

HEAT TRANSFER TO CYLINDERS IN A CONFINED

JET AT HIGH TEMPERATURE

BY

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A

THESIS

BY

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Submitted to the Faculty of Graduate Studies
and Research of McGill University in partial
fulfillment of the requirements for
the degree of Master of Engineering

McGill University

April 11, 1967

ACKNOWLEDGEMENTS

The author wishes to express his gratitude to those who have contributed both directly and indirectly to the completion of this project. He is especially grateful to :

Le Conseil des Arts du Canada for the award of a Fellowship in two consecutive years;

The Pulp and Paper Research Institute of Canada for its technical and financial support in this study as well as for the extensive use of equipment, machine shops, library and computer facilities;

Mr. R.A. Lindsay of the P.P.R.I.C. for his helpful assistance in the solution of his electronic-equipment problems.

Dr. G.R. Kubanek for his collaboration at the beginning of this work and for his valuable suggestions.

Mr. K. Satyanarayana for his assistance and valuable help during the course of the experimental work;

His wife for her patience, understanding and encouragement without which this work could not have been completed, as well as for her direct contribution in the typing of this thesis;

His parents for their tangible and intangible assistance.

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GENERAL INTRODUCTION

Interest in high-temperature chemical processing was stimulated about ten years ago when high temperature generating equipment, particularly plasma torch devices, became commercially available. High temperature chemical processing provides a new method of obtaining extremely rapid reaction rates (resulting in smaller equipment) and, in addition, enables the chemical engineer to achieve reactions which are only feasible at such energy levels. Because of the high energy input involved in these processes, maximum recovery is necessary from an economic standpoint.

In view of these considerations, it is of importance to know whether or not the empirical equations predicting the transfer of heat and mass at low or moderate temperatures can be extrapolated to conditions prevailing at higher temperature levels. In this context, Kubanek (1) constructed a reactor in which transport and chemical processes could be studied, using a direct-current plasma torch as the heat source. While his investigation was concerned principally with forced convection to stationary spheres, it was decided to extend his study to other blunt bodies. The present aims were first to obtain heat transfer data for different flow configurations

and second, to check if a particular characteristic length could be obtained to predict forced convection to various shaped bodies from a standard relationship. The first part of the thesis consists of a critical review of the literature pertaining to heat transfer to cylinders. The second part deals with the experimental aspects of the investigation. The latter section has been made complete in itself, in that it has a separate Introduction, Nomenclature and Bibliography. This makes it possible to publish this part of the thesis with little modification, while presenting the work in a clearer light and facilitating its interpretation.

SURVEY OF THE LITERATURE

INTRODUCTION

Kubanek, working in this laboratory, has already presented a comprehensive literature survey as part of his Ph.D. Thesis (1), in which most of the aspects of the field of plasma jets and their applications are reviewed, i.e. :

- Developments in high temperature techniques;
- Measurement techniques in gases at elevated temperature;
- Advances in plasma jet technology;
- Properties of confined jets.

Accordingly, the present literature review has been restricted to the specific topic of heat transfer to cylinders in cross-flow. Even so, this restriction has left considerable ground to cover : for example, Morkovin (2) reported 155 references to studies concerned with the flow of fluid in the wake region of a cylinder alone. It was, therefore, deemed beyond the scope of the present study to review all the papers published in this area and an attempt has been made to report only those which are directly pertinent to the topic of this thesis. Emphasis has been given to the various problems arising from the seemingly simple flow and heat transfer situations prevailing around a cylinder in the light of the limited knowledge of the mechanisms involved.

I. ANALYTICAL PREDICTION OF LOCAL HEAT
TRANSFER BY FORCED CONVECTION AROUND A
TWO - DIMENSIONAL BODY

INTRODUCTION

The differential equations which govern the flow of a fluid around a two-dimensional body are well known. Attempts to solve them have been made with the objective of predicting the local heat transfer rates from the characteristics of the flow field. Unfortunately, two major obstacles have limited these attempts : the transfer mechanism in the wake region is not known and secondly, the characteristics of the flow at any point in the fluid are difficult to predict accurately.

It should be noted, however, that in the front part of a cylinder immersed in a flowing fluid, where the transfer of the momentum and heat are controlled through very thin layers, several authors (3-11), and more recently Perkins and Leppert (12), have quite successfully applied the boundary layer theory to the prediction of local heat transfer in this region.

ANALYTICAL PREDICTIONS IN THE LAMINAR BOUNDARY LAYER REGION

At any time and any point in a fluid, the momentum, continuity and energy equations must be satisfied. Whereas in the

wake region the characteristics of the fluid are strongly time-dependent due to the turbulent nature of the flow, in the boundary layer region of a cylinder a simpler approach to the prediction of heat transfer is possible, due to the steadiness of the flow and to a better understanding of its characteristics.

The main assumptions usually made in the development of the analytical predictions are as follows :

- The Reynolds number is large enough, such that boundary layer approximations apply ($Re^* > 1$) ;
- The fluid is incompressible ($M < 1$);
- The temperature difference between the main stream and the surface is small enough so that the physical properties of the fluid are constant throughout the boundary layer;
- Contribution of natural convection is neglected ($Gr/Re^2 \ll 1$);
- The heat due to friction dissipation is negligible with respect to convective heat transfer ($Ec \ll 1$);
- The density of the fluid is high enough so that it may be treated as a continuum ($Kn \ll 1$);
- The flow outside the boundary layer is inviscid and irrotational;

* all symbols and abbreviations are defined in the Nomenclature at the end of the literature review.

- The flow upstream of the cylinder is uniform;
- The boundary layer is laminar and no effect of free-stream turbulence is considered;
- The effect of curvature of the wall is ignored so that δ/R is assumed to be very small;
- The surface temperature or the heat flux is uniform around the cylinder.

Under these conditions, the velocity and temperature profiles in the boundary layer are obtained from the simultaneous solutions of the continuity, momentum and energy equations :

- Momentum,

$$u(\partial u / \partial x) + v(\partial u / \partial y) = U_{\max} (\partial U_{\max} / \partial x) + \nu (\partial^2 u / \partial y^2) \quad (1)$$

- Continuity , $(\partial u / \partial x) + (\partial v / \partial y) = 0 \quad (2)$

- Energy, $u(\partial T / \partial x) + v(\partial T / \partial y) = (k / \rho C_p) (\partial^2 T / \partial y^2) \quad (3)$

From these equations it can be noted that under the restrictions mentioned (small temperature difference and buoyancy forces negligible) the velocity field does not depend upon the temperature field but the converse is not true, and secondly the temperature equation is linear, unlike the equation of motion. This leads to considerable simplifications in the process of integrating, and superposition of known solutions becomes possible.

The momentum and continuity equations are first solved

Ref. (13) gives some of the methods employed. Suffice it to say that a velocity distribution through the boundary layer is assumed and the Von Kármán integral equation used. Moreover, the variation of U_{\max} with x is very often assumed to be given by potential flow theory if the flow upstream of the cylinder is uniform ($U_{\max} = 2U_{\infty} \sin \theta$). Hence the energy equation is solved and the temperature profile obtained.

Because at the surface heat is only transferred by molecular conduction, the heat transfer equation must satisfy the following :

$$dq = -k(\partial T / \partial y)_{y=0} \quad (4)$$

$$= h(T_{\infty} - T_w) \quad (5)$$

and therefore :

$$h = -k(\partial T / \partial y)_{y=0} / (T_{\infty} - T_w) \quad (6)$$

$$Nu = hD/k = -(\partial T / \partial y)_{y=0} / (T_{\infty} - T_w) / D \quad (7)$$

As already mentioned a number of analytical solutions (3-11) have been presented. Chapman and Rubesin (3) worked out a solution in the case of zero pressure gradient in the direction of flow and with arbitrary surface temperature. Seban (7) predicted local heat transfer for an arbitrary free-stream velocity and surface temperature distribution. Klein and Tribus (6) took into account some additional effects due to surface discontinuity, such as strips and wires. Levy and Seban (5) and Levy (4) solved for uniform surface temperature,

whereas, Lighthill (9), Seban and Chan (8) solved for the constant heat flux case. Perkins and Leppert (12) applied to a cylinder the method developed by Brown et al. (14) for sphere, assuming a 4th order polynomial thermal and velocity profiles across the boundary layer. In this last publication, a comparison of the authors' experimental values of local heat transfer coefficients with the predictions of various authors (8, 9, 11) is given.

It is interesting to note that, first, although these predictions have been set up for different temperatures around the cylinder, all have the same shape and lie within 4% of one another, and secondly, departure of 30% between the experimental values and the predicted ones can be explained by turbulence effects in the experiment, which are not considered in the predictions. It can be concluded, therefore, that in spite of the complexity of the problem and of the assumptions made, the agreement between predicted values of the local heat transfer coefficients and the measured ones is surprisingly good, as long as no turbulence effects are introduced.

Recently, Libby et al. (15) proposed a two-layer model to accommodate the law of the wall and the wake-like characteristic of the outer layer to the case of heat transfer through a turbulent incompressible boundary layer in presence of a free stream pressure gradient. So far, this model has been successfully applied to flat plates only, but it is hoped that it will

be extended to the two-dimensional cylinder case.

PREDICTION OF HEAT TRANSFER FROM ANALOGIES BETWEEN MOMENTUM AND HEAT TRANSFER

Because the transfer of momentum and heat are analogous processes, relationships between them have been developed for the purpose of utilizing friction data in the prediction of heattransfer rates. The so-called "Reynolds Analogy", which holds for the case where the thermal and velocity boundary layers have the same thickness, i.e. for $Pr = 1$, usually refers to the equation :

$$St = h/C_p G = c_f/2 \quad (8)$$

More complicated expressions have been developed to include boundary layer effects and there now exist various other so-called analogies which try to escape the limitation of the Prandtl number. The best known are : the Prandtl, Von Kármán, Colburn, Martinelli, Seban and Shimazaki, and Lyons analogies.

Usually, these analogies have been successful in the case of flow parallel to a plane surface or flow inside a tube but they apply rather badly to cylinders in cross-flow (13, 16, 17). However, Purves and Brodkey (18) found a reasonable agreement between the Colburn analogy and the McAdams correlation for Reynolds numbers less than 100. Similarly, Davies (19), and Davies and Fisher (20) developed an analytical correlation for heat transfer from small electrically-heated wires founded

upon the analogy between momentum and heat transfer. The results, obtained in an uniform air stream, covered flow Mach numbers up to 2, with densities from atmospheric down to free molecular flow conditions and with surface temperatures up to 1470°F. ; they agreed reasonably well with the theoretical predictions at Reynolds numbers up to 1000.

For cylinders in cross-flow, any form of analogy breaks down as soon as turbulence effects take place. Giedt (21) proved that for both the laminar and turbulent boundary layer regions, the intensity of the free stream turbulence had a different effect on the heat transfer and the skin friction, the former being much more affected by it than the skin friction, and therefore, no simple relationship could be derived. Kestin (22) stressed that the Reynolds analogy was a limiting law in the case of no turbulence intensity effect.

Silver (23) pointed out that, in the case of turbulent flow, a more systematic study of the Reynolds flux would improve the understanding of the analogy, because it is the first parameter from which \underline{h} and \underline{c}_f depend. Moreover the use of the Reynolds flux is completely consistent with existing qualitative ideas of turbulence and could be predicted by some statistical treatment of the latter. It is worthy of mention that Chi and Spalding (17) have successfully established a relationship between the Stanton number and the skin friction coefficient in the case of the heat transfer to a flat plate

through a turbulent boundary layer in air. Hence, it can be concluded that so far, analogies between momentum and heat transfer do not provide accurate methods to predict heat transfer rates from skin friction data, except in the low Reynolds number range.

SIMILARITY OF TWO NON-ISOTHERMAL SYSTEMS

Two non-isothermal systems will have similar solutions if both the conditions of dynamic similarity and similarity of the variations with temperature of the physical properties of the fluid are satisfied.

The condition of dynamic similarity requires identity of the parameters which are involved in the dimensionless momentum and energy equations, namely : Re, Pr, Ec, Gr. In the case of forced convection, the heat transfer coefficient in the Nusselt form may be expected to be a function of Re, Pr, Ec, if natural convection is neglected ($Gr / Re^2 \ll 1$). If in addition the heat dissipated by friction is also negligible compared with the forced convective heat transfer, the Eckert number is also ignored. Therefore, two non-isothermal systems will have dynamically-similar solutions for pure convective heat transfer if Pr and Re are the same and hence :

$$Nu = f (Re, Pr) \quad (9)$$

Regarding the second condition of similarity, it seems interesting to quote the remark made by Douglas and Churchill (24).

"Jakob pointed out that for complete dynamic similarity between two non-isothermal systems, the same ratios must exist between the significant physical properties at geometrically equivalent points. A lack of similarity will necessarily exist for any two fluids whose properties do not vary identically with temperature. A liquid and a gas fail in this respect, but two gases provide reasonable similarity. For all gases, the variations of μ , k , and ρ with temperature are quite similar".

The expression suggested by Jakob to account for the variations of properties with temperature is as follows :

$$(\mu_2/\mu_1) = (k_2/k_1) = (\rho_2/\rho_1)^{-e} = (T_2/T_1)^e = (C_{p_2}/C_{p_1})^j \quad (10)$$

II. FLOW AND HEAT TRANSFER AROUND A CYLINDER IMMERSED IN A STEADY TURBULENCE -FREE STREAM

This chapter presents a qualitative outline of the flow and of the local heat transfer distribution around a cylinder when the main flow can be considered as steady and uniform. No attempt will be made here to take into account such effects as free-stream turbulence or large temperature gradients on the local flow pattern or heat transfer, these effects being discussed later.

FLOW AROUND A CYLINDER

At very low Re , ($Re < 1$), the relative motion between a blunt body and a fluid is governed by viscous forces. A laminar boundary layer starts from the front stagnation point and thickens as it proceeds further downstream. Past the equator the unfavorable pressure gradient has negligible effects and no separation of the layer occurs. Outside the layer, the flow can be considered as inviscid and is well described by the potential flow theory.

As Re increases ($5 < Re < 40$) the combined effects of the viscous forces and the unfavorable pressure gradient in the rear half of the cylinder slow down the particles in the layer to such an extent that they are stopped and a reverse flow is

initiated, forcing the boundary layer to separate from the surface. Beyond the separation point, symmetrical and stable fixed vortices are bounded by steady streamline surfaces.

In the range $50 < Re < 140$, the laminar wake becomes unstable and starts to oscillate laterally. Because of this instability, the vortices break away alternately forming the so-called vortex street. The vortex motion is laminar and persists for a long time before being dissipated through viscous diffusion. The shedding process is characterized by the frequency at which vortices are carried away and is usually defined in dimensionless form by the Strouhal number Sr . In this range of Re , Sr increases with Re . Moreover, because of the increasing effectiveness of the positive pressure gradient at the rear, the transition point moves forward, increasing the width of the wake. For the region $140 < Re < 300$, the detached boundary layer originated from each separation point becomes turbulent before rolling up into vortices. The point where transition to turbulence in the vortex street takes place depends on Re . The vortices formed in this range soon diffuse as they move downstream, making it difficult to determine the shedding frequency.

In the subcritical region ($300 < Re < 150,000$) a definite shedding frequency returns which is surprisingly constant over the whole range of Re . According to Roshko (25), the wake becomes fully turbulent about 50 diameters downstream of the cylinder, all traces of upstream periodicity having disap-

peared. As Re is increased, the transition to turbulence in the free vortex layer takes place closer and closer to the cylinder. In the front part of the cylinder, the separation point moves upstream. However, the experiments by Fage and Falkner (26) and Schmidt and Wenner (27) showed that for Re above 8,000 the maximum negative pressure normal to the surface of the cylinder occurred at about 70° from the stagnation point and that all separation points were between 80 and 85° .

At the critical Re the laminar boundary layer becomes turbulent. Transition to turbulence causes the separation point to move downstream because of the ability of turbulent flow to overcome much larger adverse pressure gradients. This can be explained by the increase in the lateral momentum transfer due to turbulent mixing and therefore a particle in the turbulent boundary layer entering a region of rising pressure is more able to draw energy from the external flow and to preserve its forward motion for a greater distance. This shift downstream of the separation point restricts the wake width. Besides, the pressure distribution becomes more like that of an inviscid flow and this explains the marked decrease in the drag coefficient at the critical Reynolds number. Moreover, the point of maximum negative normal pressure moves further downstream (21, 26).

At the critical point, the shedding frequency develops a marked change in behaviour, increasing abruptly. In the super-

critical range, no specific change in the flow is observed, except that the transition point of the attached boundary layer moves upstream when Re increases.

HEAT TRANSFER DISTRIBUTION AROUND A CYLINDER

In the low range ($Re < 1,000$) the main measurements of local heat transfer around a cylinder have been carried out by Eckert and Soehngen (28). Their plots of Nu versus θ at three values of the Reynolds number ($Re = 23, 224, 597$) show the heat transfer coefficient to decrease from the front stagnation point to a minimum at θ between 120 and 130° (which corresponds to the separation point) and then to rise steadily up to the rear stagnation point where it is always smaller than at the front. In this range, Nu in the attached boundary layer increases much more rapidly with increasing Re than in the wake, and therefore, the contribution of the wake is small compared to that of the front.

Extensive experimental studies of the local heat transfer (21, 27, 29 to 31) have covered the subcritical range ($500 < Re < 150,000$). Although, the results of these studies may differ quantitatively, all of them agree on the V-shape of the heat transfer distribution. As Re is increased, the wake contribution becomes more and more important. Schmidt and Wenner (27) pointed out that the heat transfer at the front and rear stagnation points has the same value at $Re = 25,000$ and at $Re = 54,000$ the wake accounts for 50% of the total heat transfer.

In the critical Re region (150,000-300,000), the local heat transfer undergoes a marked change. The heat transfer distribution (21, 26, 27, 32, 33) similarly to the skin friction distribution, displays two minima between the two stagnation points, one at about 95° and the other at 150° , while a maximum between them occurs around 115° . Several conflicting opinions have been advanced regarding the interpretation of these curves. Fage and Falkner (26) postulated that the first minimum corresponded to a transition from a laminar to a turbulent boundary layer and no separation would occur before the maximum ($\theta = 118^\circ$). Schmidt and Wenner (27) agreed with Fage and Falkner, particularly when they noted that their lowest local heat transfer coefficient in the critical region occurred at about the same position as the minimum in Fage and Falkner's frictional intensity curves and a similar correspondence occurred for the maximum at $\theta = 115^\circ$.

Another explanation has been developed by Giedt (21), Zapp (32) and Seban (33). Giedt noted the coincidence of the location of the first minimum in the heat transfer coefficient and the zero value of the skin friction coefficient and suggested that a laminar separation would occur, rather than a transition in the nature of the boundary layer. Zapp (32) basing himself upon Linke's (34) and Schubauer's experiments (35) which showed that a laminar boundary layer can exist as a free laminar layer following separation and that, as the free layer

thickens, transition to turbulent flow can occur, concluded that the first minimum, the new maximum and the second minimum in the heat transfer curves could quite conceivably be caused by the separation of a laminar boundary layer, the re-attachment of a turbulent boundary layer and its subsequent separation, respectively. Thus a laminar "separation bubble", as encountered on airfoils, may exist between the first separation point and the point of re-attachment. This view was supported later by Seban (33) and Wadsworth (36). Knudsen and Katz (13), on the other hand, believe that the first minimum is due to transition and no separation occurs before the second minimum. Kestin (37) supports the same view and adds : "further increase in the Re has little effect on the point of separation of the turbulent boundary layer but causes the point of transition to move upstream".

For supercritical Re the experiments of Schmidt and Wenner (27) at $Re = 426,000$ showed that the heat transfer distribution displays the same behaviour as in the critical regime. It is interesting to note however, that the value of the second maximum ($\theta = 115^\circ$) is twice as high as the value at either stagnation point.

III. EFFECTS OF VARIOUS FACTORS ON HEAT TRANSFER

EFFECT OF TURBULENCE ON HEAT TRANSFER

It has long been known that turbulence increases the rate of heat transfer. In early investigations, Reiher (38), and Griffiths and Awberry (39) noted increases of 50 and 100%, respectively, by placing a turbulence generator upstream of the test section. However, it seems that the first quantitative investigation was carried out by Comings et al. (40) in 1948. Since then, other contributions have provided additional data, but they are usually not consistent quantitatively and no theory exists at present to predict accurately the effect of turbulence on heat transfer.

This inconsistency between the experimental results has been very logically explained by Kestin and Maeder (22) who pointed out the difficulty of obtaining similarity between two turbulent systems : "In the elementary derivation of the laws of similarity which applied in forced convection, the external flow is always described by specifying only U_∞ . This constitutes an adequate description in cases when the external flow is laminar, i.e. $I = 0$. However, when the flow is turbulent, the laws of similarity imply, in addition, a similarity in the ran-

dom fluctuations in the stream. Present day experimental evidence seems to show that an adequate degree of similarity is achieved when the intensity of turbulence I and the scale of turbulence L are fixed in value".

As pointed out by the authors, even this description of the turbulent free-stream disregards the frequency of the random fluctuations and Kestin (37) recently suggested that the frequency spectrum should be described as well. Beside this lack of similarity, no standard method to measure I and L has been defined. Some researchers measure I outside the boundary layer with the model in place (frequently at some unspecified distance upstream of the cylinder), others measure it in the empty test section. Disagreement between the different studies can also be caused by secondary effects such as blockage or large temperature gradients.

Whatever inconsistencies may exist, qualitative, and in certain cases semi-quantitative, arguments may be obtained which will be discussed in turn.

A) Influence on Local Heat Transfer

The measurements of local heat transfer coefficients around a cylinder in presence of a turbulent free-stream have been performed by Giedt (21), Zapp (32), Perkins and Leppert (12), Seban (33) and Kestin, Maeder and Sogin (41). Sogin and Subramanian (42), carried out experiments of mass transfer.

Giedt (21) studied the effect of I on both the skin friction coefficient and the local heat transfer coefficient below and above the critical Re . He found that I will force the transition to turbulence to take place at much lower critical Reynolds number. The magnitude of the skin friction coefficient was not appreciably affected by I whereas the local heat transfer exhibited a marked increase, primarily in the boundary layer region. For instance, when I was increased from 1 to 4%, at comparable values of the main free-stream velocity, the heat transfer coefficients at the front stagnation point were approximately 25% higher and the ratios of heat transfer at the front and rear halves were increased from 0.85 to 1.1, respectively. Zapp (32) measured the local Nu for three values of Re (39,000, 71,500 and 110,000) at three levels of turbulence (3, 9 and 11.5%) and his results agree with the previous ones.

Kestin and Maeder (22) wanted to verify the assumption that I not only promotes an earlier transition to turbulence in the laminar boundary layer but has an effect on the local heat transfer as well. Because only the average heat transfer coefficient was measured, the transition point had to be fixed on the cylinder by means of a tripping wire located at 60° from the oncoming stream, so that some conclusions could be drawn. The turbulent boundary layer separation point being unaffected by I , the surface of the cylinder was then divided into three definite regions at a given Re . Their results in the critical region $140,000 < Re < 240,000$ fully confirmed their hypothesis but it was impos-

sible to discern which of the three regions, the laminar boundary layer, the turbulent boundary layer or the wake, was affected the most by the turbulence. Subsequent measurements (41) of the local heat transfer in the same flow conditions showed that the overall increase was mainly due to that of the laminar boundary layer region. In this Re range, Nu at the stagnation point was increased by 80% when I varied from 0 to 2.6%.

