MULTI-PLATE PENETRATION TESTS TO DETERMINE SOIL STIFFNESS MODULI

by

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ABSTRACT

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Bekker's pressure-sinkage equation, which characterizes the soil property of vertical deformation pertinent to off-road locomotion, has long been under discussion as to its validity. The variability of the soil parameters involved in this equation is a major issue. It is stressed in this study that these variations are attributable to the localized inhomogeneity in soil samples, which is inevitable even under strictly controlled laboratory conditions. The study has shown that the conventional two-plate soil penetration test cannot be regarded as an accurate technique for the parameter extraction, and should be substituted by a multi-plate test. This suggestion was evaluated by a series of small plate penetration tests on dry sand. The soil parameters were assessed by the regression methods, and the pressure-sinkage relationship which was predicted by using these parameters compared well with the measured one. Recommendations are formulated for the minimum number of plates required for a desired level of accuracy in the results.

RESUME

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LES ESSAIS DE PENETRATION DU SOL AFIN DE DETERMINER LE MODULES DE RIGIDITE

L'équation de "pression-enfoncement" de Bekker qui caractérise la propriété de déformation verticale du sol pertinente aux véhicules tout-terrain, a été longuement discutée quant à sa validité. La variabilité des paramètres du sol impliquées dans l'équation est le problème principal. Il est souligné dans cette étude que ces variations sont dues à la non-homogénéité localisée des échantillons de sol, qui est inévitable même dans les conditions de laboratoire entièrement contrôlées. Ainsi, le test conventionel de pénétration à deux plateaux ne peut être considéré comme une technique adéquate pour l'extraction des paramètres et doit être substitué au test à multiples plateaux. Cette suggestion a été évaluée par une série de tests à pénétration de petits plateaux dans du sable sec. Les paramètres du sol ont été évalués par la méthode de régression. La relation de pression-enfoncement, obtenue en utilisant ces paramètres, est consistante par rapport à celle mesurée. Des recommendations sont formulées pour les tests à nombre minime de plateaux d'enfoncement requis pour un niveau désiré de précision quant aux résultats.

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LIST OF SYMBOLS

A, B, C	constants
b	rectangular footing width, or radius of a circular footing
c	coefficient of cohesion
D	diameter of a circular footing
E	modulus of elasticity
g	acceleration due to gravity
K	Bernstein's modulus of soil deformation
k_c	Bekker's modulus of soil deformation
k_ϕ	Bekker's modulus of soil deformation
M, N	number of data points
n	exponent of soil deformation
n_c	exponent of soil deformation (averaged)
P	pressure
q	surcharge pressure
R_b	bulldozing resistance
R_c	compaction resistance
R_f	flexing hysteresis resistance
Z	sinkage
Z_o	maximum sinkage
γ	specific gravity
ν	Poisson ration
ρ	density
ϕ	angle of soil internal friction

CHAPTER I

INTRODUCTION

Terrain-vehicle mechanics is a branch of modern applied mechanics developed in the last forty years. Its objective is to study the interaction between the terrain and vehicles in order to formulate guiding principles for the rational development, design and evaluation of off-road vehicles (Bekker, 1969). In recent years, the growing concern over energy conservation and environment protection has further stimulated the studies in this field. In addition to be based on a good engineering design in the traditional sense, an off-road vehicle is now expected to attain a high level of energy efficiency and not to cause damage to its operational environment, such as excessive soil compaction in agriculture or the tearing of surface vegetation on tundra and muskeg in northern transportation (McKyes et al., 1978; Wong et al., 1979; Taylor and Gill, 1984). The increasing activities in the exploration and exploitation of natural resources in new frontiers, including remote areas and the seabed, have also given much new impetus to the development of off-road transportation studies (Yong and Harrison, 1978; Falbo, 1984).

The terrain-vehicle mechanics developed by Dr. M. G. Bekker represent one of the main schools of thought in this field (Wills, 1966). Bekker's work is the first and so far a very influential effort in the systematic development of principles of land locomotion mechanics (Soehne, 1981). Bekker's methods have been frequently used by engineers in the off-road vehicle industry during the last 15 to 20 years (Kogure et al., 1983; Volfson, 1984).

Bekker's theory on soil deformation by vehicles depends entirely upon the fitting of the soil pressure-sinkage data with an empirical equation

$$P = (k_c/b + k_\phi) Z^n$$

with k_c, k_ϕ and n being construed as real soil constants (Reece, 1964a). Pressure P is integrated along the terrain-vehicle interface of width, b, over the sinkage, Z, from zero to maximum sinkage, Z_o, to predict the soil motion resistance of a vehicular running gear with a width of b, i.e.

$$R_c = b \int_0^{Z_o} P \, dZ$$

Bekker's approach has inspired many investigators to proceed along this line of reasoning. The effectiveness of Bekker's method was demonstrated fully by its excellent performance in the development of the Lunar Roving Vehicle during the Apollo missions in the U.S. space program. An accuracy of a few percent was achieved in the prediction of power consumption (Costes et al., 1972).

This notwithstanding, the validity of Bekker's equation has not been left unquestioned. The variability of the three soil parameters is a main issue under discussion (Reece, 1964b; Wills, 1966; Karafiath and Nowatzki, 1978;

Wong, 1980; Youssef and Ali, 1982; Volfson, 1984). If these values are, as asserted by some investigators, dependent on the width of the loading area, b, then the commonly adopted practice, i.e. using n, k_c and k_ϕ obtained from small plate sinkage tests to predict the behaviour of a large vehicular running gear, would not be justifiable, and the application of Bekker's method would be inaccurate (Hegedus, 1965).

Presumably, there are two possible reasons which might give rise to the variations of the soil parameters in Bekker's equation. The size of loading area might have an effect on the soil parameters. In this case the variation of these parameters should be systematic and could be formulated as a function of footing size, b. On the other hand, the reported deviations might be due to the random variations of physical states of the soils under test. It has been noticed that soil variability prevails in penetration tests even under controlled laboratory conditions (Reece, 1964a; Janosi, 1965; Bekker, 1969). And the conventionally adopted two-plate penetration test makes the assessment of the soil parameters most susceptible to the soil variability.

While the first hypothesized reason is advocated or quoted by some investigators, the arguments are found based solely on observations of the variation of the soil parameters, without reporting any systematic patterns of the variation. In the meantime, the second possibility, i.e. the soil variability inducing the claimed "size effect", is found to be overlooked in the cited literature.

It is suggested in this investigation that the assessment of k_c and k_ϕ might be grossly affected by the soil variability, especially in the conventional two-plate penetration test. It is suggested that the multi-plate penetration test is conducive to a more dependable evaluation of the parameters in Bekker's pressure-sinkage equation. Such tests were conducted with a family of small model plates on loose dry sand. The ordinary and weighted least squares methods were used to assess k_c and k_ϕ. The pressure-sinkage relationship of a large plate was predicted with the soil parameters thus obtained, and was checked with the measured P(Z) relation.

CHAPTER II

LITERATURE REVIEW

2.1 Necessity of assessing the soil pressure-sinkage relation in terramechanics

The need to study terrain mechanical properties in relation to vehicle performance is evident from experience in the areas of agricultural traction, military transport, and civil transport and construction. A truck might be stalled in a swamp. A pneumatic tire might lose half of its power efficiency when shifting from concrete road to farm land (Gill and Vanden Berg, 1968). All of these are related to the weak strength and large deformation of soil under vehicular load. To predict, evaluate and improve the performance of an off-road vehicle, the mechanical properties of soils have to be understood and assessed.

The mechanical properties of terrain can be defined in several ways, depending on the analytical approach to the mechanism of the running gear-soil interaction. As an empirical method, the mechanical properties of the soil could be lumped into an integrated parameter, such as cone penetrometer index, in an attempt to correlate this single parameter with the performance of a wheel or track (Knight and Freitag, 1962; Wismer and Luth, 1973). When theoretical approaches are adopted to interpret terrain-vehicle interaction, the soil mechanical properties could be defined in terms of the angle of internal friction (ϕ), cohesion (c), and density (ρ) for the limited equilibium analysis (Yong and Windisch, 1970; Hettiaratchi and Reece, 1974; Karafiath and Nowatzki, 1978), or in terms of the modulus of elasticity (E), Poisson's ratio (ν) and the friction parameter (ϕ) for the finite element method (Yong et al., 1984).

Bekker took a semi-empirical approach to mobility problems. He separated the machine-soil interaction into two parts; soil vertical deformation giving rise to wheel or track rolling resistance, and soil horizontal deformation mobilizing thrust. Soil mechanical properties were characterized by the vertical stress-strain curve and horizontal shear stress-strain curve. By integrating these two curves, rolling resistance and thrust were calculated.

According to Bekker's (1975) theory, rolling resistance consists of three parts; compaction resistance (R_c), bulldozing resistance (R_b) and, for pneumatic tires only, flexing hysteresis resistance (R_f). Only the compaction resistance part was attributed to the soil vertical deformation. On medium strong soil, compaction resistance is the main force against the machine moving forward.

When a vehicle runs on unpaved ground, soil is compressed under the vehicle load. The inclined leading edge prevents the wheel or track from moving forward. To keep the wheel or track going ahead, energy has to be spent on compacting the soil into a rut. Bekker assumed that, on an infinitesimal segment of the machine-soil interface, the compaction pressure, no matter in which direction it acted, was identical to the compaction pressure under a flat sinkage plate at the same depth. In other words, the contact pressure is a function of only sinkage, Z. By integrating the pressure-sinkage relation, P(Z), from the ground surface to the maximum sinkage, Z_o, the compaction resistance is obtained for a running gear of width b as,

$$R_c = b \int_0^{Z_o} P(Z)\, dZ \tag{2.1}$$

To implement this method, the P(Z) relation must be assessed first. This is done by a plate sinkage test. Flat plates, usually in circular shape, are forced into the soil under investigation, and the P(Z) relation is recorded in graphical or numerical form. This experimentally assessed relation is used as the input for the calculation of R_c. Thus the assessment of the soil pressure-sinkage relation becomes an indispensable part of Bekker's terramechanics theory.

A few important facts are neglected in simulating the running gear-soil interaction with the flat plate-soil interaction. Firstly, under the action of the circular rim of a running gear, soil particles are not pushed only downward, but also forward. Secondly, the shear stresses developed in the machine-soil interface also contribute to the sinkage.

In fact, these factors are considered separately in Bekker's theory. The resistance from forward movement of soil particles is partially assessed as the bulldozing resistance. The slip sinkage is calculated as an additive part to the pressure related sinkage when the deformation is large. For moderate loads and deformation, slip sinkage can be neglected with negligible error (Bekker, 1960).

The experimental P(Z) relation recorded in graphical or numerical form cannot be conveniently used in engineering calculations of compaction resistance and the prediction of sinkage. Attempts to formulate the P(Z) relation are discussed in the next section.

2.2 Bekker's pressure-sinkage equation

The idealized pressure-sinkage process occurring in a homogeneous plastic soil mass at a shallow depth can be roughly represented by two lines, OA and AB in Figure 2.1 (Bekker, 1969). Line OA refers to the initial portion where elastic deformation or mere relocation of soil particles, or both, prevail. Point A corresponds approximately to the ultimate bearing capacity of soil. In most soils, a sinkage Z smaller than 6 mm lies within this elastic range (Road Research Lab., 1964). Line AB describes a sinkage due to soil failure through plastic flow (Terzaghi and Peck, 1948). The pressure-sinkage relation in this range is governed by the changing geometry of the soil failure zone. The "surcharge effect" of soil above the wheel-soil contact plane requires additional

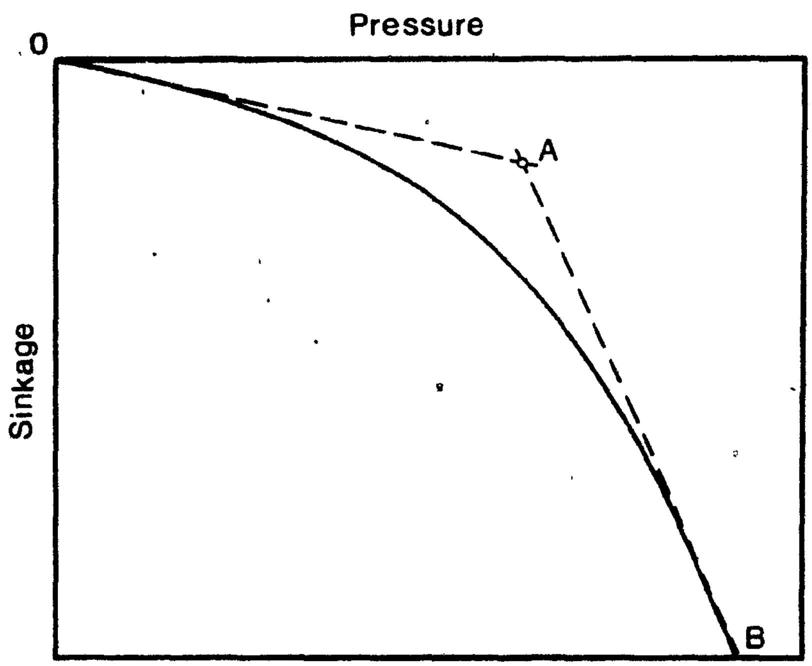

Figure 2.1 Load-penetration curve in a "plastic" homogeneous soil obtained by means of a circular or rectangular plate (Bekker, 1969)

pressure to maintain the plastic flow. This additional pressure is directly proportional to the sinkage, Z, in the Equation (2.2).

$$q = \rho g Z \qquad (2.2)$$

where q is the surcharge, g is the acceleration due to gravity, and ρ is the density of the soil.

The real line representing an actual P(Z) relation is a curve, also shown in Figure 2.1. Since two kinds of physical process are involved in sinkage, an accurate mathematical model tends to be too complex to be used in the integration for compaction resistance (Reece, 1964a). What is needed is a simple equation capable of approximating the real pressure-sinkage process to meet the requirements of practical engineering applications.

Goriatchkin and his co-workers (1936) found that the plate sinkage data could be fitted into an exponential curve. Plotting these data in log-log scales, the data points would fall roughly on a straight line with a slope of n and an intercept of K on the ordinate of log P. This suggests a function in the form

$$P = K \times Z^n \qquad (2.3)$$

where the exponent n might take any value between zero and approximately one. A specific case of this equation with n equal to 0.5 was found by Bernstein in 1913 for agricultural soils (Bekker, 1969).

Between the two parameters n and K, n is commonly agreed to be independent of the size and the form of loading area (Bernstein, 1913; Goriatchkin et al., 1936; Bekker, 1960; Wills, 1966). Thus n is taken as a true soil constant. The value of K was found to be dependent both on the soil and on the sizes of plates used in a test, so K is not a soil constant. Using Equation (2.3) to predict the motion resistance of rigid wheels, researchers had to determine the value of K for each wheel using a plate with the same width as that of the real wheel. The predictions thus obtained proved satisfactory (Bekker, 1969).

With its parameter K bound to the size of the vehicle running gear, the practical usage of Equation (2.3) is very limited. When a running gear is small and easy to fabricate, it does not make much sense to get a less reliable prediction from a penetration test using a similarly sized plate. When a running gear in design is large and costly, the penetration test demanded is also inconvenient. Without relating the performance of a running gear to the parameters of vehicle configuration, Equation (2.3) offers no link to the design or selection of the running gear. Both Bernstein and Goriatchkin failed in their efforts to identify more fundamental parameters to replace K. Progress in this direction was made by Bekker (1955).

Bekker noticed that, in civil engineering soil mechanics, the ultimate bearing capacity of soils could be expressed in the form

$$P = (A/b + C) \times Z \qquad (2.4)$$

where parameters A and C are regarded independent of b, the smaller dimension of the loading area. This formula is valid for small sinkages where Z is less than b (Taylor, 1948). Bekker combined the two-term expression of K with the curvilinear relation between pressure P and sinkage Z, and proposed a new equation.

$$P = (k_c/b + k_\phi)\ Z^n \qquad (2.5)$$

where k_c, k_ϕ and n were all perceived as empirical parameters without specific physical meanings attached.

Through his intensive study, Bekker found that, for practical purposes, the parameters in Equation (2.5) were constants independent of the size and form of the loading area. Bekker checked the generality of Equation (2.5) by fitting experimental data from tests of his own and those published by other workers. Results with a "reasonable degree of accuracy" were achieved (Bekker, 1960). According to Bekker(1977), Equation (2.5) is valid in the following cases:

1. Fine-grained and coarse mineral soils,
2. Frictional and cohesive soils, or a mixture thereof,
3. Homogeneous or stratified ground,
4. Organic soil such as turf, muskeg, peat moss, etc.,
5. Snow.

2.3 The two-plate sinkage test and data processing

A penetration test with a single plate cannot yield all the three parameters, k_c, k_ϕ and n. To solve for k_c and k_ϕ, at least two K values from plates of two sizes are needed. Bekker believed that two plates were enough to extract k_c, k_ϕ and n reliably for homogeneous, unstratified terrain.

