

1 Spatio temporal analysis of precipitation and 2 temperature in the Basin of Mexico

3

4 J. J. Carrera-Hernández*, S. J. Gaskin

5 *McGill University, Department of Civil Engineering and Applied Mechanics, 817*
6 *Sherbrooke Street West, Montreal QC, H3A 2K6, Canada*

7 Abstract

8 The spatial distribution of climatological variables such as rainfall and temperature
9 is needed whenever hydrological modelling is undertaken at the watershed scale.
10 These models can be used to simulate hydrological processes at a daily or hourly
11 time step and the interpolation of climatological variables (in particular precipita-
12 tion) at this time scale poses a particular problem due to its large spatial variation.
13 This work analyzes the temporal variation of both minimum and maximum temper-
14 ature and rainfall, its correlation with elevation and whether or not this relation-
15 ship should be used when daily data are interpolated. In order to achieve this, the
16 monthly distribution of these variables is derived from daily interpolations, which is
17 compared to their monthly accumulated value for each climatological station. The
18 interpolation methods used to undertake the analysis were Ordinary Kriging (OK),
19 Kriging with External Drift (KED), Block Kriging with External Drift (BKED),
20 Ordinary Kriging in a local neighborhood (OK_l) and Kriging with External Drift
21 in a local neighborhood (KED_l). This analysis used daily climatological data from
22 approximately 200 stations located in the Basin of Mexico for June 1978 and June
23 1985, from which accumulated monthly data were derived. The results of this anal-
24 ysis show that the interpolation of daily events is improved by the use of elevation
25 as a secondary variable even when these variables show a low correlation.

26 *Key words:* Rainfall, temperature, interpolation, Kriging, Mexico City, Basin of
27 Mexico

1 Introduction

The spatial distribution of climatological variables is needed as input in distributed hydrological models and different authors have undertaken interpolations in different geographical locations and for different time periods. Temperature has been interpolated at a daily time step (Jarvis and Stuart, 2001), while rainfall has been interpolated using averaged values ranging from daily (Kyriakidis et al., 2001), monthly (Lloyd, 2005; Hudson and Wackernagel, 1994) or annual (Hofierka et al., 2002; Goovaerts, 2000; Martinez-Cob, 1996; Phillips et al., 1992; Dingman et al., 1988; Tabios and Salas, 1985) aggregation levels.

The analysis of the spatial distribution of daily rainfall is difficult mainly because of intermittence and large variability. The use of daily climatological data is needed in (semi)arid regions when developing water balances or when studying aquifer recharge, as annual evapotranspiration may greatly exceed rainfall. Thus in order to improve the estimates of aquifer recharge in these regions, daily data should be used.

Spatial interpolation can be undertaken through the use of various algorithms, and their evaluation has been addressed by several authors among them Tabios and Salas (1985) who compared Kriging with Thiessen, Inverse Distance Weight (IDW), Polynomial trend surfaces and inverse square distance, and by Jarvis and Stuart (2001) who compared Ordinary Kriging (OK), partial thin plate splines, inverse distance weighting and trend surface analysis. These studies concluded that the Kriging method yields a more realistic spatial behaviour of the climatological variable of interest. However, Kriging comprises different interpolation methods which can be differentiated by whether or not they use an external variable and if this variable is used in a global or local neighborhood. Among the Kriging methods that do not make use of an external variable are the following: Simple Kriging (SK), Ordinary Kriging (OK), Kriging with varying local means (Klm) and Block Kriging (BK), while among the methods that make use of a secondary variable are Factorial Kriging (FK), Kriging with External Drift (KED) and Cokriging

* Corresponding author.

Email addresses: jaime.carrera@mail.mcgill.ca (J. J. Carrera-Hernández), susan.gaskin@mcgill.ca (S. J. Gaskin).

1 (Cok). The performance of different Kriging methods has been reported in
2 previous works such as Lloyd (2005) who used Moving Window Regression
3 (MWR), IDW, OK, SKlm and KED to interpolate monthly precipitation val-
4 ues in England for 1999 for which the use of elevation as an auxiliary variable
5 through the application of KED provided more accurate estimates from March
6 to December.

7 The use of auxiliary variables in order to improve the spatial interpolation of
8 climatological variables has been analyzed by Jarvis and Stuart (2001). When
9 interpolating minimum and maximum temperatures they concluded that in-
10 cluding values recorded at nearby stations provides greater accuracy than the
11 selection of auxiliary or guiding variables. The only study that uses daily
12 rainfall data is the Spatial Interpolation Comparison of 1997, summarized in
13 Dubois (1998) and which consisted of the comparison of different interpo-
14 lation techniques applied to a set of 100 rainfall measurements in Switzer-
15 land on May 8th, 1986. The interpolation methods applied were Ordinary
16 and Indicator Kriging (Atkinson and Lloyd, 1998), Inverse Distance Weight-
17 ing (Tomczak, 1998), Linear and Zone Kriging (Saveliev et al., 1998), Neu-
18 ral Network Residual Kriging (Demyanov et al., 1998), Multiquadratic func-
19 tions (Thieken, 1998) and Probability Class Kriging (Allard, 1998). In addi-
20 tion, Hofierka et al. (2002) applied the Regularized Spline with Tension (RST)
21 method of Mitasova and Mitas (1993) to the SIC 97 data set, comparing their
22 results with those obtained by the previously mentioned authors using the
23 Root Mean Square Error (RMSE) as a benchmark. The RMSE values ranged
24 from 5.20 (mm) to 6.14 (mm) (Hofierka et al., 2002); the lowest RMSE value
25 was obtained by the Regularized Spline with Tension method, without using
26 elevation as an auxiliary variable.

27 Unfortunately, a clear answer can not be found on whether or not the use of
28 elevation as a secondary variable can improve the spatial interpolation of daily
29 rainfall. According to some authors (Lloyd (2005); Goovaerts (2000)) although
30 the use of elevation as an auxiliary variable improves the spatial interpolation
31 of monthly rainfall data, the relationship between rainfall and elevation is less
32 useful when interpolating daily data.

33 This study examines whether or not the relationship between three different
34 climatological variables (rainfall, minimum and maximum temperature) and

1 elevation should be used when interpolating daily climatological data using
 2 data for four different days, two days in June 1978 and two days in June
 3 1985 thus analyzing the effect of temporal variations in the relationship be-
 4 tween elevation and the climatological variable of interest. In addition, the
 5 accumulated monthly values for June 1978 and 1985 obtained from the daily
 6 interpolations are analyzed. Five different methods are investigated: Ordinary
 7 Kriging (OK), Kriging with External Drift (KED), Block Kriging with Ex-
 8 ternal Drift (BKED), Ordinary Kriging on a local neighborhood (OK_l) and
 9 Kriging with External Drift on a local neighborhood (KED_l). Kriging with
 10 External Drift was chosen among the methods that consider elevation as a sec-
 11 ondary variable as Goovaerts (2000) found that this method provides slightly
 12 better results than cokriging while not being as computationally demanding.
 13 Although other methods such as RST (Mitasova and Mitas, 1993) can also be
 14 used, it was decided to use only Kriging methods, in order to analyze a method
 15 that can be automatically applied to large datasets (e.g. daily interpolations
 16 for long term analysis). Those such as RST need additional parameters to be
 17 “tuned” (Hofierka et al., 2002).

18 2 Kriging

19 This section presents a brief overview of the interpolation methods used. It is
 20 based on Wackernagel (2003), Deutsch and Journel (1998), Goovaerts (1997)
 21 and Isaaks and Srivastava (1989), which provide an in depth view of the Krig-
 22 ing method along with its variants. Kriging methods use the available data
 23 $z(\mathbf{u}_\alpha)$ at n points in a specified search neighborhood in order to determine the
 24 values at unsampled locations $z(\mathbf{u})$, where the $z(\mathbf{u})$ value is a realization of
 25 a stationary Random Function (RF) that comprises $n + 1$ Random Variables
 26 (RV). The $z(\mathbf{u})$ values are estimated through a linear estimator $Z^*(\mathbf{u})$ in the
 27 following way (Goovaerts, 1997):

$$Z^*(\mathbf{u}) - m(\mathbf{u}) = \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u}) [Z(\mathbf{u}_\alpha) - m(\mathbf{u}_\alpha)] \quad (1)$$

28 where $\lambda_\alpha(\mathbf{u})$ is the weight assigned to datum $z(\mathbf{u}_\alpha)$, interpreted as a realiza-

1 tion of the RV $Z(\mathbf{u}_\alpha)$ while $m(\mathbf{u})$ and $m(\mathbf{u}_\alpha)$ are the expected values of the
 2 RVs $Z(\mathbf{u})$ and $Z(\mathbf{u}_\alpha)$. The Kriging methods aim to minimize the estimation
 3 variance $\sigma_E^2(\mathbf{u}) = Var(Z^*(\mathbf{u}) - Z(\mathbf{u}))$ where the RF $Z(\mathbf{u})$ is decomposed into
 4 a residual ($R(\mathbf{u})$) and trend ($m(\mathbf{u})$) component:

$$Z(\mathbf{u}) = m(\mathbf{u}) + R(\mathbf{u}) \quad (2)$$

5 where the residual component is modeled as a stationary RF with zero mean
 6 and covariance $C(\mathbf{h})$. In order to model $Z(\mathbf{u})$ the Kriging methods use the
 7 semivariogram ($\gamma(\mathbf{h})$), which considers the spatial relation of data and which
 8 is related to the covariance by:

$$\gamma(\mathbf{h}) = C(0) - C(\mathbf{h}) \quad (3)$$

9 where $C(0)$ is the covariance when $\mathbf{h} = 0$, or the variance of Z . In order
 10 to apply the Kriging method, a theoretical variogram model ($\gamma(\mathbf{h})$) is fitted
 11 to the experimental variogram ($\hat{\gamma}(\mathbf{h})$) which is computed by considering the
 12 difference between observations separated by a distance \mathbf{h} :

$$\hat{\gamma}(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} [z(\mathbf{u}_\alpha) - z(\mathbf{u}_\alpha + \mathbf{h})]^2 \quad (4)$$

13 Three Kriging variants can be distinguished according to the way in which
 14 the trend $m(\mathbf{u})$ (eq. 2) is handled (Goovaerts, 1997): Simple Kriging (SK)
 15 which considers the trend to be known and constant on the study area, Ord-
 16 inary Kriging (OK) which considers that the trend is unknown and constant
 17 on a specified search neighborhood and Universal Kriging or Kriging with a
 18 Trend model (KT) which considers that the local mean varies within each
 19 local neighborhood and on which the trend is modelled as a function of coor-
 20 dinates. Kriging with External Drift is an extension of KT, but in this case
 21 the trend is a function of one or more secondary variables (such as elevation)
 22 (Deutsch and Journel, 1998).

1 *2.1 Ordinary Kriging*

2 Ordinary Kriging (OK) accounts for local variation of the mean as it uses a
 3 local neighborhood $w(\mathbf{u})$ centered on the location (\mathbf{u}) being estimated. This
 4 Kriging technique considers the trend component $m(\mathbf{u})$ to be stationary, thus
 5 the linear estimation is expressed as a linear combination of the $n(\mathbf{u})$ RVs
 6 $Z(\mathbf{u}_\alpha)$ and the mean value m :

$$Z_{OK}^*(\mathbf{u}) = \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha^{OK}(\mathbf{u}) Z(\mathbf{u}_\alpha) \quad (5)$$

7 where

$$\sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha^{OK}(\mathbf{u}) = 1 \quad (6)$$

8 These equations yield the Ordinary Kriging system in terms of the semivari-
 9 ogram (Goovaerts, 1997; Wackernagel, 2003):

$$\begin{cases} \sum_{\beta=1}^{n(\mathbf{u})} \lambda_\beta^{OK}(\mathbf{u}) \gamma(\mathbf{u}_\alpha - \mathbf{u}_\beta) - \mu_{OK}(\mathbf{u}) = \gamma(\mathbf{u}_\alpha - \mathbf{u}) \text{ for } \alpha = 1, \dots, n(\mathbf{u}) \\ \sum_{\beta=1}^{n(\mathbf{u})} \lambda_\beta^{OK}(\mathbf{u}) = 1 \end{cases} \quad (7)$$

10 where $\mu(\mathbf{u})$ is a Lagrange parameter used to constraint the weights and $\gamma(\mathbf{u}_\alpha -$
 11 $\mathbf{u})$ represents the semivariogram for different lags, as described by (4). The
 12 term $\gamma(\mathbf{u}_\alpha - \mathbf{u})$ expresses the dissimilarities between each data point (\mathbf{u}_α) and
 13 the estimation point (\mathbf{u}) , while $\lambda_\beta^{OK}(\mathbf{u})$ represents the weight values obtained
 14 by solving (7), which are then used in (5) to determine Z_{OK}^* .

1 *2.2 Block Kriging*

2 Block Kriging is an extension of Ordinary Kriging and uses a moving neigh-
 3 borhood or block of given dimensions to estimate the mean. For a block V of
 4 known dimensions, on which \mathbf{u}_i samples are found, the block mean $z_v(\mathbf{u})$ can
 5 be computed as:

$$z_v(\mathbf{u}) = \frac{1}{N} \sum_{i=1}^N z(\mathbf{u}_i) \quad (8)$$

6 The block ordinary system is written as (Goovaerts, 1997):

$$\begin{cases} \sum_{\beta=1}^{n(\mathbf{u})} \lambda_{\beta v}(\mathbf{u}) \gamma(\mathbf{u}_\alpha - \mathbf{u}_\beta) + \mu_v(\mathbf{u}) = \gamma(\mathbf{u}_\alpha, V(\mathbf{u})) \text{ for } \alpha = 1, \dots, n(\mathbf{u}) \\ \sum_{\beta=1}^{n(\mathbf{u})} \lambda_{\beta v}(\mathbf{u}) = 1 \end{cases} \quad (9)$$

7 *2.3 Kriging with External Drift*

8 Kriging with External Drift (KED) (sometimes called Universal Kriging) should
 9 be used when a secondary variable is highly correlated with the variable of in-
 10 terest (Hudson and Wackernagel, 1994).