Seban's conclusions (33) agree with those of Giedt, Zapp, Kestin et al., and may be summarized as follow :

- The heat transfer coefficient in the laminar boundary layer increases with a maximum gain at the point of largest pressure gradient and a minimum at the smallest (hence, he suggested that the effect of I could be strongly dependent on the pressure gradient.);
- The critical Reynolds number is reduced to lower values as I is increased;
- The intensity of turbulence seems to have no appreciable effect on the heat transfer in the turbulent boundary layer region;
- The intensity of turbulence affects the character of the separated flow.

This last point has been discussed by Richardson (43) who mentioned however that the wake is much less affected than the laminar boundary layer.

Sogin and Subramanian (42) studied the local mass transfer

from cylinders in cross-flow in the critical region. Mass transfer rates were increased by 50% in the boundary layer region when I was changed from 0.8 to 2.4%. Besides, their mass transfer distribution clearly demonstrates that, at least in the boundary layer region, an almost complete correspondence exists between the transfer of mass and that of heat.

B) Effect of Intensity of Turbulence on the Average Heat Transfer Coefficient

The more important investigations dealing with the average heat transfer coefficient have been performed by Comings, Clapp and Taylor (40), Maisel and Sherwood (44), Van der Hegge Zijnen (45) and Kestin and Maeder (22, 41, 46, 47).

Comings et al. (40) worked with cylinders in the lower range of Re (from 400 to 20,000) and varied the turbulence level widely (from 1.8 to 22%). At $Re = 5,800$, Nu was increased by 25% when I varied from 1.8 to 7%, while a further increase in the turbulence level up to 22% resulted in another increase of only 5%. This tendency of Nu towards a limiting value as I is increased seems to indicate that at a given Re , I is more effective in the low range of turbulence. Moreover, the higher the Re , the more effective is I , the second having almost no effect at low Re ($Re < 500$).

Maisel and Sherwood (44) however, did not confirm the type of relationship obtained by Comings, Clapp and Taylor. Whereas the latter found a definite flattening of the curve Nu versus I

for high values of \underline{I} , Maisel and Sherwood obtained a reverse curvature in the case of spheres.

Van der Hegge Zijnen (45) seems to be the only one who made a systematic investigation of the combined influence of the scale of turbulence, \underline{L} , and of the intensity of turbulence, \underline{I} , on the transfer of heat from a cylinder in cross-flow. In his extensive measurements, the scale of turbulence varied from values much greater than, to values comparable to the cylinder diameter ($0.3 < L/D < 240$). The range of \underline{Re} covered extends from 60 to 25,800 and that of the turbulence intensity, from 0 to 14%. His conclusions from his experimental results for \underline{Nu}^* , defined as the ratio of $(\underline{Nu})_{\text{turb}}$ over $(\underline{Nu})_{\text{lam}}$ are :

- 1) At constant value of the ratio $\underline{L/D}$, \underline{Nu}^* increases with $(\underline{Re} \times \underline{I})$, the rate of increase being highest for low values of the product; when $(\underline{Re} \times \underline{I}) > 100$ the rate of increase is constant.
- 2) At constant value of the product $(\underline{Re} \times \underline{I})$, heat transfer either increases or decreases with increasing scale ratio, the maximum being reached when $(\underline{L/D})$ is about 1.5 to 1.6.
- 3) Any variation in \underline{I} or \underline{L} is more effective at higher \underline{Re} .

Moreover, he noted that :

- 4) In the range $60 < \underline{Re} < 600$, no turbulence effect on heat transfer was found up to a turbulence level of 13%.

- 5) The optimum value of $(L/D) = 1.6$, may correspond to a condition of resonance between free-stream fluctuations and eddy shedding. This resonance would reinforce the oscillation of the eddies in the wake and cause the observed maximum in Nu^* . This hypothesis has been supported by Fand and Cheng (48).

The author assumed that the following equation represents well the turbulence and scale effects on heat transfer :

$$Nu^* = [1 + \varphi(Re \times 1) \psi(L/D)] \quad (11)$$

where φ and ψ are two empirical functions.

Although this investigation was carried out carefully, some results are quite inconclusive : at $Re = 10,000$, Van der Hegge Zijnen's curves predict that, for (L/D) less than 1.6, increasing the scale would increase the effectiveness of the turbulence, whereas Taylor's theory predicts the opposite and Dryden's data seem to confirm Taylor's point of view. Besides the scale of turbulence has an effect even at (L/D) exceeding ten.

Kestin and Maeder (22) carried out experiments at high Re ($140,000 < Re < 300,000$). Their results are presented in the form of curves of Nu , at constant value of I (varying from 0.8 to 2.5%), which indicate that I is much more effective at low values of Re in this range. An attempt to correlate the variation of Nu at constant Re and Pr with Taylor's parameter did not lead to a useful result.

Kestin and co-workers (47, 49) studied the combined effects of the pressure gradient and \underline{I} upon the heat transfer to a flat plate. In the case of zero pressure gradient (47) no effect of \underline{I} was detected in either the laminar or turbulent boundary layer. \underline{I} simply forces transition to take place earlier.

In the case of a moderate pressure gradient (49) a slight increase in free stream turbulence was found to increase heat transfer through a laminar boundary layer, but nothing happened in the case of a turbulent boundary layer.

In spite of the failure to find a correlation, it is possible to summarize the results of the investigations reported above as follows :

- Free-stream turbulence may affect heat (and mass) transfer rates both locally and through flow configuration.
- The local effect is much greater on a laminar boundary layer than on a turbulent boundary layer or the wake.
- Since the skin and drag coefficient are much less affected by turbulence than the heat transfer coefficient, it seems that the temperature profile in the thermal boundary layer is much more sensitive to variations of \underline{I} than the velocity profile.

C) Effect of Turbulence on Transition

This topic being of particular interest to the experimental part of this thesis, it will be discussed in some details.

The first theory dealing with instability of the laminar boundary layer is the well known Tollmien-Schlichting theory. This theory correctly predicts the growth or decay of small disturbances in the boundary layer but cannot predict where transition will occur. Moreover, it applies only at low levels of turbulence intensity ($I < 0.2\%$). For higher I , Taylor (50) and Wieghardt (51) postulated that pressure fluctuations coupled with I are mainly responsible for transition and they established that the (Re) crit. depends on the single parameter $\Lambda = I (D/L)^{1/5}$ usually called Taylor's parameter. Dryden and Schubauer (52) successfully applied this parameter, which reduced the scatter in their data in the transition zone. However, it cannot be applied to commonly occurring non-isotropic flow and its validity at higher I levels and lower Re cannot be assumed.

Torobin and Gauvin (53 to 58) in their extensive review of the fundamental aspects of solid-gas flow, fully discussed the three causative factors which are mainly responsible for transition namely, the pressure gradient, the roughness of the surface and the disturbance in the free-stream. It is recognized to-day that a favorable pressure gradient will postpone transition whereas an unfavorable one will promote it.

In their experimental study on freely-moving spheres at low Re (400 - 3,000) in turbulent air stream (the relative intensity of turbulence in the case of a freely-moving part-

icle can reach values as high as 40%) no particular diameter effect on the transition phenomenon was detected. Besides, all the results at $I > 1\%$ confirmed that the magnitude of the turbulent disturbance rather than its frequency was the first causative factor for transition. They proposed that transition will occur if the ratio E_T/E_V (E_T = turbulent energy of the oncoming fluid; E_V = viscous damping energy) reaches a critical value which allows sufficient penetration of the free-stream disturbances into the laminar boundary layer in order to promote turbulence in it. In mathematical terms, this criterion reduces to :

$$I^2 (Re)_{crit.} = \text{constant} \quad (12)$$

Their experimental data on spheres agreed with this correlation quite well, which seems to confirm that, at least for high turbulence levels, the scale of turbulence is a minor parameter. This small degree of dependence is corroborated by the fact that a power of only $1/5$ was taken to account for scale effects in the Taylor parameter.

Van Driest and Blumer (59) formulated a criterion for the transition of the boundary layer, by considering that the ratio of the local inertial stress to the local viscous stress must reach a limiting value somewhere in the flow. With that assumption they derived a transition equation given as :

$$1,690/(Re)_{x,t} = 0.321/(m+0.11)^{0.528} + 0.73 \eta^2 I^2 (Re)_{x,t}^{0.5} \quad (13)$$

where subscript x refers to the distance from the stagnation point where transition t to turbulence takes place; m is defined by $U_{\max} \propto x^m$ where U_{\max} is the velocity at the outer edge of the boundary layer; and η is a given function of m only. This equation fits flat plate ($m = 0$) transition data well.

Recently, Talmor (60) reported a heat transfer investigation to throat tubes in high-temperature transonic cross-flow with combustion-induced turbulence which indicates that the boundary layer around small (61) and large (62) diameter throat tubes is turbulent essentially from the stagnation point or very close to it ($\theta = 10^\circ$). Then, because he was not able to measure I directly, Talmor solved Van Driest and Blumer's equation for I and found that transition took place at the observed points on the cylinder for values of I ranging from 15 to 20%. These values being quite reasonable, it seems that the method could provide an indirect method of measuring I .

Recently, Kozlov (63), while reassessing some of Taylor's assumptions, derived the following relationship for a flat plate :

$$(Re)_{crit.} = (Re)_m + C(12 + \lambda')^{1/2} / I^{5/4} \quad (14)$$

where λ' is a parameter depending on the pressure gradient and $(Re)_m$ is the minimum Re below which no transition could take place. Tetervin (64) predicted a minimum value of 44 whereas Van Driest (65) suggested 268.

As already discussed, knowledge of the flow pattern about

cylinders at the point or region where transition occurs is still incomplete.

In addition to the contradictory interpretations of the local heat transfer curves around a body, the specific studies, dealing with transition do not shed more light. Wadsworth (36) assumes momentary separation of the laminar boundary layer to be a necessary condition, whereas Kramer (66) refers to experiments in which transition occurred without separation. Taylor, Wieghardt, Van Driest and Blumer and others workers assume the pressure fluctuations and the free-stream turbulence to cause separation, whereas Sternberg (67) disagrees with the above assumptions and supposes that turbulence acts on the laminar boundary layer by creating large Reynolds shear stresses in it.

It can be concluded that the laminar boundary layer reacts to free-stream fluctuations in a very complicated manner and that, at present, no satisfactory theory is available to explain and predict the mechanism of transition.

EFFECT OF LARGE TEMPERATURE DIFFERENCES

Large temperature differences pose two main problems : how they affect the value of h , and how to take into account the resulting large variations in physical properties. As discussed by Audiutori (68), the so-called "Newton's law of cooling" is a very useful concept if, and only if :

$$dh / d (\Delta T) = 0 \quad (15)$$

It seems that equation (15) is clearly satisfied for moderate ΔT , but at higher values this is not so. Hilpert (69) found that h was about 6% higher for $\Delta T = 1,000^{\circ}\text{C}$. than for $\Delta T = 100^{\circ}\text{C}$.

McEligot et al. (70) observed that large temperature differences between a gas heated (while in turbulent flow through a tube) and the wall reduced both the heat transfer and the skin friction in comparison with small ΔT behaviour at the same Re , whereas Peturkhov et al. (71) came to the opposite conclusion for heat transfer.

Churchill and Brier (72) showed that existing correlations based upon small ΔT do not adequately represent data for large ΔT because of the lack of similarity in the variation of physical properties of fluids and the impossibility of finding a proper temperature basis.

However, in many gas studies involving a large ΔT , satisfactory representation of the data for both overall and local heat transfer rates was found possible by the introduction of the parameter (T_b/T_w) . For liquids, a better representation is achieved by a viscosity ratio factor with an appropriate exponent. The choice of the appropriate temperature for the evaluation of physical properties in the case of large temperature differences will be discussed later.

Eckert (73, 74) has shown that when the temperature varia-

tion through the boundary layer is such that C_p varies considerably, the temperature difference should be replaced by the enthalpy difference. This is particularly important under conditions in which thermal equilibrium does not exist and a single temperature cannot properly describe the state of the gas.

EFFECT OF COOLING OR HEATING ON HEAT TRANSFER

Schlichting (75) showed from the boundary layer equation that the heating or cooling of a body should have some effect on the stability of the boundary layer. For cooling a gas, the boundary layer should be destabilized because the viscosity increases with temperature. Perkins and Leppert (12) while studying the pressure distribution around a cylinder when the latter cools or heats a fluid, did not find significant differences over the laminar boundary layer region. However, their data seem to indicate, in agreement with Schlichting's suggestion, that cooling or heating may affect both the pressure distribution and the transition point when Re is near $(Re)_{crit.}$

Smith and Kaups (76) found, in their study on laminar boundary layers in water, that heating the wall had a strong, favorable effect on the boundary layer stability. In adverse pressure gradients, a heated wall appeared to delay separation.

It has been observed experimentally that cooling and

heating data fall on two different lines. Douglas and Churchill (24) however, achieved a reasonable agreement when they replotted all the data for heat transfer to cylinders in cross-flow on the basis of the film temperature.

Nevertheless, several authors (72) (77) recommend the use of two different lines, this being more consistent with the principle of similarity of two nonisothermal systems.

EFFECT OF BODY SHAPE ON HEAT TRANSFER

According to the principle of hydrodynamic similarity, it is obvious that the shape and the orientation of the body with respect to flow constitute the most important parameters which affect the flow and the heat transfer. Therefore, one could expect as many relationships for heat transfer as there exist different shapes of particles in different orientations. However, some attempts have been made to find a standard characteristic length which could allow prediction of heat transfer for any shape of particle from a standard heat transfer equation. Although the problem is probably too complex for this simplified approach, some of these attempts have succeeded reasonably well.

In 1953, Krischer (78) defined a "flow length" L_k , as the length of the path covered by the streaming medium along the surface. Subsequent investigations on heat and mass transfer (79, 80) on fourteen shapes showed that L_k may be considered as useful for two dimensional shapes only.

Polonskaya and Mel'nikova (81) suggested that $A^{0.5}$ should be used instead (A is the surface area).

Pasternak and Gauvin (82) studied the convective heat and mass transfer for twenty shapes suspended in various orientations in a hot turbulent air stream at moderate Re (400 - 8,000). They derived a characteristic length L'' from qualitative boundary layer considerations which takes into account both the body shape and the orientation of the particle and which was defined by :

$$L'' = A / P_m \quad (16)$$

where P_m is the mid cross-section perimeter normal to the flow. A standard correlation was obtained with a scatter of only $\pm 15\%$. This concept was also applied to freely-moving bodies. (83).

Shchitnikov (84) tried to apply the above characteristic lengths in the case of pure convective heat transfer at much higher Re (10,000 to 140,000) and found better correlation of his data by using the reduced perimeter over the mid-body cross-section (P_m/π).

SURFACE EFFECT ON HEAT TRANSFER

Surface roughness may affect the heat transfer by creating in the boundary layer some disturbances which under certain circumstances, may grow to be large enough to destabilize it and promote transition. Artificial roughness can be achieved by means of strips or tripping wires, and relevant studies

are found in ref. : (13, 22, 27, 56, 85).

The surface may also have a chemical action on the fluid, and heat generation may be involved as well. This is the case for tests in arc-heated streams of dissociated nitrogen where recombination of the atoms takes place at the surface. For such conditions Winkler and Sheldahl (86) reported that errors up to 20% had been experienced in the measurement of free-stream enthalpy by not maintaining a highly catalytic surface on their probe. The rate at which heat is evolved during recombination of nitrogen atoms at the surface has been investigated by Chludzinski, Kadlec and Churchill (87) and it was found that the contribution from the heat of reaction could be as important as that of the convective heat transfer itself. In that case, any modification in the state of the catalytic surface affects the heat transfer measurements to a large degree.

EFFECT OF MACH NUMBER ON HEAT TRANSFER

The formation of the boundary layer is due to frictional effects between the surface and the viscous nature of the flowing fluid. Fluid friction is an irreversible process and therefore generates heat which must be considered in the energy equation. As already mentioned, however, for velocities corresponding to $M < 0.5$, the influence of frictional dissipation is negligible with respect to the convective heat transfer. For velocities beyond this, it gradually becomes of major

importance and references (19) (20) and (88) have studied this effect on heat transfer, in the low Re range ($0.02 < Re < 1,000$) for continuum flow or free molecular flow. The experimental results clearly show that :

- At a given Re , Nu decreases when M increases;
- This effect is larger the lower Re is.

EFFECT OF A JET FLOW ON HEAT TRANSFER

Kubanek (1) has fully reviewed the hydrodynamic properties of free and confined jets. Emphasis should be placed here on some aspects of heat transfer to a body immersed in a jet flow. The changes in the properties of a jet in both the axial and radial directions of the flow issuing from an orifice causes flow problems which are not encountered in studies involving a duct or a wind tunnel.

Schuh and Persson (89) investigated the effect of a two-dimensional jet of limited initial thickness upon the average heat transfer of a cylinder in cross-flow placed at different distances, z , from the nozzle. The Reynolds number based on D and V at the nozzle exit, ranged from 20,000 to 50,000. Surprisingly, the maximum heat transfer coefficient was obtained for a jet with an initial thickness of $1/8$ the cylinder D and with the cylinder at a distance from the nozzle exit of about two to eight times the jet thickness. This result was explained by the ability of thin jets to adhere to curved surfaces (Coanda effect).

Gardon and Akfirat (90) showed that some seemingly anomalous heat transfer phenomena with impinging jets could be explained as effects of the intense and spatially-varying turbulence inherent in jets. As a matter of fact, turbulence in jets is generated by the jets themselves while in a wind tunnel a more uniform distribution of turbulence is artificially obtained over the test section. Besides, I has been found to be uniquely determined by the jet Re and the dimensionless jet length z/d (where d is the diameter of the nozzle).

EFFECT OF BLOCKAGE ON HEAT TRANSFER

When a cylinder blocks off a large proportion of the channel cross-section, the pressure and velocity distributions, the separation and transition points, and the local heat transfer will be affected. Perkins and Leppert (12), using both potential flow theory and pressure measurements, investigated and correlated the effect of blockage on the hydrodynamic behaviour over the front half of a cylinder. Blockage in this region was found to force velocities to increase, separation to occur further downstream and velocity distributions to be more like those predicted by potential flow theory. These authors and more recently Talmor (60) made an extensive review and comparison of the existing methods for correcting the local or average heat transfer data for blockage effects. Therefore, no further discussion will be presented here, except for stressing that these methods are purely empirical and correct the effect only partially. Studies (12, 60, 61, 62, 91) invol-

ving high blockage ratios should be examined with circumspection when they are to be compared with other heat transfer investigations in which this problem is absent.

IV. CONVECTIVE HEAT TRANSFER CORRELATIONS

FOR A CYLINDER IN CROSS - FLOW

THE CIRCULAR CYLINDER

Under the condition of pure convective heat transfer, it has been shown in the section on similarity of two non-isothermal systems that the heat transfer coefficient was a function of Re and Pr alone :

$$Nu = f (Re, Pr) \quad (17)$$

The available evidence indicates that a form of this function is :

$$Nu = a + b (Re)^n (Pr)^m \quad (18)$$

where a is the value of the Nu in the case of an almost stagnant fluid and has values of 2.0 and 0.32 for a sphere and a cylinder, respectively.

An analogous equation exists for mass transfer :

$$Sh = a' + b' (Re)^n (Sc)^m \quad (19)$$

It is usually admitted from both experimental observation and theoretical consideration that $m = 1/3$. Under conditions where heat transfer and mass transfer are analogous processes, equations (18) and (19) are identical.

A very clear example for this is given by Sogin and Subramanian's work (42), who found an excellent identity between the local Nu and Sh in the boundary layer region of a cylinder. For relatively large Re , the contribution of a is negligible and the pure convective heat transfer equation becomes :

$$Nu = h(Re)^n (Pr)^{1/3} \quad (20)$$

Most of the correlations for cylinders are reported in this form.

Colburn (16) proposed a general method of correlating heat transfer data by plotting :

$$J_H = (St) (Pr)^{2/3} = (t_1 - t_2) / \Delta t \quad (S/A) (Pr)^{2/3} \quad (21)$$

versus Re , where h in the Stanton number is the film coefficient. The advantages of using this dimensionless number are twofold : J_H contains the data which are generally found in experiments and which are used in the design of heat exchangers, and secondly, $J_H = Pr/2$ when the Colburn's analogy between heat and momentum transfer holds. Moreover, Colburn pointed out that the usual way of plotting $(Nu)/(Pr)^{1/3}$ versus Re was the same as plotting $(J_H) (Re)$ versus Re , which thus involves plotting a function against itself. Because it is usually admitted that the exponent of Re is 0.5 in the subcritical range, Kestin (37) found it more convenient in the study of particular effects, e.g. turbulence, blockage etc, to present the heat transfer results by the Froessling

number ($Fr = Nu/(Re)^{1/2}$). In all the relationships which will be reviewed, it must be kept in mind that small differences in n (which occur in measuring the slope of the straight line on log-log coordinates representing equation (20)) cause large changes in the value of the constant b .

A considerable divergence of opinion exists about the temperature basis at which the fluid properties must be evaluated. Some authors (16, 19, 71, 72, 77) recommend the use of the bulk temperature T_b ; others (16, 24, 45, 92) the film temperature T_f defined as $(T_b + T_w)/2$, and others, e.g. Kestin and Maeder (22) and Hilpert (69), rather than a temperature, calculate the integral mean value of the physical properties through the boundary layer, that is :

$$\bar{\nu} = 1/(T_b - T_w) \int_{T_w}^{T_b} \nu dT \quad (22)$$

Talmor (60) used for reference temperature the arithmetic average of the adiabatic wall temperature (equal to the stagnation temperature for $Pr = 1$) and the actual wall temperature i.e. $(T_{aw} + T_w)/2$.

Davies and Fisher (20), from both experimental evidence and theoretical considerations, strongly recommend the evaluation of the thermal conductivity k of the fluid at the temperature of the wall.

McEligot, Magee and Leppert (70) present a table comparing

twenty different investigations on convective heat transfer inside a tube, from which it can be seen that the film basis T_f is usually employed for the average heat transfer coefficient, whereas T_b is used for the local heat transfer coefficient. Recent experimental studies on convective heat transfer with high temperature differences inside (70) or outside (72) a cylinder or on a flat plate (71) seem to indicate that T_b is a better basis, whereas T_f is restricted to low or moderate temperature differences. Unless otherwise stated, T_f will be assumed as the basis in the following discussion.

