ANALYTICAL PREDICTIVE REQUIREMENTS FOR PHYSICAL PERFORMANCE OF MOBILITY

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SUMMARY:

This paper examines the mechanics of vehicle-soil interaction in off-road mobility problems with a view to evaluation of their relevance and rationality of the physical behaviour characteristics demanded by the analyses and theories, both for predictive and evaluation purposes.

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INTRODUCTION

The study of vehicle-soil interaction from the viewpoint of ground-bearing and forward motion capability has traditionally followed along the lines of pressure-sinkage relationships for static or quasi-statical conditions. It is not surprising therefore to arrive at relationships for resistance to forward motion of wheels in terms of bearing capacity parameters not unlike those in geotechnical engineering stability analyses. Similarly, considerations in grouser studies yield analytical expressions with almost identically corresponding parameters.

Since the intent of vehicle-soil interaction studies is to provide a basis for understanding and analysis of relevant parametric influences and control, it becomes obvious that the physical base for formulation of working relationships must closely resemble the appropriate "field" situation. It is apparent that whilst static pressure-penetration tests [as performed previously] assess bearing stability, their ability to reveal like patterns of mobility can be critically questioned.

This paper examines some of the fundamental physical interaction characteristics established between vehicle contact units [e.g. wheel, grouser, etc.] and soil - with a view to establishing the necessary components for the analytical model. Compatibility and similarity between physical and analytical models are seen to be the requirements for informed analyses and successful predictions.

GENERAL ANALYTICAL REQUIREMENTS

In arriving at a method for analysis of a typical wheel-soil interaction problem, the primary assumptions can include the fact that tire deformability can be suitably accounted for by associated changes in contact area and pressure. The interaction problem can thus be analyzed by considering the stability of an element of the supporting material. The essential requirements reduce to:

1] Conservation of linear momentum - i.e. the rate of change

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of momentum of a body is equal to the force acting on the body. The resultant formulations provide the equations of motion, [differential equations of equilibrium, etc.].

2] Constitutive relationship - i.e. Mohr-Coulomb or alternate appropriate failure criterion.

3] Boundary conditions.

4] Continuity.

Some or all of the essential requirements may be used to seek a solution to the interaction problem - dependent on the requirements for sophistication, flexibility and accuracy consistent with the demands of the problem. The complexity of the interaction problem is due in part to the number of factors and parameters involved, and also in part to the interdependent relationships established between many of the factors, e.g. -

1] Physical conditions - Wheel load, wheel surface, wheel surface, wheel diameter, tire shape and stiffness, treads, etc.

2] Soil parameters - Strength, density, type, constitutive behaviour, moisture content, etc.

3] Interaction parameters - Translational velocity, slip, dynamic sinkage, driving characteristics, etc.

Dependent on assumptions made, analyses can be simple or rigorous. By neglecting soil deformation and inertia terms, and considering only rigid body forces, classical theories of soil mechanics may be used. Resultant formulations are thus obtained by studying the static equilibrium of some assumed geometrical configuration which satisfies the Mohr-Coulomb condition. Extension of the static analysis into bearing stability considerations provides for a higher order of accuracy - albeit still not necessarily precise or rigorous.

With assumptions of plastic behaviour of soil, limit equilibrium and bounded analyses may be used. By and large, all of the essential requirements listed previously in terms of field mechanics and constitutive relationships are needed in arriving at a tractable solution. The specification of boundary and initial conditions becomes the crucial element in the solution of the problem. The various constraints attendant to the stated constitutive performance of the soil cannot be overlooked in the choice of this method of analysis.

Departure from rigid requirements in classical plasticity solutions begin with specification of non-linear work hardening theories for the constitutive performance of soil. This approach to reality is particularly desirable if numerical techniques [finite element or finite difference] are to be used in seeking a solution to the interaction problem. The requirement for limit equilibrium is thus obviated with a known and specifiable constitutive relationship.

It is obvious that regardless of method of analysis chosen, i.e. -

1] Static equilibrium - relying on simple conservation of energy.

2] Quasi-static analyses.

3] Limit equilibrium - with solution methods in terms of upper and lower bounds.

4] Exact methods - characteristic solution.

5] General field mechanics formulation - with solutions sought in terms of approximate numerical techniques [finite difference], or through variational theorems and solutions obtained with finite element techniques.

accuracy of analysis and prediction can be obtained through compatibility and consistency of field performance with analytical model.

COMPATIBILITY AND CONSISTENCY

Wheel-Soil Energy Losses

Consider the wheel-soil interaction phenomenon shown in Figure 1.