11 KED evaluates the correlation between the climatological variable and the
 12 secondary variable (in this case, elevation) within neighborhoods, providing
 13 information about the primary trend at location \mathbf{u} (Goovaerts, 2000). The
 14 trend $m(\mathbf{u})$ is modeled as a linear function of one or more secondary variables
 15 $y(\mathbf{u})$:

$$m(\mathbf{u}) = a_0(\mathbf{u}) + a_1(\mathbf{u})y(\mathbf{u}) \quad (10)$$

16 The trend coefficients $a_0(\mathbf{u})$ and $a_1(\mathbf{u})$ are constant within the search neigh-

1 borhood. In order to apply this Kriging technique the relation between the
 2 two variables must be linear (if not an appropriate transformation of the sec-
 3 ondary variable is needed) and the value of the secondary variable must be
 4 known throughout the modeling domain. The KED estimator is (Goovaerts,
 5 1997):

$$Z_{KED}^*(\mathbf{u}) = \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_{\alpha}^{KED}(\mathbf{u}) Z(\mathbf{u}_{\alpha}) \quad (11)$$

6 where the Kriging weights λ_{α}^{KED} are the solution of the $(n(\mathbf{u}) + 2)$ equation
 7 system (Goovaerts, 2000):

$$\left\{ \begin{array}{l} \sum_{\beta=1}^{n(\mathbf{u})} \lambda_{\beta}^{KED}(\mathbf{u}) \gamma_R(\mathbf{u}_{\alpha} - \mathbf{u}_{\beta}) + \mu_0^{KED}(\mathbf{u}) + \mu_1^{KED}(\mathbf{u}) y(\mathbf{u}_{\alpha}) = \gamma_R(\mathbf{u}_{\alpha} - \mathbf{u}) \text{ for } \alpha = 1, \dots, n(\mathbf{u}) \\ \sum_{\beta=1}^{n(\mathbf{u})} \lambda_{\beta}^{KED}(\mathbf{u}) = 1 \\ \sum_{\beta=1}^{n(\mathbf{u})} \lambda_{\beta}^{KED}(\mathbf{u}) y(\mathbf{u}_{\beta}) = y(\mathbf{u}) \end{array} \right. \quad (12)$$

8 The term $\gamma_R(\mathbf{u}_{\alpha} - \mathbf{u}_{\beta})$ represents the semivariogram of the residuals which are
 9 equal to the original values with the trend or drift removed (i.e. the semivari-
 10 ogram of the residuals).

11 **3 Study area**

12 The area under study is the Basin of Mexico, home to one of the largest
 13 metropolitan areas in the world: Mexico City and its Metropolitan Area (MCMA).
 14 The Basin has a mean elevation of 2240 meters above sea level (masl) and an
 15 approximate area of 9600 km². The analysis presented here used a rectangular
 16 area that encloses the Basin, as well as those climatological stations located
 17 near to it, as is illustrated in Fig. 1, thus using a total area of 16,800 km².
 18 This figure also shows that the Basin is surrounded by mountains and that its
 19 elevation ranges from 2000 masl up to 5500 masl on the high peaks located in

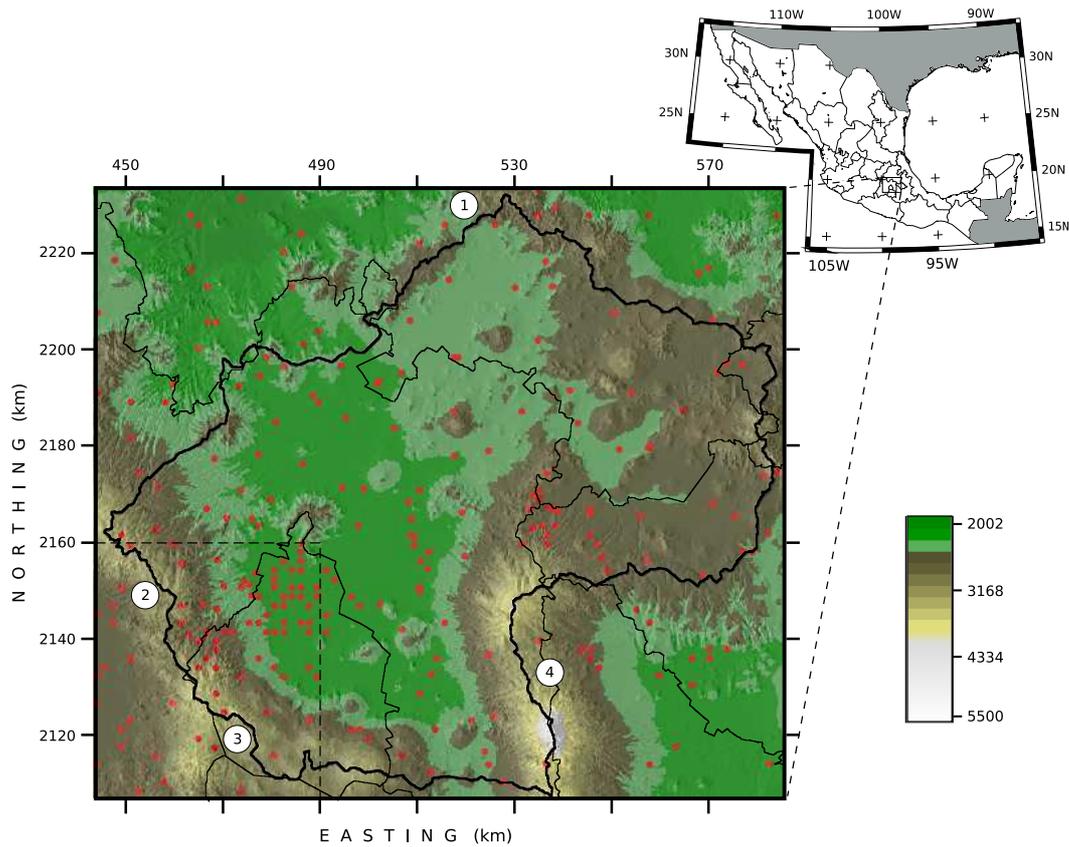


Fig. 1. Location and elevation of the study area; red dots represent the climatological stations used in the daily analysis. Circled numbers represent: 1) Sierra de Pachuca, 2) Sierra de las Cruces, 3) Sierra Chichinautzin and 4) Sierra Nevada. The dashed rectangle represents the Southwestern subarea.

- 1 the southeastern region.
- 2 The climatological data used in this study are stored in the Basin of Mexico
- 3 Hydrogeological Database (BMHDB) (Carrera-Hernández and Gaskin, 2005),
- 4 which is structured in such a way that its data can be directly used in (geo)statistical
- 5 analyzes without further processing, as illustrated in Fig. 2, which shows how
- 6 the database is accessed. The external variable used in this study (elevation)
- 7 is stored as a Digital Elevation Model in the form of a raster map in GRASS,
- 8 which is linked to R (R Development Core Team, 2005) through its GRASS
- 9 library (Bivand, 2000). The climatological variables can be interpolated using
- 10 a secondary variable through R's GSTAT library (Pebesma, 2004) which is
- 11 used to fit the experimental semivariogram and to undertake the spatial in-
- 12 terpolation; after this is achieved, the resulting distribution is written in the
- 13 GRASS database as a raster map.

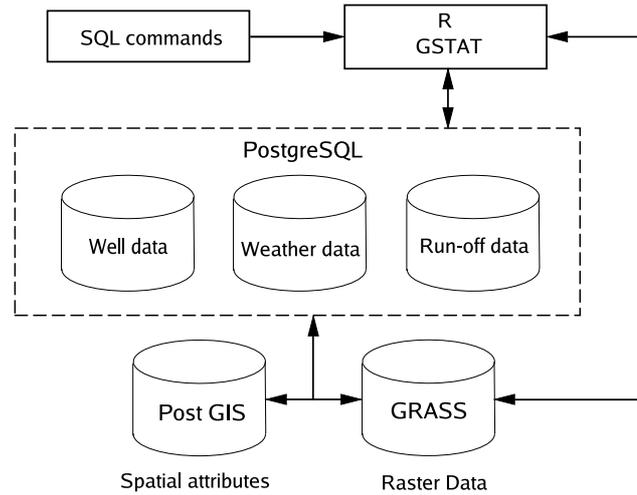


Fig. 2. Structure of the Basin of Mexico Hydrogeological Database and its interaction with (geo)statistical tools

1 The daily analysis presented used data from June 1978 and June 1985; these
 2 years were selected in order to use these results in a soil water balance, as there
 3 are other data such as land cover available for those two years. The number of
 4 stations with data within the Basin was also considered to select these years,
 5 as the quantity of climatological stations is not constant in time.

6 3.1 Data analysis

7 The spatial variation of the three climatological variables was undertaken at a
 8 grid resolution of 200 m as a resolution between 200 m and 5 km is considered
 9 to be appropriate in order to represent the variation of topographically de-
 10 pendent variables (Hutchinson and Galland, 1999). The 200 m resolution was
 11 chosen in order to account for the effects of topography on both temperature
 12 and rainfall. In order to analyze the effect of elevation as a secondary variable
 13 on spatial interpolation, daily data from June 1978 and June 1985 were used,
 14 as June is the month in which precipitation is largest in the study area. The
 15 daily correlation between the climatological variables and elevation is shown
 16 in Fig. 3(a) for June 1978 and Fig. 3(b) for June 1985. This figure shows
 17 that rainfall has a very low correlation with elevation, ranging from 0.00 to a
 18 maximum of 0.25 in 1978 and 0.18 in 1985. Maximum temperature exhibits
 19 a better correlation with elevation (in general above 0.3), although for two
 20 days this correlation is as low as 0.17 but reaches a maximum value of 0.60 on

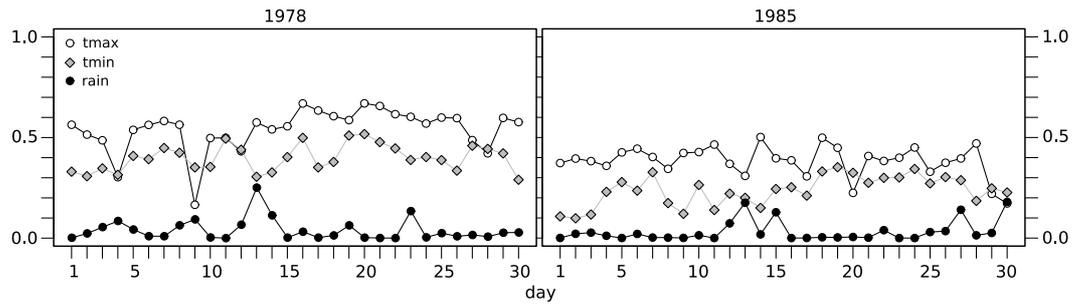


Fig. 3. Correlation of daily climatological variables for (a) June 1978 and (b) June 1985

- 1 some days. As the present study comprises a large area, the minimum value of
- 2 rainfall for all days is zero while its maximum value exceeds 100 mm in three
- 3 days during June 1978 and only once in 1985 (Fig. 4).

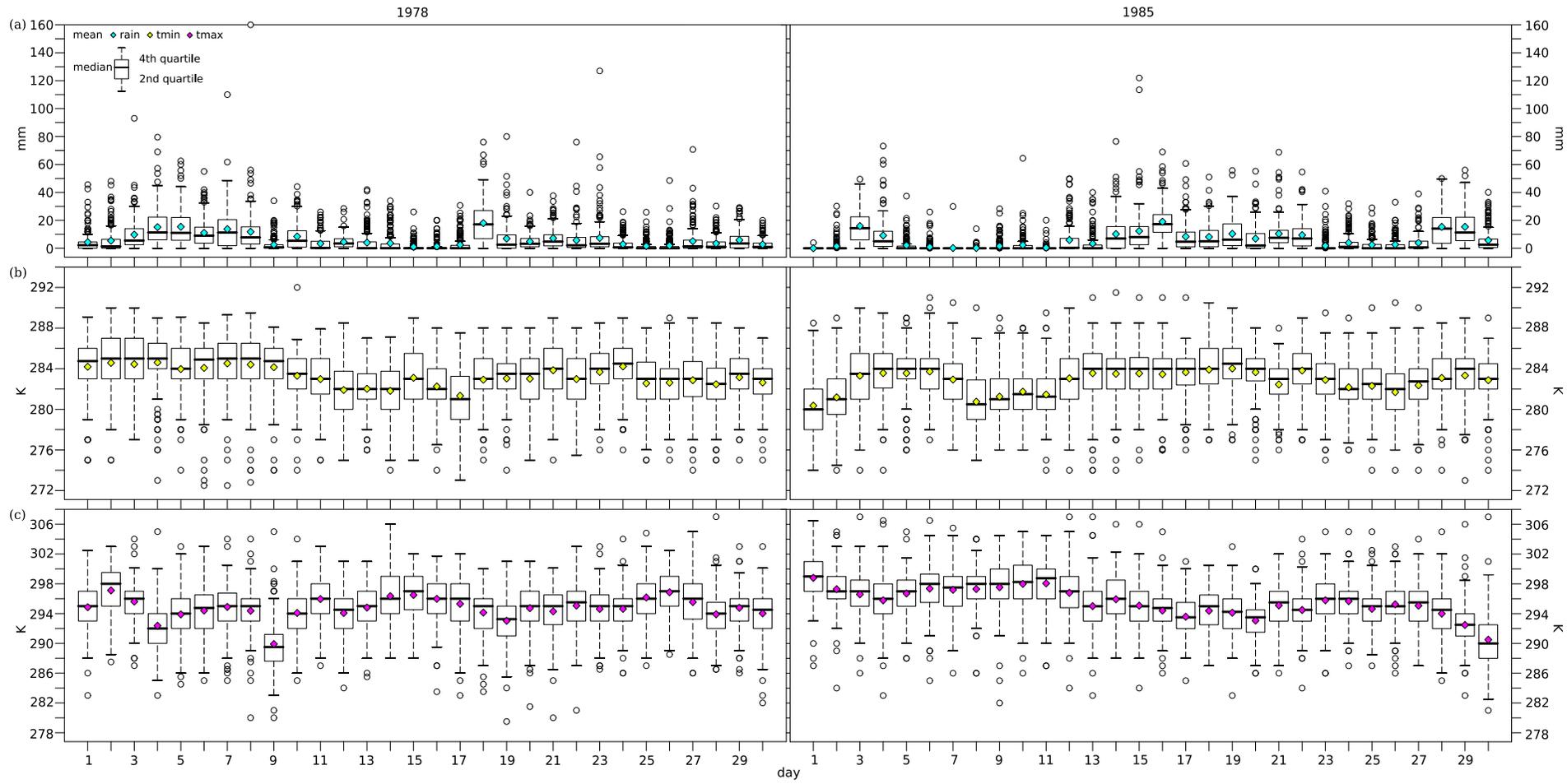


Fig. 4. Whisker plots of daily climatological variables for June 1978 and 1985: (a) rainfall, (b) minimum temperature and (c) maximum temperature

1 Before applying the Kriging algorithm, a semivariogram model needs to be
 2 fitted to the experimental semivariogram values in order to infer the model
 3 that best represents the spatial variation of the climatological variables. To
 4 this end, different models were used and fitted manually to the experimental
 5 semivariogram of the daily data; during this stage it was observed that the
 6 Bessel model was the one that best represented all three of the climatological
 7 variables. As a result, the Bessel semivariogram was used in the automatic fit-
 8 ting procedure for daily data, which is expressed as (Pebesma and Wesseling,
 9 1998):

$$\gamma(\mathbf{h}) = 1 - \frac{\mathbf{h}}{a} K_1 \left(\frac{\mathbf{h}}{a} \right) \quad (13)$$

10 where K_1 is the first order modified Bessel function of the second kind, a the
 11 range which represents the distance at which the semivariogram reaches its
 12 maximum value (sill) and \mathbf{h} represents distance.