Local heat transfer results are usually presented graphically (12, 21, 26, 27, 28, 29, 32, 37, 72, 93). Martinelli and co-workers (94) studied the data of Schmidt and Wenner (27) and proposed the following empirical equation for predicting the local Nu around a cylinder for θ up to 80° and free-stream turbulence level $I < 1\%$:

$$Nu = 1.14(Pr)^{0.40} (Re)^{0.50} [1 - (\theta/90)^3] \quad (23)$$

This equation, for $\theta = 0$, reduces to the well known Squire equation (95) at the front stagnation point:

$$Nu = 1.14(Pr)^{0.40} (Re)^{0.50} \quad 0.6 < Pr < 2 \quad (24)$$

while Perkins and Leppert (12) obtained from a theoretical approach:

$$Nu = 1.08 (Re)^{0.50} (Pr)^{0.36} \quad (25)$$

Talmor (60) recently reported heat transfer results for immersed cylinders in a high temperature transonic cross-flow with high blockage ratios. These investigations, with combustion-induced turbulence, indicated that the boundary layer around small (61) or large (62) diameter throat tubes was essentially turbulent, the transition taking place at approximately 10° from the stagnation point. A blockage-correcting correlation was proposed at the front stagnation point for $I < 1\%$:

$$(Nu) / (Pr)^{0.4} = 1.14 [K_s (Re)]^{0.5} \quad (26)$$

where $K_s = [1 + (D/W)^{0.5}]$ is a blockage-correction factor. Moreover, a relationship for the heat transfer at the stagnation point and the average around the cylinder was presented :

$$\left[\begin{matrix} (Nu) \\ \text{stag. or} \\ \text{avg.} \end{matrix} \right] (Pr)^{-0.33} \left[\frac{(M)_\infty}{(Re)_o} \right]^{0.40} \left[\frac{\mu_o^2}{\rho_o^2} \frac{D^2}{P_o} \right]^{0.20} \left[\frac{T_r}{T_o} \right]^{0.32} = 0.0155 \quad (27)$$

where $T_r/T_o = 0.5(1 + T_w/T_o)$ and the subscript o refers to stagnation conditions. It is worthwhile to mention that the same author, in a previous study (96) derived two equations whose essential feature is the prediction of the local Nu in the case of a laminar or a turbulent boundary layer from a knowledge of the angular pressure distribution data only.

Richardson (43) in examining the local heat transfer coefficient obtained by various workers (5, 21, 31, 32, 42) con-

cludes that Nu at the rear stagnation point is proportional to Re to the $2/3$ power.

For the average heat transfer coefficient the standard relationship is that of McAdams (92). He plotted all the convective heat transfer data available in 1950 between air and a cylinder in cross-flow. Re varied from 0.02 to 235,000 and the fluid properties were evaluated at T_f , except for the density which was taken at T_b . Since the $Nu - Re$ line, when plotted on a log-log scale was curved, several straight-line correlations were proposed for different ranges of Re :

$$0.1 < Re < 1,000 \quad Nu = 0.32 + 0.48(Re)^{0.52} (Pr)^{0.33} \quad (28)$$

$$1,000 < Re < 50,000 \quad Nu = 0.27 (Re)^{0.60} (Pr)^{0.33} \quad (29)$$

$$50,000 < Re < 250,000 \quad Nu = 0.027(Re)^{0.805} (Pr)^{0.33} \quad (30)$$

McAdams recommends the correlation reported by Hilpert (69) as being the most reliable and representative of most of the available data. Richardson (97), however, recently suggested that Hilpert's results could be incorrect. This point will be discussed later. Kestin and Maeder (22), noting the coincidence of their curve at $I = 0.9\%$ with Hilpert's, suggested that this coincidence could constitute an indirect determination of Hilpert's free-stream turbulence intensity.

Churchill and Brier (72) investigated the local heat

transfer at low Re ($300 < Re_b < 2,300$) and high temperature differences ($580 < \Delta T < 1,800^\circ F.$) in a low turbulence level nitrogen stream ($I < 2\%$). The average heat transfer coefficients were about 30% higher than those predicted by McAdams and were better correlated when the physical properties were evaluated at T_b . They proposed the following relationship for the average Nu :

$$Nu = 0.60(Re)_b^{0.5} (Pr)_b^{1/3} (T_b/T_w)^{0.12} \quad (31)$$

where (T_b/T_w) accounts for the large temperature difference. However, the scatter of the points and the narrow range of (T_b/T_w) which was investigated cast doubt on the significance of this temperature ratio.

Douglas and Churchill (24) in 1956, replotted all the data available for both the cooling or the heating of a cylinder by a gas and showed that a single $Nu - Re$ curve could be obtained if all the properties were evaluated at T_f . A semi-theoretical equation was presented :

$$Nu = 0.46(Re)^{0.5} + 0.00128 Re \quad 500 < Re < 300,000 \quad (32)$$

which assumes a laminar resistance through the boundary layer and a turbulent resistance in the region of separated flow.

Several authors (12, 24, 93, 98, 99) and Richardson in particular (43), have suggested that a more realistic relationship would be obtained if, instead of postulating an

expression of the form of equation (20), the total heat transfer were expressed as the sum of the transfer of heat in the boundary layer and in the wake regions, each being assessed independently of the other. Richardson proposed the following equation :

$$Nu = [C_1(Re)^{0.5} + C_2(Re)^{\bar{r}}] (Pr)^{1/3} \quad (33)$$

where C_1 is a coefficient being affected by free-stream turbulence whereas C_2 is not significantly affected by it. His results were bounded by the two equations :

$$Nu = 0.37(Re)^{0.5} + 0.057 (Re)^{2/3} \quad (34)$$

$$Nu = 0.55(Re)^{0.5} + 0.084(Re)^{2/3} \quad (35)$$

The variation in C_1 was due, as already stated, to \underline{I} , while the variation in C_2 was attributed to experimental errors. The value $2/3$ of the exponent \underline{r} was ascribed to the wake because such an exponent was reported in many investigations on heat transfer in the wake of other blunt bodies. Other authors (12, 72, 98) have confirmed the correctness of this value while some others (24, 93, 99) postulated a value of $r = 1$.

Churchill and Brier (72) presented a very interesting plot showing the variation of \underline{n} (see equation (20)) around a cylinder. While \underline{n} was quasi-steady for $0 \leq \theta \leq 80^\circ$, varying from 0.55 to 0.50, an abrupt drop was displayed in the region of separation, followed by a marked increase up to the rear

stagnation point, for which region n varied from 0.27 to 0.9. Hence, an average value of $2/3$ seems quite sensible.

Fand (98) equated the two parts of the right hand side of equation (33) to each other at $Re = 39,000$, because McAdams reported that, at this Re , the two contributions are identical, and found $n = 0.58$.

Perkins and Leppert (12) investigated local heat transfer coefficients between a uniformly heated cylinder and water in cross-flow at high Re (2,000 to 120,000), moderate Pr (1 to 7), low ΔT (20 to 120°F) and high blockage ratios D/W (0.208 to 0.415). Their data were represented by two equations :

$$(Nu) \left(\frac{\mu_w}{\mu_b} \right)^{0.25} (Pr)^{-0.40} = 0.31 (Re^*)^{0.5} + 0.11 (Re^*)^{0.67} \quad (36)$$

$$(Nu) \left(\frac{\mu_w}{\mu_b} \right)^{0.25} (Pr)^{-0.40} = 0.57 (Re^*)^{0.5} + 0.0022 Re^* \quad (37)$$

where Re^* is the blockage-corrected Reynolds number ($Re^* = U_\infty D / \nu$). Correlation (36) appears to be more satisfactory than (37).

Fand (98) carried out similar experiments except that blockage effects were carefully avoided ($10^4 < Re < 10^5$ and $4 < \Delta T < 10^\circ F$). His results are in good agreement with McAdams' correlation :

$$Nu = \left[0.35 + 0.56 (Re)^{0.52} \right] (Pr)^{0.30} \quad (38)$$

which had been derived for liquids in the low Re range ($0.1 < Re < 200$). Fand's data were equally well represented by the equation :

$$Nu = \left[0.35 + 0.34(Re)^{0.5} + 0.15(Re)^{0.58} \right] (Pr)^{0.30} \quad (39)$$

Perkins and Leppert's results are 60% higher than those of Fand. The difference in I could not be made to account for more than 20% of this difference, and Fand has suggested that the remainder was due to the blockage and secondary flow effects.

The only correlation which exists to predict the effect of scale and intensity of turbulence of the free stream on the heat transfer to cylinders is the one proposed by Van der Hegge Zijnen (45). His work having been already examined (p. 24), his relationship will just be restated :

$$Nu^* = (Nu)_{turb} / (Nu)_{lam} = \left[1 + \varphi(Re \times I) \right] \psi(L/D) \quad (40)$$

From a collection of known data, the same author (99) derived the following equation for cylinders in air :

$$Nu = 0.35 + 0.5(Re)^{0.5} + 0.001 Re \quad (41)$$

This relationship has been confirmed by Van Meel (93), who developed an original technique to measure the local heat transfer, whose essential feature is that only the temperature distribution on the surface has to be known.

Heat transfer to fine wires from rarified air streams has been studied by Baldwin (100) and Davies (19). The former found that under slip-flow condition ($Kn > 0.01$) the exponent of Re shifts from 0.5 to 1. Davies, from theoretical developments, derived a relationship relating Nu to the skin friction coefficient :

$$Nu = (c_f) (Re) (Pr) / \pi \gamma \quad (42)$$

where $\gamma = C_p / C_v$. Several equations are presented to evaluate c_f .

Heat transfer to thermocouples is a matter of importance in the determination of conduction and radiation corrections in hot gas streams. This subject has been reviewed in Ref.(1) and the relationship of Scadron and Warshawsky (101) is given below :

$$(Nu)_t = 0.478 (Re)_t^{0.5} (Pr)_t^{0.3} \quad (43)$$

$$250 < (Re)_t < 30,000$$

$$0.1 < M < 0.9$$

where the physical properties of the gas are evaluated at the total temperature.

Mention must be given to Chludzinski et al. (87) who investigated the heat transfer to a thermocouple immersed for periods less than 0.1 sec. in a 5-Kw. radio-frequency argon-nitrogen plasma jet. The coefficients they obtained

were more than twice as high as those predicted by equation (43), due to atom recombination at the surface, and were correlated by the following correlation :

$$h = 1.05(k_b/D) (Re)_b^{0.5} (Pr)_b^{0.3} + (0.20/12D)^{0.5} \left[1.72 \cdot 10^{-20} C_{A+} + 1.11 \cdot 10^{-20} C_{N+} + 7.5 \cdot 10^{-20} C_N \right] \quad (44)$$

THE NON-CIRCULAR CYLINDER

Correlations to predict heat transfer to non-circular cylinders can be said to be non-existent. It seems that only Hilpert (69), and Knoblauch and Reiher (102) have investigated the field. Results are reported in the form of equation (20) :

$$Nu = b (Re)^n \quad (45)$$

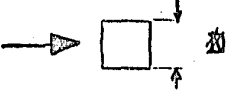
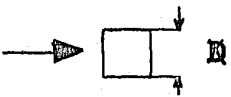
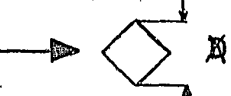
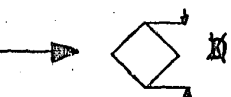
This equation has been applied to gases only, and values of b and n are given in Table I.

Recently, however, Harlow, and Fromm (103) studied the dynamics and the heat transfer in the Von Kármán wake of a rectangular cylinder ($25 < Re < 400$). The heat transfer was measured from the heated rear of the cylinder and their results, after blockage correction, showed that :

- at very low Reynolds number ($Re < 100$), the Nusselt number is proportional to Re at the $1/2$ power :

$$Nu = 0.36 Re^{1/2} \quad (46)$$

TABLE I.HEAT TRANSFER TO SQUARE CYLINDERS

Flow direction and profile	Re		n	b	Observer
	from	to			
	5,000	100,000	.675	.092	(69)
	2,500	8,000	.699	.160	(102)
	2,500	7,500	.624	.261	(102)
	5,000	100,000	.588	.222	(69)

Note The characteristic length used in table (i) is the diameter of a circular cylinder of equal surface area.

- for $100 < Re < 150$ transition in the streakline pattern takes place;
- for $Re > 150$, Nu is proportional to Re at the $2/3$ power :

$$Nu = 0.135 Re^{2/3} \quad (47)$$

indicating "a tendency to universal behaviour for heat transfer in the region of separated flow", as previously suggested by Richardson (43).

N O M E N C L A T U R E

ROMAN SYMBOLS

a	- constant
A	- surface area, ft.^2
b	- constant
c	- velocity of sound, ft./sec.
c_f	- skin friction coefficient
C_1, C_2	- constants
C_p	- specific heat at constant pressure, $\text{B.t.u./lb.}^\circ\text{F.}$
C_v	- specific heat at constant volume, $\text{B.t.u./lb.}^\circ\text{F.}$
D	- diameter, ft.
D_M	- molal diffusivity, lb.-mole/hr. ft.
D_v	- mass diffusivity, $\text{ft.}^2/\text{hr.}$
e	- exponent
f	- function
g_c	- gravitational constant.
G	- mass velocity lb./sec. ft.^2
h	- heat transfer coefficient, $\text{B.t.u./hr. ft.}^2 \text{ }^\circ\text{F.}$
j	- exponent
k	- thermal conductivity, $\text{B.t.u./hr. ft.}^2 \text{ }^\circ\text{F./ft.}$
k_G	- mass transfer coefficient, $\text{lb.-mole/hr. ft.}^2$
L	- Eulerian macroscale of turbulence, ft.

m	- constant
n	- constant
p	- pressure, lb./ft. ²
P _m	- mid cross-section perimeter, ft.
q	- heat flux, B.t.u. /hr. ft. ²
r	- constant
R	- radius, ft.
S	- cross-sectional area, ft. ²
T	- temperature, °F. or °R.
u	- velocity component in x direction, ft./sec.
u'	- fluctuating component of velocity, ft./sec.
U	- average velocity, ft./sec.
U _{max}	- velocity at the outer edge of the velocity boundary layer, ft./sec.
v	- velocity component in y direction, ft./sec.
W	- duct width, ft.

GREEK SYMBOLS

β	- coefficient of volume expansion, (°R.) ⁻¹
δ	- boundary layer thickness, ft.
λ	- mean free path of gas, ft.
μ	- viscosity, lb./ft.hr.
ν	- kinematic viscosity, ft. ² /hr.
π	- 3.1416
ρ	- density, lb./ft. ³

DIMENSIONLESS GROUPS

E	- Eckert number, $[U^2 / c_p (\Delta T)]$
Gr	- Grashof number, $[D^3 g_c \beta (\Delta T) / \nu^2]$
I	- intensity of turbulence, $[(\overline{u'^2})^{0.5} / U]$
Kn	- Knudsen number, (λ / D)
M	- Mach number, (U / c)
Nu	- Nusselt number, (hD / k)
Pr	- Prandtl number, $(c_p \mu / k)$
Re	- Reynolds number, $(DU \rho / \mu)$
(Re) _{crit.}	- critical Reynolds number
Sc	- Schmidt number, $(\mu / \rho D_v)$
Sh	- Sherwood number, $(k_G D / D_M)$
St	- Stanton number, $(h / c_p G)$

SUBSCRIPT

aw	- adiabatic
b	- bulk or free-stream
f	- film
o	- stagnation
w	- wall
∞	- free-stream

OPERATORS

Δ	- difference
\propto	- proportionality
∂	- partial derivative
d	- derivative

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EXPERIMENTAL PART

EXPERIMENTAL PART

INTRODUCTION

Research in the field of high-temperature chemical and metallurgical processing is rapidly gaining momentum, principally as a result of recent progress in the design of direct-current plasma torches.

Kubanek (1), working in this laboratory, studied the pure convective heat transfer from a nitrogen plasma jet to water cooled stationary spheres. His results showed that, in the low range of Reynolds numbers investigated (630 - 4300), the boundary layer was turbulent in nature.

The purpose of the present work was to study, under similar conditions, the behaviour of circular cylinders and of a cylinder with a square cross-section in two orientations.

HEAT TRANSFER TO CYLINDERS

Heat transfer by forced convection to cylinders in cross-flow has been extensively investigated in the case of circular cylinders (2 - 19) whereas an almost negligible number of studies have dealt with other cross-sectional geometries, such as elliptic (9, 20, 21), square (16, 20), rectangular (22)

or hexagonal (16). All these studies have been conducted at low or moderate temperature differences (less than 350°F.) in streams of relatively low turbulence levels. For circular cylinders, however, a few experiments (1, 16, 23-30) were carried out with temperature differences ranging from 1000 to 5000°F. In some of them (16, 29, 30) the wall was raised to elevated temperature (1000 to 1800°F.) while the gas was at room temperature. Hilpert (16) observed a 6% increase in $\frac{hD}{k_m}$ as the surface temperature was increased from 100 to 1000°C.

A considerable divergence of opinions exists concerning the temperature basis at which the physical properties must be evaluated. Some authors (1, 25, 31, 32, 33) recommended the bulk temperature, others, the film temperature defined as $0.5 (T_b + T_w)$ by (3, 14, 34, 35) or as $0.5 (T_{aw} + T_w)$ by Talmor (36). Kestin and Maeder (5) and Hilpert (16) integrated the physical properties through the boundary layer.

The effect of the intensity of turbulence on the local heat transfer coefficient or the average coefficient has been investigated by (2, 4, 8, 9, 37) and (5, 12, 14, 38), respectively. Although a considerable disagreement exists between the results of the various workers, general conclusions may be summarized as follows :

- Free-stream turbulence affects the heat transfer rate both locally and through flow configuration;

- The local effect is much greater on a laminar than on a turbulent boundary layer or on the wake.

Van der Hegge Zijnen (14) seems to be the only one who made a systematic investigation of the combined influence of the scale and intensity of turbulence. He assumed that the functional relationship between heat transfer and these parameters was of the form :

$$Nu^* = (Nu)_{\text{turb.}} / (Nu)_{\text{lam.}} = \left[1 + \psi(Re, I) \psi(L_x / D) \right] \quad (1)$$

From his experimental results he concluded that :

- At constant value of (L_x/D) , Nu^* increased with the product $(Re \cdot I)$, the rate of increase being highest for low values of $(Re \cdot I)$; when $(Re \cdot I) > 100$ the rate of increase was constant.
- At constant value of $(Re \cdot I)$, Nu^* either increases or decreases with increasing scale ratio, the maximum being reached when (L_x/D) is about 1.5 to 1.6.
- Any variation in I or L_x is more effective at higher Re .

Although his experiments were carefully conducted, it should be noted that his data were quite scattered and that some of his conclusions are opposed to those predicted by the Taylor parameter.

Turbulence causes transition in the boundary layer to occur at much lower critical Reynolds number. While Taylor (39) and

Wieghardt (40) demonstrated that $(Re)_{crit.}$ depends on the so-called Taylor parameter $\Lambda = I (L_x/D)^{0.2}$ for isotropic and low turbulence streams. Torobin and Gauvin (41), in their experimental study on freely-moving spheres at low Re (400 - 3000) and high relative intensity of turbulence (1 to 40%), found that Λ was the first causative factor of transition, and obtained a criterion predicting transition :

$$I^2 (Re)_{crit.} = \text{constant} = 45 \text{ for spheres} \quad (2)$$

Van Driest and Blumer (42) proposed an equation which predicts, at transition, the location of the transition point at the surface. Recently, Talmor (43) applied the equation in reverse to obtain the turbulence intensity in a combustion-induced turbulent stream from a knowledge of the point of transition.

As the calculated intensity was of the expected magnitude, Talmor proposed that this method may provide a useful means of indirect intensity determination in situations where direct measurements are impossible.

Several characteristic lengths have been proposed to predict convective heat (and mass) transfer to various particles from a standard relationship. Krischer and Loos' characteristic length L' (44) was found to be applicable to two-dimensional bodies only. Pasternak and Gauvin (45) studied convective heat and mass transfer for twenty shapes suspended in various orientations, in a hot turbulent air stream (10%) at moderate

Re (400 - 8000). They derived a characteristic length L'' from qualitative boundary layer considerations which takes into account the body shape and the orientation of the particle and which was defined as :

$$L'' = A / p_m \quad (3)$$

Shchitnikov (46) tried to apply the characteristic length of Pasternak and Gauvin and that of Polonskaya and Mel'nikova (47) (defined as $A^{0.5}$) to the case of pure convective heat transfer at much higher Re (10,000 to 140,000) and found a better correlation of his data by using the reduced perimeter over the mid-body cross-section, (p_m / π) .

Relationships to predict force-convective heat transfer to circular cylinders for low and moderate temperature differences are abundant. They can be obtained in the following references (3, 10, 13, 14, 16, 17, 19, 35) and (2, 18, 35) for gases and liquids, respectively.

The heat transfer investigations at high temperature, which are particularly pertinent to this work, will now be examined.

Scadron and Warshawsky (27) have studied heat transfer to bare thermocouples in considering the hot junction as consisting of two cylinders. They obtained the following relationship :

$$(Nu)_t = 0.478 (Re)_t^{0.5} (Pr)_t^{0.3} \quad (4)$$

in the ranges : $250 < (Re)_t < 30,000$; $0.1 < M < 0.9$. The subscript t stands for total temperature. This relationship has been verified by Glawe and Johnson (48) at temperatures up to 3000°F .

Churchill and Brier (25) investigated the local heat transfer coefficient at low Re, ($300 < (Re)_b < 2,300$) and relatively high temperature differences ($580 < \Delta T < 1800^\circ\text{F}$.) in a low-turbulence level nitrogen stream ($I < 2\%$). Their average coefficients were 30% higher than those predicted by McAdams' correlation. They proposed :

$$(Nu)_b = 0.60 (Re)_b^{0.5} (Pr)_b^{1/3} (T_b/T_w)^{0.12} \quad (5)$$

Douglas and Churchill (34) replotted the data available for both the cooling and heating of a cylinder by a gas and showed a single Nu - Re curve could be obtained if all the properties were evaluated at T_f. A semi-empirical equation was presented :

$$(Nu)_f = 0.46(Re)_f^{0.5} + 0.00128(Re)_f \quad (6)$$

$$500 < (Re)_f < 300,000.$$

Chludzinski et al. (26) measured the heat transfer to a thermocouple immersed for periods less than 0.1 sec. in a 5-Kw. radio-frequency argon-nitrogen plasma jet. The coefficients they obtained were more than twice greater than those predicted by Scadron and Warshawsky's equation, due to atom

recombination at the surface, and were correlated by the following correlation :

$$h = 1.05 (k_b/D) (Re)_b^{0.5} (Pr)_b^{0.3} + (0.20/12D)^{0.5} \left[1.72 \cdot 10^{-20} C_{A+} + 1.11 \cdot 10^{-20} C_{N+} + 7.5 \cdot 10^{-20} C_N \right] \quad (7)$$

The first term in equation (7) is the convective contribution while the second is the reaction contribution. It is worthy of mention that the gas velocity was not measured, and an average value, based on the total flow rate was used for calculating Re which ranged from 2 to 10.

Recently, Talmor (36) reported heat transfer data for cylinders immersed in a high temperature (5,000°F.), high blockage ratio ($D/W=0.75$), transonic stream which consisted of combustion products. Under these conditions, the boundary layer was found to be essentially turbulent, transition taking place at about 10° from the front stagnation point.

A relationship for the heat transfer at the stagnation point or averaged around the cylinder was presented :

$$\left[(Nu)_{\text{stag. or avg.}} / (Pr)^{-1/3} \right] \left[M_{\infty} / (Re)_{o,\infty} \right]^{0.40} \left[\mu_o^2 / g_c \rho_o^2 p_o \right]^{0.20} (T_r/T_o)^{0.32} = 0.0155 \quad (8)$$

where $T_r/T_o = 0.5(1 + T_w/T_o)$ and the subscript o refers to the stagnation conditions.

