

FLOW PAST A THIN  
INFLATED LENTICULAR AEROFOIL

by



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### SUMMARY

A theoretical and experimental investigation has been conducted on a simulation of a long inflated building.

Thin aerofoil theory using sources and sinks, combined with the equilibrium condition for the membrane, yields an integral equation. This has been solved by using a Fourier Sine series and the shape, membrane tension and pressure distribution are predicted.

Related experiments were carried out for various combinations of the independent parameters, Reynolds number ( $2.87 \times 10^5 - 9.77 \times 10^5$ ), inflation pressure coefficient ( $-0.12 - 3.22$ ), and ratio of excess length of membrane to chord length ( $0.56\% - 5.2\%$ ). Results for tension in particular are in good accord with theory for values of height-to-chord up to 0.1. The tension coefficient is shown to be a unique function of a parameter which combines the inflation pressure coefficient and the excess length ratio.

## RESUME

Une étude théorique et expérimentale a été conduite sur un long bâtiment gonflé.

La théorie des ailes fines, utilisant une distribution de puits et de sources, et l'analyse de tension de membrane ont été employées conjointement pour former une équation intégrale. Cette dernière a été résolue à l'aide des séries Sinus de Fourier.

Des expériences ont été organisées avec une combinaison des paramètres indépendants, comme: nombre de Reynolds ( $2.87 \times 10^5 - 9.77 \times 10^5$ ), coefficient de pression (-0.12 - 3.22) de gonflement, et un rapport "excès de longueur de la membrane sur la corde"  $((1-c)/c)$  (0.56% - 5.2%). Ces résultats ont été en accord avec la théorie, spécialement pour le coefficient de tension jusqu'à valeur de hauteur/corde = 0.1. Le coefficient de tension était fonction unique de la combinaison de la pression de gonflement et du rapport excès de longueur sur corde.

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## NOMENCLATURE

$A_n$	Fourier Sine coefficient
$B_s$	Solid blockage
$B_t$	Total blockage
$B_w$	Wake blockage
$c$	Chord length
$C_D$	Drag coefficient
$C_p$	Static pressure coefficient at the surface of the inflated aerofoil
$C_{p_1}$	$\frac{P_1 - P_\infty}{\frac{1}{2} \rho U_\infty^2}$ inflation pressure coefficient
$C_T$	$\frac{T}{qc}$ induced tension coefficient
$C_{p_1} \left( \frac{L-c}{c} \right)^{-1}$	Normalized inflation pressure coefficient
$d$	Portion of the envelope extends outside the tunnel roof
$F_c$	Total end correction force
$F_L$	Total measured force on the leading wedge
$F_T$	Total measured force on the trailing edge
$g(t)$	Strength of source or sink per unit length
$h$	$t/2$ , maximum height of the envelope
$H$	Tunnel working section height

$l$	Length of membrane in chordwise direction
$m$	$-\frac{\delta^2}{\nu} \frac{dU}{ds}$ , laminar flow separation parameter
$N$	Number of Fourier coefficient (also, equals to the number of chordwise position)
$P$	Local absolute static pressure at the inflated aerofoil surface
$P_a$	Atmospheric pressure
$P_i$	Inflation absolute pressure
$P_{ig}$	Inflation gauge pressure
$P_r$	Tunnel reference pressure
$P_s$	Free stream static pressure without test model in the working section
$P_t$	Total pressure of the flow
$q$	$\frac{1}{2} \rho U_\infty^2$ , free stream dynamic pressure
$R$	Radius of curvature
$Re$	Reynolds number
$s$	Distance along the surface, measured from leading edge
$S_f$	Shape factor
$S_s$	Laminar separation point
$S_t$	Transition point



$t$	Maximum thickness of the inflated aerofoil
$T$	Induced membrane tension per unit length in span direction
$T_c$	Induced tension per unit length by the circular arc profile of the envelope
$u$	Local wind velocity at the surface
$u'$	Perturbation velocity
$U_\infty$	Free stream velocity
$U$	Free stream just outside the boundary layer
$U_t$	Velocity outside the boundary layer at transition
$U_{ts}$	Velocity outside the boundary layer at turbulence separation
$W$	Tunnel working section width
$x, y$	Cartesian coordinate

#### Greek Symbols

$\delta_2$	Momentum thickness
$\theta$	Chordwise position
$\theta_0$	Leading or trailing edge angle (Theory)
$\theta_L$	Leading edge angle (measured)
$\theta_T$	Trailing edge angle (measured)
$\theta_c$	The circular arc edge angle

$$\delta_{2t} \left( \frac{U_t \delta_{2t}}{\nu} \right)^{\frac{1}{2}}$$

Ⓜ

ρ

Free stream density

♦

Angle subtained by the circular arc element which is  
along the surface of the envelope

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## 1. INTRODUCTION

The use of air structures could be said to date back to the time of the Roman civilization when inflated animal skins were used as floats for crossing rivers [Ref. 1]. However, it was not until the eighteenth century that air structures were used in transportation. In 1783, the Montgolfier brothers introduced the hot air balloon made of paper and linen, and at the same time Meusnier suggested a design for an airship [Ref. 2]. These devices marked the beginning of pneumatic air-transportation.

Initial development of pneumatic buildings benefitted considerably from techniques used in balloon and airship design. The first known architectural attempt to apply these techniques to an earthbound structure was by the English engineer, Lanchester [Ref. 3]. In his patent of 1917 for a field hospital, the basic principles for an air supported structure were developed.

During the second world war, Steven, an American engineer, tried to promote Lanchester's idea; but the real break-through was achieved only after the war, at the Cornell Aeronautical Laboratory, by Bird who designed and built successful spherical radomes for early warning radar [Ref. 4]. His work was supported by research, and in particular was the wind tunnel testing of models. Since then, similar work has been done elsewhere especially in Germany, Britain, Japan and Sweden [Ref. 5]. In Germany, Beger and Macher

undertook wind tunnel investigations of model buildings of spherical and cylindrical shape, [Ref. 6] and similar experiments had been made by Niemann [Ref. 7].

The stress distribution in a flexible air structure, is a function of the pressure distribution over the surface. For two-dimensional cases, under uniform inflation pressure, and without wind loading, equal stresses are developed at every point on the envelope. However, the varying pressure differential resulting from airflow across the surface of the unit causes a non-uniform loading and consequently causes deformation of the unit. As the membrane distorts, the wind pressure distribution varies with the change in shape until an equilibrium condition is reached or an instability may arise. The wind pressure distribution is therefore different from that over a solid body with the same profile as the flexible body with wind off.

In general, it was found that for thickness to chord ratios greater than or equal to half, it is necessary to relate the inflation gauge pressure  $P_{ig}$  to the free stream dynamic pressure  $q$  to avoid vibration or even collapse. For instance, a  $3/4$  spherical shape requires  $P_{ig}/q > 1$ , a hemispherical shape requires  $P_{ig}/q > 0.7$ , and a cylindrical shape with  $1/4$  spherical ends requires  $P_{ig}/q > 0.6$  [Ref. 7].

Recently, Hoxey and Wells [Ref. 8] did a full scale wind pressure measurement on a twin-span greenhouse inflated-roof, the roof was a thin lenticular aerofoil. They pointed out it was



impractical to measure the shape. However, they predicted the shape indirectly from the pressure distribution. They also observed that a roof of increased slope was easily deformed by wind pressure.

The use of air structures has grown rapidly in recent years because they are light weight, portable, easily erected, and can provide column-free cover over a large area. However, it is a relatively new building technology and many technical problems remain to be solved.

The objective is the idealization of flow across an inflated building in two-dimensional flow with the effect of the boundary layer removed.

In the present work, a theoretical and experimental study was conducted on a thin inflated lenticular aerofoil in two-dimensional flow at zero flow incidence. Thin aerofoil theory [Ref. 9] and membrane stress analysis were employed together to predict the shape, surface pressure distribution and induced membrane tension.

A preliminary smoke tunnel investigation at  $Re = 5 \times 10^3$  gave an indication of the maximum value of  $L/c$  for which flow separation would be avoided. The experiments were carried out in a blower wind tunnel of closed working section of dimensions 0.762 m wide by 0.432 m high. Model sizes were restricted by consideration of wind tunnel blockage and flow separation on model. Methods of measuring pressure distribution and induced membrane tension were developed. Experiments were carried out for various combinations of the independent parameters: Reynolds number, inflation pressure coefficient and the excess length

of membrane over the chord length. The Reynolds number varied from  $2.87 \times 10^5$  to  $9.77 \times 10^5$ , the inflation pressure coefficients varied from -0.12 to 3.22, and the excess length to chord ratio varied from 0.56% to 5.2% which approximately corresponded to wind-off height-chord ratios of 4.6% to 14% respectively.

## 2. THEORY

### 2.1 Thin Symmetrical Aerofoil at Zero Flow Incidence in Potential Flow

In potential flow theory, flow past a closed surface may be obtained by considering a number of sources and sinks located in a uniform stream as for instance in the Rankine oval and its degenerate case the circular cylinder. In these examples, modelling is done by introducing singularities of finite strength located at isolated points within the body.

To simulate the flow past a thin aerofoil at zero incidence, a continuous distribution of sources and sinks of strength  $g(x)$  per unit length located along the chordline is considered [Ref. 9, 10]. Referring to [Fig. 12], orthogonal axes are used: the origin is at the leading edge with  $x$  the distance along the chordline measured from the leading edge, and  $y$  the distance perpendicular to the chordline. The aerofoil is symmetrical about the  $x$ -axis. Thus an element of length ' $dx$ ' has associated with it a source of strength  $g(x)dx$  on the  $x$ -axis.

If the aerofoil is sufficiently thin the slope of the surface is small and for the purposes of determining the source distribution the flow velocity inside the aerofoil may be taken as  $U_\infty$ .

Within the profile, the difference between the quantity of fluid flowing at  $x_1$  and  $x_1 + dx_1$  is equal to that emanating from the sources contained within this interval.

Hence,

$$g(x)dx_1 = 2 U_\infty (y_1 + \frac{dy_1}{dx_1} dx_1) - 2 U_\infty y_1$$

$$g(x) = 2 U_\infty \frac{dy_1}{dx_1} \quad (2.1.1)$$

Thus the required source distribution is simply determined by the shape of the aerofoil. The slope for the first half of the body is positive, which implies a continuous source distribution. The second half body has negative slope, which implies a continuous sink distribution.

If the thickness is small, the velocity induced at a point  $(x, y)$  on the surface of the profile by a fluid source or sink at the point  $(x_1, 0)$  is approximately equal to that which would be induced at the point  $(x, 0)$  by the same source or sink [Fig. 12]. Then the perturbation velocity at point  $(x, y)$  due to source at  $(x_1, 0)$  is given by:

$$du' = \frac{g(x_1)dx_1}{2\pi(x-x_1)} \quad (2.1.2)$$

and the perturbation due to the combined effect of source and sink along the chord length is,

$$u' = \int_0^c \frac{g(x_1)dx_1}{2\pi(x-x_1)}$$

Using equation (2.1.1) gives

$$\frac{u'}{U_\infty} = \frac{1}{\pi} \int_0^c \frac{\frac{dy_1}{dx_1} dx_1}{(x-x_1)} \quad (2.1.3)$$

and hence the total velocity at  $(x, y)$  is

$$u = u_{\infty} + u'$$

$$\text{i.e. } \frac{u}{u_{\infty}} = 1 + \frac{1}{\pi} \int_0^c \frac{\frac{dy_1}{dx_1} dx_1}{(x-x_1)} \quad (2.1.4)$$

## 2.2 Formulation of Inflated Lenticular Aerofoil

In [Fig. 13] let the chordwise tension per unit span of the inflated aerofoil be  $T$ . Due to ideal flow, the skin friction is zero, and tangential equilibrium requires that tension  $T$  must be constant throughout the membrane [Ref. 11, 12, 13].

Thus,

$$T = \text{constant} \quad (2.2.1)$$

The condition of normal equilibrium is,

$$2T \sin(\phi/2) + \Delta p \cdot 2R \sin(\phi/2) = 0 \quad (2.2.2)$$

where  $\Delta p = p_1 - p$

Hence

$$\Delta p = -T/R \quad (2.2.3)$$

where  $R$  is the radius of curvature and is given by:

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} \quad (2.2.4)$$

with the assumption  $y$  is small from thin aerofoil theory,  $\frac{dy}{dx}$  is small. Therefore  $\left(\frac{dy}{dx}\right)^2$  can be neglected when comparing to unity and consequently,

$$\frac{1}{R} = \frac{d^2 y}{dx^2} \quad (2.2.5)$$

combine (2.2.3) and (2.2.5),

$$\Delta p = -T \frac{d^2 y}{dx^2} \quad (2.2.6)$$

$$\frac{\Delta p}{\frac{1}{2} \rho U_\infty^2} = \frac{(p_1 - p_\infty)}{\frac{1}{2} \rho U_\infty^2} - \frac{(p - p_\infty)}{\frac{1}{2} \rho U_\infty^2}$$

i.e.  $\frac{\Delta p}{q} = C_{p_1} - C_p \quad (2.2.7)$

where  $q$  is the free stream dynamic pressure,

$C_{p_1}$  is the inflation pressure coefficient.

$C_p$  is the local surface pressure coefficient.

By Bernoulli's equation,

$$p_\infty + \frac{1}{2} \rho U_\infty^2 = p + \frac{1}{2} \rho u^2$$

hence,

$$C_p = 1 - \left( \frac{u}{U_\infty} \right)^2 \quad (2.2.8)$$

substitute (2.1.4) into (2.2.8) and neglecting the second power of perturbation, gives,

$$C_p = -\frac{2}{\pi} \int_0^c \frac{\frac{dy_1}{dx_1} dx_1}{(x-x_1)} \quad (2.2.9)$$

and consequently combine (2.2.7) and (2.2.9),

$$\frac{\Delta p}{q} = C_{p_1} + \frac{2}{\pi} \int_0^c \frac{\frac{dy_1}{dx_1} dx_1}{(x-x_1)} \quad (2.2.10)$$

Normalize equation (2.2.6) by  $q$ , and combine with (2.2.10),

$$-\frac{T}{q} \frac{d^2 y}{dx^2} = Cp_1 + \frac{2}{\pi} \int_0^c \frac{dy_1}{dx_1} \frac{dx_1}{(x-x_1)} \quad (2.2.11)$$

Equation (2.2.11) is the master equation for analysis; it relates the induced membrane tension, inflation pressure, and flow condition.

In order to solve this integral equation, Fourier Series and numerical matrix techniques are employed [Ref. 12].  $x$  and  $y$  are transformed into parametric equations involving  $\theta$  with  $y$  expressed as a Sine Series.

hence,

$$x = \frac{c}{2} (1 - \cos \theta) \quad (2.2.12)$$

$$y = f(x) = c \sum_{n=1}^{\infty} A_n \sin(n\theta) \quad (2.2.13)$$

where  $0 \leq \theta \leq \pi$

$\theta = 0$  at leading edge

$\theta = \pi$  at trailing edge.

$A_n$ 's are the non-dimensional Fourier coefficients and  $c$  is the chord length. The use of Sine Series satisfies the boundary conditions at the leading and trailing edges, which require  $y$  equal to zero.

Taking the differential of (2.2.12) and (2.2.13), gives,

$$dx_1 = \frac{c}{2} \sin \theta_1 d\theta_1$$

$$dy_1 = c \sum_{n=1}^{\infty} n A_n \cos(n\theta_1) d\theta_1$$

Substitute them into equation (2.1.3),

$$\frac{u'}{U_\infty} = \frac{2}{\pi} \int_0^\pi \sum_{n=1}^{\infty} \frac{n A_n \cos(n\theta_1)}{(\cos \theta_1 - \cos \theta)} d\theta_1 \quad (2.2.14)$$

It is shown in the reference by Glauert

(Ref. 14)

$$\int_0^\pi \frac{\cos(n\theta_1) d\theta_1}{(\cos \theta_1 - \cos \theta)} = \pi \frac{\sin(n\theta)}{\sin \theta} \quad (2.2.15)$$

With the above integral result, equation (2.2.14) reduces to:

$$\frac{u'}{U_\infty} = 2 \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin \theta} \quad (2.2.16)$$

hence,

$$\frac{u}{U_\infty} = 1 + 2 \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin \theta} \quad (2.2.17)$$

Consequently, equation (2.2.9) and (2.2.11) give,

$$C_p = -4 \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin \theta} \quad (2.2.18)$$

$$-\frac{T}{q} \frac{d^2 y}{dx^2} = C_{p_i} + 4 \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin \theta} \quad (2.2.19)$$

It can be shown that

$$\frac{d^2 y}{dx^2} = -\frac{4}{c} \left[ \sum_{n=1}^{\infty} n A_n \frac{\cos(n\theta) \cos(\theta)}{\sin^3 \theta} + \sum_{n=1}^{\infty} n^2 A_n \frac{\sin(n\theta)}{\sin^2 \theta} \right] \quad (2.2.20)$$

Substitute (2.2.20) into (2.2.19) and rearrange,

$$\sum_{n=1}^{\infty} \left\{ \begin{array}{l} C_T [n \cos(n\theta) \cos(\theta) + n^2 \sin(n\theta) \sin \theta] \\ - n \sin(n\theta) \sin^2 \theta \end{array} \right\} \left( \frac{A_n}{C_{p_i}} \right) = \frac{T}{4} \sin^3 \theta \quad (2.2.21)$$

where  $C_T = \frac{T}{qc}$



It can be shown that the shape is symmetrical about the centre line of the chord [Appendix I], and this requires that for  $n$  even  $A_n = 0$ .\*

Equation (2.2.21) is the governing equation, it must satisfy the requirement of sharp edges (i.e. leading and trailing edge). Thus at the edges it is required that

$$\frac{dy}{dx} = \frac{2 \sum_{n=1}^{\infty} n A_n \cos(n\theta)}{\sin \theta} \quad n \text{ is odd} \quad (2.2.22)$$

be finite.

At the leading edge ( $\theta = 0$ ),  $\sin(\theta) = 0$  and for  $\left(\frac{dy}{dx}\right)_{x=0}$  to be finite, requires

$$\sum_{n=1}^{\infty} n A_n = 0 \quad n \text{ is odd} \quad (2.2.23)$$

For the trailing edge ( $\theta = \pi$ ), it gives the same end conditions as (2.2.23).

The length of the surface in chordwise direction is given by:

$$L = \int_0^c \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}} dx \quad (2.2.24)$$

this can be linearized, and yields the excess length to chord ratio  $(L-c)/c$

$$\frac{L-c}{c} = \frac{1}{2G} \int_0^c \left( \frac{dy}{dx} \right)^2 dx \quad (2.2.25)$$

\* From now on, it is understood that all summation signs are for  $n = 1, 3, 5, 7, \dots, \infty$  (odd terms only).

To find the relation of the edge angle as a function of the tension parameter  $C_T$ , equation (2.2.19) is used,

$$-C_T c \frac{d^2 y}{dx^2} = C_{p_i} + 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} n A_n \frac{\sin(n\theta)}{\sin\theta} \quad (2.2.26)$$

integrating once [Appendix II].

$$\frac{1}{C_{p_i}} \frac{dy}{dx} = \frac{1}{2C_T} \left[ 1 + 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right] - \frac{x}{C_T c} \quad (2.2.27)$$

at the leading edge,  $x = 0$ , this gives

$$\left. \frac{1}{C_{p_i}} \frac{dy}{dx} \right|_{x=0} = \frac{1}{2C_T} \left[ 1 + 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{A_n}{C_{p_i}} \right] \quad n \text{ is odd} \quad (2.2.28)$$

since  $\left. \frac{dy}{dx} \right|_{x=0}$  = slope at leading edge  
 $= \tan(\theta_0)$

where  $\theta_0$  is the leading edge angle.\*

Rearranging (2.2.28) gives

$$\frac{C_{p_i}}{\tan\theta_0} = 2 \left[ 1 + 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{A_n}{C_{p_i}} \right]^{-1} C_T, \quad n \text{ is odd} \quad (2.2.29)$$

To obtain the relation of excess length as a function of the tension coefficient  $C_T$ , substitute (2.2.27) into (2.2.25) and carry out the integral [Appendix III]. It gives

$$\frac{C_{p_i}}{\sqrt{\frac{L-c}{c}}} = \sqrt{24} \left\{ 1 - 24 \sum_{n=1}^{\infty} \frac{1}{(n^2-4)} + 48 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \frac{A_m}{C_{p_i}} \right) \cdot \left( \frac{A_n}{C_{p_i}} \right) \left[ \frac{1-n^2-m^2}{[(1-n)^2-m^2][(1+n)^2-m^2]} \right] \right\} C_T \quad (2.2.30)$$

where  $m, n$  are odd.

\* Due to symmetry leading and trailing edge angles are the same (Theory).

Eliminate  $C_T$  in (2.2.29) and (2.2.30),

$$\frac{dp_i}{\tan \theta_0} = \frac{1}{\sqrt{6}} \left\{ \frac{1-24 \sum_{n=1}^{\infty} \frac{A_n}{Cp_i} \frac{1}{(n^2-4)}}{\left[ 1 + 4 \sum_{n=1}^{\infty} \frac{A_n}{Cp_i} \right]} + 48 \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{A_m}{Cp_i} \frac{A_n}{Cp_i} \frac{1-n^2-m^2}{[(1-n)^2-m^2][(1+n)^2-m^2]}}{\left[ 1 + 4 \sum_{n=1}^{\infty} \frac{A_n}{Cp_i} \right]} \right\} \times \frac{Cp_i}{\sqrt{\frac{l-c}{c}}} \quad (2.2.31)$$

### 2.3 Numerical Solution and Discussion

The determination of the Fourier's coefficients involves the transformation of the master equation (2.2.21). Equation (2.2.21) is valid at any chordwise position. Thus a set of linear simultaneous equations can be obtained by specifying a number of chordwise positions in the equation (2.2.21).

Due to symmetry of the shape about the chordline centre, all positions can be chosen within  $\theta = 0$  and  $\theta = \pi/2$ . The sharp edge condition requires that  $\theta = 0$  must be included, and symmetry about the chordline centre requires  $\theta = \pi/2$  be included. The rest of the positions are chosen to be equally spaced between 0 and  $\pi/2$  [Ref. 14]. The magnitude of the interval is then equal to  $\frac{\pi}{2} \frac{1}{(N-1)}$ , where  $N$ , which is also the number of Fourier's coefficients chosen, is the number of chordwise positions. The use of equal intervals of  $\theta$  was found to produce a well behaved and non-singular matrix. Some trials with non-equal intervals produced a nearly singular matrix and numerical inaccuracy.

In matrix form, equation (2.2.21) is rewritten as:

$$C_{n,m} \dot{S}_n = D_m \quad (2.3.1)$$

$$\text{where: } C_{n,m} = C_T [n \cos(n\theta_m) \cos(\theta_m) + n^2 \sin(n\theta_m) \sin(\theta_m)] \\ - n \sin(n\theta_m) \sin^2(\theta_m)$$

$$S_n = A_n / C p_i, \quad \theta_m = \frac{\pi}{2} \frac{(m-1)}{(N-1)}$$

$$D_m = \frac{1}{4} \sin^3(\theta_m)$$

$$\text{and } n = 1, 3, 5, 7, \dots (2N-1)$$

$$m = 1, 2, 3, 4, \dots N$$

In order to find the sufficient number of coefficients which gives acceptable accuracy, the work by Nielsen [Ref. 12] was used as a guide.

For a given  $C_T$ , the matrix was solved for different numbers of  $N$ 's, say  $N_1 < N_2 < N_3$  etc. Then the following comparisons were made: (1) the difference between each corresponding coefficient; (2) the difference in maximum thickness which was evaluated from the corresponding set of coefficients; (3) the difference in excess length to chord ratio. For comparison (2) within a practically acceptable tolerance, say 0.01% (Nielsen), the smallest value of  $N$  which gives such tolerance would be accepted as the optimum number of coefficients for analysis.

The matrix had been solved by truncation with  $N$  equal to 10, 15, 20 and 25 for various values of the tension parameter  $C_T$ .

Results had been compared and presented in Tables 1-7. It was seen that the successive coefficients decrease rapidly in magnitude, and only the first two coefficients are dominant.

Assuming for the moment that the 'true' values are given for  $N = 25$ , comparison for  $N = 20$  shows that the percentage difference for  $(h/c)/C_{p1}$  is about 0.005% and for the excess length to chord ratio  $\left(\frac{l-c}{c}\right)/C_{p1}^2$  is about 0.05% for all values of  $C_T$  greater than 0.5. It is concluded that  $N = 20$  gives a nearly asymptotic solution and provides a sufficiently accurate solution. \*

Once the Fourier coefficients are known, all the characteristics of the inflated aerofoil can be obtained.

It is interesting to note that the numerical solution gives values of  $A_n/C_{p1}$  and not  $A_n$ . [Fig. 14], shows  $C_T$  is a unique function of  $C_{p1}\left(\frac{l-c}{c}\right)^{-\frac{1}{2}}$  as given by equation (2.2.30). The value of  $C_T$  equals to about 0.5 separates positive and negative value of  $C_{p1}$ .  $C_{p1}\left(\frac{l-c}{c}\right)^{-\frac{1}{2}}/C_T$  achieves the asymptotic value  $\sqrt{24}$ , which corresponds to wind off when  $C_T$  reaches very large value.

In [Fig. 15], for  $C_T$  greater than 0.5, the normalized shape  $(y/c)/C_{p1}$  starts with cusps at the edges for low value of  $C_T$ , and tends to become a circular arc profile as  $C_T$  increases.

In [Fig. 16], for values of  $C_T$  less than about 0.5, the normalized shapes  $(y/c)/C_{p1}$  are negative, except for  $C_T = 0.2$  and  $C_T = 0.125$  which cross over the zero line. In the range of  $C_T$  between 0.49847 and 0.28438, the shapes are only different in amplitude

\* All theoretical results are obtained for  $N = 20$ .

They are geometrically similar with only one peak at the centre. At  $C_T$  equals to 0.28438, the shape has zero slope at the leading and trailing edges. For  $C_T$  from 0.28438 to 0.2, the shapes start to cross over the zero line near the edges. The amplitude of the cross over as well as the width grow rapidly, and have their maximum case at  $C_T$  equals to 0.2. Further decrease of  $C_T$  from 0.2 to 0.17625, changes the shapes in such a way that they are inverse of the case for  $C_T$  equals to 0.2 but with smaller amplitudes. When  $C_T$  reaches 0.125, it crosses over the zero line again with more ripples and smaller amplitude. For  $C_T$  less than 0.125 until 0.05 the shape becomes negative, and once again more ripples and smaller amplitude. \*

In [Fig. 17], this provides a direct estimate of the tension in the membrane for a given flow speed, inflation pressure and height-to-chord ratio. It shows that the smaller the height-to-chord ratio, the higher the induced membrane tension. However, the percentage of increase is larger for large height-to-chord ratio.

In [Fig. 18], as a practical example, the change of shape for a particular  $\ell$  corresponding to wind off height-to-chord ratio 0.14,  $\left(\frac{\ell-c}{c} = 0.05\right)$ , has been determined for various values of  $C_{p_i}$ . Their corresponding induced tension is also given in the figure. As the inflation pressure coefficient goes from large to small, the corresponding shapes are once again seen to change from almost a circular arc to a cusp shape.

\* The wavy shapes in Fig. 16 were not achieved experimentally.

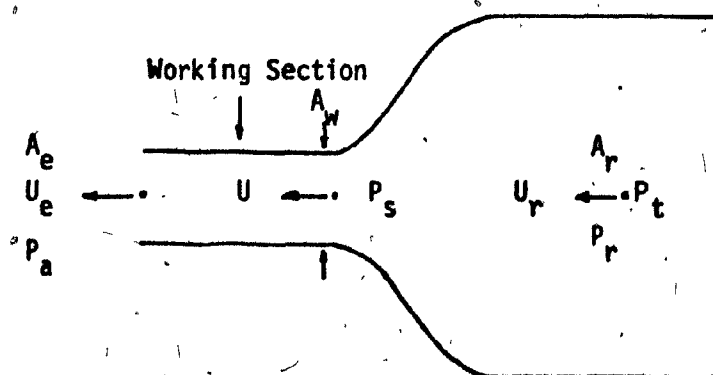
### 3. EXPERIMENT

#### 3.1 Wind Tunnel Description and Calibration

The general arrangement of the blower wind-tunnel is as shown in [Fig. 1]. The tunnel is of the blower type, and has an open jet exit of 0.762 m wide by 0.432 m high (30" x 17"). The contracting section which leads to the working section [Fig. 2, 3] is a two-dimensional type having a contraction ratio of 6:1 [Ref. 15].

The test model extended through the roof and floor, located at the centre of the working section. The model was aligned in flow direction by means of pressure taps at front of leading edge wedge. A device which carried a static tube for the measurement of surface pressure distribution was attached to the plexiglas window. It could be moved parallel to flow direction.

The tunnel calibration involved two pressure taps located on each side of the vertical walls of the settling chamber, one foot from the upstream end. These tubes were connected to give an average pressure  $P_r$  for the determination of wind speed.



$$\frac{A_r}{A_w} = 6$$

$$A_w = A_e$$

Pitot-static tube, which was located at the centre of the working section, was used to measure the values of  $(Pr-Pa)$ ,  $(Pt-Pa)$ , and  $(Ps-Pa)$  simultaneously. Inclined alcohol (S.G. 0.8) manometers were used for these measurements. Results were presented by plotting  $(Pt-Ps)/(Pr-Pa)$  and  $(Ps-Pa)/(Pr-Pa)$  versus  $[(Pr-Pa)/\rho v^2]^{\frac{1}{2}}$ , where  $v$  is the kinematic viscosity [Fig. 10]. The value  $[(Pr-Pa)/\rho v^2]^{\frac{1}{2}}$  when multiplied by unit length is a measurement of the Reynolds number.

The calibration curves had been fitted by linear regression method to give the following expressions.

$$\frac{Pt-Pa}{Pr-Pa} = 1.029$$

$$\frac{Pt-Ps}{Pr-Pa} = [9583.052 + 0.575324 \times 10^{-4} Re + 0.495353 \times 10^{-10} Re^2 - 0.21716 \times 10^{-6} Re^3] \times 10^{-4}$$

$$\frac{Ps-Pa}{Pr-Pa} = [750.296 - 0.184041 \times 10^{-3} Re + 0.49051 \times 10^{-10} Re^2 - 0.274852 \times 10^{-17} Re^3] \times 10^{-4}$$

where  $Re = [(Pr-Pa)/\rho v^2]^{\frac{1}{2}}/\text{metre}$

and  $2.5 \times 10^5 < Re < 18 \times 10^5/\text{metre}$

### 3.2 Model Design and Accessory Measurement Devices

The chord length was chosen to avoid separation ahead of the trailing edge. The method to determine laminar and transition was by the following equations [Ref. 16].



$$\frac{U\delta_2^2}{\nu} = \frac{0.45}{U^5} \int_0^c U^5 ds \quad (3.2.1)$$

$$m = -\frac{\delta_2}{\nu} \frac{dU}{ds} \quad (3.2.2)$$

where  $U$  is the free stream velocity just outside boundary layer

$\nu$  is the kinematic viscosity

$\delta_2$  momentum thickness

$s$  is the distance along the surface,

and together with [Fig. 11] of "boundary layer Reynolds number at transition plotted against the pressure parameter", which was obtained from L.F. Crabtree [Ref. 17]. Laminar separation occurs when  $m = 0.09$ .

When no laminar separation occurs; the procedure of calculating the transition point is as follows:  $m$  and  $Re\delta_2$  at points along the surface are calculated and plotted on Crabtree's diagram. So long as this curve lies to the left of Crabtree's critical curve, the boundary layer is laminar; transition occurs where the curves cross.

After determining the transition point, the turbulent separation is found approximately by Spence's method,

$$1.829 U_{ts}^2 = U_t^2 - 0.00135 \int_{S_t}^{S_s} \frac{U^2}{\mathbb{H}} ds \quad (3.2.3)$$

where  $\mathbb{H} = \delta_{2t} \left( \frac{U_t \delta_{2t}}{\nu} \right)^{\frac{1}{2}}$ ,

and  $U_t$ ,  $U_{ts}$  are the velocity outside the boundary layer at transition and the turbulence separation points.  $S_t$ ,  $S_s$  are their corresponding locations along the surface.

Based on the above consideration, and avoiding the merge of the boundary layer growth at the roof and floor along the surface, the chord was chosen about 0.38 m ( $h/c$  was assumed 0.05). It was found that this dimension of chord was satisfactory for experiments.

Only solid and wake blockage were considered for determining the model sizes. The blockages can be estimated by [Ref. 18 and 19],

$$\text{Total blockage} \quad B_t = B_s + B_w \quad (3.2.4)$$

$$\text{Wake blockage} \quad B_w = \frac{1}{2} \left( \frac{c}{w} \right)^2 C_D \quad (3.2.5)$$

$$\text{Solid blockage} \quad B_s = \frac{\pi^2}{12} \left( \frac{t}{w} \right)^2 S_f \quad (3.2.6)$$

$$\text{Shape factor} \quad S_f = \frac{4}{\pi} \int_0^c \frac{u}{U_\infty} y \frac{ds}{t^2} \quad (3.2.7)$$

$$\text{Corrected velocity} \quad u_c = u_t (1 + B_t) \quad (3.2.8)$$

where  $w$  width of tunnel

$t$  maximum thickness of object

$c$  chord length

$y$  ordinate of the surface measured from the chordline

$u$  local velocity of the surface at a distance  $s$  from the leading edge

$C_D$  measured drag coefficient. (It was assumed 0.01 from NACA0010-35)

$u_t$  measured tunnel wind velocity

$u_c$  corrected tunnel wind velocity

Total blockage was estimated for a few chord to thickness

ratios, for instance,

when  $c/t = 10$ ,  $B_t = 0.01$

$c/t = 5.5$ ,  $B_t = 0.02$

$c/t = 4$ ,  $B_t = 0.027$

They provided a guide for determining the  $c/t$  ratio. These were applied for the correction of measured inflation pressure coefficient ( $C_p$ ), tension parameter ( $C_T$ ), and Reynolds number ( $Re$ ).