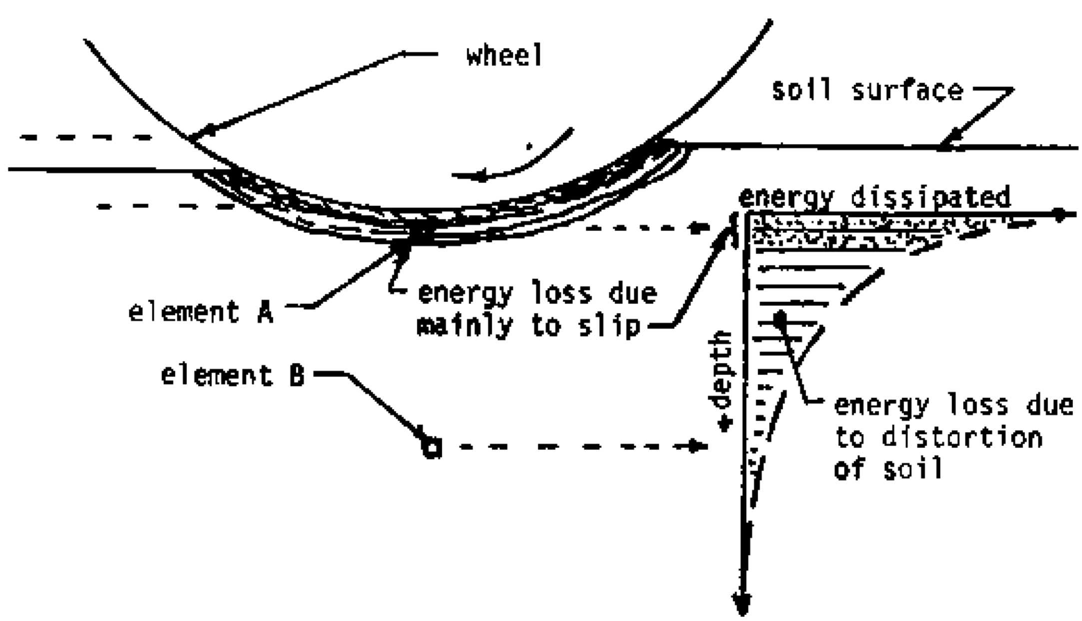

FIGURE 1 - ENERGY DISSIPATION IN SUBSTRATE DUE TO INTERACTION

It is evident from Figure 1 that above the self propelled point, total energy loss becomes sensitive to slip considerations. The two elements A and B in Figure 1 show that two separate mechanisms of shear distortion are operative - in the case of slip performance. For element A, high shear distortion exists if slip between wheel and soil occurs. Limit shear depends either on soil/wheel adhesion or on soil-soil shear - whichever is lesser. The end result is analogous to a viscous shear resistance.

Element B on the other hand is subject to shear distortion arising from applied deviatoric stresses. The governing criterion is either a limit shear condition or an operative shear below the failure value. A work hardening constitutive model appears to be most applicable.

In cases where slip is not particularly large, the performance of element A is not unlike that of B. The transition of elemental performances distinguishing A from B is neither abrupt nor distinct. Slip energy loss dies very rapidly at distances not far removed from the wheel surface.

In speculating on the kind of analytical model that might be used to describe the interaction shown in Figure 1, it would be instructive to recall some direct performance characterics. Confirmation of the physical model shown in Figure 1 is demonstrated in Figure 2.

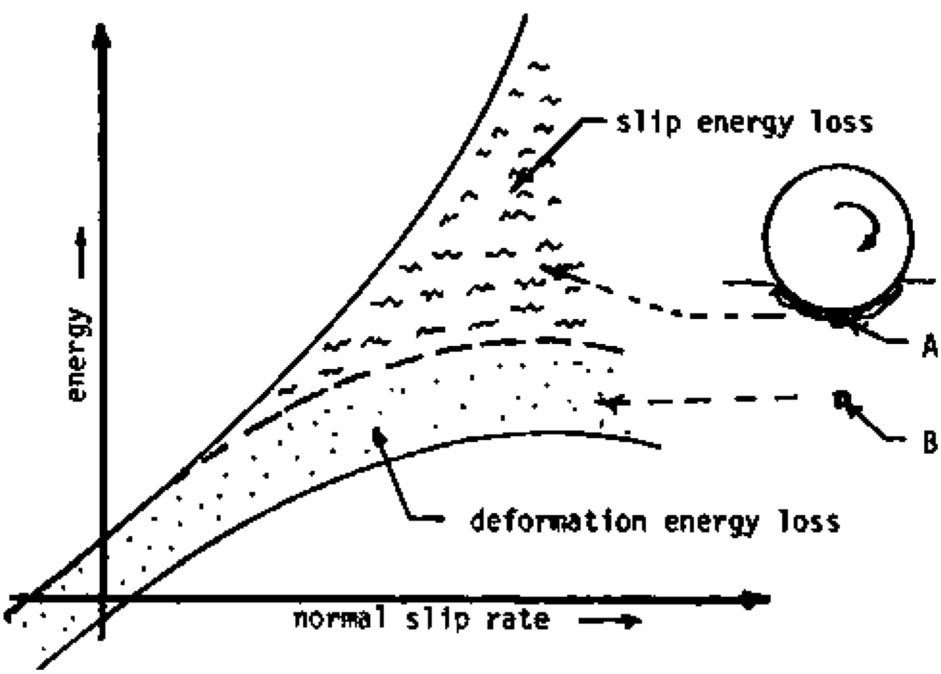

FIGURE 2 - COMPONENTS OF PARASITIC ENERGY

Yong and Webb [1969] have shown from controlled soil bin tests on soft clay that parasitic energy components as shown in Figure 2 can be evaluated directly from measurements of substrata deformation and distortion. This confirms both the mechanistic formulations shown in the schematic diagram of Figure 1, and the fact that direct interdependence relationships between aboveground and in-ground parameters do exist to control final output performance. The relevance and importance of interaction and interdependencies are best demonstrated in terms of: [a] contact pressure, [b] dynamic sinkage [rebound], and [c] substrata performance.

Wheel-Soil Interface Performance

In physical measurements of contact pressures and stresses, two systems are used.