In a two-plate penetration test, two rectangular plates of width b_1 and b_2, or two circular plates of radii b_1 and b_2, are pressed at constant speed into the soil under investigation. Plotting the two sets of P(Z) data in log-log scales, two K values, K_1 and K_2, are available as the intercepts of the plotted lines on the ordinate of log P. If the two transformed P(Z) lines were straight and with identical slope, n, the two K's, K_1 and K_2, could be written in the form suggested in Equation (2.5),

$$K_1 = (k_c/b_1 + k_\phi)$$
$$K_2 = (k_c/b_2 + k_\phi) \qquad (2.6)$$

k_c and k_ϕ could then be solved graphically by plotting K against 1/b. Drawing a straight line through $(1/b_1, K_1)$ and $(1/b_2, K_2)$, k_c and k_ϕ would be available as the slope and K-intercept of the line (Figures 2.2, 2.3). Usually, k_c and k_ϕ are solved by rearranging the Equations (2.6) as follows,

$$k_c = (K_2b_2 - K_1b_1)/(b_2 - b_1)$$
$$k_\phi = (K_1 - K_2)\ b_1b_2 / (b_1 - b_2) \qquad (2.7)$$

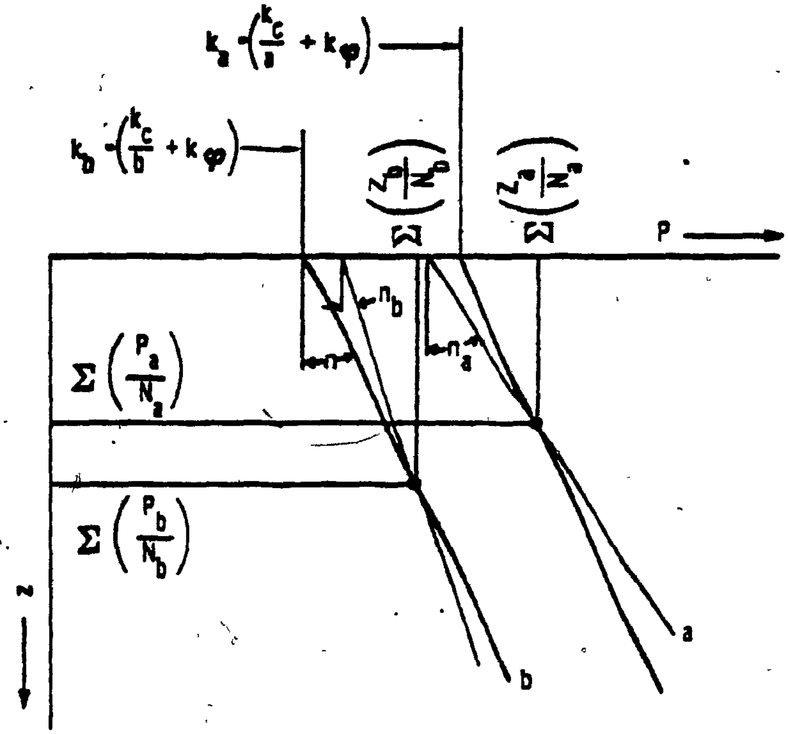

Figure 2.2 Interpolation of k_c, k_ϕ and n values from tests performed with two plates having radii a and b, or width 2a and 2b respectively. $n = (n_a + n_b)/2$. N_a and N_b denote numbers of test points related to each plate (Bekker, 1966)

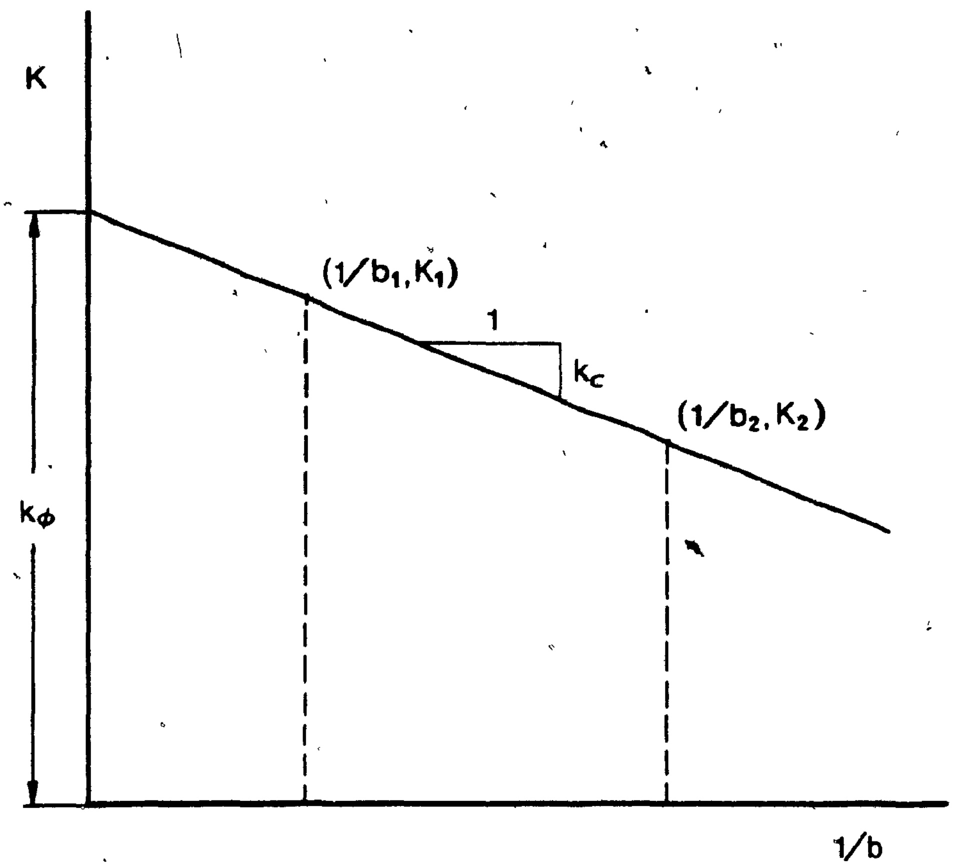

Figure 2.3 Solution for k_c and k_ϕ graphically

The graphical method to straighten out the experimental P(Z) curve does not need a great deal of calculation, and seems easy and fast. However, because several adjustments have to be made to the lines in practice, the real process might not be so easy to handle. The first problem is that the transformed P(Z) line is seldomly straight. This is because, essentially, the exponential equation is only an approximation to the real P(Z) relation. Moreover, even if the exponential equation is an accurate model of the pressure-sinkage process of soil, the variability of soil would distort the experimental curve, and hence deflect the transformed line. A best fit line has to be found from the raw data.

Secondly, the slopes of the two transformed P(Z) lines are usually not the same. Even though n is regarded as a constant for a given soil, the soil itself is not constant. Homogeneity in theory is an idealization. In practice, it is a relative concept, referring to an average effect on the whole. The variability of soils will be discussed in Section 2.5. In this case, the values of n have to be averaged to get a common value of n. Subsequently, the centers of gravity of the two straightened transformed lines should be found as pivots about which to reorient the two lines, in accordance with the common value of n, in order to assess the intercepts, K_1 and K_2.

In making these adjustments, it is difficult to reach the desired objective by visual inspection. Meanwhile, a small difference in making these adjustments might lead to a large difference in the final results. It is not uncommon that different investigators draw totally different conclusions from the same set of P(Z) data (Wong, 1980, 1983). One cannot verify these conclusions without a knowledge of the error involved in their curve fitting step.

Aiming towards an objective judgement, various methods have been proposed in the past to find the best fit line. The minimum error method, proposed by Reece in 1964, was most frequently quoted. In executing this method, three parallel straight lines are drawn, the first passing through two pre-determined limit data points on the log-scaled P(Z) curve, the second tangent to this log-scaled curve, and the third mid-way between the first two straight lines. The third is regarded as the best fit line. Besides being objective, this method is superior to the previous graphical method since it enables the estimation of the error involved in the fit (Reece, 1964a). Such an estimate informs the investigator as to how well the exponential P(Z) function matches the experimental data. If the matching is poor, the subsequent prediction based on the exponential function will be less reliable.

Through the foregoing discussion, it is clear that two goals have been sought for in the improvement of the processing method of penetration data, namely objectivity and a measure of error. Without these two criteria, it is hard to reconcile the resulting of different conclusions drawn from the same set of measurements.

To this end, a numerical method of curve fitting can function more reliably. At the cost of long and tedious calculations, this type of method leaves less room for subjective influence, and gives more likelihood of a precise estimate of soil parameters. However, because this method involves an analysis of the experimental curves and numerous calculations, it was once not favoured by some researchers as "impractical" as an engineering method (Reece, 1964a; Wills, 1966). This was true before electronic computing machines became popular.

The invention of the micro-processor in the mid-1970's and its proliferation thereafter have opened a new dimension in various fields of research. Micro-processor based automatic data acquisition and processing is replacing the traditional techniques. Aided by these new devices, numerical curve fitting yields faster and more accurate results for terrain investigations (Wong, 1980). The numerical method, which was once avoided because of its association with computing machines, now becomes preferable for just the same reason.

The ordinary least squares method is the basic method in fitting pressure-sinkage data to straight lines in log-log scales. Meanwhile, the weighted least squares method was also introduced into terramechanics studies to accomodate certain situations. In field penetration tests, the unevenness of the terrain surface usually causes large data scatter and makes the data from shallow penetrations less reliable. In the curve fitting , a correction could be made by increasing the weight of data of deep penetration depths and decreasing the weight of data of shallow penetration depths. To this effect, the values of pressure, P, could be selected as the weight in the regression (Wong, 1980, 1983).

In this investigation, as explained in later sections, all the merits offered by numerical curve fitting, namely objectivity, accuracy and estimate of error, are of assistance. This method will not only be used in the fitting of log Z vs. log P data, but also in the evaluation of the K-1/b relation. In the second assessment, a weighted least squares method might be necessitated.

2.4 Validity of Bekker's equation

Soil pressure-sinkage relations and two-plate penetration tests, along with Bekker's study in shear stress-strain relations, are the corner-stone of Bekker's terramechanics study. On this basis, a comprehensive theory has been developed and refined. Bekker's work has inspired many workers to follow his direction or work hard to explore new roads. Since he proposed his pressure-sinkage equation, more than twenty years have passed. The equation has been used, checked and questioned. Discussion as to the validity of the equation has never stopped.

Some of the questions concern the rationality of the whole strategy of Bekker's approach. Karafiath and Nowatzki (1978) argued that the two different mechanical processes (elastic and plastic deformation of soils) involved in the plate penetration test cannot be represented by a smooth exponential function in the forms of Berstein-Goriatchkin or Bekker.

Whenever referring to the pressure-sinkage equation, Bekker would stress that this equation was acceptable "for practical purposes" as an approximation, instead of an accurate mechanical model. Whether the approximation is acceptable should be judged by the practicality and usefulness of the results which it produces in the solution of engineering problems (Wong, 1984). In its practical application, Bekker's method has achieved a certain record of success (Costes et al., 1972).

However, in the practical aspect, Bekker's pressure-sinkage equation is not free from question. Many workers found that k_c and k_ϕ seemed not to be real constants, but plate-size related (Wills et al., 1965, 1966). As mentioned in Section 2.1, the purpose of introducing two parameters k_c and k_ϕ to replace K is to eliminate the size effect on the basic parameters of the equation. Only parameters immune from the influence of plate size can be used in the prediction of the sinkage of an arbitrarily selected loading area (Hegedus, 1965). If k_c and k_ϕ do not remain constant, then they are not ultimately suitable to replace K.

Wills (1966) conducted penetration tests to check the variability of the soil parameters in Equation (2.5). An important part of his investigation was to test with a family of plates covering a large range of sizes, from 5 - 25 cm. Wills found that k_c and k_ϕ "vary appreciably even over a limited range of footings". As for the exponent n, Wills observed "significant variation" but concluded that "n is in fact approximately constant".

The results of the investigation by Wills is illustrated in Figure (2.4). "To evaluate k_c and k_ϕ," Wills noted, " two model footing tests are now required." Proceeding in this way, the variations of k_c and k_ϕ are apparent when different pairs of data points are used to extract k_c and k_ϕ (Figure 2.4).

However, Wills' conclusion with reference to the considerable variation in k_c and k_ϕ, when plate size changes, is only one way to interpret the phenomenon presented in Figure 2.4. And interpreting Figure 2.4 is the main issue here. A more direct description of Figure 2.4 is that the (1/b, K) points have not all fallen onto a straight line. The points might have exhibited another systematic relation between 1/b and K. But they did not, otherwise Wills would have reported it. The data points simply did not fall onto a straight line and had no systematic pattern.

Now, if the relation between 1/b and K is exactly the same as that suggested by Bekker, i.e.

$$K = k_c / b + k_\phi \tag{2.8}$$

is it possible for all the data points of (1/b, K) to fall exactly onto a straight line, as suggested in Equation (2.8)? If the mechanical properties of soils are

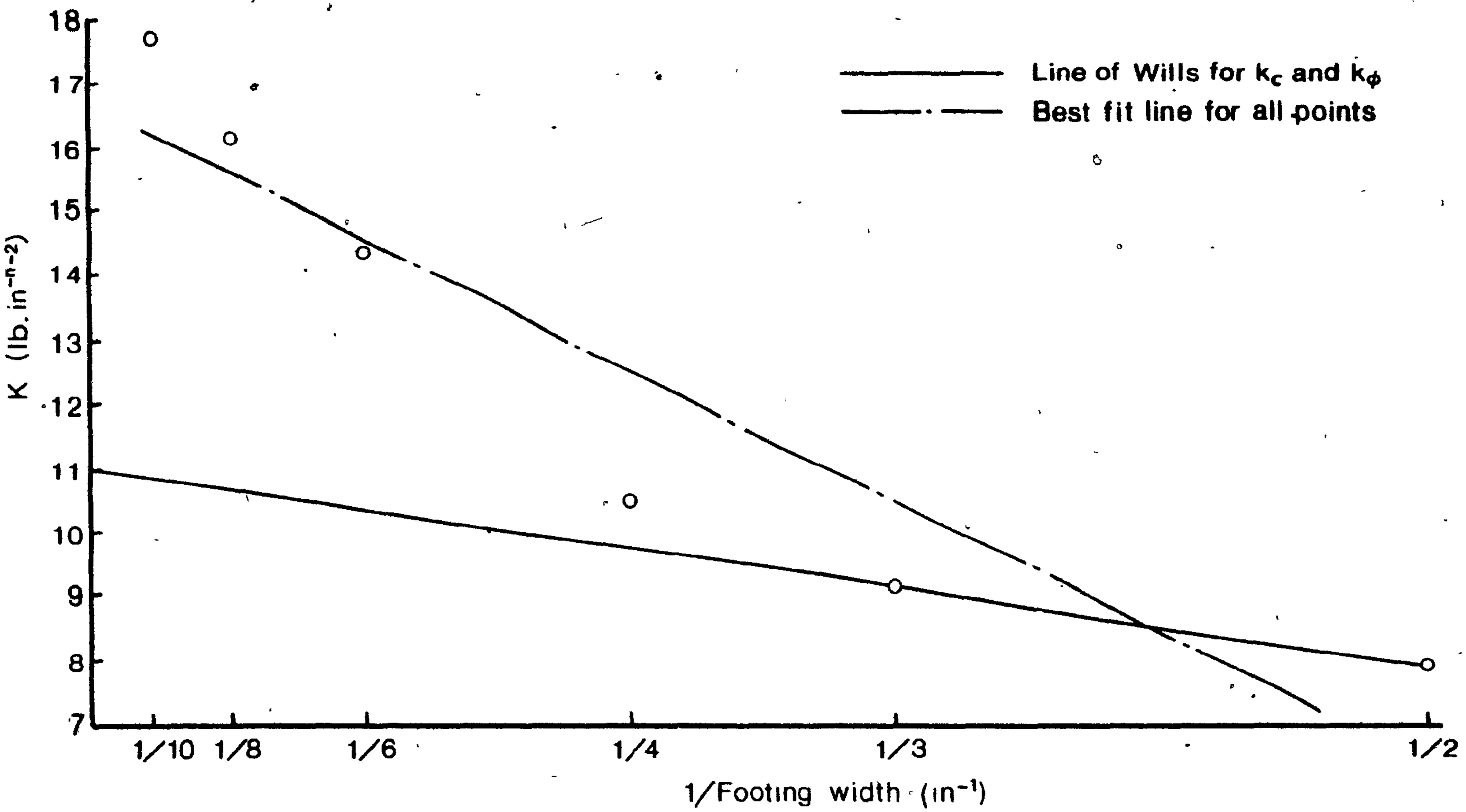

Figure 2.4 Experimental and theoretical pressure sinkage moduli (Wills, 1966)

deterministic, free from any kind of variation, the answer is yes. Joining any two of the (1/b, K) points would make no difference to k_c and k_ϕ. On the other hand, if soil variability is inevitable, the points will also inevitably fall off the straight line. Large differences might exist among the slopes and intercepts of the lines drawn through different pairs of (1/b, K) points, as demonstrated in the next section. But in the first place, the two-point technique is not a proper method in this case.

So the question is whether the relationship between 1/b and K should be treated deterministically or probabilistically. This will be discussed in the next section.

2.5 Variability of soil samples

Equation (2.5) holds only for homogeneous soils. But soils are complex in composition, changeable in texture and weak in strength, so their mechanical charactersitics are vulnerable to any small disturbance. "Soil is never uniform in the field or even in a laboratory soil bin" (Janosi, 1964).

Because of soil variability, "in determining P(Z) functions, an individual penetration test is unreliable". "Even with an almost perfectly graded granular dry soil sieved from a constant height under controlled humidity, changes in density and particle configuration can cause enough error in penetration tests that a number of measurements must be evaluated." However, "when repetitive tests are made in a number of spots of the sample area, natural soil may display such a scatter of loads for the given sinkage that any size error may be expected, and even data from controlled laboratory tests, ..., may indicate soil nonhomogeneity similar to that prevailing in the field". In other words, "even the best laboratory methods applied to the 'easiest' soil, dry sand, to reproduce its original condition cannot prevent the scatter of measured data" (Bekker, 1969).

From a single test arrangement, i.e. without changing plate size, one gets a set of curves spreading in a band (Reece, 1964a). According to Bekker, the width of the band is about 20% of the average load for wet loam and well graded pumice when the sinkage is larger than 4 cm. In the sinkage range less than 4 cm, the scatter is even larger, possibly near 37% of the average load.

All these individual curves from the same plate are different in their values of K and n. K and n are not deterministic. They are random variables. "Soil behavior, and consequently, vehicle behavior are both statistical". "Even the most extensive scatter of P(Z) data is not an obstacle in locomotion evaluation, ..., for the 'mean' soil characteristic produces the 'mean' vehicle performance; and this is the only observable parameter in any field and laboratory exploration" (Bekker, 1969). By repetitive penetration, a pair of K and n are available as sample means. They are better estimates of the population means of K and n. But they are still estimates.

2.6 Evaluation of k_c and k_ϕ

From his tests and analysis, Bekker recognized that K is a function of plate size b, and this function could be expressed, for practical purposes, as Equation (2.5),

$$P = (k_c/b + k_\phi)\, Z^n$$

From here on, this function will be taken as a verified approximation of the K-b relation. Meanwhile, the results presented by Wills (1966) are also verified facts, i.e. the mean n and mean K are still random variables. Wills' finding stressed one thing: Two plates are not enough to evaluate k_c and k_ϕ.

In fact, the scatter of K values from repetitive penetrations of one plate tends to be comparable to the scatter of P(Z) curves, which is, as mentioned before, generally large. Comparing such two P(Z) curves at the same sinkage level Z, the relation between the deviation of P and the deviation of K is

$$\Delta P/P = \Delta K/K + \ln Z \Delta n \qquad (2.9)$$

where ΔK, Δn and ΔP are deviatives of corresponding variables respectively. As noticed by Wills, the deviations of K and n often take opposite directions. When this happens, there is

$$\Delta K/K > \Delta P/P \qquad (2.10)$$

The repetitive penetrations can bring the sample means of K and n nearer to the population means, but may not eliminate the deviations. In the two-plate approach to k_c and k_ϕ, the deviations of K are transferred into k_c and k_ϕ without correction. In fact, the deviations might become much larger in these two values.

Suppose, in Figure 2.3, the value of K_1 (from the larger plate b_1) varies a small amount ΔK_1 from its population mean, K_2 (from the smaller plate b_2) equals exactly its population mean, the values of k_c and k_ϕ would change respectively by Δk_c and Δk_ϕ,

$$\Delta k_\phi = \Delta K_1 \;/\; (1 - b_2/b_1) \qquad (2.11)$$

$$\Delta k_c = \Delta K_1 \, b_2 \;/\; (1 - b_2/b_1) \qquad (2.12)$$

The ratio of b_2/b_1 is normally close to one due to the following reasons. The smaller plate is usually larger than 10 cm in diameter to avoid excessive influence of the localized soil nonhomogeneity. The upper limit of size of the larger plate is subjected to the limitation of loading capacity of the test equipment.

The near unity ratio of b_2/b_1 causes a larger deviation of k_ϕ, as suggested in Equation (2.11). The deviation of k_c is affected by the ratio b_2/b_1, and is also proportional to b_2. Increasing the plate size might be a method to subdue the effect of localized soil nonhomogeneity, but it might bring more error to k_c, as suggested in Equation (2.12).

The relative changes of k_c and k_ϕ are also high:

$$\Delta k_c / k_c = \Delta K_1 / (K_1 - K_2) \qquad (2.13)$$

$$\Delta k_\phi / k_\phi = \Delta K_1 / (K_1 - K_2\, b_2 / b_1) \qquad (2.14)$$

They are both larger than $\Delta K_1 / K_1$.