13 In order to undertake the daily interpolations, the automated procedure pro-
 14 vided by GSTAT (Pebesma, 2004) which uses weighted least squares was used
 15 to fit the Bessel semivariogram. The semivariograms fitted in this way are
 16 shown in Fig. 5, which also shows the semivariograms of the residuals. The
 17 semivariograms of the residuals show the effect of removing the trend due
 18 to elevation on the experimental semivariogram. It can be noticed from this
 19 figure that when the climatological variable presents higher correlation with
 20 elevation (Fig. 3) the semivariogram of the residuals has a lower sill than the
 21 original values; this effect is quite obvious for maximum temperature in June
 22 23, 1978 (Fig. 5(c) which has a correlation of 0.60. For this variable on this par-
 23 ticular date, the fitted semivariogram of the residuals shows a range of nearly
 24 5 km and a sill of approximately 3 K², while the semivariogram fitted to the
 25 temperature data shows an asymptotic range. The difference between these
 26 semivariograms indicates that temperature has a drift caused by elevation,
 27 which would not be the case if the semivariograms were identical. The effect
 28 of removing the drift caused by elevation for maximum temperature had the
 29 same effect on June 15, 1985. It can be noted that the semivariograms fitted to
 30 both rainfall and its residuals are almost identical (Fig. 5(a)). The difference
 31 between the semivariogram of the variable and of its residual shows that the

1 climatological variables exhibit a trend due to elevation; this is obvious for
 2 both minimum and maximum temperature (Fig. 5(b) and 5(c)) but not for
 3 rainfall (Fig. 5(a)). From this same figure, it can be seen that the automatic
 4 fitting procedure yielded an interesting theoretical semivariogram for rainfall
 5 in June 15, 1985 and for maximum temperature in June 3, 1985 as closer val-
 6 ues show larger semivariances which is the opposite behavior of the theoretical
 7 semivariogram chosen. This situation might be improved if a smaller range is
 8 chosen for both the experimental and the theoretical semivariogram; however,
 9 the range was not modified as an automatic procedure is needed to undertake
 10 the large amount of data used in this work.

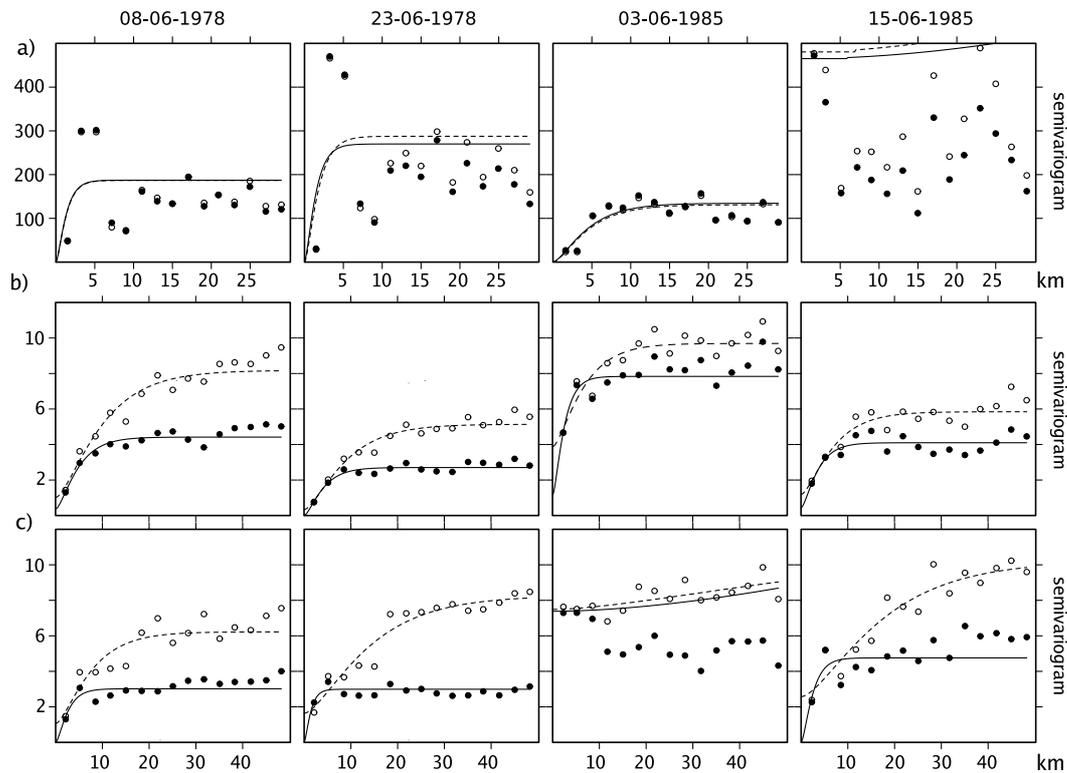


Fig. 5. Semivariograms of daily climatological variables: (a) rainfall, (b) minimum temperature and (c) maximum temperature. Circles represent the experimental semivariogram of the variables while the dashed line is the fitted semivariogram to these values. Black dots represent the semivariograms of the residuals and the solid line its fitted semivariogram

11 The spatial distribution of the climatological variables was developed using:
 12 Ordinary Kriging (OK) in a global neighborhood (Fig. 6), Kriging with Ex-
 13 ternal Drift (KED) in a global neighborhood (Fig. 7), Block Kriging with
 14 External Drift (BKED) using 30 x 30 km² blocks (Fig. 8), Ordinary Kriging
 15 on a local neighborhood (OK_l) using a 20 point neighborhood (Fig. 9) and

1 Kriging with External Drift on a local neighborhood, using 20 points as well
2 (Fig. 10). Although daily interpolations were undertaken for both June 1978
3 and June 1985, the aforementioned figures show the spatial distribution of the
4 climatological variables for only four different dates: two days in June 1978
5 and two in June 1985. For June 1978, days 8 and 23 were selected because
6 they exhibit the largest precipitation values, with 160 and 127 mm respec-
7 tively (Fig. 4); this was the same criterion used to select June 23, 1985 (122
8 mm), while June 3, 1985 was selected based on the fact that the maximum
9 rainfall depth for that day shows the smallest difference with its mean (Fig.
10 4). It should be noted that negative values were produced by the interpolation
11 algorithms at some points, which were considered to have a value equal to
12 zero.

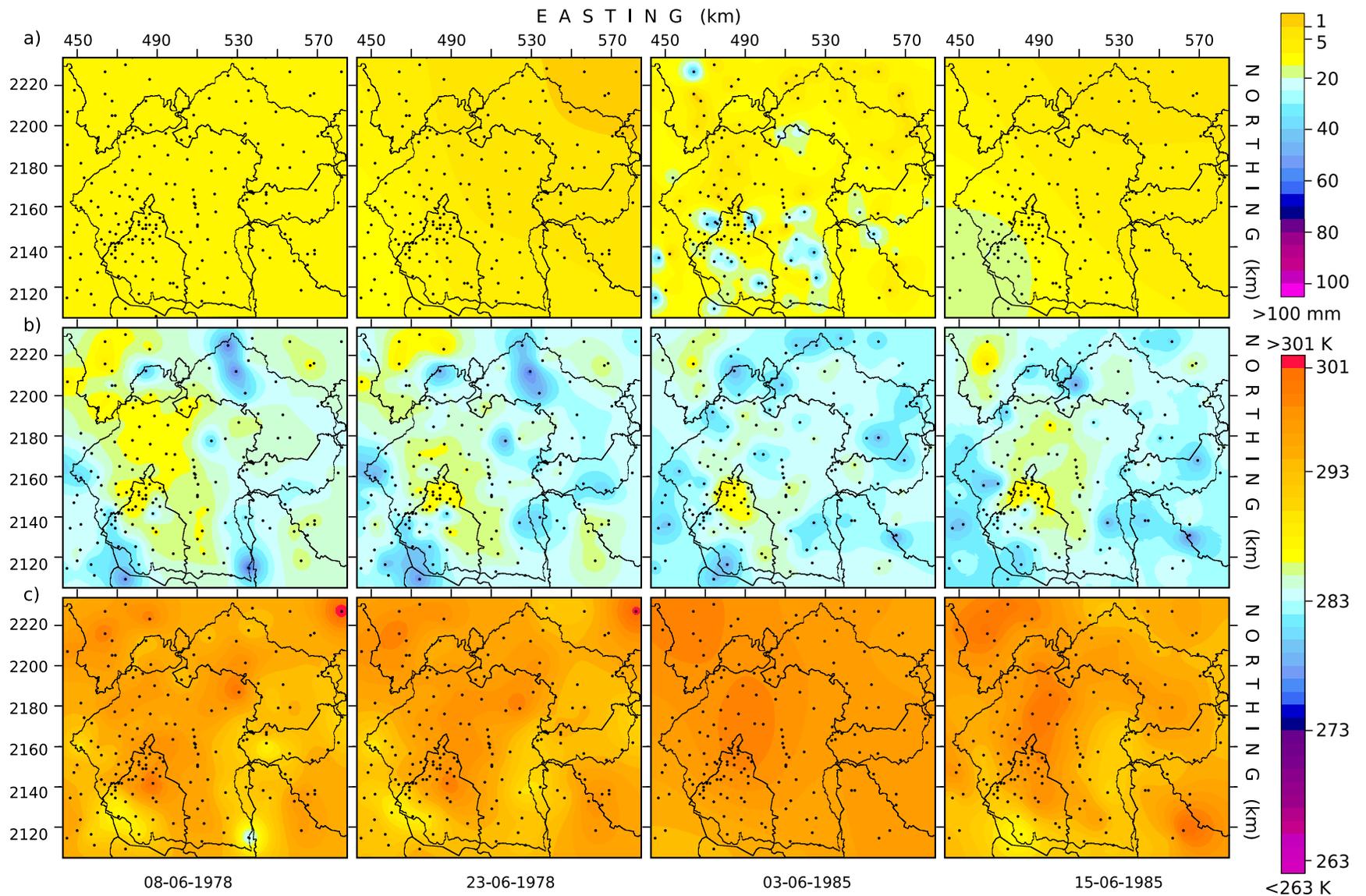


Fig. 6. Interpolation of daily climatological variables using Ordinary Kriging: (a) rainfall, (b) minimum temperature and (c) maximum temperature

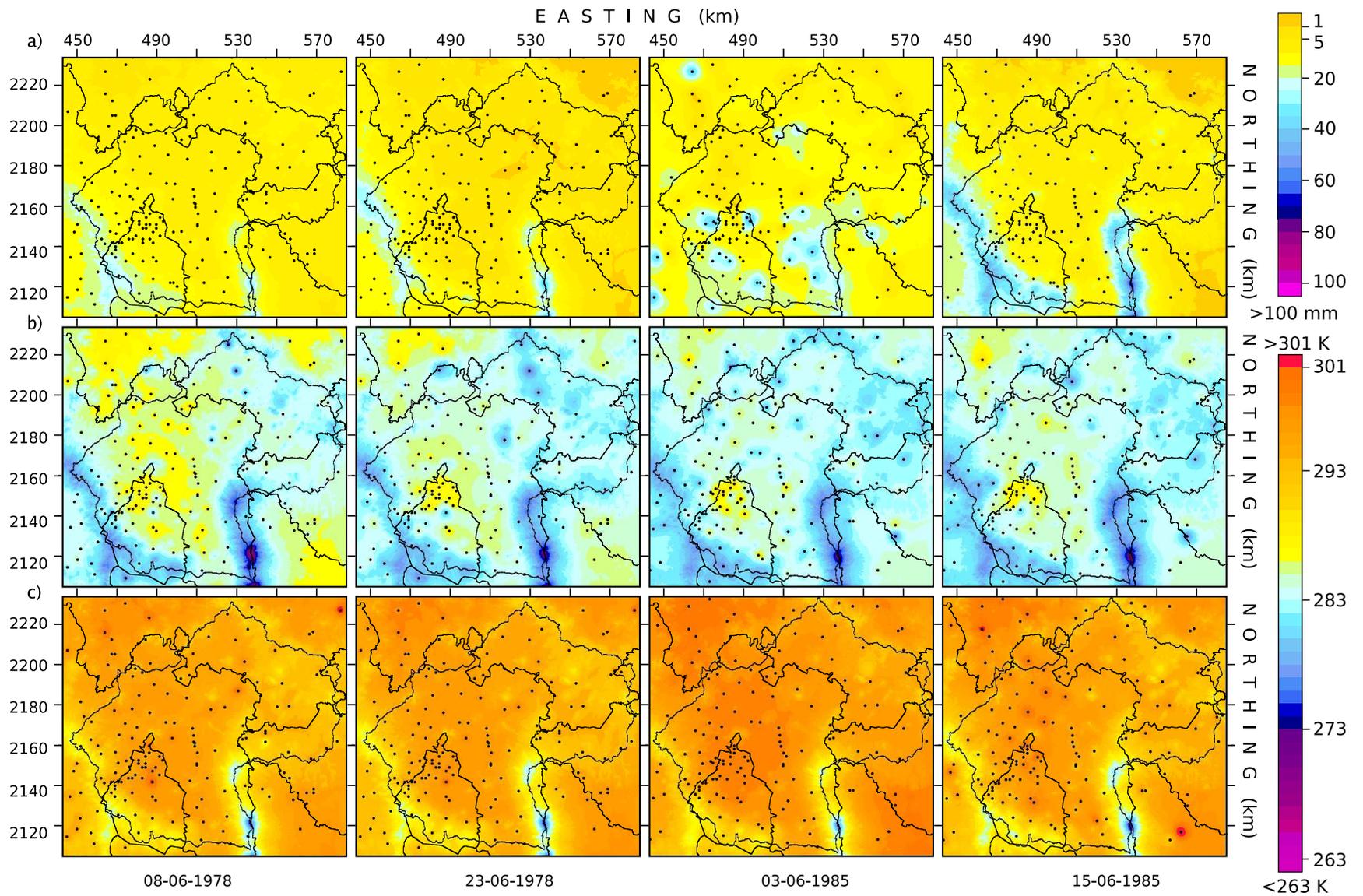


Fig. 7. Interpolation of daily climatological variables using Kriging with External Drift: (a) rainfall, (b) minimum temperature and (c) maximum temperature

