**DEVELOPMENT OF A BROADBAND MICROWAVE
INTERFEROMETER FOR DIAGNOSTIC MEASUREMENTS
OF DETONATIONS**

**by
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ABSTRACT

This thesis describes a feasibility study on the use of a novel coaxial waveguide configuration to perform detonation velocity measurements using the microwave Doppler interferometry technique. With this method, continuous velocity histories of the detonation wave propagating in an explosive gaseous mixtures can be obtained. Existing microwave diagnostic techniques are mostly based on the use of the detonation tube as a hollow waveguide. Consequently, the phase and group velocity of electromagnetic waves propagating in the waveguide have a strong frequency dependency, rendering swept frequency and other broadband techniques impractical. The principle advantage of using the coaxial configuration is the possibility of operating in the TEM (waves in the transverse electro-magnetic) mode, where the phase and group velocities are frequency independent, permitting the use of broadband remote sensing radar techniques.

In the present study, the microwave interferogram is digitized and analyzed using computer software developed to facilitate the extraction of velocity information from the microwave Doppler interference signals by applying digital signal processing techniques.

The waveguide or detonation tube of the present microwave interferometer consisted of a copper tube 38.4 mm in diameter, 3.5 m long, with a thin wire stretched along the center axis acting as a center conductor for the coaxial configuration. The system was tested at microwave frequencies of 6.70GHz and 9.21GHz by performing a number of detonation experiments in explosive gaseous mixtures of $\text{C}_2\text{H}_2+2.5\text{O}_2$ and $\text{C}_3\text{H}_8+5\text{O}_2$, at low initial pressures (6torr to 80torr). Average velocity measurements obtained by the microwave method generally agreed with independent photodetector measurements to within 2%.

In this study, the present technique has been used to explore unstable detonations near the limit. This demonstrates that the improved microwave Doppler interferometer is particularly well suited for unstable detonations where the large scale velocity fluctuations must be monitored continuously. It may be concluded that the present coaxial configuration microwave Doppler interferometry technique shows promise as a useful diagnostic tool for studying unstable detonations.

RESUME

Ce travail de thèse décrit une configuration de guide d'onde coaxial mise au point pour la mesure de la célérité de propagation des détonations par effet Doppler en ondes centimétriques. Les mesures antérieures de célérité utilisant une onde électromagnétique se sont faites dans un tube qui servait de guide d'onde pour propager les modes TE ou TM. Mais pour ces modes, la fréquence doit être limitée à une gamme très étroite pour éviter la propagation simultanée de plusieurs modes. De plus, les vitesses de phase et de groupe de ces ondes varient avec la fréquence, excluant ainsi l'usage de diagnostics hyperfréquence tels que le radar adaptif et le balayage de fréquence qui nécessitent une large gamme de fréquence. L'avantage principal du système coaxial à deux conducteurs proposé dans ce rapport est la possibilité d'utiliser le mode TEM pour la propagation des ondes. Dans ce mode, la vitesse de propagation ne varie pas avec la fréquence, donc les diagnostics pour lesquels une large gamme de fréquence est requise peuvent être employés.

Dans cette étude, un programme sur ordinateur exploitant les techniques de traitement digital du signal a été conçu pour extraire du signal Doppler l'information permettant d'obtenir la célérité de l'onde de détonation.

Le tube à détonation coaxial est constitué d'un tube en cuivre de 3.5m de long et de diamètre intérieur égal à 38.4mm dans lequel un fil de cuivre très fin est tendu suivant l'axe du tube. Le fil joue le rôle de conducteur central dans la configuration coaxiale. Le système fut mis à l'épreuve en utilisant des fréquences de 6.70GHz et 9.21GHz sur des détonations dans des mélanges gazeux de $C_2H_2+2.5O_2$ et de $C_3H_8+5O_2$ à basse pression (0.800 à 10.66 kPa). Les mesures de célérité moyenne obtenue par la méthode Doppler sont en accord avec des mesures indépendantes de célérité obtenue par la chronométrie par sonde optique avec une différence de moins de 2% entre les deux.

Quelques expériences préliminaires sur des détonations instables démontrent que la technique Doppler est particulièrement bien adaptée à la mesure de l'évolution de la célérité des détonations dont les variations de vitesse sont importantes sur une distance de propagation très longue. Nous pouvons conclure que le dispositif Doppler constitue un diagnostic utile pour la mesure de l'évolution de la célérité des détonations dans les mélanges gazeux.

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As the McGill group has traveled abroad to attend various conferences, it has come to be called the McGill mafia. This is perhaps because, more than any other research group, the McGill one is perceived as a "big family". It is true. We are a big family, and I have enjoyed all the time I have spent working as a part of it. This said, I would like to dedicate this thesis to all my relatives.

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CHAPTER I. INTRODUCTION

The subject of this thesis is the measurement of the velocity of detonations using electromagnetic waves. The Doppler interferometry technique as used in continuous wave radar systems can provide a useful diagnostics tool for measuring the speed of propagation of detonations in gaseous media. These detonations are an explosive burning phenomenon that can occur in a combustible medium such as a flammable gas mixture, and they typically propagate at very rapid speeds of about 2000 m/s in fuel-air mixtures. Interest in the study of detonations has been fueled by the need to understand situations in which detonations may occur, such as accidental explosions of flammable substances used in chemical industries or burning of gasoline in a car. Since a large part of our energy is provided by the burning of fossil fuels, the situations in which they may occur are numerous.

Detonation in a gaseous medium can be obtained by energetically igniting combustible gas mixtures. For practical and safety reasons, detonation experiments are most often conducted in confined environments, and the most common type of confinement consists of a long metallic tube or channel. These tubes usually have a rectangular or circular cross section that can vary in diameter from approximately one centimeter to more than a meter. Various diagnostic probes are mounted on the walls of the tube in order to obtain velocity and pressure measurements of the detonation as it travels down the tube.

The most widespread methods of measuring detonation velocity depend on the determination of the time interval between the passage of the detonation front over two or more detectors which are sensitive to the change of state across the front. Such methods can only determine the average velocity between the positions of the detectors, which are usually widely separated so as to reduce the errors in the measurement. In contrast to these methods, the Doppler interferometry technique can provide continuous velocity measurements of the detonation wave as it propagates along a tube.

1.1 Historical

Doppler interferometry consists of reflecting waves from a moving object (sound waves in the case of sonar, and radio waves in the case of radar) and inferring the speed of the object from the Doppler frequency shift of the reflected wave. In 1957, this technique was used by Hey et al.¹ to measure the velocity of highly ionized shock waves in a tube. In his experiments, Hey obtained a continuous velocity history of a strong shock wave travelling inside a metallic tube filled with argon. After being compressed and heated to a very high temperature by the shock, the gas is ionized forming a sharp plasma boundary. As the shock travels through the gas, the plasma boundary moves with it. An electromagnetic wave in the microwave range can readily reflect off the plasma boundary and generate a reflected wave with a Doppler frequency shift. The Doppler shift is measured and used to calculate the velocity of the shock. This procedure forms the basis of the Doppler interferometry technique.

During the 50's, early work in detonation research was growing and many detonation experiments were being carried out in confinements identical to the shock tube. The similarity between the shock wave and detonation wave experiments prompted Edwards et al.² to apply Doppler interferometry to velocity measurements of gaseous detonations for the first time in 1962. He successfully used microwaves to measure the velocity of detonations in a gaseous hydrogen-oxygen mixture at low initial pressure. In a short article on this work², Edwards immediately recognized the usefulness of the method for studying near-limit detonations, i.e. detonations that occur in situations where the initial conditions are close to the limit beyond which combustion can no longer occur in the detonation mode. In fact, he later performed some experiments on near-limit detonations and published his results in 1970³. He also exposed some pertinent problems such as the effects of non uniformity in the detonation front on the microwave signal, and the effect of the electron density in the detonation front on the reflected signal amplitude. The former refers specifically to the possible modulation of the microwave signal due to periodic instabilities in the detonation front known as transverse waves. The latter refers to the weakening of the reflected signal when the ionization in the detonation front is low.

Early work on velocity measurements of detonations using microwaves was actually done in the 50's by a number of researchers ^{4,5,10}. These studies involved a variety of detonation phenomena including unconfined detonations and detonations in solid explosives such as TNT rather than in gaseous mixtures. For the case of solid explosives, the confinements were usually destroyed due to the high pressures involved, and the ionized gases behind the wave were consequently highly turbulent, rendering the microwave technique difficult to use. However, the fundamental ideas behind the method were similar, i.e. by treating the ionized front of the detonation as a moving reflector of microwave energy, Doppler interferometry could be used to determine its velocity. One important point brought up in a paper by Cook⁴ was the problem of the microwaves having multiple waveguide modes propagating simultaneously, thus complicating the reflected signal. Researchers subsequently took great care in avoiding multiple mode propagation when using this technique.

As previously mentioned, the first use of the microwave technique in a shock tube with gaseous detonations was by Edwards et al. ² in 1962. In that work, 3cm wavelength microwaves were used inside a stainless steel tube, and the interference pattern obtained by combining the transmitted and reflected microwaves was recorded. The detonation velocities obtained from these interference patterns or interferograms agreed very well with the theoretical Chapman-Jouguet velocities as well as with independent measurements using ionization probes. However, for H₂-O₂ mixtures at low initial gas pressure, Edwards encountered some unexpected results: an irregular amplitude fluctuation in the reflected signal that he described as a gradual appearance of a low frequency component in the interferogram. He also found that, contrary to theory, the reflected microwave power was greater for detonations at low initial gas pressure than for high initial gas pressure.

Brossard et al. ⁶ later used a similar set-up to observe detonations in a nitrogen diluted propane- oxygen mixture. His gaseous mixtures consisted of C₃H₈+xO₂+yN₂ at atmospheric pressure where, by varying the proportion of the diluent N₂ and the oxidant O₂, he was able to obtain near limit detonations. The transverse wave structure for detonations in this mixture is known to be quite irregular and Brossard observed amplitude fluctuations in the microwave interferograms similar to Edwards' results. He also noted the

possibility of the microwaves propagating in more than one waveguide mode simultaneously, interfering with each other and distorting the signal.

In the early 70's, Edwards et al. ⁷ undertook a study on a type of near-limit detonation known as a galloping detonation and because of a need to monitor the velocity continuously, he opted once again to use the microwave interferometry technique. This time the experiments were performed inside a rectangular X-band microwave waveguide, using a frequency of 9GHz to ensure that the microwaves propagated only in the TE₁₀ mode. In this work, only qualitative observations of the velocity fluctuations of the detonations were made. The interferograms were recorded on an oscilloscope screen and photographed. One might speculate that the velocity information had to be extracted from the interferogram by painstakingly measuring its period on the photograph; a tedious process that might account for the lack of detailed quantitative velocity information presented in the results.

Around the same time, Saito et al. ^{8,9} performed very similar microwave interferometry experiments with C₂H₂+2.5O₂ inside an X-band waveguide. In addition to recording the power of reflected microwaves from the front of the incoming detonation, Saito also measured the power of the reflected microwaves from the back of the detonation and recorded the phase of the reflected microwaves. An interesting result in his work was that for low initial gas pressures, the detonation velocity can also be measured from the microwave reflection off the back of the detonation wave. This was possible because for the particular pressure range that Saito worked in, the ionization behind the detonation front falls off at a constant rate and possibly constitutes a more uniform plasma boundary than the front of the detonation. It should also be noted that for acetylene-oxygen mixtures, the ionization zone behind the detonation front is particularly well defined. This behavior generally does not occur in other combustible gaseous mixtures, so Saito's variation on the microwave Doppler technique is only of limited use.