Since K is recognized as a random variable, there is no special reason to cling to the two-plate approach. By including more plates in penetration tests and obtaining more data points of (1/b, K) as Wills did, k_c and k_ϕ can be assessed more reliably by the regression method. Just as an individual penetration curve is unreliable in determining the P(Z) function, the two-plate approach is also unreliable in determining K(1/b) parameters. The principle of statistical evaluation of the general trend should not only be applied to the P(Z) function, but also to the K(1/b) function. In the two-plate approach, the values of k_c and k_ϕ reflect more the relative magnitude of two sample means, K_1 and K_2, rather than the general trend. In the multi-plate approach, unless the K(1/b) relation is not linear, the resulted k_c and k_ϕ would not be so misleading.

Another important merit of the multi-plate penetration test is, along with determining k_c and k_ϕ, that the error involved in the estimate is measurable. By reference to the goodness of fit, one might know how well the pressure-sinkage equation might perform in the prediction of the P(Z) relation of a larger plate. In the two-plate approach, only one pair of k_c and

k_ϕ values are calculated without any knowledge of accuracy.

The K(1/b) data reported by Wills (1966) in Figure (2.4) were examined in light of the above considerations by this author. Data (1/b,K) were read from the reported curves and fitted into a simple linear model with 1/b as the independent variable and K as the dependent variable, $K = k_c \, 1/b + k_\phi$. Thus k_c and k_ϕ were estimated as the regression coefficients.

$$k_c = -51.8 \ \text{kPa/m}$$

$$k_\phi = 1617.5 \ \text{kPa/m}$$

The correlation coefficient is -0.93. In a footing size range of 5 - 25 cm, the linear relation of Equation (2.8) matches the data to a substantial extent. For purposes of engineering application, the linear model could be regarded as an acceptable approximation of the relationship between K and 1/b.

2.7 Summary

Bekker's pressure-sinkage equation, although formulated empirically, functions well within the framework of his terramechanics theory. However, in the conventional two-plate sinkage test, large variations of k_c and k_ϕ were observed. Recognizing the above facts, analysis shows that all the parameters in the pressure-sinkage equation are random variables and should be estimated correspondingly. The regression method should be applied throughout the processing of penetration data, including both the extraction of K and n, and the extraction of k_c and k_ϕ. For this treatment, multi-plate penetration tests are necessitated.

CHAPTER III

LABORATORY PENETRATION TESTS

3.1 Objectives and scope

From the literature review in the last chapter, it is recognized that Bekker's pressure-sinkage equation is a well established approximation to the P(Z) relation relevant to terramechanics studies. The exponential form of that equation had been found before Bekker's study, but the two-term expression of K is a result of Bekker's extensive experimental observations. Analysis on data of Wills' (1966) multi-plate investigation suggests that, over a range covering small model plates of 5 cm diameter to large plates of 25 cm diameter, the composite form of K approximates the relation between 1/b and K substantially, with a correlation coefficient of 93%.

It is also recognized that all the empirical parameters involved in the pressure-sinkage equation are statistical in nature. In the conventional two-plate approach, while n is always estimated on the basis of a large number of sinkage versus pressure data points, k_c and k_ϕ, which are also prone to large variation, are calculated usually by only two data points.

It is believed that multi-plate penetration tests would facilitate more accurate estimates of the soil parameters in question. By implementing regression precedures in the process of parameter extraction, the performance of the pressure-sinkage equation in prediction could be assessed objectively.

The objectives of this investigation are:

(1) To observe the pattern of variation of Bernstein's modulus of soil deformation, K with plate size b.

(2) To observe the variations of Bekker's soil moduli of soil deformation, k_c and k_ϕ in the two-plate approach.

(3) To evaluate the feasibility and benefits of a multi-plate approach in penetration tests.

A family of plates ranging from 3.5 cm to 6.5 cm in diameter was planned as model plates. The pressure versus sinkage relation of a plate of 15 cm diameter was to be predicted and measured. The penetrations were performed on loose dry sand.

3.2 Penetration equipment

The soil tank used was 2 meters long, 0.7 meters wide and 0.5 meters high. Within each soil preparation, five model plates (one group of 3.5 cm, 4.0 cm, 4.5 cm, 5 cm and 5.5 cm in diameter; and another group of 4.5 cm, 5.0 cm, 5.5 cm, 6.0 cm, and 6.5 cm in diameter) were tested down to 4 cm sinkage without a wall or bottom effect due to the tank size.

The penetration shaft was a rack of 2.54 x 2.54 cm^2 cross section, driven by a 1.5 kW DC motor through a pinion. To safeguard the equipment, the vertical movement of the penetration shaft was limited by the preset control arms and the corresponding limit switches.

This assembly was mounted on a carriage, which, supported by four 3.5 cm diameter wheels, could be moved freely by hand along the rails flanking the soil tank. The height of the rail was 1.5 m measured from the tank bottom. Before each penetration, four bolts fastened the carriage to the rails to ensure a rigid connection between the two. During the penetration the supporting wheels were suspended above the rails and did not sustain any load.

To this movable carriage, a plywood board with a width the same as that of the soil tank was clamped, serving as a scraper. After each soil preparation, the soil surface was levelled longitudinally by the scraper hand-pushed along the rails.

The vertical load on the penetration plate was sensed by a linear variable differential force transducer (Daytronic Model 152A-500) mounted at the lower end of the penetration shaft. The penetration plate was connected to the LVDT by a rod of 25 cm length and 2.54 cm diamter.

The sinkage of the plate was measured by a linear potentiometer mounted beside the penetration shaft. This measurement was also checked by a ruler fixed on the side of the penetration shaft.

The sinkage and the load were recorded simultaneously on a X-Y chart recorder. Calibrations on the load and the sinkage measurements were done before each test.

The small circular plates used as models were 3.5 cm, 4.0 cm, 4.5 cm, 5.0 cm, 5.5 cm, 6.0 cm and 6.5 cm in diameter. A large plate of 15 cm diameter was used to get a datum P(Z) relation for comparison with the predicted P(Z) relation. All the plates were chamfered to prevent side wall friction.

The experimental equipment is shown in Figures 3.1 to 3.3.

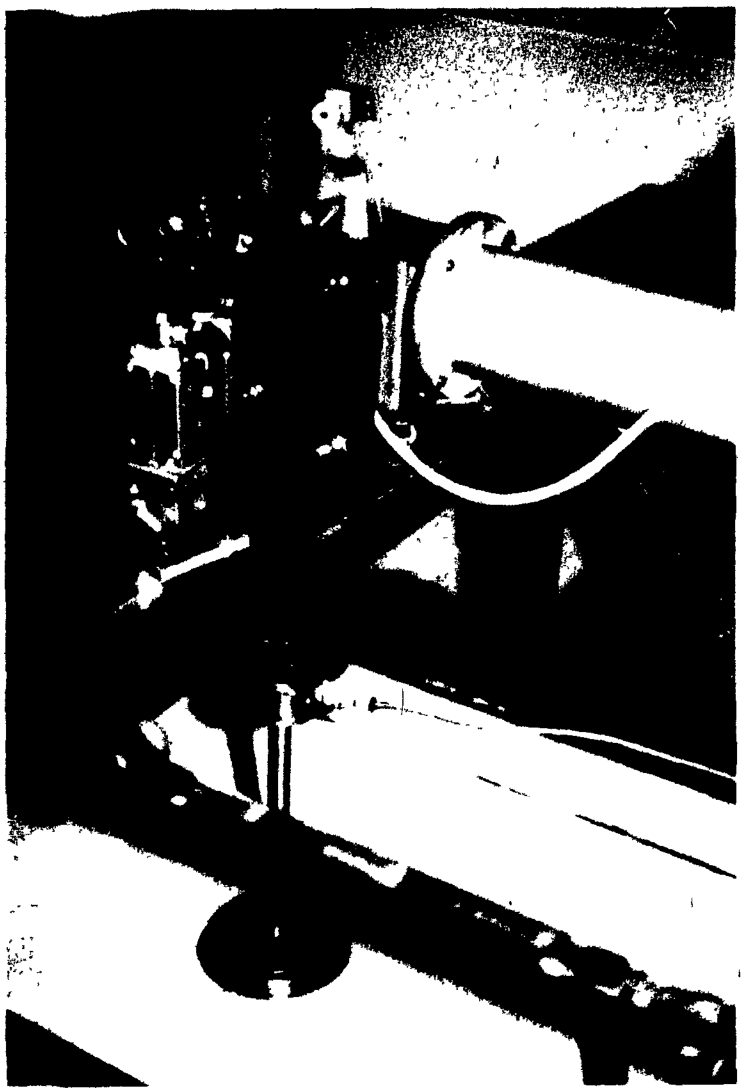

Figure 3.1 Penetrometer

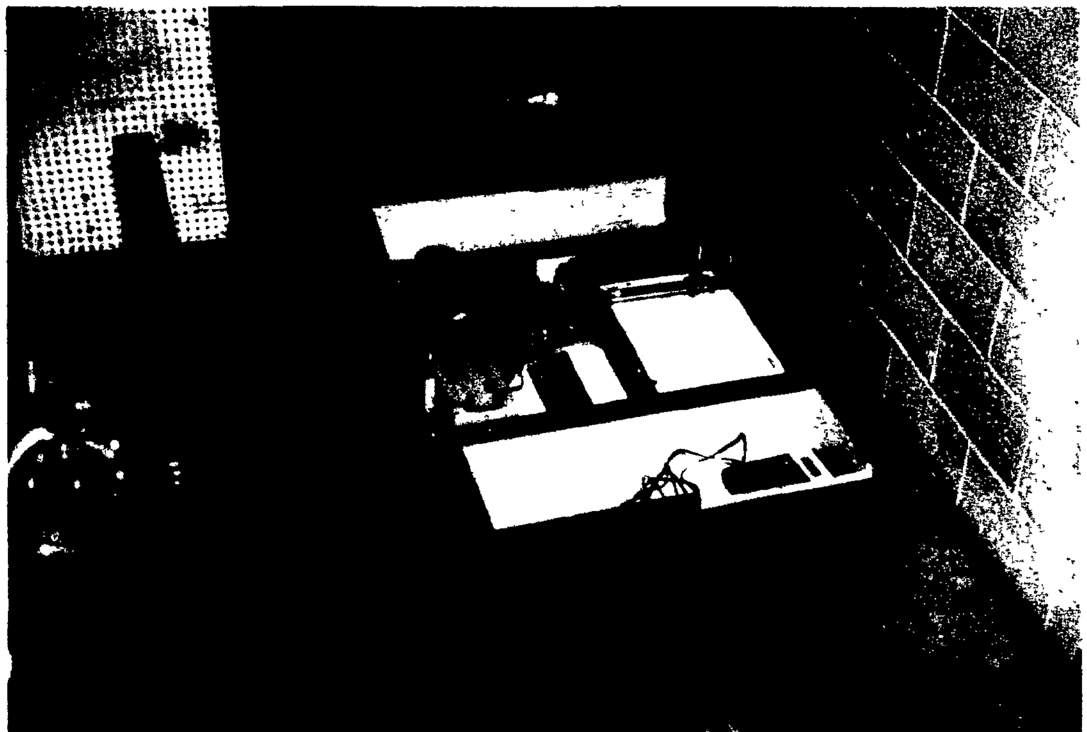

Figure 3.2 Control and recording equipment

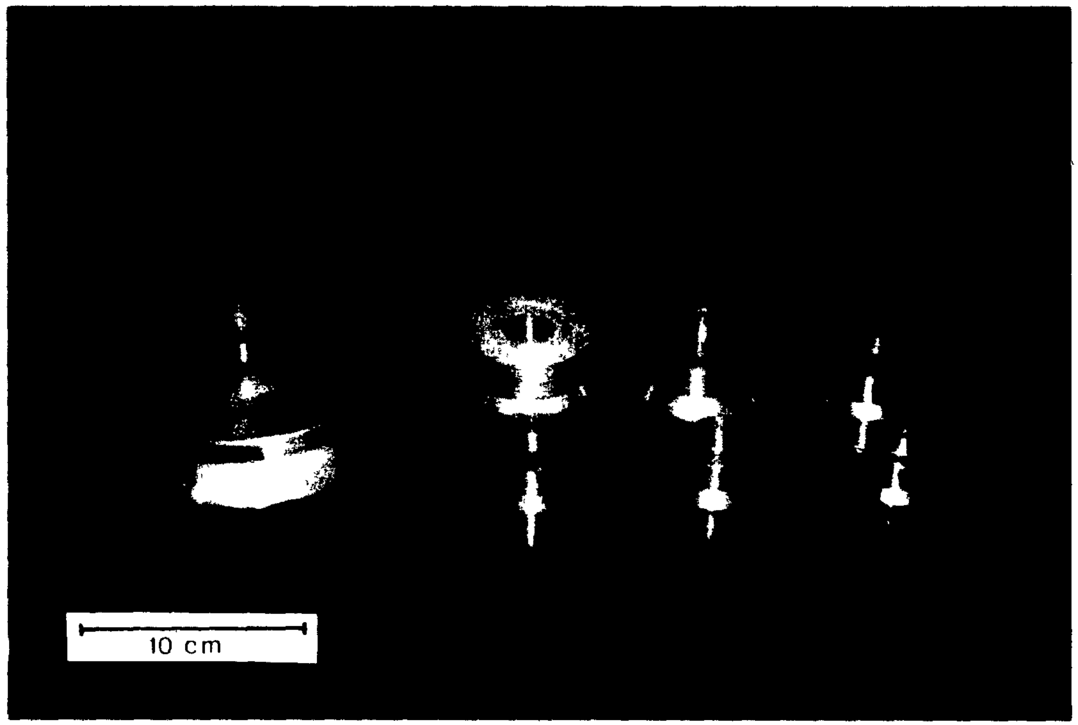

Figure 3.3 Plates for penetration tests

3.3 Soil characterization and experimental procedure

A dry medium-fine sand of particle density 2600 kg/m^3 was used in the tests. The angle of repose of the sand was 30°. The internal friction angle ϕ was measured by a Soiltest Sheargraph to be 26.2°. The grain size distribution is shown in Figure 3.4.

Prior to each run of tests, the sand was shovelled thoroughly to the full depth with uniform action. Then the sand was levelled with a rake for a fixed time. Finally, the scraper built on the carriage was pushed across the tank to bring the sand surface level.

Along the length of the prepared sand sample, a run of penetration tests with five small plates was carried out once. Five runs formed a series. There were four series, denoted by Series 1, 2, 3 and 4, in this investigation.

For the large plate of 15 cm diameter, penetration could be repeated three times in each sand preparation.

In agricultural machinery operation on soils of suitable consistency, wheel penetration is usually not greater than 3 cm (McKyes, 1980). Penetration in this investigation was limited to this range, and the penetration rate was kept constant at 1 cm/sec..

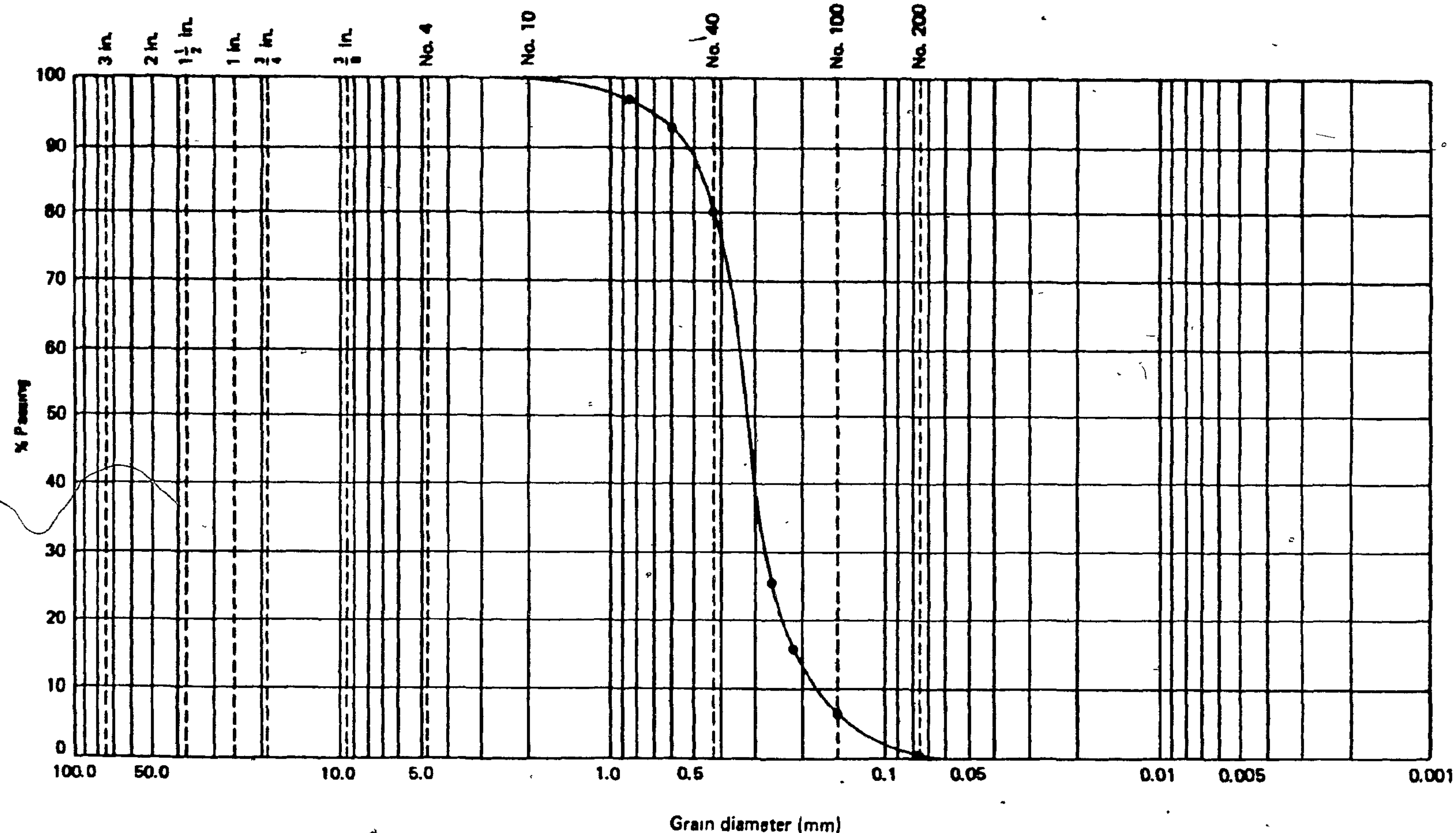

Figure 3.4 Distribution of particle size of test sand

CHAPTER IV

RESULTS AND DISCUSSION

4.1 Data processing

In this investigation, each test series included five runs of soil preparation and penetration. In each run, each of the five plates was used once. So five curves for each of the five plates were obtained in each test series.

Load readings were taken from the experimental curves at sinkage increments of 2 mm down to 20 mm. Ten data points were read from each curve. The loads were transformed into pressure by dividing by the loading area of the plate. For each plate a mean curve was obtained as an average of the five P(Z) curves. The results are shown in the Appendix.

The logarithms of the sinkage and pressure, (ln Z, ln P), were fitted to the equation

$$\ln P = \ln K + n \ln Z \qquad (4.1)$$

by the ordinary least squares method,

$$n = \frac{\Sigma(\ln Z \ln P) - (\Sigma \ln Z \Sigma \ln P)/N}{\Sigma(\ln Z)^2 - (\Sigma \ln Z)^2/N} \qquad (4.2)$$

where n is the slope of the best fit line, and is also the exponent of the pressure-sinkage equation,

$$P = K Z^n \qquad (4.3)$$

N is the number of data points used in the fitting, equal to 10 in this case.

