AN EXPERIMENTAL STUDY OF THE THERMAL WAKE INTERFERENCE BETWEEN CLOSELY SPACED WIRES OF A X-TYPE HOT-WIRE PROBE

69-7

by

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Summary

In constructing an X-type hot-wire probe it has been the policy of a number of experimenters and manufacturers to place the two wires forming the X close to each other to assure that they are both measuring in effectively the same plane. A number of important experiments have been made using such a probe design.

Recent experiments at the University of British Columbia have shown that conventional X-wire probes which have two wires almost in the same plane, are quite sensitive to movements of the velocity vector out of that plane (defined as pitching motion). This has been attributed to the influence on one wire of the hot wake produced by the other. In this note a Disa X-type probe is tested and found to have a sensitivity to small angles of pitch, which is very significant for low wire Reynolds numbers (< 5) but becomes small for Reynolds number greater than 10. A modified probe, having the wires one wire length apart, is suggested and when tested found to have no pitch sensitivity.

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Contents

Notations

A	constant (in Equation (15))
A_1	constant (in Equation (A2))
B	constant (in Equation (15))
B_1	constant (in Equation (A2))
a, b, c	sensitivity coefficients defined by Equations (4) and (5)
a_1, b_1	sensitivity coefficients defined by Equation (A6)
a_2, b_2	sensitivity coefficients defined by Equation (A8)
d	diameter of wire (= 5 micron)
D	defined in Figure 1a
E	bridge D.C. voltage
e	instantaneous bridge voltage fluctuations
k	longitudinal cooling correction factor (in Equation (A2))
l	length of a hot-wire
l_o	characteristic width of the wake
Re	Reynolds number $(= \frac{Ud}{\nu})$
s	spacing between hot-wires (see Figure 1a)
T	temperature in wake of a hot-wire
T_o	scaling temperature in wake temperature profile
U	mean flow velocity
u, v, w	instantaneous fluctuating velocity components in x, y and z directions respectively
x, y, z	orthogonal co-ordinates (x in direction of mean flow)
θ	angle of pitch
ψ	angle of yaw
ν	kinematic viscosity

1. Introduction

In constructing an X-type hot-wire probe it has been the policy of a number of experimenters and manufacturers to place the two wires forming the X close to each other to assure that they are both measuring in effectively the same plane. A number of important experiments have been made using such a probe design.

Recent measurements of Reynolds stresses made by Jerome (Burling 1969) in the natural boundary layer formed by wind flow over the earth's surface, showed significant disagreement between results given by a Disa constant temperature X-type hot wire probe having its wires almost in the same plane (.006" apart; see fig. 2a), and those taken with a sonic anemometer. Subsequent wind tunnel tests demonstrated that when a X-wire was pitched - i.e. the wind vector was moved out of the plane parallel to and midway between the two wires (as compared to yaw which is movement of the vector in that plane) - the downstream wire required <u>less</u> power than it did at zero pitch angle to maintain itself at constant temperature. The upstream wire, behaved as expected and required <u>more</u> power due to the additional cooling caused by a larger normal component of velocity. Jerome's experiments intimated that the sensitivity to pitch is caused by the hot wake, created by the upstream end of one wire which impinges on the downstream end of the other and reduces its heat transfer to the air flow. The influence of the hot wake will obviously depend on the distance between the wires (s in fig. 1a) and must be determined independently for each probe to be used.

Burling has noted that pitching or yawing a X-wire will cause one of the wires to sense a decreasing cooling rate relative to the other and that this similar behaviour leads to errors in the interpretation of turbulence data. Hitherto all analyses of X-wire measurements have been based on the assumption that the two wires behave as independent sensors which, in pitch, should experience identical heat loss. If this notion is used to analyse turbulence data the contribution of w, to the signal of a X-wire in the u-v plane, will be attributed to v. The magnitude of the resulting error is examined in the next section.

The experimental portion of this note is aimed at determining the static sensitivity to pitch of a Disa X-wire probe having its wires 0.006" apart and comparing it to that of a probe modified by moving the wires further apart (s increased).

2. Calculation of Reynolds stresses if pitch sensitivity included:-

This section is included to provide an indication of the relative importance of pitch sensitivity in the analysis of turbulence data.

If pitch sensitivity is included, the response of one of the wires in a X-wire array is written generally as

$$E = E\ (U,\ \theta,\ \psi) \tag{1}$$

where θ and ψ are the pitch and yaw angles respectively. For small changes

$$dE = \frac{\partial E}{\partial U}\, dU + \frac{\partial E}{\partial \psi}\, d\psi + \frac{\partial E}{\partial \theta}\, d\theta \tag{2}$$

and by definition

$$\begin{aligned} dU &= u \\ d\psi &= \frac{v}{U} \\ d\theta &= \frac{w}{U} \end{aligned} \tag{3}$$

and

$$e = \left(\frac{\partial E}{\partial U}\right) u + \left(\frac{1}{U}\,\frac{\partial E}{\partial \psi}\right) v + \left(\frac{1}{U}\,\frac{\partial E}{\partial \theta}\right) w \tag{4}$$

The bracketed terms are the sensitivity coefficients and for convenience the above relation is written as

$$e = au + bv + cw \tag{5}$$

In figure 1a, for positive w only wire no. 2 will be influenced by the hot wake and equation (5) is written respectively for each wire as

$$\underline{+\text{ ve } w} \qquad e_1 = au + bv$$
$$e_2 = au - bv - c|w| \qquad (6)$$

Similarly when w is negative only wire no. 1 will sense the wake:

$$\underline{-\text{ ve } w} \qquad e_1 = au + bv - c|w|$$
$$e_2 = au - bv \qquad (7)$$

In the above equations e is a positive quantity. The difference between the instantaneous signals can be written

$$e_1 - e_2 = (e_1 - e_2)_{+ve\ w} + (e_1 - e_2)_{-ve\ w}$$
$$= (2bv + c\ |w|\ )_{+\ ve\ w} + (2bv - c\ |w|)_{-\ ve\ w}$$
$$= 2bv + cw \qquad (8)$$

and their sum as

$$e_1 + e_2 = 2\ au - c\ |w| \qquad (9)$$

To obtain $\overline{v^2}$ equation (8) is squared and time averaged

$$\overline{(e_1-e_2)^2} = 4b^2\ \overline{v^2} + 4bc\ \overline{vw} + c^2\ \overline{w^2} \qquad (10)$$

and in <u>a two-dimensional flow</u> where $\overline{vw} = 0$

$$\overline{(e_1 - e_2)^2} = 4b^2\ \overline{v^2} + c^2\ \overline{w^2} \qquad (11)$$

or

$$\overline{v^2} \text{ measured} = \overline{v^2} \text{ actual } \left(1 + \frac{c^2}{4b^2}\ \frac{\overline{w^2}}{\overline{v^2}}\right) \qquad (12)$$

One method of obtaining the shear stress $\overline{uv}$ is to take the time averaged product of the instantaneous signals

$$\overline{(e_1 + e_2)\ (e_1 - e_2)} = \overline{(2bv + cw)(2au - c\ |w|)} \qquad (13)$$

$$= 4ab\ \overline{uv} + 2ac\ \overline{uw} - 2bc\ \overline{v|w|} - c^2\ \overline{w|w|}$$

The quantity $|w|$ represents the fully rectified w signal and is composed of a mean and fluctuating part and the correlation $\overline{w\ |w|}$ is not zero unless the signal is exactly symmetrical (e.g. a sine-wave). In a two dimensional flow $\overline{uw} = 0$ but the quantity $\overline{v\ |w|}$ will not be zero because it is probable that a particular <u>sign</u> of v should be associated with a particular <u>magnitude</u> of w. Hence equation (13) cannot be greatly simplified and is written

$$\overline{uv}\ \text{measured} = \overline{uv}\ \text{actual}\ \left(1 - \frac{c}{2a}\,\frac{\overline{v|w|}}{\overline{uv}} - \frac{c^2}{4ab}\,\frac{\overline{w|w|}}{\overline{uv}}\right) \qquad (14)$$

which is very inconvenient to use because the corrections are not easily assessed. It is also possible to obtain $\overline{uv}$ by the method used when analysing single slanted wire data: i.e. the individual mean square signals from each wire are taken and $\overline{uv}$ is calculated from the difference $(\overline{e_1^2} - \overline{e_2^2})$. This calculation is done in Appendix I and the result is identical to Equation (14).

The experiments in this note indicated that, at low wire Reynolds numbers, $^c/b$ can reach $0\left[1\right]$ and consequently that large errors in $\overline{v^2}$ and $\overline{uv}$ are possible. It is emphasized that the experiments give the pitch sensitivity for a turbulence level which is relatively low (0.4%) compared to typical values encountered in boundary layers and free shear flows. The pitch sensitivity in these flows may conceivably be different and this can be verified by comparing single slanted and X-wire measurements at various Reynolds numbers.

3. Experimental Arrangement

The experiments were performed with a Disa X-wire probe type (55A32) in the McGill 3 ft. x 2 ft. suction wind tunnel. The wind speed in the working section can be varied from zero to about 130 ft/sec. and the turbulence level is about 0.4%. Other details of the tunnel are given by Wygnanski and Newman (1961).

The X-wire probe support was mounted on a circular turntable as shown in Fig. (1b), with the axis parallel to the x-y plane. Rotating the turntable clockwise (i.e. +ve θ) moved one end of wire no. 2 into the thermal wake of no. 1 (see Fig. 1b plan view and Fig. 1a). Similarly by rotating the turntable anticlockwise wire no. 1 was in the thermal wake of no. 2. For completeness, the pitch angle was varied through $\pm 25^{\circ}$ which extends well beyond the range necessary to calculate the "small perturbation" pitch sensitivity discussed in section 2.

The physical details of the X-wire probe and a modified version were measured with an optical comparator. The necessary details are given in Fig. (2). For the modified probe the prongs were separated approximately by a wire length.

A non-linearized calibration of the X-wire probe (see Fig. (3)) was obtained by varying the tunnel speed and keeping the X-wire plane parallel to the flow direction. No attempt was made to match the wires exactly because, for this investigation, only the relative behaviour of the wires was of interest.

4. Results

4.1 The Unmodified X-wire Probe

To establish whether or not thermal wake effects exist the upstream wire was initially left cold and output, E_2, of the downstream wire at various angles of pitch, θ, was recorded (Figs. (4) and (5)). When the upstream wire was heated, its thermal wake clearly influenced the output of the downstream wire.

The output of both wires for positive angles of pitch at Re = 8.95 is shown in Fig. (6). At large pitch angles the voltage across each wire is expected to increase for two reasons:

(i) the component of velocity normal to the wire increases (when $\theta = 90^{\circ}$ the full vector is normal to the wires),

(ii) the effect of prong and stem blockage speeds the flow over the wire (Hoole and Calvert, 1967; Gilmore, 1967; Guitton, 1968). At small angles these effects are of second order and the small perturbation sensitivity to pitch of the upstream wire is zero. The thermal wake interfering with the downstream wire forces that sensor's output to drop slightly.

A convenient method of isolating the hot-wake effect for <u>all</u> pitch angles is to consider the difference ΔE between the output of the wires. If the thermal wake effects were negligible, the difference, ΔE, between the output of both wires at various pitch angles would be constant, or zero for matched wires. Fig. (7) shows the behaviour of ΔE for positive and negative values of θ

at Re = 8.95 and 2.35. The sensitivity to pitch is the same in both directions and subsequently tests at other Reynolds numbers were, therefore, made only for positive values of θ and the results are shown in Fig. (8). In Fig. (9), the data of Figures (7) and (8) are replotted non-dimensionally. In this figure $(\Delta E)_{\theta=0}$ is the difference between the outputs of the hot-wires at zero pitch angle.

4.2 The Modified X-wire Probe

To avoid the thermal wake effects on X-wire measurements two simple modifications of the probe are possible. One of these is that suggested by Burling (1969), i.e. the prongs of a conventional X-wire are displaced in the plane parallel to the X in such a way that the X-wires take the shape of a V. The other possibility is to separate the prongs and this simpler modification was adopted in this investigation (Fig. 2). As mentioned before the prongs were separated approximately by one wire length and great care was taken to keep the wires in parallel planes.