The design of the test model consisted of an envelope made of Stablekote II Nylon ( $25 \text{ gm/m}^2$ ) [Fig. 4, 5] wrapped around two steel wedges in order to achieve aerofoil shape when inflated. Wedge angle was restricted to about  $10^\circ$ , because large wedge angle would affect the edge angle of the envelope during the experiments. The wedges together with the end edges of the envelope extended through the tunnel roof and floor via the bicircular arc holes. The holes' sizes were made to closely resemble the inflated aerofoil so as to minimize the amount of leakage of tunnel flow.

The ends of the wedges were each held by flexures which were specially designed [Fig. 6] to allow very small and parallel movement in the direction of the main stream flow when forces act on the wedges. To each flexure, a full bridge connection of strain gauges was attached for force measurement [Fig. 7, 8.] The flexures were attached to the roof and floor [Fig. 8] of the working section.

The test model was two-dimensional, and required that there should be no induced membrane tension in the spanwise direction of the envelope. Thus two plexiglas plates were used which attached the ends of the envelope and served as end plates to reduce the escape of air from the inside of the aerofoil [Fig. 4].

The inflation pressure was supplied from both ends of a 0.0127 m (0.5") diameter pipe which extended through the covers. The pipe was designed to give uniform inflation pressure. Holes were drilled in it with varying spacing so as to give a uniform distribution of pressure inside [Ref. 20].

### 3.3 Measurement

The measurement of parameters included  $C_{p_i}$ , local  $C_p$ , and the excess length-to-chord ratio.

Measurements of inflation pressure, surface pressure distribution and tunnel speed give  $C_{p_i}$  and  $C_p$ . The measurements of wedge forces together with both leading and trailing edge angles give the value of  $C_T$ .

The inflation pressure was measured using 3 small tubes (0.318 cm). Two open ended and one static tubes were inserted inside the envelope from both top and bottom covers. There was negligible difference with readings from these tubes. They were however, connected together and measured by an inclined manometer.

A device was designed [Fig. 9] which allowed a mounted static tube (0.16 cm diameter) to be rotated or moved in and out so as to align the static tube parallel to the inflated surface at any chosen chordwise position. The device itself was mounted on the plexiglas window, and could be adjusted in the chordwise direction of the aerofoil. Again, the pressures were measured on an inclined manometer.

The forces on the leading edge and trailing edge were measured by the strain gauges attached to the supporting flexures, and the complete systems were individually calibrated with dead weights (Fig. 41). The outputs of the strain gauges were picked up by a B and K type 1562 strain indicator. The leading and trailing edge angles were determined from orientation of the static tube at these locations.

Since the shape of the membrane outside the tunnel was different from that affected by the wind, it was impossible to photograph the shape directly i.e. longitudinally. Some idea of the change of shape with speed was obtained from oblique photographs.

### 3.4 Experiment Procedure

To start the experiment, it was necessary to calibrate the membrane length, induced tension versus inflation pressure with wind off, because the fabric stretched slightly under tension. This was done with wind off by measuring the thickness of the circular arc aerofoil using calipers. This information combined with the measured internal pressure enabled the radius of curvature and membrane length ( $L$ ) to be determined. The information was also used to correct the wind-on readings for the effect of the ends which were outside the tunnel. Sample calibration curves are shown in [Fig. 19A, 19B].

Ambient temperature and pressure were recorded and used to calculate the corresponding density and kinematic viscosity of air.

For each envelope and combination of tunnel speed and inflation pressure, oblique photographs of the membrane shape were taken, and the corresponding surface pressure distribution and membrane tension were recorded.

Different excess length to chord ratios were obtained mainly by changing the size of the envelope, although slight adjustments were possible by changing the chord length.

Cases with negative  $C_{p_i}$  could be run by starting initially with a positive internal pressure and then, with wind on, reducing this to a value less than that in the working section.

The correction for the end effects on the measurement of tension was necessary. Since, with wind off the envelope had a circular arc profile over the whole length, with wind on, the portion of the envelope within the tunnel changed shape and the portion outside the tunnel also did, but presumably to a less extent. To make the correction for the end effects, it was assumed that the outside portion of the envelope remained a circular arc profile when wind was on. Thus the measured force would be due to two parts, namely the tunnel portion affected by the air flow, and the outside portion which was surrounded by effectively still air.

For a given inflation pressure, the value of  $\frac{l-c}{c}$  can be found from the calibration curve [Fig. 19A]. With the value of  $\frac{l-c}{c}$  and [Fig. 19B], the tension per unit length may be determined. The total correction force is given by

$$F_c = (2 T_c \cos \theta_c) d,$$

where  $d$  is the total distance of the wedge extended outside the floor and ceiling.

$\theta_c$  is the circular arc edge angle

$T_c$  is the induced tension per unit length by the circular arc profile.

Let  $F_L$  and  $F_T$  be the total measured forces on the leading wedge and trailing wedge due to combined effect of wind and inflation pressure.

The central portion of the aerofoil affected by the tunnel air is:

$$T_L = (F_L - F_c) / (2H \cos \theta_L) *$$

$$T_T = (F_T - F_c) / (2H \cos \theta_T)$$

where  $\theta_L$ ,  $\theta_T$ , are the measured leading and trailing edge angles,

$H$  is the height of the tunnel.

It has been assumed in the theory that the tension should be uniform throughout the membrane. Thus the average of  $T_L$  and  $T_T$  was taken as the induced tension ( $T$ ) per unit length. In fact their difference was usually less than 1%.

\* Where  $T_L$ ,  $T_T$  are the induced tension per unit length in spanwise direction at the leading and trailing edge of the inflated envelope respectively.

#### 4. EXPERIMENTAL RESULTS AND DISCUSSION

##### 4.1 Comparison of Experimental Results with Theory

Comparison of equation (2.2.30) and experimental results are shown in [Fig. 20], where the tension coefficient  $C_T$  is plotted against  $Cp_i \left( \frac{\ell-c}{c} \right)^{-\frac{1}{2}}$ . The agreement is very good. Experimental results nearly all collapse convincingly on the theoretical curve within about 3%. The results are for a range of Reynolds number from  $2.87 \times 10^5$  to  $8.02 \times 10^5$ , and indicate that they are insensitive to the change of Reynolds number. In all cases with positive inflation pressure [Fig. 20] the membrane appeared to be stable.

Some results for suction cases (negative  $Cp_i$ ) were obtained also [Fig. 21]. These results, however, are less accurate because the portion of the membrane outside the tunnel was usually partially collapsed, and thus the end correction for the measured tensions were uncertain. Nevertheless, the results are still in fairly good agreement with the theoretical curve. Partial or total collapse occurred at the value of  $C_T$  less than about 0.4. The less the value of  $(\ell-c)/c$ , the more readily it is subjected to collapse. To avoid collapse, it should keep  $C_T$  greater than about 0.4, which corresponds to  $Cp_i \left( \frac{\ell-c}{c} \right)^{-\frac{1}{2}}$  about -0.45.

The actual phenomenon of the membrane behaviour for negative  $Cp_i$  was a gradual process. When the inflation pressure was reduced



to the critical value which corresponds to a given flow speed and excess length to chord ratio, the envelope oscillated with very small amplitude but high frequency. There were no prominent wavy ripples observed. With further decrease of pressure, the membrane oscillated with larger amplitude but lower frequency. Finally, when the applied suction pressure was large enough for the given conditions, the envelope collapsed in such a way that the maximum thickness propagated rearwards to the trailing edge as one simple pulse wave [Fig. 40]. After collapse, the two surfaces of the envelope contacted each other tightly. However, when the suction pressure was removed, the envelope was inflated again. Also, the envelope could recover its stable state from oscillation when the flow speed increased, which is really an increase of  $C_{p_i}$ . High flow speed is favourable for stability, even for a small amount of suction. It was true for different  $(l-c)/c$  even up to 0.038.

Edge slopes given by equation (2.2.31) as a function of the  $C_{p_i} \left( \frac{l-c}{c} \right)^{-\frac{1}{2}}$  is compared in [Fig. 22]. In theory, the curve will achieve a slope of  $(6)^{-\frac{1}{2}}$  for high value of  $C_{p_i} \left( \frac{l-c}{c} \right)^{-\frac{1}{2}}$ . There is good agreement for low value of  $C_{p_i} \left( \frac{l-c}{c} \right)^{-\frac{1}{2}}$  below 4, and is still fairly good even up to 10. However, discrepancy between theory and experiment increases as  $C_{p_i} \left( \frac{l-c}{c} \right)^{-\frac{1}{2}}$  beyond 10. Such disagreement is most probably due to measurement of angles when they are small. Similar features happened in [Fig. 23] where the edge slopes plotted against the tension coefficient  $C_T$ . The theoretical curve achieved a slope of 2 for high  $C_T$ . In both figures, disagreement between theory and experiment begins when  $C_{p_i}/\tan(\theta_0)$  is above 4.5.

Consider the pressure distributions which are shown in [Fig. 24-35]. They show that for low height to chord ratio, experiment and theory in general are in good accord [Fig. 24, 25]. However, as height to chord ratio is increased, the agreement is worse in some local regions [Fig. 28, 29]. Around the region of  $s/l$  equals 0.075 to 0.3, the theory gives lower pressures and larger values in the region 0.35 to 0.8. It may be noted that although there is considerable disagreement at  $s/l = 0.8$ , the agreement surprisingly improves towards the trailing edge. This could be due to flow separation and reattachment near  $s/l = 0.8$ . For even thicker cases [Fig. 34, 35], separation bubble has moved forward and becomes larger as would be expected. The flow pattern around the region were investigated quantitatively using a tuft attached to a hand held wand, and light tufts attached to the surface of the membrane itself. The pressure distribution discrepancy for the forward part of the aerofoil can only be partially explained as an error in the potential flow analysis. In [Fig. 36], for potential flow, the present linearized theory for a lenticular aerofoil with circular arc gives more negative pressures for  $s/l$  in the region 0.05 to 0.3 and vice versa in the region 0.4 to 0.5, as compared with Karman-Trefftz's method. The latter method is a conformal transformation of a circle and in the potential flow theory is exact. The comparison shown in [Fig. 36] indicates that the linearized theory gives satisfactory results for height to chord ratios of perhaps up to 0.1. This gives maximum reasonable  $h/c$ .

A second reason for the discrepancy over the rear half of the aerofoil could be the effect of the boundary layer build up over the surface. It would effectively thicken the shape of the aerofoil especially over the downstream half where there are unfavourable pressure gradients. Thus the pressure is reduced in these regions because of the wake. An additional effect of the lower pressure on the back of the aerofoil is to move the maximum thickness of the membrane rearwards and thus further reduce the pressure on the rear half surface. It also has the effect of increasing the pressure on the front half surface as is observed [Fig. 35].

The leading edge angle is always slightly less than the trailing edge angle.

#### 4.2 Estimation of the Error in the Measured Coefficients

From [Fig. 20], it appears that  $C_{p_1} \left( \frac{l-c}{c} \right)^{-\frac{1}{2}}$  and  $C_T$  have been measured to an accuracy of somewhere between 2% to 3%.  $C_{p_1}$  involved the measurement of internal pressure and tunnel dynamic pressure using manometers which could have been measured to an accuracy of 1%. The remains inaccuracy in  $(l-c)/c$  is therefore between 2% and 4%. This corresponds to an error in the wind off  $t/c$  of about 1% and 2%.

It is somewhat surprising that such accuracy was achieved in the measurement of  $t$  using calipers. The measurement of  $C_T$  is inaccurate due to inaccuracy in  $T$  and  $q$ . The force on the wedges was measured to an accuracy of about 1% and the edge angles were relatively unimportant

for determining  $T$  since the cosine was taken. Thus with a small additional error from tunnel dynamic pressure,  $C_T$  may well have been measured to an accuracy better than 2%.

## 5. CONCLUSION

Idealization of flow over a long low inflated building with the effect of the earth's boundary layer removed has been studied successfully both in theory and experiment.

The tension coefficient  $C_T$  is a unique function of the collapsed parameter  $C_{p_1} \left( \frac{\ell-c}{c} \right)^{-\frac{1}{2}}$  obtained theoretically and confirmed experimentally. The relation is insensitive to the Reynolds number, and applies for  $(\ell-c)/c$  less than approximately 0.05. (about  $h/c = 0.14$ , with wind off).

The membrane was found to be stable for all positive values of  $C_{p_1}$ .  $C_{p_1} \left( \frac{\ell-c}{c} \right)^{-\frac{1}{2}}$  should be greater than about -0.45 which corresponds to  $C_T$  about 0.4 to avoid partial or total collapse.

The non-dimensional shape  $[(y/c/C_{p_1})]$  of the membrane is determined uniquely by  $C_{p_1} \left( \frac{\ell-c}{c} \right)^{-\frac{1}{2}}$ , and the actual dimension is scaled accordingly to  $C_{p_1}$ .

The induced absolute tension is higher for low heights, however, the proportional increase with wind speed is less.

The measured pressure distribution broadly agrees with the theory although there appears to be some effect of the boundary layer built up on the aerofoil which results in a movement rearwards of the point of maximum thickness.

The theory is good for  $h/c$  up to 0.1 for design purposes.

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# APPENDIX I

Thin inflated lenticular aerofoil in potential flow at zero incidence is symmetrical in shape.

The governing equation for the analysis is given by equation (2.2.21).

$$\sum_{n=1}^{\infty} \begin{bmatrix} C_T [n \cos n\theta \cos \theta + n^2 \sin n\theta \sin \theta] \\ -n \sin n\theta \sin^2 \theta \end{bmatrix} \left( \frac{A_n}{C_{p_i}} \right) = \frac{1}{2} \sin^3 \theta \quad (2.2.21)$$

Replace  $\theta$  by  $(\pi - \theta)$ :

$\sin$  is unchanged

$\sin n\theta \rightarrow \sin n\theta$   $n$  is odd

$-\sin n\theta$   $n$  is even

$\cos \theta \rightarrow -\cos \theta$

$\cos n\theta \rightarrow -\cos n\theta$   $n$  is odd

$\cos n\theta$   $n$  is even

hence (2.2.21) becomes:

$$\sum_{n=1}^{\infty} \begin{bmatrix} C_T [n (\mp \cos n\theta) (-\cos \theta) + n^2 (\pm \sin n\theta) \sin \theta] \\ -n (\pm \sin n\theta) \sin^2 \theta \end{bmatrix} \left( \frac{A_n}{C_{p_i}} \right) = \frac{1}{2} \sin^3 \theta \quad (I.1)$$

If  $n$  is odd:

equation (I.1) is identical to (2.2.21) and this implies equation (2.2.21) is symmetrical.

If  $n$  is even:

$$\text{get: } \sum_{n=2}^{\infty} \begin{bmatrix} C_T [-n \cos n\theta \cos \theta - n^2 \sin n\theta \sin \theta] \\ + n \sin n\theta \sin^2 \theta \end{bmatrix} \left( \frac{A_n}{C_{p_i}} \right) = 0 \quad (I.2a)$$



Rearrange (I.2a).

$$\frac{1}{2} \sum_{n=2}^{\infty} n \left( \frac{A_n}{C_{p1}} \right) \left[ \begin{aligned} &-C_T [(n+1) \cos(n-1)\theta - (n-1) \cos(n+1)\theta] \\ &+ \sin(n\theta) - \frac{1}{2} [\sin(n-2)\theta + \sin(n+2)\theta] \end{aligned} \right] = 0 \quad (1.2b)$$

$$\text{take } \int_0^{\pi} (1.2b) \times \cos m\theta d\theta: \quad (1.3)$$

the typical terms will be:

$$\int_0^{\pi} \cos(n+1)\theta \cdot \cos m\theta d\theta = 0$$

$$\int_0^{\pi} \sin n\theta \cos m\theta d\theta = \begin{cases} \frac{1}{n+m} + \frac{1}{n-m} & m = \text{odd}, n = \text{even} \\ 0 & m = \text{even}, n = \text{even} \end{cases}$$

Hence (I.3) is reduced to

$$\frac{1}{2} \sum_{n=2}^{\infty} n \frac{A_n}{C_{p1}} \left[ \frac{1}{n+m} + \frac{1}{n-m} - \frac{1}{2} \left( \frac{1}{n+m+2} + \frac{1}{n-m+2} + \frac{1}{n+m-2} + \frac{1}{n-m-2} \right) \right] = 0 \quad (1.4)$$

for all odd  $m$

Hence  $A_n = 0$  for all  $n$  even.

Consequently, equation (2.2.21) becomes:

$$\sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \left[ \begin{aligned} &C_T [n \cos(n\theta) \cos \theta + n^2 \sin(n\theta) \sin \theta] \\ &- n \sin(n\theta) \sin^2 \theta \end{aligned} \right] \left( \frac{A_n}{C_{p1}} \right) = \frac{1}{2} \sin^3 \theta \quad (1.5)$$

## APPENDIX II

Derivation of relation between inflation pressure coefficient,  
tension coefficient and edge angle.

To obtain the relation

$$\frac{C_{p_1}}{y'(0)} = f(C_T)$$

From equation (2.2.19) and with  $C_T = T/qc$ ,

$$-C_T cy'' = C_{p_1} + 4 \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin\theta} \quad n \text{ odd} \quad (II.1)$$

$$\frac{-C_T c}{C_{p_1}} y'' = 1 + 4 \sum_{n=1}^{\infty} n \frac{A_n}{C_{p_1}} \frac{\sin(n\theta)}{\sin\theta} *$$

Integrate once from  $x = 0$  to  $x = x$

$$\frac{-C_T c}{C_{p_1}} y' + K_1 = x + 4 \left( \int_0^x \sum_{n=1}^{\infty} n \frac{A_n}{C_{p_1}} \frac{\sin(n\theta)}{\sin\theta} dz \right) \quad (II.2)$$

where  $z$  is a dummy variable,  $K_1$  is integration constant.

at  $x = 0$ ,

$$\frac{y'(0)}{C_{p_1}} = \frac{K_1}{C_T c} \quad (II.3)$$

Integrate (II.2) from  $x = 0$  to  $x = c$

$$-\frac{1}{C_{p_1}} y = -\frac{K_1}{C_T c} x \Big|_0^c + \frac{x^2}{2C_T c} \Big|_0^c + \frac{4}{C_T c} \cdot \int_0^c dx \int_0^x \sum_{n=1}^{\infty} n \frac{A_n}{C_{p_1}} \frac{\sin(n\theta)}{\sin\theta} dz + K_2 \quad (II.4)$$

\* All  $\Sigma$  from  $n=1$  to  $n=\infty$  for odd  $n$ .

At  $x = 0, y = 0, \rightarrow K_2 = 0$

At  $x = c, y = 0,$

$$\frac{K_1}{C_T} = \frac{c}{2C_T} + \frac{4}{C_T c} \cdot \int_0^c dx \left( \int_0^x \sum_n \frac{A_n}{C_{p_i}} \frac{\sin(n\theta)}{\sin\theta} dz \right)$$

$$\frac{K_1}{C_T c} = \frac{1}{2C_T} + \frac{4}{C_T c^2} \cdot \int_0^c dx \left( \int_0^x \sum_n \frac{A_n}{C_{p_i}} \frac{\sin(n\theta)}{\sin\theta} dz \right) \quad (II.5)$$

$$\text{Let } I_1 = \int_0^x \sum_n \frac{A_n}{C_{p_i}} \frac{\sin(n\theta)}{\sin\theta} \quad (II.6a)$$

$$\text{where } \theta = \cos^{-1} \left( 1 - \frac{2z}{c} \right)$$

$$\cos\theta = \left( 1 - \frac{2z}{c} \right)$$

$$dz = \frac{c}{2} \sin\theta d\theta$$

$$\text{so: } I_1 = \frac{c}{2} \sum \frac{A_n}{C_{p_i}} \int_0^{\cos^{-1}(1-\frac{2x}{c})} n \sin(n\theta) d\theta$$

$$= \frac{c}{2} \sum \frac{A_n}{C_{p_i}} \left[ 1 - \cos(n \cos^{-1}(1-\frac{2x}{c})) \right] \quad (II.6b)$$

Substitute  $I_1$  back into II.5:

$$\begin{aligned} \frac{K_1}{C_T c} &= \frac{1}{2C_T} + \frac{4}{C_T c^2} \cdot \int_0^c dx \frac{c}{2} \sum \frac{A_n}{C_{p_i}} \left[ 1 - \cos(n \cos^{-1}(1-\frac{2x}{c})) \right] \\ &= \frac{1}{2C_T} + \frac{2}{C_T} \sum \frac{A_n}{C_{p_i}} - \frac{2}{C_T} \sum \frac{A_n}{C_{p_i}} \int_0^c \cos(n \cos^{-1}(1-\frac{2x}{c})) dx \quad (II.7) \end{aligned}$$

$$\text{Let } I_2 = \sum \frac{A_n}{C_{p_i}} \int_0^c \cos(n \cos^{-1}(1-\frac{2x}{c})) dx \quad (II.8a)$$

also let:

$$u = n \cos^{-1} \left( 1 - \frac{2x}{c} \right),$$

$$\text{at } x = 0 \quad u = 0$$

$$x = c \quad u = n\pi$$

$$\cos\left(\frac{u}{n}\right) = \left(1 - \frac{2x}{c}\right)$$

$$dx = \frac{c}{2n} \sin\left(\frac{u}{n}\right) du$$

$$\begin{aligned} I_2 &= \sum \frac{A_n}{C p_1} \frac{c}{2n} \int_0^{n\pi} \cos(u) \sin\left(\frac{u}{n}\right) du \\ &= \frac{c}{2} \sum \frac{A_n}{C p_1} \frac{1}{n} \left[ -\frac{1}{2} \left( \frac{\cos\left(\frac{1}{n} - 1\right)u}{\frac{1}{n} - 1} + \frac{\cos\left(\frac{1}{n} + 1\right)u}{\frac{1}{n} + 1} \right) \right]_0^{n\pi} \\ &= c \sum \frac{A_n}{C p_1} \frac{(-1)}{4n} \frac{1}{\left(\frac{1}{n^2} - 1\right)} \left[ -\frac{2}{n} + \frac{1}{n} \left( \cos(\pi - n\pi) + \cos(\pi + n\pi) \right) \right. \\ &\quad \left. + \cos(\pi - n\pi) - \cos(\pi + n\pi) \right] \\ &= c \sum \frac{A_n}{C p_1} \frac{-n^2}{4n(1-n^2)} \left[ -\frac{2}{n} + \frac{2}{n} \cdot \cos\pi \cdot \cos n\pi + 2 \cdot \sin\pi \cdot \sin n\pi \right] \\ &= c \sum \frac{A_n}{C p_1} \frac{-n^2}{4n(1-n^2)} \left[ \frac{-2}{n} \cdot (1 + \cos n\pi) \right] \\ &= c \sum \frac{A_n}{C p_1} \frac{(1 + \cos n\pi)}{2(1-n^2)} \end{aligned}$$

but  $n$  is odd  $\rightarrow \cos n\pi = -1$

So  $I_2 = 0$

(11.8b)

Substitute  $I_2 = 0$  back into II.7,

$$\frac{k_1}{C_T c} = \frac{1}{2C_T} + \frac{2}{C_T} \sum \frac{A_n}{C p_1}$$

(II.9)

substitute (II.9) into (II.3),

$$\frac{y'(0)}{Cp_1} = \frac{1}{2C_T} \left[ 1 + 4 \sum \frac{A_n}{Cp_1} \right]$$

$$\text{i.e. } \frac{Cp_1}{y'(0)} = 2 \left[ 1 + 4 \sum \frac{A_n}{Cp_1} \right]^{-1} C_T \quad (\text{II.10})$$

where  $y'(0) = \tan \theta_0$ , and  $\theta_0$  is the edge angle.

### APPENDIX III

Derivation of relation between excess length to chord  
ratio, inflation pressure coefficient and tension coefficient.

To obtain the relation

$$\frac{C_{p_i}}{\sqrt{\frac{L-c}{c}}} = f(C_T)$$

From (II.2) and (II.9), get

$$-\frac{1}{C_{p_i}} y' = -\frac{1}{C_T} \left( 1 + 2 \sum \frac{A_n}{C_{p_i}} \right) + \frac{x}{C_T c} + \frac{4}{C_T c} \cdot \int_0^x \sum_{n=1}^{\infty} n \frac{A_n}{C_{p_i}} \frac{\sin(n\theta)}{\sin\theta} dz \quad (\text{III.1})$$

from appendix II, it has been shown that:

$$I_1 = \int_0^x \sum_{n=1}^{\infty} n \frac{A_n}{C_{p_i}} \frac{\sin(n\theta)}{\sin\theta} dz = \frac{c}{2} \sum \frac{A_n}{C_{p_i}} \left[ 1 - \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right]$$

Substitute  $I_1$  into (III.1)

$$-\frac{1}{C_{p_i}} y' = -\frac{1}{2C_T} + \frac{x}{C_T c} - \frac{2}{C_T} \sum \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \quad (\text{III.2})$$

$$\begin{aligned} \left( \frac{y'}{C_{p_i}} \right)^2 &= \left( \frac{1}{2C_T} - \frac{x}{C_T c} \right)^2 + \frac{4}{C_T^2} \left[ \sum \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right]^2 \\ &\quad + 2 \left( \frac{1}{2C_T} - \frac{x}{C_T c} \right) \frac{2}{C_T} \sum \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \\ &= \frac{1}{4C_T^2} + \frac{x^2}{C_T^2 c^2} - \frac{x}{C_T^2 c} + \frac{4}{C_T^2} \left[ \sum \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right]^2 \\ &\quad + \frac{2}{C_T^2} \sum \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) - \frac{4}{C_T^2} \frac{x}{c} \sum \frac{A_n}{C_{p_i}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \end{aligned} \quad (\text{III.3})$$

The equation of a length is given by

$$L = \int_0^c \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{III.4})$$

Since  $\frac{dy}{dx}$  is small, then by binomial expansion and rearrangement

$$\left(\frac{L-c}{c}\right) \frac{1}{C_{p1}^2} = \frac{1}{2c} \int_0^c \left(\frac{1}{C_{p1}} \frac{dy}{dx}\right)^2 dx \quad (\text{III.5})$$

Substitute (III.3) into (III.5),

$$\begin{aligned} \left(\frac{L-c}{c}\right) \frac{1}{C_{p1}^2} &= \frac{1}{2c} \cdot \int_0^c \left\{ \frac{1}{4C_T^2} + \frac{-x^2}{C_T^2 c^2} - \frac{x}{C_T^2 c} + \frac{4}{C_T^2} \left[ \sum \frac{A_n}{C_{p1}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right]^2 \right. \\ &\quad \left. + \frac{2}{C_T^2} \sum \frac{A_n}{C_{p1}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right. \\ &\quad \left. - \frac{4}{C_T^2} \frac{x}{c} \sum \frac{A_n}{C_{p1}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right\} dx \\ &= \frac{1}{24C_T^2} + \frac{2}{C_T^2 c} \cdot \int_0^c \left[ \sum \frac{A_n}{C_{p1}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right]^2 dx \\ &\quad + \frac{1}{C_T^2 c} \cdot \int_0^c \sum \frac{A_n}{C_{p1}} \cos(n \cos^{-1}(1 - \frac{2x}{c})) dx \\ &\quad - \frac{2}{C_T^2 c} \cdot \int_0^c \sum \frac{A_n}{C_{p1}} \frac{x}{c} \cos(n \cos^{-1}(1 - \frac{2x}{c})) dx \quad (\text{III.6}) \end{aligned}$$

From (II.8a)

$$I_2 = \int_0^c \sum_{n=1}^{\infty} \frac{A_n}{c p_1} \cos \left( n \cos^{-1} \left( 1 - \frac{2x}{c} \right) \right) dx = 0$$

$$\text{Let: } I_3 = \int_0^c \sum_{n=1}^{\infty} \frac{A_n}{c p_1} \frac{x}{c} \cos \left( n \cos^{-1} \left( 1 - \frac{2x}{c} \right) \right) dx \quad (\text{III.7})$$

$$\text{Let } u = n \cos^{-1} \left( 1 - \frac{2x}{c} \right)$$

$$\text{At } x = 0 \quad u = 0$$

$$x = c \quad u = n\pi$$

$$\cos\left(\frac{u}{n}\right) = \left(1 - \frac{2x}{c}\right)$$

$$dx = \frac{c}{2} \frac{1}{n} \sin\left(\frac{u}{n}\right) du$$

Hence

$$I_3 = \sum \frac{A_n}{c p_1} \int_0^{n\pi} \frac{1}{2} \left( 1 - \cos \frac{u}{n} \right) \cdot \cos u \cdot \left( \frac{c}{2n} \right) \cdot \sin \frac{u}{n} du$$

$$= \frac{c}{4} \sum \frac{A_n}{c p_1} \frac{1}{n} \int_0^{n\pi} \left( \sin \frac{u}{n} \cos u - \sin \frac{u}{n} \cos \frac{u}{n} \cos u \right) du$$

$$= \frac{c}{4} \sum \frac{A_n}{c p_1} \frac{1}{n} \int_0^{n\pi} \sin \frac{u}{n} \cos u du - \frac{c}{8} \sum \frac{A_n}{c p_1} \frac{1}{n} \int_0^{n\pi} \sin \frac{2u}{n} \cos u du$$

$$\int_0^{n\pi} \sin \frac{u}{n} \cos u du = 0$$

$$\int_0^{n\pi} \sin \frac{2u}{n} \cos u du = \frac{4n}{4-n^2} \quad (\text{with } n's \text{ are odd})$$



Hence

$$I_3 = -\frac{c}{8} \sum_n \frac{A_n}{cp_1} \frac{1}{n} \left( \frac{4n}{4-n^2} \right)$$

$$I_3 = \frac{c}{2} \sum_n \frac{A_n}{cp_1} \left( \frac{1}{n^2-4} \right)$$

$$\text{Let: } I_4 = \int_0^c \left[ \sum_n \frac{A_n}{cp_1} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right]^2 dx \quad (\text{III.8})$$

$$\text{i.e. } I_4 = \int_0^c \left[ \sum_m \frac{A_m}{cp_1} \cos(m \cos^{-1}(1 - \frac{2x}{c})) \right] \left[ \sum_n \frac{A_n}{cp_1} \cos(n \cos^{-1}(1 - \frac{2x}{c})) \right] dx$$

$$= \sum_m \sum_n \frac{A_m}{cp_1} \frac{A_n}{cp_1} \cdot \int_0^c \cos(m \cos^{-1}(1 - \frac{2x}{c})) \cos(n \cos^{-1}(1 - \frac{2x}{c})) dx$$

$$\text{Let: } u = \cos^{-1}(1 - \frac{2x}{c})$$

$$\text{At } x = 0 \quad u = 0$$

$$x = c \quad u = \pi$$

$$\cos u = (1 - \frac{2x}{c})$$

$$dx = \frac{c}{2} \sin u \, du$$

Hence

$$I_4 = \frac{c}{2} \sum_m \sum_n \frac{A_m}{cp_1} \frac{A_n}{cp_1} \cdot \int_0^\pi \cos mu \cos nu \sin u \, du$$

by applying:  $2 \sin A \cos B = \sin(A - B) + \sin(A + B)$ ,

$$I_4 = \frac{c}{4} \sum_m \sum_n \frac{A_m}{cp_1} \frac{A_n}{cp_1} \cdot \int_0^\pi (\sin(1-n)u \cos mu + \sin(1+n)u \cos mu) \, du$$

by:  $\int \sin px \cos qx \, dx = -\frac{1}{2} \left[ \frac{\cos(p-q)x}{(p-q)} + \frac{\cos(p+q)x}{(p+q)} \right]$

and all  $m, n$ 's are odd.

$$I_4 = \frac{c}{4} \sum_m \sum_n \frac{A_m}{Cp_1} \frac{A_n}{Cp_1} \left[ \frac{2(1-n)}{(1-n)^2 - m^2} + \frac{2(1+n)}{(1+n)^2 - m^2} \right]$$

$$I_4 = c \sum_m \sum_n \frac{A_m}{Cp_1} \frac{A_n}{Cp_1} \left[ \frac{1-n^2-m^2}{[(1-n)^2 - m^2][(1+n)^2 - m^2]} \right]$$

Substitute  $I_2, I_3, I_4$  back into (III.6),

$$\begin{aligned} \left( \frac{L-c}{c} \right) \frac{1}{Cp_1^2} &= \frac{1}{24C_T^2} + \frac{2}{C_T^2 c} \left\{ c \sum_m \sum_n \frac{A_m}{Cp_1} \frac{A_n}{Cp_1} \left[ \frac{1-n^2-m^2}{[(1-m)^2 - m^2][(1+n)^2 - m^2]} \right] \right\} \\ &+ \frac{1}{C_T^2 c} \cdot [0] - \frac{2}{C_T^2 c} \left( \frac{c}{2} \sum_n \frac{A_n}{Cp_1} \frac{1}{(n^2-4)} \right) \\ \left( \frac{L-c}{c} \right) \frac{1}{Cp_1^2} &= \frac{1}{24C_T^2} \left[ 1 - 24 \sum_n \frac{A_n}{Cp_1} \frac{1}{(n^2-4)} + 48 \sum_m \sum_n \frac{A_m}{Cp_1} \frac{A_n}{Cp_1} \frac{1-n^2-m^2}{[(1-n)^2 - m^2][(1+n)^2 - m^2]} \right] \end{aligned}$$

(III.9a)

or:

$$\star \frac{Cp_1}{\sqrt{\frac{L-c}{c}}} = \sqrt{24} \left[ 1 - 24 \sum_n \frac{A_n}{Cp_1} \frac{1}{(n^2-4)} + 48 \sum_m \sum_n \frac{A_m}{Cp_1} \frac{A_n}{Cp_1} \frac{1-n^2-m^2}{[(1-n)^2 - m^2][(1+n)^2 - m^2]} \right]^{-\frac{1}{2}} C_T$$

(III.9b)

from (II.10):

$$\frac{Cp_1}{y'(0)} = 2 \left[ 1 + 4 \sum_n \frac{A_n}{Cp_1} \right]^{-1} C_T$$

Eliminate  $C_T$  between (II.10) and (III.9b)

$$\frac{C_{p_1}}{y'(0)} = \frac{1}{\sqrt{6}} \left[ 1 - 24 \sum \frac{A_n}{C_{p_1}} \frac{1}{(n^2-4)} + 48 \sum_m \sum_n \frac{A_m}{C_{p_1}} \frac{A_n}{C_{p_1}} \frac{1-n^2-m^2}{[(1-n)^2-m][ (1+n)^2-m^2]} \right] \frac{1}{2} \\ \cdot \left[ 1 + 4 \sum \frac{A_n}{C_{p_1}} \right]^{-1} \cdot \frac{C_{p_1}}{\sqrt{\frac{L-c}{c}}}$$

(III.10)

## APPENDIX IV

### Computer Program

This program solves for the coefficients ( $A_n/C_{p_i}$ ) from the master equation (2.2.21), and hence the rest of the characteristics of the inflated lenticular aerofoil can be found by using equations (2.2.13), (2.2.17), (2.2.18), (2.2.19), (2.2.30) and (2.2.31).