As the value of n is different from curve to curve, a common n for the five mean curves of each test was calculated by taking an average of the five n's.

$$n_c = \sum_{n=1}^{5} n_i / 5 \qquad (4.4)$$

Then the value of K for the mean curve of each plate was calculated by

$$K = e^{((\Sigma \ln P - n_c \Sigma \ln Z)/N)} \qquad (4.5)$$

In each test series, five pairs of (1/b, K) values were obtained, where b is the radius of each plate. They were fitted into the composite form of K,

$$K = k_c/b + k_\phi \qquad (4.6)$$

to estimate k_c and k_ϕ. Both the ordinary least squares method (OLS) and the weighted least squares method (WLS) were used in this fitting. In the WLS fitting, (1/b) was selected as the weight. The consequences of WLS fitting will be discussed later.

The ordinary least squares method is as follows,

$$k_c = \frac{\Sigma(K/b) - \Sigma K \, \Sigma(1/b) \, / \, M}{\Sigma(1/b)^2 - (\Sigma(1/b))^2 \, / \, M} \qquad (4.7)$$

$$k_\phi = (\Sigma K - k_c \, \Sigma(1/b)) \, / \, M \qquad (4.8)$$

where M is the number of data points used in the fitting.

The weighted least squares method is

$$k_\phi = \frac{\Sigma(K \, b^2) - \Sigma(K \, b) \Sigma b \, / \, M}{\Sigma(b^2) - (\Sigma b)^2 \, / \, M} \qquad (4.9)$$

$$k_c = (\Sigma(K \, b) - k_\phi \, \Sigma b) \, / \, M \qquad (4.10)$$

The k_c and k_ϕ and n_c values were used in the pressure-sinkage equation,

$$P = (k_c/b + k_\phi) \, Z^{n_c} \qquad (4.11)$$

to predict the P(Z) relation of a large plate of 15 cm diameter. The predicted

P(Z) curve was compared with the mean curve from six penetration tests using the plate of 15 cm diameter.

4.2 Experimental P(Z) curves

For each plate, the five runs of penetration in each test series yielded a set of five curves (Figure 4.1). Compared at the same sinkage level, the standard deviation of the pressures readings from these five curves is less than 10% of the mean pressure (Table 4.1). This deviation decreases with the increase in sinkage depth. The width of the curve envelope is about 20% of the mean pressure, and also decreases with the increase in sinkage depth. Bekker (1969) reported that the spread of P(Z) curves of a single plate was about 20% of the average pressure at depths below 4 cm, and wider at less than 4 cm depth. The soil variation in this investigation is less than or comparable with those results.

The P(Z) relation in logarithmic plotting is not exactly a straight line (Figure 4.2). However, the goodness of fit is generally high, with $R^2 > 0.99$, which indicates that the linear relation approximates the ln Z - ln P relation substantially. The measured mean P(Z) curve is compared with the predicted curve in Figure 4.3. They are different in shape, but in the range of regression, the difference in the areas under the curves is very small.

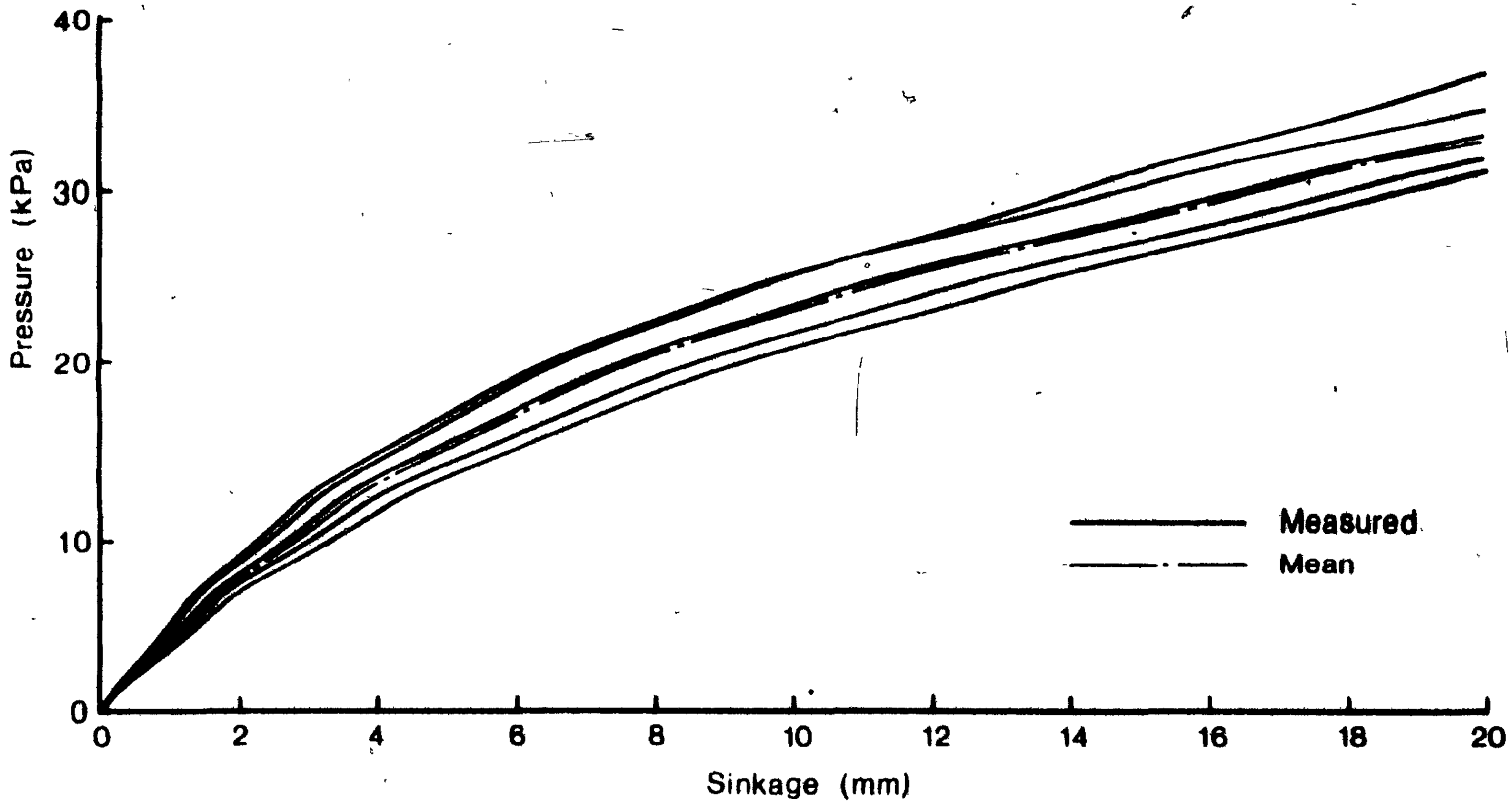

Figure 4.1 Experimental pressure-sinkage curves (Series 1, D=4.5 cm)

Table 4.1 Measured pressure vs. sinkage

(Series 1, D = 4.5 cm)

Sinkage (mm)	Pressure (kPa)							
	S1-1	S1-2	S1-3	S1-4	S1-5	mean	SE	C.V.
2	9.2	7.4	7.4	8.6	6.5	7.8	1.1	13.8
4	14.9	13.3	12.3	14.4	11.3	13.3	1.5	11.1
6	19.1	17.4	16.0	18.9	15.2	17.3	1.7	10.0
8	22.2	20.5	19.0	22.3	18.4	20.5	1.8	8.8
10	24.6	23.2	21.6	24.6	20.8	23.0	1.7	7.6
12	27.0	25.3	23.9	27.7	23.0	25.4	2.0	7.8
14	29.0	27.2	25.9	29.9	25.3	27.5	2.0	7.2
16	30.8	29.3	27.9	32.2	27.2	29.5	2.0	6.9
18	32.7	31.3	29.9	34.4	29.2	31.5	2.1	6.6
20	34.5	33.4	32.0	36.5	31.2	33.5	2.1	6.2

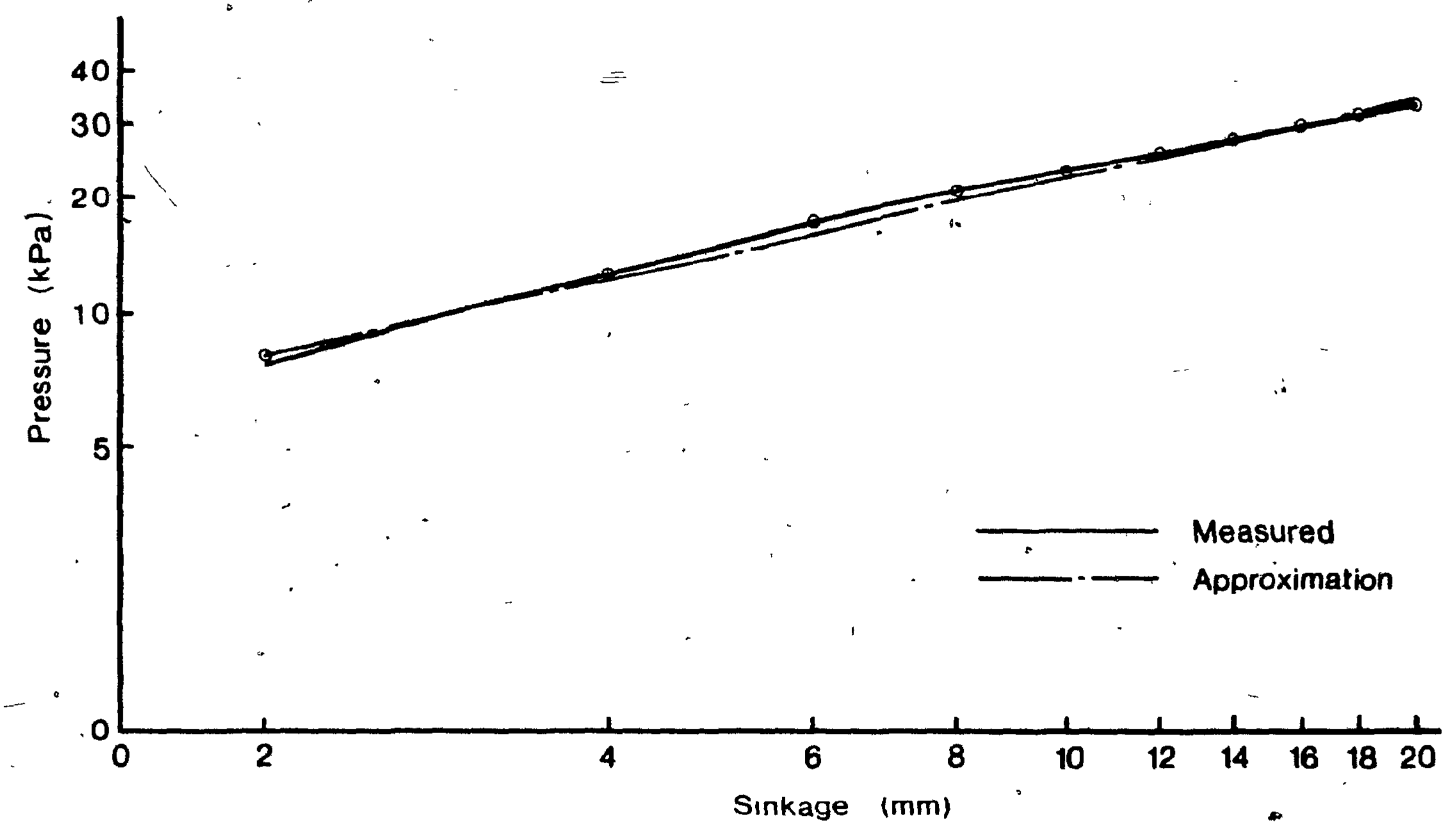

Figure 4.2 Natural logarithmic plotting of a mean curve (Series 1, D=4.5 cm) and its linear approximation

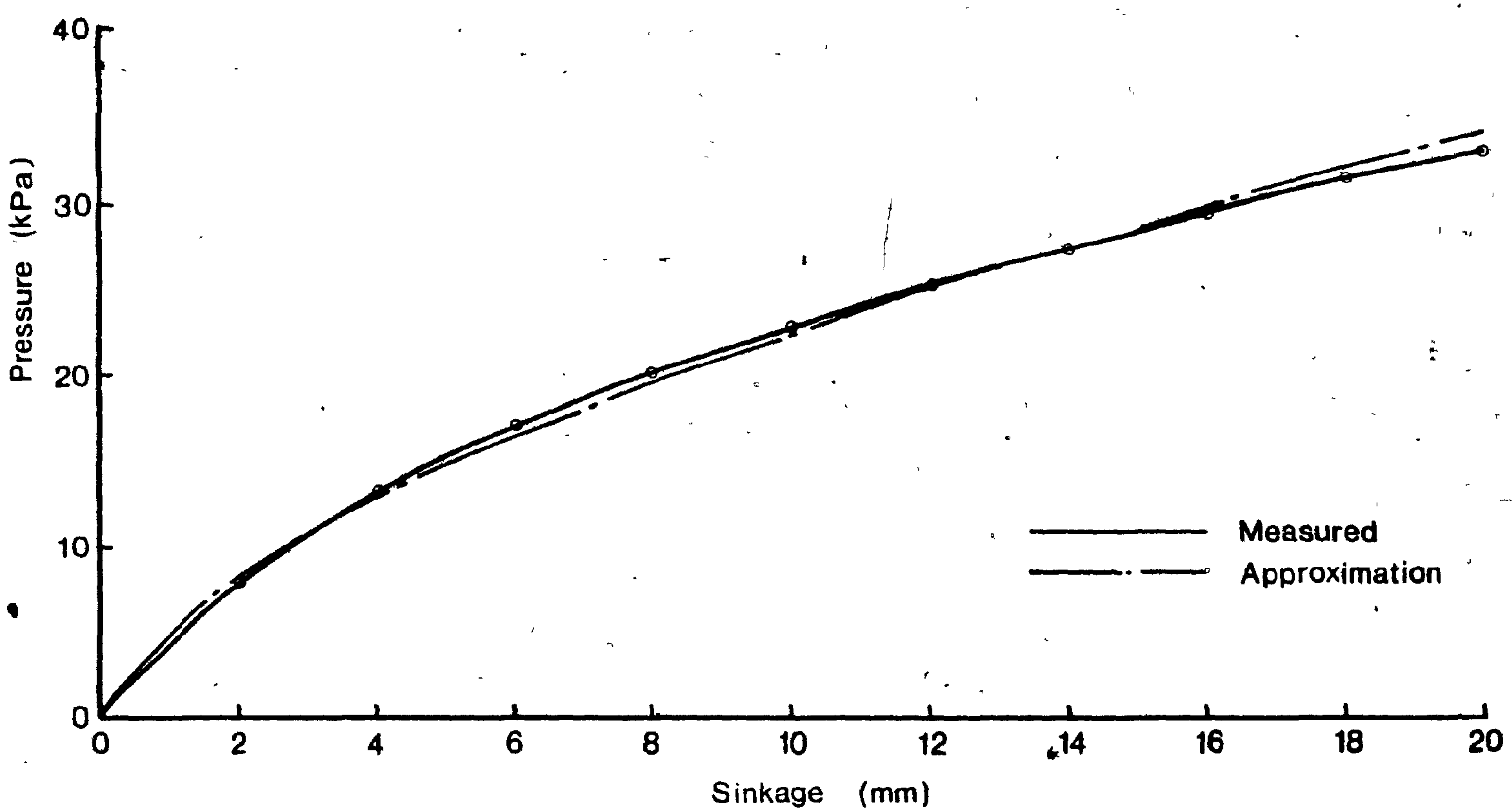

Figure 4.3 A mean pressure vs. sinkage curve (Series 1, D=4.5 cm) and its exponential approximation

The error of loads predicted is always less than 4% of the measured values. For practical purposes, the real P(Z) relation can be approximated quite accurately by the exponential equation $P = K\ Z^n$.

It is noticed that the least squares method is used here only as a convenient tool to assess the best fit line with which to approximate the true relationship between ln Z and ln P. If inferences about the regression line are to be made, a few assumptions about the raw data, i.e. the assumptions of normality, homogeneity, and additivity, have to be met. Here we have known that the transformed mean P(Z) curve deviates slightly but systematically from a straight line. The slight deviation makes it possible to approximate the true relation with a simple linear relation. The systematic deviation makes any further inferences about the regression line theorectically incorrect (Mead, 1983). This flaw in applying the least squares method in fitting of pressure-sinkage data had been mentioned by Reece (1964) without acknowledging that it is the inference part where problems occur . So even though "the method of least squares analysis is quite robust in that small or minor violations of the underlying assumptions do not invalidate the inferences or conclusions drawn from the analysis in a major way" (Chatterjee and Price, 1977), it is prefered to retain the theoretical integrity by refraining from making further inferences.

The results of fitting of the mean P(Z) curves are presented in Table 4.2. Since K values will be calculated by the adjusted n, and the adjusted n values vary with the number of (1/b, K) points used (see next section), Σln P,

$\Sigma \ln Z$ are presented in the table instead of the unadjusted $\Sigma \ln K$ for the convenience of reference.

Table 4.2 Exponential approximation of mean P(Z) curves
(Series 1)

Plate (D: cm)	n	Correlation coefficient	$\Sigma \ln P$	$\Sigma \ln Z$
3.5	0.6821	0.9993	29.25	-47.04
4.0	0.6167	0.9979	30.16	-47.04
4.5	0.6162	0.9965	30.51	-47.04
5.0	0.6008	0.9946	31.24	-47.04
5.5	0.6020	0.9961	31.27	-47.04

4.3 Estimation of k_c and k_ϕ

To examine the variation of Bekker's moduli of soil deformation, k_c and k_ϕ, in relation to the number of plates used in the parameter assessment, the five plates used in each test were combined into sub-groups of two plates, three plates and four plates. A common n_c was obtained for each of these sub-groups by averaging the n's in the sub-group. For example, in Series 1, the sub-group of plates of 5.5 cm diameter and 5.0 cm diameter has a common n of (0.6008 + 0.6020) / 2 = 0.6014; for the sub-group of plates of 5.5 cm, 5.0 cm and 4.5 cm diameter, the common n is (0.6008 +0.6020 +0.6162) / 3 = 0.6063.