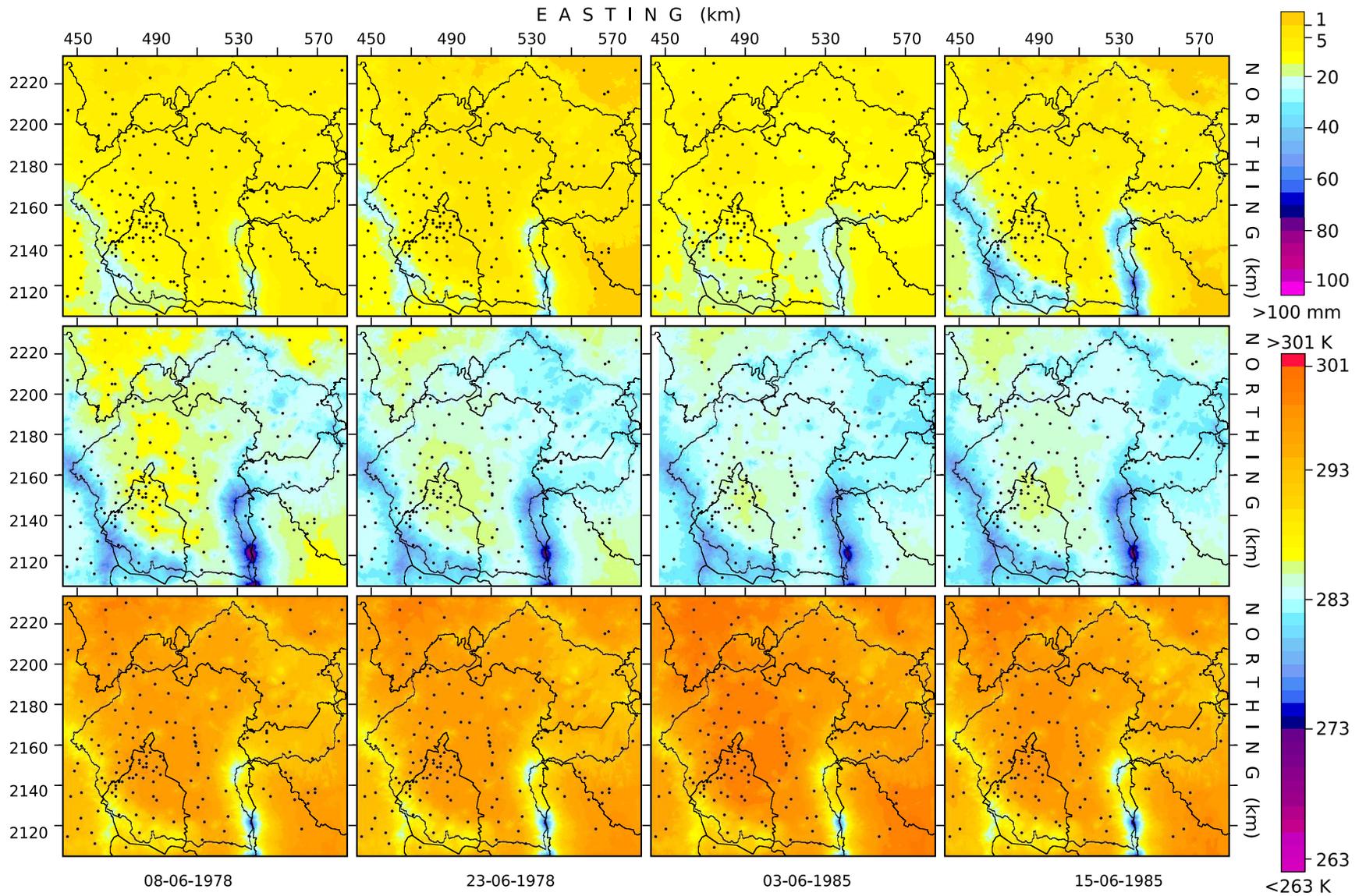


Fig. 8. Interpolation of daily climatological variables using Block Kriging with External Drift: (a) rainfall, (b) minimum temperature and (c) maximum temperature

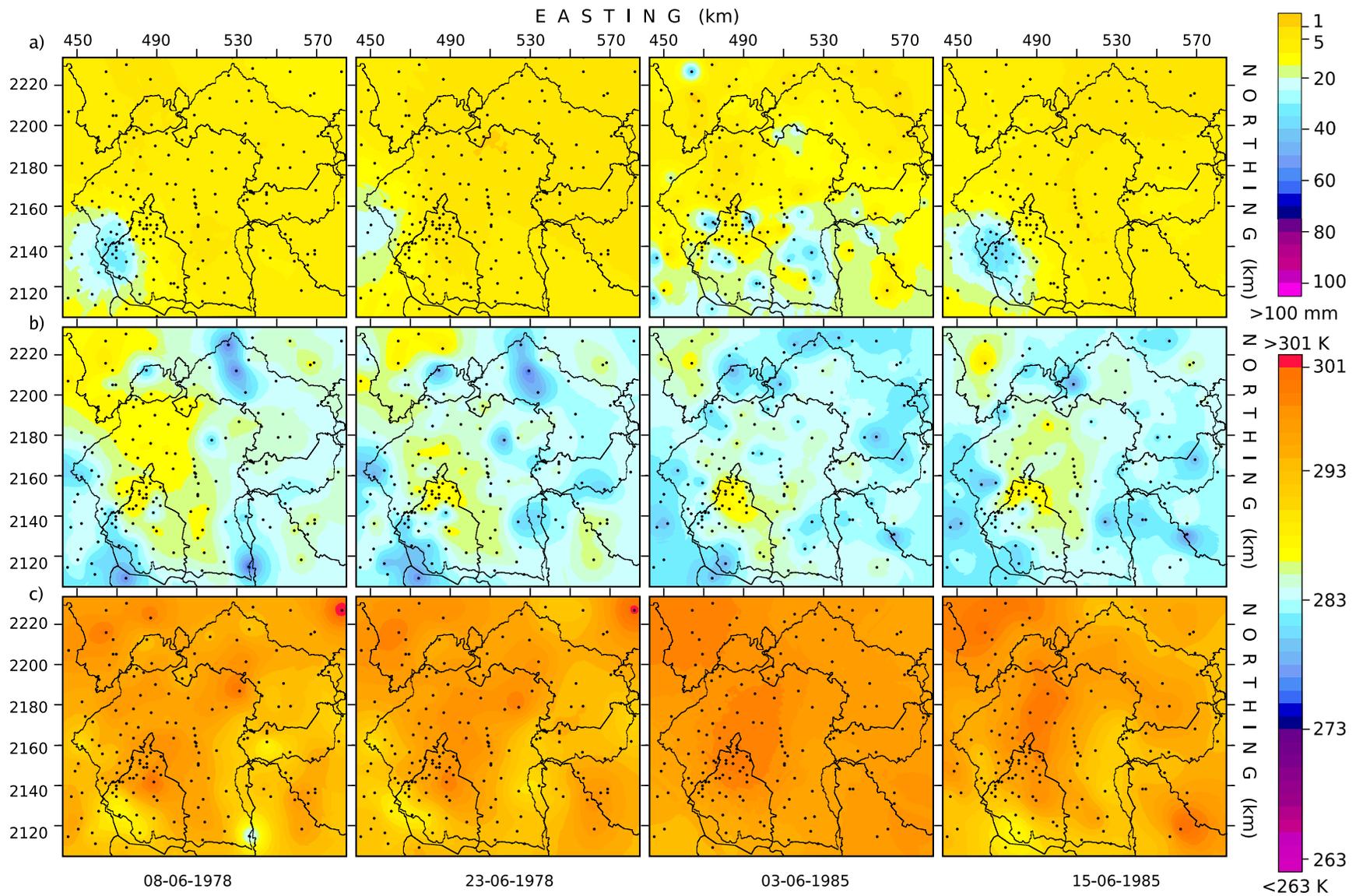


Fig. 9. Interpolation of daily climatological variables using Ordinary Kriging on a local neighborhood: (a) rainfall, (b) minimum temperature and (c) maximum temperature

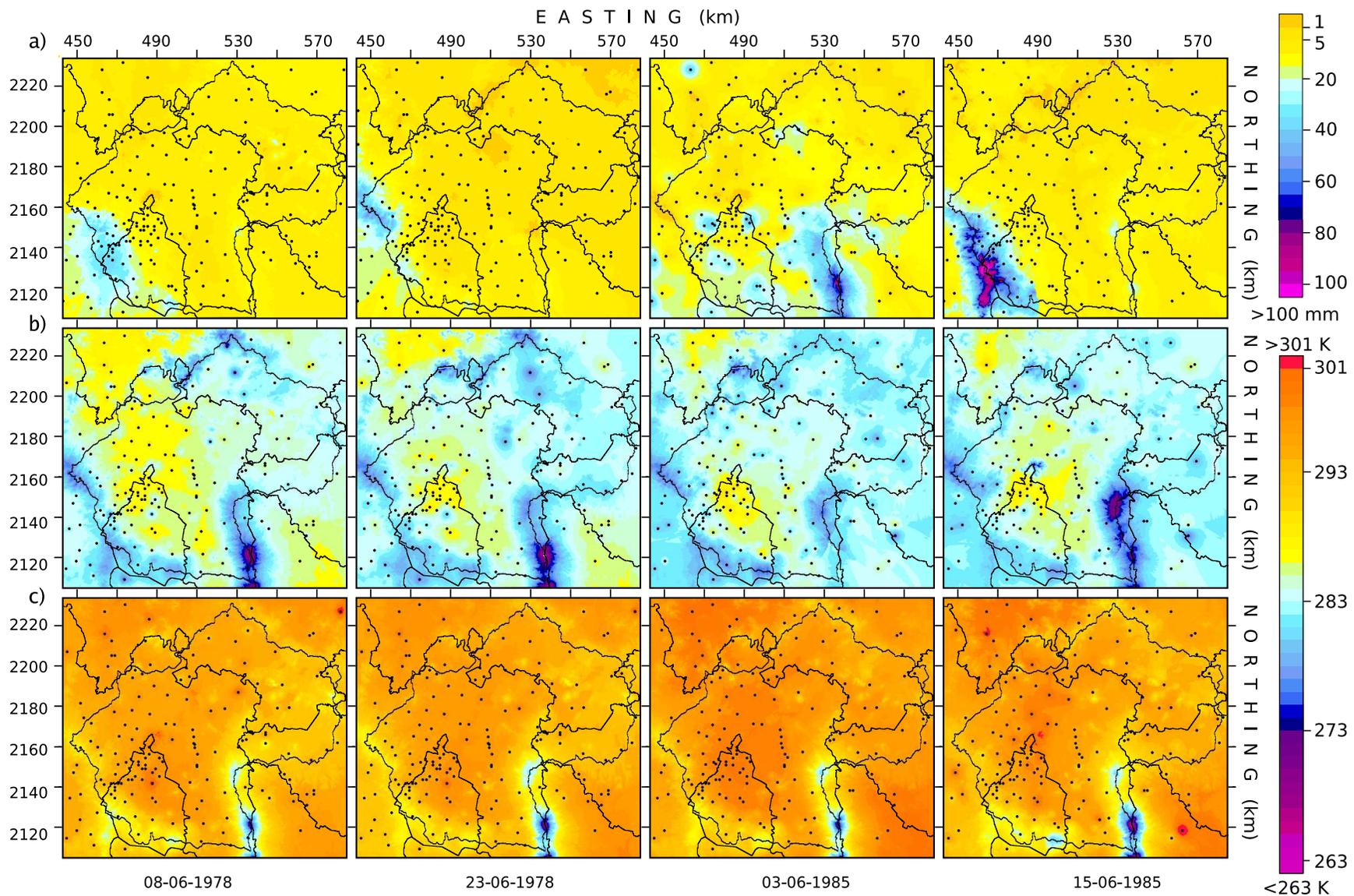


Fig. 10. Interpolation of daily climatological variables using Kriging with External Drift on a local neighborhood: (a) rainfall, (b) minimum temperature and (c) maximum temperature

1 4 Discussion

2 4.1 Visual inspection of spatial interpolation

3 4.1.1 Rainfall

4 The spatial variation of rainfall obtained by Ordinary Kriging (OK) (Fig. 6(a))
5 does not resemble the spatial pattern in the original data, which is expected
6 due to their large variation and by their non-stationarity throughout the study
7 area. For June 8, 1978 OK yields a uniform spatial distribution of rainfall,
8 while its distribution in June 23, 1978 and June 15, 1985 shows an increasing
9 trend towards the southwestern region of the Basin. The rain distribution
10 for June 23, 1978 shows some scattered points where precipitation values are
11 above 20 mm. The interpolated maps are quite different when Kriging with
12 External Drift is used (Fig. 7(a)), as precipitation values are larger in the
13 mountainous regions located southwards except for June 3, 1985 for which the
14 use of elevation as an external variable produced a more continuous surface for
15 precipitation values between 20 and 25 mm without increasing them over the
16 mountainous areas.

17 The rainfall maps produced with Block Kriging with External Drift (BKED)
18 (Fig. 8(a)) are very similar to those produced with KED, except for June 3,
19 1985 on which the larger precipitation spots located towards the Basin's centre
20 are not reproduced and larger precipitation is shown at higher elevation areas
21 (e.g. Sierra Nevada). When using Ordinary Kriging on a local neighborhood
22 (Fig. 9(a)), the resulting distribution of rainfall is quite different than the one
23 obtained by using the previous three methods, in particular for June 8, 1978
24 and June 15, 1985. These results should be expected by considering the large
25 differences between maximum and mean rainfall for those dates (Fig. 4) as
26 OK_l uses the mean of the 20 climatological stations closest to the point being
27 interpolated while OK considers the mean of the entire domain. For a day
28 in which the difference between the mean and maximum value is smaller the
29 outcome of OK in a local neighborhood is similar to that of OK on a global
30 neighborhood. This is the case for June 3, 1985 which is the day on which the
31 mean and maximum value of rainfall show the smallest difference.