Tests similar to those performed on the unmodified X-wire were also made on the modified X-wire. Since the thermal wake interference was severe at low Reynolds numbers for the unmodified probe, the modified sensor was tested at Re = 1.13 with and without heating the upstream wire. The results of these tests are given in Fig. (10) and no thermal wake effect is apparent. The variation of ΔE at Re = 1.13 and 11.92 when both wires are heated is shown in Fig. (11) and again no pitch sensitivity is discerned.

For interest the modified X-wire probe was calibrated by two methods and this is discussed in Appendix 2.

5. Discussion

The relative importance of the X-wire's sensitivity to pitch can be assessed as shown in section 2 by determining the ratios $\frac{c}{a}$ and $\frac{c}{b}$. For the non-linear response

$$E^2 = A + B\ U^{0.4} \tag{15}$$

the sensitivity coefficients a and b are:

$$a = \frac{0.4B}{2E\ U^{0.6}} \tag{16}$$

and

$$b = a \cot \psi \tag{17}$$

if longitudinal cooling effects, which are of second order, are neglected. The value of c:

$$c = \left(\frac{1}{U}\frac{\partial E}{\partial \theta}\right)$$

accounting for the thermal wake interference can be obtained by measuring the slopes of the ΔE vs. θ plots at $\theta = 0$. (It could also be obtained from the E vs. θ curve of the downstream wire because the effects due to blockage and increasing normal component have zero slope at $\theta = 0$.)

Fig. (12) shows the variation of $^c/b$ with wire Reynolds number (at a fixed Prandtl number). The ratio $^c/a$ is not shown because for $\psi \simeq 45^\circ$, a $\simeq$ b. The decrease of $^c/b$ with Re is due to the dependence of both 'b' and the width of the thermal wake on Re. From equations (15) and (16) and for the present X-wire probe calibration

$$a \propto \frac{1}{Re^{0.6}\,(1 + 1.31\,Re^{0.4})^{1/2}}$$

As an X-probe is pitched, the farthest end of the downstream wire is first affected by the hot wake (fig. 1a) and as the angle increases from zero this end moves towards the centre or high temperature region of the wake. It is evident from fig. (1a) that; (i) increasing θ lengthens the portion of the downstream wire which lies in the thermal wake and (ii) the affected segment 'sees' along its length, a different wake temperature. For the purpose of obtaining the dependence of c on Reynolds number it is convenient to assume that the effect in (ii) can be accounted for by an average thermal wake profile of the form

$$T = T_o\, f\left(\frac{z}{l_o}\right) \tag{18}$$

where T_o is a suitable mean maximum temperature and l_o a characteristic width of the wake (see Fig. (1a)). This assumption is equivalent to a simple 2-D analysis of the problem in which two parallel normal wires separated a distance D are considered. At constant U and ψ then,

$$E = E(T)$$

and

$$\frac{\partial E}{\partial \theta} = \frac{\partial E}{\partial T}\,\frac{\partial T}{\partial \theta}$$

$$= \frac{\partial E}{\partial T} (D \frac{\partial T}{\partial z}) \tag{19}$$

from Fig. 1a

Since

$$\frac{\partial T}{\partial z} \propto \frac{1}{l_o}$$

then

$$\frac{\partial E}{\partial \theta} \propto \frac{1}{l_o} \tag{20}$$

At a given Prandtl number the characteristic width of the laminar wake is written

$$l_o \propto (\frac{1}{Re})^{1/2}$$

and therefore

$$c = \frac{1}{U} \frac{\partial E}{\partial \theta}$$

i.e. $$c \propto (\frac{1}{Re})^{1/2} \tag{21}$$

Combining this with the expression for 'a' yields

$$\frac{c}{b} \propto \frac{1}{Re^{1.1} (1 + 1.31 Re^{0.4})^{1/2}} \tag{22}$$

which is plotted in Fig. (13). The linear relationship between the parameters is fairly well substantiated.

6. Conclusions

(1) A X-wire probe having the ratio of wire separation to wire length, $^s/l = 0.15$ was tested for pitch sensitivity in a wind tunnel having a turbulence intensity of 0.4%. In this relatively low turbulence field the ratio of pitch sensitivity to yaw sensitivity ($^c/b$) varied strongly with the wire Reynolds number. This is explained in part by the dependence of yaw sensitivity (b) on Re, and also to a laminar thermal wake width which is proportional to $Re^{-1/2}$. At $Re \sim 0[1]$ the ratio $^c/b \sim 0[1]$ and decreases to $0[0.1]$ at $Re \sim 0[10]$. Some results of Jerome (1969) suggest that the pitch sensitivity is unchanged in large scale turbulence.

(2) Modifying a X-wire probe by moving the wires apart by approximately one wire length, in the direction <u>perpendicular</u> to the plane of the X removes the thermal wake interference.

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Appendix 1

Calculation of $\overline{uv}$ from X-wire data using the method applied to single slanted wires

From section 2: (using subscript + and - to associate a parameter with a particular +ve or -ve w)

$$\overline{e_1^{\,2}} = \overline{(e_{1_+} + e_{1_-})^2}$$

$$= \overline{e_{1_+}^{\,2}} + \overline{2\, e_{1_-} e_{1_+}} + \overline{e_{1_-}^{\,2}}$$

$$\overline{e_{1_+}^{\,2}} = a^2\, \overline{u^2}_+ + 2ab\, \overline{uv}_+ + b^2\, \overline{v^2}_+$$

and

$$\overline{e_{1_-}^2} = a^2\, \overline{u_-^2} + 2ab\, \overline{uv}_- + b^2 \overline{v_-^2} - 2bc\, \overline{v\,|w|}_- - 2ac\, \overline{u\,|w|}_-$$

$$+ c^2\, \overline{|w|_-^2}$$

Similarly

$$\overline{e_2^2} = \overline{(e_{2_+} + e_{2_-})^2}$$

$$\overline{e_{2_+}^2} = a^2\, \overline{u_+^2} - 2ab\, \overline{uv}_+ + b^2\overline{v_+^2} + 2bc\, \overline{v|w|}_+ - 2ac\, \overline{u|w|}_+ + c^2\overline{|w|_+^2}$$

and

$$\overline{e_{2_-}^2} = a^2\overline{u^2}_- - 2ab\, \overline{uv}_- + b^2\, \overline{v_-^2}$$

Hence

$$\overline{e_1^2} - \overline{e_2^2} = 2ab\, \overline{uv} - 2bc\, \overline{v|w|} + 2ac\, \overline{uw} + c^2\left[\, \overline{|w|_-^2} - \overline{|w|_+^2}\, \right]$$

Now

$$c^2\left[\, \overline{|w|_-^2} - \overline{|w|_+^2}\, \right] = -\, c^2\left[\, \overline{w|w|}_- + \overline{w|w|}_+ \right]$$
$$= -\, c^2\left[\, \overline{w|w|}\right]$$

In 2-D flow $\overline{uw} = 0$

$$\overline{e_1^2} - \overline{e_2^2} = 4ab\, \overline{uv} - 2bc\, \overline{v|w|} - c^2\, \overline{w|w|}$$

i.e. $4ab\,\overline{uv}_{measured} = 4ab\,\overline{uv}_{actual} - 2bc\,\overline{v|w|} - c^2\,\overline{w|w|}$

$$\overline{uv}_{measured} = \overline{uv}_{actual}\left[1 - \frac{c}{2a}\,\frac{\overline{v|w|}}{\overline{uv}} - \frac{c^2}{4ab}\,\frac{\overline{w|w|}}{\overline{uv}}\right]$$

Appendix 2

For interest the modified X-wire probe was calibrated in two ways: (a) by varying the mean flow velocity and keeping the wires at a known fixed angle of yaw to the mean flow (the probe axis was parallel to the free stream velocity vector) Fig. (14), and, (b) by determining directly the yaw sensitivity by varying the yaw angle at constant mean flow velocity, Figs. (15) and (16).

The calibration by method (a) can be expressed as:

$$E^2 = A + BU^{0.4} \tag{A1}$$

The non-linearized response equation including longitudinal cooling effects could be written as:

$$E^2 = A_1 + B_1 \left[\sin^2\psi + k^2 \cos^2\psi \right]^{0.2} U^{0.4} \tag{A2}$$

where k represents the longitudinal cooling effects.

Comparing equations (A1) and (A2),

$$B = B_1 \left[\sin^2\psi + k^2 \cos^2\psi \right]^{0.2} \tag{A3}$$

For fluctuations in velocity about the mean, equation (A2) gives:

$$\begin{aligned} 2EdE = {} & 0.4B_1 \left[\sin^2\psi + k^2 \cos^2\psi \right]^{0.2} U^{-0.6} \, dU \\ & + 0.2B_1 \left[\sin^2\psi + k^2 \cos^2\psi \right]^{-0.8} \left[2\sin\psi\cos\psi(1-k^2)U^{0.4}d\psi \right] \end{aligned} \tag{A4}$$

For small fluctuating components u and v,

$$dE = e, \quad dU = u \quad \text{and} \quad Ud\psi = v \tag{A5}$$

Substituting equations (A5) and (A3) into equation (A4) gives,

$$e = \left(\frac{0.2B}{EU^{0.6}}\right) u + \left(\frac{0.2B}{EU^{0.6}}\right)\left(\frac{1-k^2}{1+k^2\cot^2\psi}\right) \cot \psi \; v$$

$$\therefore \quad e = a_1 u + b_1 v \qquad \text{(A6)}$$

where $a_1 = \dfrac{0.2B}{EU^{0.6}}$

and $b_1 = \left(\dfrac{0.2B}{EU^{0.6}}\right)\left[\dfrac{1-k^2}{1+k^2\cot^2\psi}\right] \cot \psi$

For no longitudinal cooling effects (i.e. $k = 0$) the above equation reduces to that derived by Patel (1963).

The response of hot-wire in method (b) may be written as:

$$E = E\ (U,\psi) \qquad \text{(A7)}$$

Hence $dE = \dfrac{\partial E}{\partial U}.\ dU + \dfrac{\partial E}{\partial \psi}.\ d\psi$

$$\therefore \quad e = a_2 u + b_2 v \qquad \text{(A8)}$$

where $a_2 = \dfrac{\partial E}{\partial U}$

and $b_2 = \dfrac{1}{U} \dfrac{\partial E}{\partial \psi}$

The results of method (a) yields:

$$Re = 1.8 \begin{cases} a_1 = 0.0491 \\ b_1 = 0.0409 \end{cases} ; \quad Re = 11.9 \begin{cases} a_1 = 0.0121 \\ b_1 = 0.0101 \end{cases}$$

while method (b) gives:

$$Re = 1.8 \begin{cases} a_2 = 0.0491 \\ b_2 = 0.0410 \end{cases} ; \quad Re = 11.9 \begin{cases} a_2 = 0.0121 \\ b_2 = 0.0105 \end{cases}$$

The parameter k^2 could be evaluated by noting that the calibration technique (b) automatically includes longitudinal cooling effects. Hence, since $b_1 = b_2$,

$$\frac{1}{U}\frac{\partial E}{\partial \psi} = a_1 \cot\psi \left[\frac{1-k^2}{1+k^2\cot^2\psi}\right]$$

But $\frac{\partial E}{\partial \psi}$ must be obtained graphically and the limited accuracy of this technique precludes it as a means of obtaining the small quantity k^2. A preferable approach is to use equation (A2) directly.

