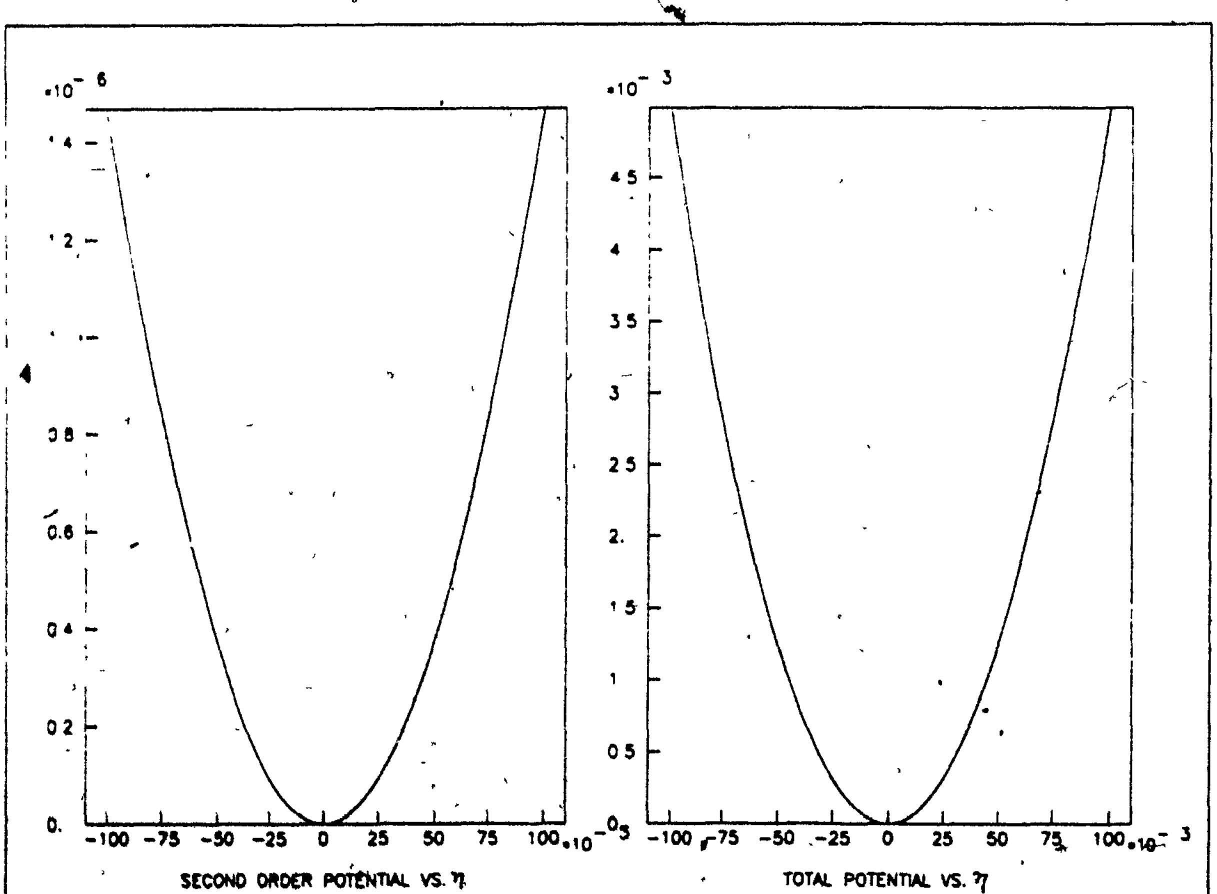
In the next pages are gathered the graphs representing the renormalized effective potential of SQED at orders zero, one and two in $\hbar$ as well as the total of all contributions as given in equation (262)

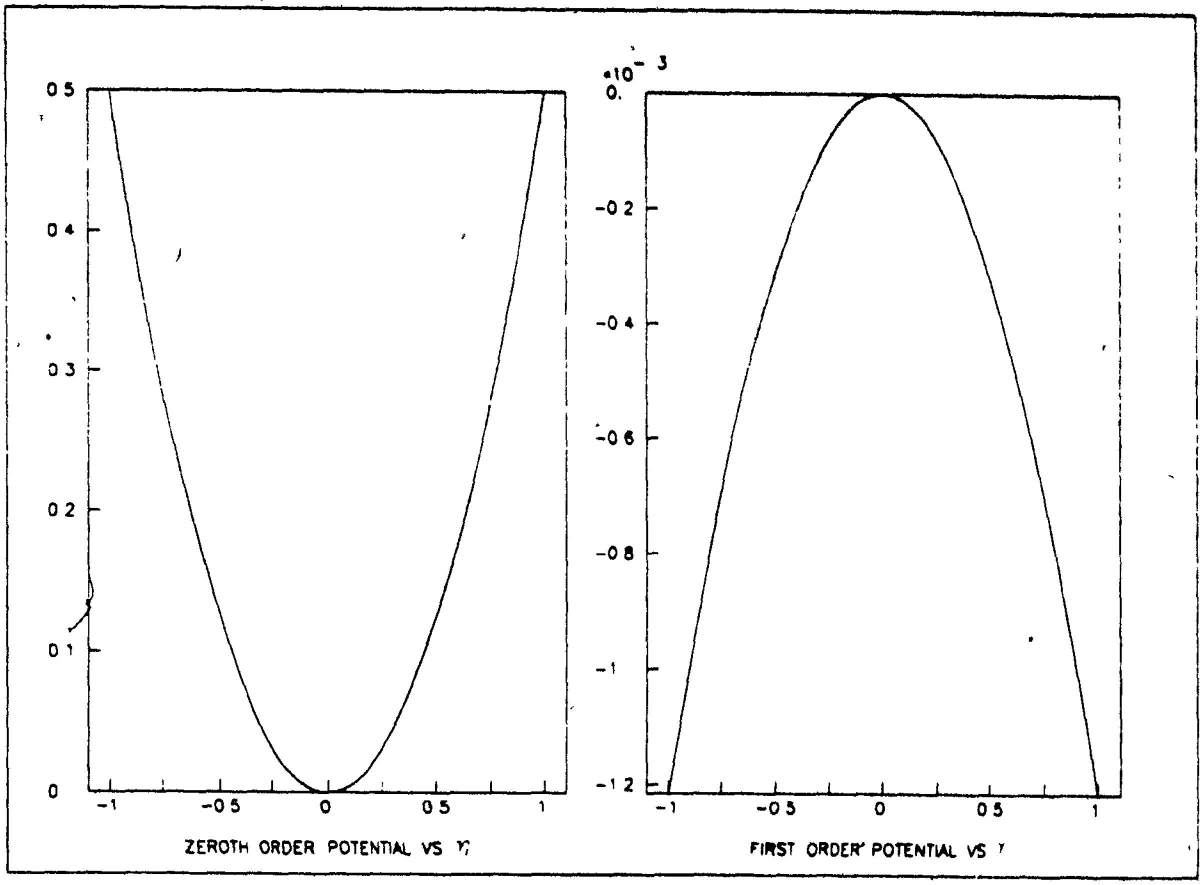
In the first four pages, the effective potential is given as a function of η while σ is kept fixed. Finally, a three-dimensional representation of U is drawn on the last page of this appendix.