1.2 Difficulties

The microwave Doppler technique has been repeatedly tried in the past by a number of researchers who recognized its benefits as a non-intrusive (it does not physically obstruct or perturb the detonation), continuous velocity

measurement technique. In these previous studies, the microwave measurements on detonations have been performed while adhering as much as possible to the guidelines of ideal microwave propagation in waveguides, i.e. smooth, conducting tube walls with no discontinuities, and tube dimensions small enough to ensure single mode propagation at a certain fixed frequency. In general, the focus of the research has naturally been on the detonation phenomenon, and relatively little effort has been put into developing the microwave diagnostic technique itself.

One of the challenges in the microwave technique is the difficulty in interpreting the microwave interferogram. If the reflecting surface were a perfectly plane and smooth conducting mirror, the interferogram would be a sine wave. However, it appears that inhomogeneities and fluctuations of the electron density in the detonation front complicate the reflected microwave signal and cause it to depart from an ideal sinusoidal shape rendering its analysis and interpretation more difficult.

Several other problems have not been systematically investigated previously. For example, the effects of microwave propagation in two or more waveguide modes inside the detonation tube has only been reported in some early work involving detonations in solid explosives ^{4, 10} and in Brossard's ⁶ paper. In studies of gaseous detonations in tubes, researchers have generally restricted the tube dimension and the microwave frequency to ensure single mode propagation. It would be useful in certain cases to change the frequency of the microwaves without having to restrict the diameter of the tube. One important advantage of using a higher microwave frequency is that the spatial resolution of the velocity measurements is improved, but the use of high frequencies automatically raises the problem of multi-mode electromagnetic wave propagation.

Another electromagnetic waveguide propagation problem is the susceptibility of the microwave signal to discontinuities in the detonation tube. Any crack or protrusion on the wall of the tube can constitute a source of reflection for the microwaves, which can in turn result in a large attenuation of the signal amplitude due to impedance mismatch and in signal distortions. It is often the case in detonation experiments that the tube cross section is not constant, or that obstacles are placed inside the tube, so it is of interest to investigate how such non-uniformities in the tube affect the microwave propagation.

It is also important to investigate the dependence of the amplitude of the reflected signal on the level of ionization in the detonation front. In order to ensure a strong reflected signal, the microwave frequency must be restricted to a range below or close to the plasma frequency in the ionization zone (which depends on the level of ionization) in the detonation front. For example, detonations with low ionization density (such as hydrogen oxygen detonations) would necessitate a low microwave frequency since the plasma frequency decreases as the level of ionization decreases. The challenge of this problem is that the electron density inside the detonation front is not uniform and difficult to estimate. This inhomogeneity could account for anomalous results such as those observed by Edwards et al. ², who found that the power of the reflected microwaves increases rather than decreases at low initial pressures of the gaseous mixture. Edwards suggested that small localized regions of very high ionization can be found at the triple points. These points are the intersection points between the main shock and the transverse shocks in the detonation and form part of its cellular structure. The presence of these highly ionized triple point regions would then be largely responsible for the unexpectedly large reflected microwave signal.

A final problem of note is the spatial resolution of the microwave technique. The velocity measurements are not truly continuous since the resolution is limited to one half of the wavelength of the microwaves inside the tube. The resolution is typically of the order of a couple centimeters, which is adequate for studying detonation velocity fluctuations that occur over several meters of propagation, such as in galloping detonations. Smaller fluctuations may be difficult to resolve using the microwave method.

In the following section, the various advantages of microwave diagnostics which provide motivation for the present study will be discussed.

1.3 Motivation

In the course of the evolution of detonation diagnostic measurements using the microwave interferometry technique, the problem of multiple mode propagation was avoided by using a single, fixed microwave frequency in the appropriate range. This constituted somewhat of a limitation to the technique, since many microwave diagnostic measurements such as swept

frequency plasma measurements and adaptive radar ^{11, 12} require broadband operation. The dominant mode in two conductor wave guiding systems is the transverse electromagnetic (TEM) mode, which in the ideal case, offers an infinite bandwidth with constant phase and group velocity. By using a two conductor coaxial configuration in a detonation tube, the opportunity for broadband operation presents itself. The feasibility of such an arrangement opens the door to many newly developed techniques such as adaptive radar, which has become a powerful diagnostic tool following the advent of microprocessor controlled solid state microwave sources. By transmitting several microwave frequencies simultaneously and observing the frequency spectrum of the reflected signal, the adaptive radar is capable of providing information on the location, size, shape, and velocity of a target object. Broadband operation expands the range of microwave diagnostic measurements possible in a detonation tube, and the coaxial configuration proposed in this thesis work can provide this feature.

The continuous velocity measurement feature of the microwave interferometry technique also offers important advantages in certain types of detonation studies. In 1962, Edwards ² was the first to recognize the potential of this method for studying near limit detonations where the velocity fluctuates greatly and requires continuous velocity measurements. Thirty years later, near limit detonations are still not understood, and the microwave technique can provide a more complete record of the velocity fluctuations of these unstable phenomena than other methods.

A practical problem in detonation research has always been the definition of detonation limit criteria as well as the determining of other dynamic parameters such as the direct initiation energy and the critical tube diameter ^{13, 14}. Up until now, experimental work has been oriented towards trying to determine the conditions for which a detonation will propagate or not ¹⁵. As a result, there exists a vast amount of empirical data on detonation limits, and practically no predictive theoretical models. Because the detonation propagation mechanisms themselves remain to be understood, the existing models have been successful only in a few very restricted cases. It is therefore important to study near limit detonations more closely in order to attempt to correlate their behavior to other dynamic parameters of the phenomenon.

Being an inherently unstable phenomenon, detonations at the limit do not display a sharp cut off (i.e. a simple go/no go behavior). In fact, it is difficult

to determine the conditions for which a combustible mixture will burn in the detonation mode or in another mode such as the quasi-detonation or the deflagration mode. Detonations have long been known to be unstable, and an understanding of instabilities in the reaction zone behind the detonation front is crucial to understanding the propagation mechanisms of detonation waves ^{16, 17}. Near limit detonations offer a means of studying these instabilities because they display chaotic behavior on a large scale which is easier to observe. In the near limit region, the combustion phenomenon is extremely sensitive to perturbations and its parameters such as velocity, pressure, and temperature show large unpredictable fluctuations. Measurement of these fluctuations can perhaps provide insight into the chaotic nature of detonations.

The two main characteristics of near limit detonations are the erratic nature of their velocity fluctuations, and the long propagation distance over which they occur. This large scale chaotic behavior is difficult to monitor using traditional methods, but is well suited to the microwave Doppler technique. The microwave technique can provide a detailed record of the complicated velocity fluctuations of a near limit detonation over a large distance. The acquisition of these long detonation velocity histories opens the door to the many newly developed methods of characterizing chaotic data such as measuring the fractal dimension or the Lyapunov exponent ^{18, 19}. These analysis methods, which generally require large data sets to be applied, are capable of extracting hidden patterns in seemingly random behavior. They could perhaps be used to quantify the chaotic behavior of the velocity fluctuations of detonations.

Although continuous velocity measurement has always been a principle advantage of the microwave technique, there is room for much improvement in the extraction and analysis of the velocity data from the interferograms. Past methods of analyzing these signals have been limited to measuring the period of interferograms from photographs of oscilloscope traces and subsequently calculating the corresponding velocities. Because of the tediousness of this procedure, the continuous velocity information contained in the microwave signals have never been completely exploited. The advent of computers and of digital signal processing techniques have greatly simplified the task of reducing and analyzing large quantities of data, and by applying these high power tools to the interpretation of microwave

interferograms, complete plots of the velocity histories of detonations can readily be obtained. A significant part of the effort in developing the microwave technique used in this work was devoted to the development of software to perform digital analysis of the signal using a computer system.

1.4 Thesis work

This thesis is a feasibility study of the use of a novel coaxial configuration for the measurement of the detonation velocity using the microwave Doppler technique. The problem of setting up a microwave interferometer is approached from a microwave propagation point of view. By stretching a thin wire along the axis of a metallic tube, a two conductor coaxial waveguide is obtained. Since the wire is thin, it does not disturb the flow of gas and the detonation can propagate unhindered inside the tube. This variation on the arrangements used in previous studies results in infinite bandwidth capability and frequency independent propagation velocity.

The investigation in this thesis also treats the problem of analyzing the microwave interferogram using modern digital techniques. Since the extraction of continuous velocity measurements of the detonation from the interferogram involves a frequency analysis of the signal, computers provide a useful tool for processing the signal automatically. A computer program making use of several standard digital signal processing techniques was designed for this task.

1.5 Thesis objectives

The objectives of this thesis work are to:

- explore the feasibility of a microwave Doppler interferometry system based on a coaxial configuration to propagate the microwave energy inside the detonation tube,
- develop the software for the data analysis,
- test the broadband operation of the coaxial test section,
- perform some preliminary velocity measurements of near-limit detonations using this coaxial system.

CHAPTER II. THEORY

In developing the theory behind the measurement of detonation velocities using Doppler interferometry, three areas are important to consider: Doppler interferometry, the propagation of electromagnetic waves in a coaxial configuration, and the reflection of electromagnetic waves by a detonation front. The first topic, Doppler interferometry, is a technique whereby waves reflected off a moving object can be used to infer the speed of the object. It forms the heart of the method described in this work. The second topic discusses the properties of electromagnetic waves travelling inside a coaxial wave guide, since the detonation measurements are conducted in an apparatus made to resemble a typical coaxial transmission line. The third topic treats the difficult problem of electromagnetic wave reflection from a detonation front. Even though no adequate theoretical treatment of this problem exists, some simple approximations have been made.

2.1 The Doppler interferometry technique

If a continuous wave (unmodulated) train of electromagnetic energy is transmitted towards an object that is moving relative to the transmitter, the waves that reflect off the object will be shifted in frequency due to the Doppler effect. There is a direct relationship between the magnitude of the frequency shift and the velocity of the moving object, hence the reflected wave or echo can provide velocity information on the object. A diagram of a typical Doppler interferometry system is shown in fig.1 (shown on the next page).

Continuous wave electromagnetic energy is transmitted at a frequency F_0 towards the moving object. Upon reflection, the Doppler effect causes it to shift frequency by $+f_D$ for an incoming object or by $-f_D$ for a receding object. The reflected signal or echo is sensed by a probe antenna and is heterodyned in the detector with a portion of the transmitted signal to provide the isolated Doppler beat frequency f_D (the sign of f_D is lost in the process). In the final stage, the signal can be analyzed or displayed with the appropriate instrumentation.

Fig. 1. Schematic of a Doppler interferometry system

The relation between the Doppler shift can be simply derived as follows. If R is the distance between the target object and the transmitter/detector, then the total path travelled by the electromagnetic waves to and from the object is $2R$. The number of wavelengths λ contained in the path is $2R/\lambda$, and the angular excursion ϕ made by the wave is $4\pi R/\lambda$ since each wavelength corresponds to an angular excursion of 2π . Since the object is in motion, the separation distance R , and therefore phase ϕ , are constantly changing, and the frequency corresponding to the phase change is the Doppler frequency shift. The Doppler angular frequency is given by ²⁰

$$\begin{aligned}\omega_d &= \frac{d\phi}{dt} \\ &= \frac{4\pi}{\lambda} \frac{dR}{dt} \quad \text{since } \phi = \frac{4\pi R}{\lambda}, \\ &= \frac{4\pi v_r}{\lambda}\end{aligned}$$

where v_r is the relative velocity of the object and λ is the wavelength of the electromagnetic wave.