Kubanek (1) studied the heat transfer to spheres in a confined nitrogen plasma jet. In the Re range covered (600 - 4,300), with temperatures up to 5,000°F., the boundary layer was found to be turbulent as indicated by the value close to 0.8 of the exponent of the Reynolds number. The correlation proposed was :

$$\text{For spheres : } (\text{Nu})_b = 0.118 (\text{Re})_b^{0.757} (\text{Pr})_b^{0.33} \quad (9)$$

It is worthwhile to mention the hydrodynamic and heat transfer investigation carried out by Harlow and Fromm (22) in the von Kármán wake of a rectangular cylinder at low temperature difference in the Re range 25 to 400. Their results for heat transfer, corrected for blockage effects, showed that for $\text{Re} < 100$, Nu was proportional to Re to the 1/2 power, whereas for $\text{Re} > 150$, Nu was proportional to Re to the 2/3 power. The latter Nu - Re dependency is in agreement with the "universal behaviour" for heat transfer in the region of separated flow suggested by Richardson (13).

EXPERIMENTAL

APPARATUS

The Plasma-Generating System

The nitrogen stream was brought to high temperature by a direct-current plasma torch manufactured by Thermal Dynamics Corporation. The overall system consisted of four units: the plasma torch, the control console, the power supply and the cooling system. The power was supplied by a selenium rectifier, model TDC IA-40. The input voltage was 3-phase, 60 c.p.s., 757 volts, while the output open circuit could provide 80, 160 or 320 volts.

The direct current could be adjusted up to 1000 Amp. during operation. The cooling system, model H-20, maintained the torch parts at a reasonable temperature by circulation of distilled water.

The console was equipped with all the controls to operate the starting system as well as the gas flow rate and the power input to the torch.

The plasma torch, model F-40, can be seen in Fig. 1 and 2 and Table 1. The torch was 10-in. long and had a diameter of 2.75-in.

FIGURE 1

Photograph of THERMAL DYNAMICS

Model F-40 Plasma Torch

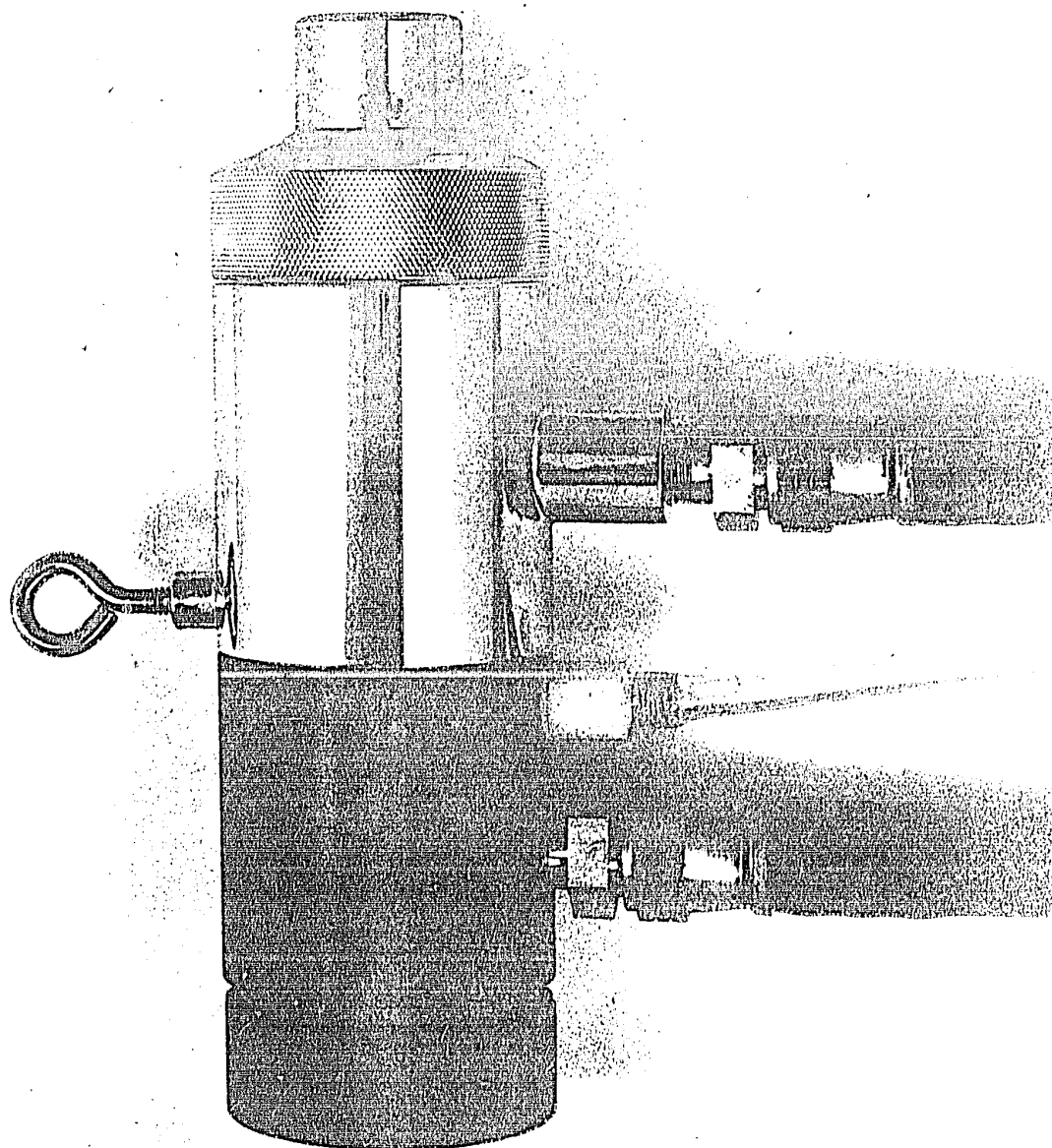


FIGURE 2**Configuration of THERMAL DYNAMICS****Model F- 40 Plasma Torch**

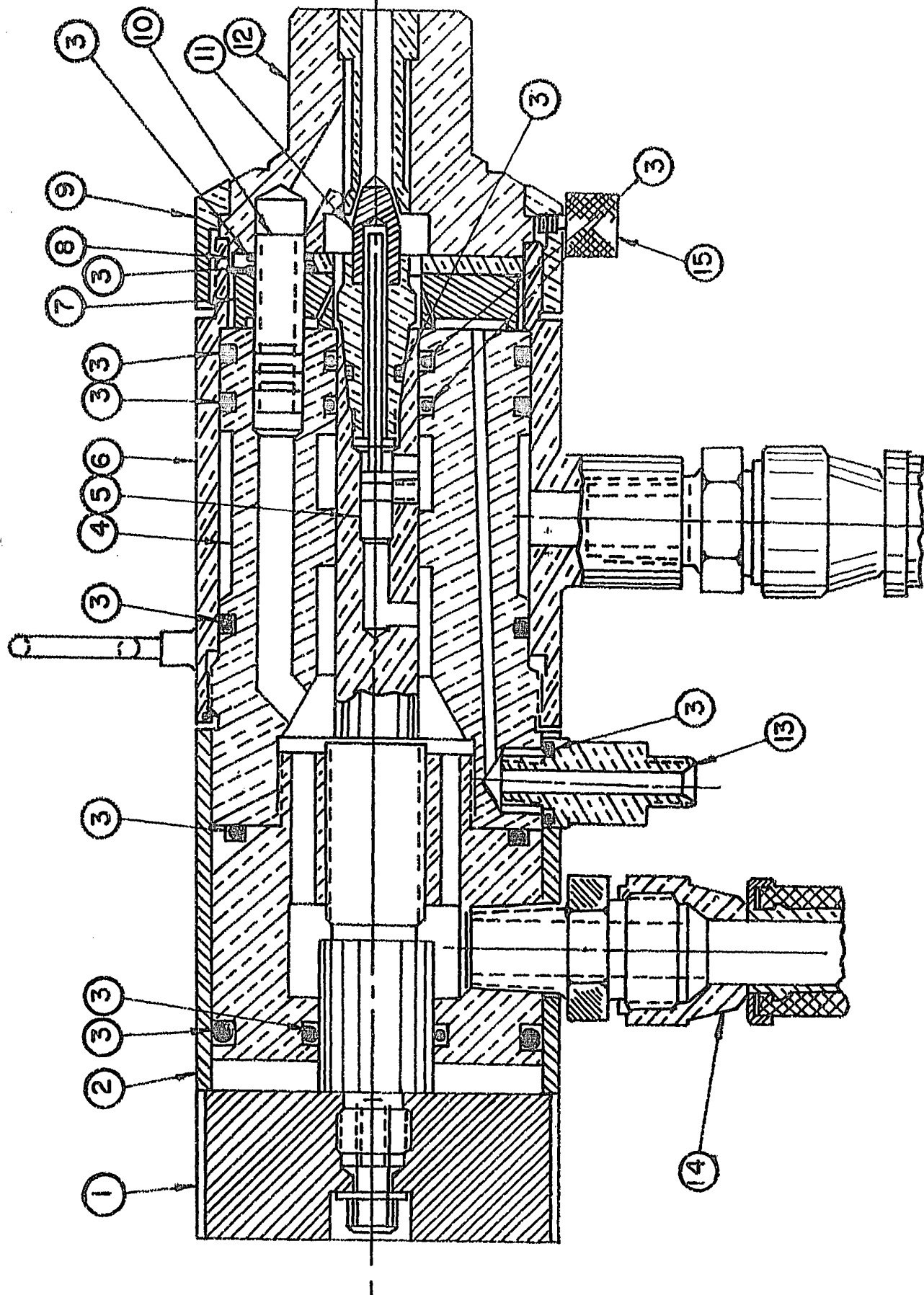


TABLE ILIST OF PARTS FOR F-40 PLASMA TORCH

<u>Item Number</u> <u>in Fig. 2</u>	<u>Description</u>
1	Electrode Knob
2	Plastic Insulator
3	'O' Rings
4	Insulated Main Body
5	Water Tube
6	Main Body Shell
7	Ceramic Gas Ring
8	Nozzle Seating Plate
9	Nozzle Retaining Nut
10	Water-Tube Stud
11	Tungsten Cathode
12	Nozzle Anode
13	Gas Inlet Fitting
14	Water-Cooled Lead
15	Nozzle-Adjusting Nut

The gas is symmetrically distributed in the chamber around the cathode by means of a circular, perforated, ceramic plate (item No. 7) before passing through the gap between the tungsten cathode and the copper anode nozzle. Both electrodes are water-cooled. The gap between them was adjusted to 1/16-in. by turning the electrode knob (item No. 1) to bring the cathode against the nozzle, followed by one turn in the opposite direction.

Two sizes of nozzle were employed (with the corresponding matching cathodes): No. 1, with a diameter of 0.219-in., and No. 3, with a diameter of 0.312-in. The alignment between the electrodes was checked visually before every run by means of a high-frequency spark issuing from the shoulder of the cathode while the gas stream flowed through the torch. Corrections could be made by means of three adjusting studs which allowed relative displacement of the nozzle with respect to the cathode. Alignment was correct when the spark was equally well distributed all around the nozzle. Misalignment as well as humidity content in the gas caused pitting and quick deterioration of the nozzle.

The ignition of the arc was achieved by the high-frequency spark which had to be maintained until the arc current was above 90 amp., so the process could sustain itself. It was noted that the higher the energy input, the more stable the arc was. Wheaton and Dean (49) have carried out a study of the arc process in a similar F-40 torch operating with nitrogen. They observed

that the arc had the shape of an inside-out umbrella issuing from the very tip of the cathode and proceeding down, from the entrance of the nozzle to its exit where it extinguished. This is a cyclic process and was reported to occur at a frequency of about 10,000 c.p.s. and to induce on the efflux some non-homogeneities known as "pockets of plasma", representing regions of high local temperature. These have also been observed in an argon plasma jet by Watson et al. (50)

The Graphite Chamber and the Cooling System

The plasma jet was confined in a vertical ARG graphite tube, 71.35-in. long, 8-in. and 10-in. in inside and outside diameters, respectively (Fig. 3 and 4). Six graphite ports were press-fitted into the chamber in three diametrically opposed pairs with their centres at distances of 12, 36 and 60 inches from the top. They were 19-in. long, 0.625-in. thick and had inside diameter of 1.25 in. Flanges, allowing insertion of probes, could be screwed on them.

The top of the chamber was closed with a press-fitted, sealed 1.0-in. thick graphite lid which was covered by a cooling plate to protect the plasma torch and its leads from the heat of the reactor. Holes of 1.375-in. in the graphite and of 2.75-in. in the plate, centred on the axis of the chamber, were provided for the insertion of the torch. An off-centre 0.50-in. i.d. graphite purge tube was also available.

FIGURE 3

**Photograph of Graphite Chamber with Top Cooling
Plate and Plasma Torch in Position**

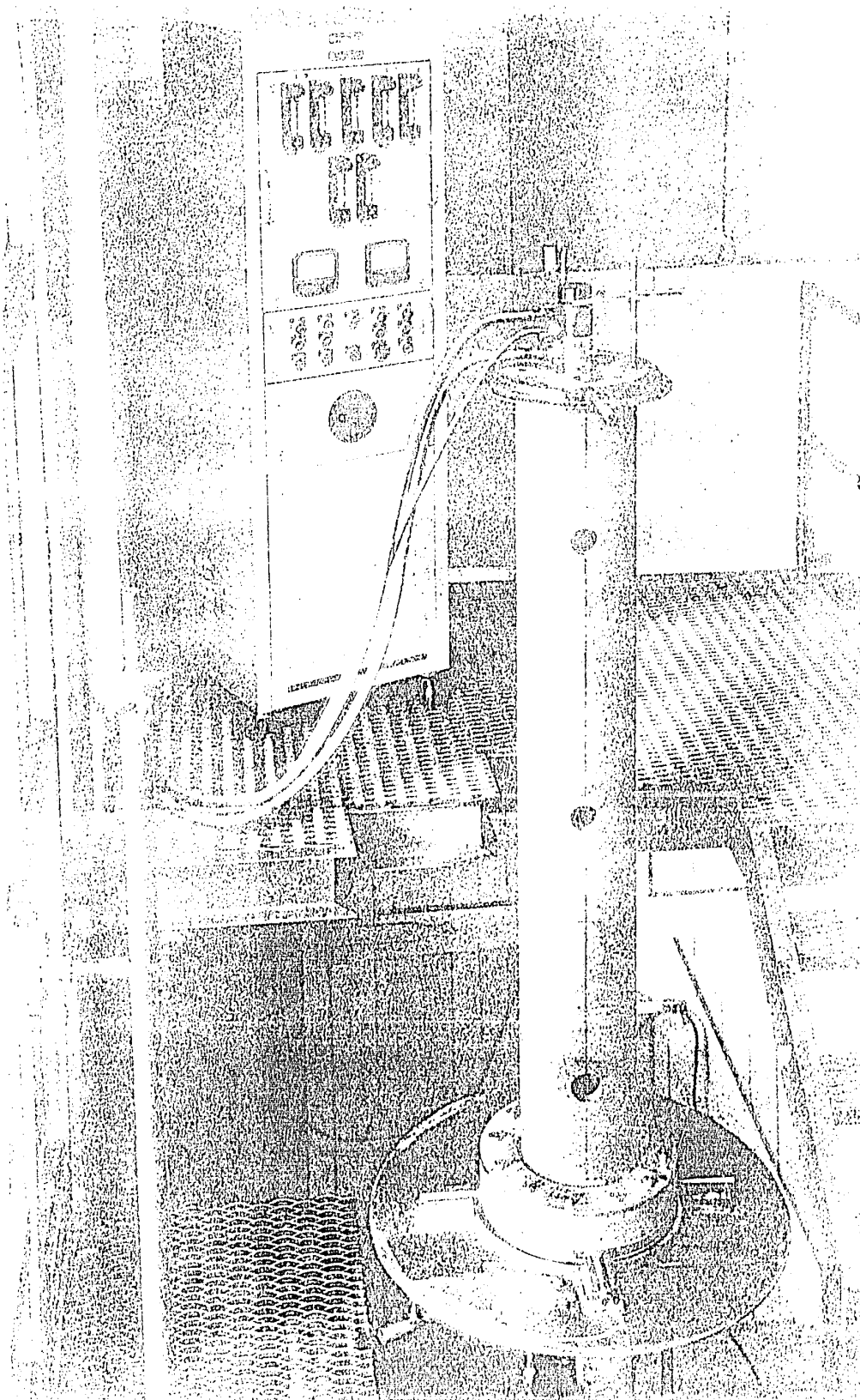
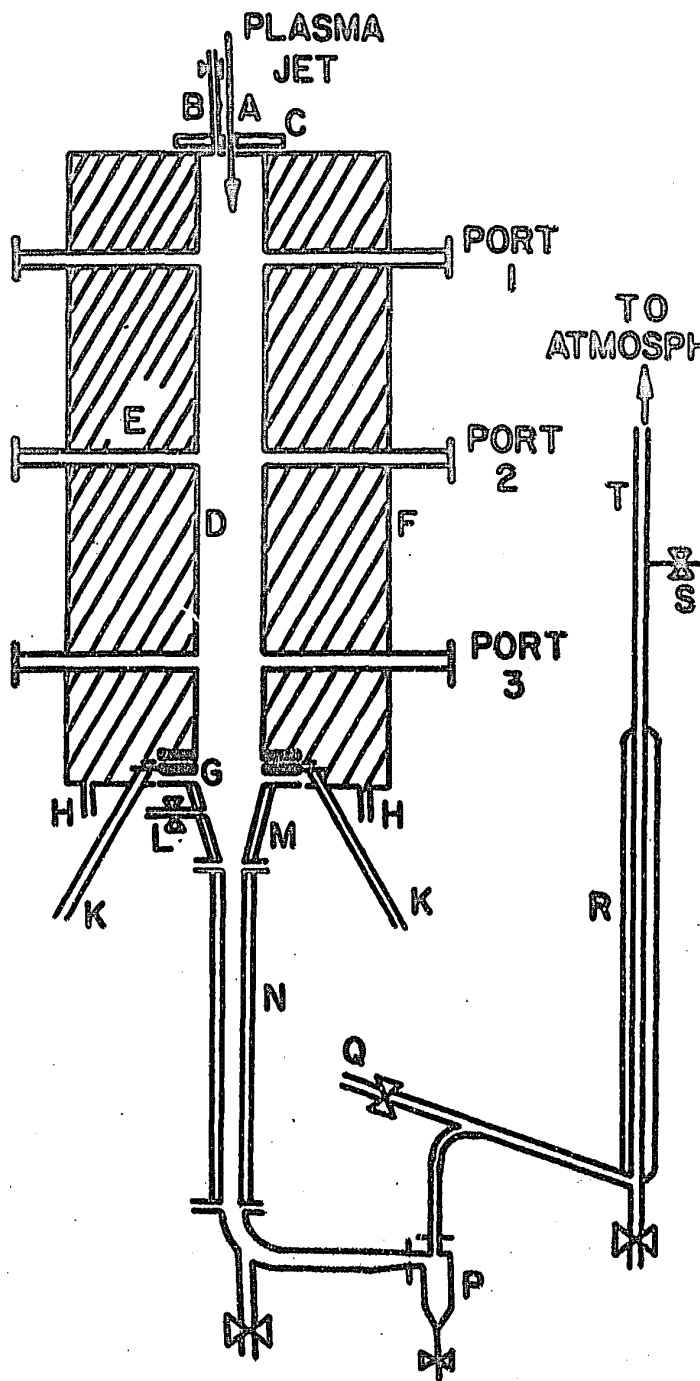


FIGURE 4

**Schematic Diagram of Graphite Chamber
and Cooling System**



- A - PLASMA TORCH
- B - REACTOR NITROGEN PURGE
- C - COOLING PLATE
- D - GRAPHITE REACTOR
- E - INSULATION
- F - STEEL SHELL
- G - SUPPORTING FLANGE
- H - SHELL NITROGEN PURGE
- K - I - BEAM SUPPORTS
- L - RELIEF LINE
- M - COOLING CONE
- N - DOUBLE-PIPE COOLER
- P - CYCLONE SEPARATOR
- Q - NITROGEN INJECTOR
- R - DOUBLE-PIPE COOLER
- S - SAMPLING LINE
- T - FLAME ARRESTER

At its base, the graphite tube had a 3-in. thick graphite screwed flange which rested on a water-cooled supporting flange. The latter was supported at 85 inches above the floor by two 4-in. x 4-in. I-beams. The gas leaving the chamber was passed through cooled tubes before exhausting to the atmosphere. The exhaust line was also provided with a cyclone separator, a nitrogen injector, and a flame arrester for operations involving fine particle injections into the torch, or hydrogen as working gas.

The graphite chamber was insulated with twelve inches of "Fiberfrax", enclosed in a steel shell.

A purge line was provided to flush the oxygen present in the insulation material when the chamber wall was brought to elevated temperatures. To measure the wall temperatures, nine chromel-alumel 1/16-in. o.d. sheathed thermocouples were distributed along the chamber and were connected to a Minneapolis-Honeywell 12-point strip chart recorder.

TECHNIQUE OF MEASUREMENTS

a) Temperature and Velocity Measurements

Measurements of the gas temperature were made with a bare-wire, 0.005-in. diameter tungsten -5% rhenium, tungsten -26% rhenium thermocouple. The sheath and insulation material were inconel and magnesia, respectively, with an outside diameter of

0.040 inch. The thermocouple was enclosed inside a water-cooled 1/4-in..o.d. brass tube with a conical end, out of which the bare tip was made to protrude 1/4-in. Extension wires from the thermocouple junction were connected to a Minneapolis-Honeywell strip chart recorder.

Velocity was measured by means of a total-head probe which was specially constructed by soldering a 0.064-in. o.d., 0.023-in. i.d. and 3/8-in. long type 316 stainless steel tube to the opening of a water-cooled aspirating probe supplied by Thermal Systems Inc. A reference pressure, close to the static pressure in value, was taken inside port No.1, and the dynamic pressure was read on an inclined manometer with methanol as the working fluid. Fluctuations were damped out by means of short lengths of capillary tubing.

Radiation and conduction corrections were applied to the thermocouple readings. Radiation corrections were calculated by the Scadron and Warshawsky (27) equation :

$$\Delta T_r = \frac{\sigma \epsilon_{tc} D_{tc} T_b^4 [1 - (T_d/T_{tc})^4]}{Nu_b k_b [1 + (4\sigma \epsilon_{tc} D_{tc} T_b^4)/(Nu_b k_b T_b)]} \quad (9)$$

where T_b , T_d , T_{tc} are the absolute temperatures of the bulk gas, enclosing duct, and the thermocouple junction, respectively;

D_{tc} is the thermocouple wire diameter;

σ is the Stefan-Boltzmann constant;

ϵ_{tc} is the thermocouple wire emissivity from Ref. (51)

$(Nu)_b$ is given by :

$$(Nu)_b = 0.32 + 0.478 (Re)_b^{0.5} (Pr)_b^{0.33} \quad (10)$$

Conduction corrections are evaluated by the solution of the following system of equations :

$$\Delta T_c = \psi (T_{tc} - T_{sh}) / (1 - \psi) \quad (11)$$

$$\psi = \text{sech}(\eta L_{tc} / 2) \quad (12)$$

$$\eta = \left[4 (Nu)_b k_b / D^2 k_{tc} \right]^{0.5} \quad (13)$$

where T_{sh} is the absolute temperature of the wire at the sheath;

L_{tc} is the length of the thermocouple wire outside the sheath;

k_{tc} is the thermocouple wire thermal conductivity.