The non-linearized response equation (A2) can be rewritten as:

$$(E^2 - A_1) = B_1\, U^{0.4}\left[\sin^2\psi + k^2\cos^2\psi\right]^{0.2} \qquad \text{(A9)}$$

If the output of a hot-wire at various angles of yaw is measured in a constant mean velocity field then the above equation can be rearranged as:

$$\sin^2\psi = \left(\frac{B_2}{1-k^2}\right)\left[E^2 - A_1\right]^5 - \left(\frac{k^2}{1-k^2}\right) \qquad \text{(A10)}$$

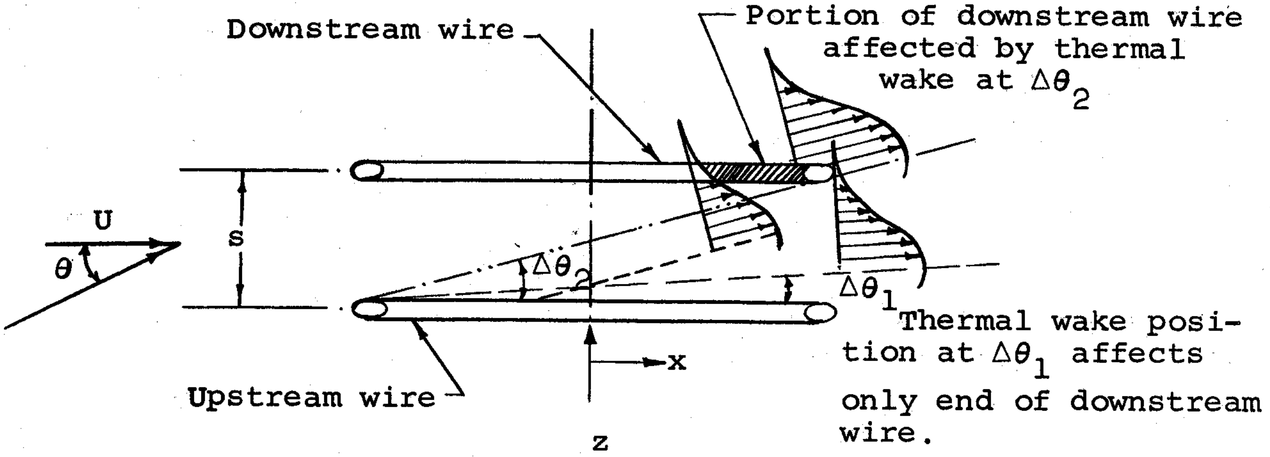
where

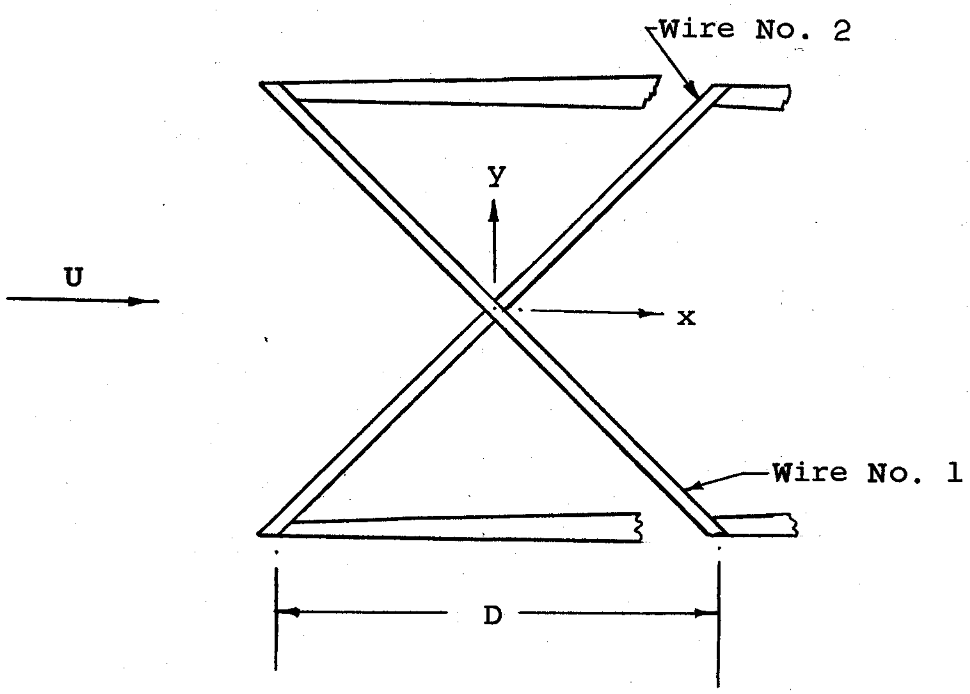
$$B_2 = \left(\frac{1}{B_1 U^{0.4}}\right)^5$$

Hence a plot of $\sin^2\psi$ vs. $(E^2-A_1)^5$ would give a straight line with an intercept, on $\sin^2\psi$-axis, equal to $-\left(\frac{k^2}{1-k^2}\right)$.

For wire no. 2, Fig. (17) shows the results of Fig. (15) replotted in accordance with equation (A10) and a best straight line is fitted to the experimental points. Note that $A_1 = 18.2$ and is obtained from Fig. (14). The intercept on the $\sin^2\psi$-axis is -0.035 and hence $k^2 = 0.0338$. From Champagne's (1965) and Patel's

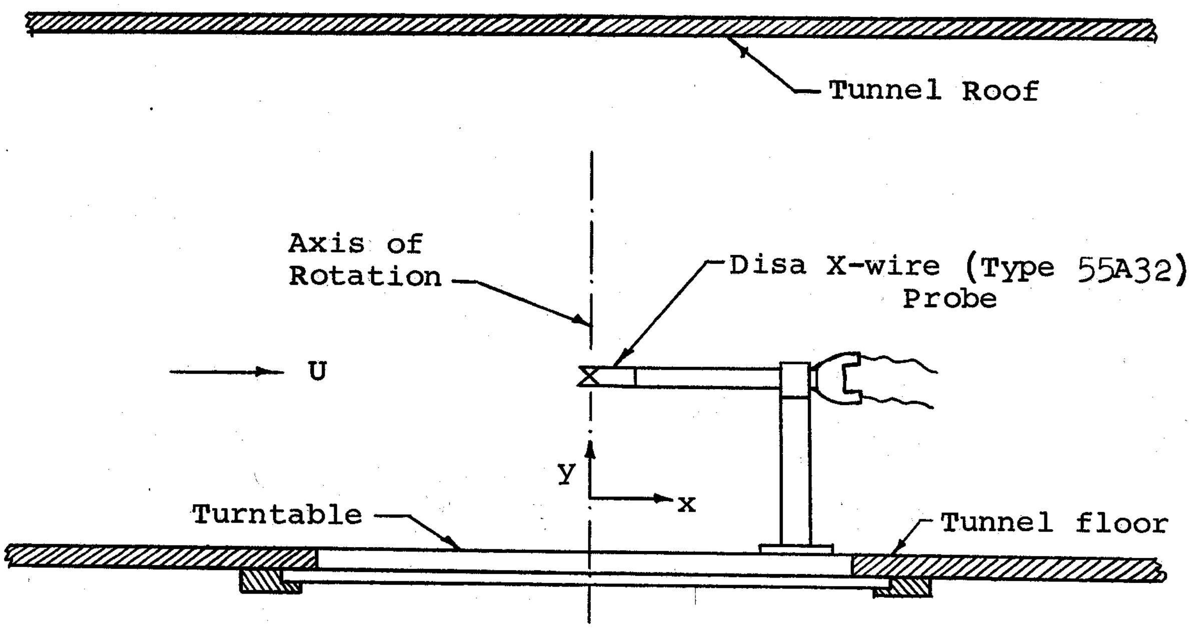
(1968) investigations on hot-wires having length to diameter ratio of about 200 it was found that $k^2 \approx 0.04$. The value of k^2 obtained by the above method is not inconsistent with the values suggested by other investigators.



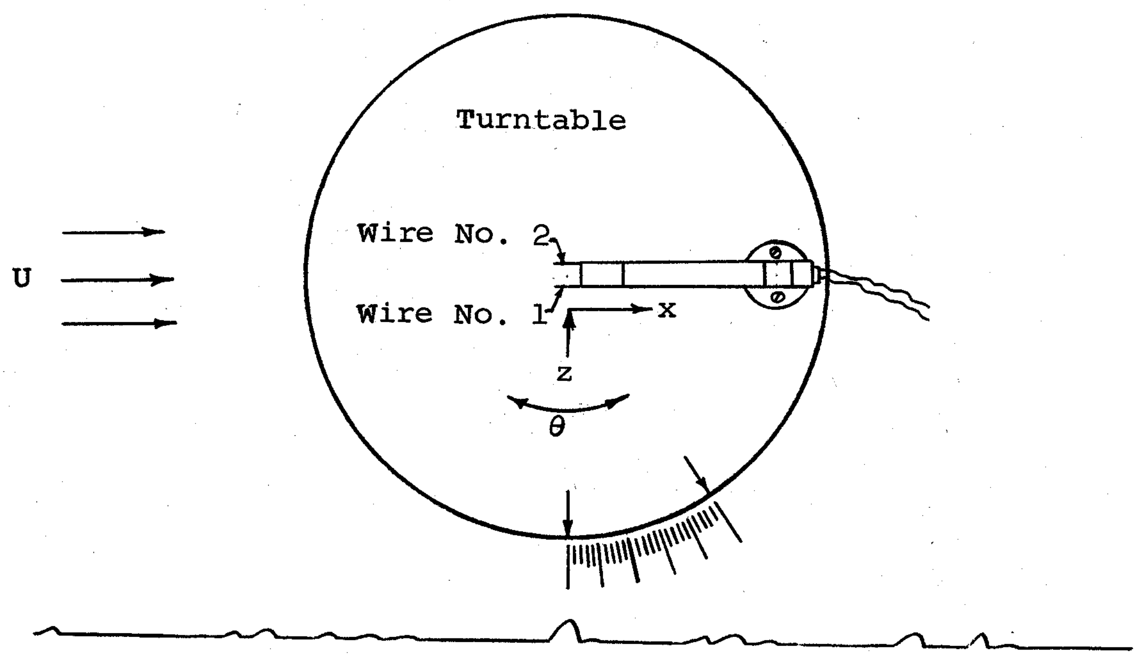


SKETCH SHOWING THERMAL WAKE POSITION
ON A CONVENTIONAL X-WIRE PROBE

Fig. 1b

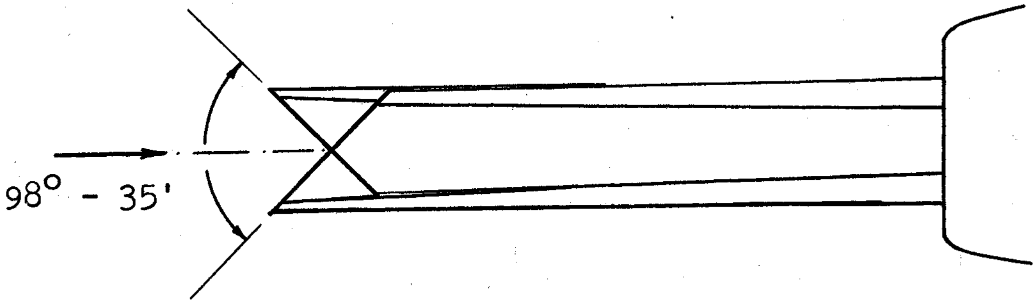


ELEVATION

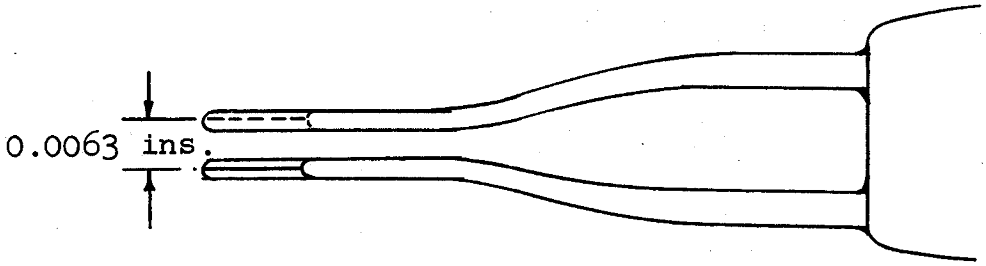


PLAN VIEW

SKETCH OF THE EXPERIMENTAL ARRANGEMENT

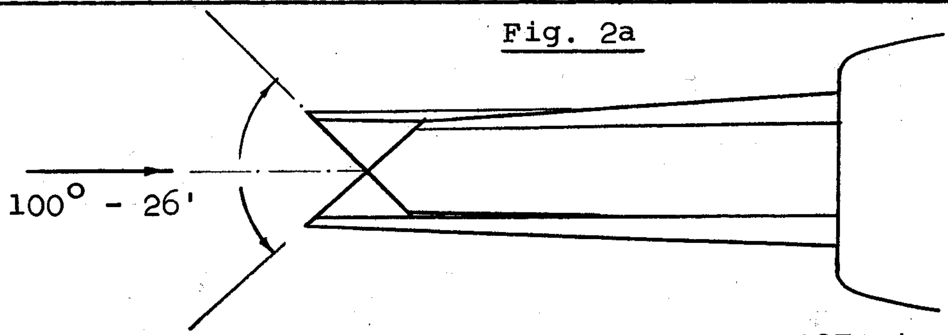


Length of wire no. 1 = 0.040 ins.
"   "   " no. 2 = 0.0384 ins.
Spacing between prongs = 0.0018 ins.
Diameter of prong tip = 0.0045 ins.
Distance between wires = 0.0063 ins.