Since

$$f_d = \frac{\omega_d}{2\pi},$$

the Doppler frequency can be written

$$\begin{aligned} f_d &= \frac{2V_r}{\lambda} \\ &= \frac{2V_r f_o}{c} \end{aligned}$$

where f_o is the transmitted frequency of the wave and c is the velocity of propagation.

Thus if the wavelength of the electromagnetic waves (which must be calculated using waveguide mode theory) is known, the instantaneous velocity can be simply calculated from the relation:

$$V_r = \frac{f_d \lambda}{2} \quad (\text{equ. 2.1.1})$$

where f_d is the Doppler frequency and λ is the wavelength of the electromagnetic wave.

This analysis is in a simplified form, since relativistic corrections are not included in the equation for the velocity V_r . This is justified because the detonation speed which is typically on the order of 2×10^3 m/s, is still more than five orders of magnitude smaller than the speed of an electromagnetic wave which is equal to the speed of light ($c = 3 \times 10^8$ m/s). Since the ratio of the detonation speed to the electromagnetic wave speed is small (6.67×10^{-6}), relativistic effects are negligible.

2.2 Wave propagation in a coaxial shock tube

The experimental apparatus used in this study consists essentially of a long metallic tube. In order to achieve a coaxial configuration, a center conductor was added by stretching a thin wire inside the tube along the axis. The shock tube is thus made into an air filled, two conductor coaxial transmission line wave guide (fig. 2 shown on the next page).

The dominant mode of propagation of electromagnetic energy inside an ideal two conductor system is the TEM (transverse electric and magnetic) mode. In this mode, the electric and magnetic field components are normal to the

Fig. 2. Coaxial configuration

direction of propagation, and the electromagnetic wave behaves much like a plane wave in free space. Unlike a hollow wave guide, a two conductor transmission line has no cut off frequencies for the TEM mode, so its wave parameters such as impedance and phase velocity do not contain the expression

$$\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

As a result, the wave parameters for TEM wave inside an ideal lossless coaxial line, are inherently constant and frequency independent. For example, the group and phase velocities are both equal to the speed of light, and the characteristic impedance for an air filled coaxial guide (discounting imperfect conductors and dielectric losses) is given by ²¹ :

$$Z_0 = 60 \ln\left(\frac{b}{a}\right)$$

where a =the radius of the inner conductor, and b = the radius of the outer conductor.

The wavelength in the guide is simply:

$$\lambda = \frac{c}{f}$$

where c =the speed of light (3×10^8 m/s) and f = the frequency of the electromagnetic wave in Hz.

Since the non-ideal conditions in the apparatus used in this work are small, it is highly probable that almost all of the electromagnetic wave energy will propagate in the TEM mode, and only a very small fraction would be imparted to spurious TM and TE modes. For example, the dielectric constant ϵ_r of the gas in the tube which is assumed to be equal to 1, is very close since $\epsilon_r = 1.0013$ for C_2H_2 and $\epsilon_r = 1.0005$ for O_2 . The tube walls and the center conductor are made of copper, which is close to ideal since its resistivity is very low (resistivity=1.72 microhm-centimeter). Thus discounting dimensional perturbations (see sections 2.2.3 and 3.1.1.5), the TEM mode is likely to propagate unhindered.

2.2.1 Field structure

The electric and magnetic field structures for the TEM mode are shown in fig.3.

Fig. 3. Electric and magnetic field lines in a coaxial configuration. 22

The magnitude of the radial electric field is given by ²²:

$$E_r = \frac{V}{r \ln\left(\frac{b}{a}\right)}$$

where V=the potential difference between the conductors, r=the radial distance from the center axis, a=the radius of the inner conductor, and b= the radius of the outer conductor.

From the above equation, we note that the intensity of the field decays as the inverse of the radial distance r. The rate of decay of the field also depends on the dimensions of the inner and outer conductors (due to the $\ln(b/a)$ term). For the coaxial apparatus used in this work, the field decays quite rapidly. At a radial distance of only 2mm. from the surface of the center conductor where the field is at a maximum, the intensity has already decreased to 10% of its highest value (appendix A).

Since most of the electromagnetic energy is concentrated in a small region surrounding the center conductor, it follows that this region is the most sensitive to the presence of plasma. A possible consequence of this is that inhomogeneities in the ionization level across the detonation front may cause fluctuations of the reflected microwave signal. Small regions of highly ionized plasma in the front can move into the sensitive region near the center wire, causing sudden increases in the amplitude of the reflected signal. The degree to which the reflected signal fluctuates due to this effect depends on the level of ionization of the small regions of ionized plasma and their spatial distribution across the detonation front. In general, the length scale of the structure of the ionization inhomogeneities is small for detonations at high initial gas pressure, so the overall ionization level is more evenly spread out across the front and the detonation closely resembles a smooth reflecting surface. In this case, the reflected microwave signal is quite smooth. When the initial gas pressure is low, the length scale of the ionization inhomogeneities in the detonation becomes larger, and the unevenness of the plasma density becomes more apparent. The reflected signal becomes more irregular. Since relatively little is known about the exact distribution of the ionization across the detonation front, the consequent reflected signal fluctuations can not be predicted. If they are small enough, however, they can

be filtered out using the appropriate signal processing techniques and the signal can still be properly interpreted.

A more beneficial consequence of the field being localized around the center conductor is that the coaxial system is not sensitive to small physical disturbances such as slight displacements of the wire from its ideal central position and small discontinuities on the inner wall of the tube. In other words, the coaxial assembly is tolerant to small amounts of sag and misalignment of the center wire as well as to small cracks or protrusions in areas such as joints and instrumentation ports on the tube. If small enough, these non ideal conditions do not affect the operation of the Doppler interferometry system.

2.2.2 Higher order modes

In addition to the dominant TEM mode, other modes of propagation called higher order modes can also exist in a coaxial waveguide. These modes are generally known as TE (transverse electric) and TM (transverse magnetic) modes and can only propagate at frequencies above their respective cutoff frequencies in the waveguide. These cutoff frequencies depend on the dimensions of the coaxial structure.

The cutoff frequencies of the higher order modes in a coaxial configuration can be obtained by solving the (Helmholtz) generalized wave equation in cylindrical coordinates. By applying the appropriate boundary conditions in the TE and TM modes, transcendental equations are obtained, and the cutoff frequencies and of the modes can be obtained from the roots of these equations.

The TM_{mn} modes, where m and n are indices identifying the order of the mode, have cutoff wavelengths λ_c given by ²²:

$$\lambda_c = \frac{2\pi}{(c-1)\chi_{mn}} (b-a)$$

where $c=b/a$ and χ_{mn} = the n^{th} roots of $J_m(cx)N_m(x) - J_m(x)N_m(cx) = 0$ where J_m and N_m are the m^{th} Bessel and Neumann functions.

The TE_{mn} modes have cutoff wavelengths λ_c given by ²²:

$$\lambda_c = \frac{2\pi}{(c+1)\chi_{m1}'} (a+b)$$

for $n=1$, and by

$$\lambda_c = \frac{2\pi}{(c-1)\chi_{mn}'} (b-a)$$

for $n=2,3,4,\dots$

where $\chi_{mn}' =$ the n^{th} roots of $J_m'(cx')N_m'(x') - J_m'(x')N_m'(cx') = 0$ where J_m' and N_m' are the derivatives of the m^{th} Bessel and Neumann functions.

The cutoff wavelengths and frequencies for the first seven high order modes of the coaxial apparatus used in this work are calculated (appendix B) and shown in the table below.

Mode	λ_c (cm)	f_c (GHz)
TE ₁₁	6.66	4.50
TM ₀₁	4.37	6.86
TE ₂₁	4.01	7.48
TE ₀₂ /TM ₁₁	3.20	9.38
TE ₃₁	2.92	10.29
TM ₂₁	2.39	12.55

Table 1. Cutoff frequencies of the higher order modes of the coaxial tube

2.2.3 Excitation of high order modes

Higher order modes can be excited at any sharp bend or discontinuity in the coaxial line. In the vicinity of a sharp discontinuity, an infinite number of modes is theoretically present, but only the ones that are above their respective cutoff frequencies can propagate. The modes that are below or close to their cutoff frequency are rapidly attenuated and appear only as a localized reactive loss. When a mode is only slightly above its cutoff frequency, it suffers a large attenuation due to resistive losses in the inner and outer conductors ^{22, 23}.

In order to suppress high order mode excitation, the walls of the apparatus used in this work were kept as smooth and as free of discontinuities as possible. One possible source of excitation that cannot be eliminated however is the detonation front itself. Indeed, the highly irregular ionization profile presented by the front can provide lossy current paths and discontinuities capable of exciting any number of higher order modes. The interaction between the detonation front and electromagnetic waves has not been fully explored as yet.

In general, higher order modes have only a small chance of propagating unless the conditions are ideal, i.e. perfectly conducting conductors, and the microwave frequency far above the cutoff frequency of the mode. The propagation of higher order TM and TE modes are more of a secondary effect in two conductor systems as opposed to hollow (one conductor) waveguides where the risk of multi-moding is inherently present. However, if they do propagate, they may create a signal distortion that can complicate the interpretation of the microwave Doppler interferogram.

2.2.4 Mode interaction

Once higher order modes have been excited in a wave guiding system, a number of problems can occur, causing a distortion of the reflected microwave signal. Since each mode has a different propagation constant, they can interact with each other and result in beats, lossy mode coupling, power losses, and other problems. The simplest interaction is mode beating, where modes having different phase velocities combine and form beats. For example, take two waves E_1 and E_2 having phase constants β_1 and β_2 :

$$E_1 = E_{10} e^{j\beta_1 z}$$

$$E_2 = E_{20} e^{j\beta_2 z}$$

where the time variation is suppressed and z is the axis in the direction of propagation.

Summing these waves yields the resulting wave E_r , given by:

$$\begin{aligned}
 E_r &= E_1 + E_2 \\
 &= E_{10} e^{-j\beta_1 z} + E_{20} e^{-j\beta_2 z} \\
 &= e^{-j\beta_m z} (E_{10} e^{-j\Delta\beta z} + E_{20} e^{j\Delta\beta z}) \quad \text{where } \beta_m = \frac{\beta_1 + \beta_2}{2} \\
 &\quad \Delta\beta = \frac{\beta_1 - \beta_2}{2}
 \end{aligned}$$

Thus we have a wave with a phase constant β_m modulated by a standing wave beat frequency with a phase constant $\Delta\beta$. If E_1 and E_2 are equal in amplitude (i.e. if $E_{10}=E_{20}$), E_r simply reduces to:

$$E_r = A e^{-j\beta_m z} \cos(\Delta\beta z) \quad \text{where } A = E_{10} = E_{20}.$$

Hence the presence of a second mode of propagation in the apparatus causes beating of the Doppler interference signal. If additional higher order modes also propagate in the apparatus, the signal is further distorted by the wave interaction.

If resistive wall losses are important, then cross coupling between the power of the interacting modes can occur. This phenomenon can cause additional losses of power inside a wave guiding structure. Mode coupling however is generally small except for degenerate modes (modes having identical phase constants). This is discussed in more detail by Collin ²⁴.

2.3 Reflection of electromagnetic waves from a detonation front

The reflection of the microwave energy occurs at the material property boundary discontinuity caused by the presence of a plasma region in the detonation front. The important plasma parameters are the electron density, the electron density profile at the boundary, the distribution of the electron density across the front, and the collisional frequency. If these parameters are known, then the problem of calculating the reflected microwave signal is simplified. However, the detonations' reaction zone which constitutes the reflective plasma region has been found to be completely turbulent and unpredictable. Because of the time-dependent inhomogeneities of the plasma

region, the calculation of the electromagnetic wave reflection from it is difficult to carry out. Simplifying the problem by considering the reaction zone as a flat, homogeneous plasma slab has yielded approximations that are only valid in certain cases. However, this simplified theoretical treatment has yielded useful qualitative results.