List of symbols used in the computer program and sample program are shown on the following pages.

List of symbols used in the computer program

A(I, J)	to replace COA(M, N) i.e. COA(M, 2N-1) → A(M, N)
BS	solid blockage
BW	wake blockage
BT	BS + BW total blockage
C	chord length (cm)
C1	$-24 \sum (An/Cp_i) (n^2 - 4)^{-1}$
C2	$48 \sum \sum (An/Cp_i) (An/Cp_i) \frac{1-n^2-m^2}{[(1-n)^2-m^2][(1+n)^2-m^2]}$
C <sub>D</sub>	drag coefficient (0.01)
CF2	C1 + C2
CF1	$4 \sum (An/Cp_i)$
CT	tension coefficient
CPI	inflation pressure coefficient
CC(M, N)	$n \sin n\theta_m \sin^2 \theta_m$
CL(M, N)	$n \cos(n\theta_m) \cos \theta_m + n^2 \sin(n\theta_m) \sin \theta_m$
COA(M, N)	$= C_T * CL(M, N) - CL(M, N)$
DXDYO	$Cp_i / y'(o)$
EST	$Cp_i (\frac{\ell-c}{c})^{-\frac{1}{2}}$ theoretical value
ESA	$Cp_i (\frac{\ell-c}{c})^{-\frac{1}{2}} = \sqrt{24} C_T$ , asymptotic value
ITE	= 1: if both theoretical and experimental results desired i.e. U <sub>o</sub> , C, Vis, inputs must have some value = 0: if only theoretical results desired i.e. U <sub>o</sub> , C, VIS, inputs are zero
NDS	number of data sets inputed

NR	number of coefficients ( $A_n/C_{p_i}$ ) desired
NT	$2 * NR$
RCP	$C_p/C_{p_i}$
RV	$(U/U_o - 1)/C_{p_i}$
RYC	$(y/c)/C_{p_i}$
RE	Reynolds number base on chord length
RCT	$c/t$
RCW	$c/w$
RTW	$t/w$
S(M)	$\frac{1}{2} \sin^2(\theta_m)$
SF	shape factor
SL	$l$ - membrane length (cm)
SOL(N)	solution of ( $A_n/C_{p_i}$ )
T	t inflated aerofoil thickness
TCP	$C_p$
TRV	$u/U_o$
TRYC	$y/c$
UO	free stream velocity
VIS	$\nu$ kinematic viscosity ( $m^2/s$ )
W	tunnel width
SLEQN	this subroutine solves a system of linear equations by pivoting method
CHANGE	this subroutine interchanges the values of the variables
ESLOPE	this subroutine solves equation (2.2.29)
SLNGT	this subroutine solves equation (2.2.30)

BLOCKG        this subroutine to find the blockages

RXYUCP        this subroutine to solve equations (2.2.13), (2.2.17),  
(2.2.18)

All the rest are dummy variables

### Sample Input

## Input data

control card



\*BATCH WATFIV ME62000 M.C.TSE-7411929 (90,90)

\*\*\*\*\*

SWATFIV X.TIME=90.PAGES=90

UUUU

DATA INPUT

```

1      IMPLICIT REAL*8 (A-H,O-Z)
2      DOUBLE PRECISION CC(25,50),CL(25,50),COA(25,50),A(25,25),
3      *S(25),SOL(50)
4      W=76.250
5      READ(5,101) NDS
6      101 FORMAT(I2)
7      DO 555 NTIME=1,NDS
8      READ(5,100) NR,CT,CPI,UO,C,VIS,ITE
9      100 FORMAT(I2,1X,D22.16,1X,D16.8,1X,D6.3,1X,D6.3,1X,D11.5,1X,I1)
10     WRITE(6,200) CT,NR
10     200 FORMAT(1H,////////11X,'CT' = 'D22.16,///11X,
11     *NO. OF COEFF. = ',12,/)

```

UUUU

TO GENERATE THE COEFF. OF A(N)/CPI N=ODD

```

11     PI=DATAN(1.00)*4.00
12     Q=DFLOAT(NR-1)
13     D=(PI/2.00)/Q
14     X=0.000
15     NT=NR+NR
16     DO 1 M=1,NR
17     S(M)=0.2500*DSIN(X)**3
18     DO 2 N=1,NT,2
19     CC(M,N)=N*DSIN(N*X)*DSIN(X)**2
20     CL(M,N)=N*DCOS(N*X)*DCOS(X)+N*N*DSIN(N*X)*DSIN(X)
21     COA(M,N)=CL(M,N)*CT-CC(M,N)
22     2 CONTINUE
23     X=X+D
24     1 CONTINUE

```

UUUU

TO RE-ARRANGE THE COEFF. AS STARTING MATRIX ELEMENTS

```

25     DO 3 I=1,NR
26     J=1
27     DO 4 N=1,NT,2
28     IF(N.EQ.1) GO TO 50
29     J=N-J
30     A(I,J)=COA(I,N)
31     GO TO 4
32     50 A(I,1)=COA(I,1)
33     4 CONTINUE
34     3 CONTINUE
35     NC=NR+1
36     DO 5 I=1,NR
37     A(I,NC)=S(I)
38     5 CONTINUE

```

UUUU

TO SOLVE THE MATRIX FOR A(N)/CPI N=ODD

```

39      CALL SLEGN(NR,NC,A,CT)
      TO RE-ARRANGE THE SOLUTION.

40      I=1
41      DO 20 K=1,NT,2
42      IF(K.EQ.1) GO TO 51
43      I=K-1
44      SOL(K)=A(1,NC)
45      GO TO 20
46      51 SOL(1)=A(1,NC)
47      20 CONTINUE
48      WRITE(6,201)
49      201 FORMAT(1H0,10X,'A( N)/CPI N=000',/)
50      WRITE(6,202) (I,SOL(I),I=1,NT,2)
51      202 FORMAT(11X,'A(',I2,')/CPI = ',D12.5)

      TO FIND MAXIMUM (Y/C)/CPI

52      ANG=PI/2.00
53      RYC=0.00
54      DO 111 N=1,NT,2
55      RYC=RYC+SOL(N)*SIN(N*ANG)
56      111 CONTINUE
57      WRITE(6,210) RYC
58      T=RYC*CPI
59      210 FORMAT(1H0,11X,'MAXIMUM (Y/C)/CPI = ',D12.5)

      TO FIND CPI/DYDX AT X/C=0

60      CALL ESLOPE(NT,SOL,CT,DXDY0,CF1)
61      WRITE(6,213) CF1,DXDY0
62      213 FORMAT(1H0,10X,'CPI 4*SUM(AN/CPI) = ',D12.5,
+//11X,'CPI/DYDX (X/C=0) = ',D12.5)

      TO FIND CPI/SORT((L-C)/C)

63      CALL SLENGT(NT,SOL,CT,CF2,EST)
64      ESA=DSORT(24,30)*CT
65      WRITE(6,214) CF2,EST,ESA
66      214 FORMAT(1H0,10X,'CF2 (C1+C2) = ',D12.5,
+//11X,'CPI/SORT((L-C)/C) T = ',D12.5,
+//11X,'CPI/SORT((L-C)/C) A = ',D12.5)

      TO FIND X/C S/L (Y/C)/CPI (U/U0-1)/CPI CP/CPI Y/C U/U0 CP

67      IF(ITE.NE.1) GO TO 999
68      RE=U0*(C/100.00)/VIS

```

```

69      SL=C*(1.D0+(CPI/EST)**2)
70      WRITE(6,206) CT,CPI,RE,U0,C,SL
71      206 FORMAT(1H1,//////15X,'CT = ',D23.16,/,14X,'CPI = ',D15.8,/,12X,
+ 'RE(C) = ',D15.8,/,10X,'U0(M/S) = ',D15.8,/,12X,'C(CM) = ',D15.8,
+/,12X,'L(CM) = ',D15.8)
72      WRITE(6,207)
73      207 FORMAT(1H0,///5X,'X/C',9X,'S/L',7X,'(Y/C)/CPI',3X,
+ '(U/U0-1)/CPI',5X,'CP/CPI',9X,'Y/C',9X,'U/U0',10X,'CP',/)
74      IT=1
75      IE=41
76      SSL=0.D0
77      RSL=0.D0
78      DO 23 I=IT,IE
79      RV=0.D0
80      RYC=0.D0
81      RCP=0.D0
82      RXC=0.025D0*I-0.025D0
83      CALL RXYUCP(NT,SOL,RXC,RYC,RV,RCP)
84      TRYC=RYC+CPI
85      TRV=1.D0+RV*CPI
86      TCP=RCP+CPI
87      XX=RXC+C
88      YY=TRYC+C
89      IF(I.EQ.1) GO TO 929
90      DYDX=(YY-VYI)/(XX-XXI)
91      XXI=XX
92      VYI=YY
93      SSL=SSL+0.025D0*C+DSQRT(1.D0+DYDX**2)
94      RSL=SSL/SL
95      WRITE(6,208) RXC,RSL,RYC,RV,RCP,TRYC,TRV,TCP
96      208 FORMAT(2X,D9.3,3X,D9.3,2X,D12.5,1X,D13.5,
+ 1X,D12.5,1X,D12.5,1X,D12.5)
97      GO TO 23
98      XXI=XX
99      VYI=YY
100     WRITE(6,208) RXC,RSL,RYC,RV,RCP,TRYC,TRV,TCP
101     23 CONTINUE

      CCCCCC
      TO FIND SHAPE FACTOR,SOLID,WAKE & TOTAL BLOCKAGE.

102     CALL BLOCKG(NT,SOL,CPI,C,T,W,SF,BS,BW,BT)
103     WRITE(6,211) CT,CPI,SF,BS,BW,BT
104     211 FORMAT(1H1,//////15X,'CT = ',D23.16,/,37X,'CPI = ',D15.8,
+ //23X,'SHAPE FACTOR (SF) = ',D12.5,/,21X,'SOLID BLOCKAGE (BS) = ',
+ D12.5,/,22X,'WAKE BLOCKAGE (BW) = ',D12.5,/,21X,'TOTAL BLOCKAGE = ',
+ (BT) = ',D12.5,/)
105     GO TO 555

      CCCCCC
      TO FIND X/C (Y/C)/CPI (U/U0-1)/U CP/CPI

106     999 CONTINUE
107     WRITE(6,300) CT
108     300 FORMAT(1H1,//////10X,'CT = ',D22.16,/,14X,'X/C',8X,
+ '(Y/C)/CPI',4X,'(U/U0-1)/CPI',6X,'CP/CPI',/)
109     IT=1
110     IE=41

```

```

111      DO 29 I=1,IE
112      RV=0.00
113      RYC=0.00
114      RCP=0.00
115      RXC=0.025D0*I-0.025D0
116      CALL RXYUCP(NT,SOL,RXC,RYC,RV,RCP)
117      WRITE(6,301) RXC,RYC,RV,RCP
118      301  FORMAT(11X,D9.3,3(3X,D12.5))
119      29  CONTINUE
120      C
121      C
122      C
123      555 CONTINUE
124      A
125      WRITE(6,212)
126      212  FORMAT(1HJ)
127      STOP
128      END
129
130      SUBROUTINE SLEQN(NR,NC,A,CT)
131      IMPLICIT REAL*8 (A-H,O-Z)
132      DOUBLE PRECISION A(25,26)
133      INTEGER IR(25),JC(25)
134      EPS=0.1D-10
135      N=NR
136      S=0.00D0
137      DO 6 K=1,NR
138      IR(K)=K
139      JC(K)=K
140      6  CONTINUE
141      DO 7 K=1,NR
142      PIV=0.00D0
143      DO 8 I=K,NR
144      DO 8 J=K,NR
145      G=DABS(A(I,J))
146      PIV=DMAX1(PIV,G)
147      IF((PIV-G).GT.EPS) GO TO 8
148      II=I
149      JJ=J
150      8  CONTINUE
151      DO 9 J=1,NC
152      9  CALL CHANGE(A(K,J),A(II,J),N,N)
153      DO 10 I=1,NR
154      10 CALL CHANGE(A(I,K),A(I,JJ),N,N)
155      CALL CHANGE(B,B,IR(K),IR(II))
156      CALL CHANGE(B,B,JC(K),JC(JJ))
157      PIV=A(K,K)
158      IF(DABS(PIV).LT.EPS) GO TO 500
159      DO 11 I=1,NR
160      DO 11 J=1,NC
161      IF(I.EQ.K) GO TO 11
162      IF(J.NE.K) A(I,J)=A(I,J)-A(I,K)*A(K,J)/PIV
163      11 CONTINUE
164      DO 12 J=1,NC
165      A(K,J)=A(K,J)/PIV
166      12 CONTINUE
167      DO 13 I=1,NR
168      IF(I.NE.K) A(I,K)=-A(I,K)/PIV
169      IF(I.EQ.K) A(K,K)=1.00D0/PIV
170      13 CONTINUE

```

```

166       7 CONTINUE
167       DO 14 K=1,NR
168       IF(IR(K).EQ.K) GO TO 14
169       DO 15 I=1,NR
170       15 IF(IR(I).EQ.K) L=I
171       CALL CHANGE(B,B,IR(K),IR(L))
172       DO 16 I=1,NR
173       16 CALL CHANGE(A(I,K),A(I,L),N,N)
174       14 CONTINUE
175       DO 17 K=1,NR
176       IF(JC(K).EQ.K) GO TO 17
177       DO 18 I=1,NR
178       18 IF(JC(I).EQ.K) L=I
179       CALL CHANGE(B,B,JC(K),JC(L))
180       DO 19 J=1,NC
181       19 CALL CHANGE(A(K,J),A(L,J),N,N)
182       17 CONTINUE
183       GO TO 222
184       500 WRITE(6,204) CT
185       204 FORMAT(1H1,//////11X,'CT = ',D22.16,////11X,'DETERMINANT = 0.0',
+////11X,'NO INVERSE MATRIX',////)
186       222 RETURN
187       END

```

```

188       SUBROUTINE CHANGE(X,Y,I,J)
189       DOUBLE PRECISION X,Y,Z
190       Z=X
191       X=Y
192       Y=Z
193       K=I
194       I=J
195       J=K
196       RETURN
197       END

```

```

198       SUBROUTINE ESLOPE(NT,SOL,CT,DXDY0,CF1)
199       IMPLICIT REAL*8 (A-H,O-Z)
200       DOUBLE PRECISION SOL(NT)
201       RANCP1=0.00
202       DO 25 N=1,NT,2
203       RANCP1=RANCP1+SOL(N)
204       25 CONTINUE
205       CF1=4.00*RANCP1
206       DXDY0=CT*2.00/(1.00+4.00*RANCP1)
207       RETURN
208       END

```

```

209       SUBROUTINE SLENGT(NT,SOL,CT,CF2,EST)
210       IMPLICIT REAL*8 (A-H,O-Z)
211       DIMENSION SOL(NT)
212       C1=0.00
213       C2=0.00
214       DO 26 N=1,NT,2
215       P=DFLOAT(N)
216       C1=C1+SOL(N)/(P*P-4.00)
217       SSAMAN=0.00
218       DO 27 N=1,NT,2

```

```

219 Q=DFLOAT(N)
220 CMN=(1.00-Q*Q-P*P)/((1.00-Q)**2-P*P)/((1.00+Q)**2-P*P)
221 SSAMAN=SSAMAN+SOL(N)*SOL(N)*CMN
222 27 CONTINUE
223 C2=C2+SSAMAN
224 26 CONTINUE
225 C1=-24.00*C1
226 C2=48.00*C2
227 CF2=C1+C2
228 EST=CT*DSORT(24.00)/DSORT(1.00+CF2)
229 RETURN
230 END

```

```

231 SUBROUTINE RXYUCP(NT,SOL,RXC,RVC,RV,RCP)
232 IMPLICIT REAL*8 (A-H,O-Z)
233 DOUBLE PRECISION SOL(NT)
234 PI=DATAN(1.00)*4.00
235 A=DARCOS(1.00-2.00*RXC)
236 IF(RXC.EQ.0.500) A=PI/2.00
237 IF(RXC.LT.1.0-10) GO TO 53
238 IF((1.00-RXC).LT.1.0-10) GO TO 53
239 DO 24 N=1,NT,2
240 DD=SOL(N)*DSIN(N*A)
241 RVC=RVC+DD
242 RV=RV+N*DD
243 24 CONTINUE
244 RV=2.00*RV/DSIN(A)
245 RCP=-2.00*RV
246 GO TO 54
247 53 DO 25 N=1,NT,2
248 RVC=RVC+SOL(N)*DSIN(N*A)
249 RV=RV+2.00*N*SOL(N)*N
250 25 CONTINUE
251 RCP=-2.00*RV
252 54 CONTINUE
253 RETURN
254 END

```

```

255 SUBROUTINE BLOCKG(NT,SOL,CPI,C,T,W,SP,RS,BW,BT)
256 IMPLICIT REAL*8 (A-H,O-Z)
257 DOUBLE PRECISION SOL(NT)
258 COT=0.01D0
259 W=76.2D0
260 PI=DATAN(1.00)*4.00
261 RCT=C/T
262 RCW=C/W
263 RTW=T/W
264 DIV=100.00
265 H=1.00/DIV
266 S=0.00
267 SVY=0.00
268 DO 1 M=1,99
269 S=S+H
270 A=DARCOS(1.00-2.00*S)
271 IF(S.EQ.0.500) A=PI/2.00
272 RYC=0.00
273 V=0.00
274 DO 2 N=1,NT,2

```

```

275      D=SOL(N)*DSIN(N*A)
276      RYC=RYC+D
277      V=V+N*D
278      2 CONTINUE
279      RYC=CPI*RYC
280      V=1.D0+2.D0*CPI*V/DSIN(A)
281      VY=V*PYC
282      IF(M.EQ.M/2*2) GO TO 111
283      SVY=SVY+4.D0*VY
284      GO TO 1
285      111 SVY=SVY+2.D0*VY
286      1 CONTINUE
287      SVY=SVY*M/3.D0
288      SF=SVY*4.D0*RCT**2/PI
289      BS=SF*RTW**2*PI**2/12.D0
290      RW=RCW**2*CDT/4.D0
291      BT=BS+RW
292      RETURN
293      END

```

DATA

CT = 0.4500000000000000 00

NO. OF COEFF. = 15

A( N)/CPI N=000

A( 1)/CPI = -0.202620 01  
A( 3)/CPI = 0.623690 00  
A( 5)/CPI = -0.205070-01  
A( 7)/CPI = 0.922560-02  
A( 9)/CPI = 0.399190-02  
A(11)/CPI = 0.227070-02  
A(13)/CPI = 0.140220-02  
A(15)/CPI = 0.924590-03  
A(17)/CPI = 0.641090-03  
A(19)/CPI = 0.462600-03  
A(21)/CPI = 0.344860-03  
A(23)/CPI = 0.264310-03  
A(25)/CPI = 0.207890-03  
A(27)/CPI = 0.123160-02  
A(29)/CPI = 0.987020-03

MAXIMUM (Y/C)/CPI = -0.267720 01

CF1 4\*SUM(AN/CPI) = -0.560420 01

CPI/DYDX (X/C=0) = -0.195470 00

CF2 (C1+C2) = 0.799020 02

CPI/SQRT((L-C)/C) T = 0.245100 00

CPI/SQRT((L-C)/C) A = 0.220450 01



CT = 0.4500000000000000 00  
 CPI = -0.7500000000-01  
 RE(C) = 0.37428590D 06  
 UO(M/S) = 0.150000000D 02  
 C(CM) = 0.390000000D 02  
 L(CM) = 0.41558204D 02

X/C	S/L	(Y/C)/CPI	(U/UO-1)/CPI	CP/CPI	Y/C	U/UO	CP
0.0000 00	0.0000 00	0.000000 00	0.141430 02	-0.282860 02	0.000000 00	-0.607230-01	0.212140 01
0.2500-01	0.2470-01	-0.138230 00	0.542510 01	-0.108500 02	0.103670-01	0.593120 00	0.813760 00
0.5000-01	0.5000-01	-0.295210 00	0.375340 01	-0.750680 01	0.221410-01	0.718490 00	0.563010 00
0.7500-01	0.7560-01	-0.462300 00	0.268400 01	-0.536800 01	0.346720-01	0.798700 00	0.402600 00
0.1000 00	0.1020 00	-0.640460 00	0.150100 01	-0.300210 01	0.480350-01	0.887420 00	0.225160 00
0.1250 00	0.1270 00	-0.819950 00	0.778960 00	-0.155770 01	0.614970-01	0.941590 00	0.116930 00
0.1500 00	0.1540 00	-0.100720 01	-0.307550 00	0.615090 00	0.755370-01	0.102310 01	-0.461320-01
0.1750 00	0.1800 00	-0.119420 01	-0.128140 01	0.256290 01	0.895640-01	0.109610 01	-0.192210 00
0.2000 00	0.2060 00	-0.137640 01	-0.202200 01	0.404410 01	0.103230 00	0.115170 01	-0.303310 00
0.2250 00	0.2320 00	-0.155620 01	-0.288340 01	0.576690 01	0.115710 00	0.121630 01	-0.432520 00
0.2500 00	0.2580 00	-0.173080 01	-0.379490 01	0.758980 01	0.129810 00	0.128460 01	-0.569230 00
0.2750 00	0.2830 00	-0.189460 01	-0.453480 01	0.906950 01	0.142100 00	0.134010 01	-0.680220 00
0.3000 00	0.3080 00	-0.204640 01	-0.516740 01	0.103350 02	0.153480 00	0.138760 01	-0.775110 00
0.3250 00	0.3330 00	-0.218630 01	-0.582260 01	0.116450 02	0.163980 00	0.143670 01	-0.873400 00
0.3500 00	0.3580 00	-0.231230 01	-0.644790 01	0.128960 02	0.173420 00	0.148360 01	-0.967180 00
0.3750 00	0.3820 00	-0.242100 01	-0.694310 01	0.138860 02	0.181570 00	0.152070 01	-0.104150 01
0.4000 00	0.4050 00	-0.251120 01	-0.732210 01	0.146440 02	0.188340 00	0.154920 01	-0.109830 01
0.4250 00	0.4290 00	-0.258280 01	-0.764180 01	0.152840 02	0.193710 00	0.157310 01	-0.114630 01
0.4500 00	0.4520 00	-0.263500 01	-0.789500 01	0.157900 02	0.197630 00	0.159210 01	-0.118420 01
0.4750 00	0.4750 00	-0.266670 01	-0.804460 01	0.160890 02	0.200000 00	0.160330 01	-0.120670 01
0.5000 00	0.4980 00	-0.267720 01	-0.809050 01	0.161810 02	0.200790 00	0.160680 01	-0.121360 01
0.5250 00	0.5200 00	-0.266670 01	-0.804460 01	0.160890 02	0.200000 00	0.160330 01	-0.120670 01
0.5500 00	0.5430 00	-0.263500 01	-0.799500 01	0.157900 02	0.197630 00	0.159210 01	-0.118420 01
0.5750 00	0.5670 00	-0.258280 01	-0.764180 01	0.152840 02	0.193710 00	0.157310 01	-0.114630 01
0.6000 00	0.5900 00	-0.251120 01	-0.732210 01	0.146440 02	0.188340 00	0.154920 01	-0.109830 01
0.6250 00	0.6140 00	-0.242100 01	-0.694310 01	0.138860 02	0.181570 00	0.152070 01	-0.104150 01
0.6500 00	0.6380 00	-0.231230 01	-0.644790 01	0.128960 02	0.173420 00	0.148360 01	-0.967180 00
0.6750 00	0.6620 00	-0.218630 01	-0.582260 01	0.116450 02	0.163980 00	0.143670 01	-0.873400 00
0.7000 00	0.6870 00	-0.204640 01	-0.516740 01	0.103350 02	0.153480 00	0.138760 01	-0.775110 00
0.7250 00	0.7120 00	-0.189460 01	-0.453480 01	0.906950 01	0.142100 00	0.134010 01	-0.680220 00
0.7500 00	0.7380 00	-0.173080 01	-0.379490 01	0.758980 01	0.129810 00	0.128460 01	-0.569230 00
0.7750 00	0.7670 00	-0.155620 01	-0.288340 01	0.576690 01	0.115710 00	0.121630 01	-0.432520 00
0.8000 00	0.7890 00	-0.137640 01	-0.202200 01	0.404410 01	0.103230 00	0.115170 01	-0.303310 00
0.8250 00	0.8150 00	-0.119420 01	-0.128140 01	0.256290 01	0.895640-01	0.109610 01	-0.192210 00
0.8500 00	0.8420 00	-0.100720 01	-0.307550 00	0.615090 00	0.755370-01	0.102310 01	-0.461320-01
0.8750 00	0.8680 00	-0.819950 00	0.778960 00	-0.155770 01	0.614970-01	0.941590 00	0.116930 00
0.9000 00	0.8940 00	-0.640460 00	0.150100 01	-0.300210 01	0.480350-01	0.887420 00	0.225160 00
0.9250 00	0.9200 00	-0.462300 00	0.268400 01	-0.536800 01	0.346720-01	0.798700 00	0.402600 00
0.9500 00	0.9450 00	-0.295210 00	0.375340 01	-0.750680 01	0.221410-01	0.718490 00	0.563010 00
0.9750 00	0.9710 00	-0.138230 00	0.542510 01	-0.108500 02	0.103670-01	0.593120 00	0.813760 00
0.1000 01	0.9950 00	-0.647950-16	0.141430 02	-0.282860 02	0.635960-17	-0.607230-01	0.212140 01

CT = 0.4500000000000000 00

CPI = -0.750000000-01

SHAPE FACTOR (SF) = 0.524150 01

SOLID BLOCKAGE (BS) = 0.432230-01

WAKE BLOCKAGE (BW) = 0.621720-03

TOTAL BLOCKAGE (BT) = 0.438450-01

CT = 0.1000000000000000D 01

NO. OF COEFF. = 20

A( N)/CPI N=000

A( 1)/CPI = 0.206250 00  
A( 3)/CPI = -0.483900-01  
A( 5)/CPI = -0.378390-02  
A( 7)/CPI = -0.149990-02  
A( 9)/CPI = -0.700130-03  
A(11)/CPI = -0.382470-03  
A(13)/CPI = -0.231480-03  
A(15)/CPI = -0.150650-03  
A(17)/CPI = -0.103490-03  
A(19)/CPI = -0.741430-04  
A(21)/CPI = -0.549270-04  
A(23)/CPI = -0.418210-04  
A(25)/CPI = -0.325770-04  
A(27)/CPI = -0.258710-04  
A(29)/CPI = -0.208900-04  
A(31)/CPI = -0.171130-04  
A(33)/CPI = -0.142020-04  
A(35)/CPI = -0.119320-04  
A(37)/CPI = -0.100620-03  
A(39)/CPI = -0.857740-04

MAXIMUM (Y/C)/CPI = 0.251890 00

CF1 4\*SUM(AN/CPI) = 0.602130 00

CPI/DYDX (X/C=0) = 0.124830 01

CF2 (C1+C2) = 0.281610 01

CPI/SQRT((L-C)/C) T = 0.250780 01

CPI/SQRT((L-C)/C) A = 0.489900 01

CT = 0.1000000000000000 01  
 CPI = 0.500000000-01  
 RE(C) = 0.499047870 06  
 U0(M/S) = 0.200000000 02  
 C(CM) = 0.380000000 02  
 L(CM) = 0.380151050 02

X/C	S/L	(Y/C)/CPI	(U/U0-1)/CPI	CP/CPI	Y/C	U/U0	CP
0.0000 00	0.0000 00	0.000000 00	-0.209580 01	0.419160 01	0.000000 00	0.895210 00	0.209580 00
0.2500-01	0.2500-01	0.203760-01	-0.469950 00	0.939900 00	0.101880-02	0.976500 00	0.469950-01
0.5000-01	0.5000-01	0.407470-01	-0.242450 00	0.484900 00	0.203740-02	0.987880 00	0.242450-01
0.7500-01	0.7500-01	0.604980-01	-0.138720 00	0.277440 00	0.302490-02	0.993060 00	0.138720-01
0.1000 00	0.1000 00	0.802770-01	0.742710-02	-0.148540-01	0.401390-02	0.100040 01	-0.742710-03
0.1250 00	0.1250 00	0.990200-01	0.817100-01	-0.163420 00	0.495100-02	0.100410 01	-0.817100-02
0.1500 00	0.1500 00	0.117440 00	0.190500 00	-0.381000 00	0.587200-02	0.100950 01	-0.190500-01
0.1750 00	0.1750 00	0.134570 00	0.241940 00	-0.483880 00	0.672830-02	0.101210 01	-0.241940-01
0.2000 00	0.2000 00	0.151150 00	0.325760 00	-0.651530 00	0.755750-02	0.101630 01	-0.325760-01
0.2250 00	0.2250 00	0.166500 00	0.381440 00	-0.762880 00	0.832510-02	0.101910 01	-0.381440-01
0.2500 00	0.2500 00	0.180690 00	0.426280 00	-0.852560 00	0.903450-02	0.102130 01	-0.426280-01
0.2750 00	0.2750 00	0.193960 00	0.486680 00	-0.973360 00	0.969810-02	0.102430 01	-0.486680-01
0.3000 00	0.3000 00	0.205800 00	0.522970 00	-0.104590 01	0.102900-01	0.102610 01	-0.522970-01
0.3250 00	0.3250 00	0.216350 00	0.555300 00	-0.111060 01	0.108180-01	0.102780 01	-0.555300-01
0.3500 00	0.3500 00	0.225750 00	0.594760 00	-0.118950 01	0.112880-01	0.102970 01	-0.594760-01
0.3750 00	0.3750 00	0.233660 00	0.619130 00	-0.123830 01	0.116830-01	0.103100 01	-0.619130-01
0.4000 00	0.4000 00	0.240140 00	0.637130 00	-0.127430 01	0.120070-01	0.103190 01	-0.637130-01
0.4250 00	0.4250 00	0.245300 00	0.657510 00	-0.131500 01	0.122650-01	0.103290 01	-0.657510-01
0.4500 00	0.4500 00	0.248980 00	0.669870 00	-0.133970 01	0.124490-01	0.103350 01	-0.669870-01
0.4750 00	0.4750 00	0.251150 00	0.675440 00	-0.135090 01	0.125580-01	0.103380 01	-0.675440-01
0.5000 00	0.5000 00	0.251890 00	0.678000 00	-0.135600 01	0.125950-01	0.103390 01	-0.678000-01
0.5250 00	0.5250 00	0.251150 00	0.675440 00	-0.135090 01	0.125580-01	0.103380 01	-0.675440-01
0.5500 00	0.5500 00	0.248980 00	0.669870 00	-0.133970 01	0.124490-01	0.103350 01	-0.669870-01
0.5750 00	0.5750 00	0.245300 00	0.657510 00	-0.131500 01	0.122650-01	0.103290 01	-0.657510-01
0.6000 00	0.6000 00	0.240140 00	0.637130 00	-0.127430 01	0.120070-01	0.103190 01	-0.637130-01
0.6250 00	0.6250 00	0.233660 00	0.619130 00	-0.123830 01	0.116830-01	0.103100 01	-0.619130-01
0.6500 00	0.6500 00	0.225750 00	0.594760 00	-0.118950 01	0.112880-01	0.102970 01	-0.594760-01
0.6750 00	0.6750 00	0.216350 00	0.555300 00	-0.111060 01	0.108180-01	0.102780 01	-0.555300-01
0.7000 00	0.7000 00	0.205800 00	0.522970 00	-0.104590 01	0.102900-01	0.102610 01	-0.522970-01
0.7250 00	0.7250 00	0.193960 00	0.486680 00	-0.973360 00	0.969810-02	0.102430 01	-0.486680-01
0.7500 00	0.7500 00	0.180690 00	0.426280 00	-0.852560 00	0.903450-02	0.102130 01	-0.426280-01
0.7750 00	0.7750 00	0.166500 00	0.381440 00	-0.762880 00	0.832510-02	0.101910 01	-0.381440-01
0.8000 00	0.8000 00	0.151150 00	0.325760 00	-0.651530 00	0.755750-02	0.101630 01	-0.325760-01
0.8250 00	0.8250 00	0.134570 00	0.241940 00	-0.483880 00	0.672830-02	0.101210 01	-0.241940-01
0.8500 00	0.8500 00	0.117440 00	0.190500 00	-0.381000 00	0.587200-02	0.100950 01	-0.190500-01
0.8750 00	0.8750 00	0.990200-01	0.817100-01	-0.163420 00	0.495100-02	0.100410 01	-0.817100-02
0.9000 00	0.9000 00	0.802770-01	0.742710-02	-0.148540-01	0.401390-02	0.100040 01	-0.742710-03
0.9250 00	0.9250 00	0.604980-01	-0.138720 00	0.277440 00	0.302490-02	0.993060 00	0.138720-01
0.9500 00	0.9500 00	0.407470-01	-0.242450 00	0.484900 00	0.203740-02	0.987880 00	0.242450-01
0.9750 00	0.9750 00	0.203760-01	-0.469950 00	0.939900 00	0.101880-02	0.976500 00	0.469950-01
0.1000 01	0.1000 01	-0.175800-18	-0.209580 01	0.419160 01	-0.879000-20	0.895210 00	0.209580 00

