ON THE DEPOSITION OF SMALL PARTICLES FROM TURBULENT STREAMS

by

P.O. Rouhiainen and J.W. Stachiewicz

(4) Report No. 69-4

January 1969

(3) Department of Mechanical Engineering (5) [Research Lab.]
(1) McGill University
Montreal
Canada

FILED AND KARDEXED
AS M.E.R.L No. 69-4

On the Deposition of Small Particles from Turbulent Streams

by P. O. Rouhiainen* and J. W. Stachiewicz**

Abstract

It is shown in the paper that the two fundamental assumptions (that of equality of particle and fluid diffusivity and that of purely inertial coasting within the viscous sublayer) on which existing deposition models are based, are untenable under most actual conditions.

The concept of Stokes stopping distance, in particular, is shown to be invalid, since the effect of shear-flow-induced transverse lift force, which heretofore has been disregarded, is not negligible when considering the passage of a dense particle through the viscous sublayer. Due to the action of this lift force much lower radial velocities are required at the edge of the sublayer to insure particle deposition on the wall than would be the case if Stokes-drag were the only force present. This explains why deposition models based on Stokes stopping distance concept must resort to the use of unrealistically high radial velocities within the sublayer to insure agreement with experimental data.

* Shawinigan Engineering Company, Montreal, Canada

** Professor, Department of Mechanical Engineering, Montreal, Canada, Mem. ASME.

NOMENCLATURE

a	Particle radius, ft
D_{eq}	Equivalent diameter = 4 floorarea/wetted perimeter, ft
d_p	Particle diameter, ft
$E(n)$	Lagrangian Energy spectrum of velocity fluctuations
F_e	External force, lbs
f	Friction factor (.079 $Re^{-0.25}$ in turbulent flow)
n	Wave number, or frequency in cycles/sec
Re	Reynolds number = $U_{ave} D_{eq.} / \nu$
S	Stokes stopping distance = $\rho_p d^2 \, v/18\mu$, ft
S^+	Stokes stopping distance = $\frac{Su*}{\nu}$, dimensionless
U_{ave}	Average stream velocity, ft/sec
$u*$	Friction velocity = $U_{ave} \sqrt{f/2}$
u^+	Dimensionless velocity u/u^*
v	Velocity in the transverse direction, ft/sec
v'	R.m.s. turbulent fluctuating velocity in the y-direction, ft/sec
y	Distance from the wall, ft
y^+	Dimensionless distance = $yu*/\nu$

Greek Symbols

ϵ	Turbulent diffusivity, ft^2/hr
η	Ratio between amplitudes of oscillation of particle and fluid
k	Magnitude of velocity gradient du/dy, sec^{-1}
μ	Absolute viscosity, lb/ft hr
ν	Kinematic viscosity ft^2/hr
ρ	Density lb/ft^3
ω	Angular frequency, rad/sec

Subscripts

f	Fluid
p	Particle

ON THE DEPOSITION OF SMALL PARTICLES FROM TURBULENT STREAMS

Existing Deposition Models

The transfer of solid particles or liquid droplets from turbulent streams to channel walls has been studied by a number of investigators both theoretically and experimentally. The theoretical treatment has generally been based on the diffusion model, in which particle flux is expressed in terms of particle diffusivity and concentration gradient. Thus,

$$N = (D + \epsilon_p) \frac{dc}{dy} \qquad \text{...(1)}$$

For very small particle sizes (in the sub-micron range) both the molecular diffusivity D and the turbulent eddy diffusivity ϵ_p must be considered, such as in the work of Lin, Moulton and Putnam (1), and Beal (2). For larger particle sizes, D is much smaller than ϵ_p and can be neglected. This is the model employed by Friedlander and Johnstone (3, 4) and Davies (5).

The first difficulty that must be overcome in order to solve this equation is the determination of particle diffusivity. Here all the authors use the same basic assumption that the diffusivities of the particles and of the fluid are identical. This assumption is reasonable for sub-micron particles, (as in Ref. (1)), but becomes questionable for particle sizes of about 1μ and is shown to be completely untenable for 30μ particles (such as those used in Ref. (3)) as will be demonstrated in detail in the present paper.