2.3.1 The simplified detonation front model

A one dimensional detonation model called the ZND (Zel'dovich, Von Neuman, and Döring) model ²⁵ was formulated during the early 40's. This model treats the detonation as an equilibrium process and disregards any three dimensional transient effects. The detonation structure is assumed to be a planar shock followed by a planar reaction zone.

For the purpose of looking at the interaction between the electromagnetic waves and the detonation front, it is also convenient to treat the detonation as possessing a one dimensional planar ZND structure. The front can be modeled by a uniformly ionized, flat slab of plasma with a certain thickness and electron density.

The theoretical treatment of electromagnetic wave reflection at a plasma boundary is fairly straightforward for a "cold" or Lorentz plasma ²⁶. In this model, the plasma is treated as a cloud of free electrons embedded in an electrically neutral environment. The determining force for the motion of the free electrons is electromagnetic and interactions with the positive ions in the plasma are neglected. Thermal motions are not taken into account, hence the qualifier "cold"(this may already be an unjustified assumption since detonation plasma are typically as high as 3000 °K). A damping or drag term " ν " called the collisional frequency is introduced into the equation of motion to account for the collisions between electrons.

The problem can be stated as a case of an electromagnetic wave propagating through an abrupt change in the medium. The propagation constants in each medium must be known, so the complex index of refraction for the plasma must be found. First, we define the plasma frequency which is the natural oscillation frequency of an electron inside an electromagnetic field. This important parameter characterizes the behavior of the plasma and is denoted ω_p . When the frequency of the electromagnetic wave incident on the plasma is below the plasma frequency, the plasma behaves roughly like a lossy

conductor. When it is above it, the plasma behaves more like a dielectric. The plasma frequency can be defined in terms of the electron density by ²⁶:

$$\omega_p = \left(\frac{ne^2}{m\epsilon_0} \right)^{\frac{1}{2}}$$

where n = the electron density (electrons/cm³), e = the electronic charge (C), m = the electron mass (g), and ϵ_0 = is the permittivity of free space.

The complex dielectric constant for the plasma is ²⁶:

$$\kappa = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} (1 + j\frac{\nu}{\omega})$$

where ω_p is the plasma frequency, ω is the frequency of the electromagnetic wave, and ν is the collisional frequency of the electrons (all in Hz).

The collisional frequency accounts for the collisions between the electrons and is used as drag term. The index of refraction is ²⁶

$$\mu = \sqrt{\kappa}$$

and the squared magnitude of the reflection coefficient is given by ²⁶

$$R = \left| \frac{1 - \mu}{1 + \mu} \right|^2$$

The reflection coefficient v.s. the microwave frequency for different collisional frequencies is shown in fig. 4 below.

Fig. 4. The reflection coefficient v.s. microwave frequency ²⁷

2.3.2 The step profile electron density assumption

In the previous derivation, the ionization profile of the plasma boundary is assumed to be a step function (fig. 5 on the next page)

Fig. 5. A step function profile for the plasma boundary.

For the case of detonations, the ionization profile can also be assumed to be a step.

This assumption is justified by direct measurements of the conductivity profile of detonations using electric probes²⁷. Figures 6a and 6b show typical conductivity profiles taken with a gap probe.

Fig. 6a. Conductivity profile of a detonation in $2\text{H}_2 + \text{O}_2$ ²⁷.

Fig. 6b. Conductivity profile of a detonation in $\text{C}_2\text{H}_2+\text{O}_2$ ²⁷.

The probes used by Edwards were operated in a voltage range where the Langmuir probe theory applies, so the current through the probe, which is directly related to the conductivity, is proportional to the ionization level. From the conductivity profiles shown in figures 6a and 6b, we see that the ionization profile does not present an ideal step function shape. The step profile assumption is justified however by theoretical studies done on the phenomenon of electromagnetic wave reflection off plasma boundaries. In a work by Albini and Jahn ²⁸ on the theoretical treatment of electromagnetic wave reflection at electron density gradients, it can be seen that the initial rise of the electron density profile has a large effect on the reflected wave. Upon studying various profiles such as linear ramps, kinked ramps, and trapezoids, the authors conclude that the wave reflection depends most strongly on the width of the transition zone at the boundary (fig. 7 on the next page). The details of the profile were found to have a small effect on the reflected wave compared to the transition zone width.

From the conductivity profiles of real detonations shown in figures 6a and 6b, we note a sharp rise at the boundary of the detonations' ionization zone. This feature appears to be characteristic of all detonations, even though the profiles differ behind the initial rise for various gaseous mixtures. Since the transition zone of the ionization profiles of detonations generally appears to

Fig. 7. A ramp profile for the plasma boundary.

be a sharp rise, the assumption of a step profile for theoretical calculations seems, in principle, to be a reasonable one.

2.3.3 Ionization level in detonations

The ionization level in detonations has been measured using several techniques such as Langmuir probes ²⁹, microwave absorption ³⁰, and magnetohydrodynamic interaction ³¹. Work on this problem until 1963 has been reviewed by Toong ³². In his review, Toong outlines the inadequacy of the commonly used one-dimensional detonation model for calculating the ionization in the front. The one-dimensional model assumes that the detonation consists of a planar shock closely followed by a chemical reaction zone. The failure of this model is shown in the large discrepancy between the theoretical and experimental results.

The theoretical calculation of the equilibrium ionization of the species in the detonation front can be done using the Saha equation ²⁷:

$$\log_{10} \alpha_i = -3.25 + \frac{5}{4} \log_{10} T - \frac{1}{2} \log_{10} p - 2.52 \times 10^3 \frac{V_i}{T} + \frac{1}{2} \log_{10} \frac{g_{e_i} g_{A^+}}{g_A}$$

where α_i is the degree of ionization contributed by each ion producing species in the reaction products, T is the temperature (°K), p is the partial pressure of the species i (atm), V is the ionization potential of i (eV), g_{A^+} , g_A and g_e are the statistical weights of the ionized atom, neutral atom and the

electron respectively. So by taking into account a sufficient number of species, the total ionization of the reaction zone can be estimated. Once the total ionization in the front and the collisional cross section of the various ions is known, the reflected microwave power can be calculated. Figure 8 shows the theoretical predictions of reflected microwave power for $C_2H_2 + O_2$ using equilibrium ionization calculations and the experimentally measured reflected microwave power.

Fig. 8. Reflected microwave power v.s. initial gas pressure P_i for $C_2H_2 + O_2$ ²⁷.

There appears to be a closer agreement between experimental and theoretical results for detonations at higher initial gas pressures. This may be attributed to the smaller scale of the transverse fluctuations in the front, causing the detonation to more closely resemble the planar, uniform structure assumed by the one dimensional model. In the worst case, the theoretical predictions can be off by a factor of ten. The cause of the higher than predicted ionization has been explained by Edwards ²⁷ among others by localized spots of highly ionized gas inside the detonation front.

Despite the discrepancy between theoretical and experimental results for microwave reflection, experimental measurements of the equilibrium

ionization some distance behind the detonation front are in close agreement with theoretical predictions ³³. This is because different mechanisms are responsible for the ionization level at the front and behind the front. The two main mechanisms responsible for ionization in detonations are thermal and chemical ionization. Thermal ionization, which can be calculated with the Saha equation, seems largely responsible for the equilibrium ionization behind the front. Chemical ionization seems to be responsible for the ionization at the front boundary. The latter is more difficult to estimate because the reaction zone is a non linear and highly turbulent medium. In addition to this, all the intermediate reactions in the detonation front (which can be quite numerous) must be known to precisely calculate the ionization.

As previously discussed, the most important region of ionization as far as microwave reflection is concerned is the transition zone at the plasma boundary. Therefore, the most important ionization zone to measure is the reaction zone, which is the thin region at the detonation front which is dominated by chemical ionization. Difficulty in measuring the ionization in this region comes from its transient and inhomogeneous nature. Traditional methods such as Langmuir probe measurements do not have the necessary spatial and temporal resolution to measure the fast moving, turbulent reaction zone.

The difficult-to-measure inhomogeneities in the front unfortunately play an important role in the ionization distribution. These irregularities depend strongly on the transverse structure of the detonation front and cannot be taken into account by a simple one dimensional model. A one dimensional model such as the Chapman Jouguet model may be adequate for calculating thermodynamic parameters, but cannot be used to describe electromagnetic wave reflection off a detonation, since the ionization profile is strongly influenced by two dimensional effects. Until better methods of measuring the ionization profile of detonation fronts are developed, the phenomenon of microwave reflection off detonations cannot be predicted quantitatively.

CHAPTER III. EXPERIMENTAL APPARATUS AND INSTRUMENTATION

3.1 Description of the experimental apparatus

The microwave Doppler interferometry system was set up inside a previously built detonation tube. No significant modifications to the apparatus needed to be performed since the microwave system could be easily installed using the ports already present on the apparatus. These ports were originally intended for more conventional diagnostics such as pressure transducers, photodetectors, or ionization probes. A schematic of the entire experimental set up is shown in figure 9.

Fig. 9. Schematic of the experimental set up

The set up consists of a test section and various diagnostics and control instruments. Each component of the arrangement will be described separately in the following sections.

3.1.1 The test section

The test section consists of four main sections: the driver/ignition section, the damping reinitiation section, the main section, and the microwave transition section. They are assembled as shown in fig.10.

Fig. 10. Schematic of the entire test section.

3.1.1.1 The driver/ignition section

This section consists of a piece of copper tubing 78 cm in length , 38.4mm in diameter. The detonation was initiated by the ignition plug at the end of the tube. In order to promote the formation of a detonation, a Schielkin spiral was inserted inside the tube to generate turbulence which greatly facilitates the transition to detonation. The spiral consisted of 3.2mm diameter copper tubing wound into a coil. The entire coil was approximately 40cm in length.

Throughout the series of tests, two different ignition methods were used: a spark initiation or a direct initiation by a strong driver.

a) Strong spark initiation

For spark initiation, the ignition plug contains two electrodes through which a capacitor bank of 8 μ F charged to 4kV is discharged. The spark discharge circuit is shown in fig. 11 (on the next page).

Fig.11. Schematic of the spark discharge circuit

b) Direct initiation with a driver section.

This method of initiation was adopted to reduce the undesirable electrical noise generated by the high voltage discharge in the spark initiation method. Instead of being initiated by a spark, the detonation in the test section is initiated by another detonation which is generated in a driver section. The idea is to generate a detonation in the driver section and to allow it to propagate directly into the test section. To do this, the driver is filled with a sensitive gaseous mixture which can be easily detonated using a low energy igniter. Instead of a strong spark, a glo-wire plug consisting of a short coil of Nichrome wire is used (fig. 12 on the next page). The glo-wire is slowly heated up by gradually increasing the current through it. When the temperature of the wire is sufficiently high, a flame is ignited inside the driver. The flame transits to a detonation as it passes through the Schielkin spiral and bursts through a diaphragm into the test section. The glo-wire does not generate any electrical noise that could be disruptive to the sensitive microwave receiving instruments used in the set up.

Fig.12. Schematic of the glo wire circuit

A diaphragm is used to isolate the driver mixture from the mixture in the test section which is usually of a different composition and at a different pressure. This diaphragm is made from a sheet of acetate with a thickness of 0.06 mm, which is thin enough so that it does not excessively impede the passage of the detonation through it. The center of the diaphragm was lightly scored with a sharp knife to further facilitate its bursting upon contact with the detonation. The diaphragm was replaced after each trial. The gaseous mixture chosen to be used in the driver was equimolar oxy-acetylene at an initial pressure of 5.33kPa (40torr), which is the most sensitive explosive gas mixture.