CT = 0.1000000000000000 01  
CPI = 0.50000000D-01  
SHAPE FACTOR (SF) = 0.66578D 02  
SOLID BLOCKAGE (BS) = 0.21601D-02  
WAKE BLOCKAGE (BW) = 0.62172D-03  
TOTAL BLOCKAGE (BT) = 0.27818D-02

CT = 0.2500000000000000 01

NO. OF COEFF. = 25

A( N)/CPI N=000

A( 1)/CPI = 0.525350-01  
A( 3)/CPI = -0.111480-01  
A( 5)/CPI = -0.132710-02  
A( 7)/CPI = -0.455070-03  
A( 9)/CPI = -0.208690-03  
A(11)/CPI = -0.112920-03  
A(13)/CPI = -0.679540-04  
A(15)/CPI = -0.440590-04  
A(17)/CPI = -0.301890-04  
A(19)/CPI = -0.215860-04  
A(21)/CPI = -0.159680-04  
A(23)/CPI = -0.121430-04  
A(25)/CPI = -0.944960-05  
A(27)/CPI = -0.749760-05  
A(29)/CPI = -0.604860-05  
A(31)/CPI = -0.495030-05  
A(33)/CPI = -0.410270-05  
A(35)/CPI = -0.343820-05  
A(37)/CPI = -0.290990-05  
A(39)/CPI = -0.248470-05  
A(41)/CPI = -0.213850-05  
A(43)/CPI = -0.185410-05  
A(45)/CPI = -0.161860-05  
A(47)/CPI = -0.177310-04  
A(49)/CPI = -0.156400-04

MAXIMUM (Y/C)/CPI = 0.626750-01

CF1 4\*SUM(AN/CPI) = 0.156050 00

CPI/DYDX (X/C=0) = 0.432510 01

CF2 (C1+C2) = 0.534120 00

CPI/SQRT((L-C)/C) T = 0.988820 01

CPI/SQRT((L+C)/C) A = 0.122470 02

CT = 0.2500000000000000 01

X/C	(Y/C)/CPI	(U/U0-1)/CPI	CP/CPI
0.0000 00	0.000000 00	-0.619800 00	0.123960 01
0.2500-01	0.570000-02	-0.124870 00	0.249740 00
0.5000-01	0.112340-01	-0.601250-01	0.120250 00
0.7500-01	0.165280-01	-0.228760-01	0.457530-01
0.1000 00	0.216380-01	0.141040-01	-0.282090-01
0.1250 00	0.264390-01	0.363320-01	-0.726640-01
0.1500 00	0.309730-01	0.549540-01	-0.109910 00
0.1750 00	0.352600-01	0.749120-01	-0.149820 00
0.2000 00	0.392220-01	0.874810-01	-0.174960 00
0.2250 00	0.429210-01	0.102200 00	-0.204400 00
0.2500 00	0.463040-01	0.113200 00	-0.226400 00
0.2750 00	0.493750-01	0.122430 00	-0.244860 00
0.3000 00	0.521610-01	0.132920 00	-0.265840 00
0.3250 00	0.545900-01	0.138170 00	-0.276340 00
0.3500 00	0.567420-01	0.146500 00	-0.293000 00
0.3750 00	0.585410-01	0.150710 00	-0.301430 00
0.4000 00	0.600250-01	0.155060 00	-0.310110 00
0.4250 00	0.611890-01	0.158980 00	-0.317960 00
0.4500 00	0.620080-01	0.160610 00	-0.321210 00
0.4750 00	0.625080-01	0.162360 00	-0.324720 00
0.5000 00	0.626750-01	0.162810 00	-0.325660 00
0.5250 00	0.625080-01	0.162360 00	-0.324720 00
0.5500 00	0.620080-01	0.160610 00	-0.321210 00
0.5750 00	0.611890-01	0.158980 00	-0.317960 00
0.6000 00	0.600250-01	0.155060 00	-0.310110 00
0.6250 00	0.585410-01	0.150710 00	-0.301430 00
0.6500 00	0.567420-01	0.146500 00	-0.293000 00
0.6750 00	0.545900-01	0.138170 00	-0.276340 00
0.7000 00	0.521610-01	0.132920 00	-0.265840 00
0.7250 00	0.493750-01	0.122430 00	-0.244860 00
0.7500 00	0.463040-01	0.113200 00	-0.226400 00
0.7750 00	0.429210-01	0.102200 00	-0.204400 00
0.8000 00	0.392220-01	0.874810-01	-0.174960 00
0.8250 00	0.352600-01	0.749120-01	-0.149820 00
0.8500 00	0.309730-01	0.549540-01	-0.109910 00
0.8750 00	0.264390-01	0.363320-01	-0.726640-01
0.9000 00	0.216380-01	0.141040-01	-0.282090-01
0.9250 00	0.165280-01	-0.228760-01	0.457530-01
0.9500 00	0.112340-01	-0.601250-01	0.120250 00
0.9750 00	0.570000-02	-0.124870 00	0.249740 00
0.1000 01	-0.721960-18	-0.619800 00	0.123960 01

CT = 0.10000D 00

NO. OF COEFF. =	N = 24	N = 20	N = 15	N = 10
MAX. (Y/C)/CPI =	-0.44885D 00	-0.44885D 00	-0.44883D 00	-0.44897D 00
CPI/SQRT ((L-C)/C) =	0.63070D 00	0.63070D 00	0.63068D 00	0.63056D 00

A ( N ) /CPI N=ODD

A ( N ) /CPI	N=24	N=20	N=15	N=10
A ( 1 ) /CPI	= -0.30634D 00	-0.30634D 00	-0.30634D 00	-0.30634D 00
A ( 3 ) /CPI	= 0.74481D-02	0.74483D-02	0.74493D-02	0.74563D-02
A ( 5 ) /CPI	= 0.68185D-01	0.68186D-01	0.68188D-01	0.68203D-01
A ( 7 ) /CPI	= 0.65025D-01	0.65025D-01	0.65027D-01	0.65041D-01
A ( 9 ) /CPI	= -0.90222D-01	-0.90222D-01	-0.90224D-01	-0.90236D-01
A ( 11 ) /CPI	= 0.37430D-01	0.37430D-01	0.37431D-01	0.37439D-01
A ( 13 ) /CPI	= -0.90847D-02	-0.90847D-02	-0.90847D-02	-0.90862D-02
A ( 15 ) /CPI	= 0.12636D-02	0.12636D-02	0.12639D-02	0.12688D-02
A ( 17 ) /CPI	= -0.23315D-03	-0.23311D-03	-0.23293D-03	-0.52105D-03
A ( 19 ) /CPI	= -0.40339D-04	-0.40302D-04	-0.40158D-04	-0.21524D-03
A ( 21 ) /CPI	= -0.39252D-04	-0.39218D-04	-0.39143D-04	
A ( 23 ) /CPI	= -0.28360D-04	-0.28331D-04	-0.28394D-04	
A ( 25 ) /CPI	= -0.21637D-04	-0.21612D-04	-0.21923D-04	
A ( 27 ) /CPI	= -0.16863D-04	-0.16847D-04	-0.12093D-03	
A ( 29 ) /CPI	= -0.13406D-04	-0.13401D-04	-0.93390D-04	
A ( 31 ) /CPI	= -0.10839D-04	-0.10854D-04		
A ( 33 ) /CPI	= -0.88927D-05	-0.89485D-05		
A ( 35 ) /CPI	= -0.73908D-05	-0.75398D-05		
A ( 37 ) /CPI	= -0.62146D-05	-0.60467D-04		
A ( 39 ) /CPI	= -0.52836D-05	-0.50525D-04		
A ( 41 ) /CPI	= -0.45448D-05			
A ( 43 ) /CPI	= -0.39737D-05			
A ( 45 ) /CPI	= -0.39828D-04			
A ( 47 ) /CPI	= -0.34464D-04			

Table 1



CT = 0.49800D 00

NO. OF COEFF. = N = 24

N = 20

N = 15

N = 10

MAX. (Y/C)/CPI = -0.27161D 03

-0.27178D 03

-0.27251D 03

-0.27687D 03

CPI/SORT((L-C)/C) = 0.24021D-02

0.24004D-02

0.23937D-02

0.23541D-02

A (L, N) /CPI H=ODD

A (1) /CPI	=	-0.20939D 03	-0.20952D 03	-0.21006D 03	-0.21349D 03
A (3) /CPI	=	-0.60899D 02	-0.60937D 02	-0.61097D 02	-0.62098D 02
A (5) /CPI	=	-0.59880D 00	-0.59912D 00	-0.60050D 00	-0.60903D 00
A (7) /CPI	=	0.10348D 01	0.10355D 01	0.10384D 01	0.10564D 01
A (9) /CPI	=	0.47227D 00	0.47259D 00	0.47395D 00	0.48274D 00
A (11) /CPI	=	0.26538D 00	0.26557D 00	0.26638D 00	0.27184D 00
A (13) /CPI	=	0.16303D 00	0.16315D 00	0.16368D 00	0.16769D 00
A (15) /CPI	=	0.10710D 00	0.10719D 00	0.10758D 00	0.11125D 00
A (17) /CPI	=	0.74054D-01	0.74120D-01	0.74429D-01	0.38352D 00
A (19) /CPI	=	0.53299D-01	0.53351D-01	0.53618D-01	0.27070D 00
A (21) /CPI	=	0.39620D-01	0.39665D-01	0.39918D-01	
A (23) /CPI	=	0.30245D-01	0.30285D-01	0.30557D-01	
A (25) /CPI	=	0.23607D-01	0.23645D-01	0.24000D-01	
A (27) /CPI	=	0.18777D-01	0.18814D-01	0.14215D 00	
A (29) /CPI	=	0.15180D-01	0.15219D-01	0.11400D 00	
A (31) /CPI	=	0.12446D-01	0.12490D-01		
A (33) /CPI	=	0.10332D-01	0.10386D-01		
A (35) /CPI	=	0.86720D-02	0.87541D-02		
A (37) /CPI	=	0.73509D-02	0.73643D-01		
A (39) /CPI	=	0.62874D-02	0.62666D-01		
A (41) /CPI	=	0.54239D-02			
A (43) /CPI	=	0.47221D-02			
A (45) /CPI	=	0.49145D-01			
A (47) /CPI	=	0.43032D-01			

Table 2

CT = 0.49850D 00

NO. OF COEFF. = N = 24

N = 20

N = 15

N = 10

MAX. (Y/C)/CPI = 0.50811D 04

0.50227D 04

0.47926D 04

0.37351D 04

CPI/SQRT((L-C)/C) = 0.12840D-03

0.12988D-03

0.13610D-03

0.17450D-03

A ( N ) /CPI N=ODD

A ( 1 ) /CPI = 0.39177D 04  
 A ( 3 ) /CPI = -0.11389D 04  
 A ( 5 ) /CPI = -0.10961D 02  
 A ( 7 ) /CPI = -0.19378D 02  
 A ( 9 ) /CPI = -0.88463D 01  
 A ( 11 ) /CPI = -0.49706D 01  
 A ( 13 ) /CPI = -0.30533D 01  
 A ( 15 ) /CPI = -0.20059D 01  
 A ( 17 ) /CPI = -0.13869D 01  
 A ( 19 ) /CPI = -0.99818D 00  
 A ( 21 ) /CPI = -0.74200D 00  
 A ( 23 ) /CPI = -0.56642D 00  
 A ( 25 ) /CPI = -0.44211D 00  
 A ( 27 ) /CPI = -0.35165D 00  
 A ( 29 ) /CPI = -0.28428D 00  
 A ( 31 ) /CPI = -0.23309D 00  
 A ( 33 ) /CPI = -0.19349D 00  
 A ( 35 ) /CPI = -0.16240D 00  
 A ( 37 ) /CPI = -0.13766D 00  
 A ( 39 ) /CPI = -0.11775D 00  
 A ( 41 ) /CPI = -0.10158D 00  
 A ( 43 ) /CPI = -0.88431D 01  
 A ( 45 ) /CPI = -0.92035D 00  
 A ( 47 ) /CPI = -0.80587D 00

0.38727D 04  
 -0.11258D 04  
 -0.10834D 02  
 -0.19156D 02  
 -0.87454D 01  
 -0.49140D 01  
 -0.30187D 01  
 -0.19833D 01  
 -0.13714D 01  
 -0.98709D 00  
 -0.73368D 00  
 -0.56031D 00  
 -0.43745D 00  
 -0.34809D 00  
 -0.28156D 00  
 -0.23107D 00  
 -0.19216D 00  
 -0.16196D 00  
 -0.13625D 01  
 -0.11594D 01

0.36950D 04  
 -0.10741D 04  
 -0.10334D 02  
 -0.18279D 02  
 -0.83461D 01  
 -0.46904D 01  
 -0.28821D 01  
 -0.18942D 01  
 -0.13104D 01  
 -0.94403D 00  
 -0.70281D 00  
 -0.53798D 00  
 -0.42253D 00  
 -0.25027D 01  
 -0.20071D 01

0.28805D 04  
 -0.83742D 03  
 -0.80388D 01  
 -0.14265D 02  
 -0.65207D 01  
 -0.36716D 01  
 -0.22647D 01  
 -0.15024D 01  
 -0.51794D 01  
 -0.36558D 01

Table 3

CT = 0.50000D 00

NO. OF COEFF. =	N = 25	N = 20	N = 15	N = 10
MAX. (Y/C) /CPI =	0.84499D 02	0.84477D 02	0.84416D 02	0.83972D 02
CPI/SQRT ((L-C)/C) =	0.77198D-02	0.77207D-02	0.77256D-02	0.77604D-02

A ( M ) /CPI M=ODD

A ( 1 ) /CPI	=	0.65181D 02	0.65167D 02	0.65114D 02	0.64790D 02
A ( 3 ) /CPI	=	-0.18919D 02	-0.18925D 02	-0.18900D 02	-0.18807D 02
A ( 5 ) /CPI	=	-0.17035D 00	-0.17029D 00	-0.17010D 00	-0.16887D 00
A ( 7 ) /CPI	=	-0.32318D 00	-0.32312D 00	-0.32291D 00	-0.32164D 00
A ( 9 ) /CPI	=	-0.14769D 00	-0.14766D 00	-0.14758D 00	-0.14714D 00
A ( 11 ) /CPI	=	-0.82958D-01	-0.82948D-01	-0.82916D-01	-0.82842D-01
A ( 13 ) /CPI	=	-0.50953D-01	-0.50949D-01	-0.50942D-01	-0.51092D-01
A ( 15 ) /CPI	=	-0.33471D-01	-0.33471D-01	-0.33478D-01	-0.33890D-01
A ( 17 ) /CPI	=	-0.23141D-01	-0.23143D-01	-0.23160D-01	-0.11682D 00
A ( 19 ) /CPI	=	-0.16654D-01	-0.16657D-01	-0.16683D-01	-0.82463D-01
A ( 21 ) /CPI	=	-0.12379D-01	-0.12383D-01	-0.12420D-01	
A ( 23 ) /CPI	=	-0.94496D-02	-0.94548D-02	-0.95068D-02	
A ( 25 ) /CPI	=	-0.73754D-02	-0.73815D-02	-0.74664D-02	
A ( 27 ) /CPI	=	-0.58661D-02	-0.58734D-02	-0.44225D-01	
A ( 29 ) /CPI	=	-0.47420D-02	-0.47509D-02	-0.35468D-01	
A ( 31 ) /CPI	=	-0.38877D-02	-0.38989D-02		
A ( 33 ) /CPI	=	-0.32270D-02	-0.32423D-02		
A ( 35 ) /CPI	=	-0.27081D-02	-0.27326D-02		
A ( 37 ) /CPI	=	-0.22951D-02	-0.22988D-01		
A ( 39 ) /CPI	=	-0.19623D-02	-0.19562D-01		
A ( 41 ) /CPI	=	-0.16915D-02			
A ( 43 ) /CPI	=	-0.14694D-02			
A ( 45 ) /CPI	=	-0.12873D-02			
A ( 47 ) /CPI	=	-0.14040D-01			
A ( 49 ) /CPI	=	-0.12363D-01			

Table 4

CT = 0.10000D 01

NO. OF COEFF. = N = 25

N = 20

N = 15

N = 10

MAX. (Y/C)/CPI = 0.25190D 00

0.25189D 00

0.25192D 00

0.25181D 00

CPI/SQRT((L-C)/C) = 0.25072D 01

0.25067D 01

0.25058D 01

0.25034D 01

A( M)/CPI M=ODD

A( 1)/CPI = 0.20625D 00  
 A( 3)/CPI = -0.48390D-01  
 A( 5)/CPI = -0.37839D-02  
 A( 7)/CPI = -0.14999D-02  
 A( 9)/CPI = -0.70011D-03  
 A(11)/CPI = -0.38246D-03  
 A(13)/CPI = -0.23147D-03  
 A(15)/CPI = -0.15063D-03  
 A(17)/CPI = -0.10348D-03  
 A(19)/CPI = -0.74132D-04  
 A(21)/CPI = -0.54916D-04  
 A(23)/CPI = -0.41809D-04  
 A(25)/CPI = -0.32564D-04  
 A(27)/CPI = -0.25856D-04  
 A(29)/CPI = -0.20872D-04  
 A(31)/CPI = -0.17091D-04  
 A(33)/CPI = -0.14172D-04  
 A(35)/CPI = -0.11882D-04  
 A(37)/CPI = -0.10060D-04  
 A(39)/CPI = -0.85939D-05  
 A(41)/CPI = -0.74005D-05  
 A(43)/CPI = -0.64206D-05  
 A(45)/CPI = -0.56123D-05  
 A(47)/CPI = -0.61373D-04  
 A(49)/CPI = -0.54101D-04

0.20625D 00  
 -0.48390D-01  
 -0.37839D-02  
 -0.14999D-02  
 -0.70013D-03  
 -0.38247D-03  
 -0.23148D-03  
 -0.15065D-03  
 -0.10349D-03  
 -0.74143D-04  
 -0.54927D-04  
 -0.41821D-04  
 -0.32577D-04  
 -0.25871D-04  
 -0.20890D-04  
 -0.17113D-04  
 -0.14202D-04  
 -0.11932D-04  
 -0.10062D-03  
 -0.85774D-04

0.20625D 00  
 -0.48390D-01  
 -0.37840D-02  
 -0.15000D-02  
 -0.70018D-03  
 -0.38252D-03  
 -0.23153D-03  
 -0.15070D-03  
 -0.10355D-03  
 -0.74209D-04  
 -0.55008D-04  
 -0.41930D-04  
 -0.32752D-04  
 -0.19420D-03  
 -0.15623D-03

0.20625D 00  
 -0.48391D-01  
 -0.37846D-02  
 -0.15005D-02  
 -0.70067D-03  
 -0.38306D-03  
 -0.23221D-03  
 -0.15176D-03  
 -0.51911D-03  
 -0.36899D-03

Table 5

CT = 0.50000D-01

NO. OF COEFF. =	N = 25	N = 20	N = 15	N = 10
MAX. (Y/C)/CPI =	0.27814D-01	0.27814D-01	0.27818D-01	0.27803D-01
CPI/SQRT((L-C)/C) =	0.22138D-02	0.22132D-02	0.22120D-02	0.22090D-02

A ( M ) /CPI M=ODD

A ( 1 ) /CPI =	0.23468D-01	0.23468D-01	0.23468D-01	0.23468D-01
A ( 3 ) /CPI =	-0.48314D-02	-0.48314D-02	-0.48314D-02	-0.48314D-02
A ( 5 ) /CPI =	-0.63414D-03	-0.63414D-03	-0.63414D-03	-0.63411D-03
A ( 7 ) /CPI =	-0.21357D-03	-0.21357D-03	-0.21357D-03	-0.21355D-03
A ( 9 ) /CPI =	-0.97472D-04	-0.97471D-04	-0.97469D-04	-0.97455D-04
A ( 11 ) /CPI =	-0.52604D-04	-0.52604D-04	-0.52602D-04	-0.52590D-04
A ( 13 ) /CPI =	-0.31608D-04	-0.31607D-04	-0.31606D-04	-0.31597D-04
A ( 15 ) /CPI =	-0.20472D-04	-0.20472D-04	-0.20470D-04	-0.20469D-04
A ( 17 ) /CPI =	-0.14017D-04	-0.14016D-04	-0.14015D-04	-0.69840D-04
A ( 19 ) /CPI =	-0.10017D-04	-0.10017D-04	-0.10016D-04	-0.49906D-04
A ( 21 ) /CPI =	-0.74067D-05	-0.74066D-05	-0.74060D-05	
A ( 23 ) /CPI =	-0.56308D-05	-0.56307D-05	-0.56307D-05	
A ( 25 ) /CPI =	-0.43806D-05	-0.43804D-05	-0.43820D-05	
A ( 27 ) /CPI =	-0.34749D-05	-0.34748D-05	-0.26029D-04	
A ( 29 ) /CPI =	-0.28029D-05	-0.28028D-05	-0.20989D-04	
A ( 31 ) /CPI =	-0.22935D-05	-0.22936D-05		
A ( 33 ) /CPI =	-0.19006D-05	-0.19008D-05		
A ( 35 ) /CPI =	-0.15925D-05	-0.15932D-05		
A ( 37 ) /CPI =	-0.13476D-05	-0.13468D-04		
A ( 39 ) /CPI =	-0.11505D-05	-0.11495D-04		
A ( 41 ) /CPI =	-0.99010D-06			
A ( 43 ) /CPI =	-0.85822D-06			
A ( 45 ) /CPI =	-0.74892D-06			
A ( 47 ) /CPI =	-0.82087D-05			
A ( 49 ) /CPI =	-0.72422D-05			

Table 6

CT = 0.10000D 02

NO. OF COEFF. =

N = 25

N = 20

N = 15

N = 10

MAX. (Y/C)/CPI =

0.13166D-01

0.13166D-01

0.13168D-01

0.13161D-01

CPI/SQRT((L-C)/C) =

0.46629D 02

0.46616D 02

0.46590D 02

0.46524D 02

A ( M ) /CPI M=ODD.

A ( 1 ) /CPI	=	0.11143D-01	0.11143D-01	0.11143D-01	0.11143D-01
A ( 3 ) /CPI	=	-0.22607D-02	-0.22607D-02	-0.22607D-02	-0.22607D-02
A ( 5 ) /CPI	=	-0.31001D-03	-0.31001D-03	-0.31001D-03	-0.30999D-03
A ( 7 ) /CPI	=	-0.10379D-03	-0.10379D-03	-0.10378D-03	-0.10377D-03
A ( 9 ) /CPI	=	-0.47267D-04	-0.47267D-04	-0.47265D-04	-0.47253D-04
A ( 11 ) /CPI	=	-0.25479D-04	-0.25479D-04	-0.25478D-04	-0.25467D-04
A ( 13 ) /CPI	=	-0.15298D-04	-0.15298D-04	-0.15297D-04	-0.15287D-04
A ( 15 ) /CPI	=	-0.99036D-05	-0.99034D-05	-0.99023D-05	-0.98932D-05
A ( 17 ) /CPI	=	-0.67784D-05	-0.67782D-05	-0.67772D-05	-0.33750D-04
A ( 19 ) /CPI	=	-0.48429D-05	-0.48427D-05	-0.48418D-05	-0.24132D-04
A ( 21 ) /CPI	=	-0.35802D-05	-0.35800D-05	-0.35791D-05	
A ( 23 ) /CPI	=	-0.27214D-05	-0.27212D-05	-0.27203D-05	
A ( 25 ) /CPI	=	-0.21169D-05	-0.21167D-05	-0.21161D-05	
A ( 27 ) /CPI	=	-0.16790D-05	-0.16789D-05	-0.12573D-04	
A ( 29 ) /CPI	=	-0.13542D-05	-0.13540D-05	-0.10141D-04	
A ( 31 ) /CPI	=	-0.11080D-05	-0.11079D-05		
A ( 33 ) /CPI	=	-0.91810D-06	-0.91798D-06		
A ( 35 ) /CPI	=	-0.76925D-06	-0.76923D-06		
A ( 37 ) /CPI	=	-0.65092D-06	-0.65044D-05		
A ( 39 ) /CPI	=	-0.55568D-06	-0.55527D-05		
A ( 41 ) /CPI	=	-0.47815D-06			
A ( 43 ) /CPI	=	-0.41442D-06			
A ( 45 ) /CPI	=	-0.36156D-06			
A ( 47 ) /CPI	=	-0.39642D-05			
A ( 49 ) /CPI	=	-0.34978D-05			

Table 7

# GENERAL ARRANGEMENT OF THE BLOWER WIND-TUNNEL

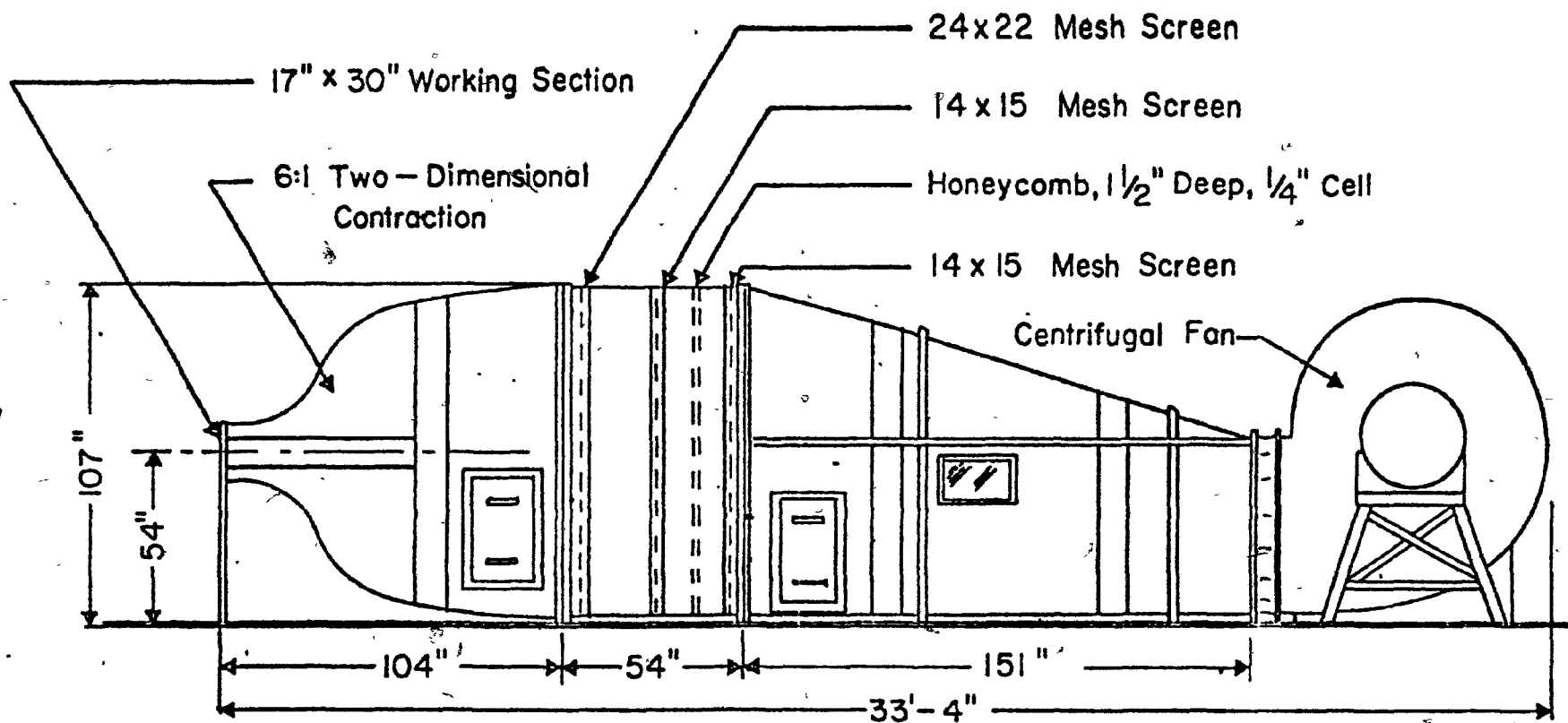
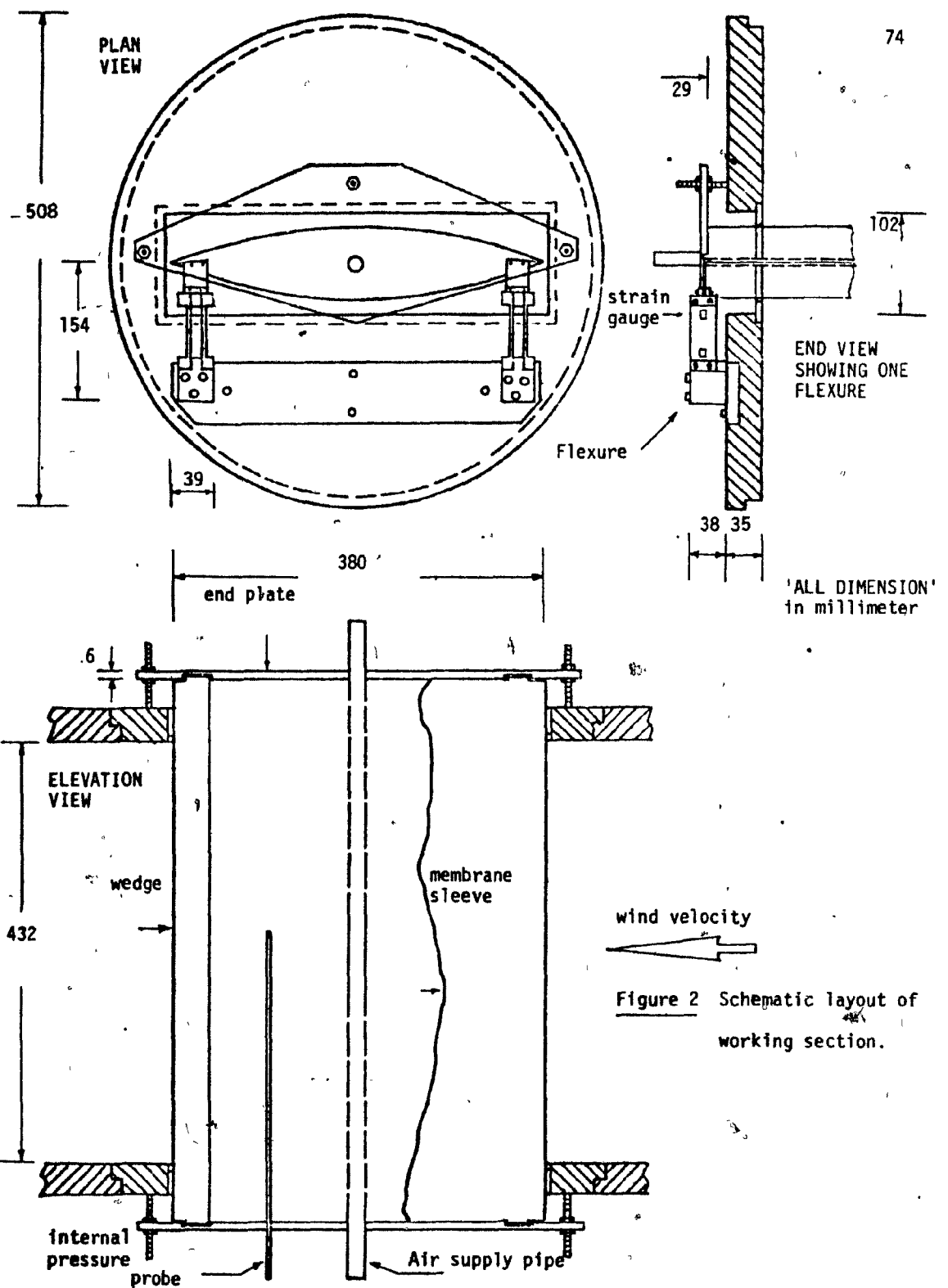


Fig. 1 General arrangement of the blower wind-tunnel





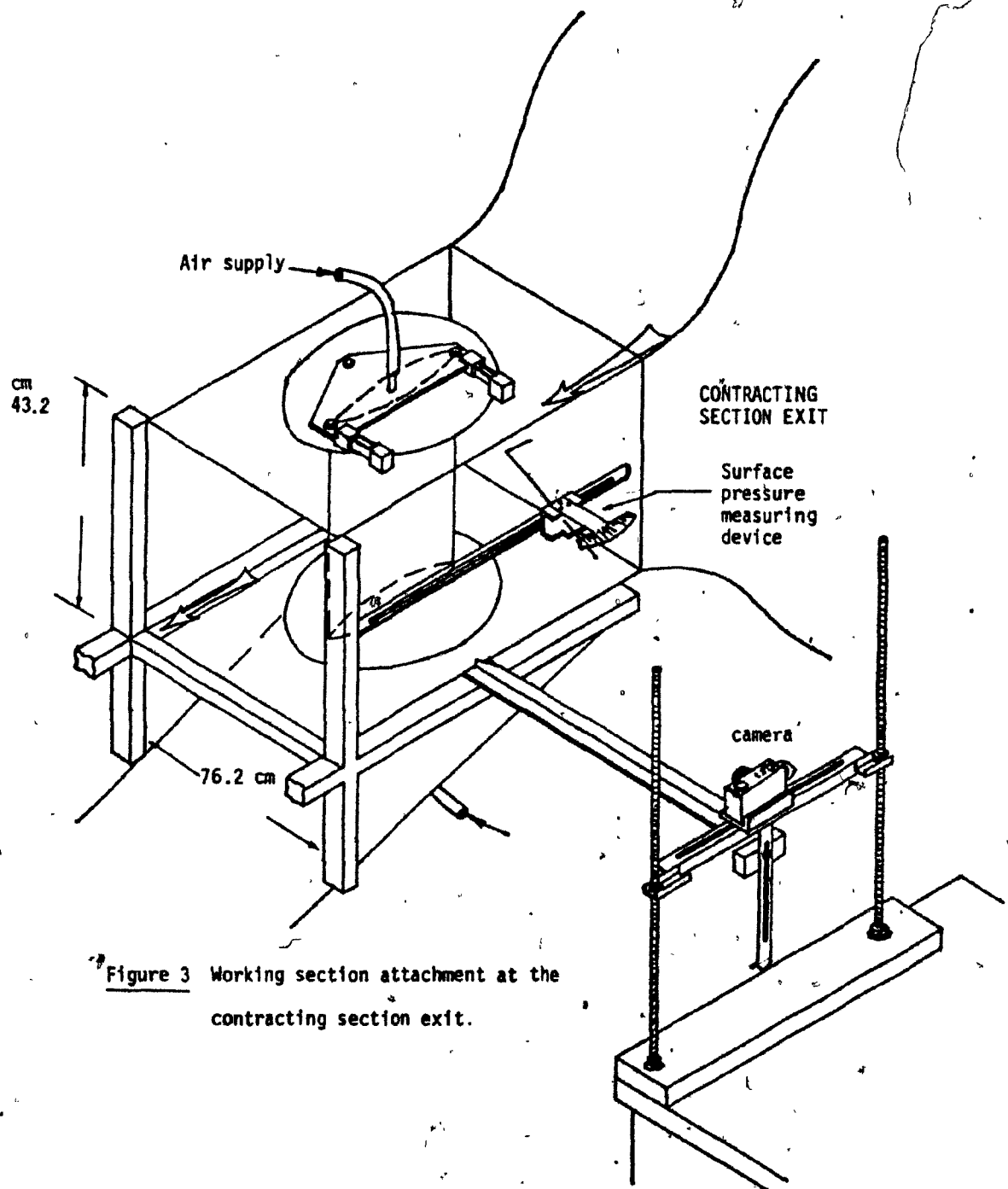


Figure 3 Working section attachment at the contracting section exit.