1] Pressure gauges embedded in the soil.

2] Pressure gauges fixed to the wheel contact surface.

Since the first system is not reliable in view of the physical displacement of the embedded gauges, the second method is more commonly used. By and large, the pressure gauges will only sense pressures acting directly on them [i.e. direct normal <u>compressive</u> pressures]. Figure 3 shows a comparison between measured and theoretically computed normal pressure at the interface - using measurements of subsoil performance [Yong and Windisch, 1970].

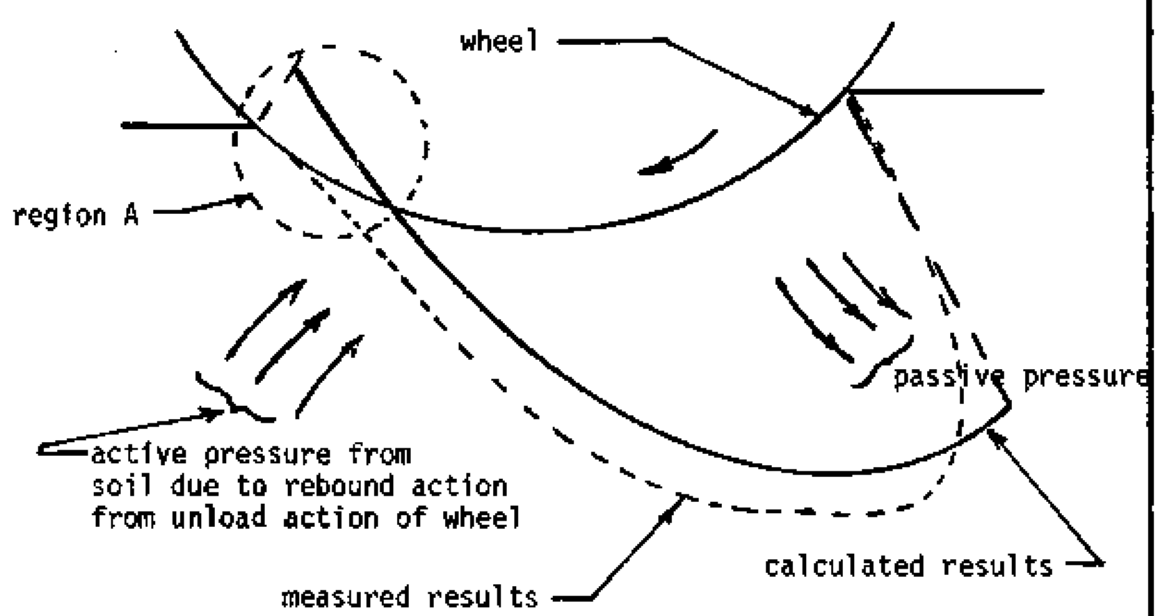

FIGURE 3 - MEASURED AND CALCULATED INTERFACE PRESSURES

Except for a clear and distinct separation of soil from the contact surface in region A as shown in Figure 3, the pressures on the wheel surface should either be positive [compressive] or negative [tension]. However, since the pressure gauges can only sense compressive performance, due either to -

[a] compressive wheel action into soil - soil is in passive state,

[b] active soil action on to contact surface in rebound action as wheel begins to unload in its forward motion,

it is evident that tension values are not recorded. The tension values can occur due to adherence of soil to wheel surface because of and in addition to the slow rebound characteristics of soil - i.e. wheel unload is faster than rebound behaviour.

From measurements of instantaneous soil deformation [compression and rebound] in soil bin tests, it has been shown [Yong and Windisch, 1970] that computed interface pressures compare well with applied pressures. The particular solution technique chosen relied on evaluation of limit equilibrium conditions with a Tresca type failure condition. Tension values are thus not excluded in region A - if such values are indeed representative of the behaviour of the subsoil during wheel unloading. It is clear therefore that the likelihood of such kinds of performance, not necessarily sensed by pressure measurements, must be built into the analytical model devised to provide an analysis of the interaction problem.

Grouser-Soil Performance

Delineation of parasitic energy components and interfacial behavioral aspects in grouser-soil interaction is not easily accomplished. Results

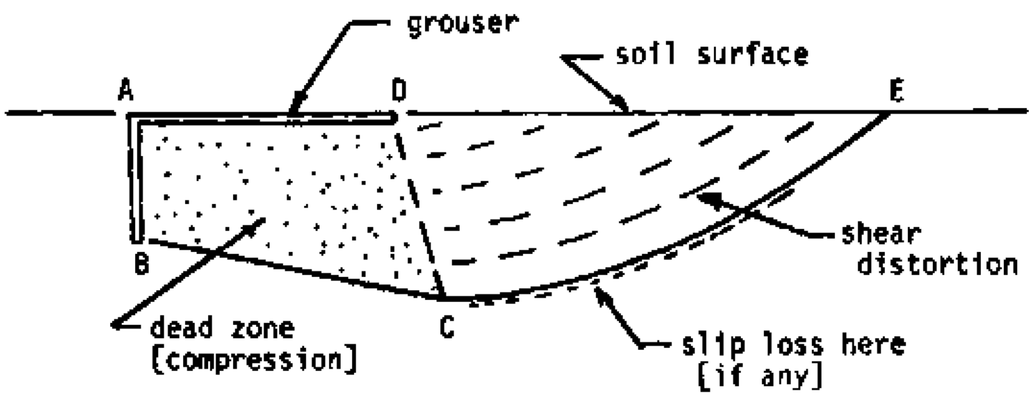

FIGURE 4 - GROUSER-SOIL INTERACTION

reported by Yong and Sylvestre-Williams (1969) indicate that compression of soil occurs in the "dead" zone prior to shear distortional failure [Figure 4].