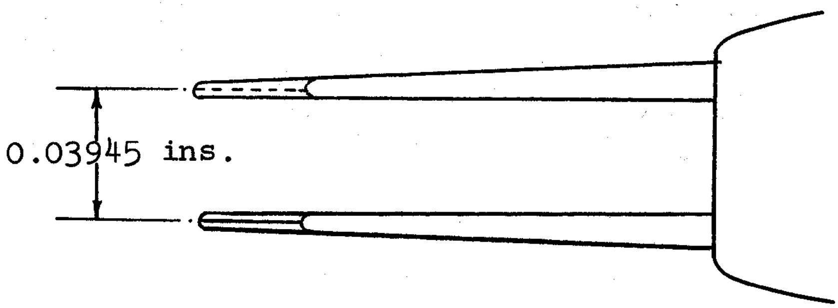


CONVENTIONAL DISA X-WIRE (TYPE 55A32) PROBE USED IN EXPERIMENTS

Fig. 2a

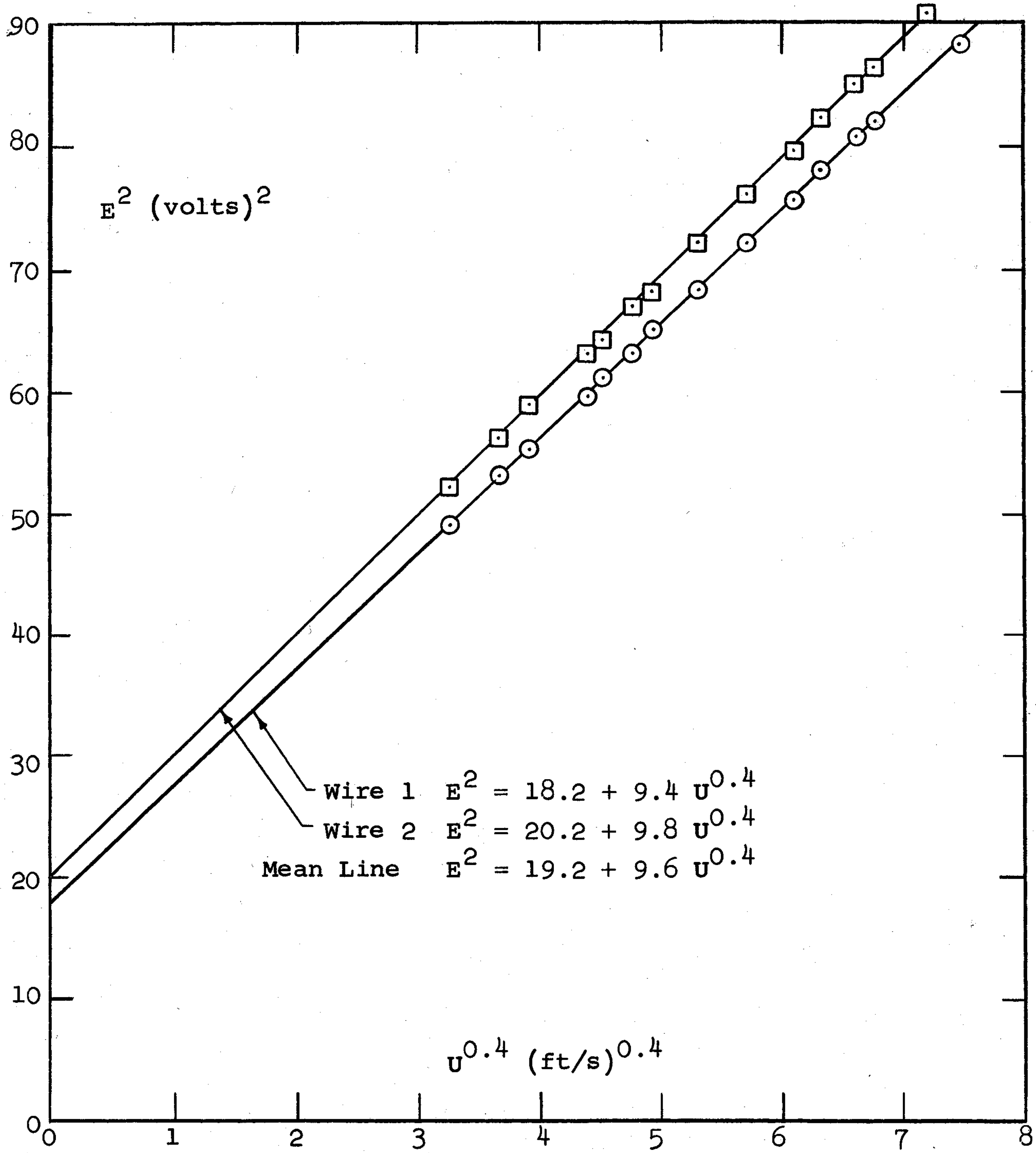


Length of wire no. 1 = 0.0372 ins.
"   "   " no. 2 = 0.0414 ins.
Spacing between wires = 0.03945 ins.
Diameter of prong tip = 0.0046 ins.