This system of ignition provided a noiseless and repeatable initiation for the detonation in the test section.

3.1.1.2 The damping transition section

The damping/transition section consists of a short section of steel tubing of a large cross sectional diameter (9.5cm.) and of a short section of copper tube 38.4mm in diameter. Both these section are 50 cm, in length. They were designed to study the phenomenon of damping and reinitiation of a detonation. They were not required for the microwave interferometry tests done in this work but were left in the set up as they did not disturb the experiments.

3.1.1.3 The main section

This section is shown in figure 10 consists of a long copper tube 38.4mm in diameter, 3.27m in length.

Photodetectors are installed in ports 7, 5, and 3. Port 8 is used for the center wire support (the center wire which comprises the center conductor of the coaxial configuration). Port 4 was used for the tests involving the antenna plug that was designed to study high order modes.

It is in this section that all the diagnostic measurements are performed for the detonation propagation.

3.1.1.4 The microwave transition section

This section is designed to provide a smooth coupling between the coaxial cable that carries the microwave energy and the test section. It consists of a brass cone with a standard SMA female jack mounted at the tip. The cone flairs out to a diameter of 38.4mm in order to match the inner diameter of the test section. The center wire inside the test section is attached directly to the back of the SMA jack (fig. 13).

Fig13. Schematic of the microwave transition

The dimensions of the cone was chosen to approximate those of an exponential horn with a lower cutoff frequency of approximately 400MHz (appendix C). A good impedance match between the coaxial cable and the larger diameter test section was thus obtained.

3.1.1.5 The monopole antenna

In order to study the effects of higher order modes, a small antenna was inserted into the test section tube in an attempt to excite them. The antenna was mounted onto a brass plug that was flush with the inner wall of the tube when inserted into a side port. The antenna extended from the plug such that it was normal to the wall, protruding towards the center of the tube, forming a small monopole.

Although higher order modes can be generated by any abrupt discontinuity in the tube, the antenna was roughly designed to excite the TE₁₁, which is the most likely mode to propagate given its low cutoff frequency. The length of the antenna was made equal to a quarter of the TE₁₁ wavelength and the orientation was in accordance with the electric field lines of the mode (appendix D).

3.1.1.6 Installation of the center wire

The center wire, which consists of a thin copper wire 0.4mm in diameter (26ga.), extends from the microwave transition at the far extremity of the tube to a support situated near the ignition end of the tube. The center wire support installed in port 8 of the main section (fig.14).

Fig. 14. Installation of the center wire

The tension of the wire was increased by tightening the tuning peg on the center wire support until the sag of the wire near the center of the test section was less than 2mm from the central axis of the tube.

3.2 Diagnostics and instrumentation

The two types of diagnostic measurements performed were photodetector velocity measurements and microwave Doppler interferometry.

3.2.1 The microwave instrumentation

Fig.15. The microwave circuit

The microwave circuit (fig. 15.) makes use of a four port circulator to channel the microwave energy to and from the test section tube. The microwave energy from the generator enters the circulator via a ferrite isolator, and is channeled to the tube via a coaxial cable. The reflected microwave signal returns through the same cable and enters the circulator, where it mixes with the leakage signal from the transmitted wave. The resulting interference wave is received by a crystal detector, and the rectified envelope of the signal is preamplified and sent to a digitizing oscilloscope. The digitized wave is then transferred to a computer for storage and analysis. The frequency and power calibration of the four port circulator and the crystal detector are shown in Appendix F.

3.2.2 The photodetector instrumentation

Photodetectors were used to provide an independent velocity measurement. These devices are capable of sensing the brief flash of light emitted by the detonation as it passes in front of a small window installed on the side of the tube. When the photodetector is illuminated, the current through it changes. This sudden current change is converted into a voltage pulse using a simple circuit (appendix E). The signals from all three photodetectors are mixed and sent to an oscilloscope which is used as a signal preamplifier, and are finally sent to the recording instruments. The three photodetectors were installed in ports 7, 5, and 3 of the tube (fig.16) so as to detect the time of arrival of the detonation at each of these ports.

Fig.16. The photodetector set up

The average velocity of the detonation between adjacent photodetector ports was subsequently calculated.

A typical photodetector signal is shown in fig.17 (at the end of sect. III).

3.3 Experimental procedure

In this section, a description of the procedure required to perform a typical trial using the experimental set up is given. Descriptions of the procedure using first the spark ignition system, then the direct initiation system will be given.

For the strong spark ignition, the entire tube, from the main section to the driver section is evacuated then filled to the desired pressure with the premixed detonable gas mixture. The tube is then sealed off and the gas is ready to be ignited. The capacitor bank is charged to the appropriate voltage using the high voltage power supply, then the knife switch is closed, discharging the stored energy of the capacitors through the spark plug in the tube, thus igniting the gas. The photodetector and microwave signals are recorded on the digital oscilloscope, then transferred to the computer for analysis and storage.

For the detonation driver initiation system, the procedure is similar except that the driver and test sections must be evacuated and filled separately with their respective gases since they are isolated from each other by a diaphragm. The ignition is triggered by slowly increasing the voltage to the glow-wire plug until the driver gas is inflamed. The data recording procedure is the same.

3.4 Experimental conditions

In this section, the initial conditions for all the trials are given.

3.4.1 Ignition

a) Spark initiation:	Capacitor bank: $8\mu\text{F}$ ($4 \times 2\mu\text{F}$)
	Voltage: 4kV
	Spark gap separation: 1mm.
b) Detonation driver initiation:	Driver gas: $\text{C}_2\text{H}_2 + \text{O}_2$ (equimolar)
	Initial pressure: $P_i = 40\text{torr}$

3.4.2 Vacuum

A vacuum of 0.0015 kPa (0.11 torr) was achieved in the tube. After 10 minutes, the vacuum was found to deteriorate to 0.0400 kPa (.20 torr) due to leaks in the tube. This vacuum was judged to be acceptable for the tests, since most of the experiments were performed at relatively high initial pressures ($\sim 4.00\text{kPa}$ or 30 torr), so the gas impurity contributed by the leak was a small percentage.

Later, upon examination of the data, it was found that the present microwave technique was able to detect a small local velocity decrease in the

detonation resulting from a contamination of the gas by a small leak at the end of the tube.

3.4.3 Gaseous mixtures

Two, well studied, extensively tested and documented explosive gas mixtures were used in this work: stoichiometric oxy-acetylene ($C_2H_2 + 2.5O_2$), and stoichiometric propane-oxygen ($C_3H_8 + 5O_2$). These mixtures were chosen for their different chemical properties and their different transverse wave structure commonly known as cellular structure. Oxy-acetylene is known to have be highly sensitive to initiation and has a fairly regular cellular structure. Propane-oxygen has a lower sensitivity to initiation and a more irregular cellular structure, and consequently, its detonation front is likely to be more unstable and inhomogenous³⁴. The ionization profiles are also likely to be different.

In order to investigate the effects of ionization inhomogeneities across the front, the scale of the transverse wave structure, which is described by a quantity known as the cell size, was varied. Using empirical relations between the initial gas pressure P_i and cell size¹⁴, a range of pressures was chosen such that the cell size varied from the millimeter to the centimeter range. Comparisons of the cell sizes and cell regularity of the various combustible mixtures can be found in detonation literature^{34,35,37}. The tests were carried out over a range of initial pressures from high P_i 's where the cell size is small compared to the tube diameter down to low P_i 's where the cell size is comparable to the tube diameter. At very low initial pressure, detonations propagate close to their limit and are consequently unstable. They present a particularly interesting phenomenon where the microwave technique offers a unique advantage over other velocity measurement techniques. The initial pressures for all the trials are tabulated below in table 2 (shown on the next page).

f (GHz)	C₂H₂ + 2.5O₂	C₃H₈ + 5O₂
6.70	0.533 to 6.66kPa (4 to 50torr)	0.800 to 10.7kPa (6 to 80 torr)
9.21	2.67kPa (20 torr)	1.07 to 6.66kPa (8 to 50 torr)
6.70 *	0.800 to 6.66kPa (6 to 50 torr)	6.66kPa (50 torr)
9.21 *	0.800 to 6.66kPa (6 to 50 torr)	6.66kPa (50 torr)

Table 2. Initial pressures for the trials (* for antenna trials)

3.4.4 Mode excitation

In an attempt to observe the phenomenon of mode beating due to interactions with higher order modes, a series of trials was done with the antenna plug installed in the tube. Trials were repeated at the same initial conditions as the trials done without the antenna, and velocity measurements were taken.

3.4.5 Microwave frequency

Since one of the main features of the coaxial configuration is its large bandwidth, tests were done at two frequencies: 6.70GHz and 9.21GHz. The number of useable frequencies was limited by the small operating frequency band of the four-port circulator (appendix F).

The microwave generator was operated at a power output of 13dBm, but less than -12dBm actually entered the test section due to impedance mismatches and various other losses in the microwave circuit. At this level, the microwave energy was more than three orders of magnitude below the ignition energy of the most sensitive gaseous mixture used in this study (C₂H₂ + O₂) consequently, there was no danger of igniting the mixture. The microwave power was too low to influence the detonation propagation in any way, since it has been found that even very high intensity electric fields do not have a significant effect on detonations ³⁶. At the power level used in

this work, the microwave energy could therefore be considered truly non-intrusive.

It was expected that at the higher frequency (9.21GHz), the amplitude of the interferogram would be reduced due to a higher transmission of the microwaves through the plasma and a smaller reflected signal. It was also thought that the higher modes would be more easily excited since the frequency would be further above the cutoff point of the modes.

Fig. 17. A typical photodetector signal. The trial used for this plot was $\text{C}_2\text{H}_2 + 2.5\text{O}_2$ at $P_i = 10.66 \text{ kPa}$ (80 torr).

CHAPTER IV. ANALYSIS OF RESULTS

The interpretation of the microwave interferometry signal was done automatically by computer. A program was written to process the signal and produce velocity versus time and velocity versus distance plots of the detonation propagation.

Since the interferogram gives the Doppler beat frequency, the velocity of the detonation at a certain point in time can be calculated from the frequency of the interferogram at that point (equ. 2.1.1 in section 2.1). The frequency of the signal, which varies in time according to the detonation velocity, is found by measuring the period of the signal at each cycle. The duration of one complete cycle is measured by simply taking the time interval between adjacent peaks, since the signal is approximately sinusoidal. By measuring the period for every cycle in the signal, a complete velocity history of the detonation propagation can be obtained.

4.1 The analysis program

4.1.1 Description of the procedures

A block diagram of the signal analysis program showing the step by step treatment of the signal is shown in fig. 18 (shown on the next page).

Digital signal processing techniques are used to filter out unwanted noise in the signal, which is subsequently fed to a peak detector. After the period of each cycle is measured, the velocity is calculated and displayed. A detailed description of the successive steps in the program is given below.

Fig. 18. Block diagram of the signal analysis program

The first step of the signal processing is the smoothing function, which reduces unwanted noise by using a moving average. The unprocessed interferogram is averaged over a certain time interval, which is then shifted along the signal one time step at a time until the end of the signal is reached.

Given a function $f[n]$, the smoothed function $f_s[n]$ can be described by:

$$f_s[n] = \frac{1}{N} \sum_{i=0}^N f\left[n - \frac{N}{2} + i\right]$$

where N = the smoothing interval.