Graphs 1 to 4 :	U as a function of η for $\sigma = 0.1$
Graphs 5 to 8 :	U as a function of η for $\sigma = 1.0$
Graphs 9 to 12 :	U as a function of η for $\sigma = 10.0$
Graphs 13 to 16 :	U as a function of η for $\sigma = 100.0$
Graph 17 :	U as a function of σ and η (3-D Sketch)

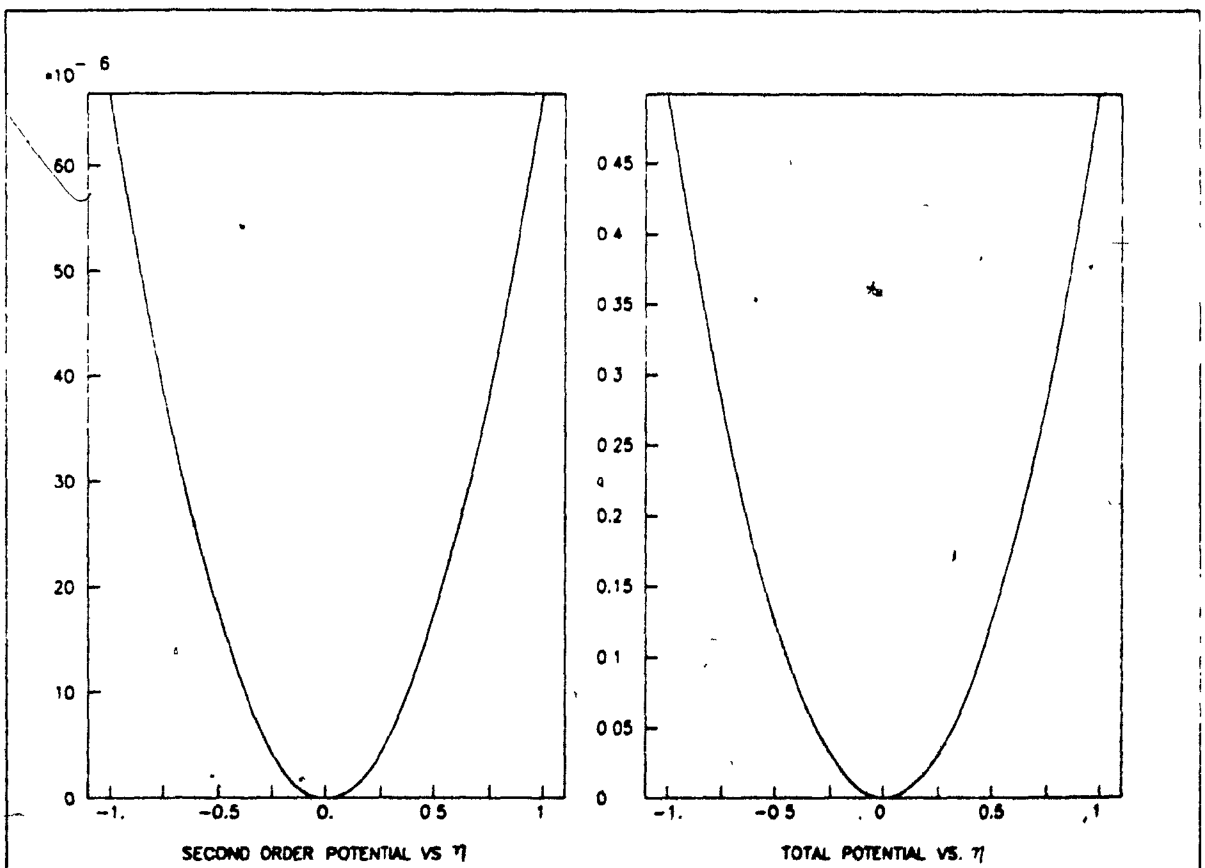

*10⁻³
*10⁻⁶
*10⁻³
ZEROTH ORDER POTENTIAL VS η