1 The local distribution of rainfall (i.e. heterogeneity) is more noticeable when
2 Kriging with External Drift on a local neighborhood is applied (Fig. 10a),
3 producing precipitation values above 100 mm in the Southwestern area of the
4 Basin for June 15, 1985 and over 70 mm in the south. The spatial distribution
5 of rainfall obtained with this last method shows larger precipitation events on
6 more elevated areas, where the effect of elevation of rainfall is accounted for
7 locally, as is evident for June 15, 1985. If the relationship between precipita-
8 tion and elevation is accounted for globally, other mountainous areas would
9 also show larger precipitation events, as is the case when using KED on a
10 global neighborhood (Fig. 7a). By visual inspection, the maps developed with
11 KED on a local neighborhood show a more realistic distribution of rainfall;
12 however, in order to obtain a more quantitative criterion, cross validation was
13 used (Isaaks and Srivastava, 1989). With this procedure, one point is removed
14 at a time and interpolation is undertaken; this is done for all points in the
15 dataset and the root mean square error (RMSE) between the interpolated
16 and the observed values is computed. This method was applied for the differ-
17 ent Kriging methods as shown in Table 1 except for BKED which was omitted
18 as it estimates the average value over a block (Goovaerts, 1997).

19 The RMSE values of the Kriging methods used (Table 1) do not show a dras-
20 tic difference and although KED_l has the lowest RMSE, these values do not
21 provide a solid basis on which to decide the better interpolation method. In
22 order to provide further insight into the performance of each method, maps of
23 monthly accumulated precipitation for June 1978 and June 1985 were devel-
24 oped by adding the daily interpolation maps (Figures 11 and 12, respectively).

Table 1
 Cross validation for Ordinary Kriging (OK), Kriging with External Drift (KED), Ordinary Kriging on a local neighborhood (OK_l),
 Kriging with External Drift on a local neighborhood (KED_l)

	8-6-1978			23-6-1978			3-6-1985			15-6-1985		
	rain	tmin	tmax	rain	tmin	tmax	rain	tmin	tmax	rain	tmin	tmax
OK	14.56	2.50	2.48	12.09	1.99	1.97	9.82	2.99	2.72	14.37	2.20	2.79
KED	14.34	2.18	2.00	11.70	1.79	1.77	9.87	2.89	2.36	13.94	2.06	2.41
OK_l	13.31	2.48	2.48	11.41	1.99	1.99	9.40	3.02	2.67	12.72	2.23	2.77
KED_l	13.65	2.18	2.09	11.49	1.76	1.86	9.47	3.01	2.42	11.76	2.13	2.36

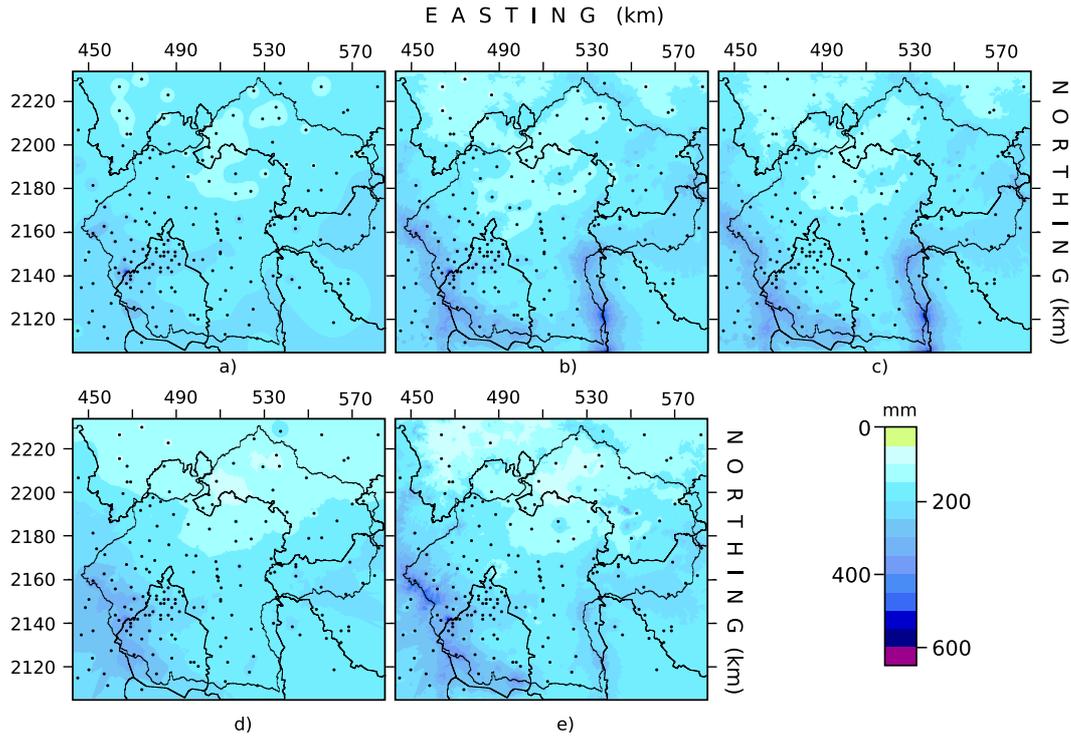


Fig. 11. Accumulated rainfall for June 1978 derived from daily interpolated maps using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Local Kriging and (e) Local Kriging with External Drift

1 The spatial distribution of rainfall obtained for June 1978 (Fig. 11) shows that
 2 for four out of the five interpolation methods, the effect of mountainous terrain
 3 can be noticed, even the map derived from OK_l , which does not consider ele-
 4 vation as an auxiliary variable shows larger precipitation values at the Sierras
 5 Chichinautzin and Las Cruces (Fig. 11d). The effect of topography on each
 6 monthly map is different according to each Kriging technique, which was also
 7 the case for the daily distributions due to the fact that OK and KED use both
 8 a global mean and global relationship between rainfall and elevation (e.g. they
 9 use all the stations to determine these values). The monthly rainfall pattern
 10 obtained with KED shows larger values as elevation increases, an effect that is
 11 observed on the southern sierras that enclose the Basin. The maps produced
 12 by KED_l (Fig. 11(e)) show larger precipitation values only at the Sierras de
 13 las Cruces and Chichinautzin which is also the case for the monthly map de-
 14 rived from the OK_l interpolations. The difference between these two maps is
 15 the maximum value of precipitation at these Sierras, as KED_l extrapolates
 16 precipitation using the local trend caused by elevation. In fact, the map de-
 17 rived from the KED_l daily interpolations shows larger precipitation values at

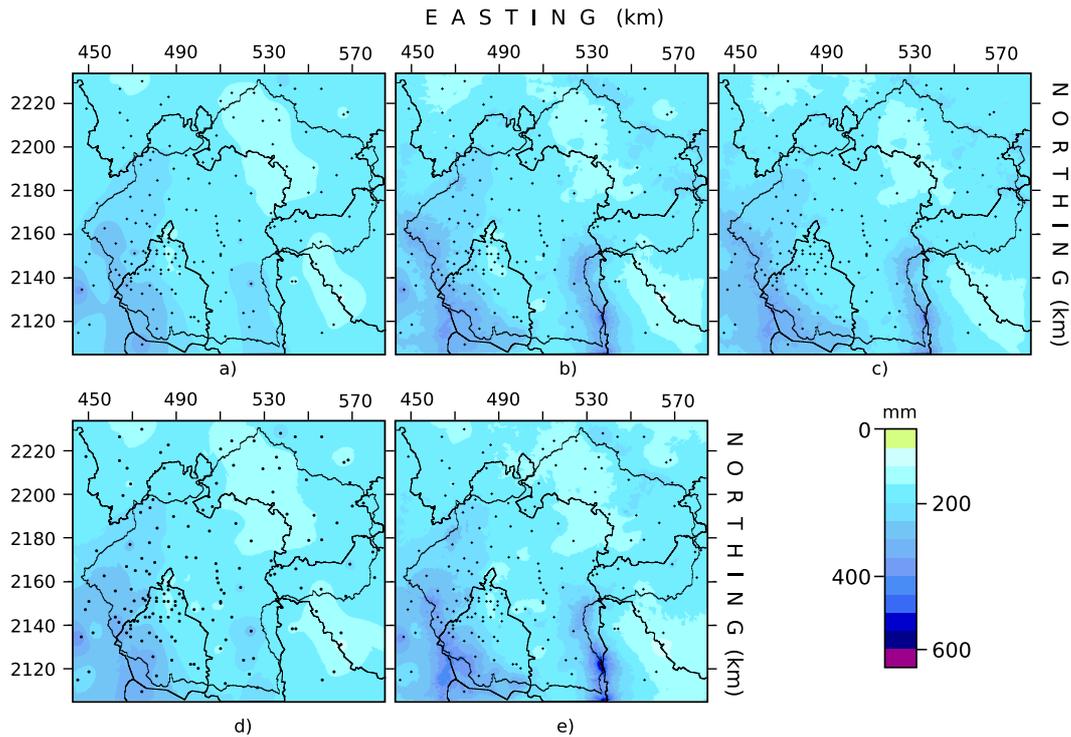


Fig. 12. Accumulated rainfall for June 1985 derived from daily interpolated maps using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift (d) Local Kriging and (e) Local Kriging with External Drift

1 the Sierra Nevada, although smaller than those at the Sierra Las Cruces. The
 2 monthly maps for 1985 (Fig. 12) show the same effect: precipitation is extrap-
 3 olated, except that the Sierra Nevada shows larger rainfall values in the map
 4 derived from the KED_l interpolations. From these two figures, it can be con-
 5 cluded that the southern Sierras are the ones that cause the orographic effect
 6 on precipitation and that each of the Kriging methods produce maps with dif-
 7 ferent rainfall distribution; however it is hard to choose a better method based
 8 on the monthly maps and on the cross validation values, as these values are
 9 not significantly different from each other. In order to provide further insight
 10 into this problem, the difference between the interpolated and observed values
 11 will be analyzed by their location and elevation after analyzing the spatial
 12 distribution of temperature.

13 4.1.2 Temperature

14 The temperature maps obtained from the different Kriging methods differ
 15 from the precipitation maps in that the only maps that show the effect of to-

1 pography are those that use elevation as an auxiliary variable. It is interesting
2 to note the effect that the presence of one particular station has on the spatial
3 distribution of temperature, which is the climatological station *Atlauta*. This
4 station is the one with the highest elevation, located at approximately 3700
5 masl in the Sierra Nevada near the southeasternmost point of the watershed
6 limit and with temperature records for only one of the four selected days.
7 The effect of the records from this station can be easily identified on the spa-
8 tial distribution of temperature using OK (Fig. 6) for June 8th, 1978, which
9 shows how both minimum and maximum temperatures decrease in the sierras
10 that surround the Basin, particularly on the Sierra Chichinautzin and Sierra
11 Nevada. However, for the remaining three days on which no data from the At-
12 lauta station are available, the southern part of the Sierra Nevada has higher
13 temperatures than the Sierra Chichinautzin. This difference of temperature is
14 not feasible, as in this area the highest peaks of the region are located with an
15 elevation above 5,000 masl and are snow-capped all year round. This anomaly
16 is due to the fact that OK does not consider the relationship between elevation
17 and the climatological variable; furthermore, the density of the climatological
18 stations in this area is not appropriate to capture the effects due to the drastic
19 change in topography.

20 The maps developed from the use of KED (Fig. 7) for both temperatures show
21 lower minimum temperatures in the Sierra Nevada than those observed in the
22 Sierra Chichinautzin and Las Cruces, reaching 273 K even in those days with-
23 out data from the Atlauta climatological station. This is the result of using
24 the relationship between temperature and elevation, as it is used to extrapo-
25 late temperature on elevations above that on which the highest climatological
26 station is located. As in the case of the maps derived by using OK, the ones
27 developed by KED show an increase in minimum temperature in the area in
28 which Mexico City is located, an effect known as the urban heat island effect.
29 The spatial extent of this effect varies according to the Kriging technique used;
30 the maps developed from BKED (8) show a smaller area with higher temper-
31 atures than those obtained from the other Kriging techniques as can be easily
32 seen for June 3, 1985. The minimum temperature maps produced by KED_l
33 (10) show lower temperatures in the Sierra Nevada, which should be expected;
34 however due to the small density of the climatological network in that area,
35 the maps show a drastic change in temperature instead of a smooth and con-

1 tinuous pattern. This is caused by the effect of using a 20 point neighborhood
2 and the areas that show this drastic change use a station that has data not
3 related to the estimation point (i.e. the datum used is too far from the estima-
4 tion point). To overcome this problem both smaller and larger neighborhoods
5 were used without improving the results; when a smaller neighborhood was
6 used, the spatial distribution of minimum temperature showed more drastic
7 changes, due to the small density of climatological stations in some areas.
8 When a larger neighborhood was used, the spatial distribution of temperature
9 was almost identical to the one obtained from KED on a global neighborhood.

10 4.2 Error analysis

11 4.2.1 Rainfall

12 In order to verify which Kriging method yields better results, the monthly
13 accumulated maps derived from daily interpolations (Figures 11 and 12) were
14 sampled at the location of each climatological station and the absolute dif-
15 ference between them and the accumulated point values was computed. The
16 statistics of these values are grouped in Table 2 which shows that for June
17 1978, KED provides the smallest mean value of errors, but that it is almost
18 equal to the mean error value obtained with KED_l (22.12 and 22.16) while
19 the correlation of the computed and observed values is also better for KED,
20 although they are also quite similar (0.83 and 0.81 respectively). KED_l also
21 yields a larger maximum difference value (145.41 mm) than KED (109.42 mm)
22 and Block Kriging is the method that yields both the largest mean and max-
23 imum difference value as well as the least correlation between observed and
24 computed monthly accumulated rainfall ($\rho = 0.45$). For June 1985, the correla-
25 tion between these monthly values is the same for OK, KED, OK_l and KED_l ;
26 while KED_l yields the smallest mean difference value, it is worth noting that
27 again, this value is not drastically different from the one obtained by OK_l
28 (18.65 and 19.43 respectively).