This averaging procedure effectively smooths out random fluctuations caused by noise. Care must be taken in choosing the smoothing interval due to the periodic nature of the interferogram. If the interval is large enough such that it is equal to the period of the signal, the smoothing function will average out a complete cycle of oscillation, thus destroying useful information. The interval must therefore be chosen to be smaller than the period of the highest frequency contained in the interferogram.

The signal is next fed to a Chebychev type digital bandpass filter which further reduces the high frequency noise and eliminates the DC bias of the signal. The high and low cut off frequencies of the filter are chosen to be far from the bandwidth of the Doppler frequency so that it does not interfere in any way with the frequency information of the interferogram. Typical values for the low and high cut off points of the filter are 500Hz and 500kHz respectively, and the Doppler frequency usually remains within a bandwidth of 70kHz to 140kHz.

The signal noise is now sufficiently reduced for frequency analysis to be performed on the interferogram. This function is executed by the peak detection procedure. This procedure detects the top of the peaks in the signal by first differentiating it and then finding the zero crossing point on the positive slope (fig.19 shown on the next page).

The time of each zero crossing is temporarily stored in memory. The period at different points in time along the signal is found by calculating the time interval between each zero crossing. Since the wavelength of the electromagnetic wave in the tube is known and the instantaneous frequency is known, the instantaneous velocity can be calculated using equation 2.1.1 in section 2.1.1.

Fig. 19. Peak detection procedure

4.1.2 Choosing the spatial resolution

As mentioned earlier, the microwave Doppler technique has a finite spatial resolution, albeit quite small compared to other techniques. The resolution limit comes about from the fact that a periodic signal must be sampled over a time interval of at least one period of the frequency in order to estimate its frequency. The Doppler signal completes one cycle of oscillation when the reflecting object, i.e. the detonation front, moves a distance equal to half of the microwave wavelength (equ. 2.1.1 in section 2.1.1). The maximum spatial resolution is therefore equal to half the wavelength of the microwave energy used in the measurement (eg. resolution $\approx 2.25\text{cm}$ for a frequency of 6.71GHz).

The main cause of error in the estimation of the Doppler frequency is uncertainty in the peak estimation, which can arise from two main sources: random noise in the interferogram and quantization error. Random noise originates in the electrical system from the usual places such as the long electrical cables, the microwave generator, the signal preamplifier etc... This noise is attenuated using the filtering techniques described above. The quantization error, also known as discretization error, is determined by the sampling rate of the digitizing equipment. The peak detector searches for local maxima in the signal, and the uncertainty in finding these points can be

no smaller than the sampling period. The digital oscilloscope used in this work was operated at a sampling period of 1.6×10^{-7} sec, which results in a velocity error of 1.6% if one period is taken for the velocity calculation. The error calculations are shown in appendix G.

The simplest way to reduce the effect of these errors is to take longer time intervals for the velocity calculations, and the simplest intervals to take are integer multiples of the period of the signal, that is, integral numbers of cycles in the interferogram are used to estimate the frequency. The computer analysis program allows one to choose the number of cycles to be taken for the frequency estimation by allowing the spatial resolution to be specified in units of half wavelengths. A spatial translation of half a wavelength corresponds to a complete cycle in the interferogram, so by selecting a resolution of N half wavelengths, the frequency is averaged over N cycles, and the uncertainty arising from peak estimation error is reduced. Unfortunately, the spatial resolution is sacrificed at the expense of reduced error in velocity calculations. The minimum resolution length is one half wavelength, but two half wavelengths were chosen for the analysis of the results in order to reduce the error. In choosing the number of wavelengths or cycles over which the average velocity is to be calculated, the desire for high spatial resolution must be weighed against the desire for precision in the calculated velocity. From a detonation point of view, the velocity measurements must be capable of resolving the fluctuations in the propagation velocity, which may be small or large depending on the phenomenon being studied. This will be discussed in a later section.

4.1.3 Calculation of the distance of propagation

In order to display the velocity history versus displacement instead of time, the distance of propagation of the detonation must be calculated. This was done by integrating the velocity versus time plot and replotting the velocity versus the distance. Because the velocity points were not calculated at uniform time intervals, the velocity curve could not be integrated by traditional methods. Instead, the trapezoidal method was used to find the distance travelled by the detonation $d[t]$. (fig. 20).

Fig.20. Integration of the velocity history plot.

This integration procedure can be described by:

$$d(t) = \sum_{n=0}^N \frac{(V_n + V_{n+1})}{2(t_{n+1} - t_n)}$$

An arbitrary point of zero propagation distance must be specified to the analysis program in order to plot the distance in the abscissa. The velocity can then be viewed with respect to the distance travelled by the detonation.

4.1.4 Operation of the analysis program

Once all the proper parameters are set and a digitized interferogram is available for analysis, the program can be run. A sample of the analysis of a trial is shown in fig. 21 (shown on the next page).

The entire program was designed using the LabVIEW software package. A LabVIEW block diagram of the program is shown in appendix H.

CHAPTER V. RESULTS AND DISCUSSION

5.1 Average velocity measurements

The average velocities were extracted from the microwave velocity history plots by averaging the segment of the plot corresponding to the time interval during which the detonation travels between the first and last photodetector. By calculating the average velocity from the first and last photodetector spikes, a direct comparison between the microwave and photodetector measurements could be made. The percent difference between the two velocity measurements is shown in figures 22 and 23 (shown at the end of sect. V). There is a good agreement to within less than $\pm 2\%$ between the photodetector and microwave results, which is close to the $\pm 1.6\%$ velocity error caused by quantization error alone. A slight increase in the difference at low P_i can be attributed to an increased distortion of the microwave interferogram due to ionization inhomogeneities. The increased distortion would cause a larger error in the peak detection procedure, which would consequently manifest itself in the velocity results.

The $\pm 2\%$ difference is larger than past microwave measurements which specified a discrepancy as small as $\pm 0.4\%$ between the microwave method and ionization probe measurements ⁶. This can be attributed to the fact that in past work, the velocity was calculated by counting a large number of cycles in the interferogram to obtain the frequency, thus reducing the error in the frequency estimation (at the expense of spatial resolution). Only two cycles are used for the frequency estimation in the experiments done in this work, resulting in a larger error in the velocity calculation but with improved spatial resolution. The trade-off between accuracy in the velocity measurement and high spatial resolution is apparent. This is problem discussed in more detail in section 5.4.

Figures 24 and 25 (shown at the end of sect. V) show the measurements of the average velocity of stable detonations propagating in oxy-acetylene and propane mixtures. There is a close agreement between the experimental velocity measurements and the theoretical Chapman-Jouguet calculations. This is in accord with past work on detonation velocity measurements which

generally specify an agreement between theoretical and experimental results to within a few percent.

This analysis of the velocity histories does not fully exploit the information contained in the plots obtained by the microwave technique. However, this simplified reduction of the data is useful for comparison with the photodetector measurements as well as with past measurements.

5.2 Microwave frequency

Testing of the broadband capabilities of the coaxial set up was limited to using only two different frequencies due to the inadequate bandwidth capability of the four-port circulator used. In addition to performing tests at 6.71GHz, a frequency well within the maximum response range of the circulator, tests were also conducted at 9.21GHz. At this higher frequency, the amplitude of the interferogram was greatly reduced due to the poor response of the circulator and possibly to a lower reflectivity of the detonation front. The low signal amplitude resulted in a lower signal to noise ratio, and consequently, the signal analysis was expected to be more problematic. The analysis program was nevertheless capable of extracting velocity information from the signal, and the percent difference between microwave and photodetector measurements for 9.21GHz was well within the scatter of the data for 6.70GHz. It can be seen in figures 22 and 23 (shown at the end of section V) that the measurements of the average velocity using the two microwave frequencies agree with each other, so for the two frequencies studied, the microwave Doppler system gives consistent results.

5.3 Detonation structure

It was mentioned earlier that the transverse wave structure in the detonation front, also known as the cellular structure, might cause fluctuations in the reflected microwave signal by creating inhomogeneities in the ionization profile. It is expected that the scale of the inhomogeneities in the detonation structure also influences the microwave signal. When the scale of the cellular structure is small compared to the dimensions of the tube

in which the detonation propagates, the regions of high ionization formed by the triple points are numerous and more or less uniformly distributed across the front. This gives the front the semblance of a flat, uniform surface, somewhat akin to the way a fly-screen door appears smooth and uniform from a distance. If the scale of the detonation structure is increased by lowering the initial pressure, the detonation cells become bigger, and the ionization inhomogeneities become significant. A number of scenarios can occur such as modulation of the microwave signal by the periodic motion of the cellular structure (eg. single head spin), or fluctuations in the signal amplitude due to important variations in the ionization level caused by abrupt changes in the scale of the cellular structure (also known as detonation mode changes) ⁷. A qualitative comparison between the microwave interferograms of high and low pressure detonations indeed show that for the $C_2H_2 + 2.5O_2$ mixture, the signal fluctuated more at low pressure. The signal for $P_i=80$ torr (figs. 26 shown at the end of sect. V) had a fairly constant amplitude and was close to an ideal sinusoidal shape whereas the signal for $P_i=4$ torr (fig. 27 shown at the end of sect. V) showed a noticeable level of amplitude fluctuation. The level of amplitude fluctuation was more difficult to compare for the $C_3H_8 + 5O_2$ mixture, as the interferograms for the high (fig. 28 shown at the end of sect. V) and low (fig. 29 shown at the end of sect. V) pressure trials both showed large irregularities. The fluctuating nature of the propane signals would seem to suggest an irregularity inherent to the mixture itself. This is discussed briefly below. What can perhaps be noted about these interferograms is that the time scale of the amplitude fluctuations in the signals is longer for the trial at $P_i=8$ torr than for the trial at $P_i=80$ torr. If these signal fluctuations are indeed caused by the transverse structure of the detonation, it follows that, as the scale of the structure increases at low pressure, the time scale of the signal fluctuations also increases.

A qualitative comparison between microwave signals for the $C_3H_8 + 5O_2$ and $C_2H_2 + 2.5O_2$ mixtures show that at a comparable cell size (cell size = 1cm), the $C_3H_8 + 5O_2$ mixture yields more irregular interferograms (figs. 30 and 31 shown at the end of sect. V). This would seem to suggest that the spurious fluctuations in the reflected microwave signal are linked with transverse structure irregularities of the detonation, since the transverse structure of $C_3H_8 + 5O_2$ is generally known to be irregular. It is reasonable to assume that transverse structure irregularities would directly affect the

distribution of the ionization density throughout the front, which in turn affects the reflected microwave signal. By analyzing the interferogram fluctuations, it may be possible to obtain information on the scale and regularity of the transverse structure of the detonation. Since the detonation structure is intimately linked with the characteristics of the chemical reaction in the front ^{37,40}, the fluctuations in the microwave signal might offer a means of studying the chemical activity of the detonation.

It is interesting to note that fluctuations in the amplitude of the interferogram do not necessarily indicate fluctuations in the velocity. The irregular interferogram shown in figure 28 (shown at the end of sect. V) actually yields a fairly smooth and flat velocity history (fig. 32 shown at the end of sect. V). The interferogram in figure 29 (shown at the end of sect. V) does however show a large velocity fluctuations (fig. 33 shown at the end of sect. V). This would seem to suggest a certain independence of the longitudinal detonation process from the transverse one.