Unlike the wheel-soil interaction problem, there appears to be very little slip energy loss vis-a-vis grouser-soil interfacial performance. [Slip loss in the track system however still exists]. The leading [vertical] edge of the grouser serves to provide the lateral thrust in the soil, thus obviating slip loss directly in the grouser-soil zone. Whatever minimal slip energy loss exists, will in all probability occur in the slip surface zone under high rates of grouser traverse.

Thus, in terms of analytic modelling, the prime requirement is in regard to a proper description of the performance of the "dead" zone. For simplicity assumptions of zero energy loss in the "dead" zone are made and limit theorems applied to seek a tractable solution. Thus, zone ABCD is considered to be rigid and zone CDE constitutes the region of analytic interest. In actual practice, three immediate problems exist -

1] Location of point C [in Figure 4].

2] Compression behaviour of soil in zone ABCD.

3] Determination and specification of the physical boundary condition - at the grouser soil interface and at DC.

By and large, because of the relative simplistic block behavioral pattern shown in Figure 4 for grouser-soil interaction, the attractiveness for static equilibrium analysis using rigid body analogies is obvious. Results show however that underprediction of thrust occurs because energy loss in distortion of soil in zones ABCD and CDE, and shear energy loss at interface CD are not accounted for.

In simulating transverse motion of the grouser, it is common practice in laboratory experimentation to control -

[a] depth of grouser travel - i.e. variable vertical force resulting therefrom,

[b] applied vertical force - i.e. variable depth of grouser motion.

The resultant thrust developed for situation [a] is obviously different from that of [b] - due to the different boundary conditions developed. Yong and Sylvestre-Williams (1969) show from experimentation and theory that situation [a] produces a greater thrust - so long as both grousers (i.e. for [a] and [b]) start off at the same equilibrium elevation. It is however, not difficult to produce higher thrusts for situation [b] as compared to [a] if initial conditions are successfully manipulated.

The actual field problem however involves -

1] Multiple grousers on a link or belt system,

2] Neither situation [a] nor situation [b] as total exclusion performance characteristics - i.e. neither vertical loads nor elevations are controlled.

Thus prediction of mobility for track systems based on single grouser analysis must suitably account for multiplicity of grouser action together with inadequacies of track morphology. Track analysis however constitutes a separate problem and is not covered within the scope of this study.

Wheel-Soil and Grouser-Soil Predictive Requirements

It is apparent that for analytical and predictive purposes, the basic requirements for the properties of the analytical model relate to -

[a] a realistic specification of the load and unload constitutive performance of soil - not necessarily confined to yield or limit equilibrium,

[b] a proper appreciation of the physical boundary conditions - i.e. interface characteristics and interaction,

[c] an informed knowledge of the physically active and passive constraints.

ANALYSIS AND PREDICTION

The distinction between analytical and predictive requirements can at times be very subtle. However, in general, rigorous analyses are demanded if successful predictions [based on the analytical model] are to be made. In the interaction study, it is assumed (and reasoned] that the forcing function [i.e. wheel or grouser load pattern] and the response function [i.e. soil response behaviour] are suitably related and modified through some correlating function. Thus, if one can describe or evaluate the response function characteristics, and if the correlation functions are known, it becomes obvious that the surficial load parameters will be identified and accounted for.

Loaded Plate Analogy

The simple analogy of a loaded plate [for simulation of wheel loading] of like contact area produces pressure-sinkage relationships not unlike those used in conventional bearing stability analyses. Figure 5 shows the required response from the application of the plate model. This is compared to direct observations [Yong and Windisch, 1970] on actual subsoil performance for towed wheels.

The one clear requirement in the plate model is that compression behaviour occurs throughout. The resultant formulation in terms of a semi-empirical plasticity solution yields the pressure relationship -