Table 2

Statistics for the difference between measured monthly accumulated values and monthly accumulated values derived from daily interpolations for Ordinary Kriging (OK), Kriging with External Drift (KED), Block Kriging (BK), Ordinary Kriging in a local neighborhood (OK_l) and Kriging with External Drift on a local neighborhood (KED_l)

var	method	June 1978					June 1985				
		ρ	mean	std. dev.	min	max	ρ	mean	std. dev.	min	max
rain	OK	0.84	24.58	20.45	0.32	128.27	0.85	20.20	19.70	0.06	131.17
	KED	0.83	22.12	19.50	0.12	109.42	0.85	20.61	18.86	0.44	137.82
	BKED	0.45	41.78	32.49	0.18	159.27	0.56	32.20	31.83	0.08	245.09
	OK_l	0.77	25.23	22.90	0.25	147.14	0.85	19.43	18.53	0.50	123.13
	KED_l	0.81	22.16	21.77	0.09	145.41	0.86	18.65	17.33	0.16	138.44
tmin	OK	0.96	10.86	9.51	0.01	51.17	0.84	20.21	20.67	0.11	115.06
	KED	0.98	7.72	6.88	0.06	40.45	0.83	20.21	21.56	0.01	134.64
	BKED	0.60	29.41	30.47	0.27	189.59	0.52	31.30	32.70	0.29	205.66
	OK_l	0.96	10.66	9.61	0.01	50.99	0.84	20.16	20.66	0.09	112.75
	KED_l	0.98	7.41	6.78	0.06	39.06	0.82	19.78	20.36	0.35	125.32
tmax	OK	0.92	19.05	15.65	0.04	89.75	0.76	30.14	31.05	0.43	174.97
	KED	0.94	18.48	16.01	0.50	88.50	0.86	23.39	22.26	0.34	151.49
	BKED	0.72	30.80	27.14	0.51	149.93	0.57	37.66	36.09	0.28	215.18
	OK_l	0.92	18.86	15.54	0.08	88.54	0.76	29.83	30.81	0.39	174.70
	KED_l	0.98	9.55	8.72	0.15	49.40	0.86	23.40	21.37	0.02	146.32

1 The values of measured and derived monthly accumulated rainfall for June
2 1978 and 1985 were plotted along with their regression line as illustrated in
3 Figs. 13 and 14 which show how for both months the regression line fitted to
4 the observed and derived values from the KED_l interpolations is closer to the
5 45° line (Figs. 13e, and 14e) and that the points show less dispersion; the 45°
6 line is shown in order to have a reference on where a perfect fit between the
7 observed and derived accumulated value should be located. The scattergrams
8 of BK show more dispersion, which should be expected by considering the
9 values of Table 2. In order to analyze whether or not the maximum errors
10 were located at higher or lower elevations, the absolute difference between
11 observed and derived monthly values are shown in Fig. 15 for June 1978 and
12 Fig. 16 for June 1985.

13 For 1978 (Fig. 15) it can be noted that errors are larger when BKED is used
14 and that when elevation is used as a secondary variable the errors for low
15 and high elevations are in general smaller than when elevation is not used;
16 furthermore, when KED_l is used (Fig 15(e)) the errors are in general lower.
17 The opposite is observed in the case of BKED as the errors are distributed
18 throughout the elevation range for both years. For 1985 (Fig. 16) it can be
19 seen that by using elevation as a secondary variable, the errors observed are
20 smaller except for BKED as previously mentioned. The difference between
21 using a local neighborhood instead of a global one is evident by comparing
22 the distribution of errors with elevation obtained from KED_l (Fig. 16(e)) and
23 KED (Fig. 16(b)) as most of the errors derived from the use of KED_l are below
24 50 [mm]. An interesting pattern can also be observed from these figures, as
25 the largest errors for four out of the five Kriging techniques are observed at
26 the same elevations (near 2600 masl) which indicates that some type of error
27 can be associated to the climatological stations located at that elevation and
28 that probably the same stations are the ones which exhibit these large errors.
29 In order to verify whether or not the errors are associated with a particular
30 climatological station, the spatial distribution of errors will also be analyzed.

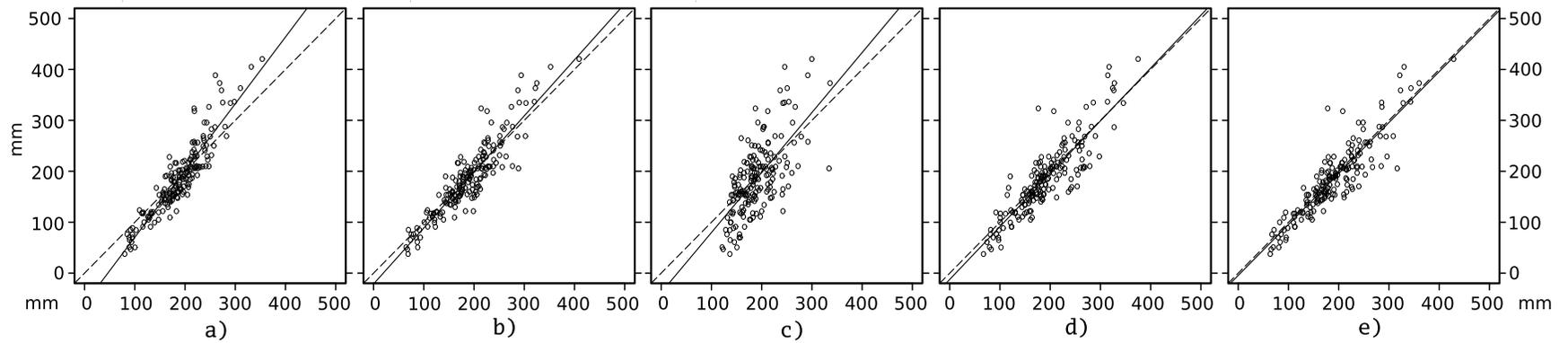


Fig. 13. Comparison between measured accumulated monthly rainfall and monthly accumulated rainfall derived from daily interpolated rainfall for June 1978 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

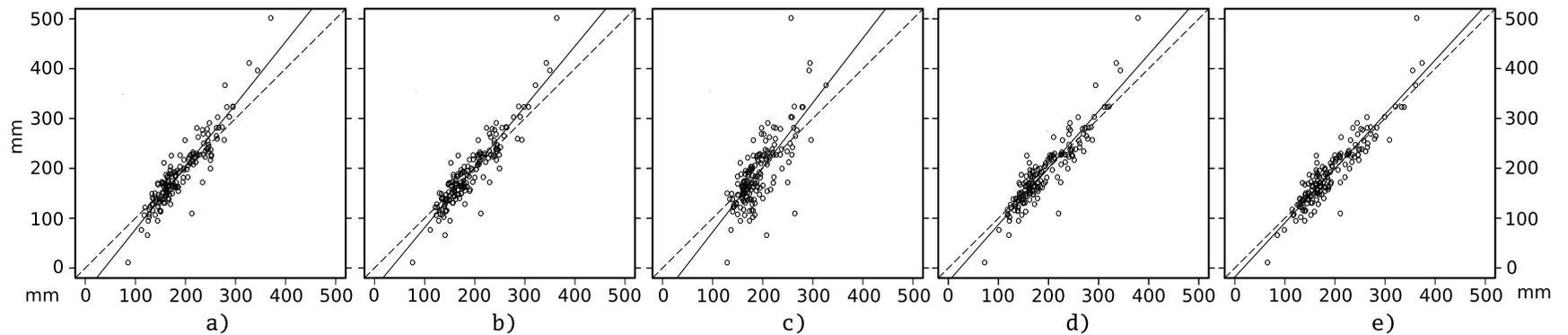


Fig. 14. Comparison between measured accumulated monthly rainfall and monthly accumulated rainfall derived from daily interpolated rainfall for June 1985 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

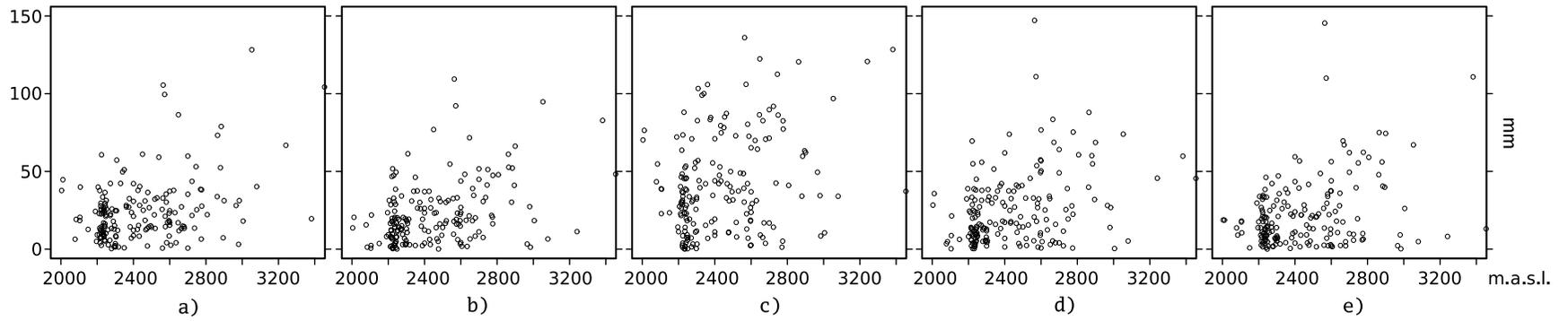


Fig. 15. Elevation of climatological stations and absolute difference between measured accumulated rainfall for June 1978 and accumulated monthly rainfall from daily interpolations using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

31

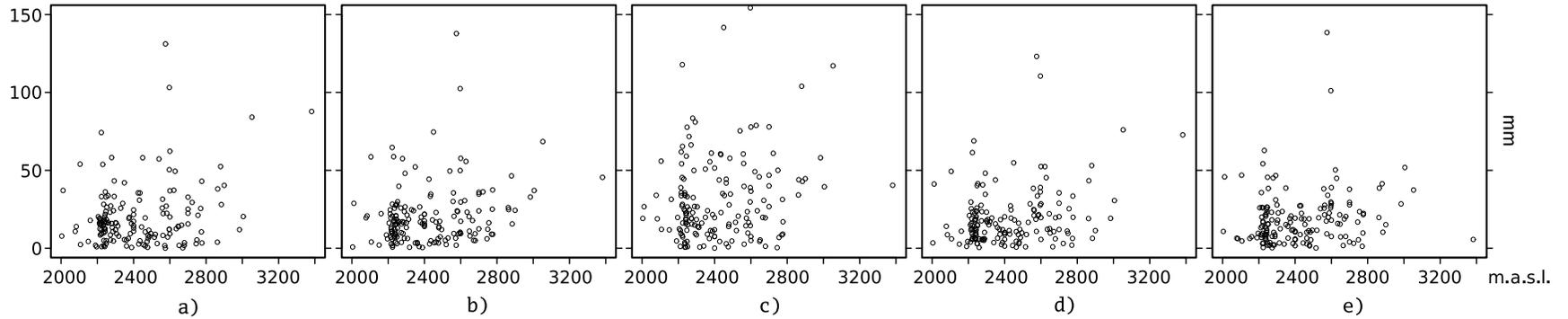


Fig. 16. Elevation of climatological stations and absolute difference between measured accumulated rainfall for June 1985 and accumulated monthly rainfall from daily interpolations using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

1 4.2.2 Temperature

2 This analysis will only compare the observed and the interpolated monthly ac-
3 cumulated value for both minimum and maximum temperature, as done in the
4 previous section for rainfall. These values were also plotted on a scattergram
5 and a regression line was fitted to them using June 1978 (Figs. 17 and 19) and
6 June 1985 (Figs. 18 and 20). The statistics of the errors are shown in Table 2 in
7 which it can be observed that the method that yields the largest mean value of
8 errors for both temperatures and for both months as well as the minimum cor-
9 relation value is BKED while the remaining four methods yield similar values.
10 For minimum temperature in June 1978, KED and KED_l exhibit the highest
11 correlation value ($\rho = 0.98$) between the accumulated observed temperature
12 and the accumulated interpolated temperature, and a similar maximum, min-
13 imum and mean value of errors which is also the case for OK and OK_l . For
14 June 1985 the methods which yield the largest correlation value are OK_l and
15 OK, although it is similar to the correlation obtained by KED and KED_l ;
16 again, the difference between the mean error value is not drastic for the four
17 Kriging methods. Based on the values shown on Table 2, the best results are
18 provided by KED_l although the statistics obtained from KED are almost iden-
19 tical to the previous method. However if the spatial pattern produced by each
20 Kriging method is considered (Figs. 6, 7, 8, 9 and 10) then KED would be
21 the preferred option due to the spatial continuity of minimum temperature
22 throughout the study area, as discussed in section 4.1.2.

23 In the case of maximum temperature, the methods that yield the largest cor-
24 relation values between the observed and interpolated accumulated values for
25 1978 are KED and KED_l (with $\rho = 0.94$ and $\rho = 0.98$ respectively) as well as for
26 1985 ($\rho = 0.86$ for both cases). These two methods provide the smallest mean
27 value of error, which is the same for both techniques in 1985, while it doubles
28 from 9.78 K for KED_l to 18.48 for KDE in 1985. As these values are similar
29 for both techniques and both years, the spatial distribution of the interpoalted
30 variable should be used as previously done for minimum temperature. Again,
31 the method that should be used is KED as the maps produced with KED_l
32 shows discontinuities in some areas due to the low density of climatological
33 stations in some areas.

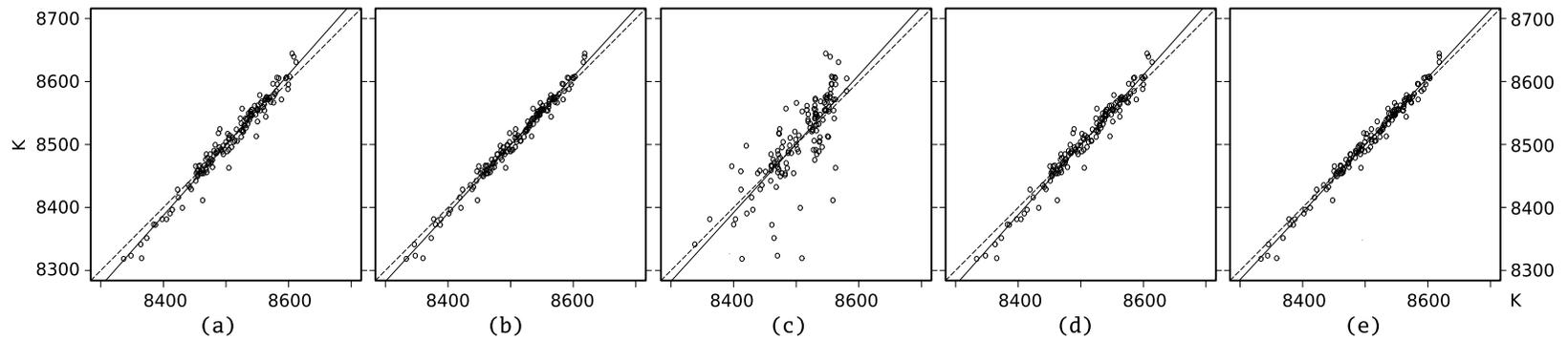


Fig. 17. Comparison between monthly accumulated minimum temperature and monthly accumulated minimum temperature derived from daily interpolated minimum temperature for June 1978 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