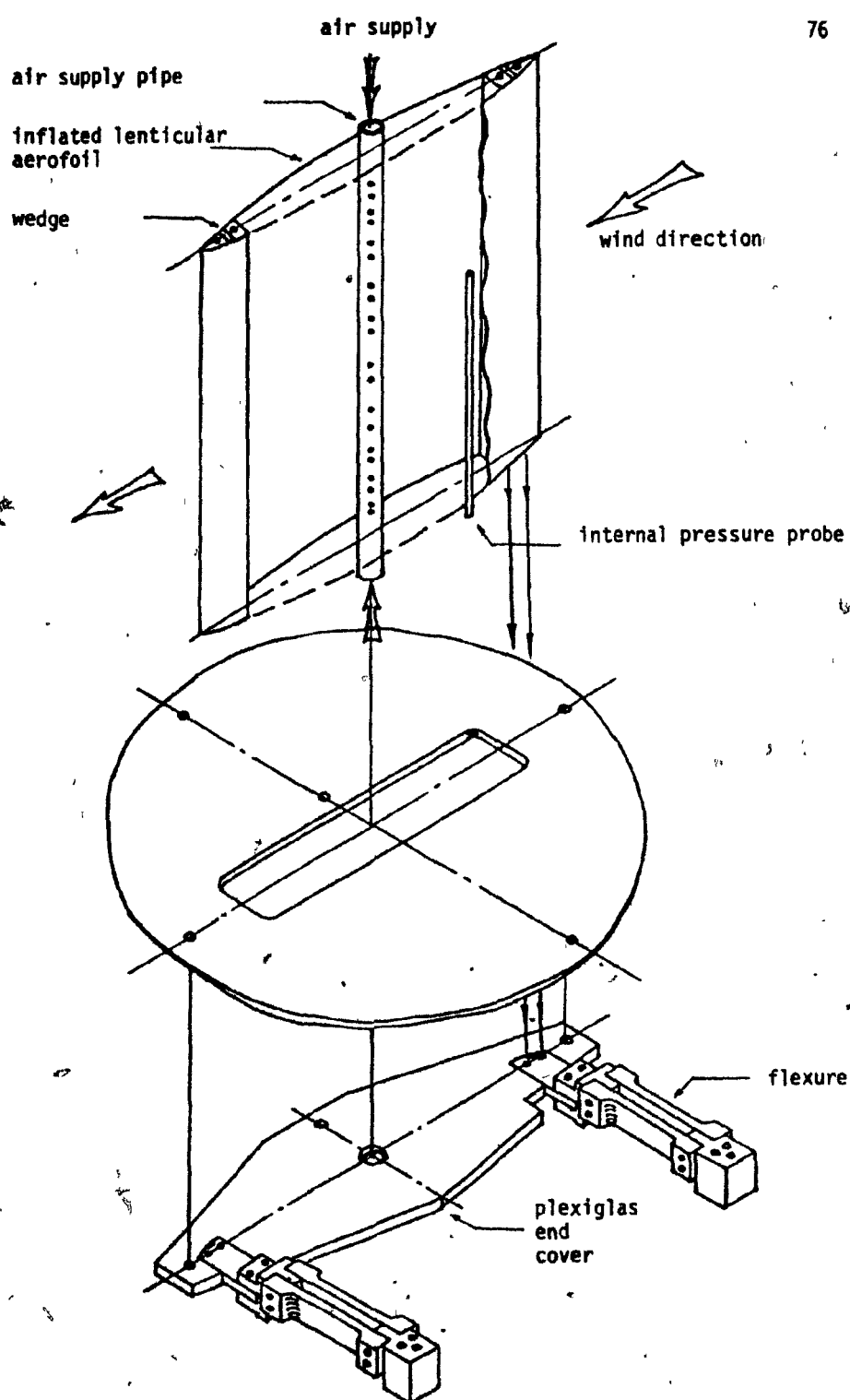


Figure 4 Design of inflated aerofoil.

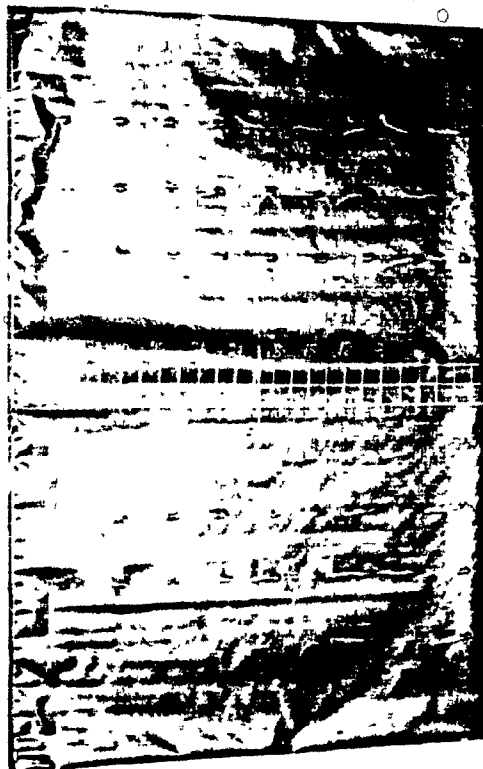


Figure 5a Envelope made of flexible  
impervious rip-stop  
nylon 25 g/m<sup>2</sup>.

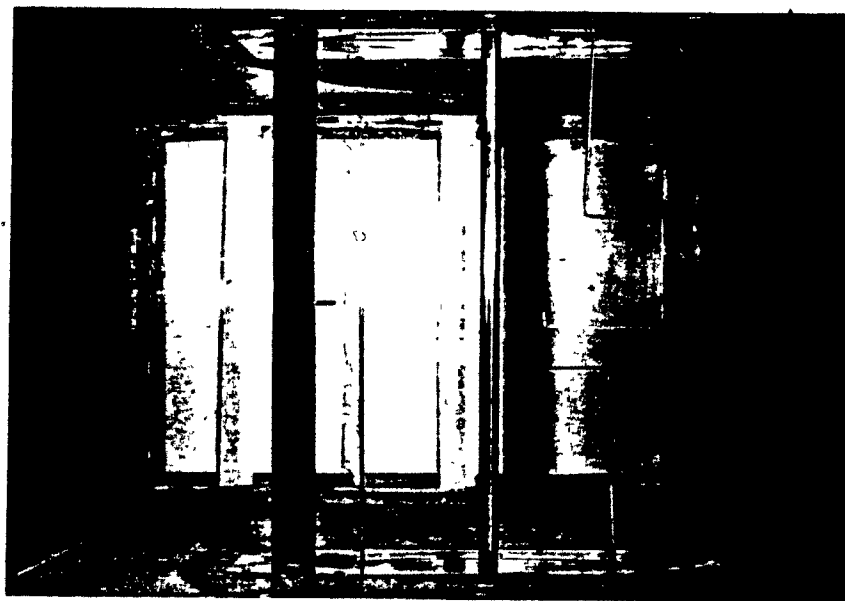


Figure 5b Configuration of wedges, air supply tube, and  
static pressure tubes.

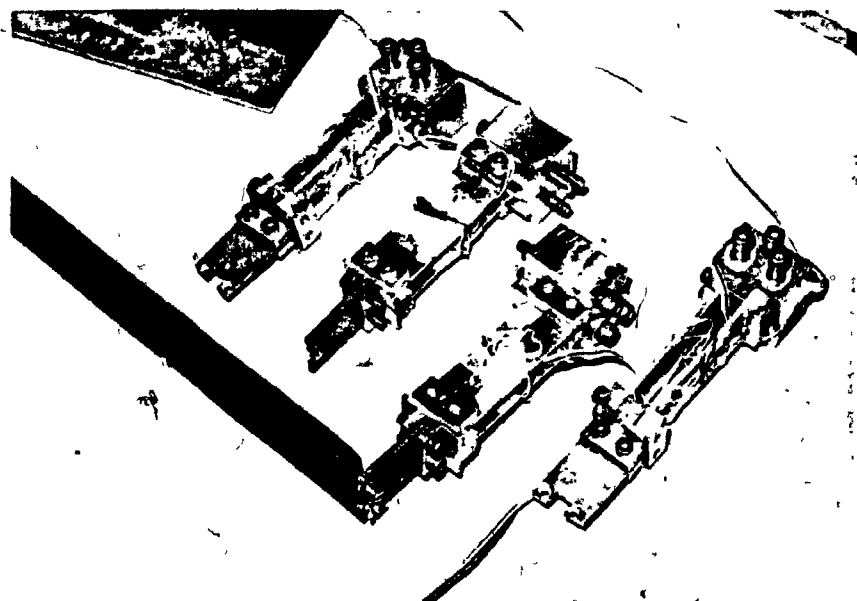


Figure 6 Flexures assembly. (Calibrated flexures, and attachment of wedge on flexure)

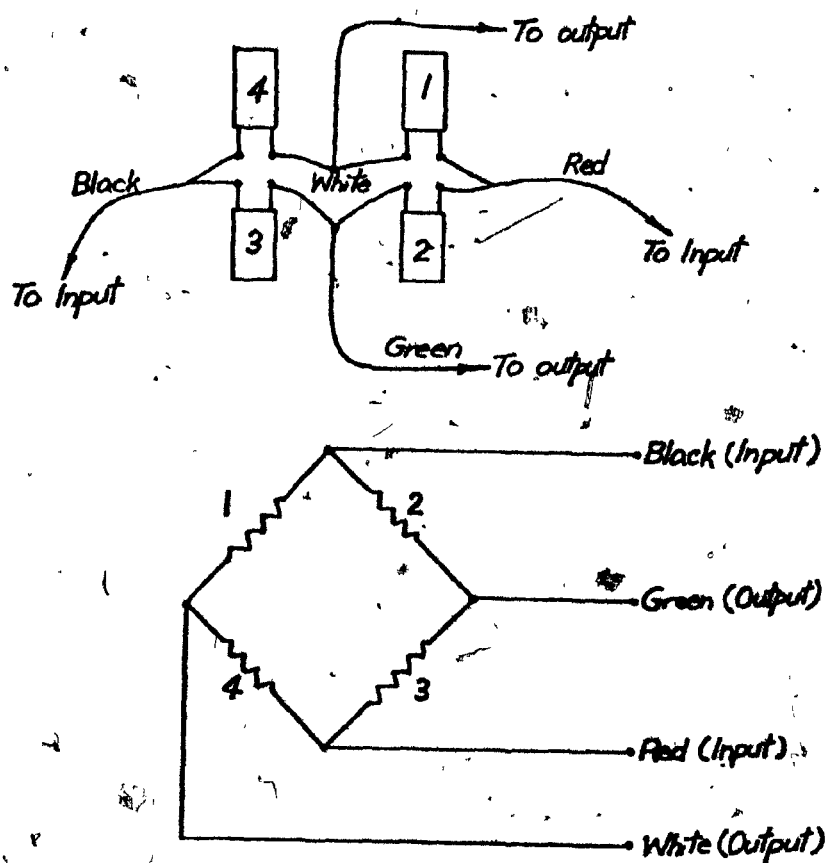
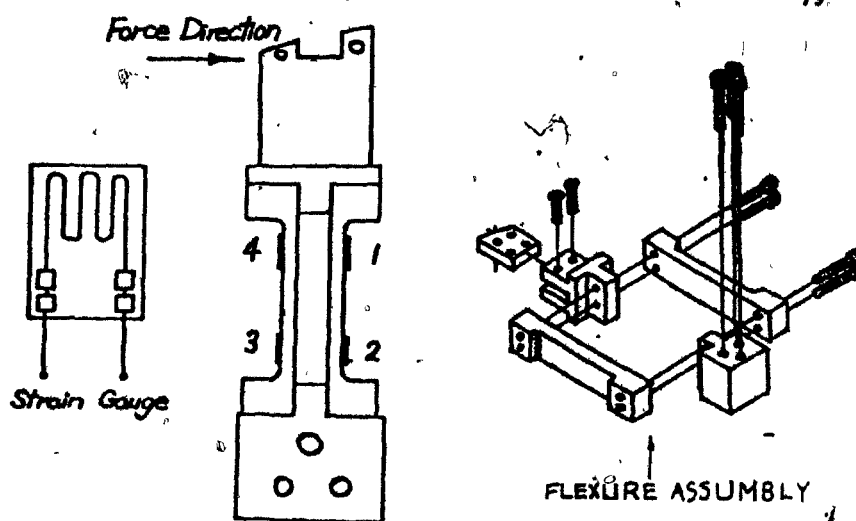


Figure 7 Schematic layout of strain gauges attachment on the flexures.

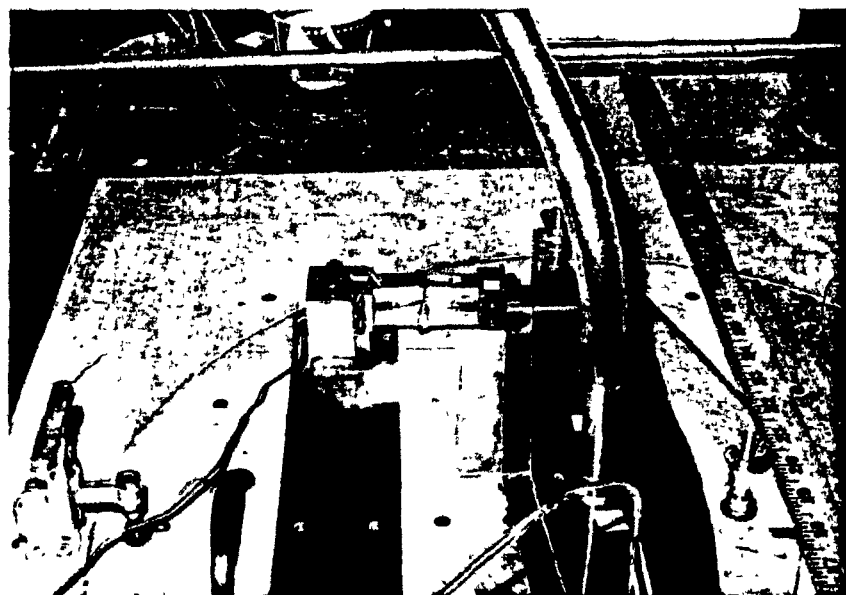


Figure 8 Flexure attachment on roof.

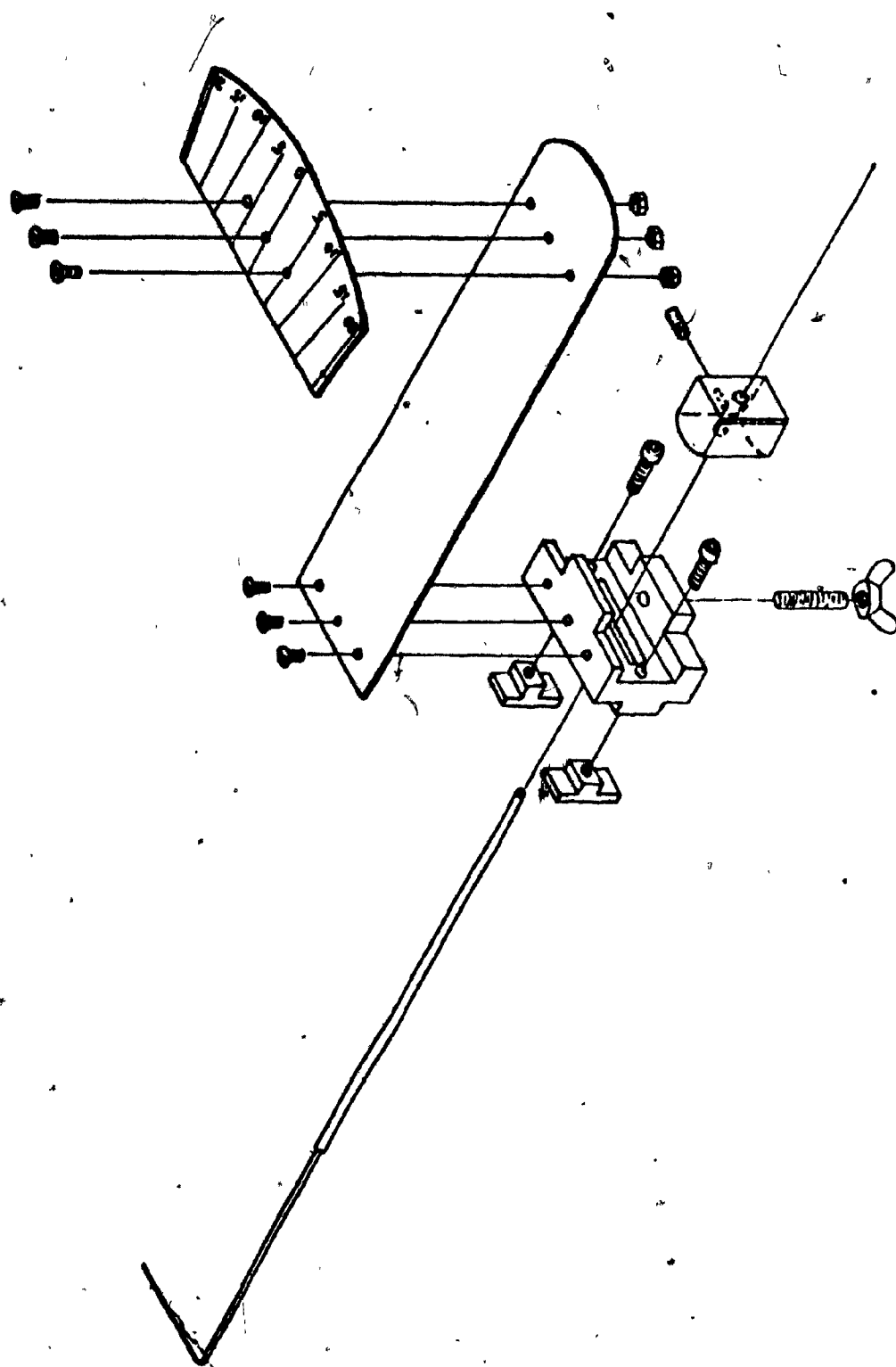


Figure 9A Surface static pressure measuring device.

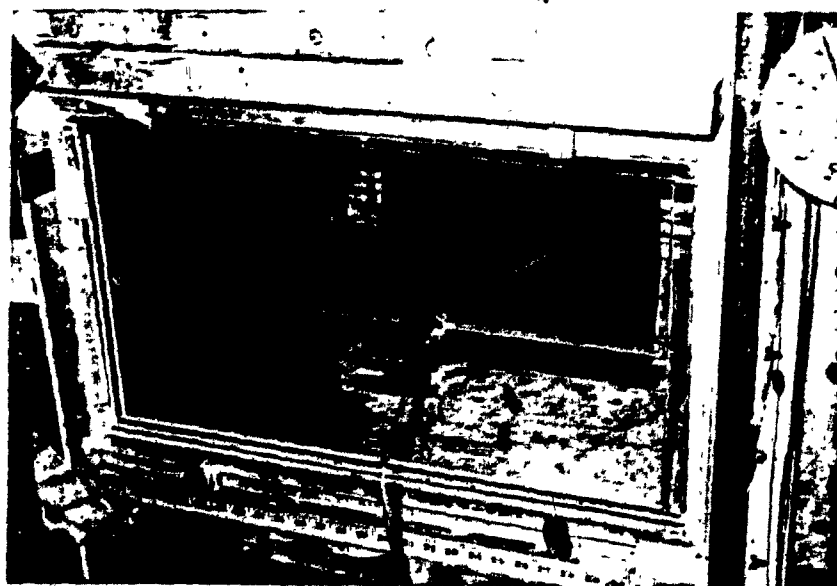


Figure 9B Surface static pressure measuring device.



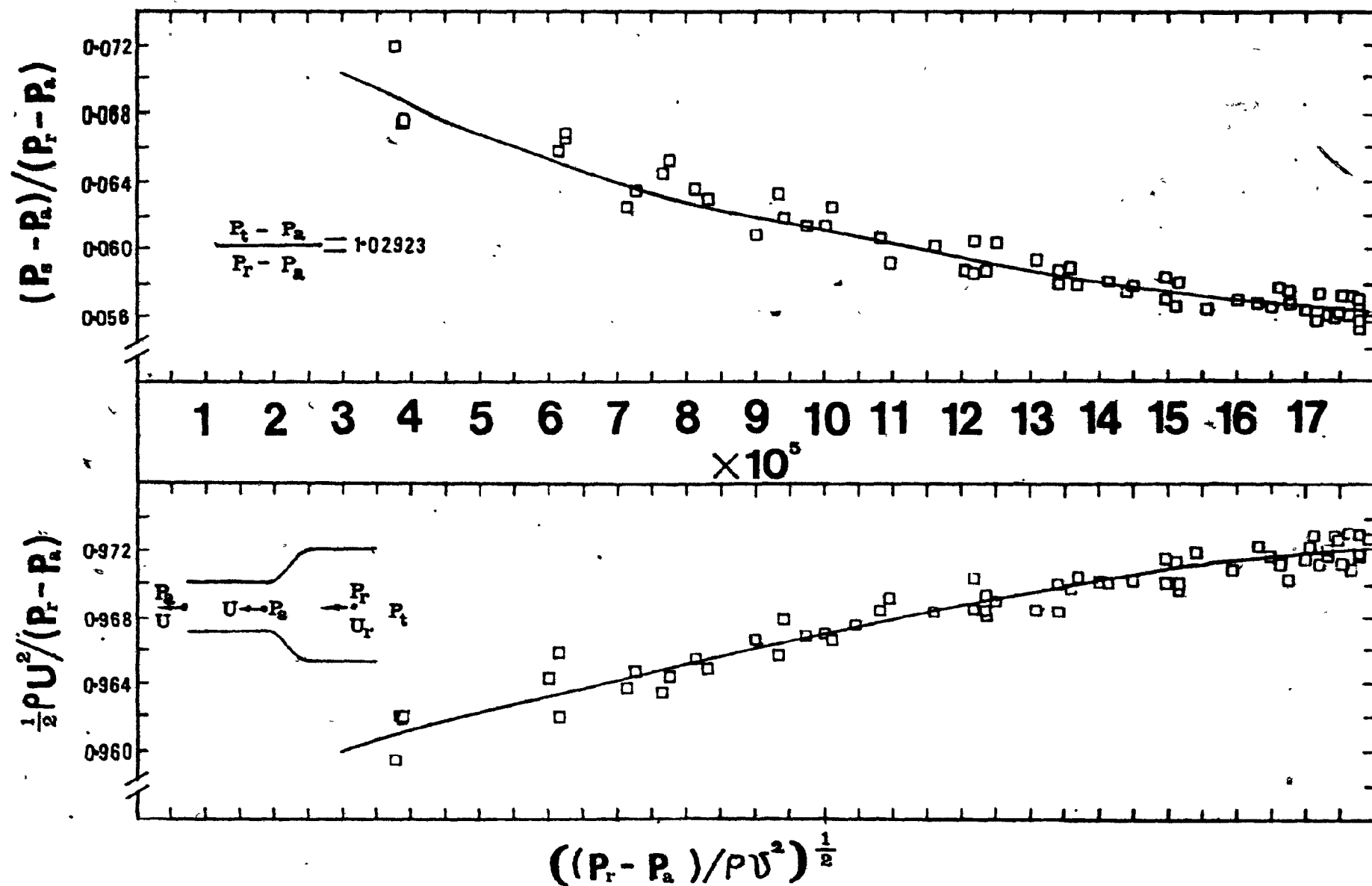


Figure 10 Wind tunnel calibration.

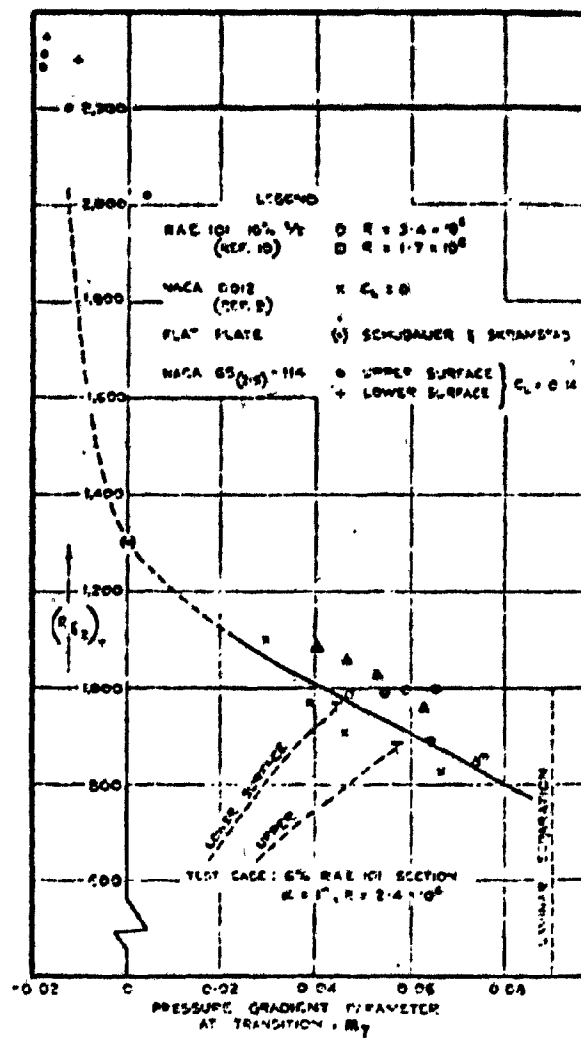


Figure 11 Boundary layer Reynolds number at transition plotted against the pressure gradient parameter.

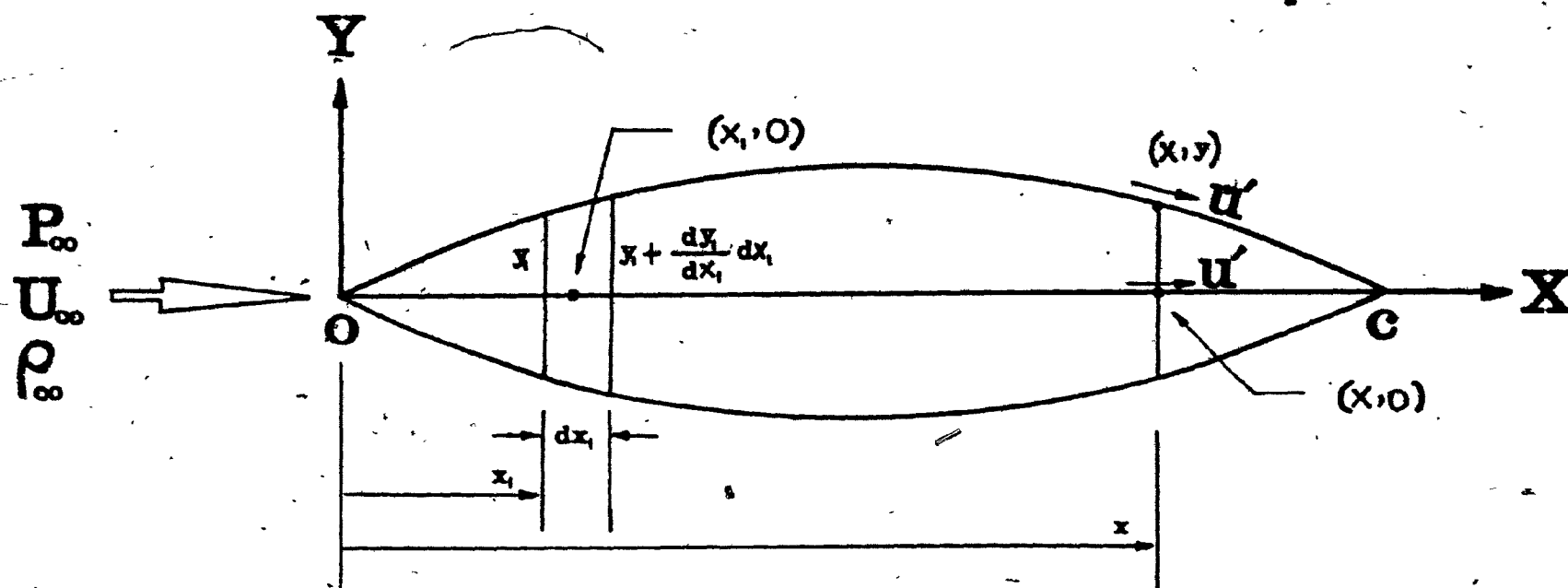


Figure 12 Inflated lenticular aerofoil at zero incidence.

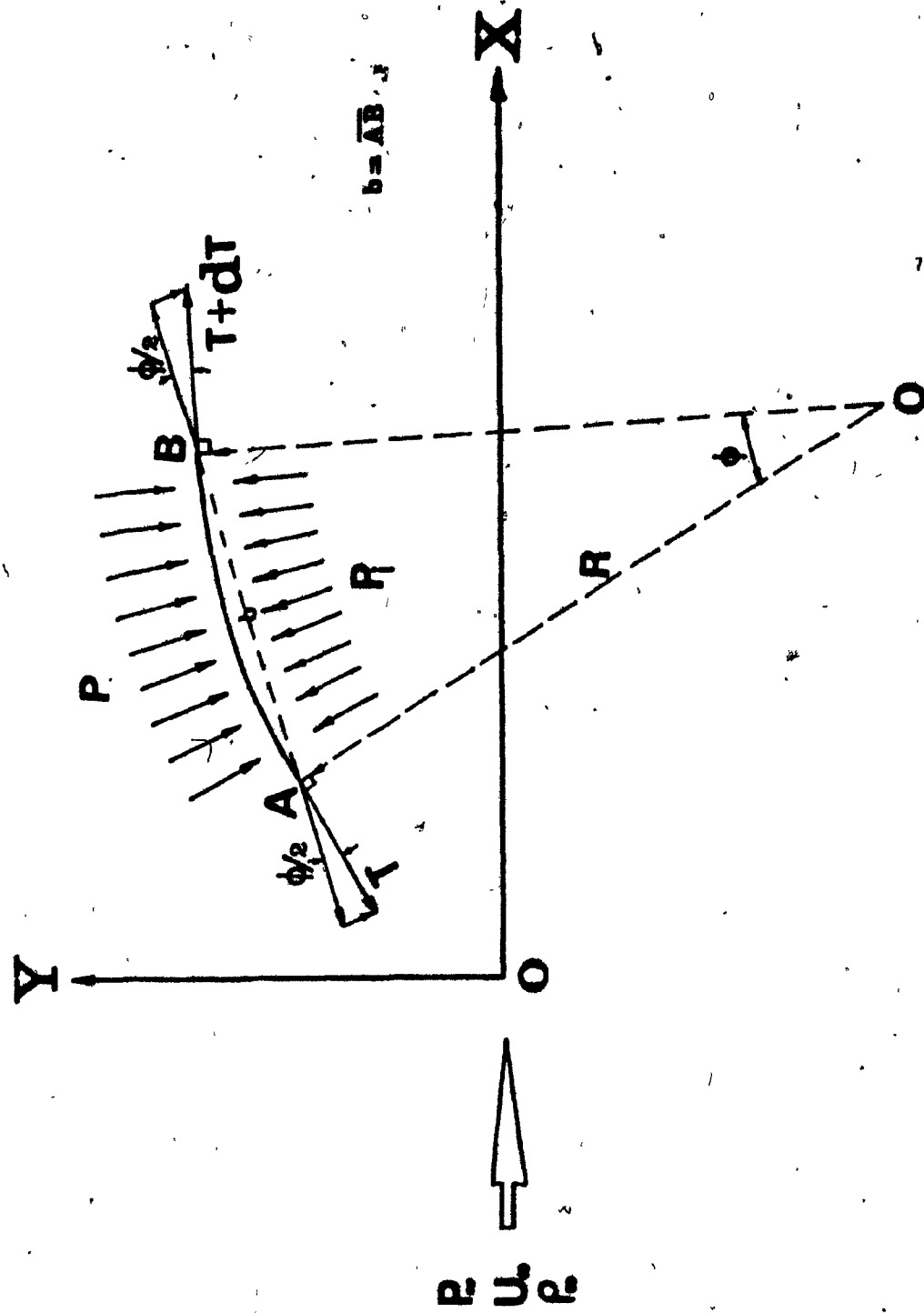


Figure 13 Force balance of the membrane element.