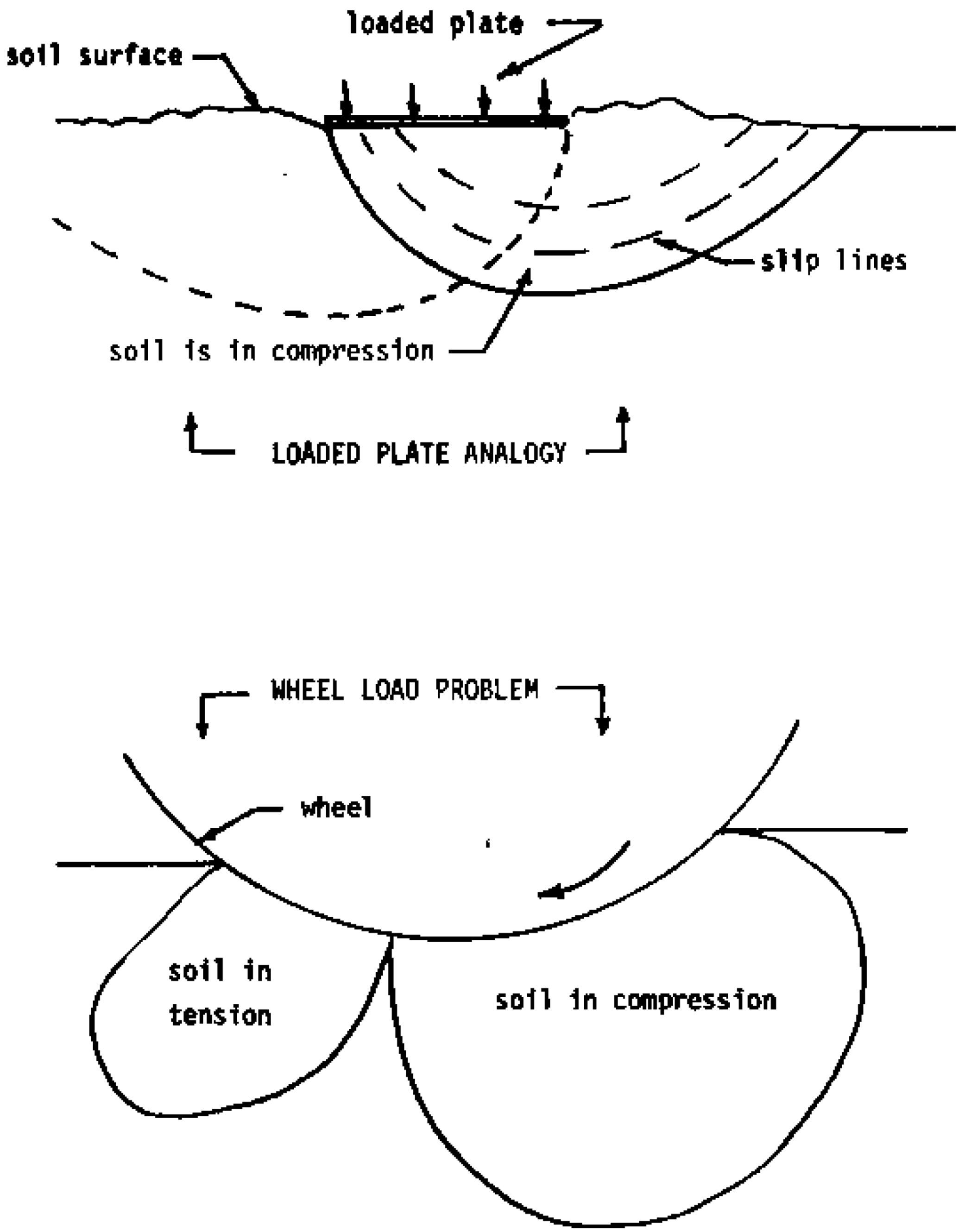

FIGURE 5 - LOADED PLATE ANALOGY AND WHEEL LOAD

$$q = 1.3C\, N_c + 0.6\, \gamma\, b\, N_\gamma + \gamma\, D\, N_q \qquad (1)$$

where

q = pressure on plate

c = cohesion

γ = soil density

b = breadth of loaded plate

D = depth of loading plane = 0 for surface loading

N_c, N_γ and N_q = bearing capacity coefficients dependent directly on internal friction ϕ of soil.

For a surface loaded plate, since $D = 0$,

$$q = 1.3C\, N_c + 0.6\, \gamma\, B\, N_\gamma \qquad (2)$$

The analogous analysis for the wheel gives [Reece, 1965]

$$p = [K_c^1 C + K_\phi^1 \gamma b]\ (\frac{z}{b})^n$$

where

p = pressure

z = sinkage

K_c^1, K_ϕ^1 and n = soil parameters

The similarity between K_ϕ^1 and K_c^1 to N_γ and N_c is obvious. Bearing in mind that solely compression behaviour is expected, the rebound performance of the soil during wheel unloading is thus not considered as part of the prediction requirement. Deviation between prediction and performance can be due in part to this aspect. For good predictability of wheel performance using the loaded plate model, it is essential that -

1] Slip is kept to an insignificant quantity.

2] Soil properties are such that rebound performance is minimal or insignificant.

3] Wheel movement is slow.

4] Wheel surface and load distributions are uniform.

From the above, it can be deduced that if slip performance can be evaluated,perhaps some degree of success can be achieved in formulating a combined model which would incorporate this loss. In addition, it would appear that both load and unload performances must also be included in the analysis. It is clear that the requirements of the plate model do not render compatibility between physical and analytical models - except for a very narrow range of wheel performance. Both forcing and response functions are so severely constrained that generalization is not possible.

Limit Equilibrium Model-Grouser

By and large the analytical models used for limit equilibrium solutions generally assume the admissibility of the Mohr-Coulomb or Tresca criteria. Soil behaviour - i.e. response function characteristic, is thus defined and analyzed in regard to the forcing function in the framework defined by the boundary conditions.

In the grouser-soil interaction problem [as shown in Figure 4], the limit equilibrium approach has been successfully used by Yong and Sylvestre-Williams - with assumptions of -

1] Rigidity of block ABCD.

2] Full failure development in the entire mass defined in zone CDE.

3] Insignificant volume change in ABCED.

The analytical model by Yong and Sylvestre-Williams (1969) is shown in Figure 6.