33

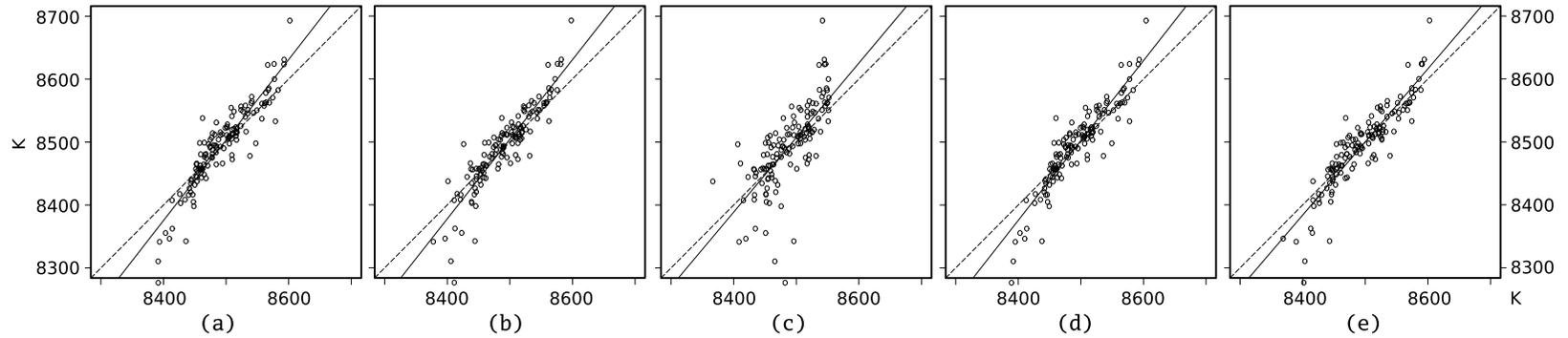


Fig. 18. Comparison between monthly accumulated minimum temperature and monthly accumulated minimum temperature derived from daily interpolated minimum temperature for June 1985 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

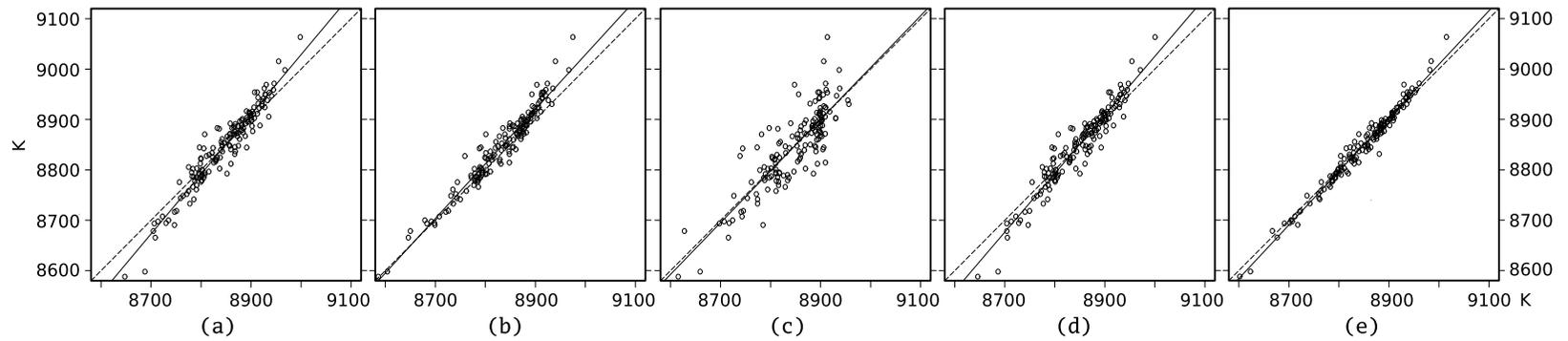


Fig. 19. Comparison between monthly accumulated maximum temperature and monthly accumulated maximum temperature derived from daily interpolated maximum temperature for June 1978 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

34

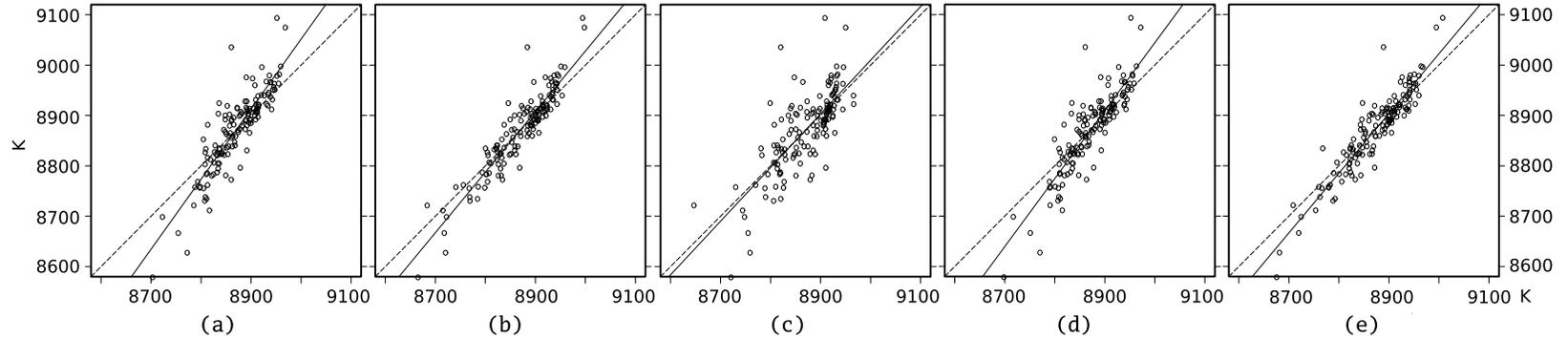


Fig. 20. Comparison between monthly accumulated maximum temperature and monthly accumulated maximum temperature derived from daily interpolated maximum temperature for June 1985 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

1 4.3 Spatial distribution of errors

2 The previous section, which analyzed the relationship between errors and ele-
 3 vation raised the question of whether or not two climatological stations yielded
 4 large interpolation errors, which might be caused by the use of erroneous co-
 5 ordinates. In order to provide further insight into this question, the spatial
 6 location of the differences obtained with each Kriging method is analyzed in
 7 this section, as shown in Fig. 21 for June 1978 and Fig. 22 for June 1985. For
 8 June 1978 all Kriging variants yield a large error (>100 mm) in particular
 9 for two locations (as expected from the previous analysis): one in the Sierra
 10 Nevada and another one just north of it (Fig. 15 and Fig. 21) in fact all Krig-
 11 ing variants produced a value that was smaller than the measured value (light
 colored circles represent larger observed values).

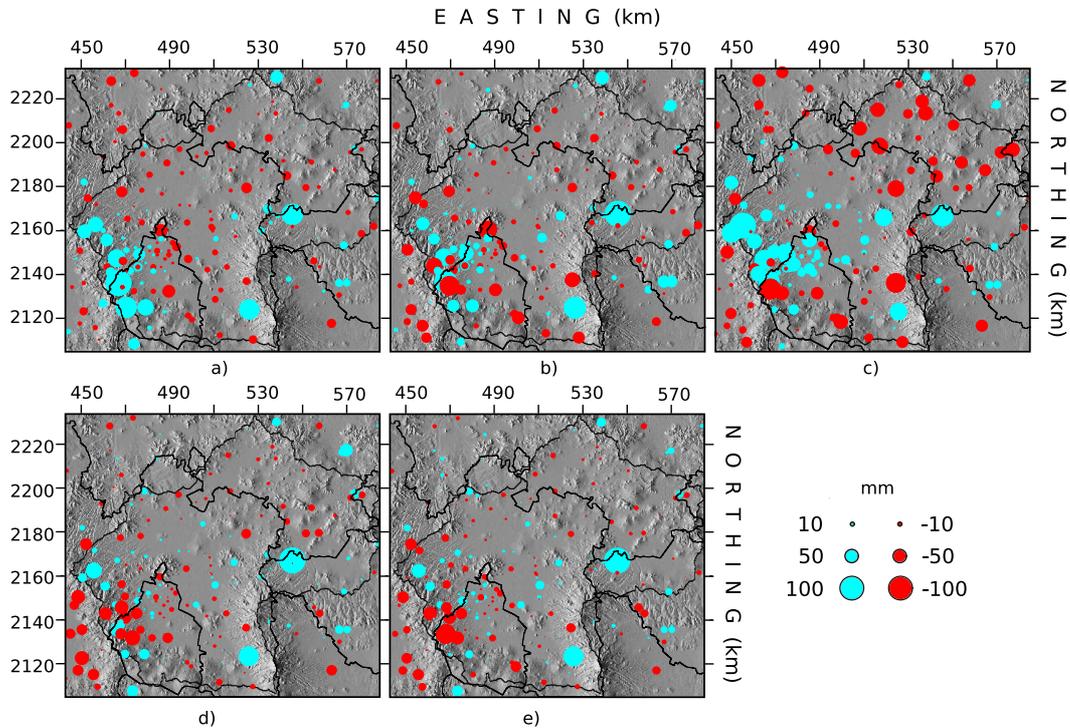


Fig. 21. Spatial distribution of difference between observed accumulated monthly rainfall and accumulated monthly rainfall derived from daily interpolations for June 1978 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

12
 13 The differences are smaller when elevation is used, except in BKED, which
 14 yielded larger errors throughout the study area (Fig 21c). It is interesting to

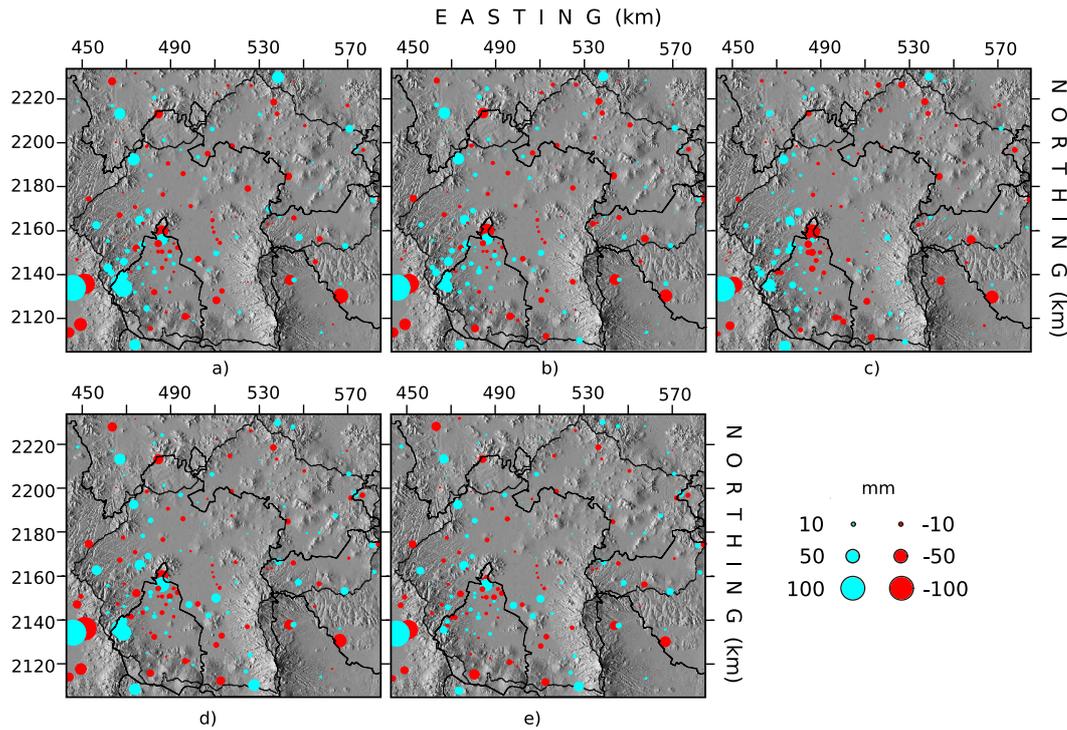


Fig. 22. Spatial distribution of difference between observed accumulated monthly rainfall and accumulated monthly rainfall derived from daily interpolations for June 1985 using: (a) Ordinary Kriging, (b) Kriging with External Drift, (c) Block Kriging using External Drift, (d) Ordinary Kriging in a local neighborhood and (e) Kriging with External Drift on a local neighborhood.

1 note that OK underpredicts rainfall in the Sierra las Cruces as well as KED;
 2 on the other hand, OK_l and KED_l overpredict monthly rainfall in this area.
 3 Analyzing the two points previously mentioned for June 1978, it can be noted
 4 that for the northernmost point, its measured monthly accumulated rainfall
 5 is 323 mm while the value recorded at its closest climatological station is
 6 196 mm; for the remaining point, its measured rainfall depth was 318 mm
 7 while its nearest station recorded 151 mm. This large heterogeneity in such a
 8 small distance causes all Kriging variants to underestimate the rainfall value at
 9 those points as the fitted semivariogram can not represent this heterogeneity.
 10 Repeating this analysis for June 1985 the same phenomenon is found, as the
 11 rainfall depth at the eastern most point is 109 mm while at the remaining point
 12 this depth is equal to 501 mm; accordingly all Kriging variants overestimate
 13 precipitation at the first point and underestimate it at the second station. This
 14 difference in precipitation values is very large, while the distance between them
 15 is 1.3 km for June 1978 and 5.5 km for June 1985; furthermore, this variation
 16 is not accounted for on the semivariogram model.

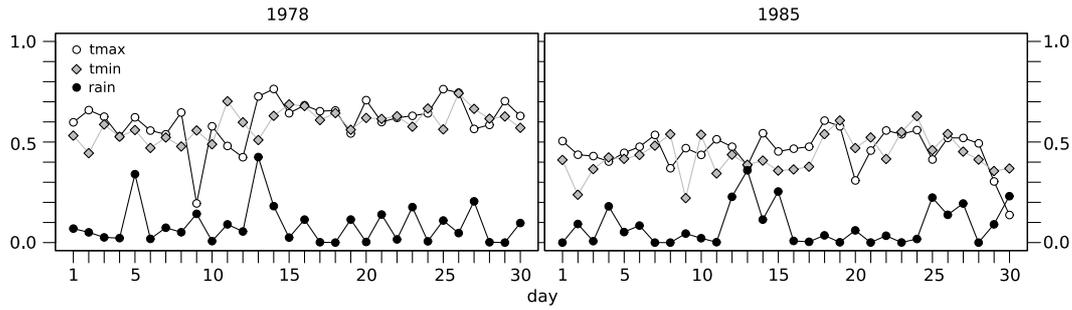


Fig. 23. Correlation of daily climatological variables with elevation in the southwestern area of the Basin for (a) June 1978 and (b) June 1985

1 4.4 Local neighborhoods

2 The previous sections discussed the effect of the different Kriging method
 3 and the difference between using a global or local neighborhood will be fur-
 4 ther discussed in this section by using the statistics of the global and local
 5 neighborhood, in the southwestern area of the Basin indicated by the dashed
 6 rectangle of Fig. 1.