FIRST ORDER POTENTIAL VS η

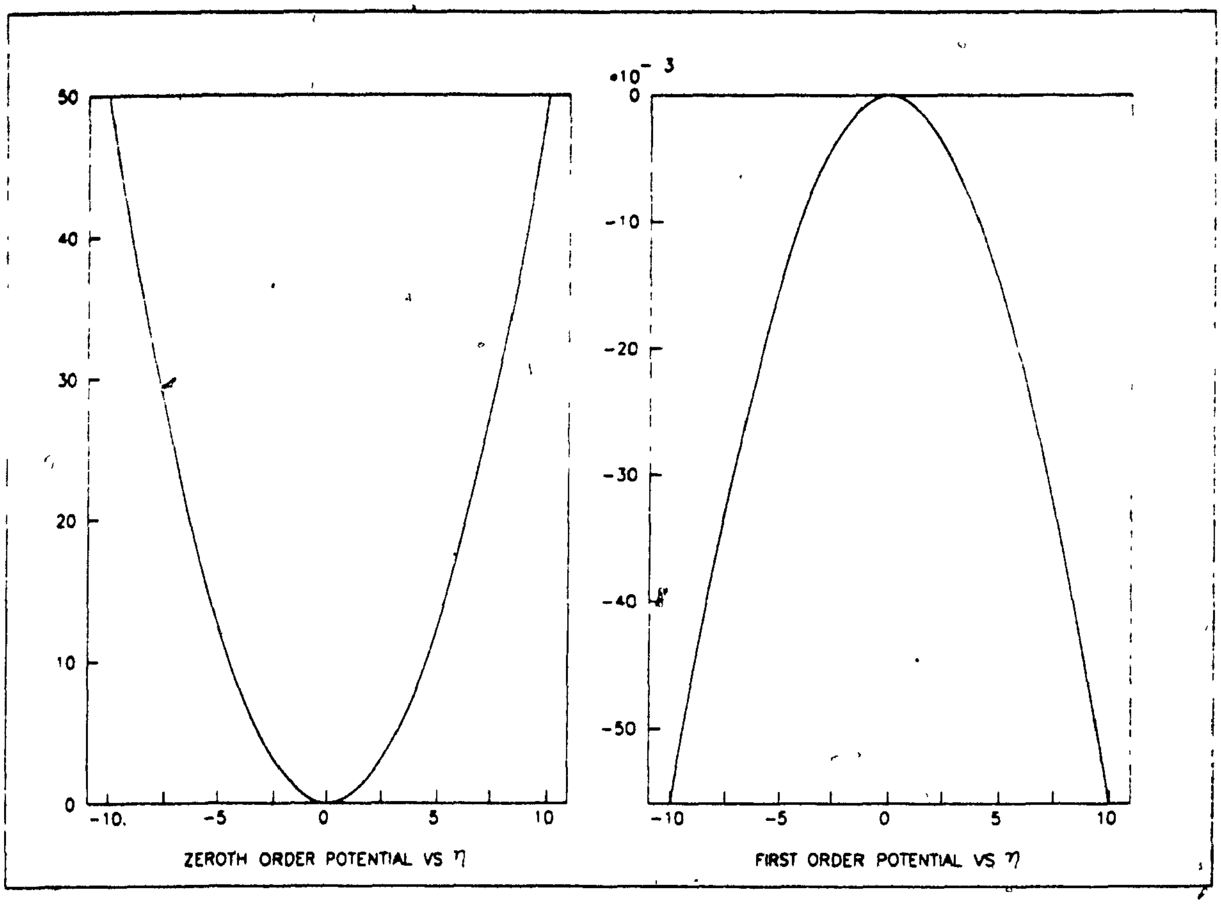
$\sigma = 0.1$


*10⁻⁶
*10⁻³
SECOND ORDER POTENTIAL VS. η
TOTAL POTENTIAL VS. η

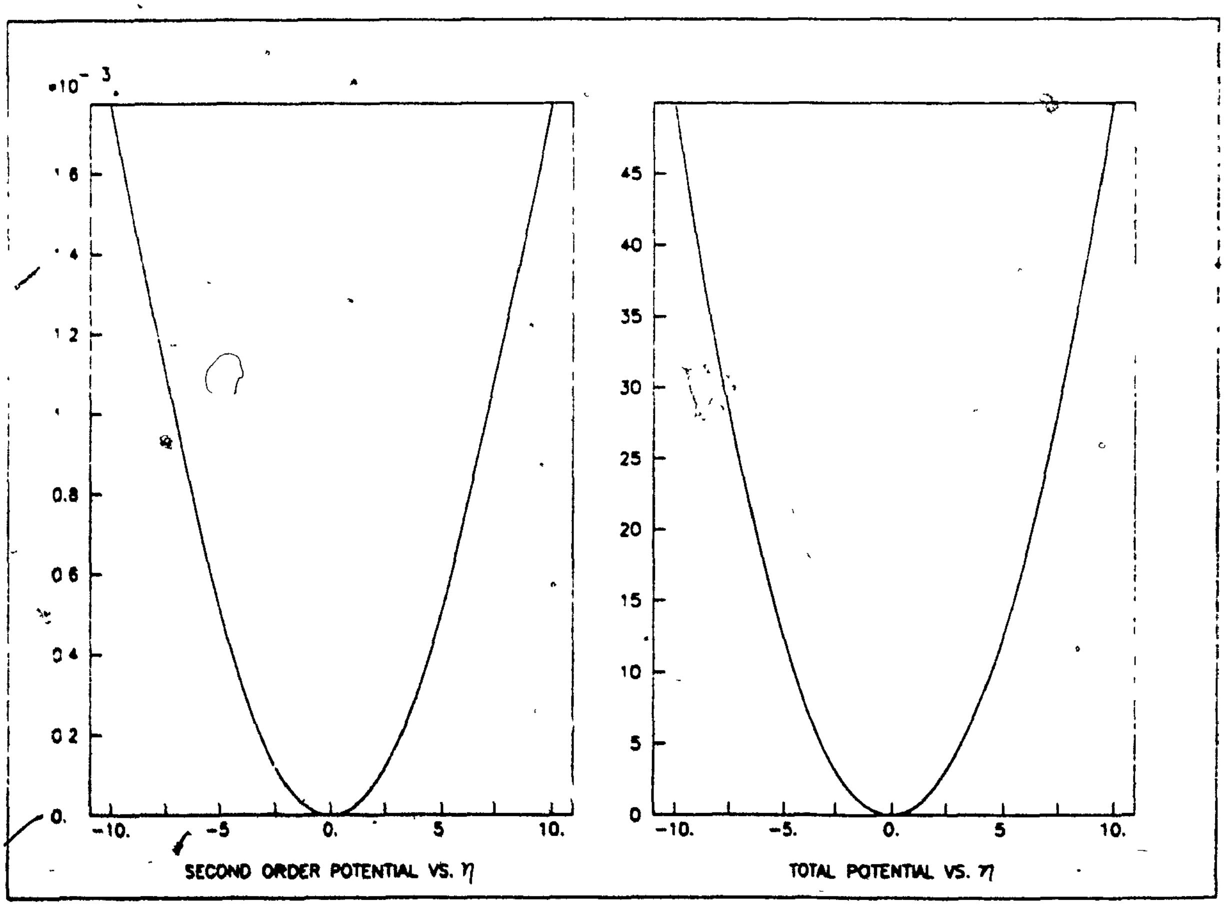

ZEROTH ORDER POTENTIAL VS η
FIRST ORDER POTENTIAL VS η
*10⁻³

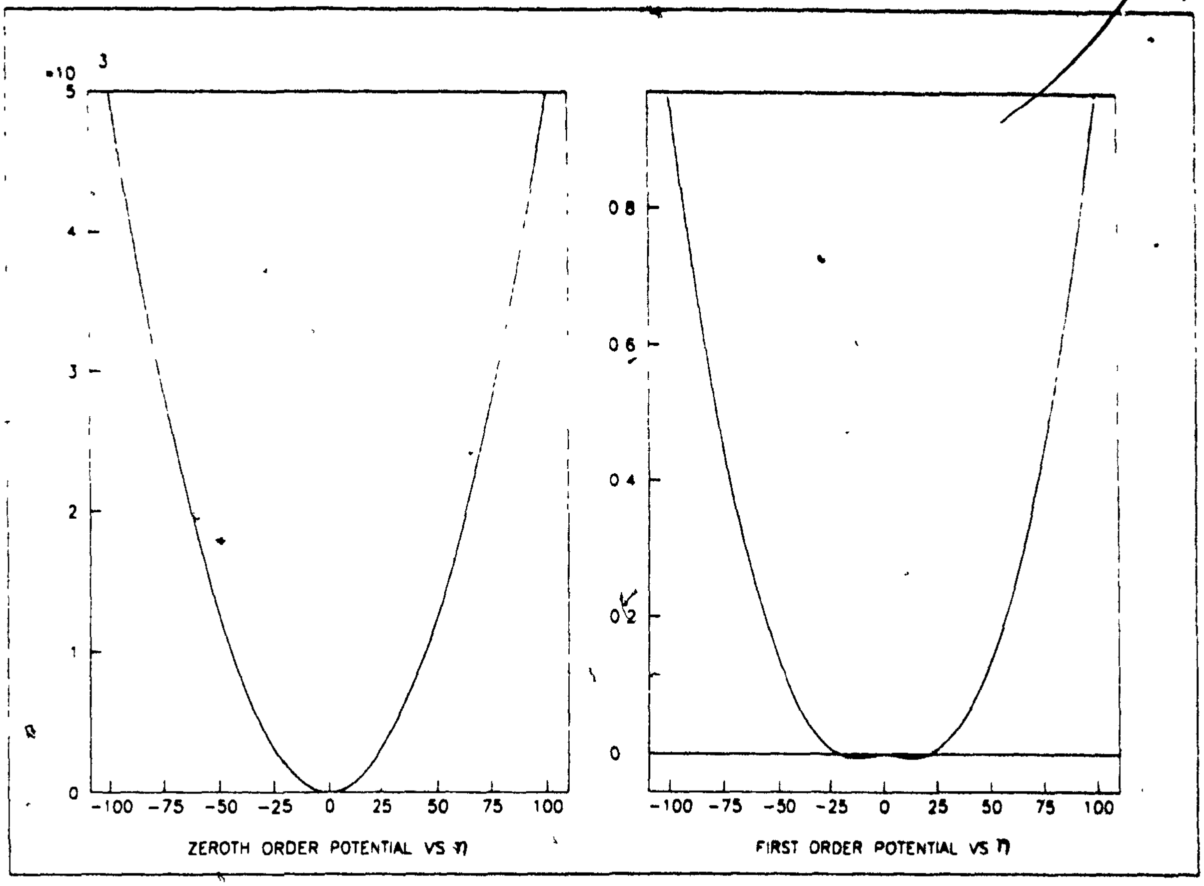
$\sigma = 1.0$


*10⁻⁶
SECOND ORDER POTENTIAL VS η