K for each plate in a sub-group was calculated using this common n_c by

$$K = e^{((\Sigma \ln P - n_c \Sigma \ln Z)/N)} \qquad (4.12)$$

where N is the number of data points used in the calculation of n, equal to 10 in this investigation. For $\Sigma(\ln P)$ and $\Sigma(\ln Z)$, see the relevant columns in Table 4.2.

k_c and k_ϕ were estimated for each sub-group as the regression coefficients by fitting the data points of (1/b,K) into the composite form of K,

$$K = k_c(1/b) + k_\phi$$

Table 4.3 k_c and k_ϕ from two plates (Series I)

Plate group (D:cm)	n	K (kPa/m^n)	k_c (kPa/m^{n-1})	k_ϕ (kPa/m^n)
5.5 5.0	0.6014	382.216	0.738	355.367
5.5 4.5	0.6091	396.314	-3.132	510.219
5.5 4.0	0.6091	396.780	-2.795	498.427
5.5 3.5	0.6421	462.758	-3.890	604.228
5.0 4.5	0.6085	397.973	-6.304	650.125
5.0 4.0	0.6088	398.441	-4.079	561.599
5.0 3.5	0.6415	464.695	-4.891	660.355
4.5 4.0	0.6165	384.053	-2.378	489.728
4.5 3.5	0.6492	447.914	-4.176	633.506
4.0 3.5	0.6494	433.914	-5.273	696.671

Table 4.4 k_c and k_ϕ from three plates (Series 1)

Plate group (D:cm)	n	K (kPa/m^n)	k_c (kPa/m^{n-1})	k_ϕ (kPa/m^n)
5.5 5.0 4.5	0.6063	391.189 393.937 366.204	-3.207	512.918
5.5 5.0 4.0	0.6065	391.496 394.246 366.492	-2.034	469.752
5.5 5.0 3.5	0.6283	433.774 436.821 357.996	-3.945	585.087
5.5 4.5 4.0	0.6116	401.065 375.449 362.536	-2.850	503.968
5.5 4.5 3.5	0.6334	444.376 415.994 366.746	-3.749	581.412
5.5 4.0 3.5	0.6336	444.724 402.001 367.034	-3.659	579.599
5.0 4.5 4.0	0.6112	403.123 374.7434 361.854	-8.106	561.533
5.0 4.5 3.5	0.6330	446.656 415.222 366.057	-4.518	622.552
5.0 4.0 3.5	0.6332	447.006 401.245 366.344	-4.697	635.242
4.5 4.0 3.5	0.6383	425.694 411.052 375.298	-4.017	606.991

Table 4.5 k_c and k_ϕ from four plates (Series 1)

Plate group (D:cm)	n	K (kPa/m^n)	k_c (kPa/m^{n-1})	k_ϕ (kPa/m^n)
5.5	0.6089	395.988	-3.172	516.318
5.0		398.769		
4.5		370.696		
4.0		357.946		
5.5	0.6253	427.645	-3.888	575.882
5.0		630.649		
4.5		400.332		
3.5		352.938		
5.5	0.6254	427.897	-3.827	575.417
5.0		430.902		
4.0		386.799		
3.5		353.146		
5.5	0.6292	435.717	-3.869	579.417
4.5		407.888		
4.0		393.859		
3.5		359.600		
5.0	0.6289	438.158	-4.378	609.177
4.5		407.312		
4.0		393.303		
3.5		359.093		

using the ordinary least squares method (Tables 4.3 - 4.5).

The coefficients of variation of values of k_c and k_ϕ were calculated for the categories of 2-plate, 3-plate, and 4-plate groups. The results are presented in Table 4.6.

Table 4.6 Variation of k_c and k_ϕ (Series 1)

Number of plates	k_c (kPa/m^{n-1})			k_ϕ (kPa/m^n)		
	mean	SE*	C.V.(%)	mean	SE*	C.V.(%)
2	-3.62	1.93	-53.4	566.02	103.79	18.3
3	-3.67	0.79	-21.6	565.02	54.17	9.6
4	-3.83	0.43	-11.2	571.21	33.76	5.9

SE* : standard deviation

Because, in the two-plate groups, the values of k_c and k_ϕ depend totally on the relative magnitude of the two K values involved, their variations are generally high, especially the variation of k_c. By increasing the number of plates to three, the variation of k_c is reduced to 40% of that of two-plate groups, and the variation of k_ϕ is reduced by 50%. When four plates are used, the variation of both k_c and k_ϕ is reduced by another 50%. For sand, if a k_c variation of less than 10% is desired, at least five plates are needed.

A common n for five plates in each test series was calculated by averaging the five n values. K value for each of the five plates was re-calculated with this common n and plotted in Figure 4.4, for test Series 1.

By visual inspection, it is obvious that a linear relation is a reasonable approximation to the relationship between K and 1/b, even if it cannot be concluded that the true relation is exactly linear.

The best fit line was calculated using both the ordinary least squares method (OLS) and the weighted least squares method (WLS) (Figure 4.4). The values of k_c and k_ϕ obtained from the two methods are only slightly different (Table 4.7).

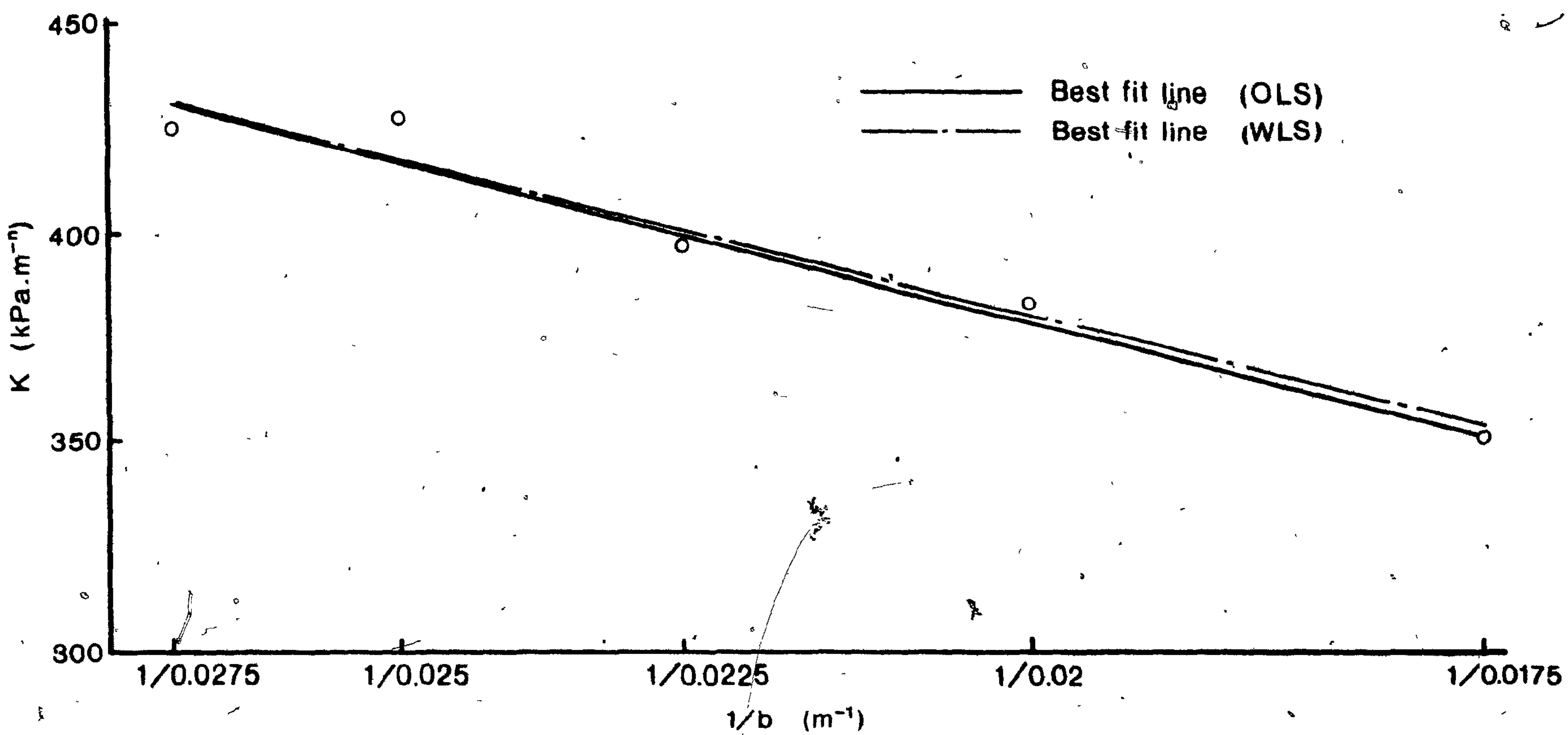

Figure 4.4 K-1/b relation fitted by the ordinary and weighted least squares methods (Series 1)

Table 4.7 k_c and k_ϕ estimated by OLS and WLS (Series 1)

Least squares method	k_c (kPa/m^{n-1})	k_ϕ (kPa/m^n)	Correlation coefficient	F
Ordinary	-3.777	568.68	0.9777	65.2
Weighted	-3.667	563.70	0.9973	553.3

Comparing the two regression lines, it can be seen that the residuals became relatively more uniform for the line of the weighted least squares method (Figure 4.4).

Bekker (1969) mentioned specifically that "smaller plates produced a wider scatter (of P(Z) curves) than the large (plates)". This is because a smaller plate is generally more susceptible to the localized soil inhomogeneity. While this phemomenon might not be always obvious in well processed soils, it is expected in most , especially soils in the field. The wider scatter of P(Z) curves of smaller plates will ultimately result in a larger variance of K values. This is also evident by examining the composite form of K, K = k_c / b + k_ϕ. Here k_c and k_ϕ are conceived as charactersitics of the soil physical state. Giving a small permutation to k_c , a larger deviation of K will be associated with a smaller b. If b is very small, e.g., at the 0.01 m level, the small permutation of k_c will be measured as a substantially larger deviation of K. Thus in general, the variance of K is inherently correlated with the plate size, b, and becomes larger when b gets smaller.

This is a type of heteroscedastic situation, in which the error variance is not constant over all the observations (Chatterjee and Price, 1977). A similar example was given by Chatterjee and Price, where in a linear relation of Y(X), the variance of dependent Y gets larger when the value of independent X gets larger (Figure 4.5). A basic assumption of the ordinary least squares method - the consistency of error variance - has been violated by the presence of unequal variances. "If the ordianry least squares method is performed on this type of data ignoring heteroscedasticity, the estimated coefficients are still unbiased, but will lack precision in a theoretical sense" (Chatterjee and Price, 1977).

The heteroscedasticity in Chatterjee and Price's example was removed by a transformation of the raw data. A new model was obtained by dividing both sides of the original model

$$Y = a X + b$$

by X to get

$$Y/X = a + b/X.$$

Then Y/X and 1/X were treated as dependent and independent variables, respectively.

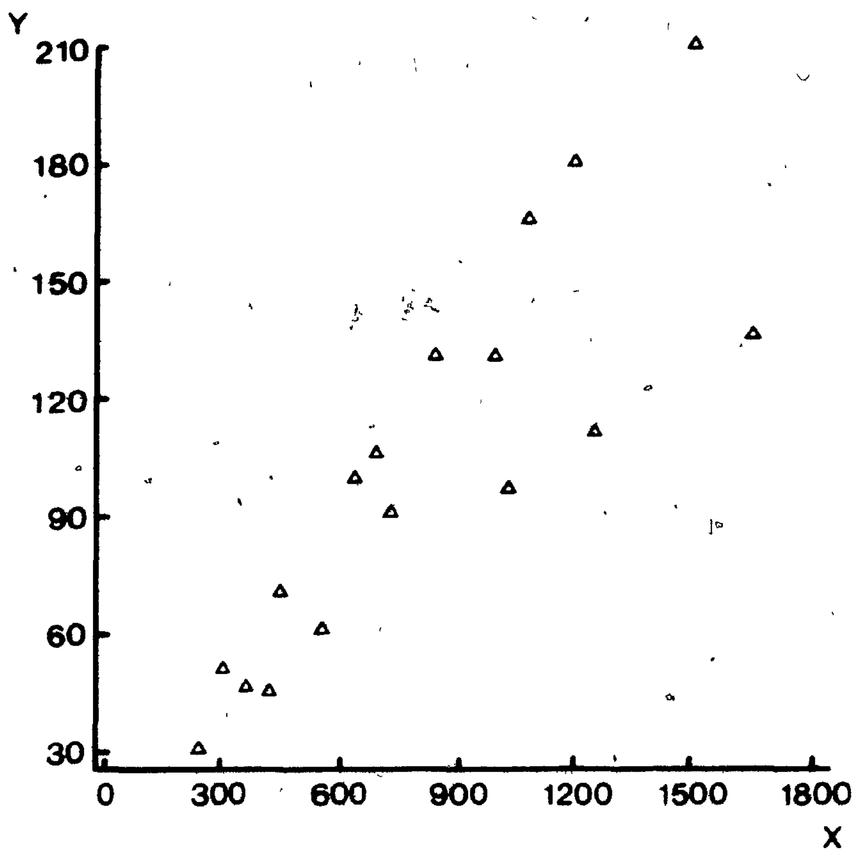

Figure 4.5 An example of heteroscedasticity (Chatterjee and Price, 1977)

The same transformation can be performed on the data of K for various plate sizes, b. Multiplying both sides by b, the equation K(1/b) becomes

$$K\ b = k_c + k_\phi\ b \qquad (4.13)$$

with (K b) and b as dependent and independent variables, respectively. The formula for k_c and k_ϕ in the weighted least squares regression (with 1/b as weight) have been presented in Section 4.1.

As was noticed in Figure 4.4 and Table 4.7, the two regression methods give almost identical results of k_c and k_ϕ. This fact suggests that the function of $K = k_c\ (1/b) + k_\phi$ is at least a good approximation of the "true" relation of K(1/b), since only when the two functions of K(1/b) and Kb(b) are algebraically equivalent could such a result be expected.

The pressure-sinkage relation for D = 0.15 m was predicted using the n, k_c and k_ϕ values obtained in the five-point regression. The resultant curve is compared with the measured pressure-sinkage curve in Figure 4.6. The maximum error of pressure prediction is 7% of the measured value. For most parts of the two pressure-sinkage curves, the discrepency is very small. Considering that the value of the parametric variable b in the prediction, 0.075m, is more than three times larger than the average b of the test small plates (b = 0.0225 m), the error of pressure prediction is quite acceptable.

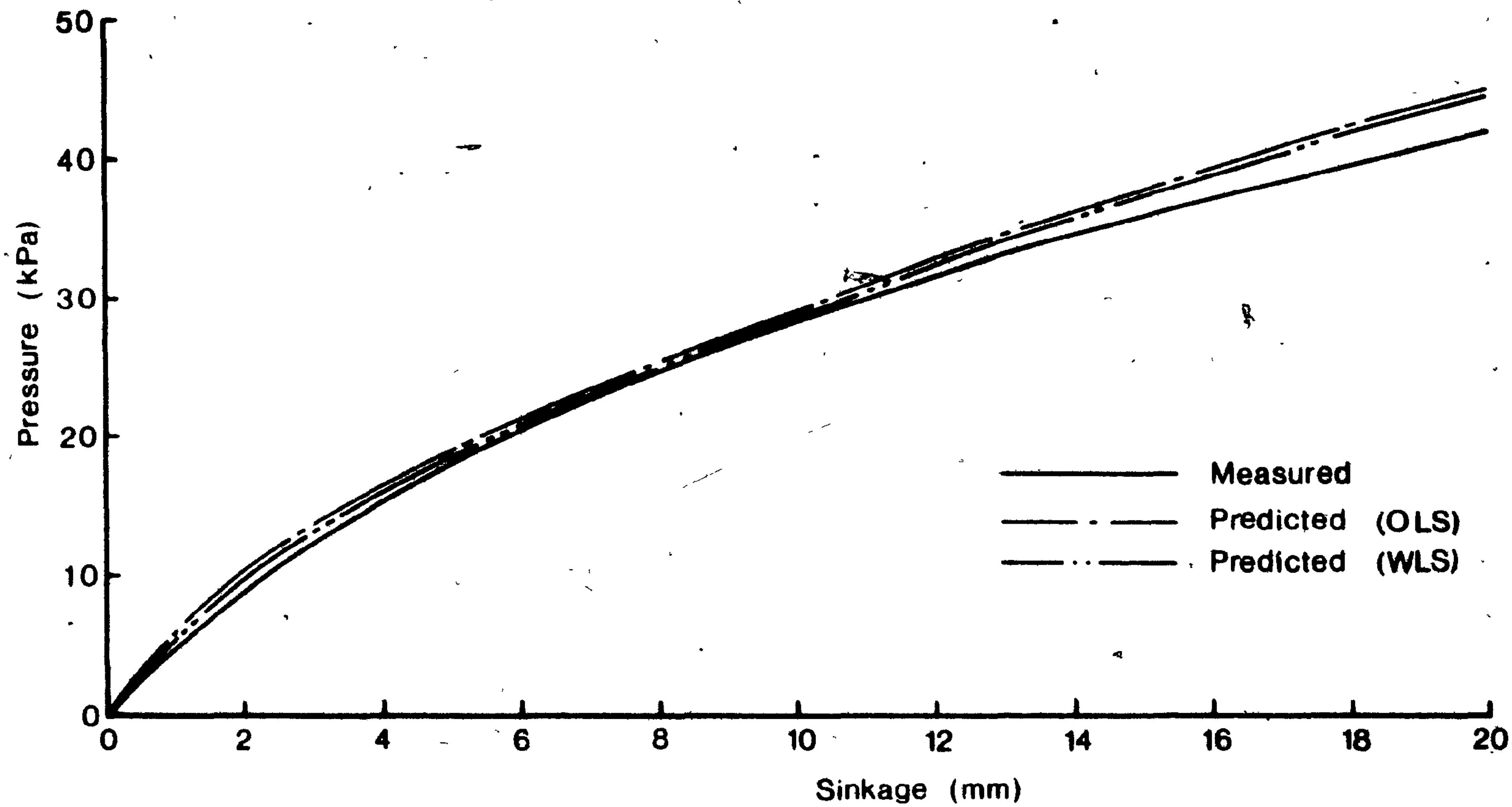

Figure 4.6 Comparison of the measured and predicted pressure vs. sinkage relationship of plate D=0.15 m (Series 1)

4.4 Results of Series 2, 3 and 4

Results of Series 2, 3 and 4 confirm the findings of Series 1. The variations of k_c and k_ϕ were high when only two data points were employed in the extraction. These varations sharply decreased when more data points were added (Table 4.8).

Table 4.8 Variations of k_c and k_ϕ (Series 2,3 and 4)

Number of	k_c variation (%)			k_ϕ variation (%)		
plates	S. 2	S. 3	S. 4	S. 2	S. 3	S. 4
2	-126.1	-52.9	-157.3	34.6	11.5	28.9
3	-32.9	-16.0	-54.5	8.5	3.4	10.0
4	-13.2	-8.4	-42.0	4.8	1.9	7.0

In Series 2 and 3, the reduction of variations of k_c and k_ϕ when the plate number changed from 2 to 3 and 4 are about 70% and 50% of the preceding values. In Series 4, the rates are 60% and 30%. Series 4 is an example of large variations in measurements. In this case, the use of five plates to determine soil properties cannot bring the variation of k_c below a value of 10%.