The velocity was calculated from Bernoulli's equation :

$$U = \left[2 g_c \Delta p / \rho_b \right]^{0.5} \quad (14)$$

Although the base of the head probe was cooled, the tip was glowing red (1400 - 2000°F.) and it was deemed that no density corrections were necessary. The assessment of the radiation and conduction corrections as well as the velocity determinations were obtained by an iterative procedure.

b) Heat Transfer Measurements

The overall heat transfer was obtained by measuring the

water temperature rise between two cross-sections (called IN and OUT and symmetrically spaced at 0.500-in. on each side of the jet axis) for a given water flow rate. The latter was simply determined by time-volumetry.

At first, the temperature rise was measured by means of two 0.005-in. diameter chromel-alumel thermocouples, each hot junction being held in a slotted circular sheath which was slipped in the tube. This system was given up because the experiments showed that the temperature difference so obtained, either by direct (differential thermocouples) or indirect (absolute thermocouples) measurements, was greater than those obtained by using one absolute thermocouple and moving the whole tube laterally so as to displace the hot junction from one cross-section to the other. The water flow being laminar ($Re < 800$), this phenomenon was attributed to either the different degrees of mixing in the orifices (the second orifice took advantage of the mixing in the first) leakage between the sheath and the tube, or both effects.

In the light of these results, T_{IN} and T_{OUT} were measured by lateral displacement of a single thermocouple junction from one section to the other.

To this end, 1/16-in. o.d. stainless steel sheathed chromel-alumel thermocouple probes, with either insulated or exposed hot junction were employed. Three inches long turbulence gene-

rators (springs or spirals) were wrapped around them to promote mixing of the fluid before it came in contact with the hot junction. It was assumed that the measured temperature was equal to the mean temperature across the section. If this were not so, it would be reasonable to expect that the temperature difference between the two cross-sections would correspond to the real temperature difference because of the identical hydrodynamic conditions at both locations.

Large fluctuations were found in the thermocouple output. Their origin was examined and the grounding of the tube was found to be very important. Although Kubanek (1) had shown that the nitrogen stream at the level of port No.1 of the chamber was in a quasi-molecular state, some ionized particles exist which can build up a voltage when they hit the ungrounded tube.

A voltage difference of 1.5 volt was measured between the tube and the ground. This noise could create fluctuations as large as the temperature difference, and therefore a perfect grounding was compulsory. Even so, the fluctuations due to both the flame instability and the imperfect mixing were still large compared to the temperature difference. To determine the effect of imperfect mixing, the ratio of the amplitude of the water fluctuations to the temperature difference was studied on a strip chart recorder when the water flow-rate was varied. This investigation showed that imperfect mixing was a partial but not the primary cause of these fluctuations. For instance, for

a heat flux of 400 B.t.u./hr. between the sections, the ratio decreased from 25 to 15% when the flow-rate was increased from 340 to 1,600 cm.³/min. (i.e. $2,900 < Re < 13,000$). The fact that these fluctuations were partially caused by the mixing was corroborated by two other observations :

- The records of water temperature displayed two kinds of fluctuations : some large in amplitude and with period of 2 to 3 seconds on which were superimposed smaller ones. The larger ones were undoubtedly due to the flame instability as they usually corresponded to a change in the sound of the torch operation.
- Cooling of the tube by a gas in a very turbulent regime ($Re = 60,000$) showed low frequency fluctuations only.

To minimize the temperature fluctuations an insulated hot junction was deemed more adequate than an exposed one, because of the integrating effect of the metallic tip of the probe. However, the difference between the two types was not appreciable, due probably to the small size of the tip and its fast response (0.28 sec.). Therefore, both kinds of probes were used.

To get the best temporal average of the fluctuation output of the thermocouple loop, several methods were tried : a strip chart recorder with low span (0 - 1 mv.) on which the averaging was done visually; a very sensitive ($0.004 \mu\text{Amp/mm.}$) critically-damped galvanometer with a natural period of oscillation of

4.5 seconds; and an integrating system. The latter was finally employed because of its accuracy and simplicity of operation. It was composed of a DYMEC voltage-to-frequency converter, model (No. 2210) and an electronic frequency counter (Hewlett-Packard model 3734A). The mean temperature could be obtained with a resolution of 0.05°C . for periods of one or ten seconds.

PROCEDURE

Eleven sets of operating conditions were selected, based on nozzle size, net power input to the gas and gas flow rate. These are listed in Table II with their identification numbers set. To avoid radiation effects from the progressively hotter graphite wall, runs were conducted over short periods of less than four minutes. Hence the wall temperature at the end of a run never exceed 200°C .

The temperature and velocity profiles around the axis being relatively flat (Fig. 5, Kubanek profiles), temperature and velocity measurements were conducted simultaneously, each probe being 1/16-in. from the jet axis. Readings were taken every fifteen seconds.

For heat transfer rate measurements, the water flow rate was chosen so as to produce an appreciable temperature difference, of the order of 2°C . The water temperature at each cross-section of the test region of the cylinder was obtained by lateral displacement of the tube-thermocouple assembly. After the

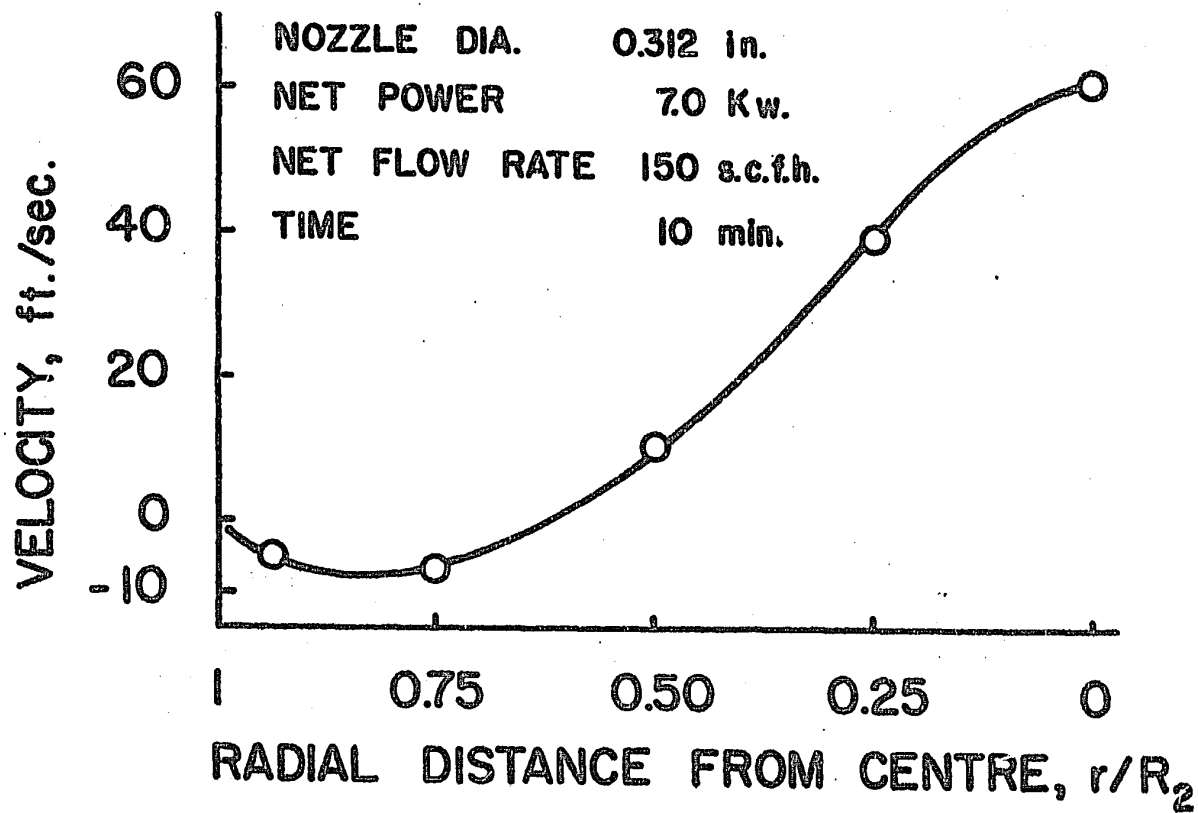
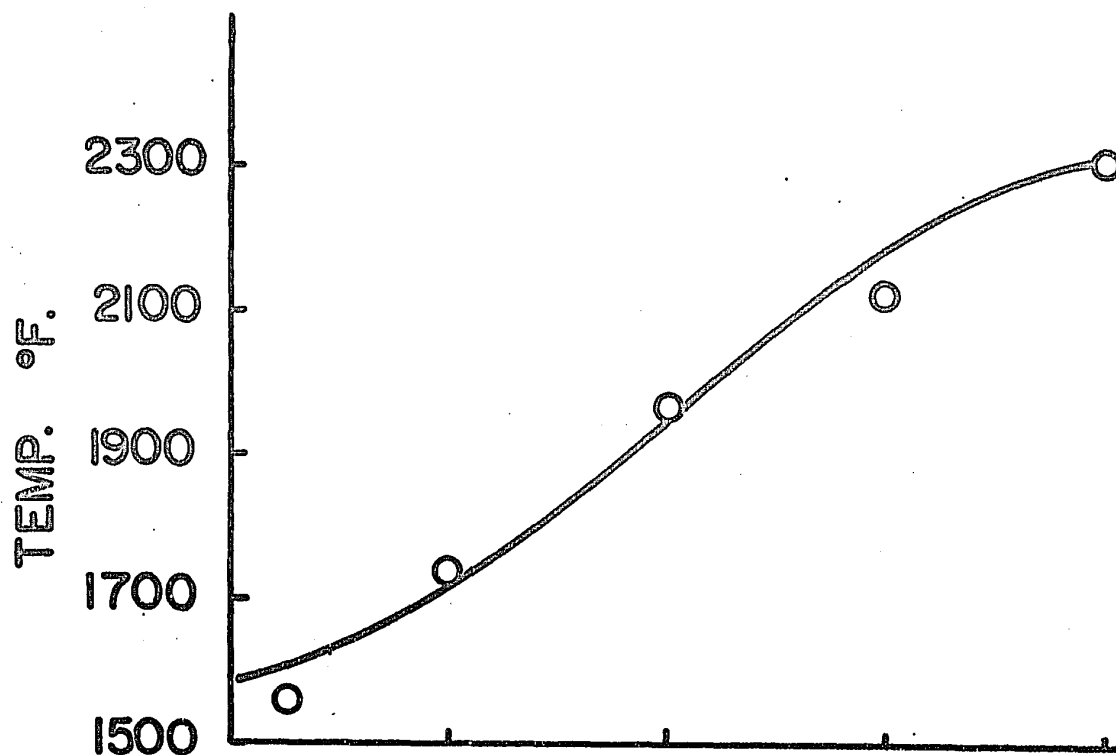
TABLE II

DESIGNATION OF THE PLASMA-TORCH OPERATING
CONDITIONS

Nozzle No.	Net Power Input to gas (Kw.)	Nitrogen flow (s.c.f.h.)	Run designation
1	11.0	100	1-11 - 100
1	11.0	150	1-11 - 150
3	11.0	150	3-11 - 150
3	11.0	200	3-11 - 200
3	14.0	100	3-14 - 100
3	14.0	150	3-14 - 150
3	14.0	200	3-14 - 200
3	19.5	150	3-19.5-150
3	19.5	200	3-19.5-200
3	23.5	150	3-23.5-150
3	23.5	200	3-23.5-200

FIGURE 5

**Temperature and Velocity Profiles
as Obtained by Kubanek (1)**



system had been positioned, the new steady-state was reached within from four to five seconds. The temperature was then measured for ten seconds. The water flow rate was determined by duplicate measurements of the time required to fill up a graduated 1000-cm.³ cylinder.

When heat transfer experiments for a cylinder were completed, the stability of plasma flame generation was checked by means of temperature measurements of the nitrogen stream.

Because slight changes took place in the temperature and velocity of the jet as the chamber was heated up, all the measured quantities were determined from a plot of these quantities vs. time and, evaluated at time $t = 3$ minutes.

STATE AND PHYSICAL PROPERTIES OF THE GAS

In the present system, the nitrogen stream experienced very large temperature changes. In the case of the highest enthalpy level run (3 - 23.4 - 150), the temperature and velocity dropped from 10,000°F. and 2,500 ft./sec. to 4,300°F and 110 ft./sec. along the axis of the jet, between the nozzle and the test section. Thus the rate of temperature change was of the order of 10^6 °F./sec. Such a rate of change approaches the lower end of the range of temperature changes ($10^6 - 10^{13}$ °K./sec.) in which heterogeneous nonequilibrium systems are formed, in which some molecules are excited to much higher energy levels than others (52). However, Kubanek (1) calculated that the

nitrogen molecules issuing from the nozzle of the torch would undergo about 10^6 collisions before reaching the test section. According to Greene et al. (53) and Blackman (54) molecular nitrogen required about 20 and $10^4 - 10^5$ collisions for rotational and vibrational relaxations, respectively. Hence it is reasonable to expect the gas at the test section to be in a state close to thermal equilibrium, since the requirements for it are almost satisfied for the most severe temperature change.

For temperatures of 10,000 (at nozzle exit) and 5,000°F. (at the test section) the equilibrium composition between the species, as given by Drellishak et al. (55), are presented in Table II which shows that the plasma at the nozzle, if at equilibrium, is in a partly molecular (70%) and partly dissociated (30%) form, while the gas is in a quasi-molecular state at the test section. Hence, the physical properties used at the test section were assumed to be those of molecular nitrogen at atmospheric pressure as proposed by John et al. (56) for the viscosity, references (56, 57, 58) for the thermal conductivity, and Drellishak et al. (55) for the density and the specific heat at constant pressure. The variations of these properties with temperature are shown in Fig. 6, 7, 8, 9.

TABLE III

EQUILIBRIUM COMPOSITION OF GAS AT NOZZLE EXIT
AND AT CENTRE OF TEST-SECTION FOR RUN 3-23.4-150

Species	Plasma composition	Centre test-section composition
Molecules (N_2) cm. ⁻³	6.8×10^{17}	2.4×10^{18}
Ionized Molecules (N_2^+) cm. ⁻³	3.3×10^{13}	1.5×10^7
Atoms (N) cm. ⁻³	4.5×10^{17}	6.1×10^{13}
Ionized Atoms (N^+) cm. ⁻³	1.0×10^{14}	1.7×10^4
Electrons cm. ⁻³	1.3×10^{14}	1.5×10^7
Total Particles cm. ⁻³	1.1×10^{18}	2.4×10^{18}