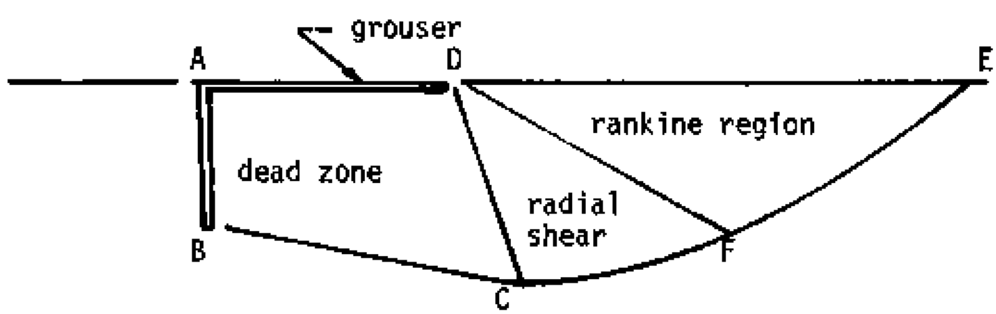

FIGURE 6 - ANALYTICAL MODEL FOR GROUSER-SOIL PROBLEM

Defining $S(\theta)$ as the stress function in relation to the mean stress σ:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} = \rho\, grS(\theta) \qquad (3)$$

where

subscripts 1 and 3 = major and minor components

ρ = density

g = gravitational acceleration

r = radial distance

ψ^1 and θ = as shown in Figure 6

The equations of equilibrium, in polar coordinate form [with origin at point D] are written as:

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = \sigma\, [1 \pm \sin\phi \cos 2\psi^1]$$

$$\tau_{r\theta} = \sigma \sin\phi \sin 2\,\psi^1 \qquad (4)$$

Combining Equation (4) with the Mohr Coulomb condition and regrouping, one obtains:

$$\frac{d\psi^1}{d\theta} = \frac{\cos\theta - \sin\phi \cos\,(2\,\psi^1 + \theta) - S\cos^2\phi}{2\,S \sin\phi\, (\cos 2\,\psi^1 - \sin\phi)} - 1$$

$$\frac{dS}{d\theta} = \frac{S \sin 2\,\psi^1 - \sin\,(2\,\psi^1 + \theta)}{\cos 2\,\psi - \sin\theta} \qquad (5)$$

Solution of Equations (5) will allow for computation of $S(\theta)$ and finally σ. The boundary conditions ascribable to the Rankine Zone at point F are:

$$S = \frac{\cos 0}{1 - \sin \phi}$$

$$\psi^1 = \pi/2 - \theta \tag{6}$$

and at C is

$$\psi_c^1 = 0.5[\pi - \delta - \text{arc} \sin \frac{\sin \delta}{\sin \phi}] \tag{7}$$

where

δ = friction angle between soil and grouser wall

The value of S at C can only be obtained through an iterative technique.

The theoretical slip line field can also be evaluated and compared with actual slip surfaces from experimentation. In applying the model shown in Figure 6, using the method of characteristics as the exact solution technique, integration of σ [from $S(\theta)$] over the length of CD will provide the thrust on the grouser. Confirmation between analytical model prediction and physical performance of a single grouser can be obtained by:

1] Matching physical failure surface in soil due to grouser action with theoretically computed failure characteristic [Yong and Sylvestre-Williams (1969)].

2] Match computed integrated value of σ with physical values.

Thus for example, the results shown in Figure 7 show individual grouser tests in sand at controlled depth and vertical stress. Comparison between limit equilibrium model predictions [McGill] with predictions from previous classical extension of loaded plate analogies [Bekker (1960)] show that the simulated inclined bearing plate relationships do not adequately predict actual performance. The standard relationships used are [Bekker (1960)]:

$$H = b\,[n_c L c + \gamma\, n_g L z + \gamma n_\gamma L^2] \sin\theta$$

$$W = b\,[n_c L c + \gamma\, n_q L z + \gamma\, n_\gamma L^2] \cos\theta \qquad (8)$$

where
- L = grouser length
- b = grouser width
- z = sinkage
- W = vertical load
- H = horizontal thrust
- c = cohesion
- n_c, n_q, n_γ = dimensionless soil parameters dependent on soil friction angle ϕ, θ and L/h
- h = grouser depth
- δ = simulated inclined footing angle dependent on sinkage.