TOTAL POTENTIAL VS. η


ZEROTH ORDER POTENTIAL VS η
FIRST ORDER POTENTIAL VS η
*10⁻ 3

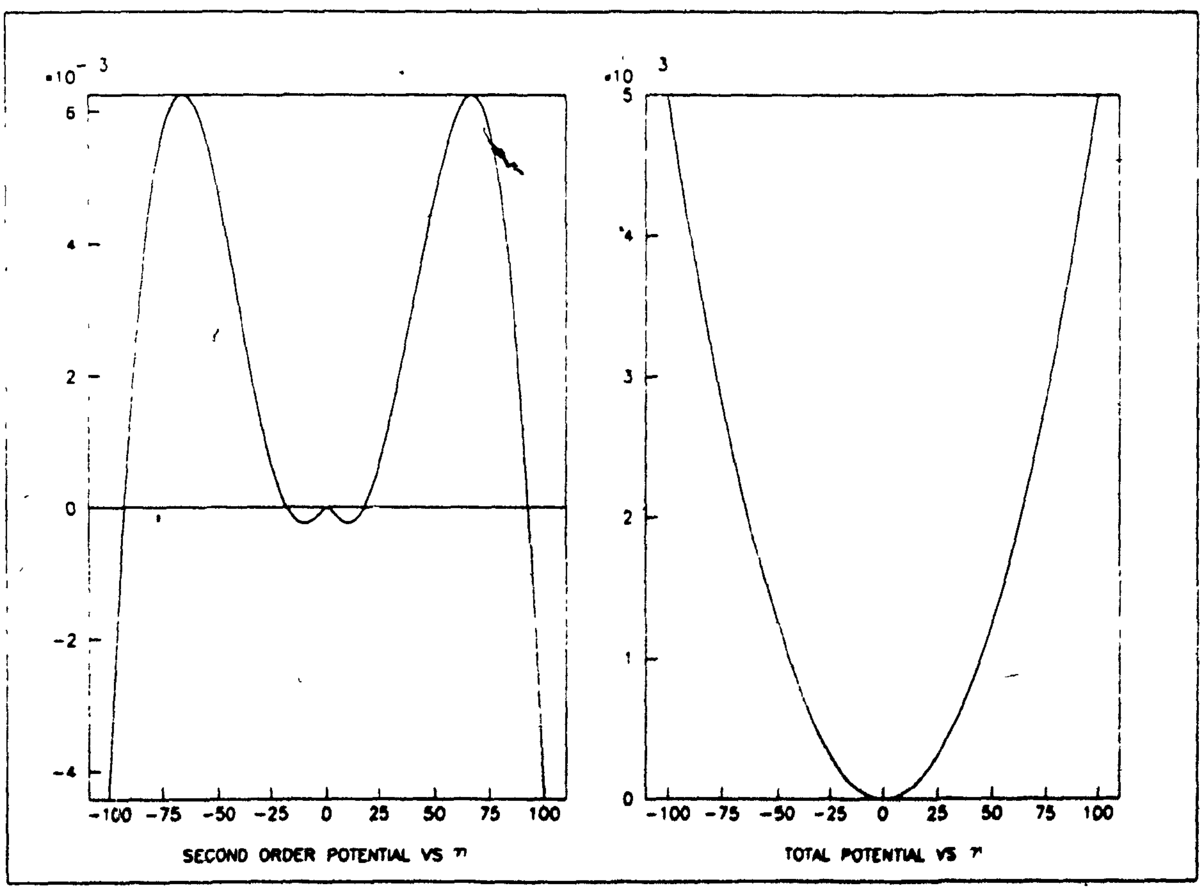
$\sigma = 10.0$


SECOND ORDER POTENTIAL VS. η
TOTAL POTENTIAL VS. η
*10⁻ 3

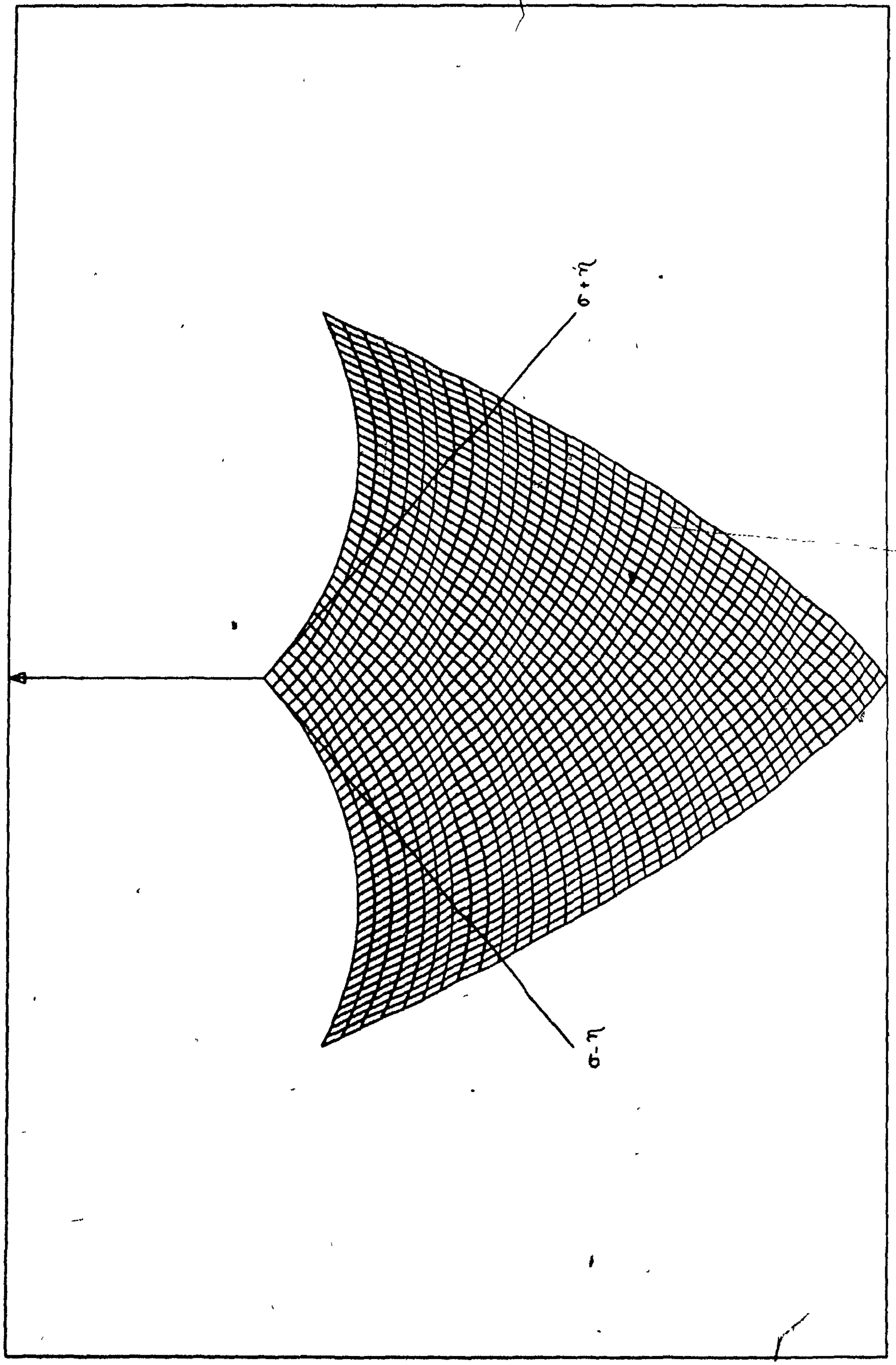


*10 3
ZEROTH ORDER POTENTIAL VS η
FIRST ORDER POTENTIAL VS η

$\sigma = 100.0$



*10⁻ 3
*10 3
SECOND ORDER POTENTIAL VS η
TOTAL POTENTIAL VS η



POTENTIAL UP TO TWO LOOP ORDER

Appendix C

GRAPHS OF THE RENORMALIZED COUPLING CONSTANT

In this appendix, one may find the graphs depicting the renormalized coupling constant, as given by equation (264). As in the case of the effective potential, the coupling constant are given as a function of m for a fixed n.