Both the ordinary and weighted least squares calculations were performed on K vs. 1/b data in the Series 2 to 4. The estimated k_c's and k_ϕ's from the two methods are almost identical (Figures 4.7, 4.8, 4.9; Table 4.9)

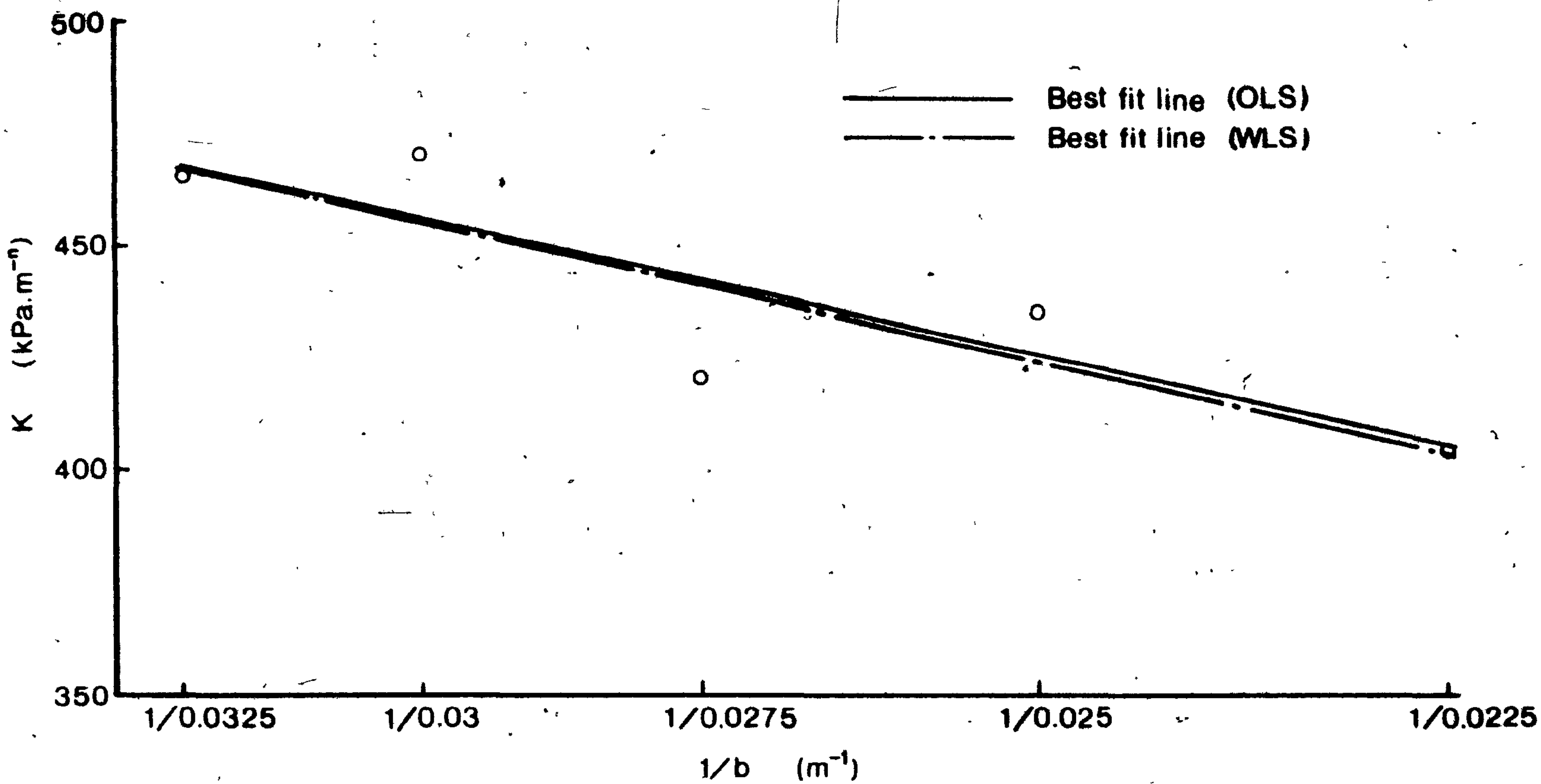

Figure 4.7 K-1/b relation fitted by the ordinary and weighted least squares methods (Series 2)

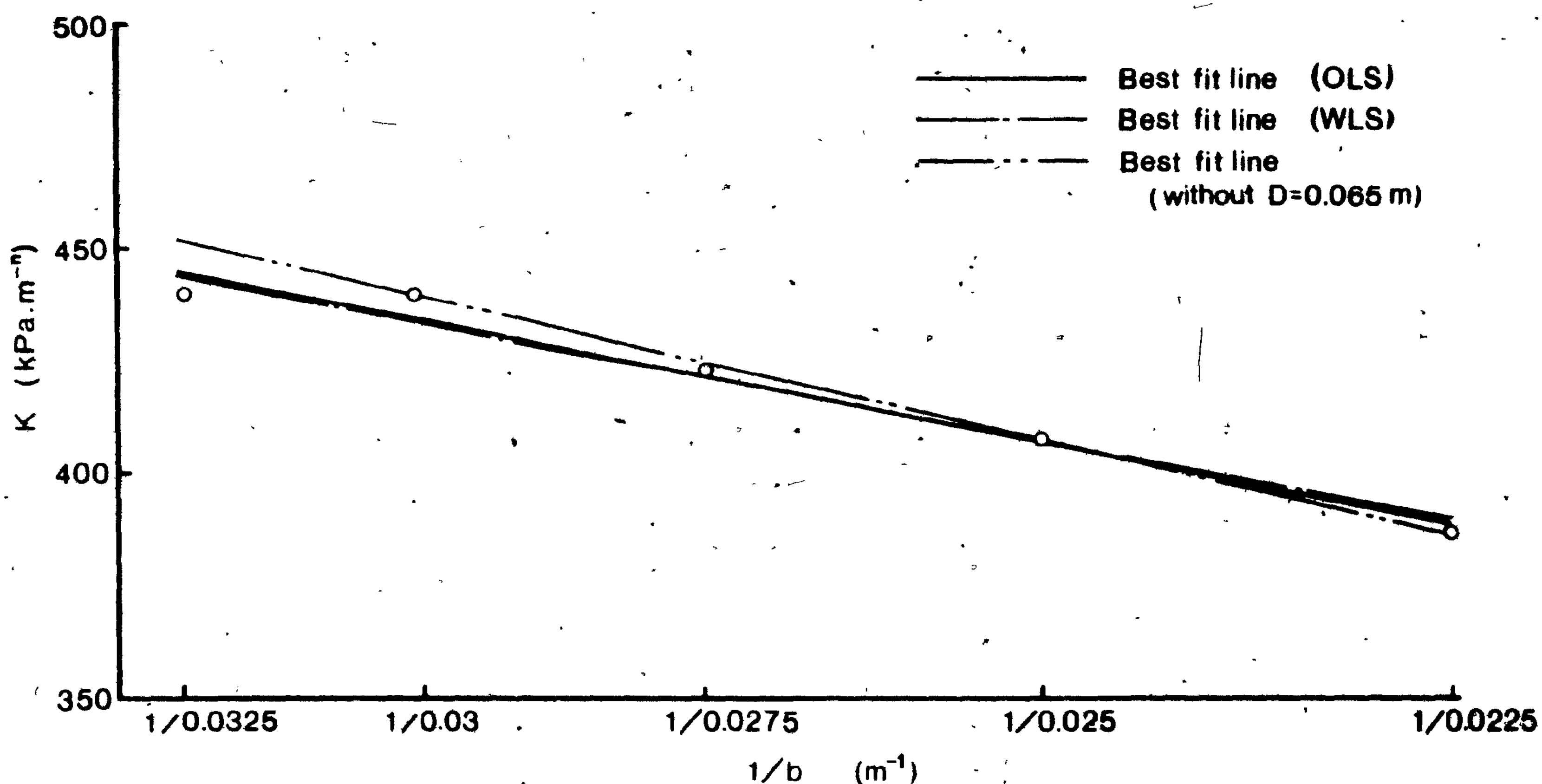

Figure 4.8 K-1/b relation fitted by the ordinary and weighted least square methods (Series 3)

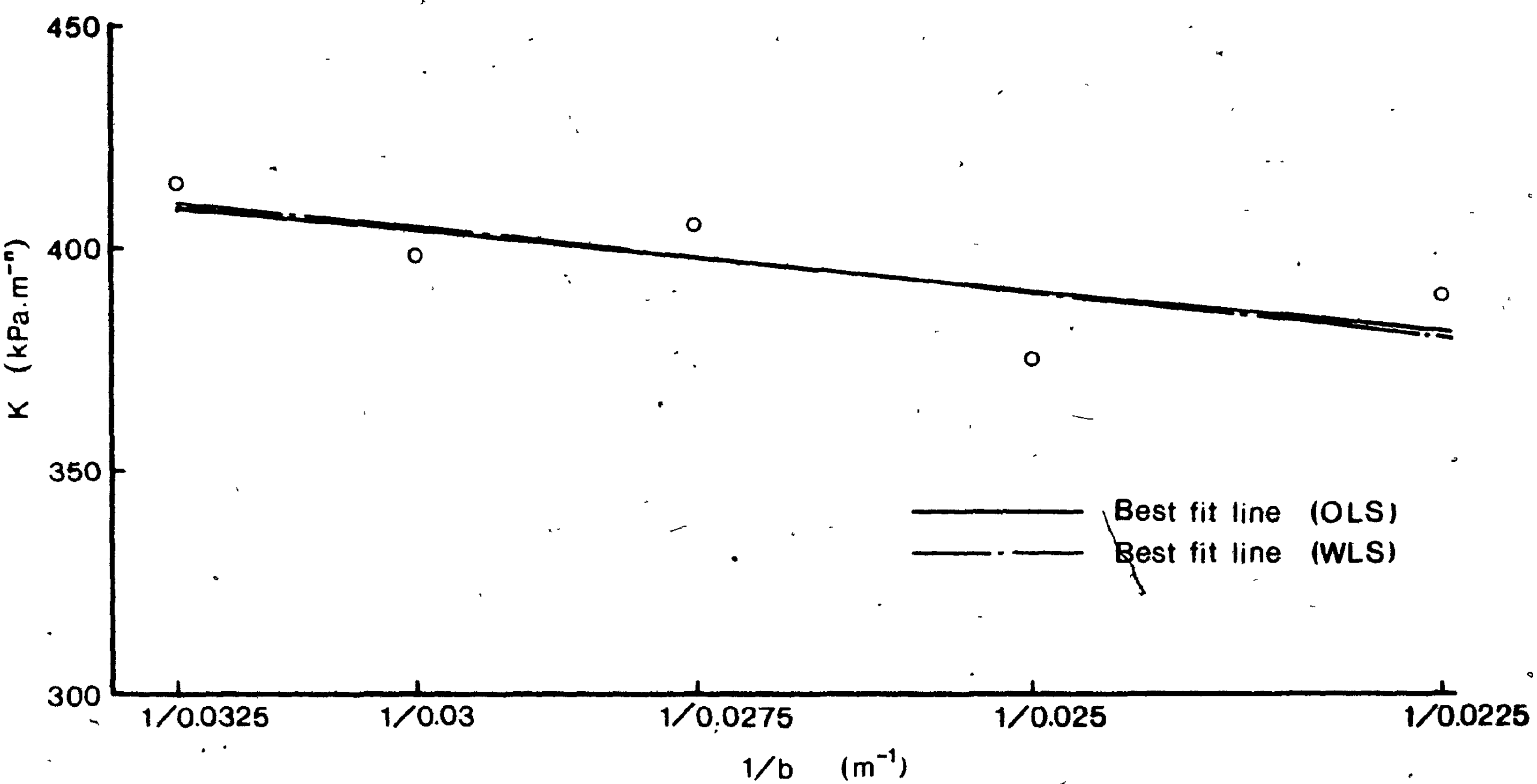

Figure 4.9 K-1/b relation fitted by the ordinary and weighted least squares methods (Series 4)

Table 4.9 k_c and k_ϕ estimated by OLS and WLS (Series 2, 3 and 4)

Series	Least squares method	k_c (kPa/m^{n-1})	k_ϕ (kPa/m^n)	Correlation coefficient	F
2	ordinary	-4.45	607.52	0.8677	9.4
	weighted	-4.62	610.23	0.9869	112.4
3	ordinary	-4.08	570.31	0.9840	91.5
	weighted	-3.96	565.97	0.9985	997.8
4	ordinary	-2.03	471.92	0.7351	3.5
	weighted	-2.22	478.73	0.9904	154.0

The predicted pressure vs. sinkage for the plate of 15 cm diameter are presented in Figures 4.10, 4.11 and 4.12. It is interesting to note that in most of the four series of tests, the P(Z) curves predicted using k_c's and k_ϕ's from WLS are nearer to the experimental curves than those from OLS, although the differences are very small.

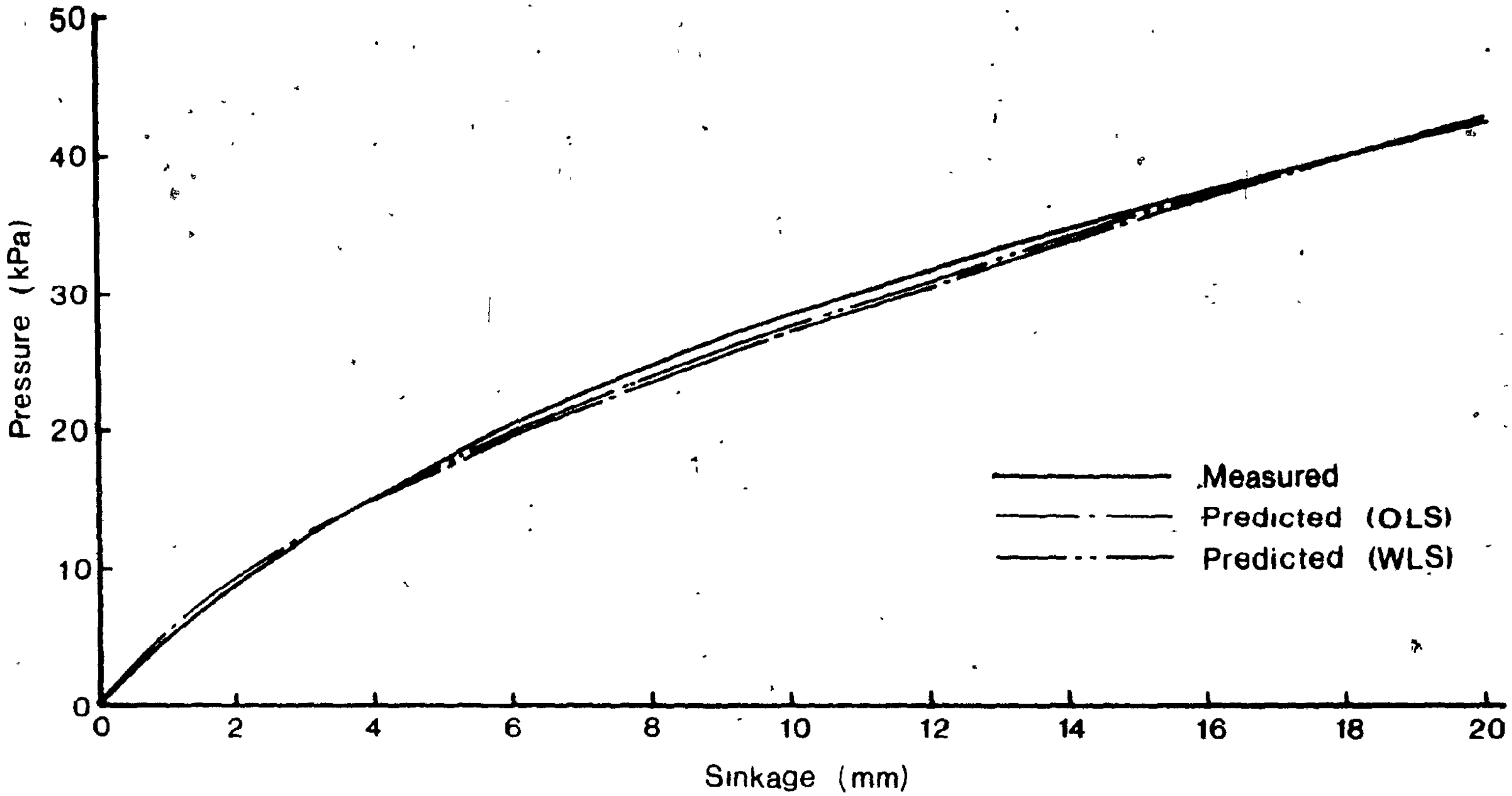

Figure 4.10 Comparison of the measured and predicted pressure vs. sinkage relationship of plate D=0.15 m (Series 2)

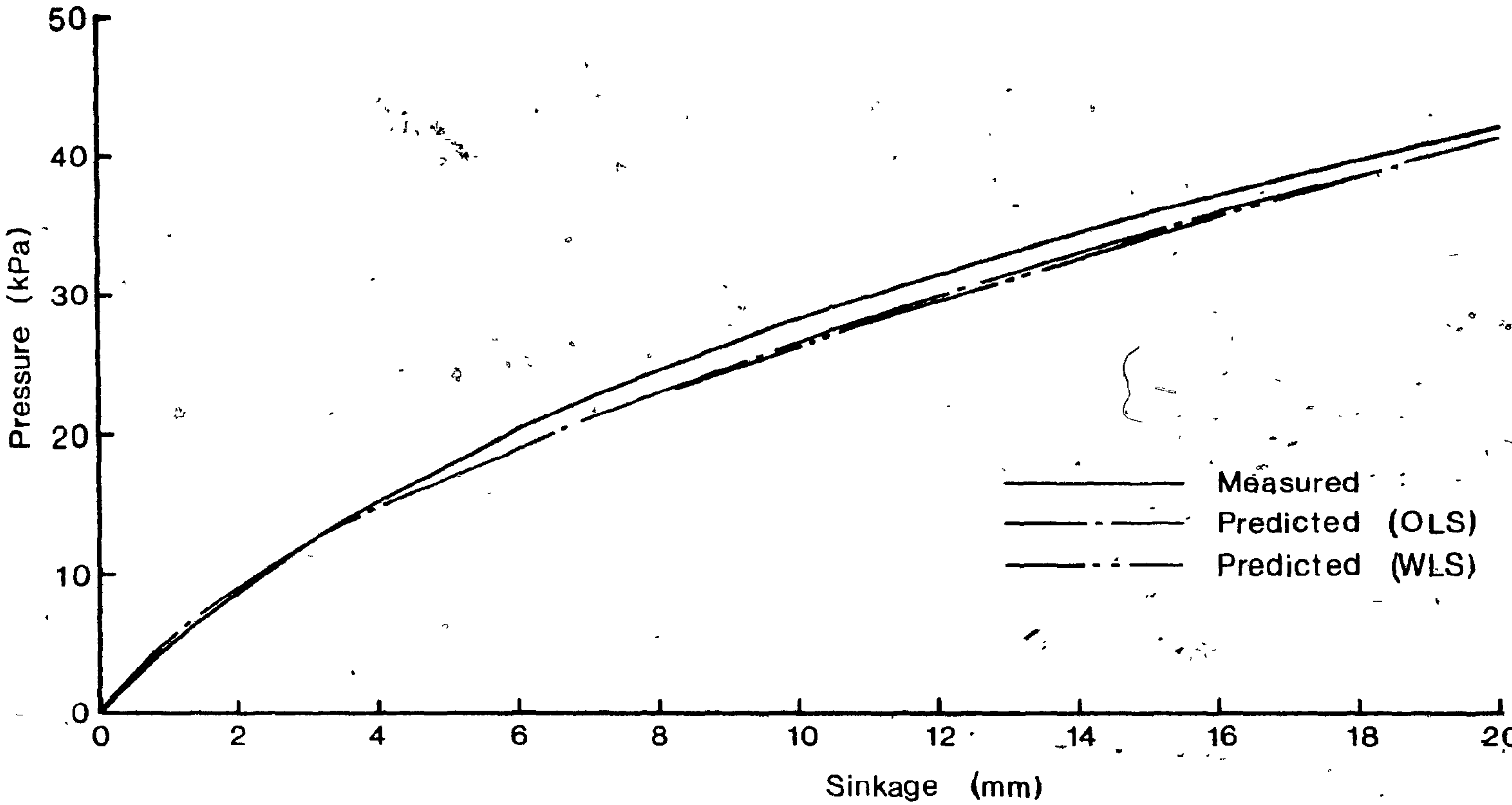

Figure 4.11 Comparison of the measured and predicted pressure vs. sinkage relationship of plate D=0.15 m (Series 3)