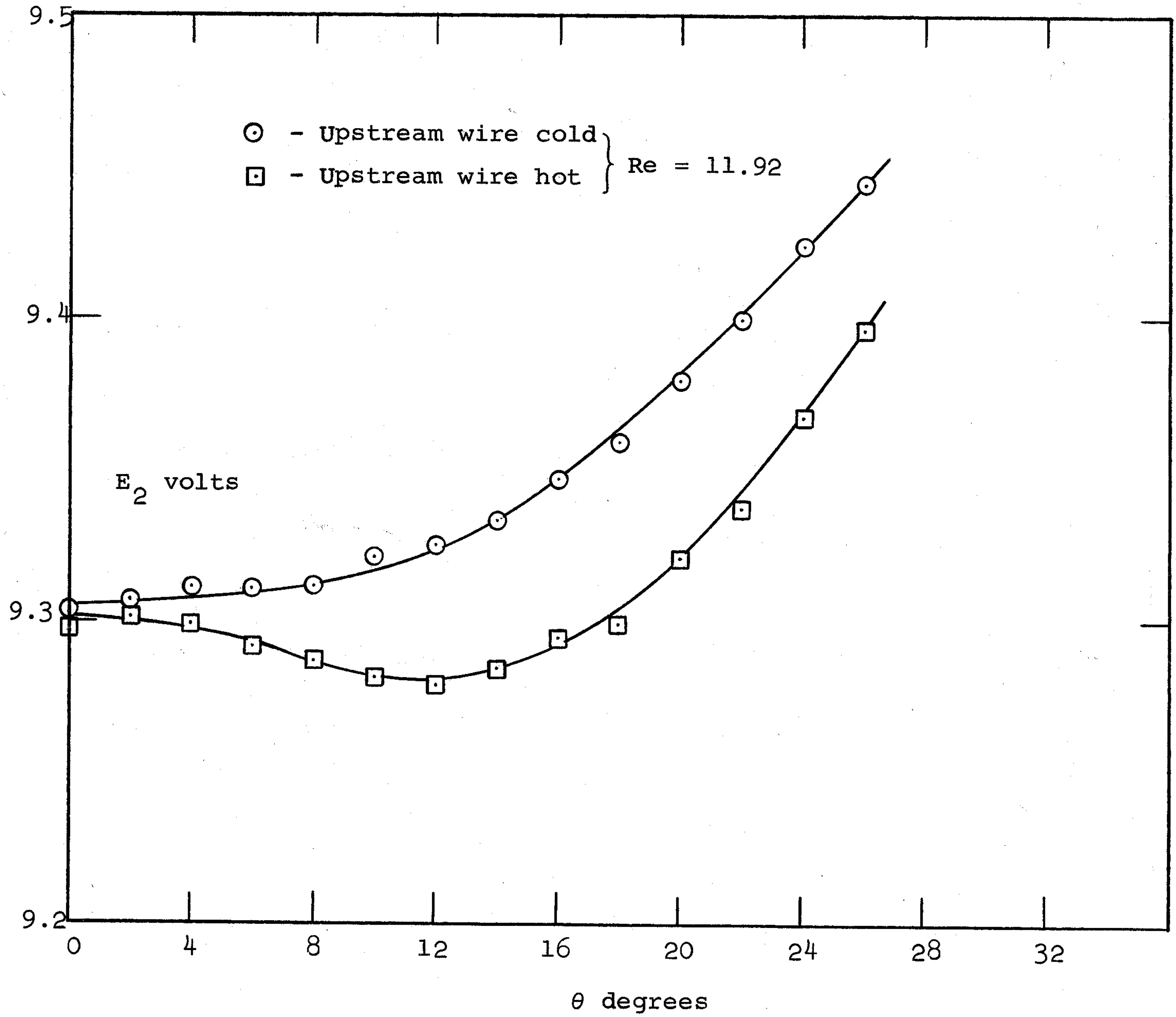


MODIFIED X-WIRE PROBE

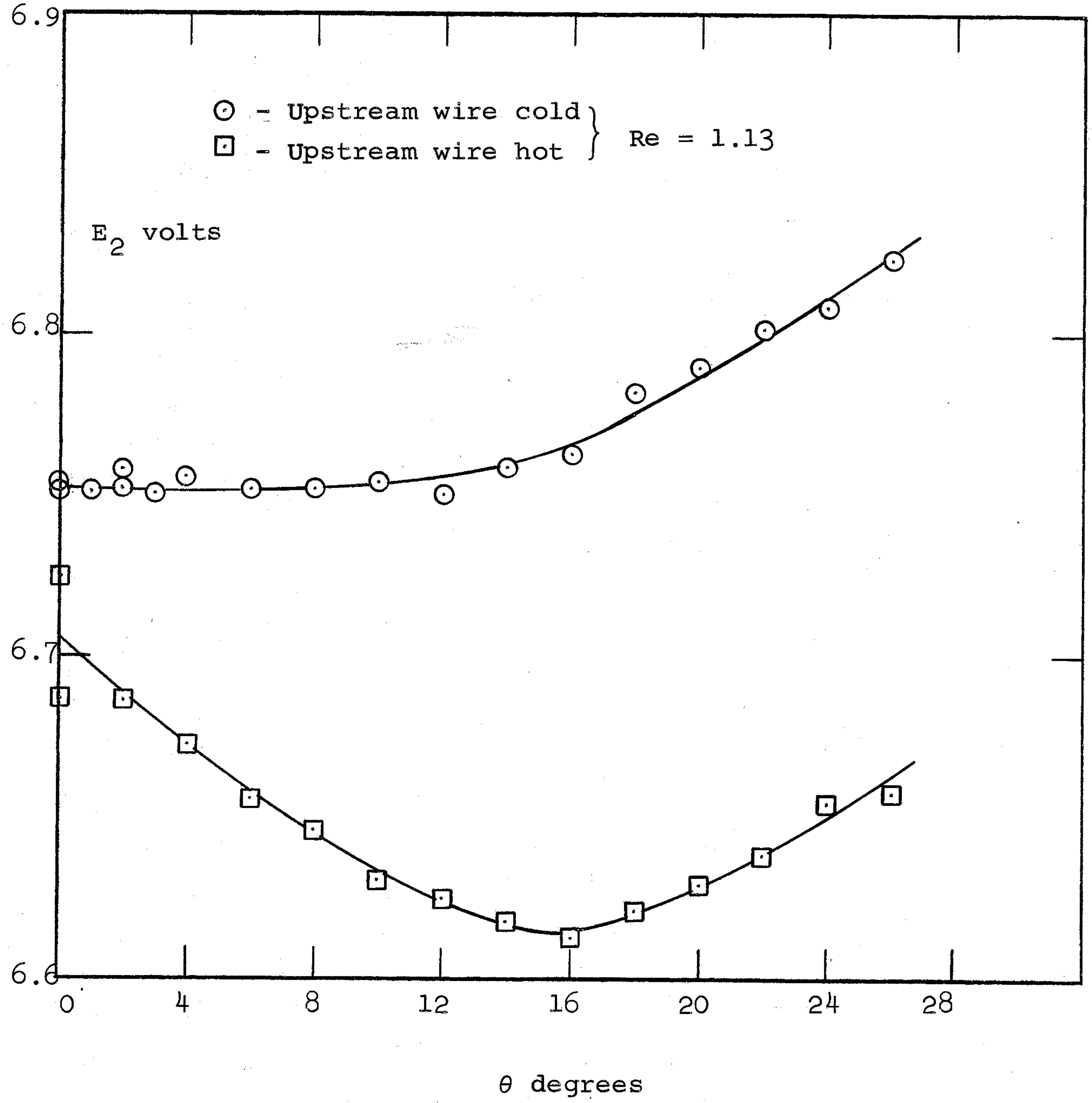
Fig. 2b



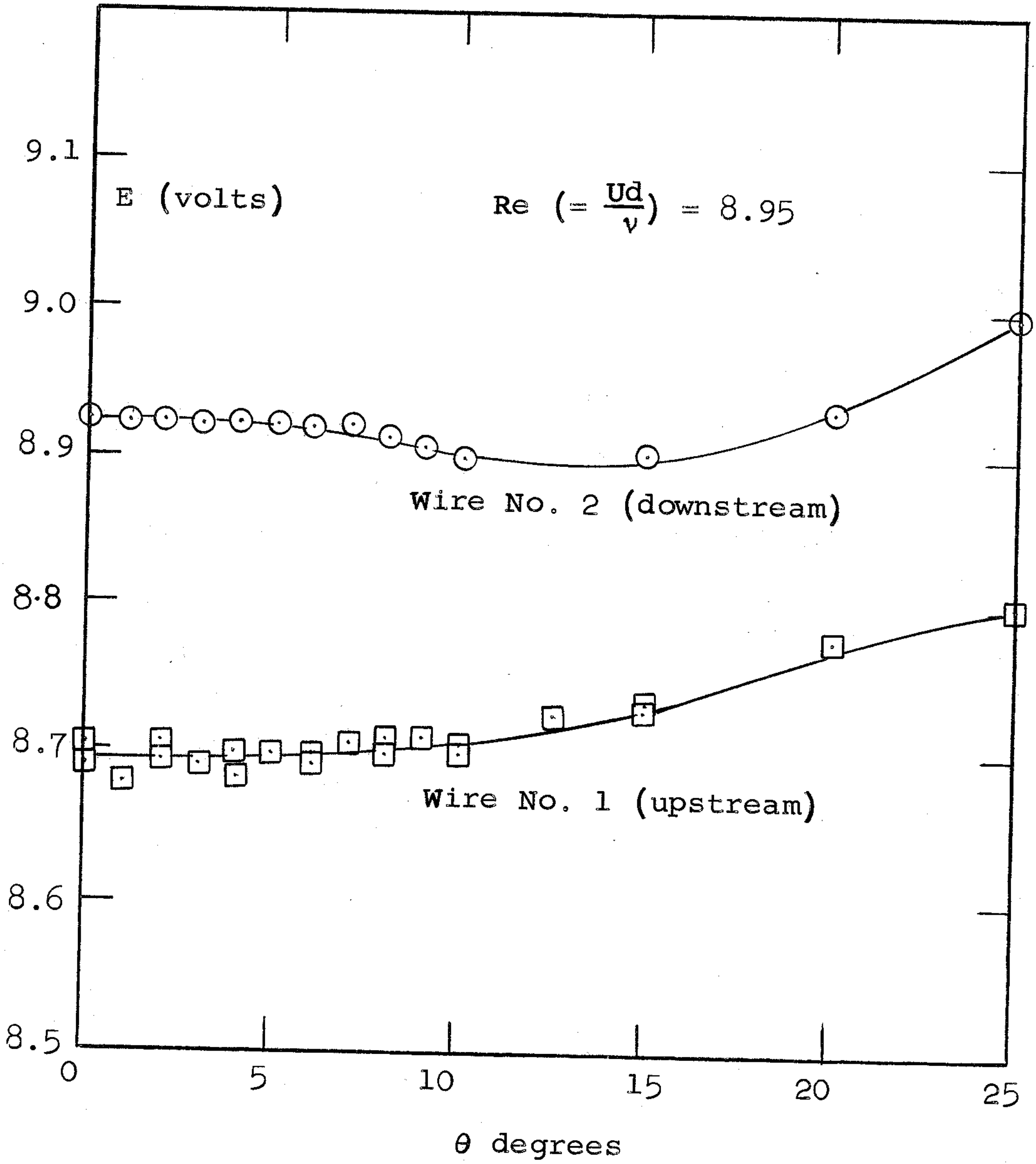
A CALIBRATION OF THE CONVENTIONAL X-WIRE (TYPE 55A32) PROBE SHOWN IN FIG. 2a



PITCH RESPONSE OF THE
DOWNSTREAM WIRE IN A CONVENTIONAL DISA X-WIRE (TYPE 55A32) PROBE WITH
AND WITHOUT A THERMAL WAKE



PITCH RESPONSE OF THE
DOWNSTREAM WIRE IN A CONVENTIONAL DISA X-WIRE (TYPE 55A32)
PROBE WITH AND WITHOUT A THERMAL WAKE



PITCH RESPONSE OF THE INDIVIDUAL WIRES OF THE CONVENTIONAL DISA X-WIRE (TYPE 55A32) PROBE OF FIG. (2a)

RELATIVE PITCH RESPONSE OF THE CONVENTIONAL DISA X-WIRE (TYPE 55A32) PROBE OF FIG. 2a

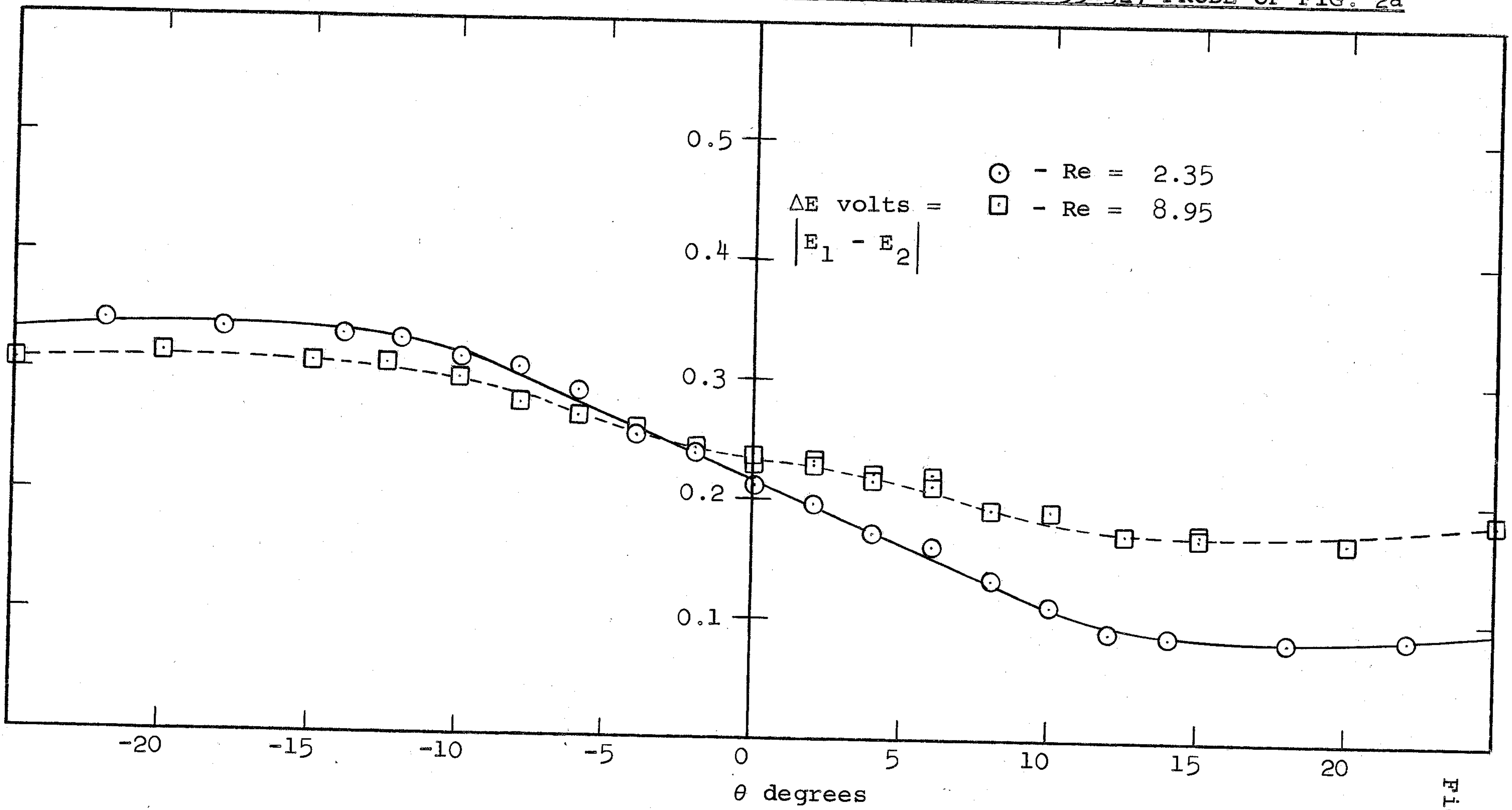
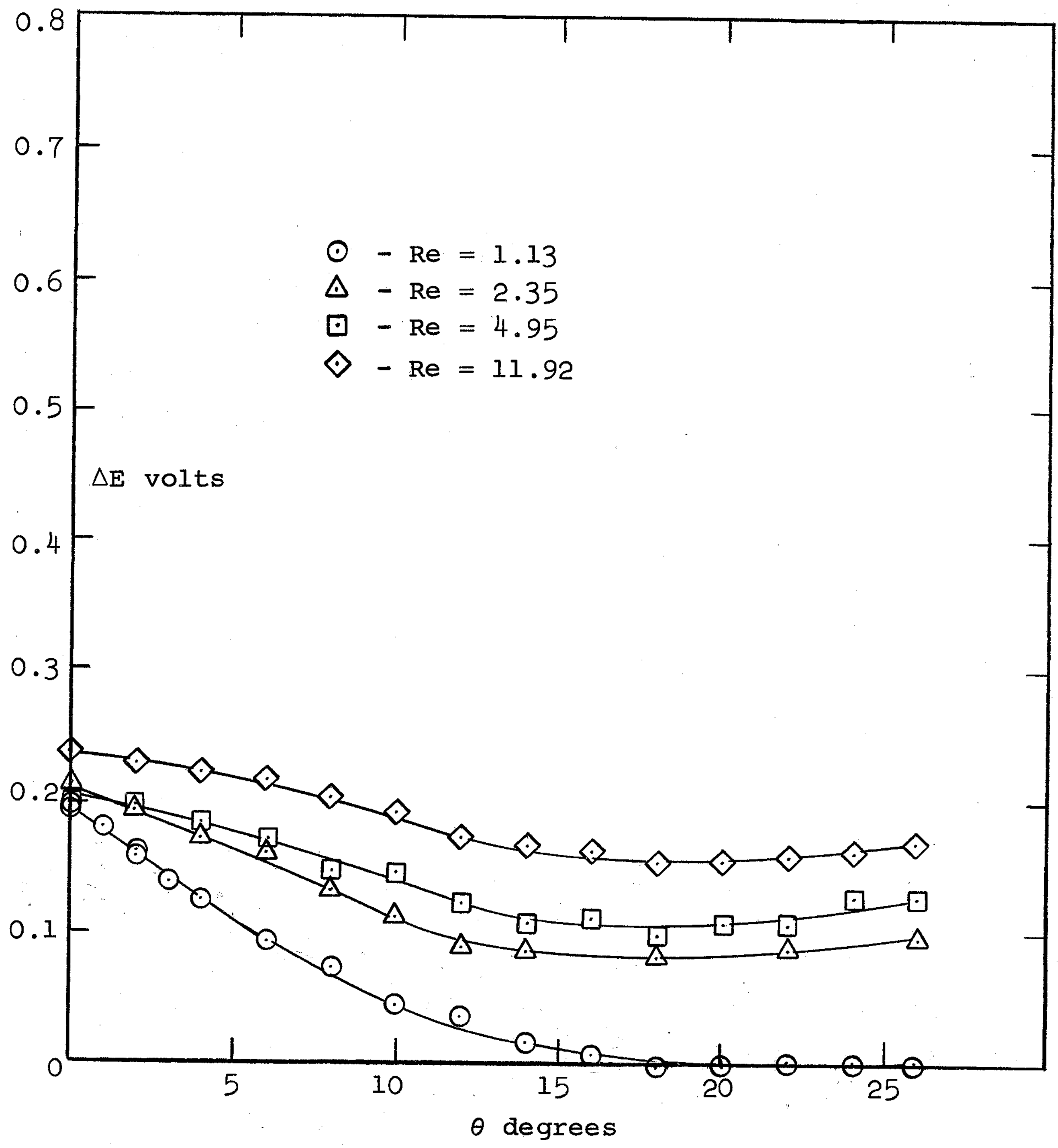