5.4 Resolution

The main factor involved in choosing a longer spatial resolution is a reduction of the fluctuations in the velocity history plots caused by the peak estimation error. By selecting longer lengths for the resolution, the time interval used for the frequency estimation of the signal is increased, and thus the precision of the estimate is improved. The effects of lengthening the resolution are investigated by using the velocity history plot of a stable detonation that propagates at a constant velocity with very small fluctuations. Since the velocity history is smooth and flat, noise fluctuations in the signal are caused by random noise and quantization error. The level of noise in the velocity plot can be described by the standard deviation of the plot. Figure 34 (shown at the end of sect. V) shows the increase in the standard deviation of the velocity plot as the resolution is lengthened in multiples of half wavelengths. This indicates that the noise is reduced as longer values of spatial resolution are taken.

There are several possible methods to improve the spatial resolution. One would be to decrease the wavelength of the microwave energy by simply increasing its frequency. The possible detrimental effect of doing this would

be the appearance of higher order modes, since increasing the frequency above lower cut off frequencies of these modes would allow them to propagate. These waves could interfere with each other, distorting the interferogram and seriously hindering the present method of analyzing the microwave signal. At its present stage of development, the technique can not be used with multi-mode electromagnetic wave propagation. Until a new signal analysis scheme is developed, the excitation of higher order modes must be avoided. An upper limit must therefore be imposed on the bandwidth of operation, limiting the useable frequencies to a range below the cut off point of the lowest TE or TM mode.

Another way of improving the resolution would be to reduce the peak estimation error, thus allowing the maximum resolution of one half wavelength to be used while maintaining the low noise fluctuations at a manageable level. Since the peak estimation error arises mainly from electrical noise in the signal and quantization error, it is these two factors that must be alleviated. Signal noise can be lessened by the usual noise reduction techniques such as shortening cable length, increasing signal strength, and also improving the quality of the test equipment. Digital techniques of noise filtering other than those used in this work can also be investigated. Possible candidates include using periodograms ³⁸ as well as a large number of specialized digital techniques ³⁹.

Quantization error can be reduced by simply increasing the sampling rate of the digitizing equipment. The cost and availability of very high speed digitizers may then become a concern. Computer memory capacity would also have to be large enough to handle the large data files (easily larger than 1 megabyte in size) resulting from high speed digitizing of long signals, and the data processing would then involve correspondingly long time periods.

In the end, the importance of high resolution depends on the nature of the detonation phenomenon to be studied. If the large scale velocity fluctuations of a galloping detonation are to be measured, then the resolution need be no better than one tube diameter, since these fluctuations evolve over distances of several tube diameters. If, however, the relatively small scale velocity fluctuations of a single head spin detonation are to be studied, then the resolution must be increased since these fluctuations typically occur within a distances of one or two tube diameters.

5.5 Higher order modes

Attempts to excite higher order modes in the apparatus used in this work were somewhat fortuitously unsuccessful in that the analysis of the interferogram was greatly simplified. This however prevented the study of the mode interaction phenomenon. Velocity results were unaffected by the presence of the antenna in the tube (fig.35 and 36 shown at the end of sect. V). All the microwave velocity results were calculated assuming the wavelength of the TEM mode of propagation, and had higher order modes with different wavelengths been present in the tube, a discrepancy would be detected in the velocity results. The results with the antenna present in the tube agreed quite well however with the independent photodetector measurements as well as with the trials performed without the antenna. Not even a local disturbance in the amplitude was detected in the interferogram. These preliminary results seem to indicate a reluctance on the part of the higher order modes to propagate. Possible explanations of this would be that the antenna did not provide enough of a source of disturbance to the microwave energy, or that oxidation and dirt on the inner walls of the tube suppressed the higher order modes by dissipating the wall currents required by the modes.

The propagation of higher order modes in the detonation tube would create the situation of two or more electromagnetic waves with different wavelengths existing simultaneously inside the tube. It can be postulated that mode beating and other mode interactions might occur, in which case the present analysis technique would be useless. New analysis techniques would have to be developed to deal with this problem, perhaps along the lines of the multi-band signal analysis techniques used in radar remote sensing. These techniques involve a statistical analysis of the different bands of frequencies.

It is also possible that the resistance of the coaxial waveguide to high order mode excitation indicates the secondary nature of this problem. The coaxial configuration may offer a stable environment for the propagation of the TEM mode even at higher frequencies above the cutoff frequencies of the TE and TM modes, and signal distortion due to mode beating can be relegated to a minor concern in microwave velocity measurements. At any rate, further experiments on these modes are necessary before their effects can be fully assessed.

5.6 Test instruments

One of the main weaknesses in the microwave Doppler interferometry system presented in this work is the poor performance of certain instruments available for the research at the present time. Some of the weak links in the system were the four port circulator which had a very poor broadband response, and the crystal detector which generated fairly high level of electrical noise. The unavailability of a microwave generator capable of producing a wide frequency range signal was also a constraint.

Fig. 22. Plot of the percent difference between the detonation velocity measurements using the microwave technique and photodetectors. The mixture for these results was $\text{C}_2\text{H}_2 + 2.5\text{O}_2$.

Fig. 23. Plot of the percent difference between the detonation velocity measurements using the microwave technique and photodetectors. The mixture for these results was $\text{C}_3\text{H}_8 + 5\text{O}_2$.

Fig. 24. Plot of average velocity measurements of detonations in $\text{C}_2\text{H}_2+2.5\text{O}_2$ using the microwave technique and photodetectors. The theoretical Chapman-Jouguet velocity is also shown.

Fig. 25. Plot of average velocity measurements of detonations in $\text{C}_3\text{H}_8+5\text{O}_2$ using the microwave technique and photodetectors. The theoretical Chapman-Jouguet velocity is also shown.

Fig.26. Interferogram for C₂H₂+2.502 at $P_i=10.66\text{kPa}$ (80torr)

Fig.27. Interferogram for C₂H₂+2.502 at $P_i=0.533\text{kPa}$ (4torr)

Fig. 28. Interferogram for C₃H₈+502 at $P_i=10.66\text{kPa}$ (80torr)

Fig. 29. Interferogram for C₃H₈+502 at $P_i=1.07\text{kPa}$ (8torr)

Fig.30 Interferogram for C₂H₂+2.5O₂ at $P_i=1.33\text{kPa}$ (10torr)
(cell size ~ 1cm.)

Fig. 31. Interferogram for C₃H₈+5O₂ at $P_i=6.66\text{kPa}$ (50torr)
(cell size ~ 1cm.)

Fig.32. Velocity history of the trial shown in fig.28
($\text{C}_3\text{H}_8 + 5\text{O}_2$ at $P_i = 10.66\text{kPa}$ or 80torr).

Fig.33. Velocity history of the trial shown in fig.29
($\text{C}_3\text{H}_8 + 5\text{O}_2$ at $P_i = 1.07\text{kPa}$ or 8torr).

Fig. 34. Plot of the standard deviation v.s. spatial resolution. The trial used for this calculation was $\text{C}_2\text{H}_2 + 2.5\text{O}_2$ at $P_i = 6.66\text{kPa}$ (50 torr).

Fig. 35. Plot of microwave velocity measurements of detonations in $C_2H_2+2.5O_2$ with and without the antenna mounted. The theoretical Chapman-Jouguet velocity is also shown.

Fig. 36. Plot of microwave velocity measurements of detonations in $C_3H_8+5O_2$ with and without the antenna mounted. The theoretical Chapman-Jouguet velocity is also shown.

VI- CONCLUSION

6.1 General conclusions

In closing, this work provides three main contributions to the use of the microwave Doppler interferometry technique for diagnostic measurements of detonations. Firstly, it introduces a new coaxial configuration that solves the problem of the narrow operating frequency bandwidth imposed by single TE or TM mode propagation. Secondly, it presents the development of computer software which facilitates the signal analysis. Thirdly, it demonstrates the immediate potential of the microwave interferometry system for original detonation research by presenting preliminary results on the velocity measurements of unstable detonations commonly known as galloping waves (section 6.2.1). The coaxial configuration allows for broadband operation of the microwave and opens the door to numerous remote sensing diagnostic techniques such as swept frequency diagnostics and other back scattering techniques. The computer software developed increases the speed of the analysis and allows more detailed quantitative data to be extracted from the abundant wealth of information contained in the microwave interferogram . The extraction of quantitative results was possible only at a very limited level in the past due to tedious analysis methods. Finally, the preliminary results shown in section 6.2.1 show that at its present stage of development, the coaxial Doppler interferometer can already provide useful results.

Present day detonation studies are focused on the chemical kinetics of the phenomenon, specifically, the highly unstable reaction zone where the chemical activity is most intense. Much effort is given to observing the interaction between this zone and the shock waves in the detonation front. Since the reaction zone essentially comprises of a bounded plasma region, it is a good candidate for microwave studies. Refinements to the application of microwave diagnostic techniques to detonation studies can further exploit the possibilities of these methods that have already proven invaluable in the fields of atmospheric and environmental remote sensing. Perhaps these techniques can be adapted to resolve the spatial structure and shape of the reaction zone.

There are several difficulties involved in adapting these microwave techniques to the study of detonation waves. The most important ones being that the plasma in the reaction zone is highly transient, inhomogeneous, and is confined inside a tube that is long and often far from smooth. The possibility of using the TEM mode presents an important step in improving the electromagnetic wave propagation problems in detonation tubes. According to mode excitation experiments performed in this thesis work, this mode fortunately appears to be surprisingly robust. The coaxial configuration improvement on the traditional single mode, hollow waveguide approach can facilitate the study of the plasma region in the detonation front by making the conditions more favorable for microwave diagnostic measurements.

Furthermore, the study conducted in this thesis has opened up a series of research activities that warrant further examination. These areas are enumerated in the following section.

6.2 Recommendations for future detonation experiments

6.2.1 Galloping detonations

As Edwards² stated in 1962, the microwave technique could be useful in the study of galloping detonations, on account of the large scale velocity fluctuations that they exhibited. These unstable detonations are still poorly understood, and further experiments using the present system would yield more detailed information on the behavior. With accurate velocity histories of the detonation propagation, it may be easier to characterize their unstable behavior and correlate it with detonation parameters such as activation energy, initial pressure etc... A study of galloping detonations using the coaxial microwave set up has currently been initiated by the shock wave physics group at McGill university. A sample of the results showing the velocity histories of a stable, fluctuating, and galloping detonation can be found in figures 37, 38 and 39 (shown at the end of sect. VI). For these trials, the gaseous mixtures was $C_2H_2+5O_2$ and the detonation tube was 9m in length. The microwave frequency used in these interferometry measurements was 9.0GHz.

6.2.2 Single head spins

By studying the velocity fluctuations of single head spin detonations, one can learn about the unstable chemical kinetics of periodically driven detonations. One dimensional numerical simulations of these types of detonations have been done and detailed experimental measurements could perhaps correlate with the numerical results, or even provide the basis for an improved analytical model. The relatively small scale velocity fluctuations of single head spins can be measured continuously with the microwave system if the resolution is increased.

6.2.3 Cellular regularity

The regularity of the cellular or transverse structure of detonation waves has recently gained considerable interest ^{35, 37, 40}. However, quantifying the level of regularity has proven to be a challenge, and until now, only qualitative comparisons between the cellular structure regularity of different gaseous mixtures has been established (eg. "excellent", "good", and "poor" regularity ³⁷). The apparent link between cellular regularity and distortions of the sinusoidal interferogram may provide a means of quantifying the regularity. If the distortions in the interferogram could be quantified, then a regularity parameter for detonations could be defined.

6.3 Recommendations for future microwave experiments

6.3.1 Improving the microwave technique

One of the most immediate improvements that should be made to the set-up is to upgrade some of the equipment used. For lack of availability, certain microwave components used were in a questionable state of operation. The four port circulator certainly hindered the performance of the system with its limited bandwidth of operation, and the crystal detector generated more noise than was specified by the manufacturer.