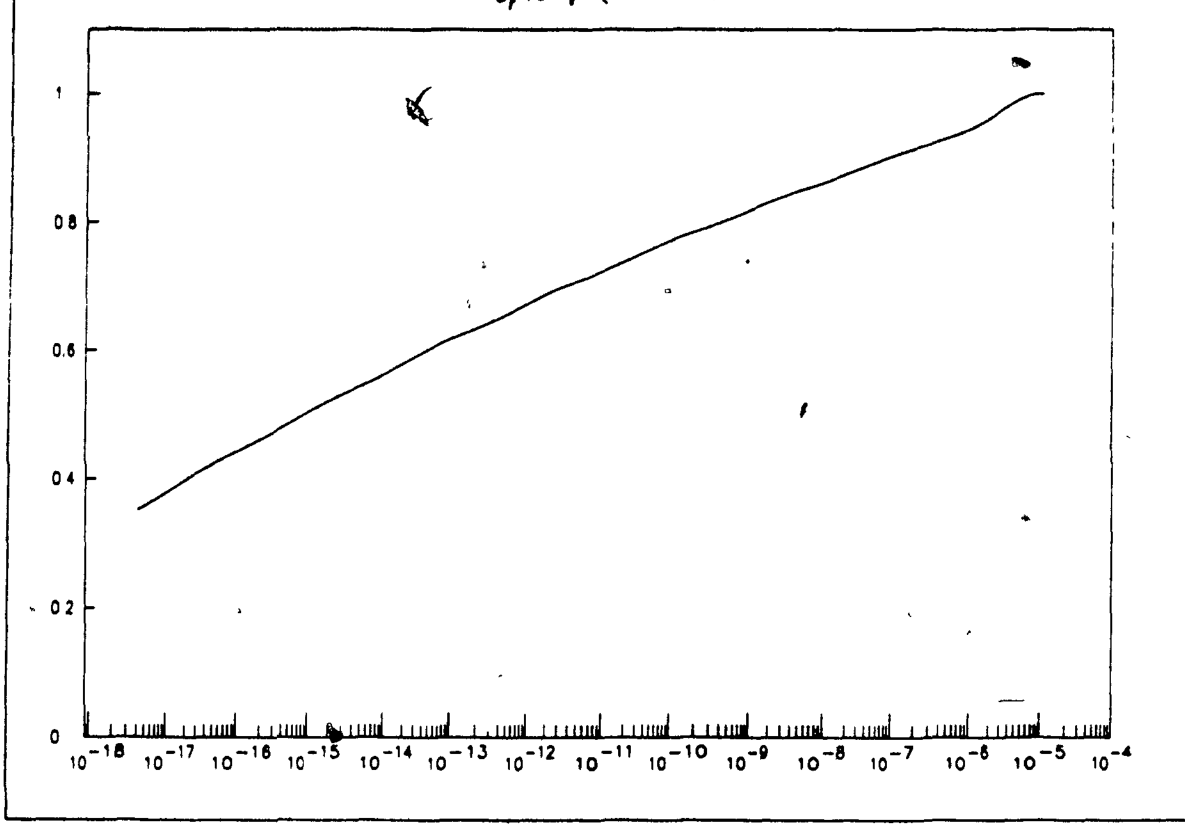
Graph 18 : e_r^2 as a function of η *for* $\sigma = 10^{-5}$

Graph 19 : e_r^2 as a function of η *for* $\sigma = 10^{-3}$

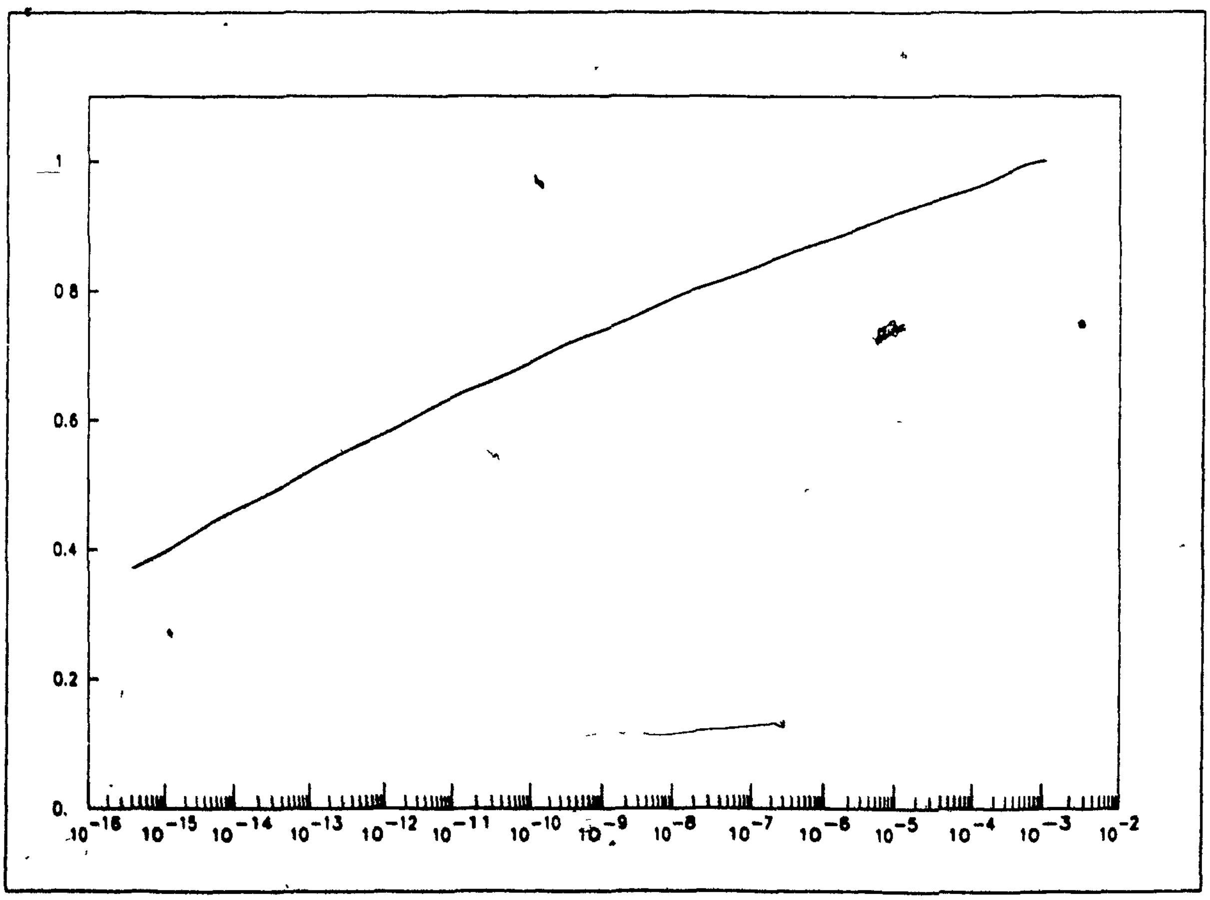
Graph 20 : e_r^2 as a function of η *for* $\sigma = 10^{-1}$