Fig. 7



RELATIVE PITCH RESPONSE OF THE CONVENTIONAL DISA X-WIRE (TYPE 55A32) PROBE OF FIG. 2a

NON-DIMENSIONAL RELATIVE PITCH RESPONSE OF THE CONVENTIONAL DISA X-WIRE (TYPE 55A32) PROBE

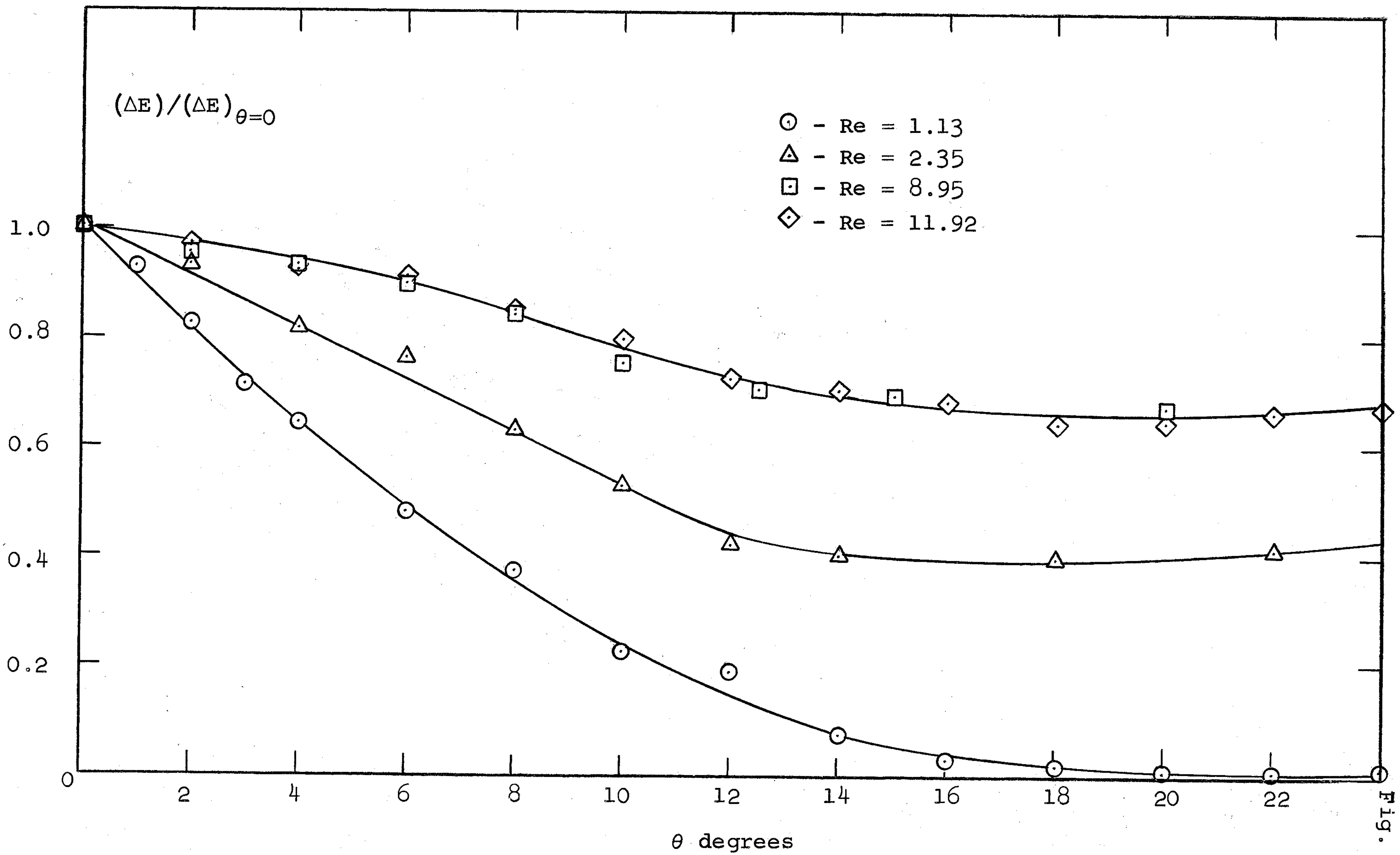
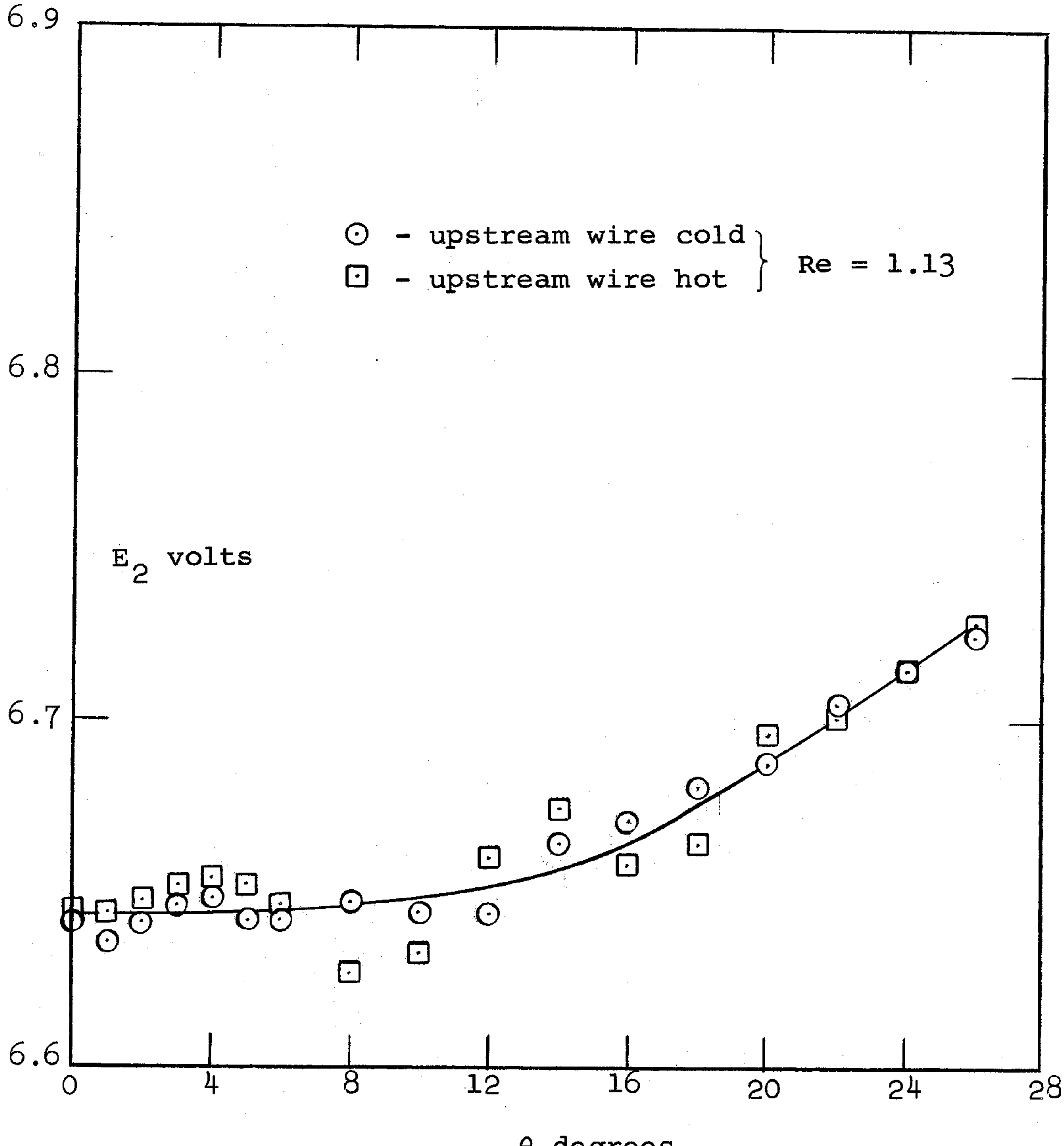
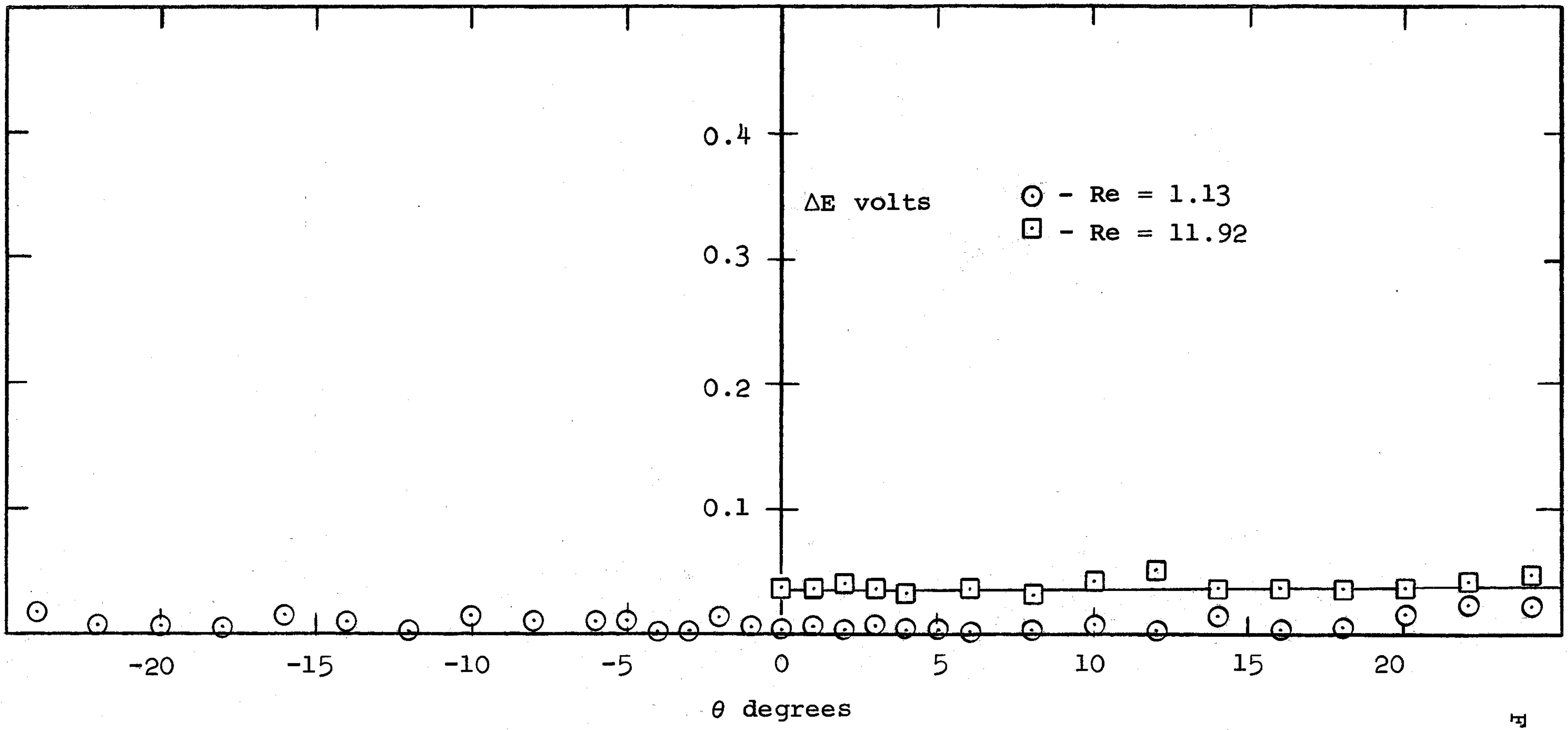