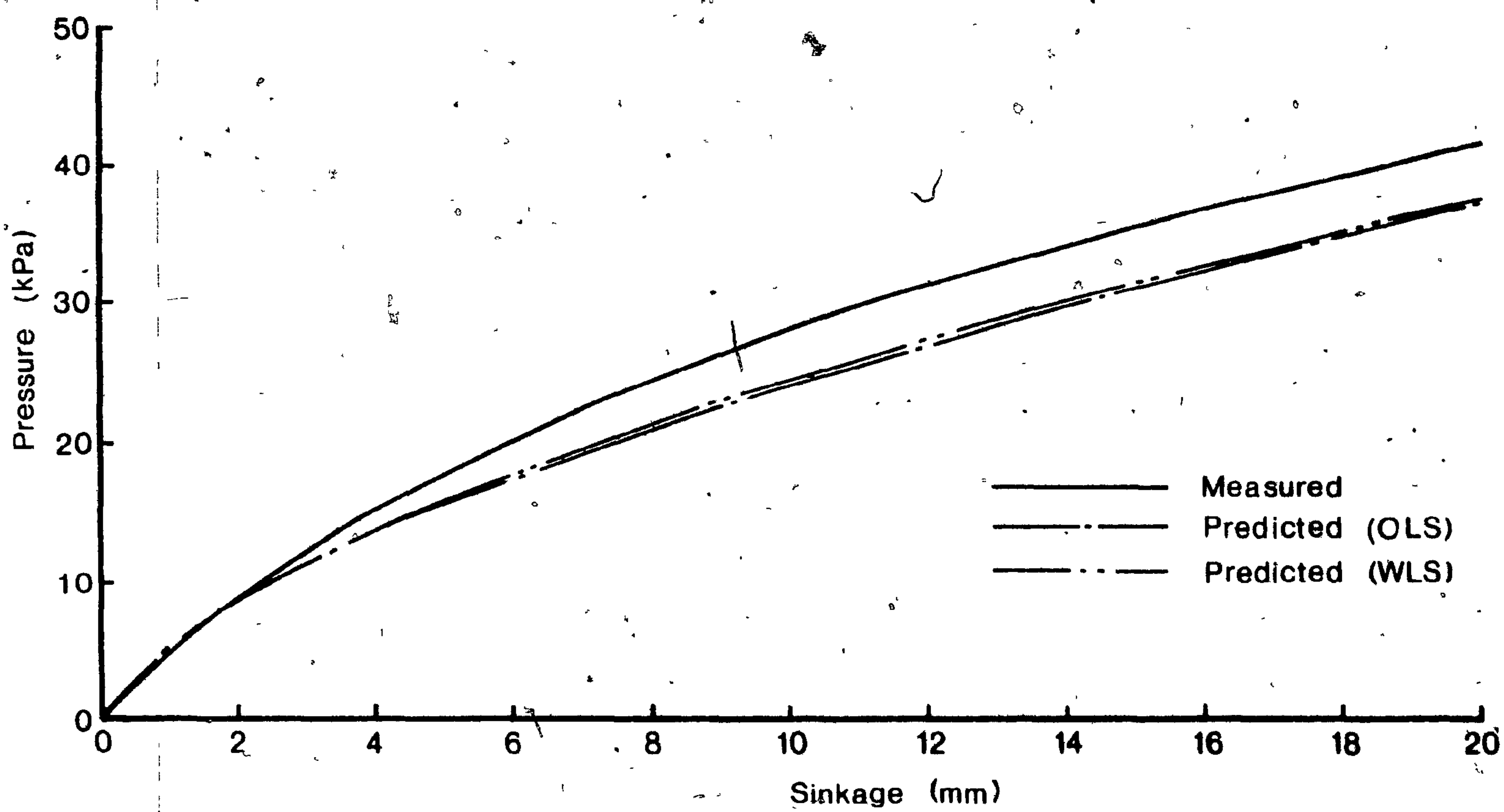

Figure 4.12 Comparison of the measured and predicted pressure vs. sinkage relationship of plate D=0.15 m (Series 4)

The results of k_c and k_ϕ from both Series 2 and 3 give good predictions of pressure. Compared with results of Series 1 to 3, the pressure prediction in Series 4 is less satisfactory, with an error around 10% of the measured loads.

Looking at the K-1/b plot of Series 4 (Figure 4.9), a typical case of K scatter increasing with b decreasing is seen, similar to that discussed in Section 4.3. If only two plates were involved in the assessment of k_c and k_ϕ, a much larger prediction error would result.

A few words of caution are necessary here. A high correlation coefficient of the K-1/b regression might be perceived as an indication that a good prediction of the P(Z) relation might result. In this connection, the results of Series 3 are a very interesting example. In spite of the fact that the five-point regression has already given a reasonably good prediction of pressure, the prediction can still be improved. Scrutinising the K-1/b plot in Figure 4.8, as shown by the double point dashed line, the four points on the right side lie almost on a single straight line, and the fifth point on the extreme left is not in this line. By excluding the outlying point from the regression, a new set of soil parameter estimates is obtained, as $n = 0.6463$, $k_c = -9.36\ kPa/m^{n-1}$, $k_\phi = 594.71\ kPa/m^n$, which gives an error of pressure prediction, $(P_{measured} - P_{predicted})/P_{measured}$, of only 2% (Figure 4.13).

But a high correlation coefficient in K-1/b regression is not the aim pursued. If such well behaved data points were always available, i.e. the data

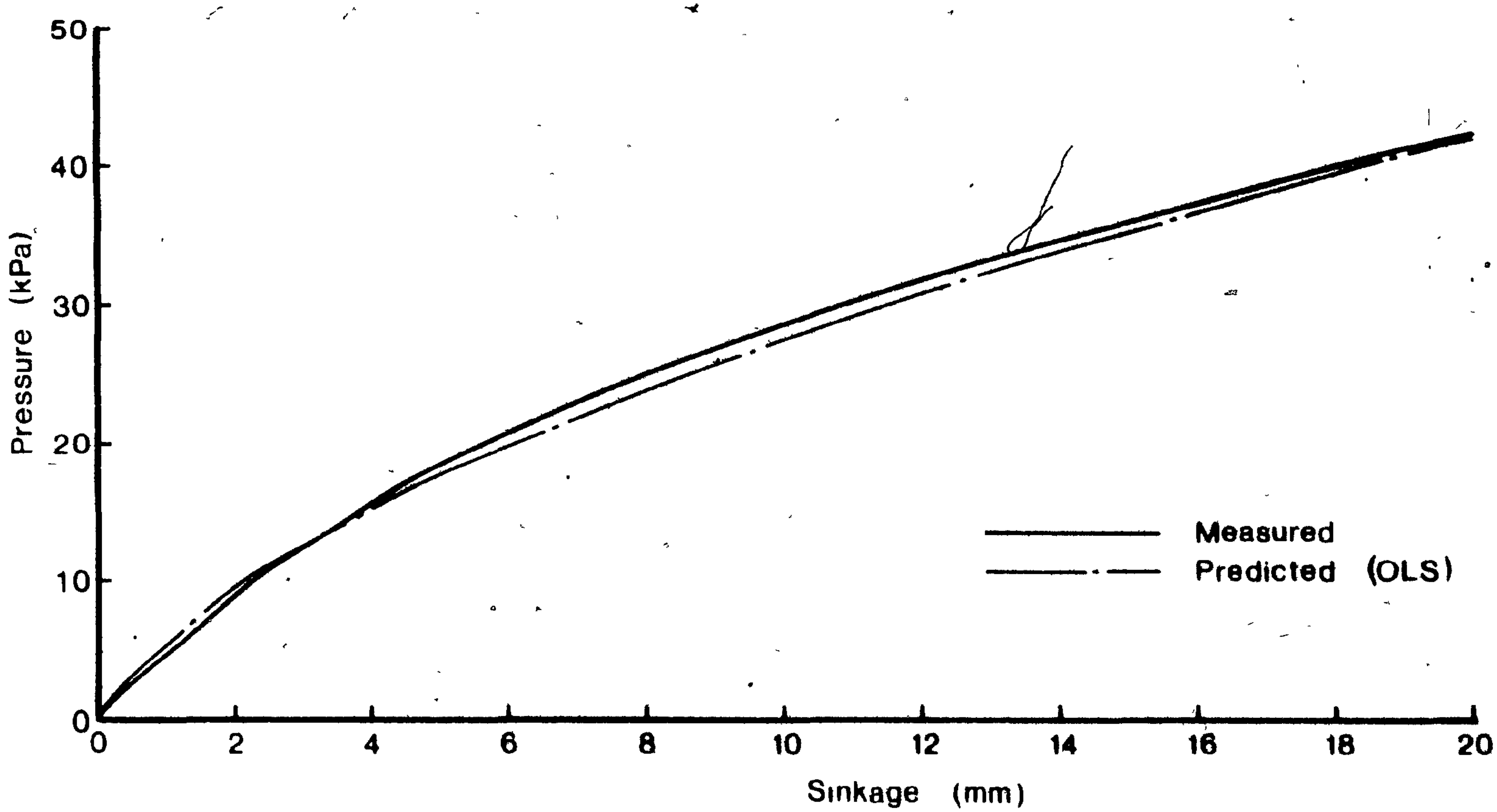

Figure 4.13 Comparison of the measured and predicted pressure vs. sinkage relationship of plate D=0.15 m (amended result of Series 3)

points always fall onto a straight line, two data points or two plates are already enough to extract k_c and k_ϕ reliably. Hence there would be no point in using more plates. In fact, because of the inevitable soil inhomogeneity, especially in field conditions, certain deviation of parameter K from its population mean is more generally expected. The merit of using more plates is that the estimate of average soil parameters will always be more accurate by using more data points. Multi-plate penetration tests also yield the information about how well the P(Z) prediction might be, because the order of magnitude of variations can be determined. This is especially valuable in the case of a larger soil variability, where large errors in prediction of pressure vs. sinkage might occur without the aid of multi-plate tests.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this investigation, multi-plate penetration tests were performed with a family of small model plates on loose dry sand. Regression methods were used to fit the data from these penetration tests to Bekker's (1955) pressure-sinkage equation, namely,

$$P = K\ Z^n$$

and

$$K = k_c\ (1/b) + k_\phi$$

to extract the parameters of k_c, k_ϕ and n in the equation,

$$P = (k_c/b + k_\phi)\ Z\ .$$

The results of the extraction were checked by comparing the predicted and measured P(Z) curves for a plate of a size about three times of the average size of the model plates.

It was found that in the range of soil sinkage investigated

(1) The measured P(Z) curves could be approximated well by the pressure-sinkage equation of Bekker (1955,1960).

(2) Even so, the exponential equation of pressure vs. sinkage is not a consistently exact mechanical model of the real penetration process.

(3) The values of n, k_c and k_ϕ reflect the soil variability and vary randomly for a given soil.

(4) To evaluate these quantities, the statistical method is very relevant. For the assessment of the k_c and k_ϕ parameters, this means that the method of repetitive penetration with each plate size should also be supplemented with additional plate sizes in order to obtain the required accuracy of determination of soil mechanical properties.

In a laboratory-prepared simple soil (dry sand), the following table summarizes the number of plates of different sizes which should be used in order to achieve desired levels of accuracy of measurement of sinkage coefficients.

Number of plates	Average variation (%)	
	k_c	k_ϕ
2	97	23
3	31	8
4	19	5

5.2 Recommendations

According to Wills (1966), the prediction performance of Bekker's pressure-sinkage equation is worse for sand than it is for clay and loam soils. According to Bekker (1969), the scatter of penetration data is more in the case of shallow sinkage depths than for deep sinkage. This investigation proved that the multi-plate penetration test can handle these most difficult situations effectively. It is therefore believed that the methods suggested in this investigation can also improve the assessment of soil properties by Bekker's method in general cases, i.e. diversified soil types, deeper penetration and field conditions. Verification of the above inference is recommended for future research.

A field investigation with small model plates is particularly recommended. A statistical estimate of soil properties is most relevant to soils in field conditions. An increase in plate number might compensate for the decrease in plate sizes in dealing with the localized soil inhomogeneity. And a decrease in plate sizes will enable rapid and dependable data acquision with portable equipment.

REFERENCES

Bekker, M.G., 1955. A system of physical and geometrical soil values for determination of vehicle performance and soil trafficability, Proc. Interserv. Symposium Stevens Inst. of Tech. and Duke Univ., Office of Ord. Res., Durham, N.C.

Bekker, M.G., 1960 Off-the-Road Locomotion. University of Michigan Press, Ann Arbor, Michigan.

Bekker, M.G., 1969. Introduction to Terrain - Vehicle Systems. University of Michigan Press, Ann Arbor, Michigan.

Bekker, M.G., 1975. Motion resistance of pneumatic tyres. Journal of Automotive Engineering, April: 6 - 10.

Bekker, M.G., 1977. Terrain-vehicles systems analysis. Special lecture held at Carleton University, Ottawa, Canada.

Bernstein, R., 1913. Problems of experimental mechanics of ploughs. Der Motorwagen, No. 16: 199 - 227.

Chatterjee, S. and B. Price, 1977. Regression Analysis by Example. John Wiley & Sons, New York, N.Y.

Costes, N.C., et al., 1972. Mobility performance of the lunar roving vehicle, NASA, TR R - 401: 60 - 66.

Falbo, D., 1984. Tecnomare activities in the field of locomotion on the seabed. Journal of Terramechanics, 21(1): 59 - 67.

Gill, W.R. and G.E. Vanden Berg, 1968. Soil Dynamics in Tillage and Traction, Agricultural Handbook No. 316. Agricultural Research Service, United States Department of Agricuturure.

Goriatchkin, B.P., et al., 1936. Theory and production of agricultural equipment, Moscow.

Hegedus, E., 1965. Plate sinkage study by means of dimensonal analysis. Journal of Terramechanics, 2(2): 25 - 32.

Hettiaratchi, D.R.P. and A.R. Reece, 1974. The calculation of passive soil resistance. Geotechnique, 24(3): 289 - 310.

Janosi, Z.J., 1965. Analysis and presentation of soil-vehicle mechanics data. Journal of Terramechanics, 2(3): 69 - 79.

Karafiath, L.L. and E.A. Nowatzki, 1978. Soil Mechanics for Off - Road Vehicle Engineering. Trans. Tech. Pub., Germany.

Knight, S.J. and D.R. Freitag, 1962. Measurement of soil trafficability characteristics, Transaction of the American Society of Agricultural Engineering, 5(1): 121 - 124, 132.

Kogure, K., Y. Ohira and H. Yamaguchi, 1983. Prediction of sinkage and motion resistance of a tracked vehicle using plate penetration test. Journal of Terramechanics, 20(3/4): 121 - 128.

McKyes, E., M. Chasse, G.S.V. Raghavan and F. Merineau, 1978. Soil compaction in an orchard of southern Quebec, Journal of Terramechanics, 15(4): 223 - 235.

McKyes, E., 1980. Agricultural and earthmoving power machinery, Lecture notes. McGill University, Montreal.

Mead, R. and R.N. Curnow, 1983. Statistical Methods in Agricultural and Experimental Biology. Methuen Inc., Toronto.

Reece, A.R., 1964a. Curve fitting technique in soil vehicle mechanics. Journal of Terramechanics 1(2): 44 - 55.

Reece, A.R., 1964b. Theory and practice of off-the-road locomotion. The Agricultural Engineer, 20(2): 45 - 50.

Road Research Laboratory, 1964. Soil Mechanics for Road Engineers. HMSO, London.

Soehne, W., 1981. We salute Greg Bekker. Journal of Terramechanics, 18(3): 125 - 129.

Taylor, W.D., 1948. Fundamentals of Soil Mechanics. John Wiley & Sons, New York, N.Y.

Taylor, J.H. and W.R. Gill, 1984. Soil compaction: state-of-the-art report, Journal of Terramechanics, 21(2): 195 - 213.

Terzagi, K. and R.B. Peck, 1948. Soil Mechanics in Engineering Practice. John Wiley & Sons, New York, N.Y.

Volfson, B.P., 1984. Determination of some soil properties for vehicle ride simulation. Transaction of the American Society of Agricultural Engineering, 27(2): 341 - 347.

Wills, B.M.D., F.M. Barret and G.J. Shaw, 1965. An investigation into rolling resistance theories for towed rigid wheels. Journal of Terramechanics, 2(1): 24 - 53.

Wills, B.M.D., 1966. The load sinkage equation in theory and practice. Proceedings of the 2nd conference of the International Society for Terrain Vehicle Systems, Quebec, Canada.

Wismer, R.D. and H.J. Luth, 1973. Off-road traction prediction for wheeled vehicles. Journal of Terramechanics, 10(2): 49 - 61.

Wong, J.Y., M.Garber, J.R. Radforth and J.T. Powell, 1979. Characterization for the mechanical properties of muskeg with special reference to vehicle mobility. Journal of Terramechanics, 16(4): 163 - 180.

Wong, J.Y., 1980. Data processing methodology in the characterization of the mechanical properties of terrain. Journal of Terramechanics, 17(1): 13 - 41.

Wong, J.Y., 1983. Evaluation of soil strength measurements. Report for National Research Council of Canada. Unpublished.

Wong, J.Y., 1984. An introduction to Terramechanics. Journal of Terramechanics, 21(1): 5 - 17.

Yong, R.N. and E. Windisch, 1970. Determination of wheel contact stresses from measured instantaneous soil deformations. Journal of Terramechanics, 7(3/4): 57 - 67.

Yong, R.N. and W.L. Harrison, 1978. On vehicle mobility in snow-covered terrain - 1. Problem development and requirements for analysis. Journal of Terramechanics, 15(4): 223 - 235.

Yong, R.N., E.A. Fattah and N. Skiadas, 1984. Vehicle Traction Mechanics. Elsevier, Amsterdam, Holland.

Youssef, A. and G.A. Ali, 1982. Determination of soil parameters using plate test. Journal of Terramechanics, 19(2): 129 - 147.