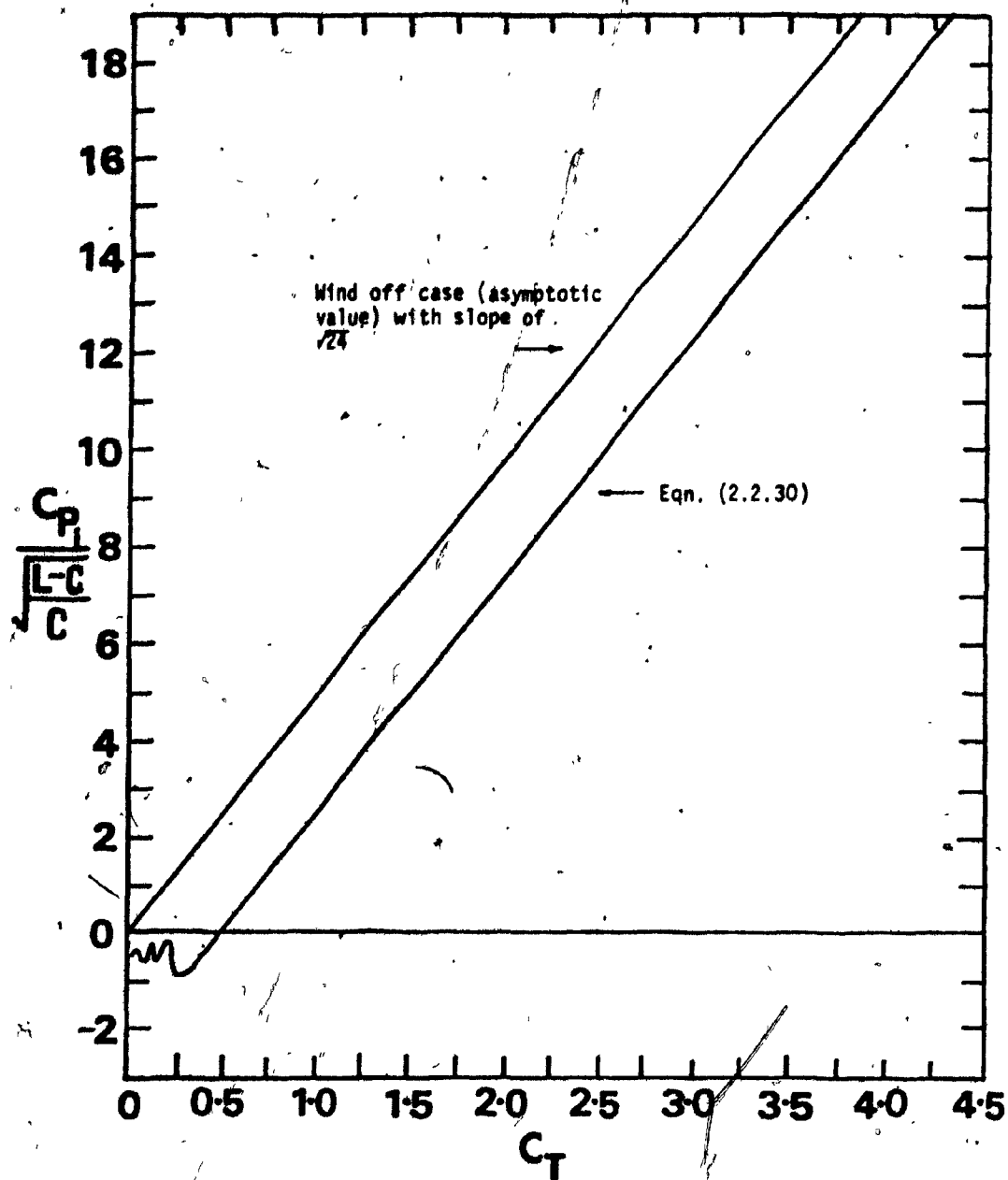


Figure 14 Normalized inflation pressure coefficient plotted against tension coefficient (Theory).

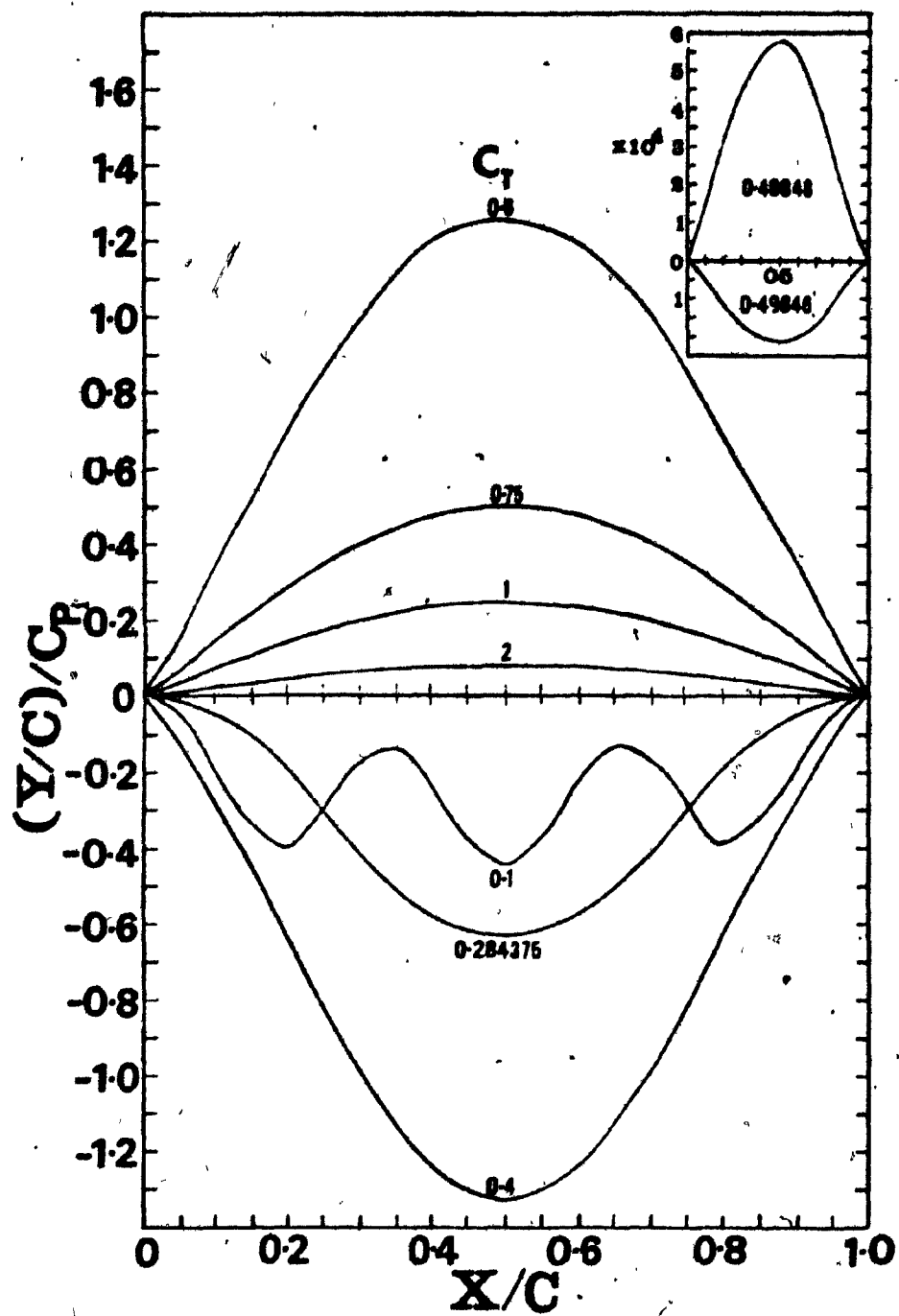


Figure 15 Theoretical membrane shapes for various  $C_T$ .

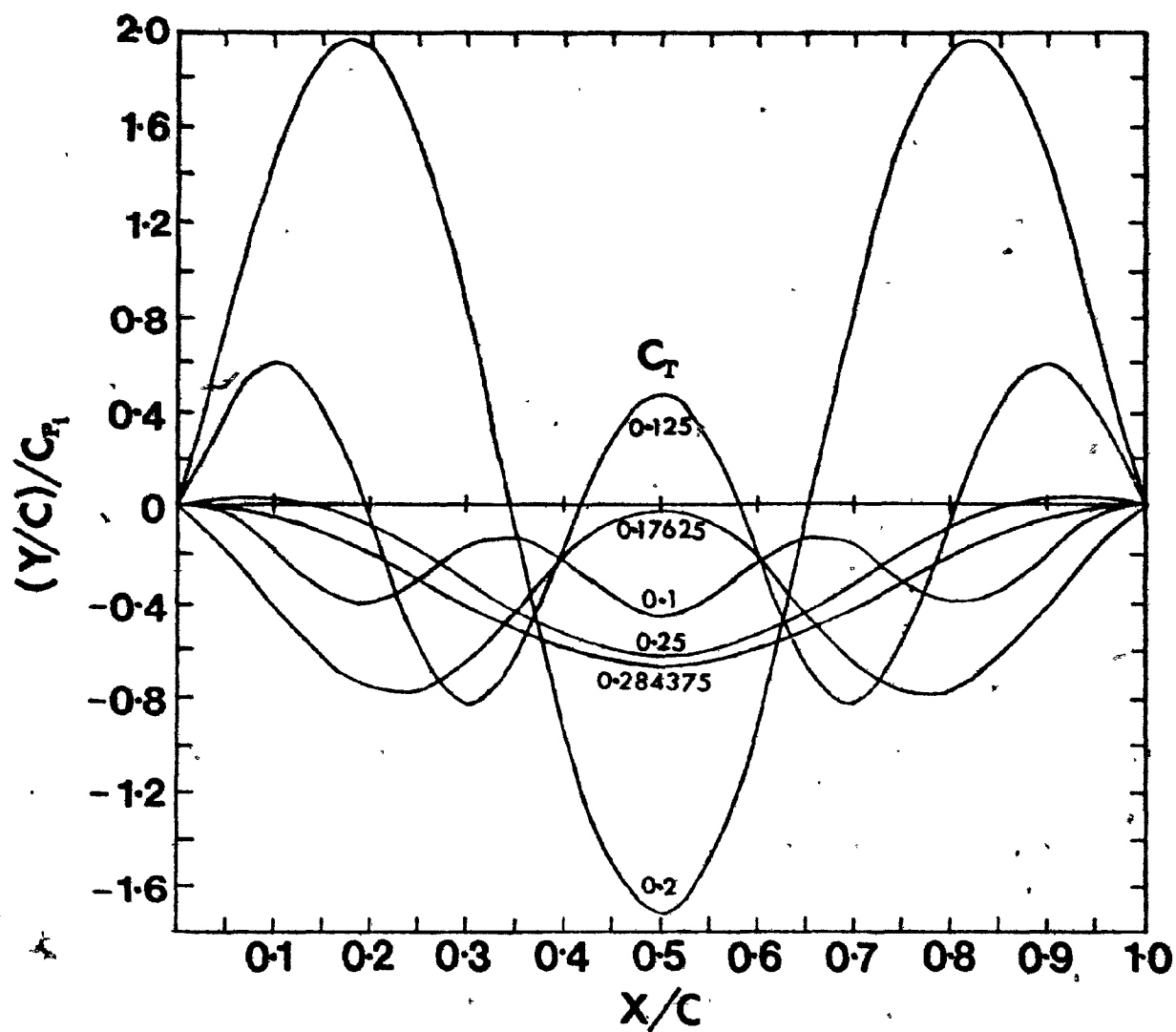


Figure 16 Theoretical membrane shapes for various  $C_T$  and negative  $C_{Pi}$ .

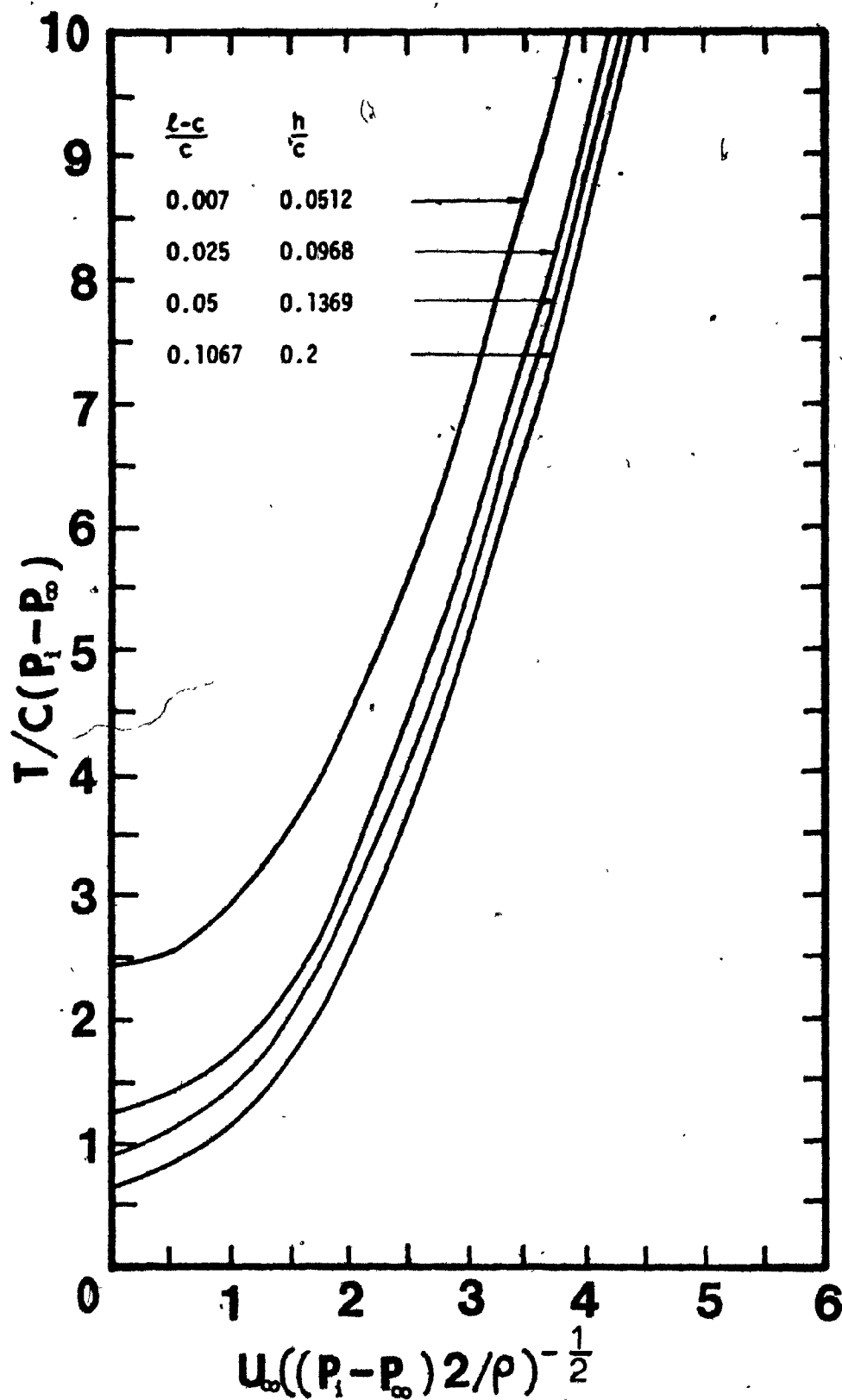


Figure 17 Induced tension as a function of flow velocity for different heights.



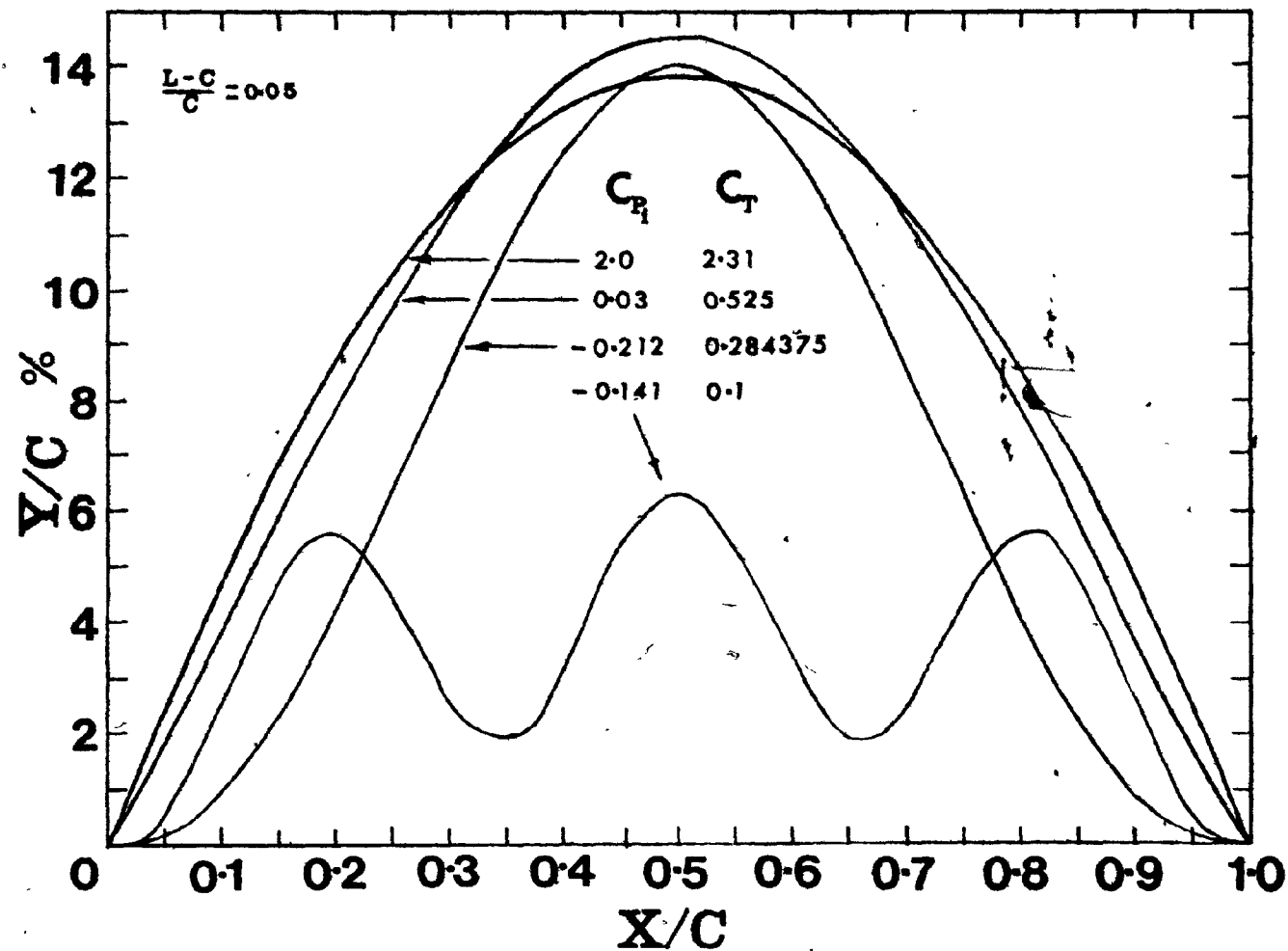


Figure 18 Theoretical membrane coordinates for a fixed membrane length.

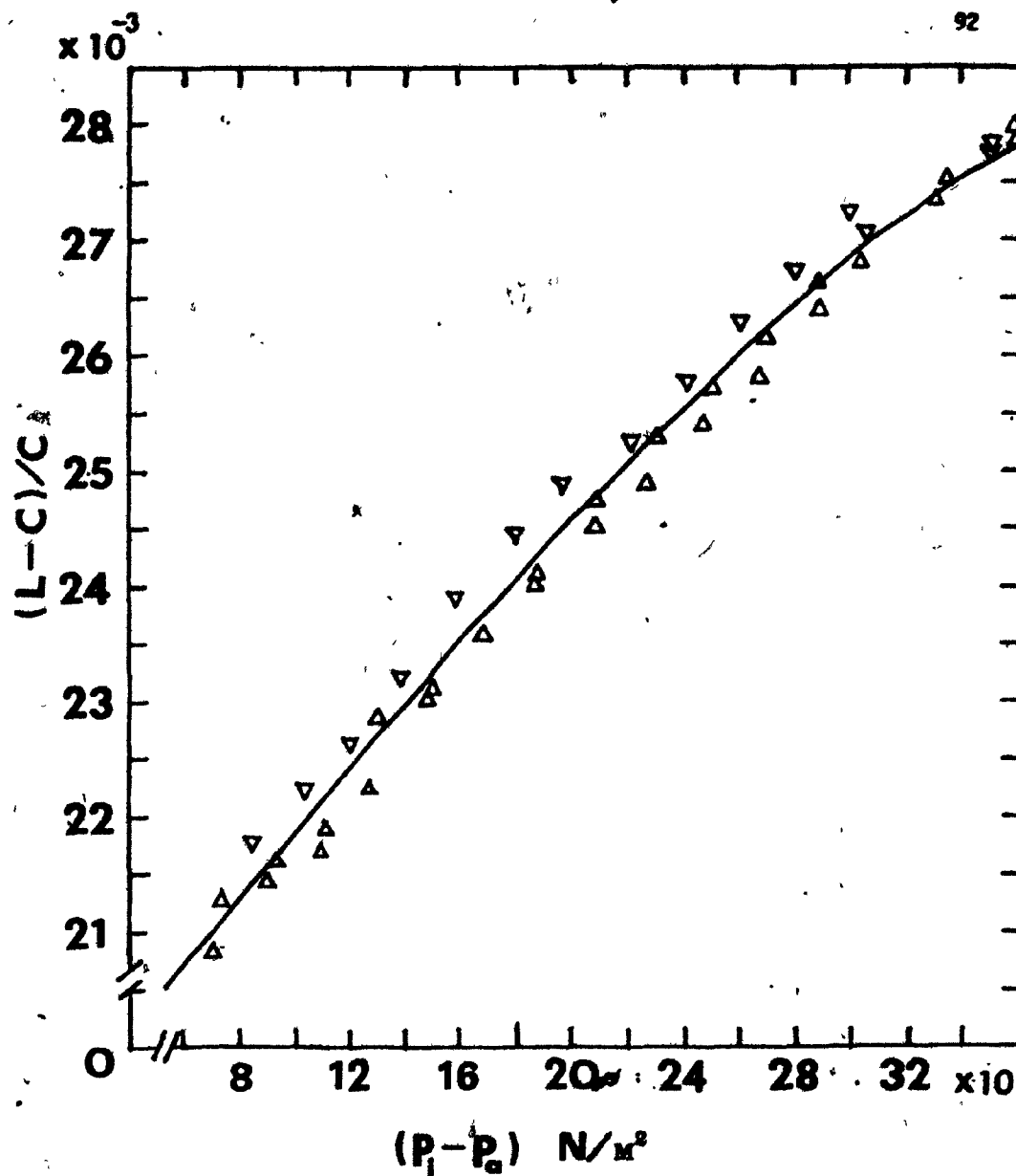


Figure 19A Calibration of excess length to chord ratio plotted against the inflation pressure at wind off.

$$\frac{L-C}{C} = (18.744 + 0.02932.07 \Delta P - 0.3393227 \times 10^{-7} \Delta P^3) \times 10^{-3}$$

$71 < \Delta P < 360$

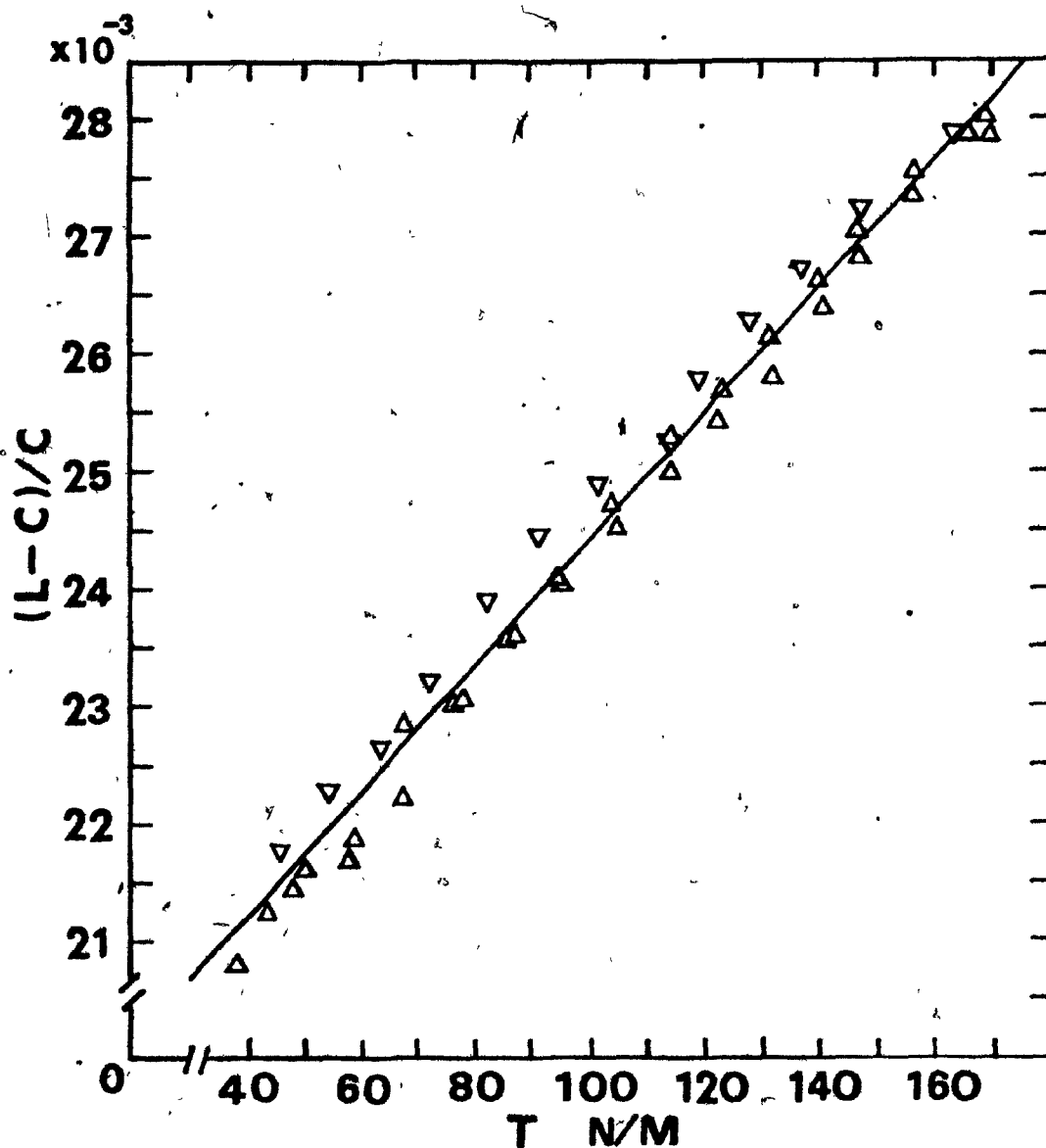


Figure 19B Calibration of excess length to chord ratio plotted against the induced tension at wind off.

$$\frac{L-C}{C} = (19.09275 + 0.0532003 T) \times 10^{-3}$$

$$38.5 < T < 169.555$$

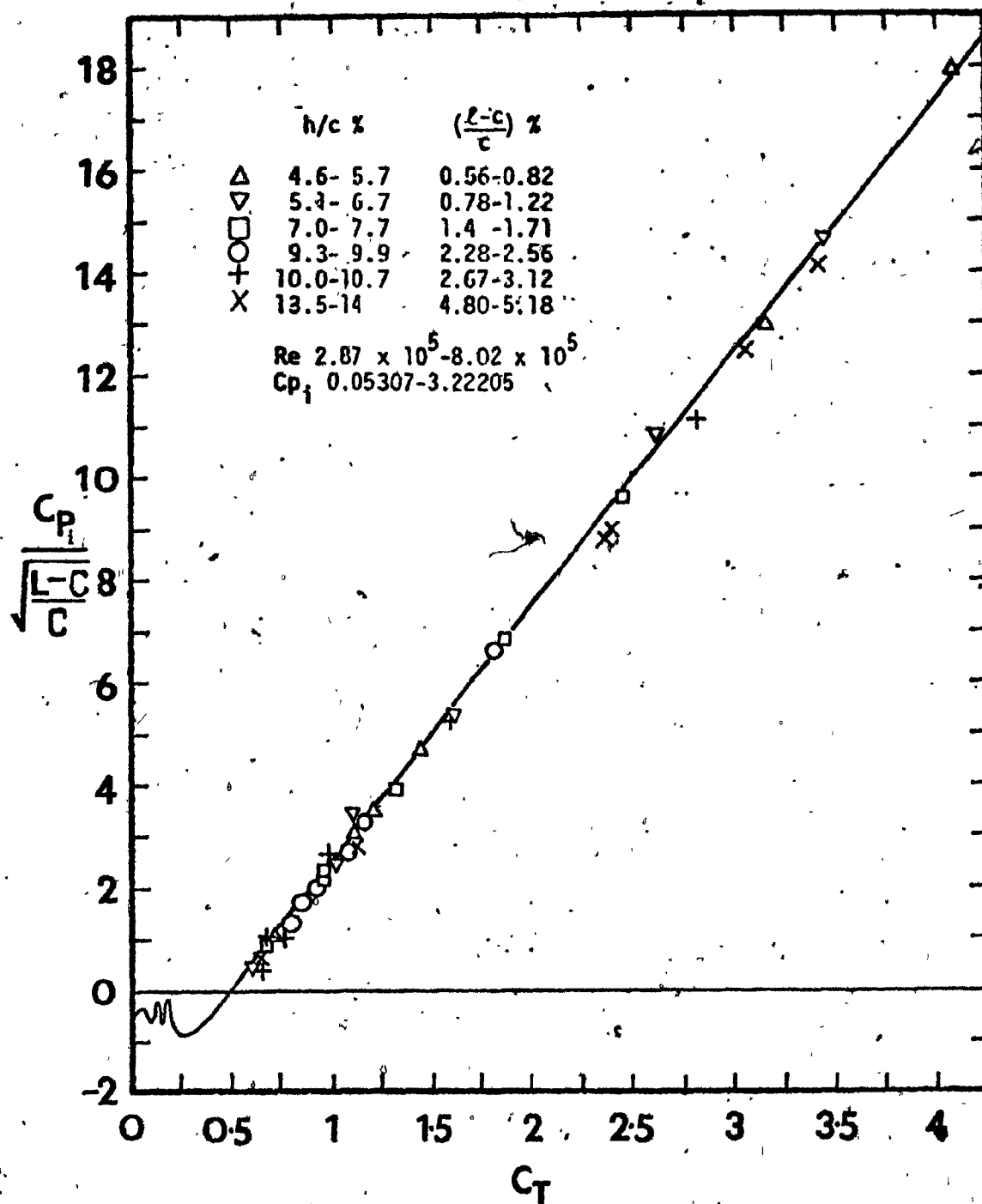


Figure 20 Comparison between theory and experiment for  $C_T$  when  $C_{p_i} > 0$ .