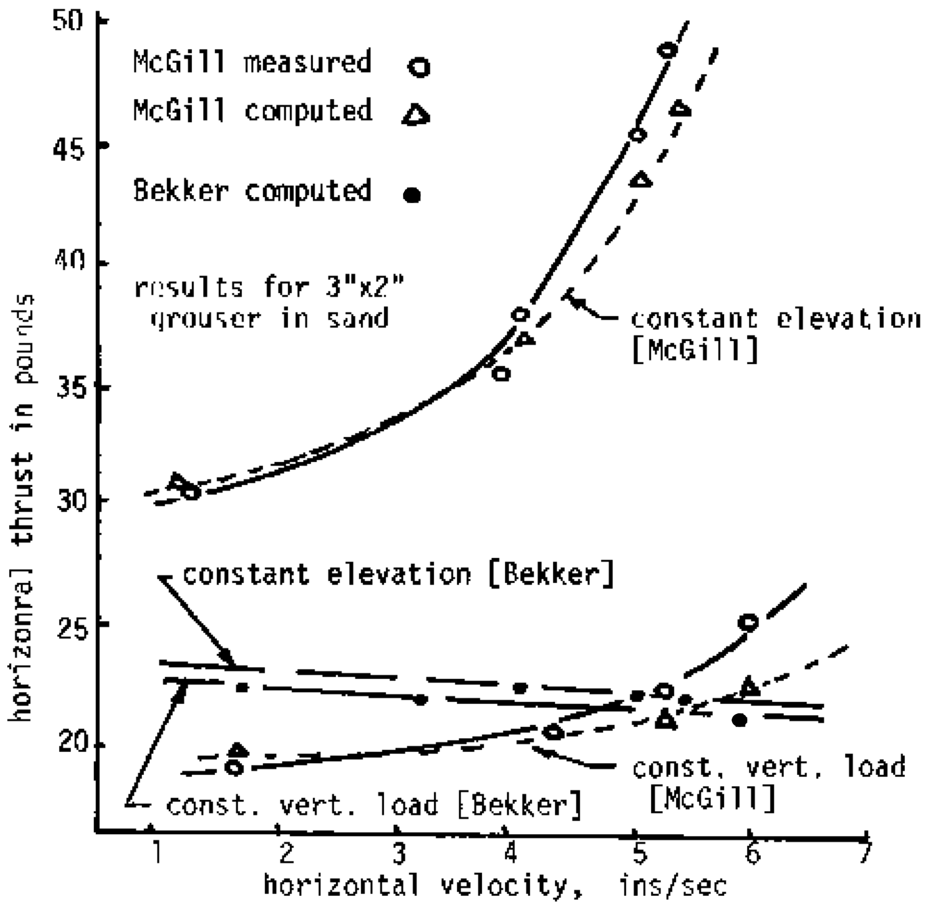

FIGURE 7 - PREDICTED AND MEASURED FORCES ON GROUSER

The dimensionless soil parameters shown in Equation (8) are in essence similar in format and philosophy to those generated for foundation stability and pressure sinkage relationships for wheels - as discussed previously. It is thus not surprising that the limit equilibrium analytical model should perform better since it has a closer appreciation of the actual physics of the grouser-interaction problem. This however does not deny the use of the inclined plate model [Equation (8)]. On the contrary, Equation (8) is useful insofar as simplicity and expedient usage are concerned.

Limit Equilibrium Model - Wheel

Much in common [in terms of philosophical formulation] with the grouser-soil interaction problem, the limit equilibrium model analytical approach to the wheel-soil interaction phenomenon treats the status of the bearing soil within the context defined by the boundary conditions. The specific constraints of this approach [Windisch and Yong (1969), Yong and Windisch (1970)] using the method of characteristics as the solution technique relate directly to:

1] Total limit equilibrium.

2] Insignificant volume change.

3] Definable boundary conditions.

The casting of analytical formulations is not unlike those developed for the grouser problem, - using the stress characteristic approach, assuming a direct relationship between stress and strain rate. By physically measuring subsoil deformation with time [as is possible in controlled soil bin tests] it is apparent that with appropriate constitutive relationships and associated flow laws, the instantaneous stress field beneath a moving wheel can be mapped. The forcing function at the boundary - i.e. wheel-soil interface can thus be obtained. Figure 3 shows the calculated pressure distribution obtained directly by strain-rate measurements in the soil [i.e. response function determination] and calculating the impulse function producing the observed strain-rates. The seemingly abrupt stress contour at the two ends is a deficiency of the analytical model - which treats the stress situation implicitly in terms of a discontinuity across the limit failure characteristic.A less abrupt contour appears to be a more realistic appreciation of stress or pressure at the interface.

Figure 3 demonstrates the possibility of tension values in soil being developed in view of unloading of the soil during passage of the wheel. As pointed out earlier, this phenomenon might not be directly detected by pressure gauges embedded in the wheel. The limitations and constraints of the analytical model are thus traceable directly to the requirements of limit equilibrium theory and the constraints associated with specification of the constitutive relationship for the subsoil.

Energy Model

Energy models for analysis and prediction of vehicle-soil interaction performance will rely on one's ability to measure [or determine] response function performance. As in the case of limit equilibrium models, the energy method utilizes forcing and response functions. In this instance, however, the two functions are fitted [multiplied] together to produce the work or energy function.

Unlike the limit equilibrium approach, the energy model does not require that limit or yield be necessarily reached in the stressed subsoil. This however demands that a proper and admissible constitutive relationship be defined. This requirement can be most demanding. Using a yield function formulation, Yong and Webb (1969) have applied the energy approach and have obtained typical results of parasitic energy as shown in Figure 2.

Finite Element Model

In recognizing the attractiveness of more realistic analytical modelling of the vehicle-soil interaction problem, tedious solution procedures have heretofore restricted implementation of what is essentially a complex boundary value problem. With the development of machine computational aids, growth in numerical solution techniques have allowed for development of more realistic modelling tools. In the finite element analysis of the wheel-soil problem for example, Yong et al. (1972) have treated the subsoil as a non-linear elastic strain hardening material subject to boundary conditions of an incremental return, thus permitting calculations to be made for description of the growth of stresses within the loaded soil from initial to final states - simulating precisely the load and unload characteristics of a moving wheel. Thus the transient loading problem of a wheel is now treated where machine computational techniques can be used with great advantage. The resultant transient stress field in the subsoil is solved as a series of equilibrium stress and displacement fields within the time span of interest.