APPENDIX

EXPERIMENTAL PRESSURE VS. SINKAGE DATA

Table A1.1 Pressure vs. sinkage data

(Series 1, D = 5.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S1-1	S1-2	S1-3	S1-4	S1-5	mean	SE	C.V.
2	8.7	8.0	9.2	8.0	8.5	8.5	0.5	6.0
4	14.4	13.0	16.2	13.8	14.4	14.3	1.2	8.2
6	18.7	16.5	16.2	13.8	14.4	18.6	1.5	8.3
8	21.7	19.4	24.3	21.4	22.3	21.8	1.8	8.1
10	24.3	21.8	27.6	24.3	21.4	24.6	2.1	8.5
12	26.4	23.9	30.4	26.7	27.6	27.0	2.3	8.6
14	28.5	25.8	32.8	29.0	29.9	29.2	2.5	8.6
16	30.5	27.7	35.0	31.0	32.2	31.3	2.6	8.4
18	32.5	29.7	37.1	33.0	34.3	33.3	2.7	8.1
20	34.6	31.7	39.4	35.0	36.5	35.4	2.8	7.9

Table A1.2 Pressure vs. sinkage data

(Series 1, D = 5.0 cm)

Sinkage	Pressure (kPa)							
(mm)	S1-1	S1-2	S1-3	S1-4	S1-5	mean	SE	C.V.
2	9.5	8.5	8.0	9.8	7.2	8.6	1.1	12.4
4	15.5	14.5	13.1	16.8	12.5	14.4	1.7	12.1
6	19.5	18.5	17.2	21.8	16.8	18.7	2.0	10.7
8	22.7	21.5	20.5	25.6	20.4	22.1	2.1	9.7
10	25.1	23.9	23.2	28.7	23.4	24.8	2.3	9.3
12	27.1	26.0	25.7	31.2	26.2	27.2	2.3	8.5
14	29.0	27.9	28.0	33.3	28.4	29.3	2.3	7.7
16	31.0	29.6	30.0	35.7	30.7	31.4	2.5	7.8
18	32.9	31.7	32.1	37.9	33.0	33.6	2.5	7.4
20	34.9	33.6	34.3	39.9	35.0	35.6	2.5	7.0

Table A1.3 Pressure vs. sinkage data

(Series 1, D = 4.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S1-1	S1-2	S1-3	S1-4	S1-5	mean	SE	C.V.
2	9.2	7.4	7.4	8.6	6.5	7.8	1.1	13.8
4	14.9	13.3	12.3	14.4	11.3	13.3	1.5	11.1
6	19.1	17.4	16.0	18.9	15.2	17.3	1.7	10.0
8	22.2	20.5	19.0	22.3	18.4	20.5	1.8	8.8
10	24.6	23.2	21.6	24.6	20.8	23.0	1.7	7.6
12	27.0	25.3	23.9	27.7	23.0	25.4	2.0	7.8
14	29.0	27.2	25.9	29.9	25.3	27.5	2.0	7.2
16	30.8	29.3	27.9	32.2	27.2	29.5	2.0	6.9
18	32.7	31.3	29.9	34.4	29.2	31.5	2.1	6.6
20	34.5	33.4	32.0	36.5	31.2	33.5	2.1	6.2

Table AI.4 Pressure vs. sinkage data

(Series 1, D = 4.0 cm)

Sinkage (mm)	Pressure (kPa) S1-1	S1-2	S1-3	S1-4	S1-5	mean	SE	C.V.
2	7.3	7.3	7.0	8.7	7.8	7.6	0.7	8.8
4	12.3	13.1	11.9	14.2	12.3	12.8	0.9	7.2
6	15.9	17.6	15.6	17.9	15.6	16.5	1.1	7.0
8	19.0	21.4	18.7	20.3	17.8	19.4	1.4	7.2
10	21.7	24.0	21.8	22.6	19.8	22.0	1.5	7.0
12	24.3	26.2	24.3	25.0	21.8	24.3	1.5	6.5
14	25.6	28.1	27.0	27.0	24.0	26.5	1.5	5.7
16	28.5	29.9	29.5	29.0	26.2	28.6	1.4	5.1
18	30.7	31.8	31.7	31.2	28.1	30.7	1.5	5.0
20	32.8	33.7	34.3	33.1	29.9	32.7	1.7	5.1

Table A1.5 Pressure vs. sinkage data

(Series 1, D = 3.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S1-1	S1-2	S1-3	S1-4	S1-5	mean	SE	C.V.
2	6.7	7.9	5.1	5.5	7.5	6.6	1.2	18.9
4	11.2	13.4	9.0	9.4	12.2	11.0	1.9	17.1
6	15.3	17.5	12.6	10.4	15.5	14.3	2.8	19.4
8	18.5	21.0	16.1	14.9	18.1	17.7	2.3	13.3
10	21.6	23.4	19.4	17.1	20.2	20.3	2.4	11.6
12	24.6	25.7	22.6	18.9	22.0	22.8	2.6	11.4
14	27.5	27.9	25.5	20.8	23.6	25.1	2.9	11.7
16	30.2	30.2	28.5	22.6	25.3	27.3	3.3	12.1
18	32.6	32.4	31.6	24.4	27.1	29.6	3.6	12.3
20	32.8	33.7	34.3	33.1	29.9	31.9	4.0	12.6

Table A2.1 **Pressure vs. sinkage data**

(Series 2, D = 6.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S2-1	S2-2	S2-3	S2-4	S2-5	mean	SE	C.V.
2	7.8	8.1	8.6	7.7	7.0	7.8	0.6	7.5
4	13.0	13.6	14.2	13.1	12.1	13.2	0.8	5.9
6	17.4	17.7	18.6	17.2	16.2	17.4	0.8	4.9
8	20.7	21.0	22.1	20.5	19.6	20.8	0.9	4.4
10	23.6	23.7	24.9	23.3	22.4	23.6	0.9	3.8
12	26.2	25.9	27.5	25.6	25.0	26.0	0.9	3.5
14	28.5	27.8	29.8	27.8	27.3	28.2	0.9	3.3
16	30.7	29.7	31.8	29.7	29.7	30.3	0.9	3.1
18	32.8	31.6	33.7	31.7	31.7	32.3	0.9	2.8
20	35.0	33.5	35.5	33.7	33.7	34.3	0.9	2.6

Table A2.2 Pressure vs. sinkage data

(Series 2, D = 6.0 cm)

Sinkage	Pressure (kPa)							
(mm)	S2-1	S2-2	S2-3	S2-4	S2-5	mean	SE	C.V.
2	7.6	9.5	6.7	7.6	9.7	8.2	1.3	16.2
4	13.9	16.3	11.8	12.5	15.3	13.9	1.9	13.4
6	18.9	19.0	15.8	16.2	19.1	17.8	1.6	9.2
8	22.9	22.0	19.0	19.3	22.2	21.1	1.8	8.5
10	26.1	24.5	21.7	21.8	24.7	23.7	1.9	8.0
12	28.8	26.9	24.0	24.3	27.0	26.2	2.0	7.7
14	31.3	28.8	26.0	26.3	29.0	28.3	2.2	7.7
16	33.6	30.7	28.1	28.4	30.8	30.3	2.2	7.3
18	35.7	32.7	29.9	30.5	32.8	32.3	2.3	7.1
20	37.9	34.7	31.7	32.4	34.7	34.3	2.4	7.0

Table A2.3 Pressure vs. sinkage data

(Series 2, D = 5.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S2-1	S2-2	S2-3	S2-4	S2-5	mean	SE	C.V.
2	6.6	6.2	4.8	7.3	6.9	6.4	1.0	15.4
4	11.6	11.6	8.2	12.4	13.0	11.4	1.8	16.2
6	16.1	15.7	11.5	16.5	18.0	15.6	2.4	15.4
8	19.6	19.1	14.2	19.4	21.9	18.8	2.8	15.0
10	22.9	22.0	16.7	21.9	25.2	21.7	3.1	14.3
12	25.8	24.6	18.9	24.1	27.9	24.2	3.3	13.7
14	28.5	26.8	20.9	26.1	30.3	26.5	3.5	13.3
16	30.9	28.9	22.8	28.0	32.6	28.6	3.7	13.0
18	33.2	31.2	24.7	30.1	34.6	30.8	3.8	12.5
20	35.5	33.3	26.6	32.0	37.0	32.9	4.0	12.2

Table A2.4 Pressure vs. sinkage data

(Series 2, D = 5.0 cm)

Sinkage	Pressure (kPa)							
(mm)	S2-1	S2-2	S2-3	S2-4	S2-5	mean	SE	C.V.
2	7.2	7.0	7.5	6.8	6.5	7.0	0.4	5.4
4	13.2	11.9	12.7	12.0	12.0	12.4	0.5	4.5
6	17.0	15.4	16.5	16.1	16.8	16.3	0.6	3.9
8	20.1	18.0	19.2	19.5	20.8	19.5	1.0	5.4
10	22.8	20.1	21.6	22.3	23.9	22.1	1.4	6.4
12	25.0	22.2	23.7	24.8	26.7	24.4	1.7	6.8
14	27.0	24.2	25.8	27.0	28.9	26.5	1.7	6.6
16	28.9	26.2	27.7	29.0	31.4	28.6	1.9	6.8
18	30.9	28.0	29.4	31.1	33.7	30.7	2.1	6.9
20	32.9	29.9	31.3	33.1	35.6	32.6	2.1	6.5

Table A2.5 Pressure vs. sinkage data

(Series 2, D = 4.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S2-1	S2-2	S2-3	S2-4	S2-5	mean	SE	C.V.
2	7.4	6.3	6.7	6.2	5.7	6.4	0.6	10.0
4	12.3	10.5	11.3	11.1	10.4	11.1	0.8	7.1
6	15.8	13.7	15.4	14.8	13.9	14.7	0.9	6.2
8	18.6	16.3	18.5	18.2	17.3	17.8	1.0	5.6
10	21.0	18.6	21.0	21.0	20.0	20.3	1.0	5.1
12	23.2	20.8	23.4	24.0	22.3	22.7	1.2	5.4
14	25.1	23.4	25.9	26.6	24.6	25.1	1.2	4.8
16	27.4	25.6	27.9	28.8	26.9	27.3	1.0	4.3
18	29.6	27.7	29.8	31.2	28.8	29.4	1.2	4.3
20	31.9	29.8	31.8	33.3	30.8	31.5	1.2	4.1

Table A3.1 Pressure vs. sinkage data

(Series 3, D = 6.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S3-1	S3-2	S3-3	S3-4	S3-5	mean	SE	C.V.
2	8.6	7.2	6.6	7.1	6.6	7.2	0.8	11.6
4	14.9	12.5	11.5	12.9	11.8	12.7	1.3	10.4
6	19.8	16.4	15.1	17.1	16.0	16.9	1.8	10.5
8	23.6	19.6	18.0	20.5	19.7	20.3	2.1	10.2
10	26.6	22.3	21.0	23.4	22.9	23.2	2.1	8.9
12	29.2	24.8	23.0	25.9	25.7	25.7	2.1	8.7
14	31.3	26.8	25.2	28.0	28.2	27.9	2.2	8.1
16	33.4	28.9	27.3	29.9	30.5	30.0	2.2	7.4
18	35.4	31.1	29.5	31.9	32.7	32.1	2.2	6.8
20	37.5	33.3	31.7	33.9	34.8	34.2	2.2	6.3

Table A3.2 Pressure vs. sinkage data

(Series 3, D = 6.0 cm)

Sinkage (mm)	Pressure (kPa) S3-1	S3-2	S3-3	S3-4	S3-5	mean	SE	C.V.
2	8.7	8.3	5.2	8.5	7.3	7.6	1.4	19.1
4	15.4	14.2	9.0	14.6	12.5	13.1	2.5	19.3
6	20.4	18.3	12.2	19.1	16.3	17.3	3.2	18.5
8	24.3	21.5	14.8	22.5	19.3	20.5	3.6	17.7
10	27.6	24.1	17.3	25.3	21.8	23.2	3.9	16.8
12	30.4	26.3	19.5	27.7	24.1	25.6	4.1	16.0
14	32.7	28.6	21.6	30.0	26.1	27.8	4.2	15.2
16	34.9	30.8	23.6	32.2	28.1	29.9	4.3	14.3
18	37.2	33.0	25.6	34.3	30.2	32.0	4.4	13.6
20	39.2	35.2	27.7	36.4	32.2	34.2	4.4	12.8

Table A3.3 Pressure vs. sinkage data

(Series 3, D = 5.5 cm)

Sinkage (mm)	Pressure (kPa) S3-1	S3-2	S3-3	S3-4	S3-5	mean	SE	C.V.
2	6.8	9.1	5.9	7.0	7.4	7.2	1.1	16.0
4	12.4	14.4	10.3	11.9	12.6	12.3	1.5	12.0
6	16.7	18.4	14.0	15.5	16.7	16.3	1.6	10.0
8	20.5	21.4	16.9	18.3	19.8	19.4	1.8	9.2
10	23.5	23.9	19.5	20.7	22.5	22.0	1.9	8.6
12	26.2	26.2	21.9	23.1	25.2	24.5	1.9	7.9
14	28.5	28.0	24.3	25.2	27.4	26.7	1.8	6.8
16	30.5	30.1	26.3	27.2	28.9	28.6	1.8	6.3
18	32.8	32.2	28.3	29.5	31.8	30.9	1.9	6.3
20	34.9	34.4	30.4	31.6	34.0	33.0	2.0	5.9

Table A3.4 Pressure vs. sinkage data

(Series 3, D = 5.0 cm)

Sinkage (mm)	Pressure (kPa) S3-1	S3-2	S3-3	S3-4	S3-5	mean	SE	C.V.
2	7.2	7.5	6.5	7.0	6.7	7.0	0.4	5.7
4	12.5	13.0	11.3	11.7	11.8	12.0	0.7	5.6
6	16.3	17.2	14.6	15.2	16.0	15.8	1.0	6.3
8	19.0	20.5	17.1	18.0	19.0	18.7	1.3	6.8
10	21.2	23.4	19.1	20.3	21.7	21.1	1.6	7.6
12	23.0	26.0	21.2	22.8	24.0	23.4	1.7	7.5
14	24.8	28.5	23.2	24.9	26.3	25.5	2.0	7.9
16	26.5	30.8	25.1	27.0	28.4	27.5	2.2	8.0
18	28.1	33.2	27.0	29.1	30.4	29.6	2.4	8.1
20	29.9	35.6	28.9	31.1	32.6	31.7	2.6	8.2

Table A3.5 Pressure vs. sinkage data

(Series 3, D = 4.5 cm)

Sinkage	Pressure[a] (kPa)							
(mm)	S3-1	S3-2	S3-3	S3-4	S3-5	mean	SE	C.V.
2	9.2	6.0	6.8	6.4	5.3	6.7	1.5	22.1
4	14.8	10.4	11.1	11.1	9.5	11.4	2.0	17.8
6	18.2	13.7	14.3	14.8	13.1	14.8	2.0	13.6
8	21.0	17.5	16.8	17.4	16.0	17.7	1.9	10.7
10	23.3	18.6	18.7	19.7	18.6	19.8	2.0	10.1
12	25.5	21.0	21.0	22.2	21.1	22.1	1.9	8.8
14	27.6	23.2	23.0	24.4	23.4	24.3	1.9	7.8
16	29.8	25.3	25.3	25.3	25.9	26.3	2.0	7.5
18	32.0	27.4	27.5	28.6	28.0	28.7	1.9	6.7
20	34.1	29.6	29.6	30.8	29.9	30.8	1.9	6.2

Table A4.1 Pressure vs. sinkage data

(Series 4, D = 6.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S4-1	S4-2	S4-3	S4-4	S4-5	mean	SE	C.V.
2	7.6	7.6	7.4	7.0	8.6	7.6	0.6	7.7
4	13.1	13.1	12.1	11.8	14.2	12.9	0.9	7.3
6	17.2	17.7	15.9	15.4	18.4	16.9	1.2	7.3
8	20.4	21.3	19.0	18.6	21.7	20.2	1.4	6.8
10	23.0	24.0	21.5	21.1	24.5	22.8	1.5	6.5
12	25.5	26.5	23.9	23.6	27.0	25.3	1.5	6.0
14	27.6	28.7	25.6	25.7	29.1	27.3	1.6	6.0
16	29.7	30.8	28.4	27.8	31.1	29.6	1.5	5.0
18	31.8	33.0	30.5	29.8	33.1	31.6	1.5	4.7
20	33.8	35.1	32.1	31.8	35.3	33.6	1.6	4.8

Table A4.2 Pressure vs. sinkage data

(Series. 4, D = 6.0 cm)

Sinakge	Pressure (kPa)							
(mm)	S4-1	S4-2	S4-3	S4-4	S4-5	mean	SE	C.V.
2	7.2	8.2	6.6	7.0	6.6	7.1	0.7	9.6
4	12.8	13.9	11.4	12.3	11.4	12.4	1.0	8.2
6	17.0	18.0	15.3	16.4	14.8	16.3	1.3	7.9
8	20.2	21.1	18.2	19.6	17.4	19.3	1.5	7.8
10	22.9	23.8	20.8	22.3	19.8	21.9	1.6	7.5
12	25.2	26.2	23.0	24.7	22.1	24.2	1.6	6.8
14	27.3	28.1	25.3	27.0	24.1	26.4	1.6	6.1
16	29.5	30.2	27.2	29.0	26.1	28.4	1.7	6.0
18	31.5	32.4	29.3	31.1	28.1	30.5	1.8	5.8
20	33.6	34.5	31.3	33.2	30.2	32.5	1.8	5.4

Table A4.3 Pressure vs. sinkage data

(Series 4, D = 5.5 cm)

Sinkage	Pressure (kPa)							
(mm)	S4-1	S4-2	S4-3	S4-4	S4-5	mean	SE	C.V.
2	7.2	7.9	7.6	7.5	7.8	7.6	0.3	3.9
4	8.2	13.4	12.8	12.8	13.3	12.1	2.2	17.9
6	16.6	17.3	16.5	16.5	16.9	16.8	0.3	2.1
8	20.0	20.4	19.6	19.4	19.8	19.8	0.4	1.9
10	22.9	22.8	22.2	21.8	22.2	22.4	0.4	2.0
12	25.5	24.7	24.6	23.9	24.3	24.6	0.6	2.3
14	28.0	27.0	26.8	26.1	26.3	26.8	0.8	2.8
16	30.2	28.9	29.0	28.0	28.3	28.9	0.8	2.9
18	32.2	30.9	31.2	30.1	30.4	30.9	0.8	2.6
20	34.3	32.8	33.2	32.2	32.3	32.9	0.9	2.6

Table A4.4 Pressure vs. sinkage data

(Series 4, D = 5.0 cm)

Sinkage	Pressure (kPa)							
(mm)	S4-1	S4-2	S4-3	S4-4	S4-5	mean	SE	C.V.
2	7.0	6.3	6.9	7.5	6.6	6.8	0.4	6.6
4	12.0	10.8	11.3	12.4	11.5	11.6	0.6	5.3
6	16.0	14.3	14.8	14.8	15.2	15.0	0.6	4.2
8	18.9	17.4	17.5	18.4	18.0	18.0	0.6	3.5
10	21.4	20.1	19.8	20.8	20.4	20.5	0.6	3.0
12	23.8	22.7	22.1	22.9	22.6	22.8	0.6	2.7
14	25.8	25.2	24.1	25.0	24.8	24.9	0.6	2.5
16	27.8	27.7	26.1	27.2	26.8	27.1	0.7	2.6
18	29.7	29.9	28.1	29.3	28.6	29.1	0.7	2.6
20	31.7	32.4	30.2	31.5	30.4	31.3	0.9	2.9

Table A4.5 Pressure vs. sinkage data

(Series 4, D = 4.5 cm)

Sinkage (mm)	Pressure (kPa) S4-1	S4-2	S4-3	S4-4	S4-5	mean	SE	C.V.
2	8.1	7.4	8.0	6.8	6.2	7.3	0.8	11.4
4	13.7	12.4	13.1	11.8	10.5	12.3	1.2	10.0
6	18.0	15.9	17.1	15.4	13.7	16.0	1.6	10.3
8	21.1	18.5	19.8	18.5	16.1	18.8	1.8	10.0
10	23.9	20.7	22.3	21.0	18.5	21.3	2.0	9.5
12	26.1	22.8	24.3	23.3	20.6	23.4	2.0	8.7
14	28.2	24.9	26.4	25.5	22.6	25.5	2.1	8.1
16	30.4	27.1	28.3	27.7	24.6	27.6	2.1	7.5
18	32.7	29.3	30.3	29.9	26.6	30.0	2.1	7.3
20	34.8	31.4	32.4	32.2	28.7	31.9	2.2	6.8