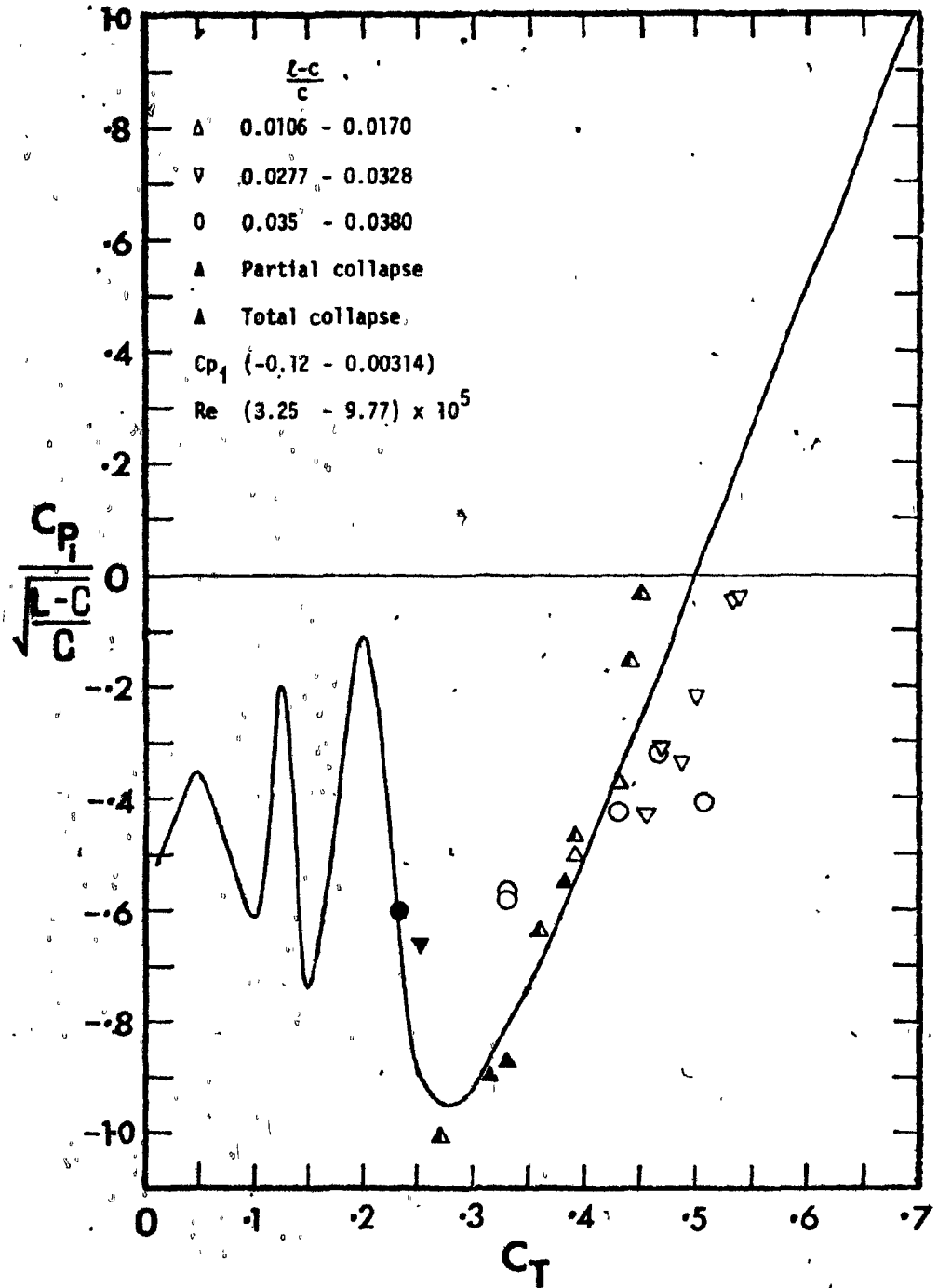


Figure 21 Comparison between theory and experiment for  $C_T$  when  $C_{p1} < 0$ .

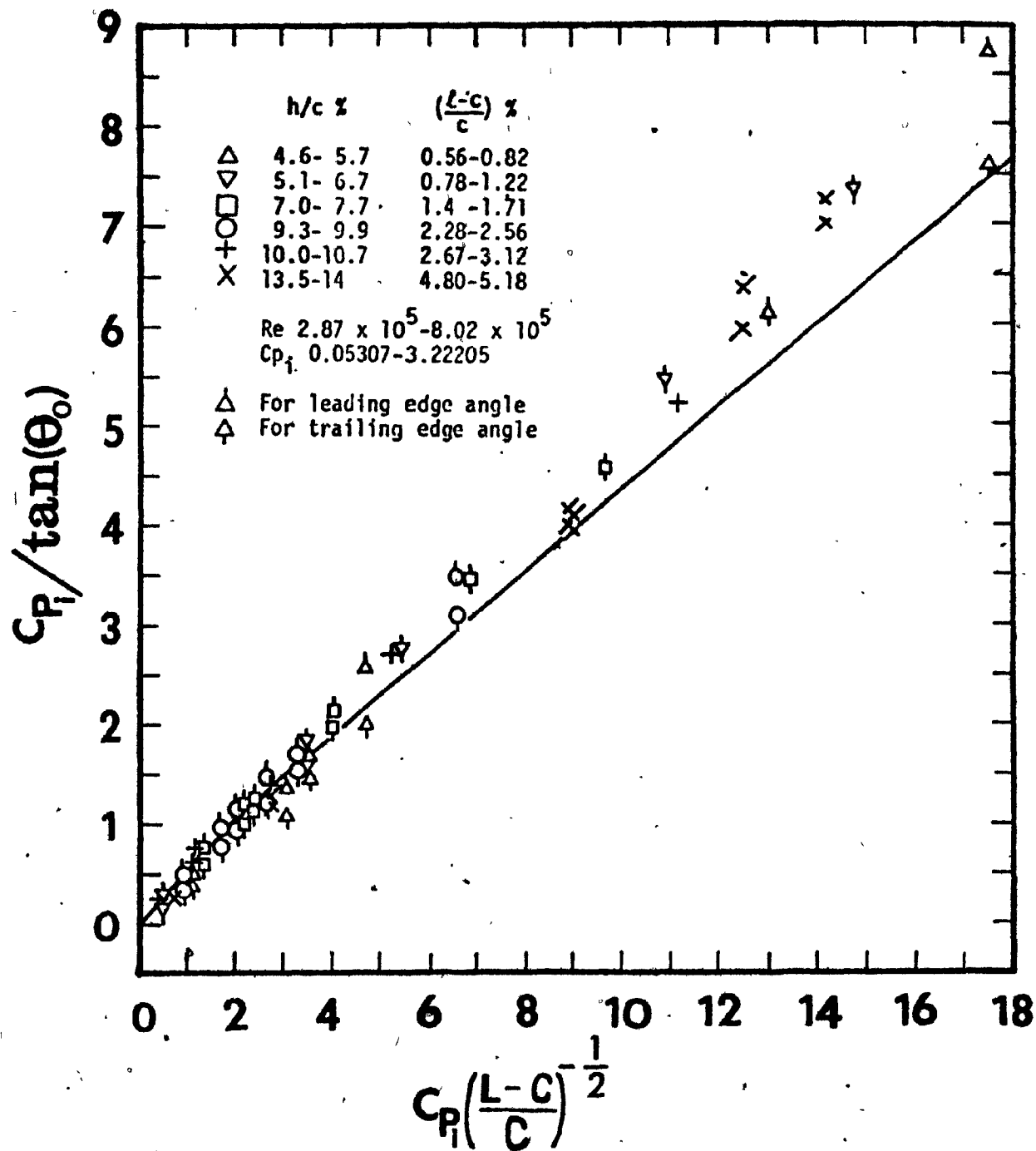


Figure 22 Comparison between theory and experiment for edge slopes at various normalized inflation pressure coefficient.

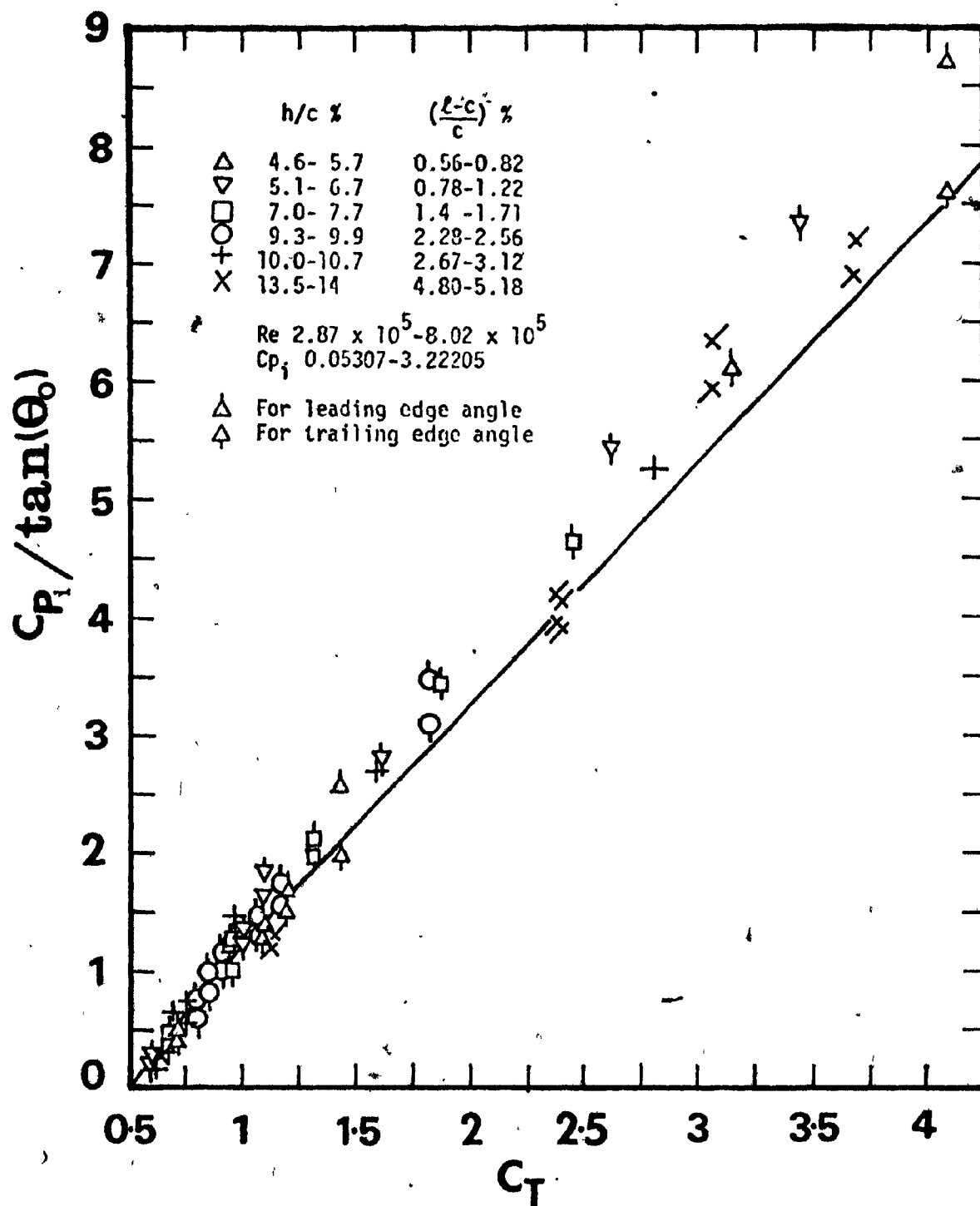


Figure 23 Comparison between theory and experiment for edge slopes at various tension coefficient.

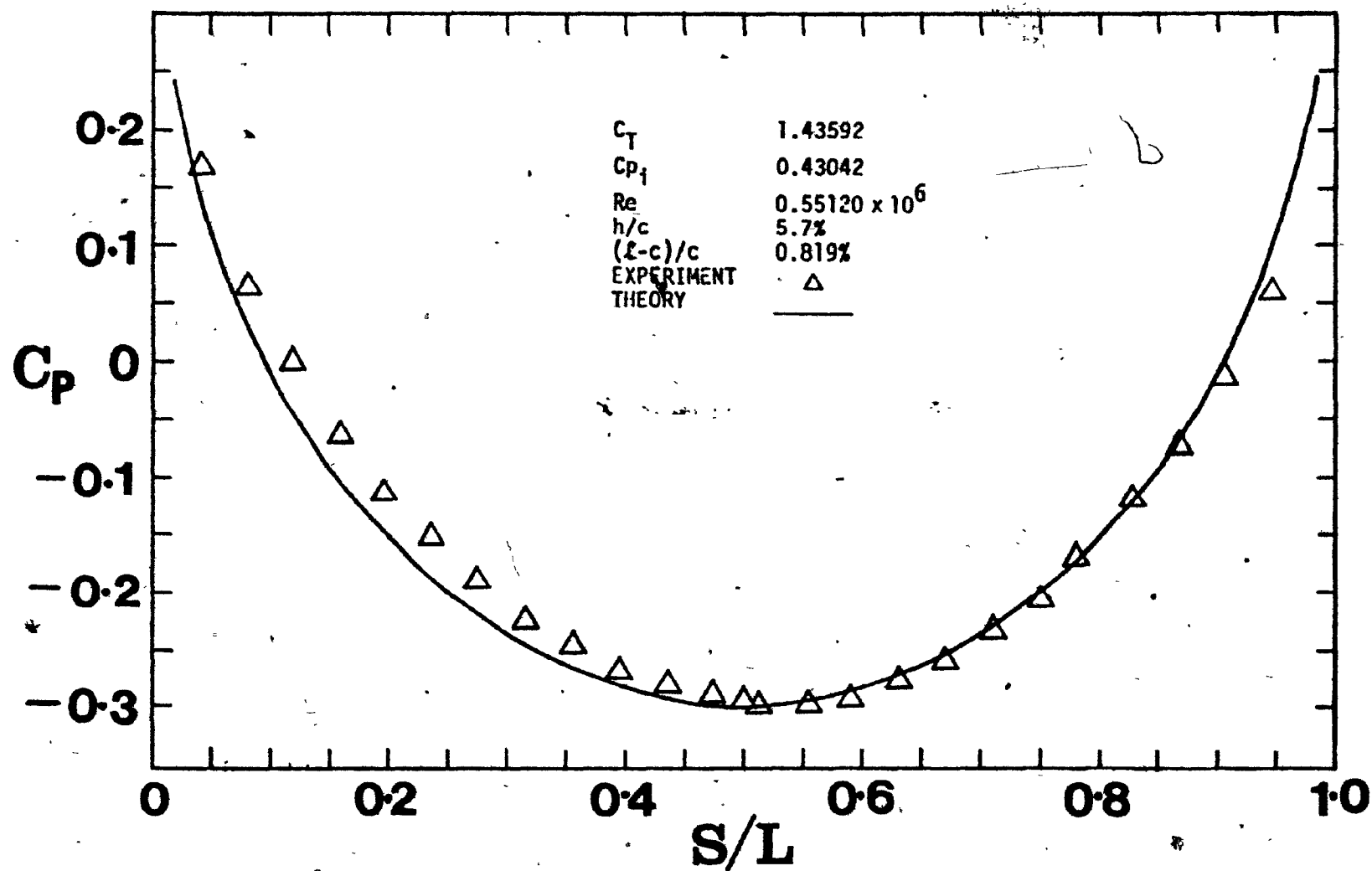


Figure 24 Comparison between theory and experiment for pressure distribution.



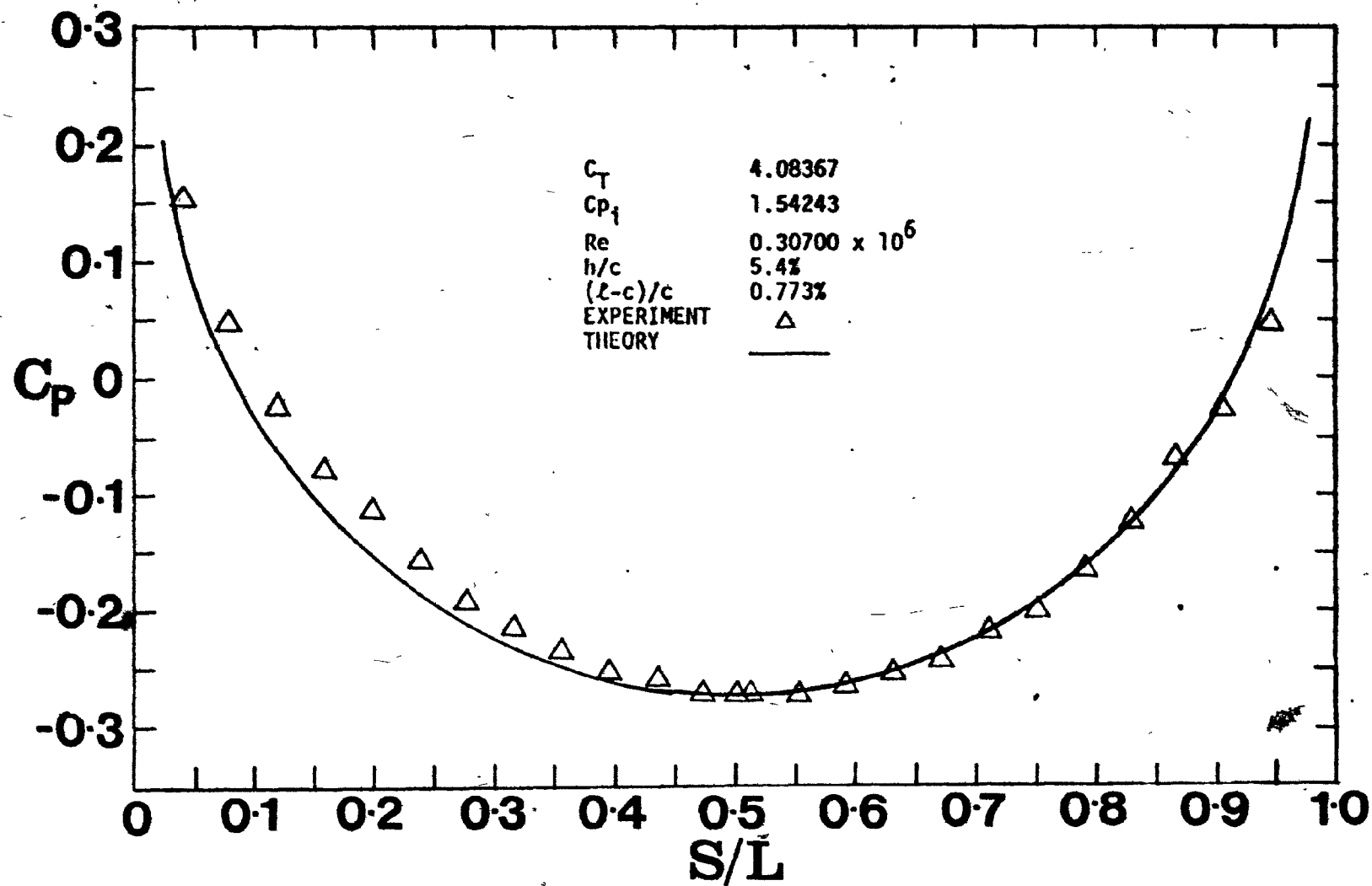


Figure 25 Comparison between theory and experiment for pressure distribution.

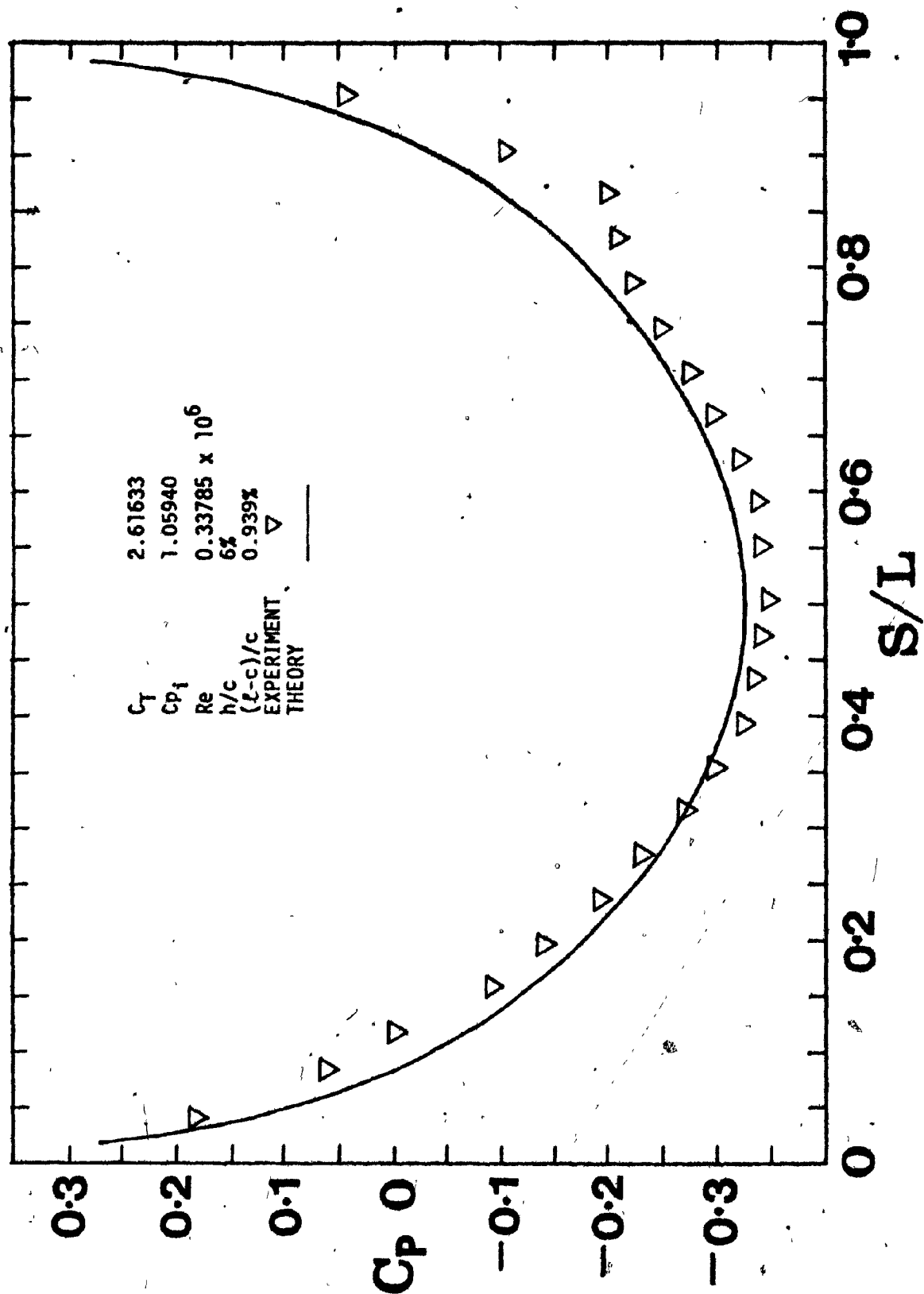


Figure 26 Comparison between theory and experiment for pressure distribution.

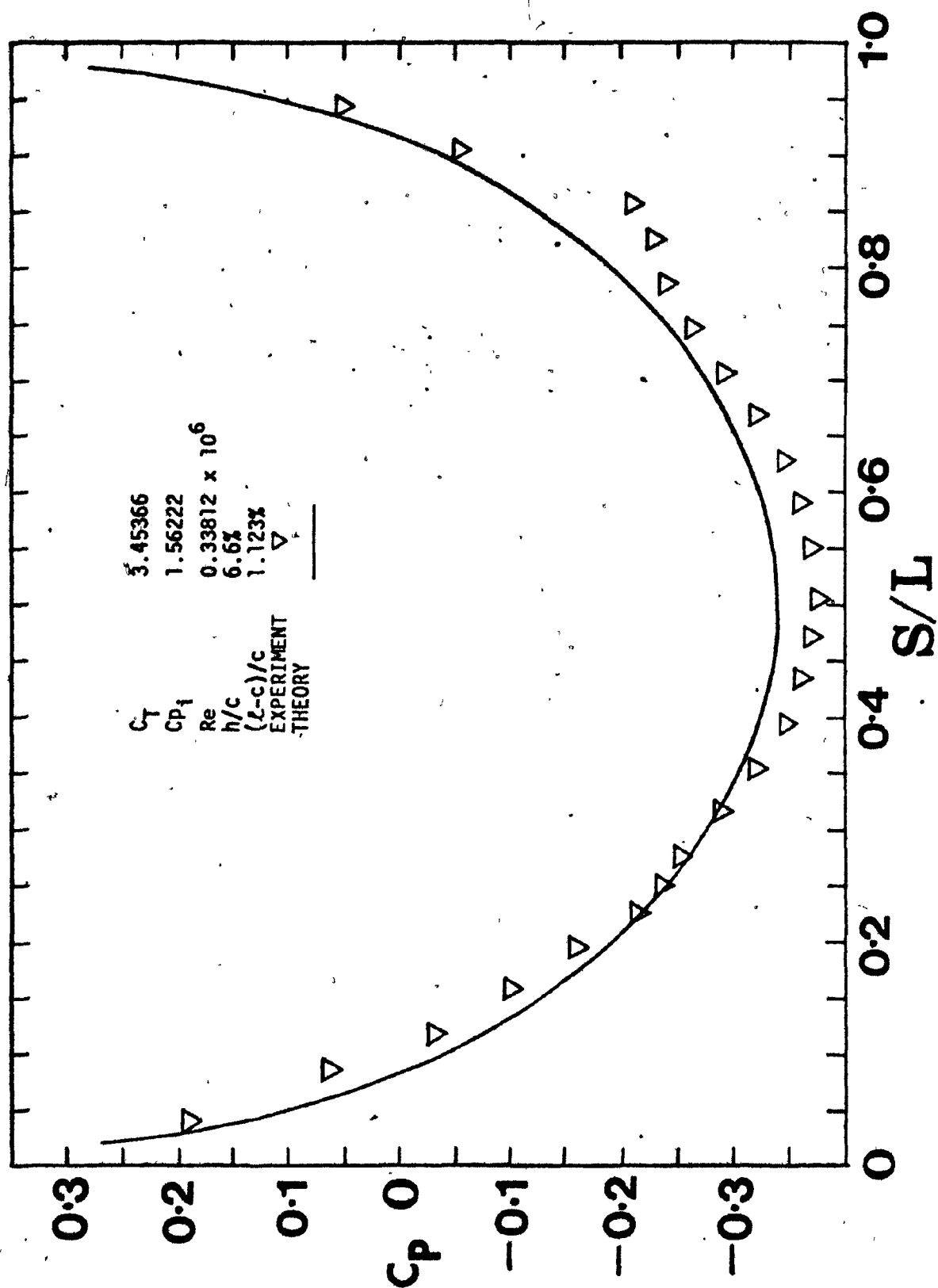


Figure 27 Comparison between theory and experiment for pressure distribution.

Figure 28 Comparison between theory and experiment for pressure distribution.

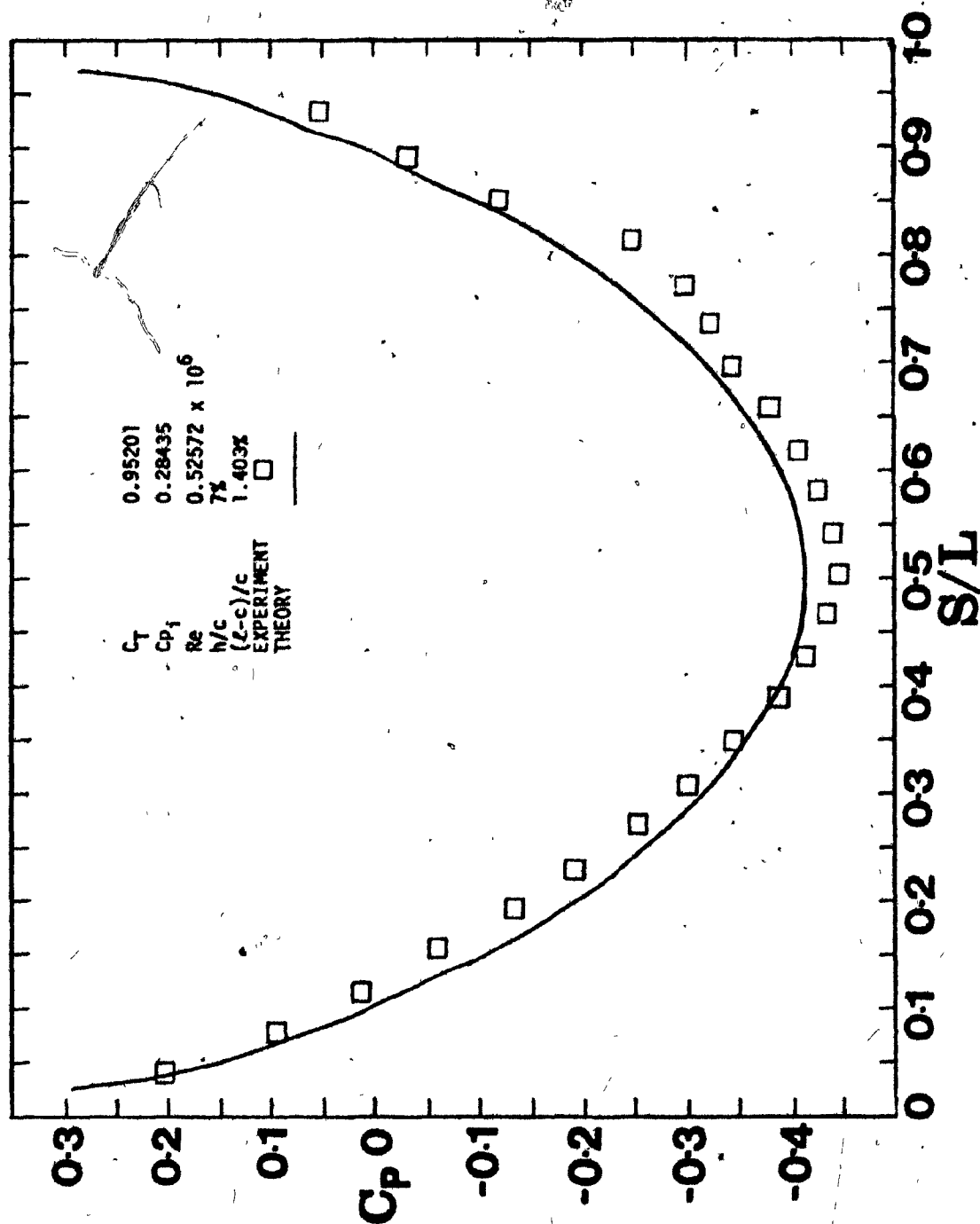
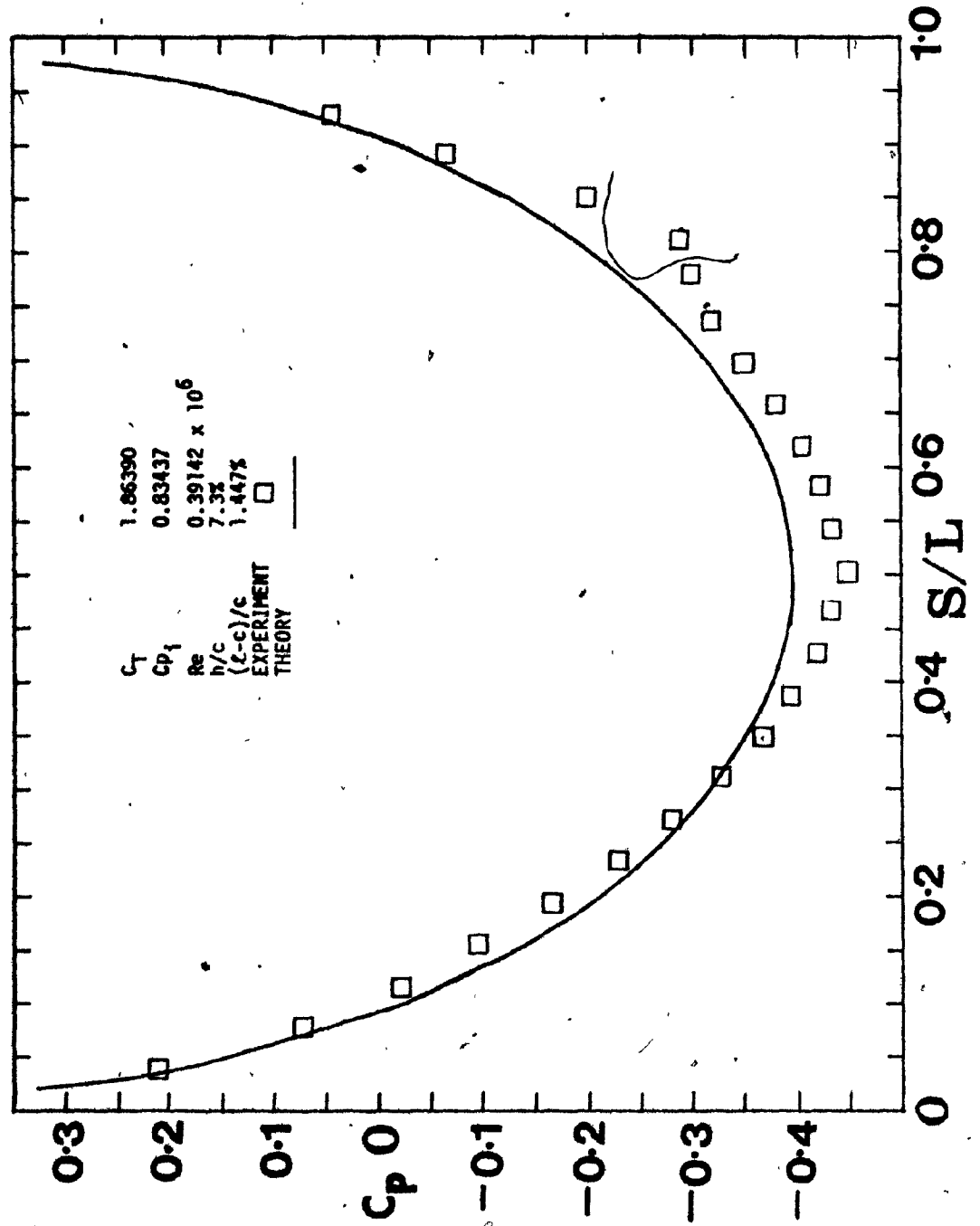


Figure 29 Comparison between theory and experiment for pressure distribution.