Perhaps the high quality of the results despite deficiencies in the quality of the equipment is testimony to the wide tolerance of the coaxial microwave interferometer. Nevertheless, a more stringent test of performance of the system would necessitate the improvement of certain poor components.

6.3.2 Broadband experiments

Adaptive radar techniques could be applied directly to detonation studies using the present coaxial set up. By transmitting a wide band of microwave energy and analyzing the reflected waves, certain physical parameters can be deduced. This is done specifically by measuring the correlation between the frequency bands in the frequency spectrum of the reflected waves. The shape of the frequency spectrum of the reflected signal depends on the shape, size, and location of the reflecting object, and as these physical parameters change, so does the spectrum. A particular object has a characteristic frequency spectrum shape which is known as the signature of the object in radar terminology.

By studying the signatures of detonations and the time evolution of these signatures, one may be able to observe the behavior of the reaction zone as it changes shape and size.

6.3.3 Investigation of higher order modes

If it is required to operate the system above the cut off frequencies of the various higher order modes (e.g. to increase the spatial resolution), a thorough investigation of their effects must be performed. It would be necessary to determine the precise conditions under which they can be excited, and once excited, the manner in which they propagate (i.e. attenuation, amplitude...). Consequently, techniques for avoiding their generation through mode suppression schemes or evading their consequences through data processing techniques provide the basis for an important investigation in this area.

Fig.37. Velocity history of a stable detonation propagating at a fairly constant velocity in a mixture of $C_3H_8+5O_2$ at $P_i=6.66\text{kPa}$ (50torr).

Fig.38. Velocity history of an unstable detonation with moderately large velocity fluctuations in a mixture of $C_3H_8+5O_2$ at $P_i=1.60\text{kPa}$ (12torr).

Fig.39. Velocity history of a failing and reinitiating detonation in a mixture of $C_3H_8+5O_2$ at $P_i=1.07\text{kPa}$ (8torr).

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APPENDIX A

Electric field inside a coaxial wave guide

The magnitude of the radial electric field in a coaxial guide is given by ²²:

$$E_r = \frac{V}{r \ln\left(\frac{b}{a}\right)} \quad \text{equ. A1}$$

where V=the potential difference between the conductors, r=the radial distance from the center axis, a=the radius of the inner conductor, and b= the radius of the outer conductor. By substituting the dimensions used in the set up:

$$a = 2.025 \times 10^{-4} \text{m.}$$

$$b = 1.95 \times 10^{-2} \text{m.}$$

into equ.A1, we obtain:

$$E_r = \frac{V}{4,57 \, r} \quad \text{for } a \leq r \leq b \quad \text{equ. A2}$$

where E_r is maximum at the surface of the center wire ($r=a$), and minimum at the surface of the inner wall of the tube ($r=b$). The normalized electric field is shown as a function of radial distance r in fig. A1 (shown at the end of this section). We can see that the electric field falls off rapidly as r increases.

The equation for the normalized field is obtained by dividing equ. A2 by the maximum:

$$E_r = \frac{V}{4,57 \, a} \quad ,$$

thus obtaining

$$E_r(\text{norm}) = \frac{a}{r}$$

When the normalized field is at 10% of its maximum ($E_r(\text{norm}) = 0.1$), we can see that the radial distance is only

$$r = \frac{a}{0.1}$$

$$\approx 2 \text{ mm.}$$

Fig. A1. Magnitude of the electric field in the tube v.s. radial distance from the center axis.

APPENDIX B

Calculation of the cut off frequencies of the higher order modes.

The cutoff frequencies of the higher order TM and TE modes are found by substituting the coaxial guide dimensions:

$$a = 2.025 \times 10^{-4} \text{ m.}$$

$$b = 1.95 \times 10^{-2} \text{ m.}$$

into the appropriate equations.

TM modes

The cutoff wavelengths for the TM_{mn} are found from 22:

$$\lambda_c = \frac{2\pi}{(c-1)\chi_{mn}} (b-a) \quad \text{equ. B1}$$

where $c=b/a$ and χ_{mn} = the n^{th} roots of $J_m(cx)N_m(x) - J_m(x)N_m(cx) = 0$ where J_m and N_m are the m^{th} Bessel and Neumann functions. The values of χ_{mn} can be found by solving for the roots using the Mathematica™ software package.

The values of $(c-1)\chi_{mn}$ are:

$n \backslash m$	0	1	2
1	2.7756	3.7932	5.0823

By substituting for a , b , and χ_{mn} in equ.B1, we obtain the three lowest TM_{mn} modes:

mn	λ_c (cm)	f_c (GHz)
01	4.37	6.86
11	3.20	9.38
21	2.39	12.55

TE modes

The cutoff wavelengths for the TE_{mn} modes are given for by 22

$$\lambda_c = \frac{2\pi}{(c+1)\chi_{m1}'} (a+b)$$

for n=1, and by

$$\lambda_c = \frac{2\pi}{(c-1)\chi_{mn}'} (b-a)$$

for n=2,3,4,...

where χ_{mn}' = the nth roots of $J_m'(cx')N_m'(x') - J_m'(x')N_m'(cx') = 0$ where J_m' and N_m' are the derivatives of the mth Bessel and Neumann functions. By solving for the roots using Mathematica™, we obtain:

m	$(c+1)\chi_{m1}'$
1	1.8599
2	3.0859
3	4.2448

for the values of $(c+1)\chi_{m1}'$, and

n \ m	0
2	3.7932

for the values of $(c-1)\chi_{mn}$. The 4 lowest TE_{mn} modes are then:

m n	λ_c (cm)	f_c (GHz)
11	6.66	4.50
21	4.01	7.48
02	3.20	9.38
31	2.92	10.29

Thus the six lowest TM and TE high order modes are calculated and tabulated in table 1 in section 2.2.2 of the theory section.

APPENDIX C

Design of the microwave transition

The microwave transition dimensions were based on an exponential horn design. An exponential horn was designed to provide an impedance match between the detonation tube and the SMA coaxial jack. To simplify construction of the horn, a simple conical horn was built to approximate the exponential one. The small and large diameters of the cone shaped transition were determined by the diameter of the SMA jack and the diameter of the detonation tube respectively, and the length of the transition was chosen to be the equal to the length of the exponential horn. Because the horn length was not critical, the transition was made to be 10cm for convenience. The calculation of the microwave parameters of this transition is presented.

Relevant parameters for the calculation:

-radius of the detonation tube $b=19.5\text{mm}$

-radius of the center wire $a=0.2025\text{mm}$

-impedance of an SMA cable $Z_{10} = 50\Omega$

-impedance of the detonation tube $Z_{hi} = 274\Omega$

since ²¹

$$Z_{hi} = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right)$$

substituting for a and b gives $Z_{hi} = 274\Omega$

-length of the horn $d=0.1\text{m}$

The impedance transformation ratio ⁴¹:

$$\begin{aligned} T &= \frac{Z_{hi}}{Z_{10}} \\ &= \frac{274\Omega}{50\Omega} \\ &= 5.5 \end{aligned}$$

Exponential horns are known to behave like high pass filters, so the lower cut off frequency of the horn must be chosen below the desired operating

frequency of the system. The lower cutoff frequency of an exponential horn is⁴⁰:

$$f_c = \frac{c \ln T}{4\pi\sqrt{\epsilon_r} d}$$

where c is the speed of light ($3 \times 10^8 \text{ m/s}$), T is the transformation ratio, ϵ_r is the relative permittivity ($\epsilon_r = 1$ for air), and d is the horn length (m). For a horn 10cm in length, the cutoff is:

$$\begin{aligned} f_c &= \frac{(3 \times 10^8) \ln(5.5)}{4\pi(0.1 \text{ m})} \\ &= 407 \text{ Hz} \end{aligned}$$

This frequency is far below the operating frequencies of 6.70GHz and 9.21GHz used in the experiments.

APPENDIX D

Design of the antenna plug

The antenna plug was designed to excite the TE₁₁ mode, since this mode had the lowest cutoff frequency and was therefore the most likely to propagate. The two design parameters of the antenna were its length and orientation. In order to avoid obstructing the detonation, a thin sewing needle was used so that it would not significantly disturb the detonation (fig. D1).

Fig.D1. The antenna plug

The length of the antenna was chosen to be a quarter of the wavelength of the TE₁₁ mode wave. The wavelength is given by ²³:

$$\lambda = \frac{\lambda_o}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

where λ_o is the free space wavelength, f_c is the cut off frequency, f is the frequency.

For

$$f = 6.70\text{GHz}$$

$$f_c = 4.50\text{GHz (from table 1)}$$

$$\lambda_o = 4.45\text{cm.}$$

$$\text{since } \lambda_o = c/f$$

$$\text{where } c = 3 \times 10^8 \text{m/s}$$

$$\text{and } f = 6.70\text{GHz,}$$

$$\text{so } \lambda_o = 4.45\text{cm.}$$

the wavelength is

$$\lambda = 6.04.$$

So the length of the quarter wavelength antenna is

$$\lambda/4 = 1.5 \text{ cm.}$$

The idea orientation of the antenna is in the direction of the electric field lines inside the wave guide. The field lines of the TE_{11} mode are shown in fig. D1.

Fig. D2. Electric and magnetic field lines in a coaxial configuration for the TE_{11} mode.²²

Since the center wire is thin, electric field lines close to the wire approximately follow a straight along the diameter of the tube. The antenna was positioned so that it would be normal to the tube wall, so that it would follow one of these field lines.

APPENDIX E

Photodetector circuit

Fig. E1. Diagram of the photodetector circuit.

Fig. E1 shows the circuit used to amplify the current pulses from the phototransistors and mix the resulting photodetector signals into one output.

APPENDIX F

Frequency response of the four-port circulator and the crystal detector

The plots of the frequency response of the 4-port circulator (fig. F1) as well as the frequency (fig. F2) and power (fig. F3) response of the crystal detector are shown on the following two pages.

Fig. F1. Frequency response of the four-port circulator at two adjacent ports (the isolation of the circulator is ≈ 10 dB).

Fig. F2. Output voltage of the crystal detector v.s. frequency.

Fig. F3. Output voltage of the crystal detector v.s. input power with a1N23E diode. (operating frequency= GHz)

APPENDIX G

Error analysis.

The equation for calculating the velocity from the microwave interferogram is:

$$V = \frac{\lambda_{1/2}}{T} \quad \text{equ. G1.}$$

where $\lambda_{1/2}$ is the half wavelength, and T is the period.

The calculation of the error in the velocity due to quantization error is found by differentiating equ. G1 on both sides.

$$\ln(V) = \ln(\lambda_{1/2}) - \ln(T)$$

$$\frac{dV}{V} = - \frac{dT}{T}$$

$$\frac{\Delta V}{V} = - \frac{\Delta T}{T}$$

So if $T \approx 1 \times 10^{-5}$ sec. and the quantization error $\epsilon = \Delta T = \pm 1.6 \times 10^{-7}$ sec, the error is:

$$\epsilon = \frac{\Delta V}{V} \approx - \frac{\Delta T}{T}$$

$$= \frac{1.6 \times 10^{-7}}{1 \times 10^{-5}}$$

$$= 1.6\%$$

APPENDIX H

LabVIEW block diagram of the analysis program

The analysis program was written using the object-based programming environment LabVIEW. In the LabVIEW block diagram representation of programs, the various procedures used by the program are shown as descriptive icons. The variable transfer between two procedures is shown as a line or "wire" connecting the two procedure icons. The complete LabVIEW diagram is shown in the following pages.

Fig. 21 Sample run of the analysis program