FIGURE 6

Nitrogen Viscosity at Atmospheric Pressure

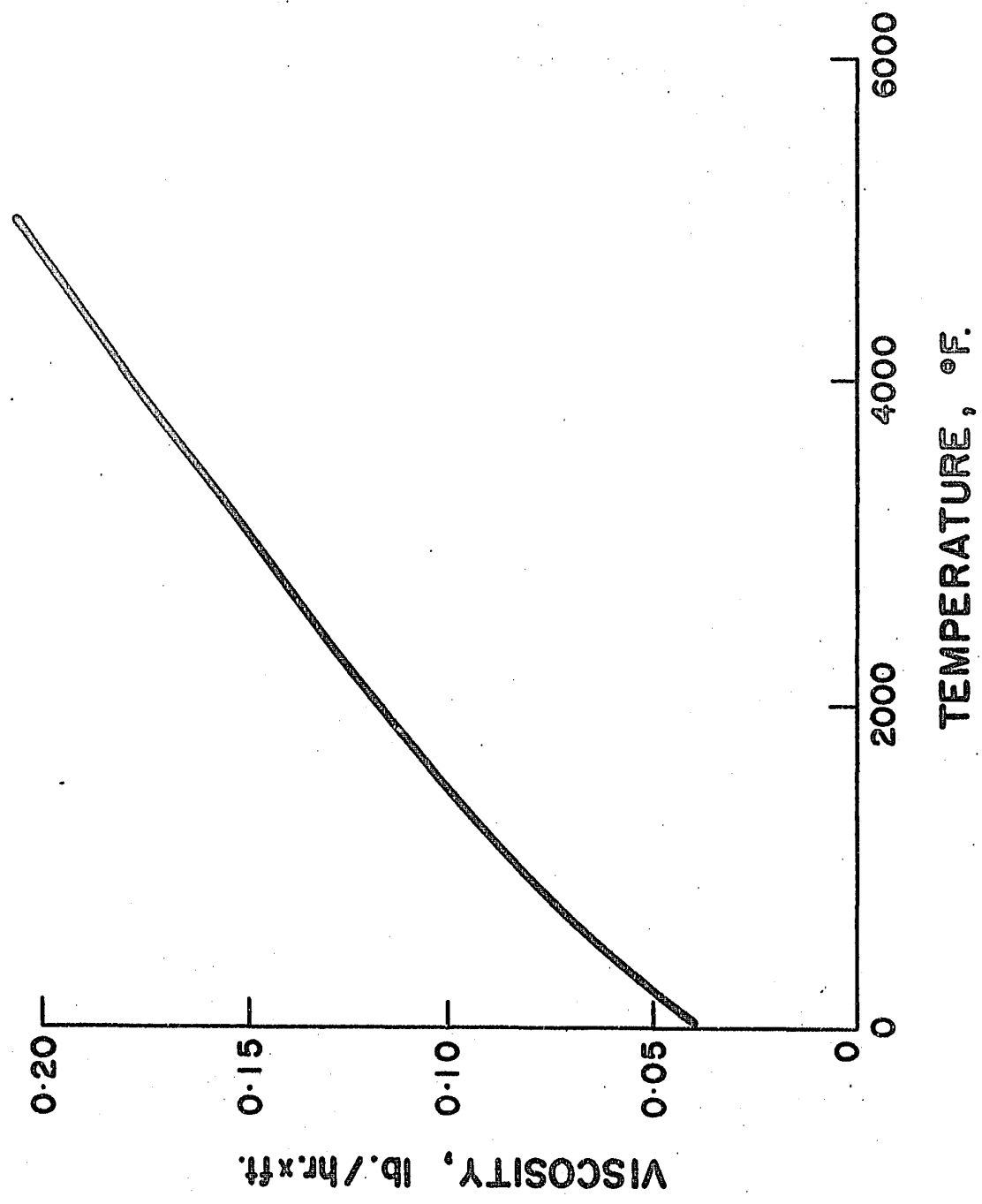


FIGURE 7

**Nitrogen Thermal Conductivity
at Atmospheric Pressure**

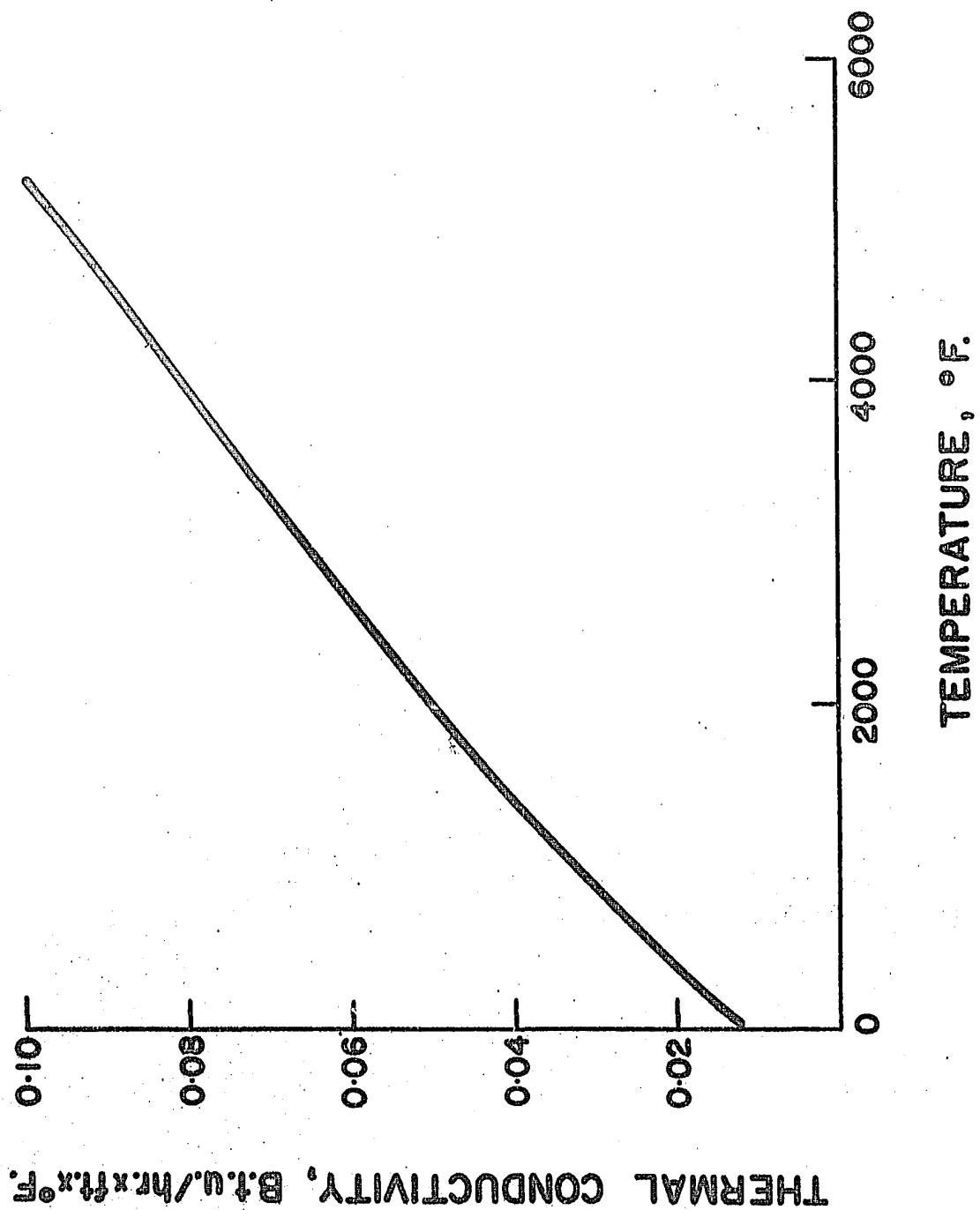


FIGURE 8**Nitrogen Density at Atmospheric Pressure**

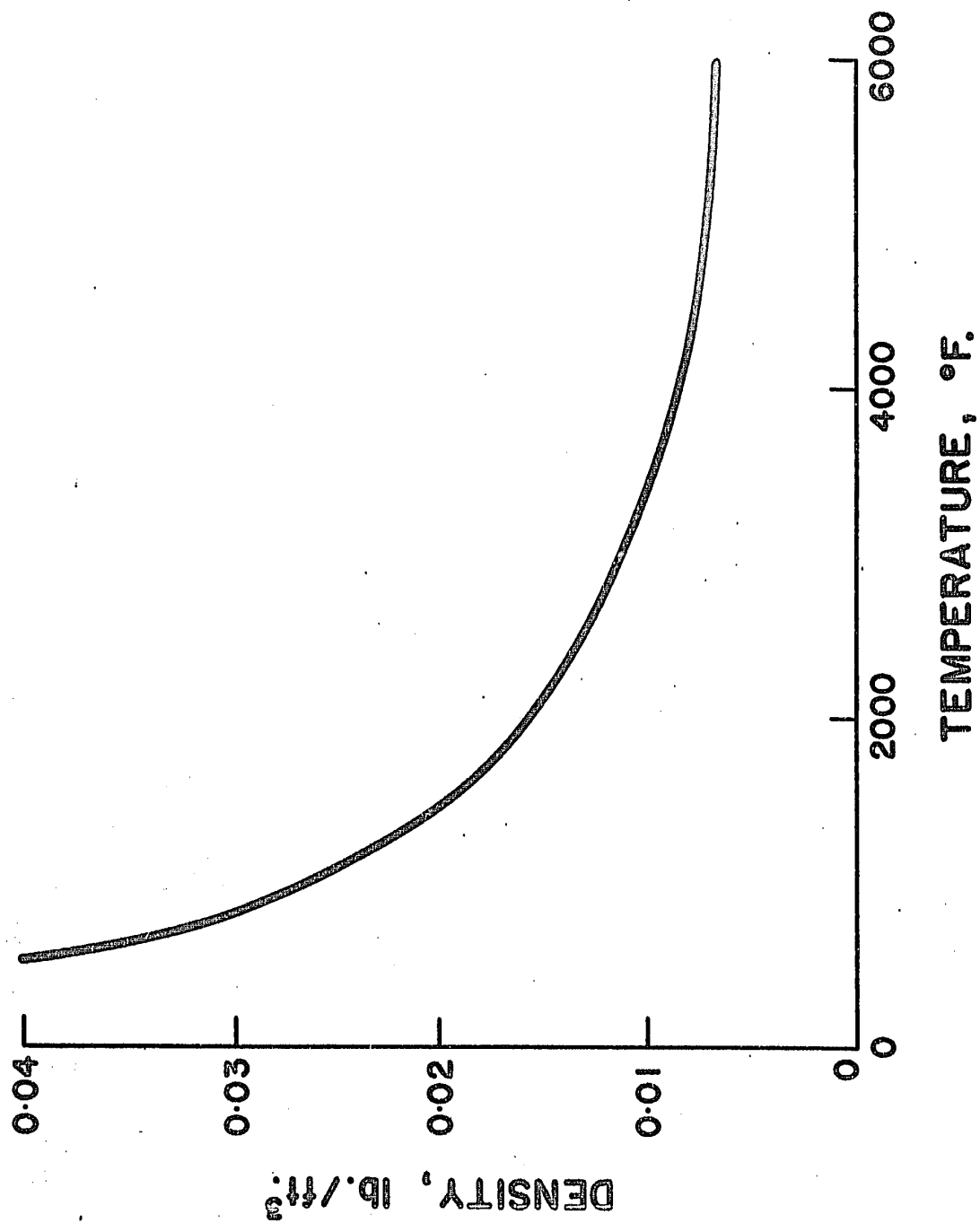
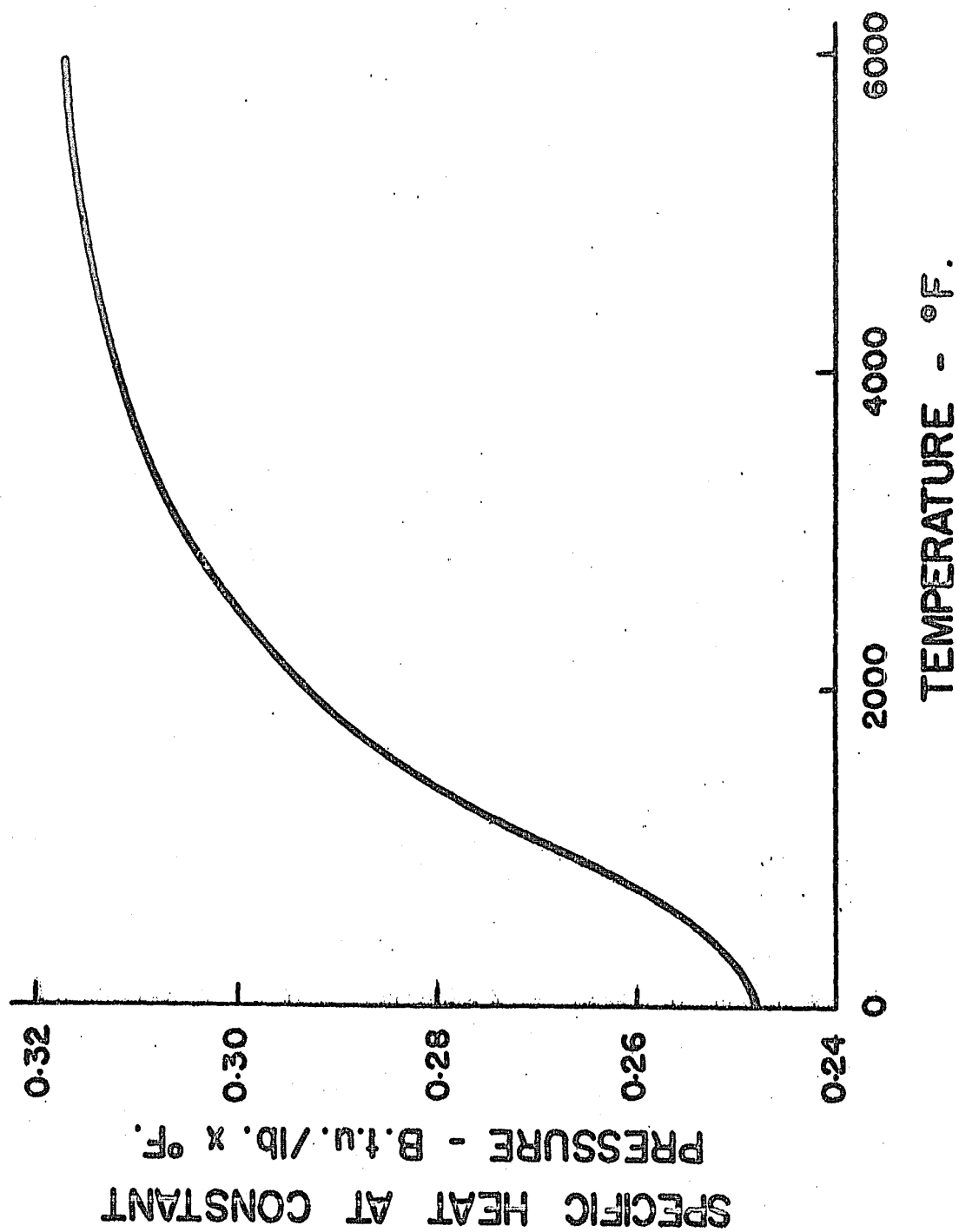


FIGURE 9

**Specific Heat at Constant Pressure
of Nitrogen at Atmospheric Pressure**



RESULTS

Velocity and Temperature Measurements

Data for simultaneous velocity and temperature measurements are presented as a table in Appendix A. Averaged temperature and velocity of the jet for each operating condition are given in Table IV along with the radiation and conduction correction for the temperature. Temperatures and velocities were found to be reproducible with a maximum deviation of 8 and 5%, respectively. Temperature fluctuations of about 100°F. were observed in the jet.

Heat Transfer Measurements

Heat transfer data are reported in the Nusselt number form using both the bulk and film temperatures as reference temperature, on the basis of the equation :

$$Nu = M c_p (T_{OUT} - T_{IN}) / A (T_b - T_w) (D/k) \quad (10)$$

They are shown in Tables V and VI for the 1/4-in. o.d. circular cylinder, Tables VII and VIII for the 1/8-in. o.d. circular cylinder, Tables IX and X for the 1/4-in. square cylinder with a face perpendicular to the jet, and Tables XI and XII for the 1/4-in. square cylinder with an edge facing the jet.

Plots of the results are presented in Fig. 10 and 11 for the circular cylinders, Fig. 12 and 13 for the square cylinder-face, and Fig. 14 and 15 for the square cylinder-edge. For

TABLE IV
TEMPERATURE AND VELOCITY RESULTS

R U N	ΔP^* avg. mm. CH ₃ OH	T _c avg. °F.	ΔT cond. °F.	ΔT rad. °F.	T _b °F.	V _b ft./sec.
1-11.0-100	202	1930	218	111	2260	77.7
1-11.0-150	279	1690	160	74	1920	85.4
3-11.0-150	129	2010	267	165	2540	64.4
3-11.0-200	151	1863	237	115	2210	63.9
3-14.0-100	102	2606	444	381	3440	66.1
3-14.0-150	152	2270	335	216	2820	73.3
3-14.0-200	194	2085	249	165	2550	80.0
3-19.5-150	197	2927	499	473	3900	96.7
3-19.5-200	290	2701	377	310	3390	105.8
3-23.4-150	227	3180	539	553	4270	108.7
3-23.4-200	312	2980	439	425	3840	121.7

* inclination at 1/25.

TABLE V. HEAT TRANSFER DATA FOR 1/4-IN. O.D. CIRCULAR CYLINDER
WITH PROPERTIES EVALUATED AT BULK TEMPERATURE

R U N	V _b ft./sec.	T _b °F.	T _b -T _w °F.	Temp. basis °F.	T _b /T _w °R./°R.	(Re) _b	(Nu) _b			
1-11.0-100	78	2260	2130	2260	4.61	650	11.10	12.30		
1-11.0-150	85	1920	1790	1920	4.03	886	15.90	15.30	17.20	
3-11.0-150	64	2540	2410	2540	5.08	452	8.10	9.45	9.75	
3-11.0-200	64	2210	2080	2210	4.52	551	10.70	12.10	11.70	12.10
3-14.0-100	66	3440	3310	3440	6.61	300	6.36	6.70	6.70	
3-14.0-150	73	2820	2690	2820	5.56	444	8.80	9.20	8.25	
3-14.0-200	80	2550	2420	2550	5.11	565	10.20	10.40	11.10	10.10
3-19.5-150	97	3900	3770	3900	7.37	361	7.56	7.20	6.80	
3-19.5-200	106	3390	3260	3390	6.52	490	10.80	9.80	9.10	
3-23.4-150	109	4270	4140	4270	8.03	351	7.68	7.80	7.4	
3-23.4-200	121	3880	3750	3880	7.29	460	8.50	8.52	8.37	8.05

TABLE VI. HEAT TRANSFER DATA FOR 1/4-IN. O.D. CIRCULAR CYLINDER
WITH PHYSICAL PROPERTIES EVALUATED AT FILM TEMPERATURE

R U N	V _b ft./sec.	T _b °F.	T _b -T _w °F.	Temp. basis °F.	T _b /T _w °R./°R.	(Re) _f	(Nu) _f			
1-11.0-100	78	2260	2130	1195	4.61	1462	16.70	18.50		
1-11.0-150	85	1920	1790	1025	4.03	1915	25.20	13.40	22.50	
3-11.0-150	64	2540	2410	1335	5.08	1049	12.30	14.30	14.80	
3-11.0-200	64	2210	2080	1170	4.52	1234	16.00	18.10	17.60	18.10
3-14.0-100	66	3440	3310	1785	6.61	758	10.00	10.50	10.50	
3-14.0-150	73	2820	2690	1475	5.56	1062	13.50	14.10	12.70	
3-14.0-200	80	2550	2420	1340	5.11	1311	15.60	15.95	17.00	15.42
3-19.5-150	97	3900	3770	2015	7.37	949	11.95	11.38	11.10	
3-19.5-200	106	3390	3260	1760	6.52	1243	16.80	15.28	14.10	
3-23.4-150	109	4270	4140	2200	8.03	943	12.20	12.40	11.80	
3-23.4-200	121	3880	3750	2005	7.29	1197	13.40	13.48	13.20	12.70

TABLE VII. HEAT TRANSFER DATA FOR 1/8-IN. O.D. CIRCULAR CYLINDER
WITH PROPERTIES EVALUATED AT BULK TEMPERATURE

R U N	V _b ft./sec.	T _b °F.	T _b -T _w °F.	Temp. basis °F.	T _b /T _w °R/°R.	(Re) _b	(Nu) _b			
1-11.0-100	78	2260	2130	2260	4.61	325	8.00	8.17		
1-11.0-150	85	1920	1790	1920	4.03	443	9.60	9.85		
3-11.0-150	64	2540	2410	2540	5.08	226	6.34			
3-11.0-200	64	2210	2080	2210	4.52	275	7.12	7.46	7.70	
3-14.0-100	66	3440	3310	3440	6.61	150	4.55	5.01	4.30	5.04
3-14.0-150	73	2820	2690	2820	5.56	222	6.46	6.62		
3-14.0-200	80	2550	2420	2550	5.11	282	6.70	7.22	7.16	
3-19.5-150	97	3900	3770	3900	7.37	180	4.86	5.12		
3-19.5-200	106	3390	3260	3390	6.52	245	6.34			
3-23.4-150	109	4270	4140	4270	8.03	175	5.14	5.32		
3-23.4-200	121	3880	3750	3880	7.29	230	6.12			

TABLE VIII. HEAT TRANSFER DATA FOR 1/8-IN. O.D. CIRCULAR CYLINDER
WITH PROPERTIES EVALUATED AT FILM TEMPERATURE

R U N	V_b ft./sec.	T_b °F.	$T_b - T_w$ °F.	Temp. basis °F.	T_b/T_w °R./°R.	$(Re)_f$	$(Nu)_f$			
1-11.0-100	78	2260	2130	1195	4.61.	731	12.00	12.22		
1-11.0-150	85	1920	1790	1025	4.03.	958	14.10	14.50		
3-11.0-150	64	2540	2410	1335	5.08.	525	9.65			
3-11.0-200	64	2210	2080	1170	4.52.	617	10.66	11.18	11.52	
3-14.0-100	66	3440	3310	1785	6.61.	379	7.17	7.90	6.76	7.95
3-14.0-150	73	2820	2690	1475	5.56.	531	9.97	10.20		
3-14.0-200	80	2550	2420	1340	5.11.	656	10.25	11.10	11.00	
3-19.5-150	97	3900	3770	2015	7.37.	475	7.69	8.10		
3-19.5-200	106	3390	3260	1760	6.52.	621	9.85			
3-23.4-150	109	4270	4140	2200	8.03.	471	8.17	8.48		
3-23.4-200	121	3880	3750	2005	7.29.	599	9.67			

TABLE IX. HEAT TRANSFER DATA FOR 1/4-IN. SQUARE CYLINDER
WITH A FACE PERPENDICULAR TO THE JET. PROPERTIES AT BULK TEMPERATURE

R U N	V_b ft./sec.	T_b °F.	$T_b - T_w$ °F.	Temp. basis °F.	T_b/T_w °R./°R.	$(Re)_b$	$(Nu)_b$		
1-11.0-100	78	2260	2130	2260	4.61	650	9.60	9.90	
1-11.0-150	85	1920	1790	1920	4.03	886	14.50	14.00	
3-11.0-150	64	2540	2410	2540	5.08	452	8.00	7.19	8.90
3-11.0-200	64	2210	2080	2210	4.52	551	10.20	9.92	
3-14.0-100	66	3440	3310	3440	6.61	300	6.60	5.90	5.50
3-14.0-150	73	2820	2690	2820	5.56	444	9.02	9.20	
3-14.0-200	80	2550	2420	2550	5.11	565	9.68	10.50	10.30
3-19.5-150	97	3900	3770	3900	7.37	361	7.00	7.35	
3-19.5-200	106	3390	3260	3390	6.52	490	9.70	8.23	
3-23.4-150	109	4270	4140	4270	8.03	351	7.76	8.00	
3-23.4-200	121	3880	3750	3880	7.29	460	9.26	10.30	10.40

TABLE X. HEAT TRANSFER DATA FOR 1/4-IN. SQUARE CYLINDER WITH
A FACE PERPENDICULAR TO THE JET. PROPERTIES AT FILM TEMPERATURE

R U N	V_b ft./sec.	T_b °F.	$T_b - T_w$ °F.	Temp. basis °F.	T_b/T_w °R./°R.	$(Re)_f$	$(Nu)_f$		
1-11.0-100	78	2260	2130	1195	4.61	1462	14.39	14.95	
1-11.0-150	85	1920	1790	1025	4.03	1915	21.30	20.62	
3-11.0-150	64	2540	2410	1335	5.08	1049	12.15	10.90	13.60
3-11.0-200	64	2210	2080	1170	4.52	1234	15.27	14.86	
3-14.0-100	66	3440	3310	1785	6.61	758	10.40	9.31	8.66
3-14.0-150	73	2820	2690	1475	5.56	1062	13.87	14.17	
3-14.0-200	80	2550	2420	1340	5.11	1311	14.84	16.10	15.75
3-19.5-150	97	3900	3770	2015	7.37	949	11.06	11.61	
3-19.5-200	106	3390	3260	1760	6.52	1243	15.10	12.80	
3-23.4-150	109	4270	4140	2200	8.03	943	12.70	12.37	
3-23.4-200	121	3880	3750	2005	7.29	1197	16.45	14.64	16.32

TABLE XI. HEAT TRANSFER DATA FOR 1/4-IN. SQUARE CYLINDER
WITH AN EDGE FACING THE JET. PROPERTIES AT BULK TEMPERATURE

R U N	V_b ft./sec.	T_b °F.	$T_b - T_w$ °F.	Temp. basis °F.	T_b/T_w °R./°R.	$(Re)_b$	$(Nu)_b$	
1-11.0-100	78	2260	2130	2260	4.61	650	11.70	12.25
1-11.0-150	85	1920	1790	1920	4.03	886	16.00	16.90
3-11.0-150	64	2540	2410	2540	5.08	452	10.20	9.10
3-11.0-200	64	2210	2080	2210	4.52	551	11.80	11.43
3-14.0-100	66	3440	3310	3440	6.61	300	8.50	8.20
3-14.0-150	73	2820	2690	2820	5.56	444	10.94	11.40
3-14.0-200	80	2550	2420	2550	5.11	565	11.60	10.46
3-19.5-150	97	3900	3770	3900	7.37	361	8.35	10.50
3-19.5-200	106	3390	3260	3390	6.52	490	12.60	10.70
3-23.4-150	109	4270	4140	4270	8.03	351	9.26	10.50 8.70
3-23.4-200	121	3880	3750	3880	7.29	460	9.58	10.80

TABLE XII. HEAT TRANSFER DATA FOR 1/4-IN. SQUARE CYLINDER
WITH AN EDGE FACING THE JET. PROPERTIES AT FILM TEMPERATURE

R U N	V_b ft./sec.	T_b °F.	$T_b - T_w$ °F.	Temp. basis °F.	T_b/T_w °R./°R.	$(Re)_f$	$(Nu)_f$	
1-11.0-100	78	2260	2130	1195	4.61	1462	17.52	18.40
1-11.0-150	85	1920	1790	1025	4.03	1915	23.51	24.88
3-11.0-150	64	2540	2410	1335	5.08	1049	15.51	13.80
3-11.0-200	64	2210	2080	1170	4.52	1234	17.66	17.12
3-14.0-100	66	3440	3310	1785	6.61	758	13.40	12.90
3-14.0-150	73	2820	2690	1475	5.56	1062	16.83	16.43
3-14.0-200	80	2550	2420	1340	5.11	1311	17.83	16.03
3-19.5-150	97	3900	3770	2015	7.37	949	13.17	18.58
3-19.5-200	106	3390	3260	1760	6.52	1243	19.60	16.90
3-23.4-150	109	4270	4140	2200	8.03	943	14.76	16.78 13.95
3-23.4-200	121	3880	3750	2005	7.29	1197	15.14	17.12