Fig. 9

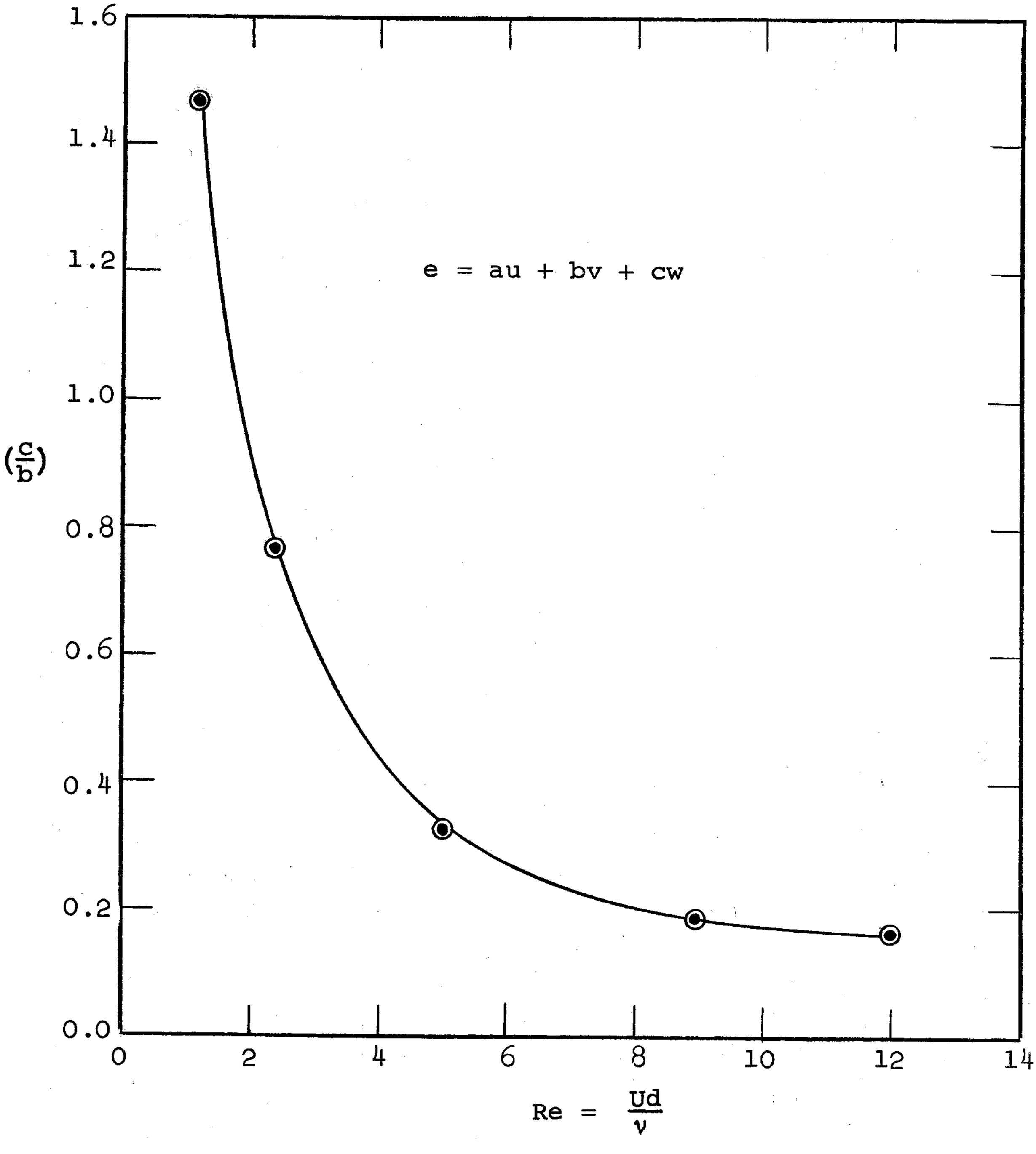


PITCH RESPONSE OF MODIFIED X-WIRE PROBE WITH AND WITHOUT A THERMAL WAKE



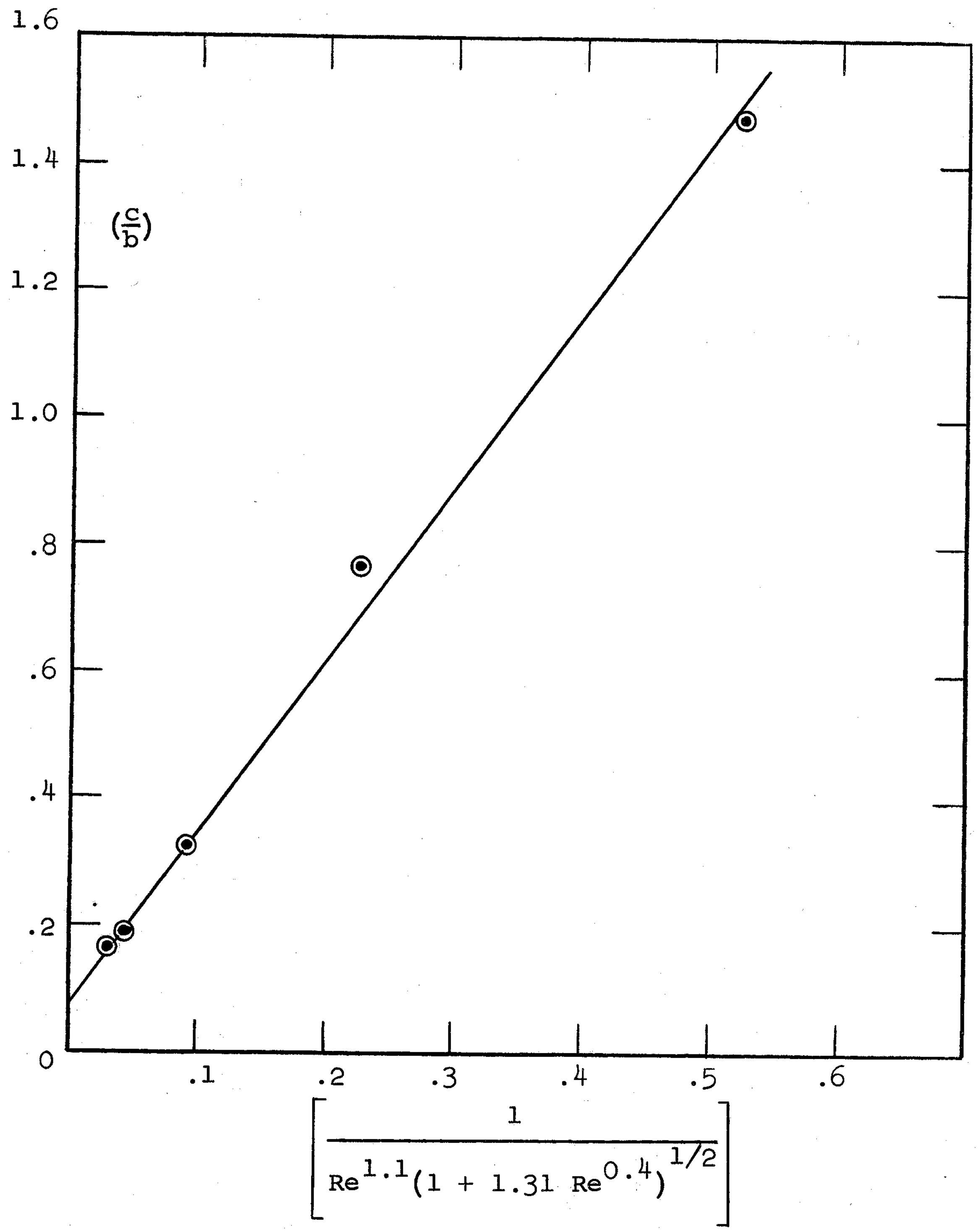
RELATIVE PITCH RESPONSE OF MODIFIED X-WIRE PROBE

Fig. 11



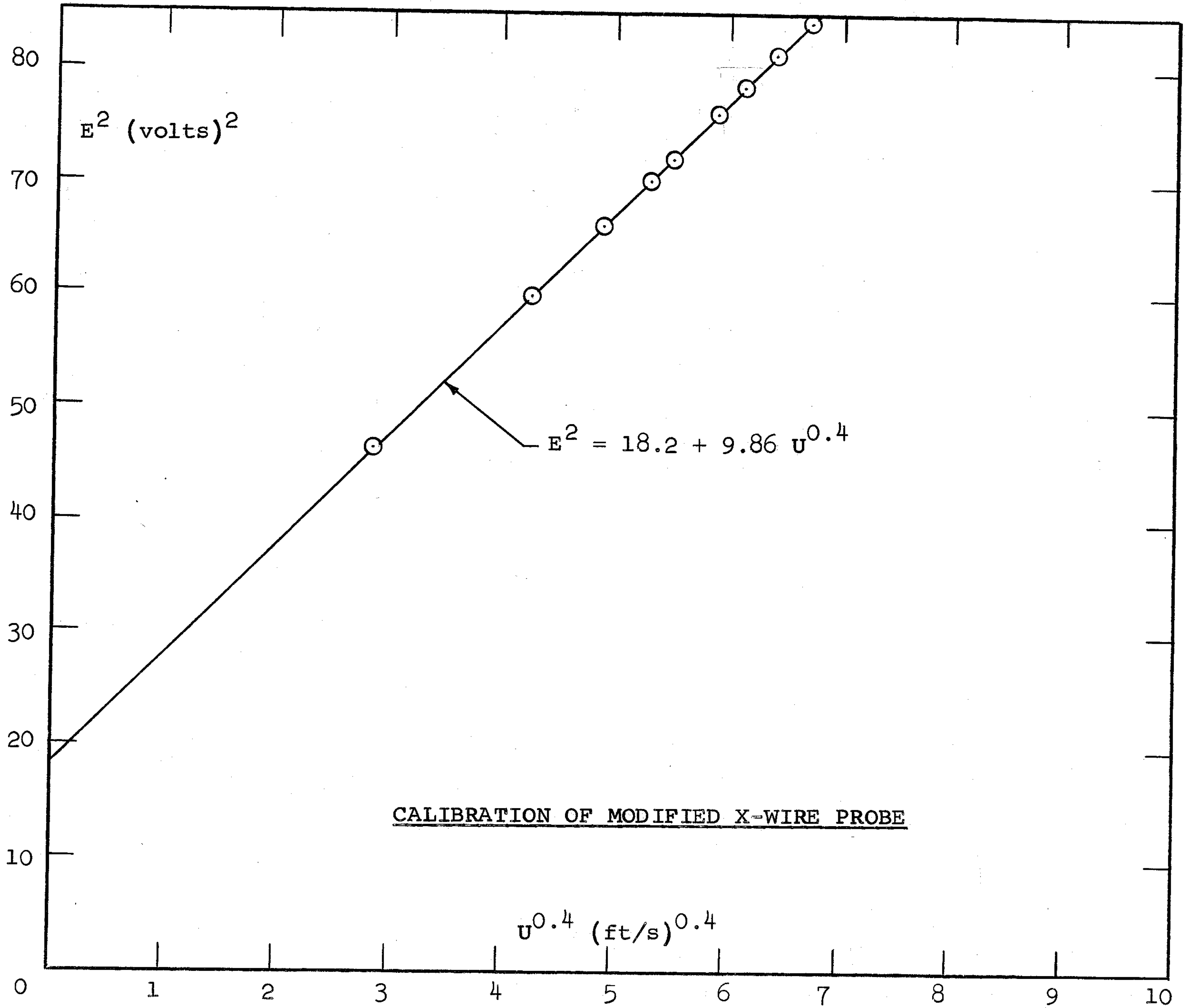
RATIO OF PITCH TO YAW SENSITIVITY FOR THE CONVENTIONAL X-WIRE vs. REYNOLDS NUMBER