7 The correlation between rainfall and elevation varies greatly on some days,
 8 e.g. June 5, 1978 changed from a global correlation of 0.04 (Fig. 3(a)) to a local
 9 one of 0.34 (Fig. 23(a)) while the correlation for June 13, 1985 changed from
 10 0.25 (Fig. 3(b)) to 0.43 (Fig. 23(a)). For the four different days discussed in
 11 the previous sections of this work, the difference between correlations was not
 12 as drastic as for June 8, 1978 the correlation was the same (0.06 for global and
 13 0.05 for local) while for June 23 1978 it changed from 0.13 to 0.18. For 1985,
 14 the correlation observed on June 3, decreased from 0.03 to 0.01 although it
 15 increased in June 15 from 0.13 to 0.25. The effect of these values is noticeable
 16 in the interpolated maps for rainfall as the day which exhibited the maximum
 17 local correlation in the southwestern area shows a large difference in the rainfall
 18 values when elevation is used (Figs. 6(a) and 10(a)).

19 The importance of using a local neighborhood is made evident by comparing
 20 the statistics between the global (Fig. 4) and local (Fig. 24) neighborhoods For
 21 June 8, 1978 the maximum value is located in the southwestern area of the
 22 Basin and the mean value of rainfall for this day changes from around 11 mm to
 23 near 22 mm when using a global or a local neighborhood respectively. The same
 24 pattern is observed for June 23 1978 and June 15 1985 as the maximum value

1 of rainfall is also located in this area and its mean value is also increased when
2 a local neighborhood is used. For June 3, 1985 the maximum value of rainfall is
3 not located in the southwestern subarea. When a global neighborhood is used
4 this value is 50 mm while on the southwestern area the mean value is near 46
5 mm, an effect that is observed in the interpolated distribution of rainfall for
6 this day by KED_t (Fig. 10).

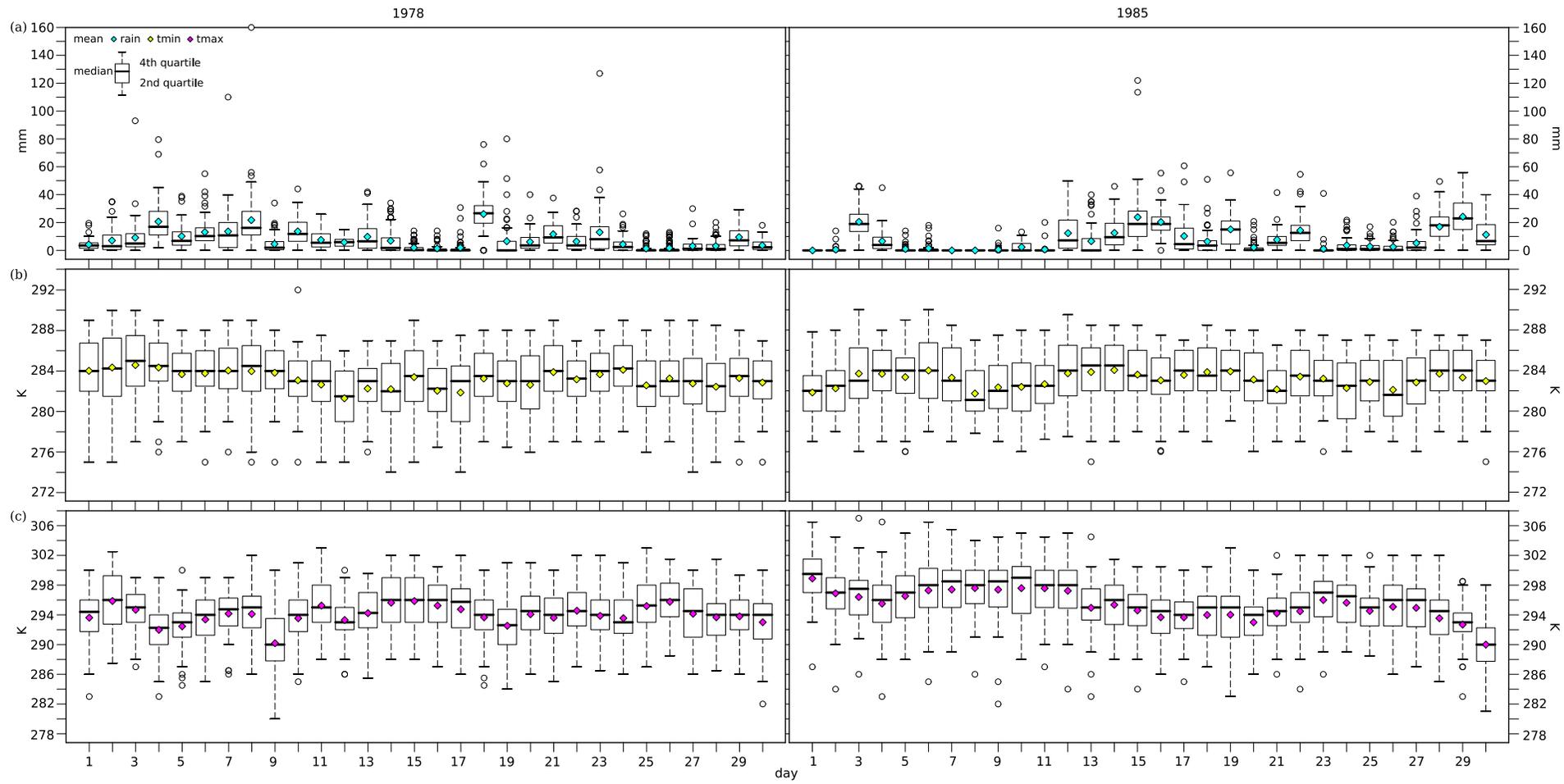


Fig. 24. Whisker plots of daily climatological variables for June 1978 and 1985 in the southwestern area of the Basin: (a) rainfall, (b) minimum temperature and (c) maximum temperature

1 In the case of temperature, the relationship between elevation and both tem-
2 peratures increases when only the southwestern area is considered for the four
3 days considered in this work, as well as for the remainder days of both months
4 (Fig. 4) and local (Fig.24). This is to be expected as the southwestern subarea
5 is predominantly mountainous (Fig. 1). The mean, maximum and minimum
6 values for both temperature fields do not change as drastically as those for
7 rainfall which also should be expected as temperature is a continuous field
8 through the study area, instead of a localized one, as rainfall is. Accordingly,
9 the spatial distribution obtained with KED and KED_l is similar (as well as
10 the errors' statistics) except in those areas which have a low density of cli-
11 matological stations. As explained in section 2, Kriging uses the mean of a
12 selected neighborhood thus in order to represent a heterogeneous field such
13 a rainfall, Kriging with External Drift in a local neighborhood is the method
14 that provides the best results as it accounts for the local relationship between
15 rainfall and elevation, as well as the way in which the mean changes with
16 a local neighborhood. Based on the errors' statistics, along with the spatial
17 distribution obtained by each interpolation method, for this study KED_l is se-
18 lected to interpolate daily rainfall, while KED is selected to interpolate both
19 minimum and maximum temperature.

20 **5 Conclusions**

21 The effect of considering elevation as a secondary variable to interpolate daily
22 rainfall, minimum and maximum temperature was analyzed in this study us-
23 ing approximately 200 climatological stations for an area of 16,800 km². In this
24 study, the use of elevation as a secondary variable improved the spatial varia-
25 tion of all climatological fields even when they exhibited low correlation with
26 elevation. According to the analyses presented, the use of Kriging with Exter-
27 nal Drift on a local neighborhood (KED_l) is recommended to undertake the
28 spatial interpolation of rainfall, (a highly heterogeneous variable) while Krig-
29 ing with External Drift (KED) is recommended to undertake the interpolation
30 of a more continuous field such as minimum and maximum temperature. In the
31 case of a more dense climatological network, KED_l would probably improve
32 the spatial pattern of minimum and maximum temperature. The selection of

1 a local neighborhood is an interactive process, in order to provide continu-
2 ous fields which capture the local behaviour of the variable of interest. To
3 summarize:

- 4 (1) Anisotropy does not need to be considered when using a local neighbor-
5 hood, as observed in the interpolated maps.
- 6 (2) The RMSE values should not be used alone in order to decide whether an
7 interpolation method yields the best interpolation. Other issues need to
8 be considered, such as the density and location of measurement points.
- 9 (3) The ability to undertake spatial interpolation is an add-on capacity of
10 many Geographic Information Systems; however, the capacity of the end-
11 user to undertake this type of interpolations is presented as a black-
12 box exercise. Knowledge about the principles behind each interpolation
13 method is required from the end-users of interpolation software in order
14 to produce reliable spatial patterns of the variable under study.

15 **Acknowledgements**

Financial support by the Mexican Council for Science and Technology (CONA-
CyT) and the Natural Sciences and Engineering Research Council of Canada
(NSERC) are acknowledged. We would also like to thank Edzer Pebesma for
reading a draft version of this paper and providing valuable feedback.

16 **References**

- 17 Allard, D. (1998). Geostatistical classification and class Kriging. *J. of Geo-*
18 *graphic Information and Decision Analysis*, 2:87–101.
- 19 Atkinson, P. M. and Lloyd, C. D. (1998). Mapping precipitation in Switzerland
20 with Ordinary and Indicator Kriging. *J. of Geographic Information and*
21 *Decision Analysis*, 2:65–76.
- 22 Bivand, R. S. (2000). Using the R statistical data analysis language on GRASS
23 5.0 GIS database files. *Computers & Geosciences*, 26:1043–1052.
- 24 Carrera-Hernández, J. J. and Gaskin, S. J. (2005). The valley of mexico hydro-

- 1 geological database (VMHDB): Implementation and basic queries. *Hydro-*
2 *geology journal*. In review.
- 3 Demyanov, V., Kanevski, M., Chernov, S., Savelieva, E., and Timonin, V.
4 (1998). Neural network residual kriging application for climatic data. *J. of*
5 *Geographic Information and Decision Analysis*, 2:34–58.
- 6 Deutsch, C. V. and Journel, A. G. (1998). *GSLIB: Geostatistical software*
7 *library and user's guide*. Applied Geostatistics. Oxford University Press,
8 New York, 2nd edition.
- 9 Dingman, L. S., Seely-Reynolds, D. M., and Reynolds, R. C. (1988). Ap-
10 plication of kriging to estimate mean annual precipitation in a region of
11 orographic influence. *Wat. Res. Bulletin*, 24:329–339.
- 12 Dubois, G. (1998). Spatial interpolation comparison 97: Foreword and intro-
13 duction. *J. of Geographic Information and Decision Analysis*, 2:1–10.
- 14 Goovaerts, P. (1997). *Geostatistics for Natural Resources evaluation*. Oxford
15 University Press, New York.
- 16 Goovaerts, P. (2000). Geostatistical approaches for incorporating elevation
17 into the spatial interpolation of rainfall. *J. of Hydrology*, 228:113–129.
- 18 Hofierka, J., Parajka, J., Mitasova, H., and Mitas, L. (2002). Multivariate
19 interpolation of precipitation using regularized spline wiht tension. *Trans-*
20 *actions in GIS*, 6(2):135–150.
- 21 Hudson, G. and Wackernagel, H. (1994). Mapping temperature using kriging
22 with external drift: Theory and and example from Scotland. *Int. Journal*
23 *of Climatology*, 14:77–91.
- 24 Hutchinson, M. F. and Galland, J. C. (1999). Representation of terrain. In
25 Longley, P. A., Goodchild, M. F., Maguire, D. J., and Rhind, D. W., editors,
26 *Geographical Information Systems*, pages 105–124. John Wiley and Sons.
- 27 Isaaks, E. H. and Srivastava, R. M. (1989). *An Introduction to Applied Geo-*
28 *statistics*. Oxford University Press.
- 29 Jarvis, C. H. and Stuart, N. (2001). A comparison among strategies for in-
30 terpolating maximum and minimum daily air temperatures. Part II: The
31 interaction between number of guiding variables and the type of interpola-
32 tion method. *J. of Applied Meteorology*, 40:1075–1084.
- 33 Kyriakidis, P. C., Kim, J., and Miller, N. L. (2001). Geostatistical mapping
34 of precipitation from rain gauge data using atmospheric and terrain char-
35 acteristics. *J. of Applied Meteorology*, 40:1855–1877.
- 36 Lloyd, C. D. (2005). Assessing the effect of integrating elevation data into

- 1 the estimation of monthly precipitation in Great Britain. *J. of Hydrology*,
2 308:128–150.
- 3 Martinez-Cob, A. (1996). Multivariate geostatistical analysis of evapotranspi-
4 ration and precipitation in mountainous terrain. *J. of Hydrology*, 174:19–35.
- 5 Mitasova, H. and Mitas, L. (1993). Interpolation by regularized spline with
6 tension: I. theory and implementation. *Math. Geol.*, 25:641–655.
- 7 Pebesma, E. J. (2004). Multivariable geostatistics in S: the gstat package.
8 *Computers & Geosciences*, 30:683–691.
- 9 Pebesma, E. J. and Wesseling, C. G. (1998). GSTAT: a program for geosta-
10 tistical modelling, prediction and simulation. *Computers & Geosciences*,
11 24(1):17–31.
- 12 Phillips, D. L., Dolph, J., and Marks, D. (1992). A comparison of geostatistical
13 procedures for spatial analysis of precipitation in mountainous terrain.
14 *Agricultural and Forest Meteorology*, 58:119–141.
- 15 R Development Core Team (2005). *R: A language and environment for statisti-*
16 *cal computing*. R Foundation for Statistical Computing, Vienna, Austria.
17 ISBN 3-900051-07-0.
- 18 Saveliev, A. A., Mucharamova, S. S., and Piliugin, G. A. (1998). Modeling
19 of the daily rainfall values using surface under tension and Kriging. *J. of*
20 *Geographic Information and Decision Analysis*, 2:62–71.
- 21 Tabios, G. Q. and Salas, J. D. (1985). A comparative analysis of techniques for
22 spatial interpolation of precipitation. *Water Resources Bulletin*, 21(3):365–
23 380.
- 24 Thielen, A. H. (1998). Estimating daily regional rainfall fields by multiquadric
25 functions: Accuracy of interpolation and decision making. *J. of Geographic*
26 *Information and Decision Analysis*, 2:86–99.
- 27 Tomczak, M. (1998). Spatial interpolation and its uncertainty using automated
28 anisotropic inverse distance weighting (IDW) - cross validation/jackknife ap-
29 proach. *J. of Geographic Information and Decision Analysis*, 2:18–30.
- 30 Wackernagel, H. (2003). *Multivariate Geostatistics: An introduction with ap-*
31 *plications*. Springer.