FIGURE 10

**Heat Transfer Results for 1/4 and 1/8-in. o.d.
Circular Cylinders. Properties at
Bulk Temperature**

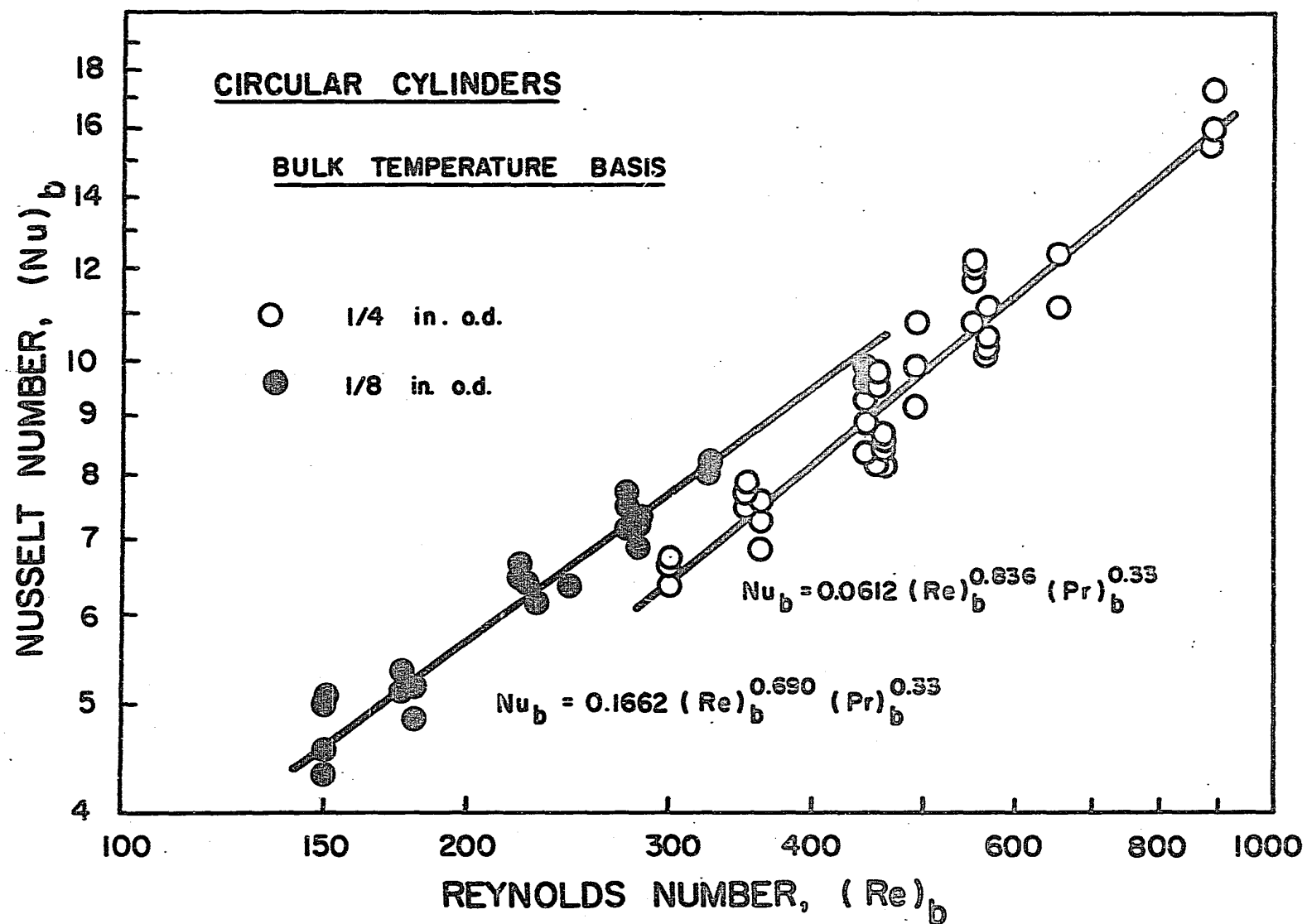


FIGURE 11

**Heat Transfer Results for 1/4 and 1/8-in. o.d.
Circular Cylinders. Properties at
Film Temperature**

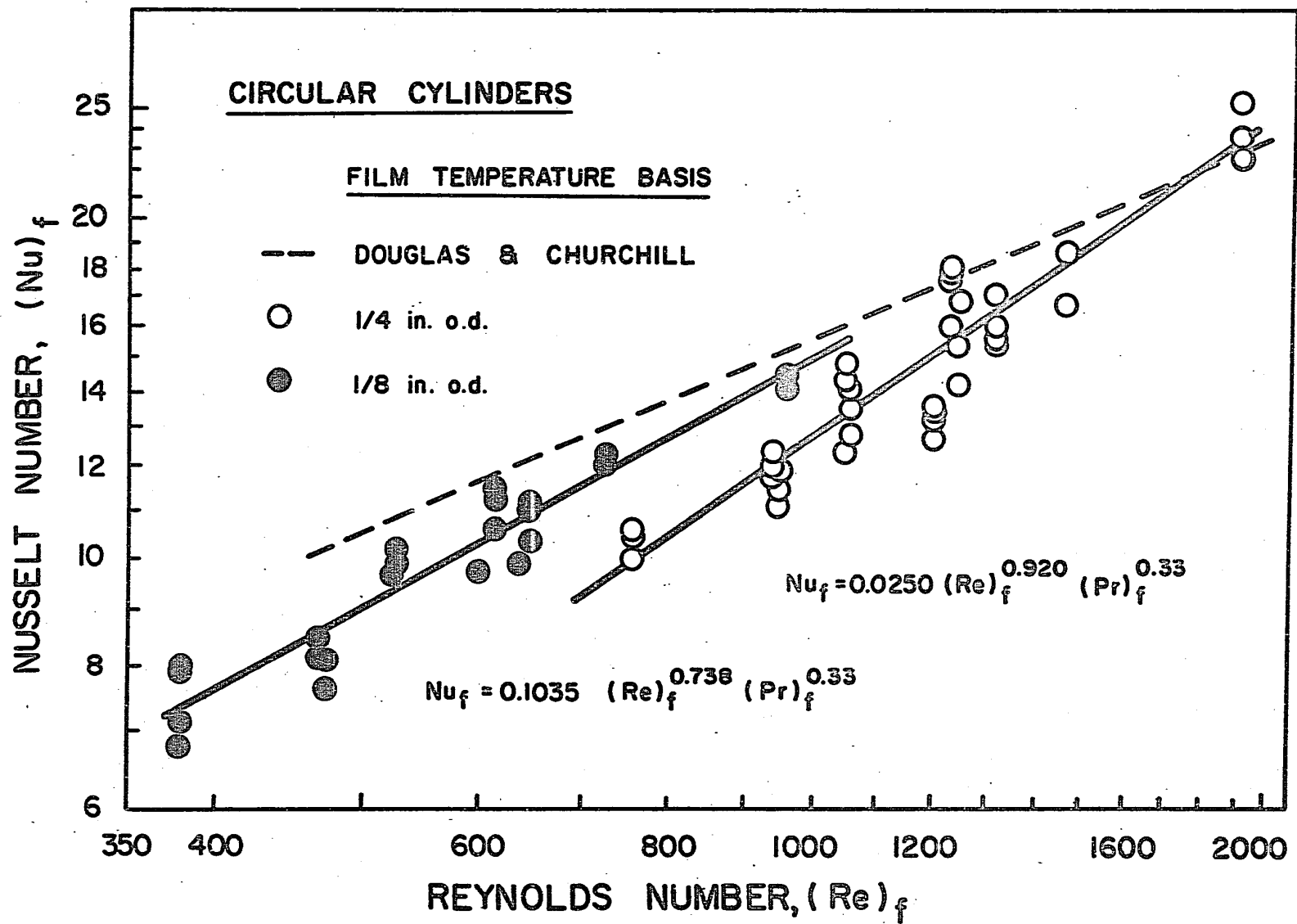


FIGURE 12

**Heat Transfer Results for 1/4-in. Square Cylinder
with a Face Perpendicular to the Jet.**

Properties at Bulk Temperature

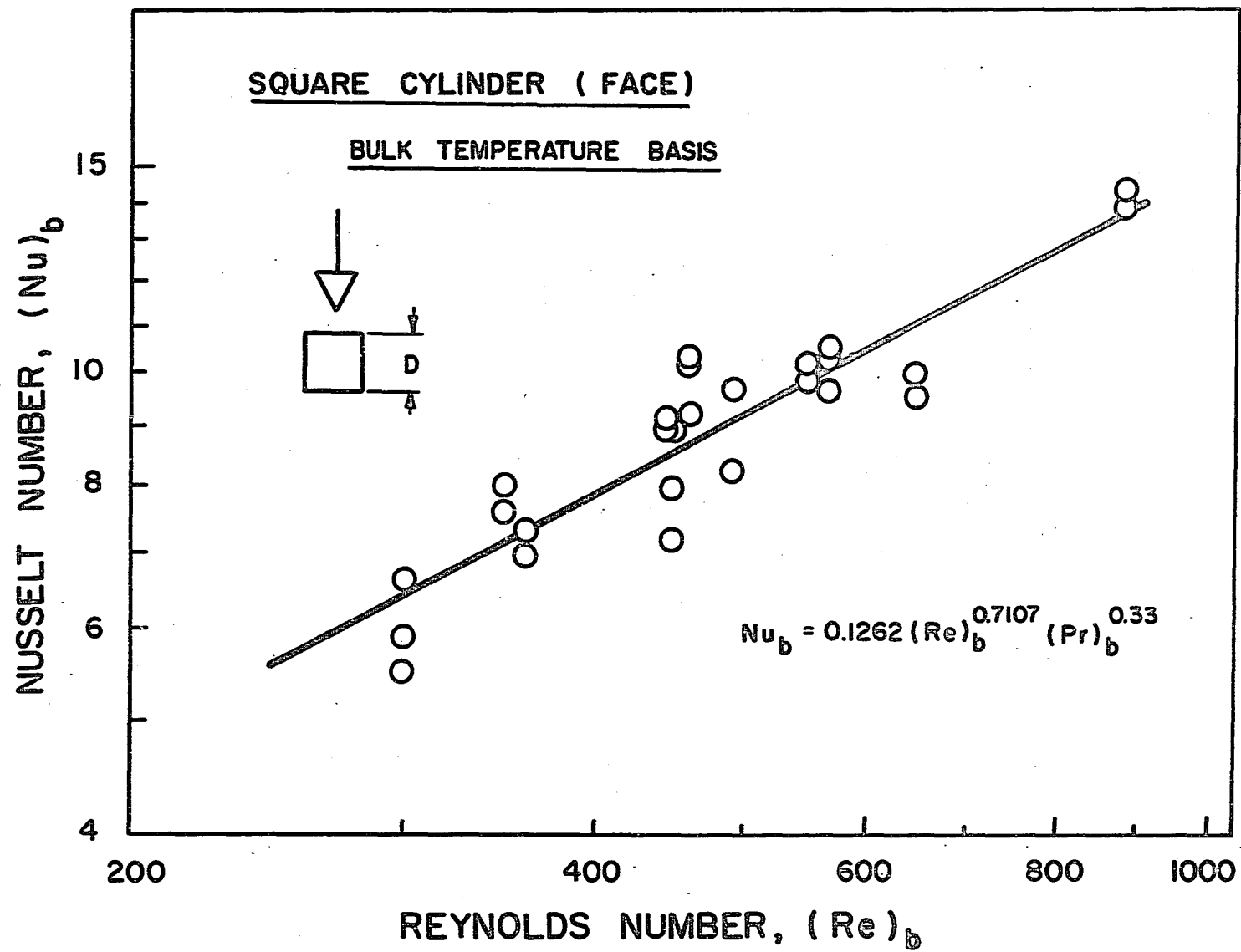


FIGURE 13

**Heat Transfer Results for 1/4-in. Square Cylinder
with a Face Perpendicular to the Jet.
Properties at Film Temperature**

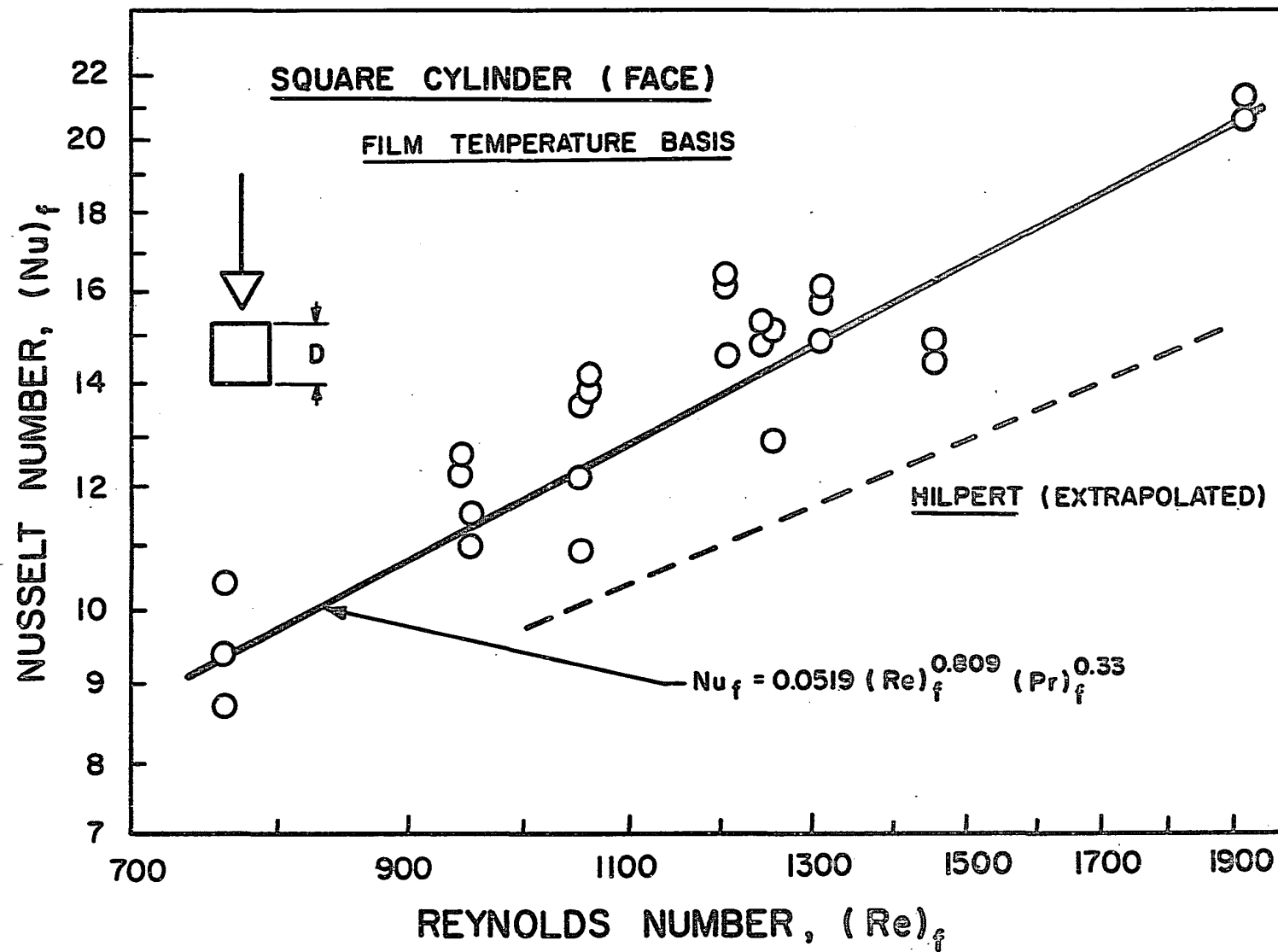


FIGURE 14

**Heat Transfer Results for 1/4-in. Square Cylinder
with an Edge Facing the Jet.
Properties at Bulk Temperature**

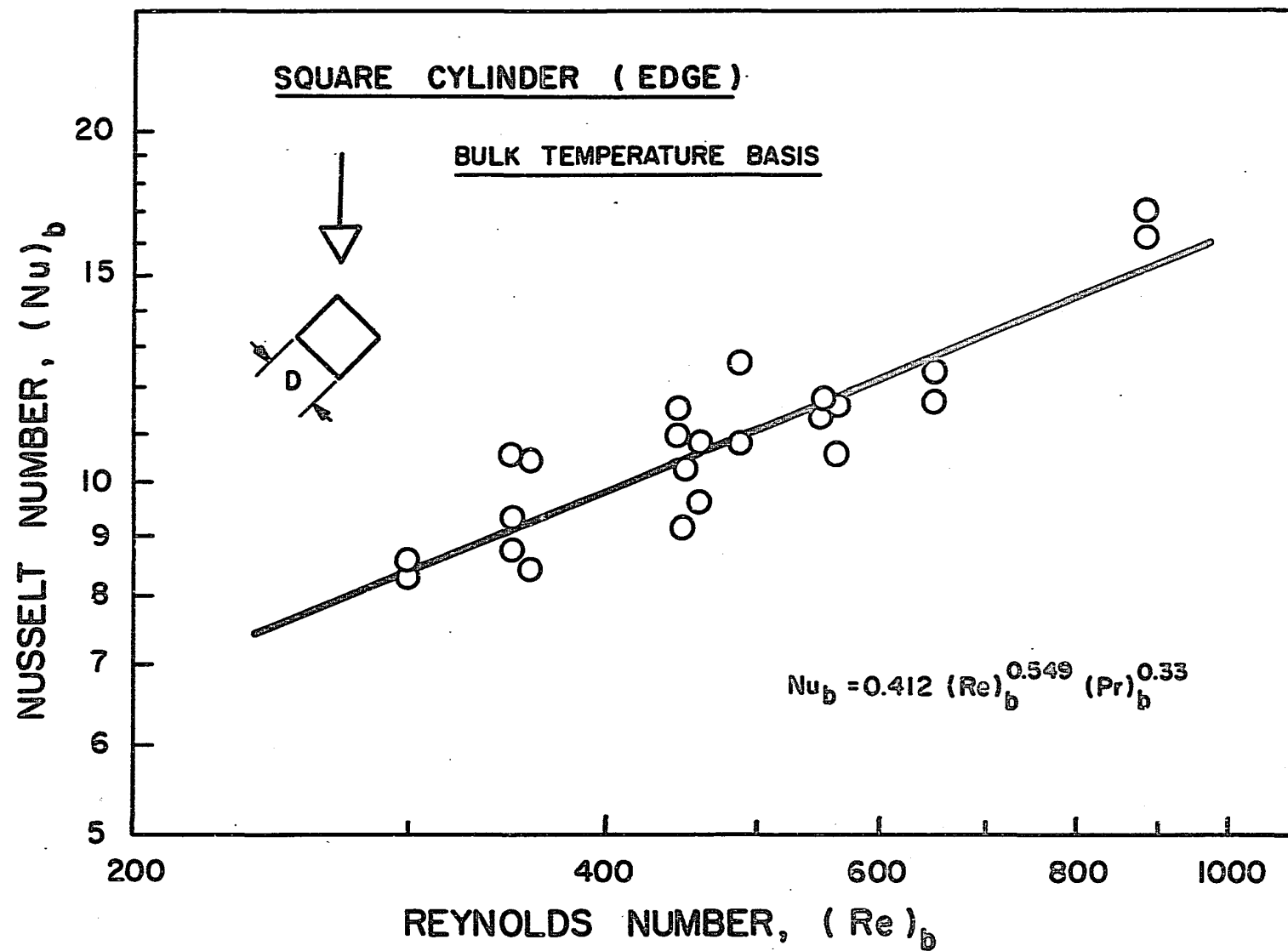
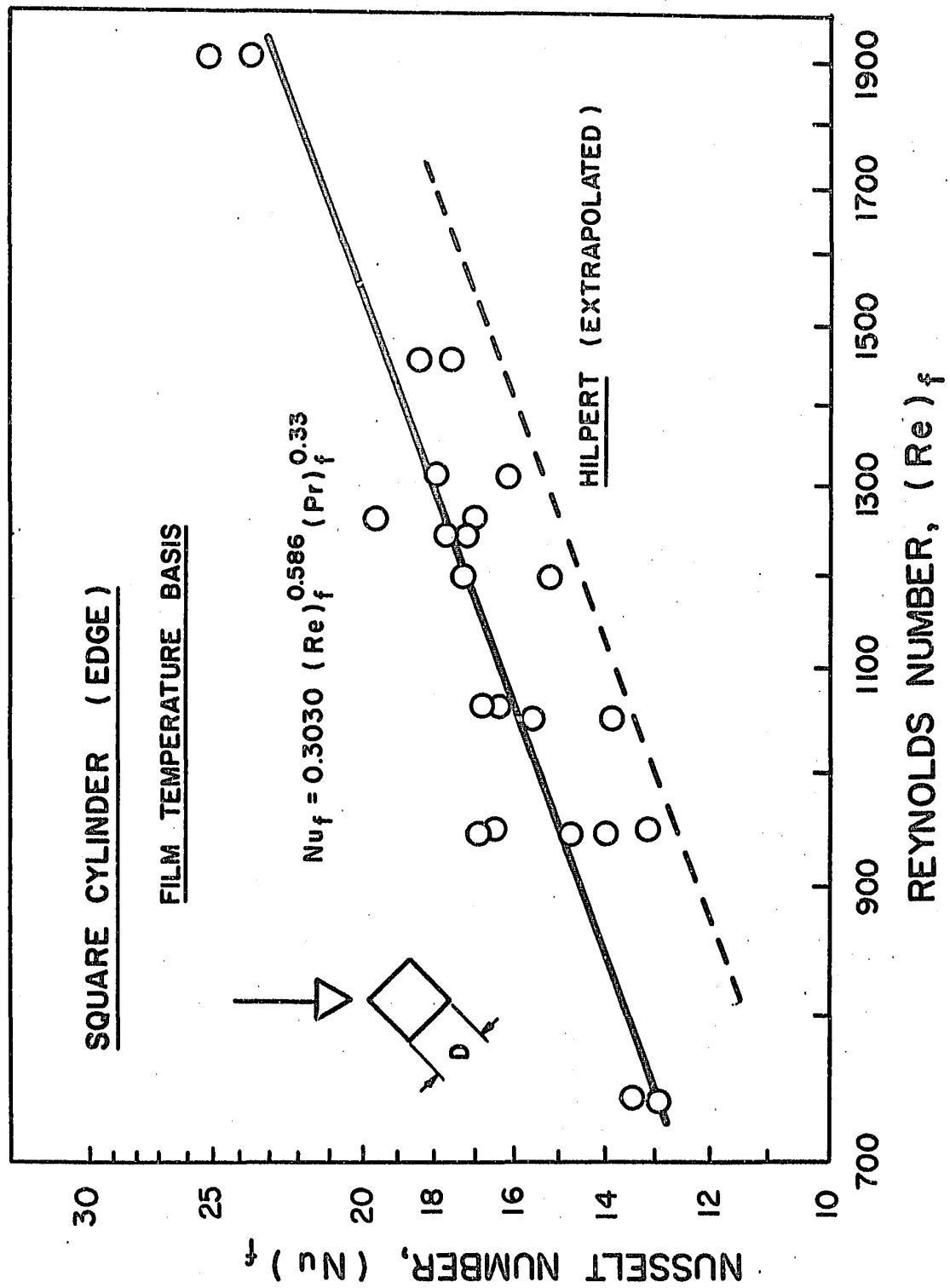


FIGURE 15

**Heat Transfer Results for 1/4-in. Square Cylinder
with an Edge Facing the Jet.
Properties at Film Temperature**



circular cylinders, comparisons of the present results with those of Douglas and Churchill and Churchill and Brier are given in Fig. 11 and 16, respectively. For the square cylinders, the line of Hilpert (16) has been extrapolated down to the Re range studied (the lower limit of the range investigated by this author was 5,000).

A comparison of the heat transfer correlations for the different geometries (including the results for spheres obtained by Kubanek (1)) is presented in Fig. 17. The values of the physical properties at bulk and film temperatures are given in Appendices B and C, respectively. It was ascertained that conduction effects along the wall of the cylinders, as well as natural convection effects ($Gr/Re^2 < 1$) were unimportant compared to the convective heat transfer.