Graph 21 : e_r^2 as a function of η *for* $\sigma = 10^{1}$

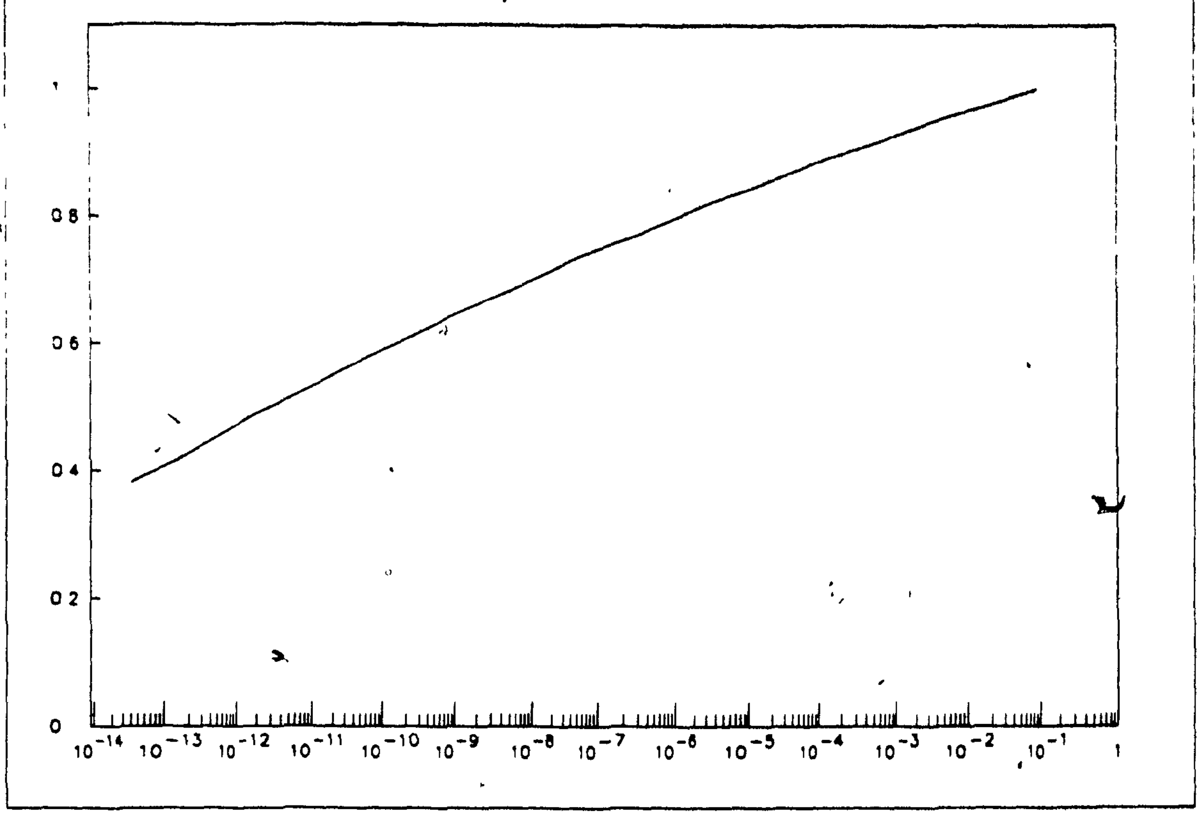
e_r^2 vs η $(\sigma = 10^{-5})$



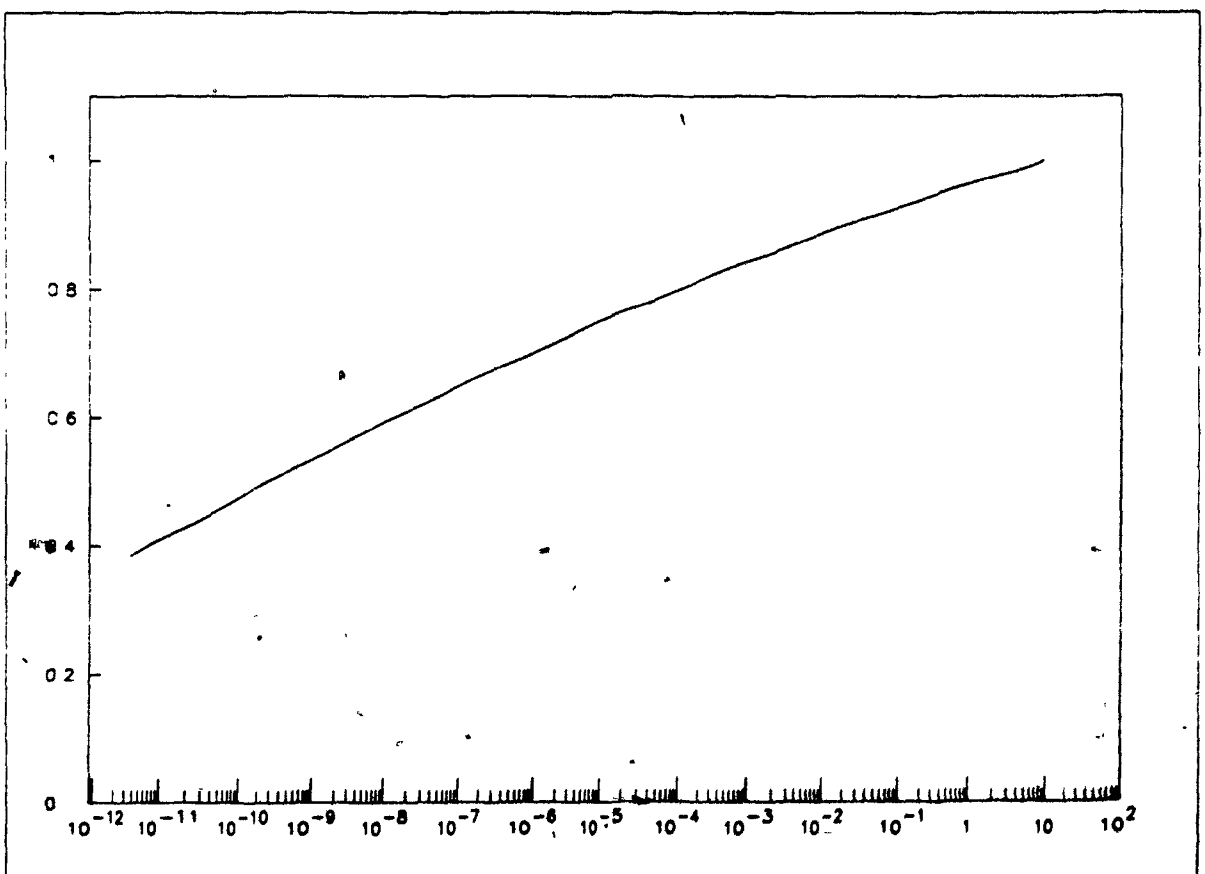
e_r^2 vs η $(\sigma = 10^{-3})$



e_r^2 vs η $(\sigma = 10^{-1})$



e_r^2 vs η $(\sigma = 10^{1})$



BIBLIOGRAPHY

1. A.Amati, K.Chou, Phys.Lett. 114B(1982)129
2. A.A.Belavin, A.A.Migdal, JETP Lett.19(1974)181
3. W.E.Caswell, Phys.Rev.Lett.33(1974)244
4. S.Coleman, *Laws of Hadronic Matter* , ed. by Zichichi, Academic Press, 1973, p.169
5. S.Coleman, E.Weinberg, Phys.Rev. D7(1973)1888
6. J.C.Collins, Phys.Rev. D10(1974)1213
7. B.De Witt, J.Smith, *Field Theory in Particle Physics* , North-Holland Personal Library, Netherland, 1986, p.305
8. I.T.Drummond, Nucl.Phys. B72(1974)41
9. F.Feruglio, J.A.Helayël-Neto, F.Legovini, Nucl.Phys. 249B(1985)533
10. S.Ferrara, B.Zumino, Nucl.Phys. B79(1974)413
11. S.Ferrara, O.Piguet, Nucl.Phys. B93(1975)261
12. G.Fogleman, K.S.Viswanathan, Phys.Lett. 133B(1983)393
13. G.Fogleman, K.S.Viswanathan, Phys.Rev. D30(1984)1364
14. S.J.Gates Jr., M.T.Grisaru, M.Roček, W.Siegel, *Superspace or One Thousand and One Lessons in Supersymmetry* , Benjamin-Cummings Publ.Co., Reading, 1983, 548pp.
15. J.L.Gervais, B.Sakita, Nucl.Phys. 34B(1971)632
16. Y.A.Gel'Fand, E.P.Likhtman, JETP Lett. 13(1971)323
17. M.T.Grisaru, F.Riva, D.Zanon, Nucl.Phys. 214B(1983)465
18. M.T.Grisaru, W.Spiegel, M.Rocek, Nucl.Phys. B159(1979)429

19. J.A.Helayël-Neto, Phys.Lett. 135B(1984)78