WIRE NO. 1



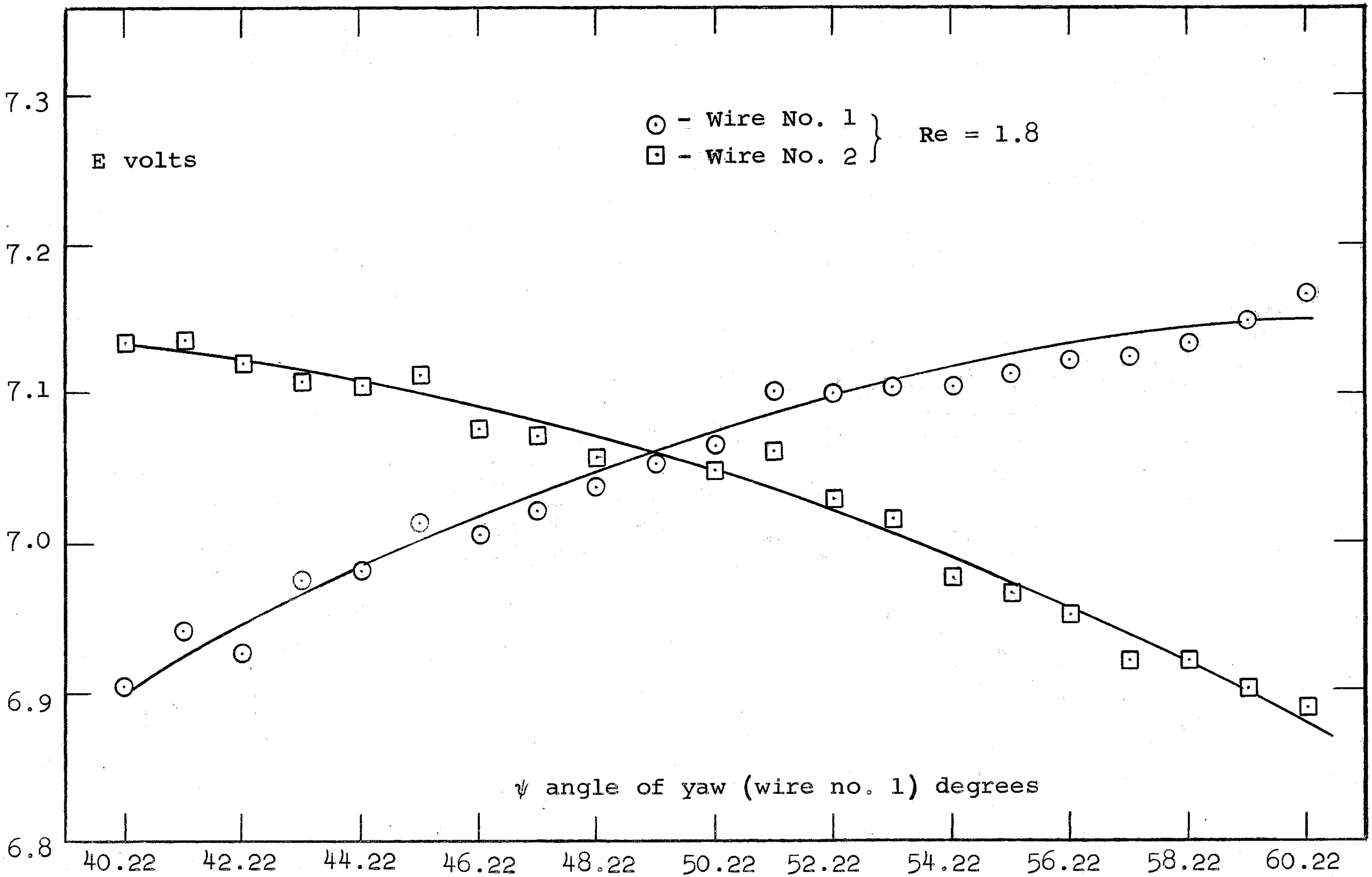
VARIATION OF $\frac{c}{b}$ IN ACCORDANCE WITH EQUATION (22)

WIRE NO. 1



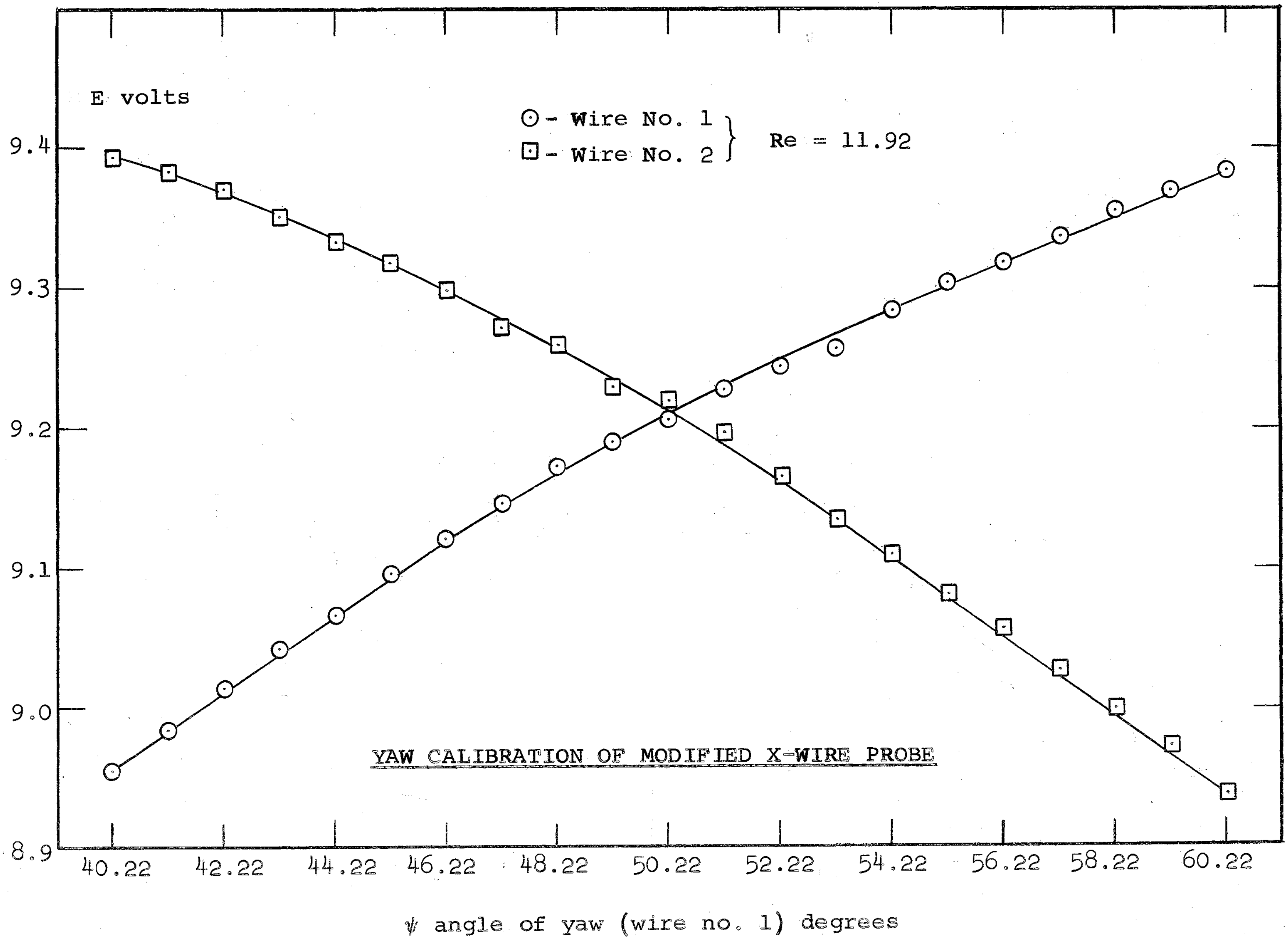
E^2 (volts)^2
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
E^2 = 18.2 + 9.86 U^0.4
CALIBRATION OF MODIFIED X-WIRE PROBE
U^0.4 (ft/s)^0.4

Fig. 14



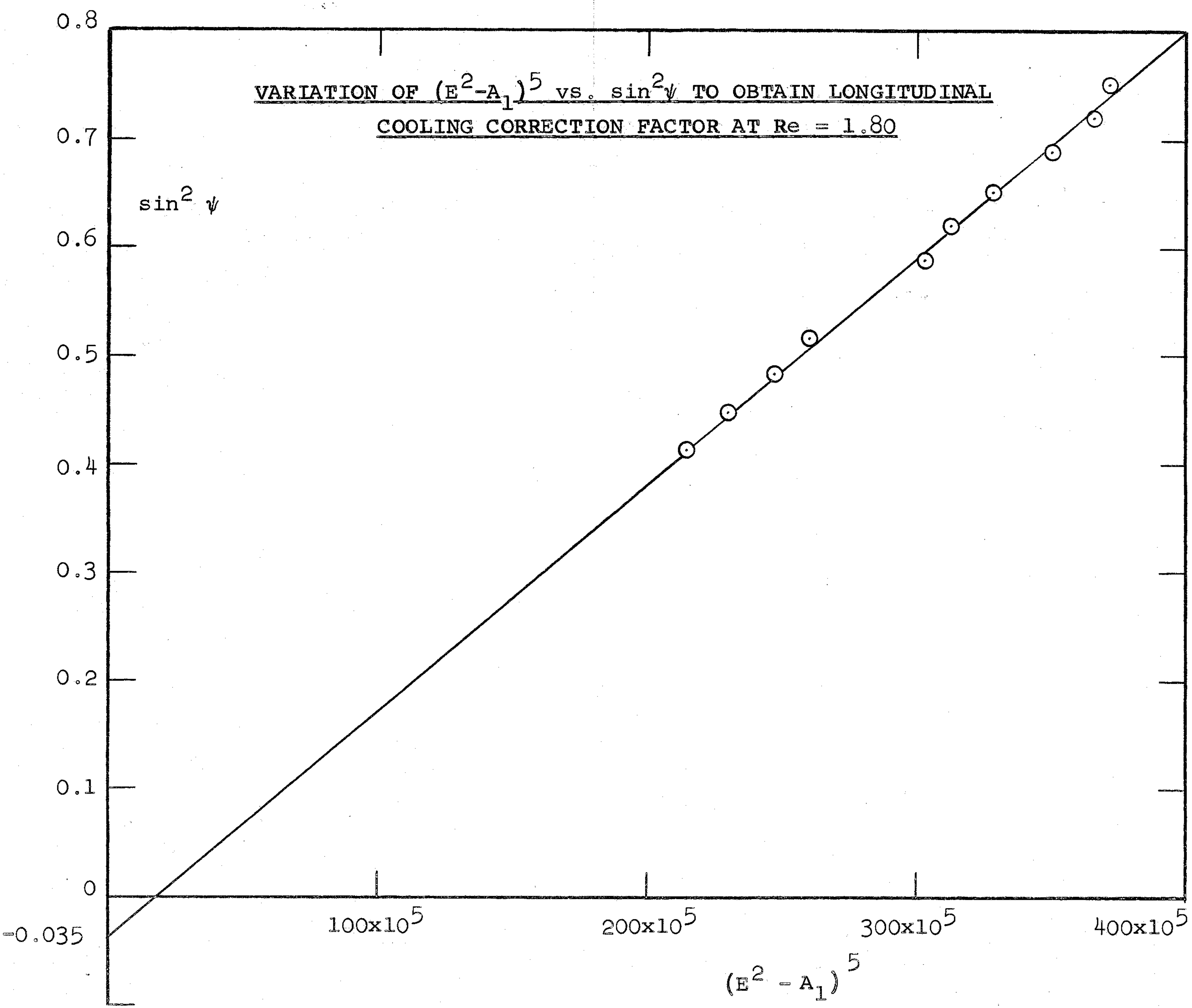
YAW CALIBRATION OF MODIFIED X-WIRE PROBE

Fig. 15



YAW CALIBRATION OF MODIFIED X-WIRE PROBE

Fig. 16



VARIATION OF $(E^2-A_1)^5$ vs. $\sin^2\psi$ TO OBTAIN LONGITUDINAL
COOLING CORRECTION FACTOR AT Re = 1.80
$\sin^2\psi$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.035
$100x10^5$
$200x10^5$
$300x10^5$
$400x10^5$
$(E^2 - A_1)^5$

Fig. 17