Treatment of the Data

Two relationships have been tried to fit the data. The first model was :

$$Nu = a Re^n \quad (11)$$

while in the second a temperature ratio (T_b/T_w) was included to account for the large temperature difference and its effect on the physical properties :

$$Nu = a' Re^{n'} (T_b/T_w)^m \quad (12)$$

Results of the least-square analysis by computer are pre-

FIGURE 16

**Comparison of the Heat Transfer Data
for Circular Cylinders with Those of
Churchill and Brier**

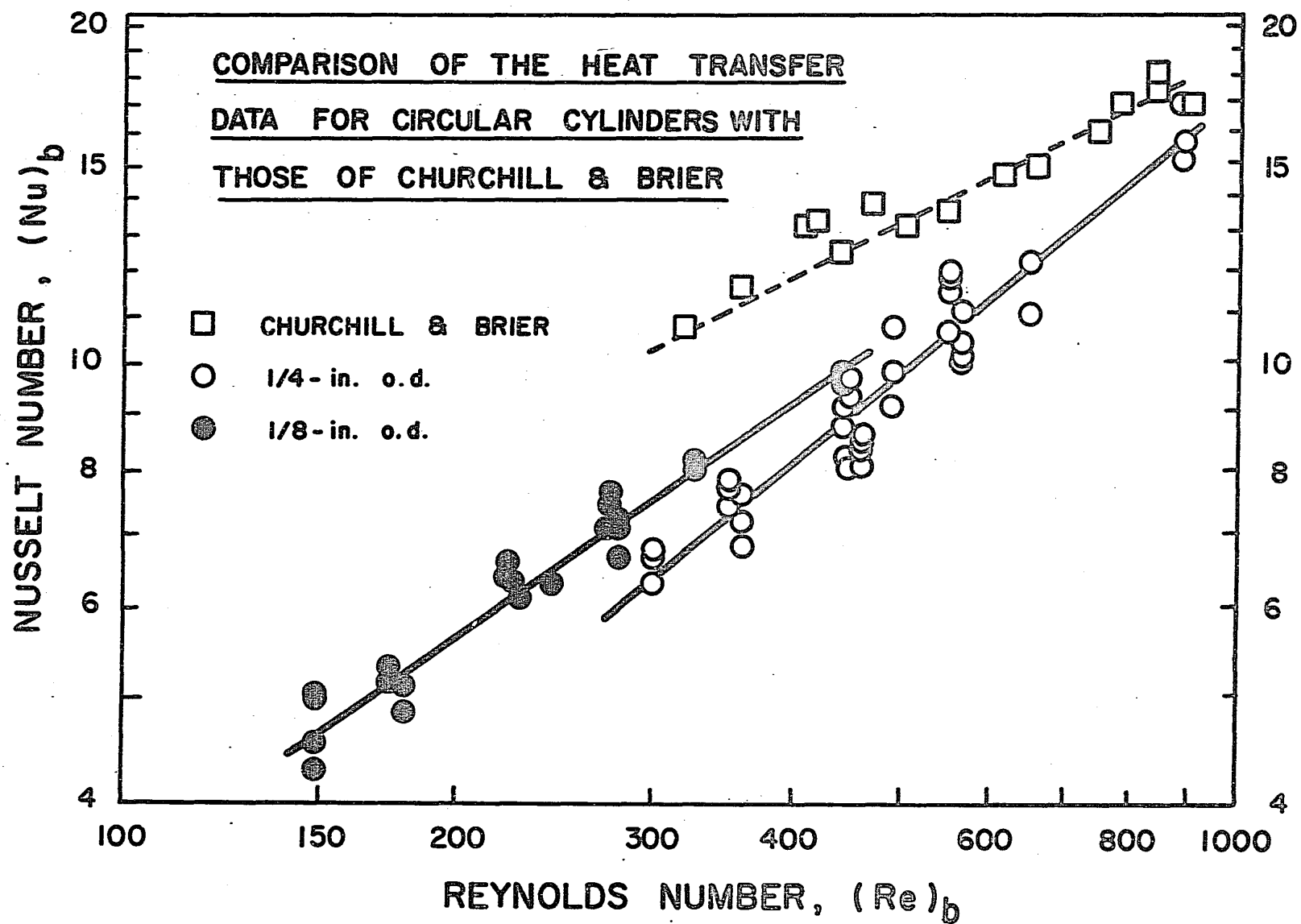
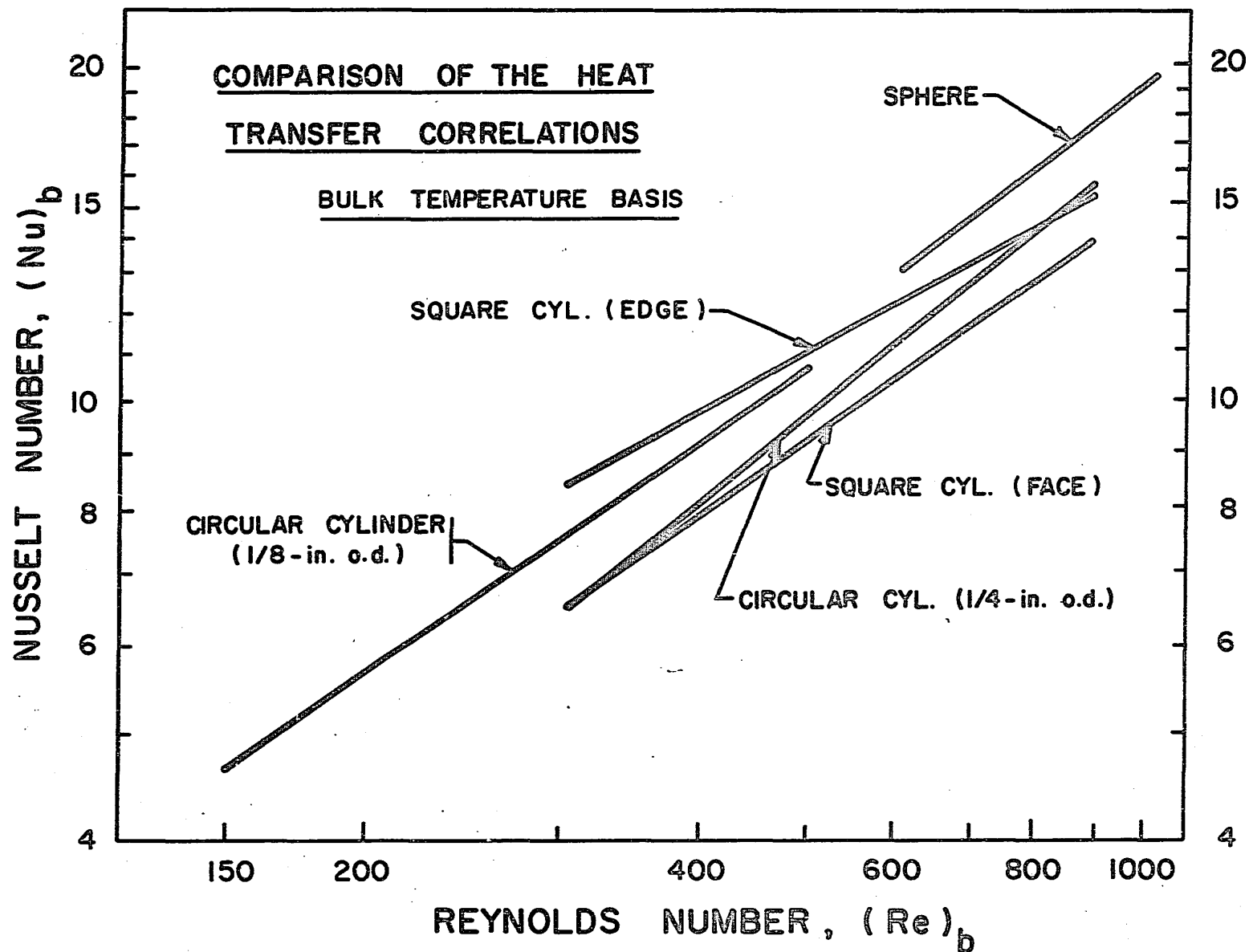


FIGURE 17**Comparison of the Heat Transfer Correlations**



sented in Table XIII.

The Prandtl number, being practically constant ($0.69 < Pr < 0.71$), was not considered during the processing of the data but was included at the end to generalize the correlations. The temperature of the wall being in the narrow range 120-140°F. an average value of 130°F., was used in all the calculations.

DISCUSSION OF RESULTS

Table XIII clearly shows that the exponent of the Reynolds number is greater than 0.5, which is interpreted as indicating that a part of the attached boundary layer is turbulent. According to Torobin and Gauvin (41), transition in the boundary layer will take place even at low Reynolds number if the turbulent energy of the free-stream is large enough (compared to the viscous damping energy in the initially-laminar boundary layer) to allow sufficient penetration of the disturbances into the boundary layer. It seems that, in the present study, transition to a turbulent boundary layer was attained at various distances downstream from the stagnation point (as reflected by the varying values of n reported in Table XIII) owing to the following considerations :

- 1) The very high turbulence level of the jet generated by the torch. According to Ref. (59) and (60) the intensity of turbulence of a confined jet is about 20 to 25% at a distance of about 40 diameters from the nozzle.

TABLE XIII

PROCESSING OF THE DATA FOR MODEL No.1 ($Nu=aRe^n$) AND MODEL No.2 [$Nu=a'Re^{n'}(T_b/T_w)^m$]




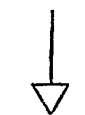
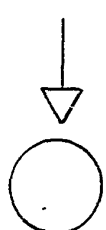
SHAPES		a or a'	SE(log a)	n. or n'	SE.(n) or SE.(n')	m	SE(m)	D.F.	SE(log.y)	Corr. Coeff. R ²
 1/4-in. o.d.	Mod. 1	0.054	0.269	0.836	0.043	-		33	0.072	0.918
	$\frac{T_b}{T_w}$ Mod. 2	0.172		0.708	0.069	-0.204	0.088	32	0.068	0.930
	Mod. 1	0.022	0.439	0.910	0.061			33	0.083	0.863
	$\frac{T_f}{T_w}$ Mod. 2	0.186		0.698	0.066	-0.324	0.068	32	0.064	0.923
 1/8-in. o.d.	Mod. 1	0.147	0.183	0.690	0.033			21	0.051	0.953
	$\frac{T_b}{T_w}$ Mod. 2	0.367		0.582	0.059	-0.187	0.086	20	0.047	0.961
	Mod. 1	0.092	0.312	0.738	0.049			21	0.061	0.915
	$\frac{T_f}{T_w}$ Mod. 2	0.458		0.560	0.059	-0.281	0.071	20	0.047	0.952
	Mod. 1	0.112	0.408	0.710	0.066			24	0.095	0.827
	$\frac{T_b}{T_w}$ Mod. 2	0.0197		0.901	0.085	0.326	0.108	23	0.082	0.876
	Mod. 1	0.0459	0.517	0.809	0.073			24	0.087	0.835
	$\frac{T_f}{T_w}$ Mod. 2			SAME AS MODEL. No. 1.						

TABLE XIII (cont'd)

PROCESSING OF THE DATA FOR MODEL No.1 ($Nu=aRe^n$) AND MODEL No.2 [$Nu=a'Re^{n'}(T_b/T_w)^m$]

SHAPES		a or a'	SE(loga)	n or n'	SE.(n) or SE.(n')	m	SE(m)	D.F.	SE(log.y)	Corr. Coeff. R ²
	Mod. 1	0.365	0.377	0.550	0.060			21	0.082	0.796
	Mod. 2	SAME AS MODEL No. 1								
	Mod. 1	0.268	0.576	0.603	0.071			21	0.079	0.775
	Mod. 2	SAME AS MODEL No. 1								
 SPHERES	Mod. 1	0.104		0.756	0.016			57	0.061	0.975
	Mod. 2	SAME AS MODEL No. 1								
	Mod. 1	0.048		0.822	0.021			57	0.068	0.964
	Mod. 2	0.106		0.749	0.033	-0.119	0.43	56	0.064	0.968

- 2) The cyclic process of the arc generates pockets of plasma about one inch long with a diameter of 0.2 to 0.3 inch (1). Due to the large axial and radial velocity gradients, these pockets will exchange energy by turbulent diffusion and decrease in size. However, this phenomenon enhances the turbulence of the jet.
- 3) The important decrease in the value of the nitrogen viscosity through the boundary layer towards the wall will decrease its stability towards disturbances in the vicinity of the latter (μ can be two to three times smaller at the wall than in the jet).

Before discussing the results in more detail, some considerations are given now concerning model No. 2.

Significance of the Temperature Ratio

Table XIII shows that the temperature ratio (T_b/T_w) does not make a consistent contribution to the heat transfer. However, it can be noted that large temperature differences seem to have a tendency to decrease the value of the heat transfer coefficient, as indicated by the negative value of most of the exponents. The only exception occurs in the case of the square cylinder results, which are more scattered than the others (owing to a possible yaw in the support when heated).

Although no significant conclusions can be drawn from Table XIII, it can be remarked that such an effect is coherent. The

effects of a large temperature difference can be expected to be reversed for the heating and the cooling of a surface by a gas, because the temperature and velocity profiles react to large ΔT in opposite ways when the direction of the heat flow is changed. The careful investigations of Hilpert (16) and Collis and Williams (61) have shown that large ΔT 's increase the heat transfer for the case of the cooling of a cylinder. Hence a decrease of the heat transfer coefficient can be expected for the heating of a cylinder by a hot gas, as in the present study. It should be noted that Churchill and Brier (25) found the reverse effect in the latter case. However, the inclusion of the temperature ratio in their relationship is not warranted, as indicated by the low correlation coefficient (less than 0.2) reported by (1) after he tested the significance of this ratio in their correlation. This may be due to the condition imposed on the Reynolds number exponent, which was fixed by Churchill and Brier at the value of 0.5.

Although the introduction of the temperature ratio may improve the correlations very slightly in the present investigation, particularly in the case of data evaluated on the film temperature basis, its significance is doubtful and model No. 2 was discarded. The correlation of the data by equation (11) is therefore adopted for the purpose of the rest of this discussion. Although the value of n was particularly affected by the experimental points at each end of the Reynolds number range investigated, its value for each geometry is consistent with the expected

flow configuration.

Circular Cylinders

Fig. 10 and 11 show that the results for the 1/4 and 1/8-in. o.d. circular cylinders fall on two lines. This was confirmed by a statistical test by the computer. Due to a lack of information about the turbulence of the jet, and more particularly about its scale, no argument is available to explain this discrepancy. Nevertheless, a possible effect of the combined action of the scale of turbulence and of the initial pockets of plasma can be suggested, the diameter of the 1/8-in. o.d. cylinder being small enough. It should be remembered that Van der Hegge Zijnen (14) has recommended the use of a (L/D) ratio. But this suggestion has never been vindicated. In addition, it should be pointed out that the results for the 1/4-in. o.d. cylinder are more reliable from an experimental point of view, as well as much more plentiful, and the preferential use of the latter results is recommended. From the high value of the exponent n on the Reynolds number, it can be inferred that the attached boundary layer is turbulent over most of its course, and that the contribution of the small wake to heat transfer (the point of separation is well in the rear of the cylinder) is not very important.

Comparison with the data of Douglas and Churchill, and of Churchill and Brier shows that the present data are slightly below their results. This can be attributed to the difference

in the temperature levels at which the experiments were conducted (the variations of the physical properties are much larger in the present investigation), also to a possible slight departure of the state of the gas from the equilibrium conditions, and mainly, to the essential difference in the boundary layer regime.

Square Cylinder with a Face Perpendicular to the Jet

This case suggests the existence of a turbulent boundary layer, as well. The layer forming at the axis plane must be in a laminar regime, but quickly becomes turbulent. The essential difference with the previous case is that a separation occurs as the upstream corners, followed in all likelihood by reattachment of the boundary layer. Final separation then takes place at the edges of the lower face. Owing to the fixed separation of the boundary layer, the wake contribution in this case (which by itself would have a value of \underline{n} of 0.6) is larger than for a circular cylinder. The relative scatter of the data for this geometry may be due to a certain yaw in the tube position.

Square Cylinder with an Edge Facing the Jet

In this geometry (which is roughly similar to a wedge body) separation points will be located at each side of the mid cross-section plane and therefore the wake contribution will occupy 50% of the total surface area. The value of \underline{n} in this case seems

to indicate that the attached boundary layer was essentially laminar. The rate of change of the pressure gradient in the front part of the body being less for this configuration (as in the case of a wedge) it is conceivable that transition, if it occurs, will take place further downstream than in the previous cases.

The quasi-laminar boundary layer region and the wake region being well delimited, a heat transfer relationship of the type proposed by Richardson (13) was attempted, namely :

$$Nu = C_1 Re^{1/2} + C_2 Re^{2/3} \quad (13)$$

However, processing of the data revealed that the correlation coefficient was low (less than 0.3) and this model was therefore given up.

C O N C L U S I O N S

Based on the data obtained, the following correlations are recommended for the various geometries studied (the results of Kubanek obtained on spheres are included for comparison), with the physical properties evaluated at bulk temperature :

SHERE

$$Nu = 0.118 (Re)^{0.757} (Pr)^{0.33} \quad (14)$$

CIRCULAR CYLINDER

$$Nu = 0.0612 (Re)^{0.836} (Pr)^{0.33} \quad (15)$$

SQUARE CYLINDER WITH A FACE PERPENDICULAR TO THE JET

$$Nu = 0.126 (Re)^{0.711} (Pr)^{0.33} \quad (16)$$

SQUARE CYLINDER WITH AN EDGE FACING THE JET

$$Nu = 0.412 (Re)^{0.549} (Pr)^{0.33} \quad (17)$$

The characteristic length in the last two cases is the side of the square.

It must be emphasized that these correlations apply only to systems of high intensity of turbulence, as studied in the present investigation. As a result, it can be inferred that the initially-laminar boundary layer will undergo transition at various points downstream of the front stagnation point

depending on the different pressure gradients imposed by the different geometries studied. In other words, because of the form of the above equations, the value of \bar{n} will reflect the combined contributions from the attached boundary layer in the front and the wake region in the back. Thus a logical increase in \bar{n} was obtained from a wedge-type body with a large fixed wake (square cylinder-edge) through a configuration yielding a smaller wake and a larger turbulent boundary layer repartition (square cylinder-face) and finally to configurations with small wakes and maximum extent of turbulent boundary layers (circular cylinder and sphere).

Under these conditions, it is not surprising that a better agreement is obtained by estimating the physical properties of the fluid at the bulk temperature. For most of the published correlations of data on heat transfer to bodies of various shapes, the mean fluid temperature has been reported to give better results. Since these studies were invariably correlated by means of $(Re)^{0.5}$, there is little doubt that the boundary layer was in a laminar regime, and an arithmetic average temperature would be expected to represent a good average for the estimation of the physical properties. When the boundary layer is turbulent, however, the intense mixing action in this region should be expected to yield an average temperature in the layer much closer to that of the free-stream.

Previous studies with the aim of finding a standard

characteristic length to correlate the convective heat transfer to bodies of various shapes were found to be fairly successful because for all of them the attached boundary layer was in an essentially laminar regime (44, 45, 46, 47). Therefore it was possible to define a characteristic length which could reasonably account for the difference in the flow pattern around the various geometries, even for fairly severe conditions of free-stream turbulence (10% in the case of reference 45), providing that transition to a turbulent boundary layer did not occur. The present study, however, clearly shows that the concept of a universal characteristic length breaks down as soon as mixed regimes in the boundary layer are involved because a similarity in the transition of the layer does not exist for the various bodies.

Although the model proposed by Richardson (13) was not successful in correlating the results obtained for a cylinder with an edge facing the jet, it is felt on the other hand that the form of the equations (14) to (17) is over-simplified and that a more complex relation, involving the separate contributions from the attached boundary layer and from the wake region (with possibly an additional interaction term) would be more successful in predicting the heat transfer rates corresponding to various geometries. Similarly, the effects of various levels of turbulence intensity should be accounted for in these equations. Many more experimented data must, however, be acquired before a better understanding of the complex mechanism of heat transfer can be obtained.

N O M E N C L A T U R E

ROMAN SYMBOLS

a	- constant
A	- lateral surface area, ft. ²
c _p	- specific heat at constant pressure, B.t.u./lb. °F.
C _{A⁺} , C _{N⁺} , C _N	- concentration of A ⁺ ions, N ⁺ ions, N atoms respectively, ft. ⁻³
D	- diameter, ft.
g _c	- gravitational constant
h	- heat transfer coefficient, B.t.u./hr. ft. ² °F.
k	- thermal conductivity, B.t.u./hr. ft. ² °F./ft.
L _x	- Eulerian macroscale of turbulence, ft.
M	- water flow rate, lb./hr.
n	- exponent
p	- pressure, lb./ft. ²
p _m	- mid cross-section perimeter, ft.
R	- correlation coefficient
T	- temperature, °F. or °R.
u'	- fluctuating component of the velocity, ft./sec.
U	- average velocity, ft./sec.
W	- duct width, ft.

ABBREVIATIONS

D.F.	- degree of freedom
SE.	- standard error

GREEK SYMBOLS

β	- coefficient of volume expansion, $(^{\circ}\text{R.})^{-1}$
μ	- viscosity, lb./ft.hr.
ν	- kinematic viscosity, ft. ² /hr.
ρ	- density, lb./ft. ³
π	- 3.1416
φ, ψ	- functions

DIMENSIONLESS GROUPS

Gr	- Grashof number, $\left[D^3 g \beta (\Delta T) / \nu^2 \right]^{0.5} / U$
I	- Intensity of turbulence, $\left[\overline{(u')^2} \right]^{0.5} / U$
M	- Mach number
Nu	- Nusselt number, (hD/k)
Pr	- Prandtl number, $(c_p \mu / k)$
Re	- Reynolds number, $(DU \rho / \mu)$
(Re) _{crit.}	- critical Reynolds number

SUBSCRIPTS

aw	- wall adiabatic
b	- bulk
f	- film
m	- mean integrated
o	- stagnation
w	- wall

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A P P E N D I C E S

APPENDIX A
DYNAMIC PRESSURE AND THERMOCOUPLE DATA

R U N	ΔP^*	T_c	ΔP^*	T_c	ΔP^*	T_c
	mm.CH ₃ OH	°F.		°F.		°F.
1-11.0-100	206	2000	203	1940	196	1860
1-11.0-150	281	1705	275	1710	271	1660
3-11.0-150	136	1900	124	2120	129	2020
3-11.0-200	169	1710	154	1860	129	2020
3-14.0-100	106	2570	107	2590	92	2660
3-14.0-150	151	2140	158	2350	147	2330
3-14.0-200	208	1975	183	2180	190	2100
3-19.5-150	202	3000	201	2975	189	2805
3-19.5-200	267	2710	274	2735	267	2660
3-23.4-150	231	3180	229	3150	223	3180
3-23.4-200	318	3035	306	2950	311	2950

* inclination at 1/25

APPENDIX B

VALUES OF THE PHYSICAL PROPERTIES - BULK TEMPERATURE

R U N	T _b °F.	$\rho \times 10^2$ lb./ft. ³	$c_p \times 10^2$ B.t.u./lb.°F.	$k \times 10^2$ B.t.u./hr.ft.°F.	$\mu \times 10^2$ lb./hr.ft.
1-11.0-100	2260	1.40	29.7	5.40	12.6
1.11.0-150	1920	1.60	29.15	4.85	11.5
3-11.0-150	2540	1.27	30.06	5.85	13.4
3-11.0-200	2210	1.43	29.65	5.32	12.4
3-14.0-100	3440	0.98	30.90	7.25	16.2
3-14.0-150	2820	1.16	30.35	6.30	14.3
3-14.0-200	2550	1.27	30.10	5.90	13.5
3-19.5-150	3900	0.87	31.15	7.90	17.5
3-19.5-200	3390	0.99	30.85	7.15	16.1
3-23.4-150	4270	0.80	31.35	8.45	18.6
3-23.4-200	3880	0.88	31.15	7.90	17.4

APPENDIX C

VALUES OF THE PHYSICAL PROPERTIES - FILM TEMPERATURE

R U N	T _f °F.	$\rho \times 10^2$ lb./ft. ³	$c_p \times 10^2$ b.t.u./lb.°F.	$k \times 10^2$ b.t.u./hr.ft.°F.	$\mu \times 10^2$ lb./hr.ft.
1-11.0-100	1195	2.30	27.45	3.60	9.2
1-11.0-150	1025	2.58	26.90	3.30	8.6
3-11.0-150	1335	2.12	27.85	3.85	9.7
3-11.0-200	1170	2.34	27.35	3.55	9.1
3-14.0-100	1785	1.70	28.90	4.60	11.1
3-14.0-150	1475	1.96	28.25	4.10	10.1
3-14.0-200	1340	2.12	27.90	3.85	9.7
3-19.5-150	2015	1.54	29.35	5.00	11.8
3-19.5-200	1760	1.72	28.85	4.60	11.0
3-23.4-150	2200	1.43	29.65	5.30	12.4
3-23.4-200	2005	1.55	29.30	5.00	11.8

CONTRIBUTION TO KNOWLEDGE

The heat transfer by forced convection between a highly turbulent jet at elevated temperature and blunt bodies of various shapes has been investigated. Correlations were obtained, which indicate that :

- 1) For such systems an attached turbulent boundary layer was prevailing, even at low Reynolds numbers;
- 2) The physical properties had to be evaluated at bulk temperature, suggesting that the choice of a temperature basis is closely linked to the nature of the attached layer;
- 3) The use of a standard characteristic length was found to fail for situations in which mixed regimes exist in the boundary layer. Hence, its value seems to be restricted to the case of laminar layers only.

SUGGESTIONS FOR FURTHER WORK

It is suggested that further investigations in the field of heat transfer to blunt bodies in high temperature surroundings could be of interest both from a fundamental and practical point of view in the followings areas :

- In the present system, the measurements of the intensity and scale of turbulence will undoubtedly shed more light on the results. More generally, researches in the mechanisms and criteria for transition in the boundary layer in highly turbulent jets will give a better understanding of the present data.
- Heat transfer investigations in the core of a jet, where turbulence effects are smaller, could provide new avenues of approach for the study of the influence of large temperature differences in the case of a heated body.
- Recent progress in the design of arc head configurations (as the dual duct or y-configuration arc head recently designed by Thermal Dynamics Corporation) have improved the performance and the stability of the arc operation. Hence, a more stable arc operation, coupled with quasi steady-state conditions provided by a free-jet will allow more accurate means to investigate the transport of heat.