It must be noted that whilst finite element analyses using the simple loaded plate analysis may be used, [e.g. Perumpral et al. (1971)] the inherent restrictive limitations associated with the plate analogy will prevail. Transient loading with slip development is the proper and necessary requirement. In applying the finite element solution technique, use of the principles of variational calculus ensures that minimization of the integral form of the governing principles would provide for the closest available solution to the problem at hand. The Lagrangian formulation for forces acting on the nodes of the described triangular elements [Figure 8] to obtain the required displacement for nodal equilibrium would be given as:

$$\{\Psi\} = \int_V \rho[S]^T\{q\}dv + \int_A [S]^T\{\bar{\rho}\}\frac{d\bar{A}}{dA}\,dA - \int_V [e]\;\{\sigma\}\quad dv \tag{9}$$

where

{ }	=	column vector
[ ]	=	matrix form
superscript T	=	transposed matrix
ρ	=	density
v	=	volume
A	=	area
superscript bar	=	deformed state
{q}	=	body forces
[S]	=	shape function defining displacement at any point within element with respect to displacement at nodal points.
[e]	=	strain displacement relation
σ	=	stress

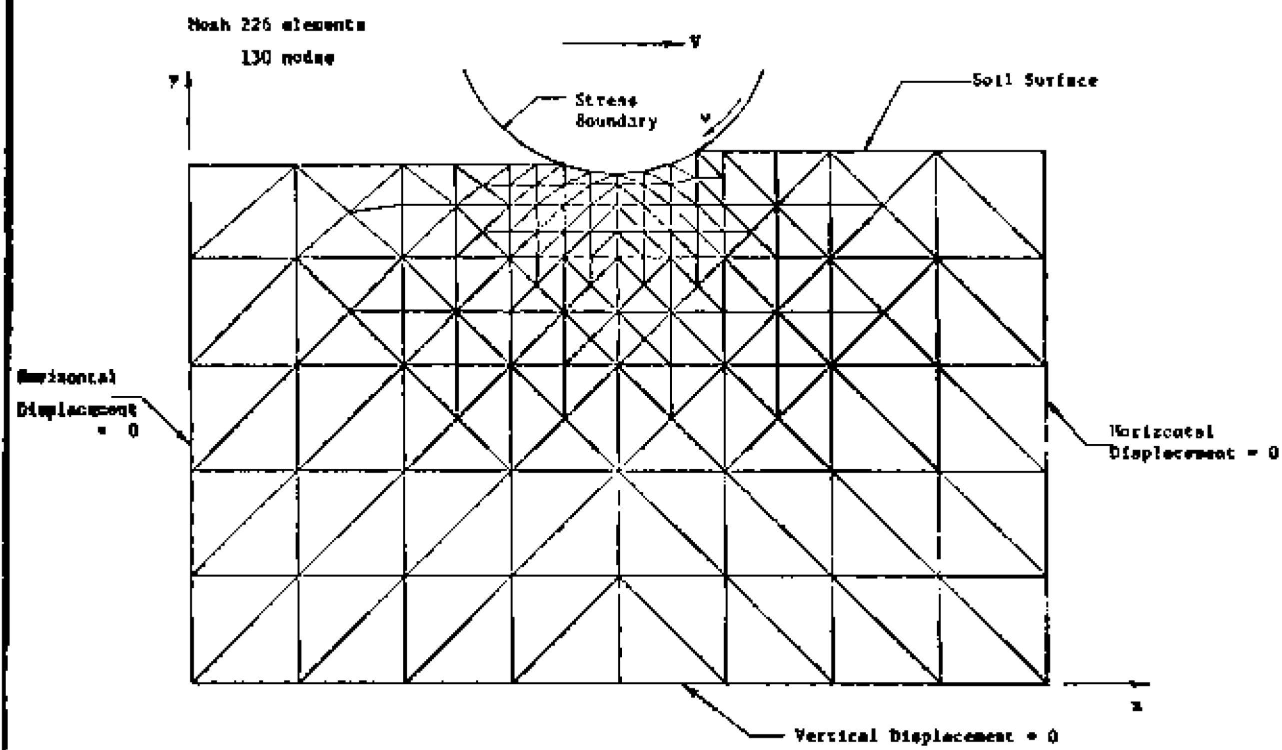

FIGURE 8 - FINITE-ELEMENT MESH FOR WHEEL-SOIL PROBLEM

Obviously, one of the key elements in Equation (9) is the strain displacement relation [e]. This is integral to the evaluation of the constitutive performance of the soil and its participation in the vehicle-soil interaction problem. Applying the appropriate variational technique, the final relationship is thus given as:

$$d\{\sigma\} = [K_T]\, d\,\{\delta\} \tag{10}$$

where

$[K_T]$ = tangential stiffness

$= ([K_o] + [K\ ] + [K_L])$

subscripts o, L and σ = small displacement, large displacement and stress level dependencies respectively.

δ = displacement

The boundary conditions applied to the system must realistically account for the entire load-unload sequence typified by the transient wheel problem. Problems of typifying interface performance characteristics can be approached in view of ideal characterization as solution techniques. The solution to the problem is presently being undertaken and will be reported at a later stage.

CONCLUSIONS

The need for a close similarity between physical and analytical models is apparent. In applying analytical modelling techniques, it is essential that proper appreciation of the constitutive performance of the material and boundary conditions be obtained and subsequently applied. The appropriate framework defined by using realistic similarities between physical and mathematical boundary conditions will ensure a higher order of predictability with the developed analytical model.

ACKNOWLEDGMENTS

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