20. J.A.Helayël-Neto, F.A.B.Rabelo de Carvalho, A.W.Smith, Nucl.Phys. B271(1986)175

21. G.'t Hooft, M.Veltman, Nucl.Phys. B44(1972)189

22. K. Huang, *Quarks, Leptons and Gauge Fields* , World Scient.Publ. , Singapore, 1984, 281 pp.

23. M.Huq, Phys.Rev. D14(1976)3548

24. M.Huq, Phys.Rev. D16(1976)1733

25. J.Iliopoulos, C.Itzyckson, A.Martin, Rev. of Modern Phys. 47(1975)165

26. R.Jackiw, Phys.Rev. D9(1974)1686

27. D.R.T.Jones,Nucl.Phys. 75B(1974)531

28. J.S.Kang, Phys.Rev. D10(1974)3455

29. T.Kreuzberger, W.Kummer, O.Piguet, A.Rebhan, M.Schweda, Phys.Lett. 167B(1986)393

30. G.Leibbrandt, Rev. of Modern Phys. 47(1975)849

31. R.D.C.Miller, Nucl.Phys. 228B(1983)316

32. R.D.C.Miller, Nucl.Phys. 229B(1983)189

33. R.D.C.Miller, Phys.Lett. 124B(1983)59

34. R.D.C.Miller, Nucl.Phys. 241B(1984)535

35. A.Neveu, J.H.Schwarz, Nucl.Phys. 31B(1971)86

36. L.O'Raifeartaigh, G.Parravicini, Nucl.Phys. 11B(1976)516

37. P.Ramond, Phys.Rev. D3(1971)2415

38. P.Ramond, *Field Theory, a Modern Primer* , Benjamin-Cummings Publ.Co., 1981, 397 pp.

39. P.P.Srivastava, Phys.Lett. 132B(1983)80

40. G. Thompson, H-L Yu, Phys.Rev. D31(1985)2141

41. D.V.Volkov, V.P.Akulov, Phys.Lett. 46B(1973)109

42. S.Weinberg, Phys.Rev. D7(1973)2887

43. J.Wess, B.Zumino, Nucl.Phys. 70B(1974)39

44. J.Wess, B.Zumino, Phys.Lett. 49B(1974)52

45. J.Wess, J.Bagger, *Supersymmetry and Supergravity* , Princeton Univ. Press, Princeton, 1983, 180p.

46. G.Woo, Phys.Rev. D12(1975)975