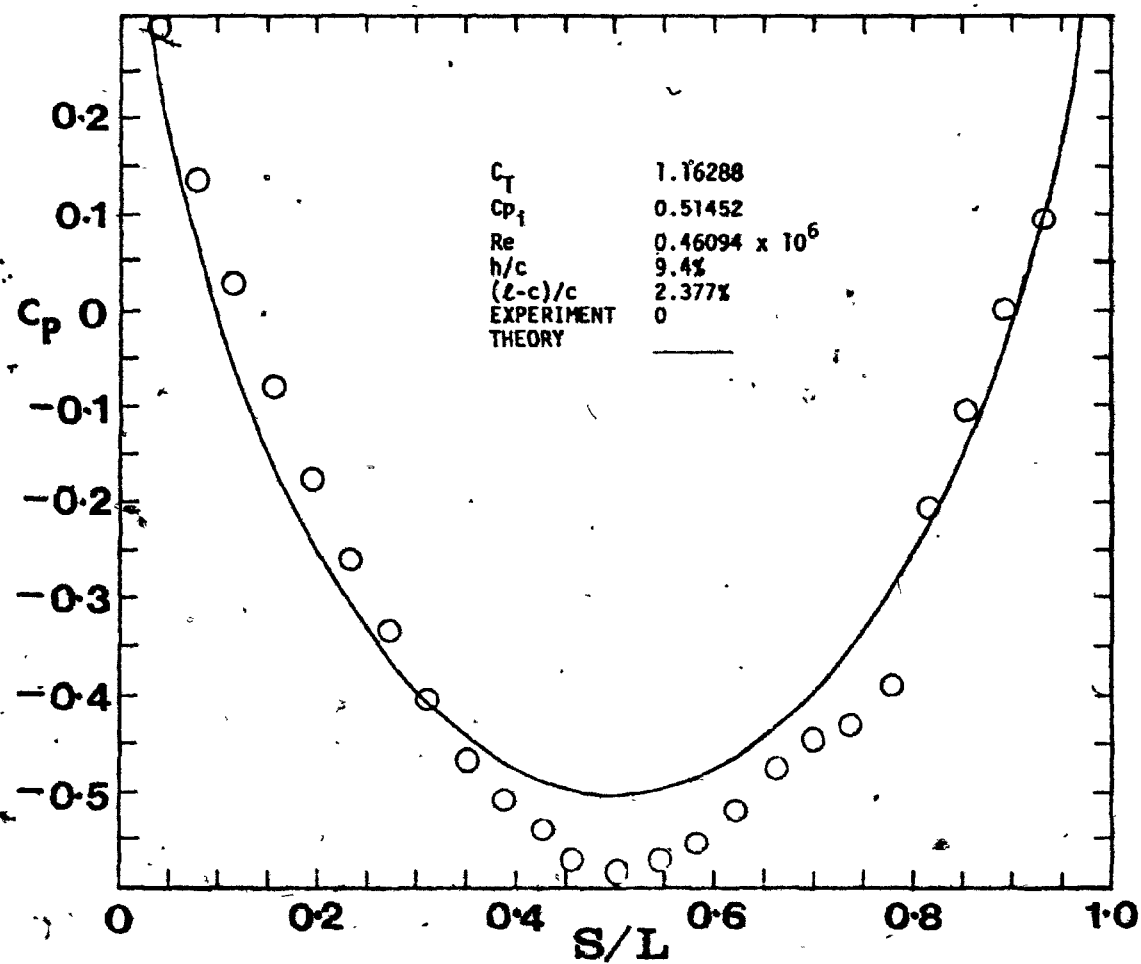


Figure 30 Comparison between theory and experiment for pressure distribution.

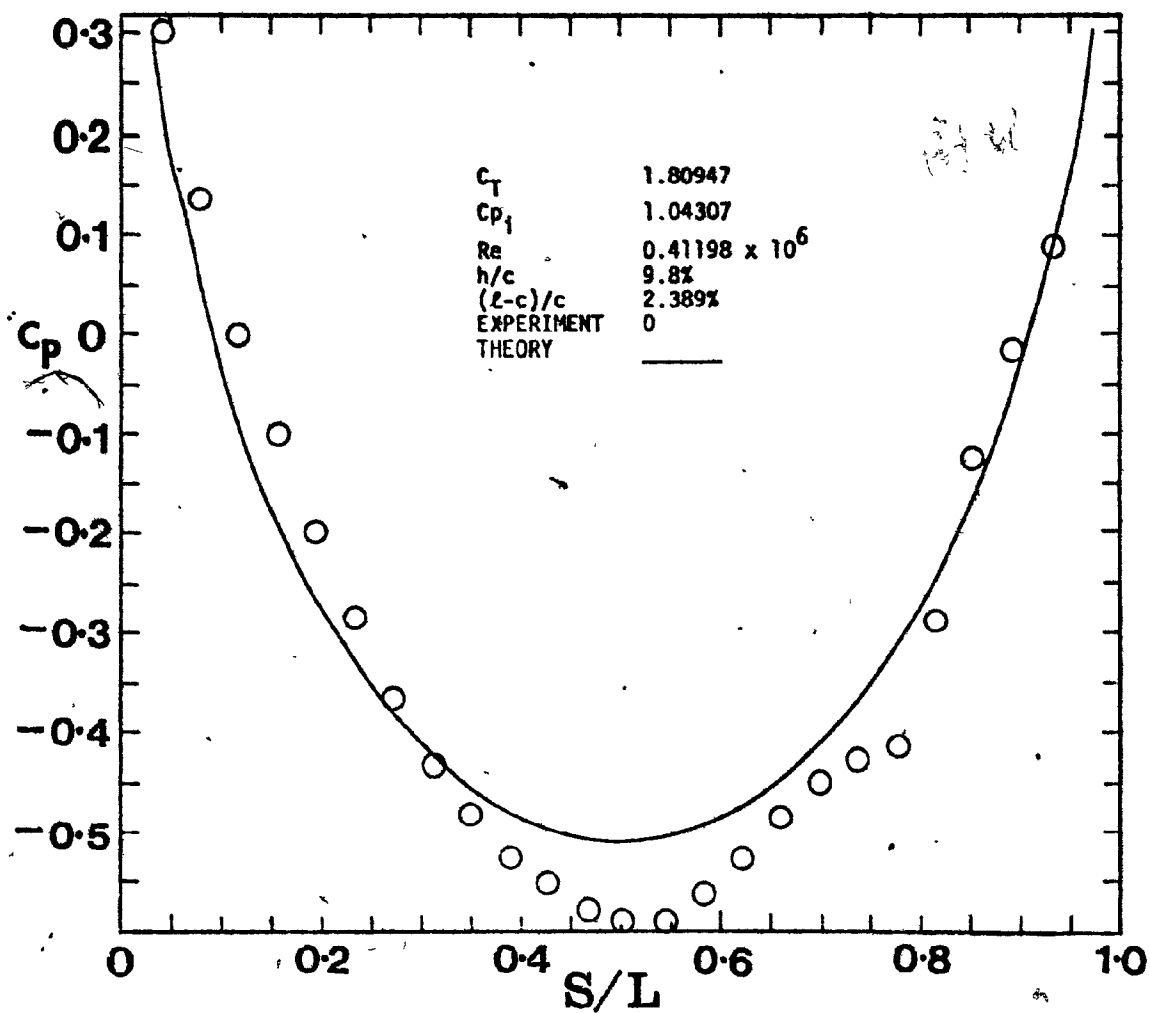


Figure 31 Comparison between theory and experiment for pressure distribution.

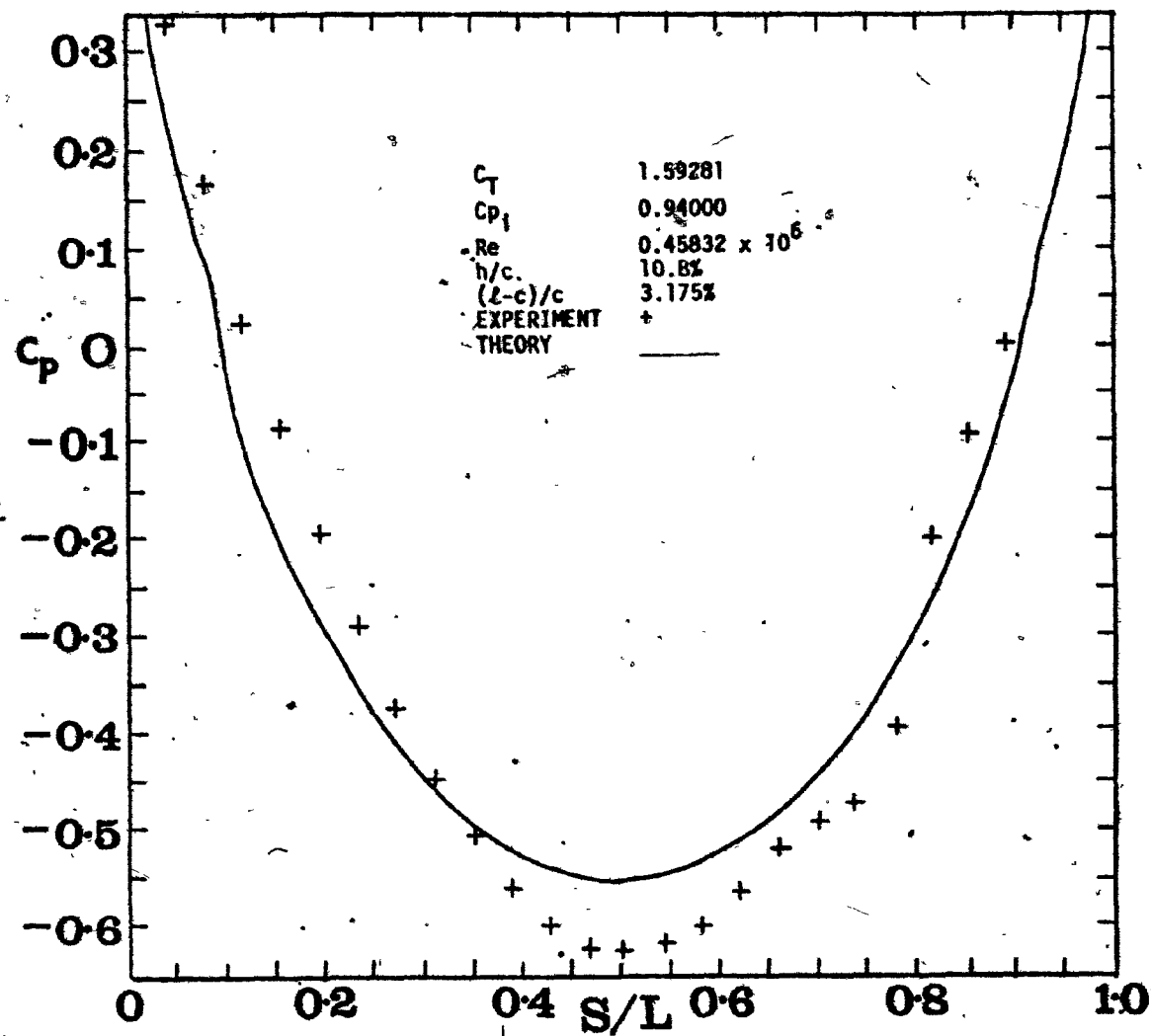


Figure 32 Comparison between theory and experiment for pressure distribution.



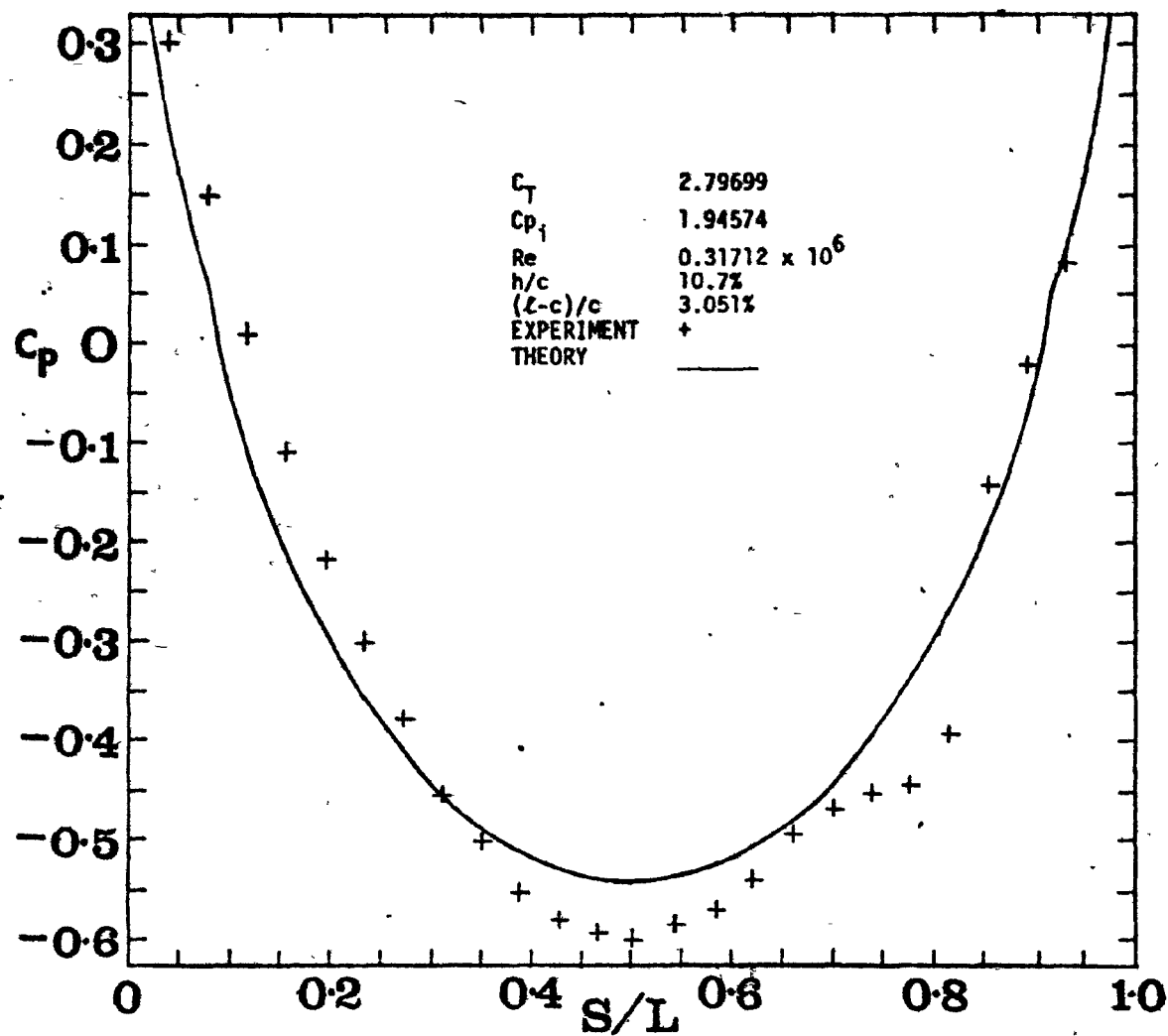


Figure 33 Comparison between theory and experiment for pressure distribution.

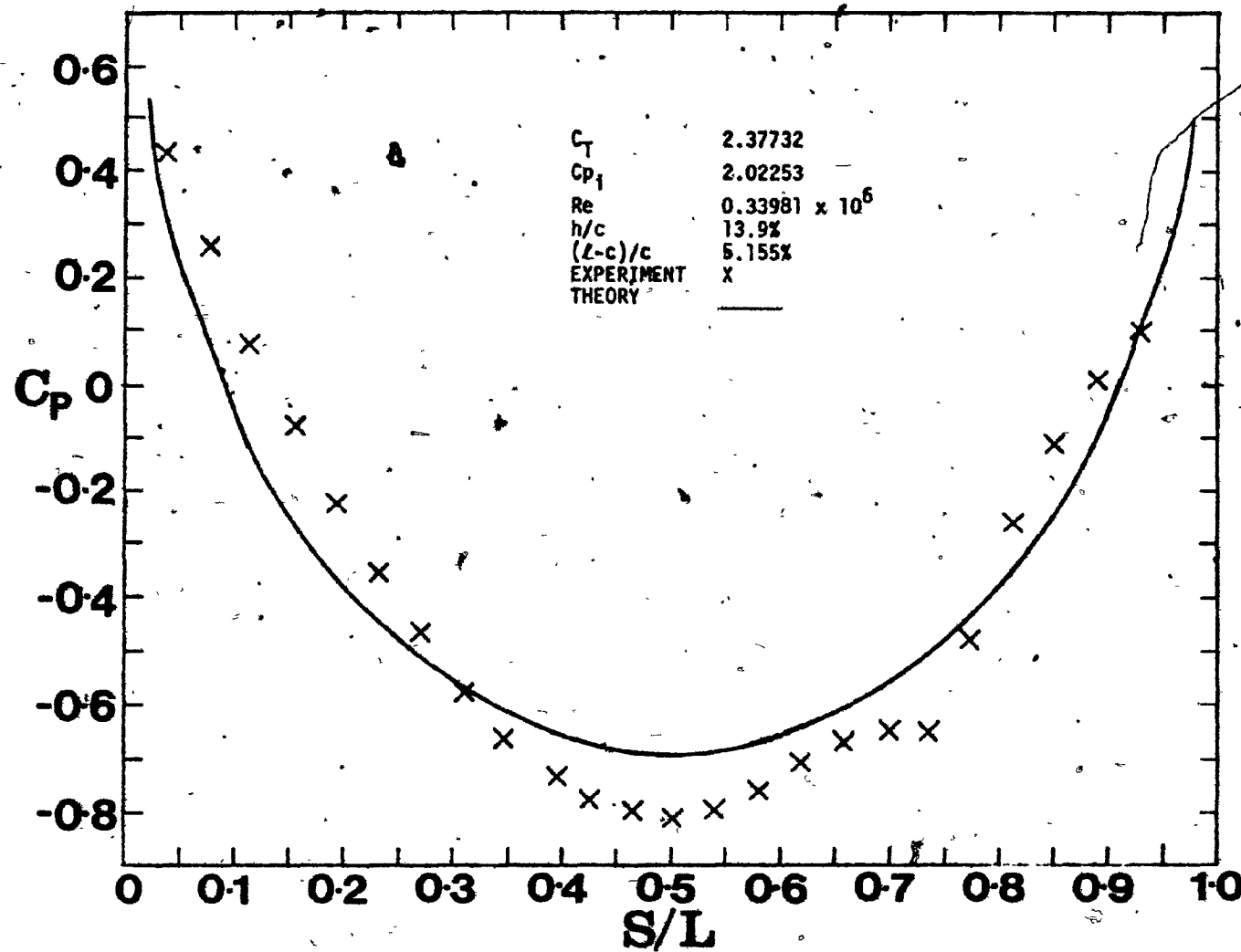


Figure 34 Comparison between theory and experiment for pressure distribution.

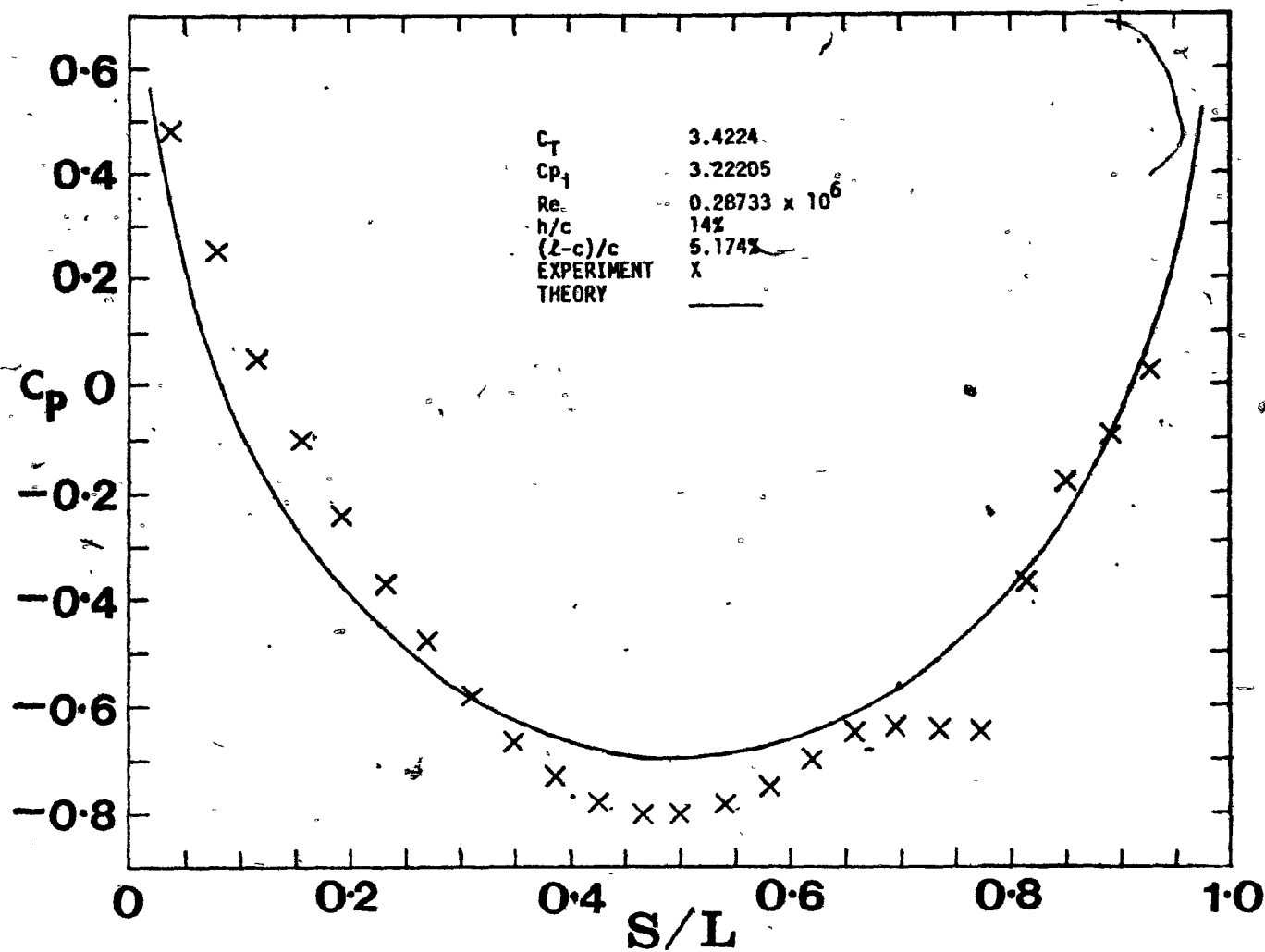


Figure 35 Comparison between theory and experiment for pressure distribution.





Figure 37a Overall wind tunnel.

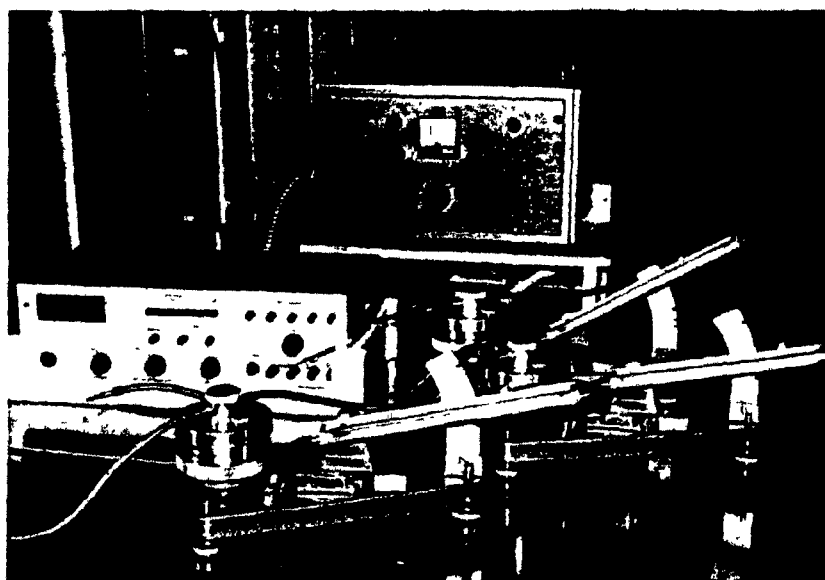


Figure 37b Measuring instruments (inclined manometers and strain indicator 1526).

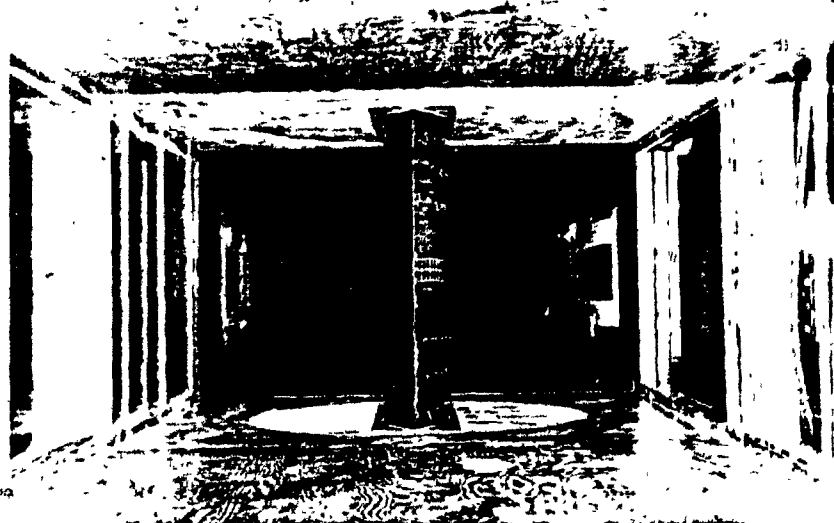


Figure 38a Inflated envelope at wind off (front view).



Figure 38b Inflated envelope at wind off (side view).



Figure 39 Inflated pressure supply pump.

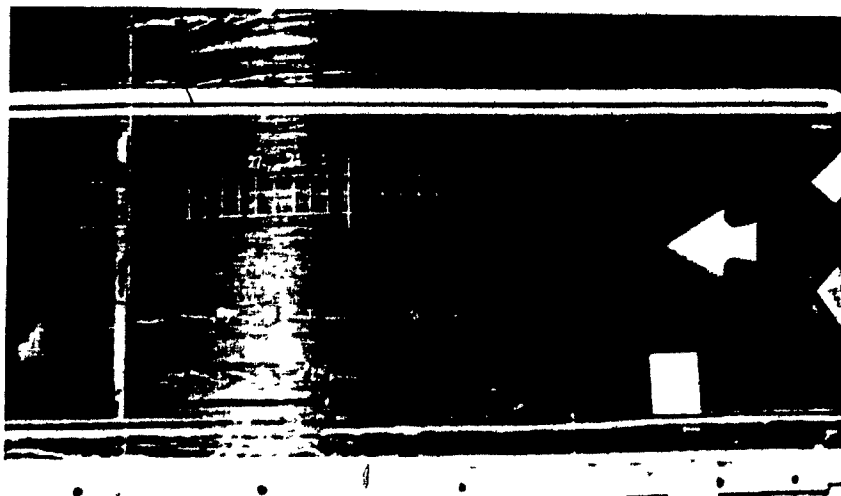


Figure 40a

$$Re = 7.34 \times 10^5$$

$$C_{p_i} = 0.015$$

$$C_T = 0.5698$$

$$t/c = 0.2 \text{ (wind-off)}$$

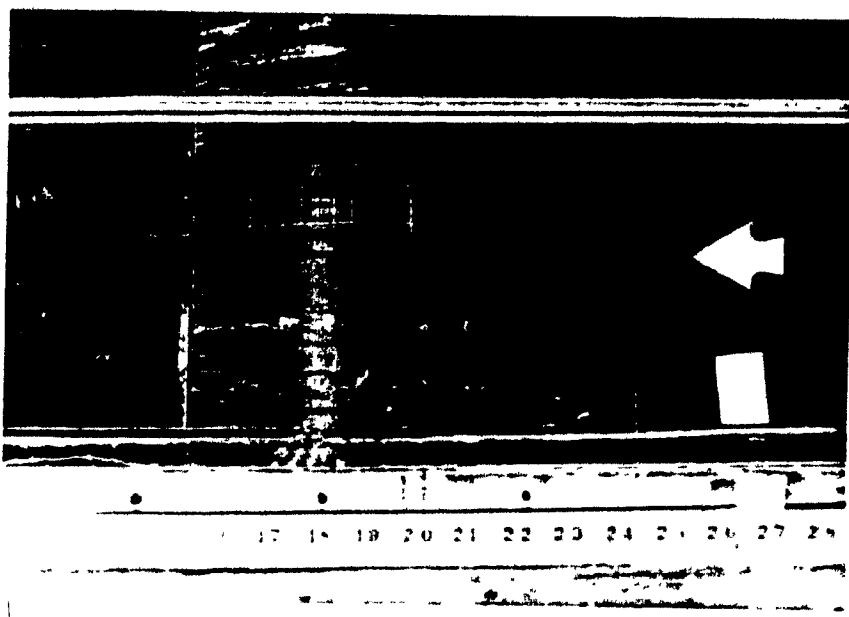


Figure 40b

$$Re = 3.77 \times 10^5$$

$$C_{p_i} = -0.09817$$

$$C_T = 0.41375$$

$$t/c = 0.2 \text{ (wind-off)}$$

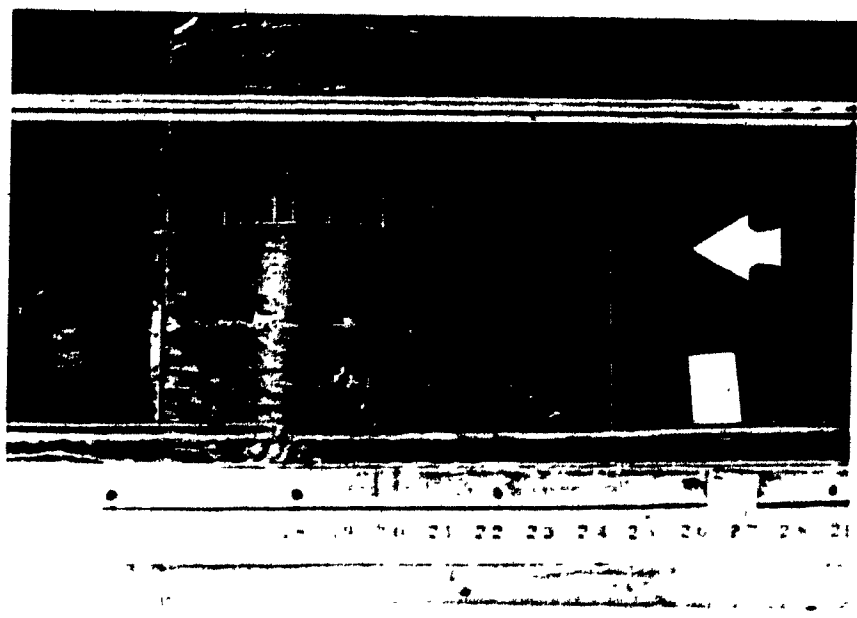


Figure 40c

$$Re = 3.28 \times 10^5$$

$$C_{p_i} = -0.10865$$

$$C_T = 0.25086$$

$$t/c = 0.2 \text{ (wind-off)}$$



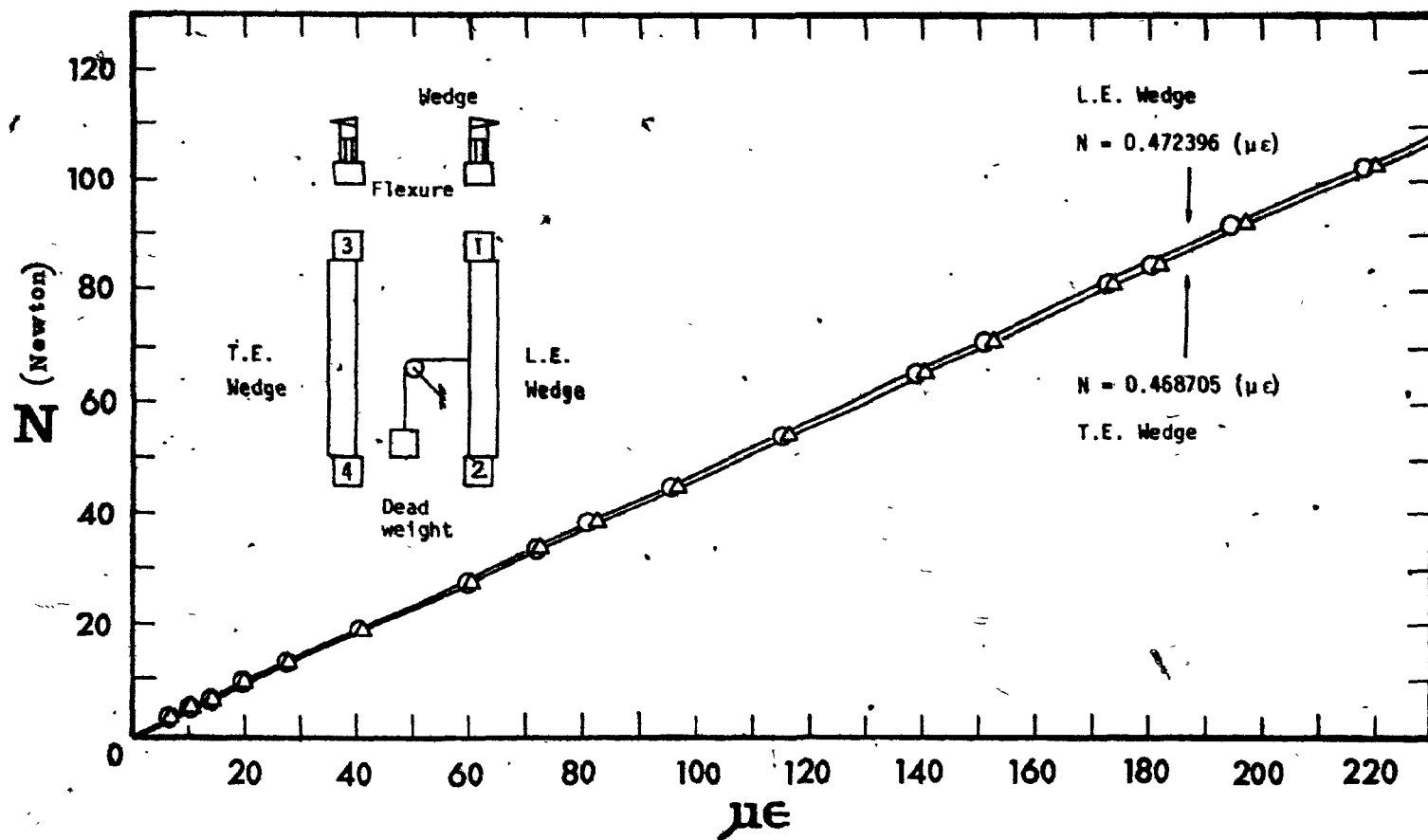


Figure 41 Wedge calibration.