Once particle diffusivities (ϵ_p) are equated to fluid diffusivities (ϵ_f), expressions for the latter must be found before Eq. 1 can be integrated.

If the classical approach of subdividing the boundary layer into three zones (laminar, buffer and turbulent) is followed, ϵ_f in the laminar sublayer should be set to zero.

Lin et al found, however, that when this was done, and Eq. 1 was integrated to obtain the deposition velocity coefficient K, the theoretical results fell considerably below the experimental ones.

In order to overcome this discrepancy they proposed the following variation of ϵ_f in the laminar sublayer :

$$\frac{\epsilon_f}{\nu} = \left[\frac{y^+}{14.5}\right]^3 \qquad \ldots(2)$$

This is a purely empirical relation used for the very pragmatic reason that "it produces the best correlation" between their model and the experimental results.

Davies (5), who did not assume a laminated structure for the boundary layer, also derived an empirical equation for ϵ_f based on his experimental results : This equation holds for values of y^+ up to 500 (at pipe Re = 10^4) and gives the same order of magnitude of ϵ_f/ν in the laminar sublayer as Eq. 2 (at $y^+ = 5$ Eq. 2 gives 0.041 while Davies Eq. gives 0.08.)

The model of Friedlander and Johnstone's (4) is similar to Lin's et al, except that the molecular diffusivity D was assumed zero not only in the turbulent core but also in the laminar and buffer layers, which is reasonable for particles of more than 1μ diameter.

According to Friedlander's model eddies carrying the particles diffuse from the turbulent core to within the stopping distance of the wall, from where the particles coast to the wall by virtue of their inertia. This stopping distance depends of course on the initial velocity of the particle and is calculated by assuming Stoke's drag relation for a sphere moving in a stagnant fluid.

The initial velocity is imparted to the particle by the transverse velocity fluctuations of the gas which carries it. The higher this fluctuating velocity the larger the stopping distance. Unfortunately, unreasonably high initial velocities are required for the stopping distance to exceed the thickness of the laminar sublayer. Friedlander used a figure of 0.9u^* which, according to Laufer exists at a distance of $y^+ = 80$ from the wall, or well into the turbulent core of the boundary layer. Even then the stopping distance was in most cases smaller than the thickness of the laminar layer.

If there were no eddies in the laminar layer no deposition should thus occur since no particles could diffuse to within one stopping distance of the wall. This conclusion leads Friedlander to employ in his analysis Lin's et al expression for ϵ_y in the sublayer (Eq. 2).

Thus according to this model, eddies from the turbulent core ($y^+ = 80$) penetrate the boundary layer and retain their momentum until they reach to within the Stokes stopping distance (S^+) of the wall (which could be as low as $y^+ = 1$ for 2μ particles.) In addition, a finite eddy diffusivity within the laminar layer must be assumed. If S^+ is calculated by using the actual value of v' at $y^+ = S^+$, transport coefficients are obtained which are some four orders of magnitude lower than those found experimentally by Friedlander and Johnstone.

In a more recent analysis Beal (2) discusses in detail the stopping distance assumption and notes that "while containing certain inconsistencies, (it) is the only one of several investigated whose results agree reasonably with the experimental data". He uses the same assumption himself. In addition, in order to calculate the deposition flux, he assumes that the radial velocity of the particles within the laminar and buffer layer is equal to half the axial velocity of the fluid at a given point y^+ from the wall. He admits this to be a somewhat unrealistic assumption and agrees that the radial velocity is probably much lower than the axial but unless such an assumption is made, analytical results again fall short of experimental data.

We are thus faced with the situation that in the best deposition models available today assumptions which cannot be defended on theoretical grounds and with which the authors themselves are somewhat unhappy, must be made in order to make the models agree with experimental data.

These assumptions will now be discussed in more detail and it will be demonstrated that the concept of Stoke's stopping distance loses much of its significance when all the forces acting on a particle traversing a laminar boundary layer are taken into account.

Relationship between eddy diffusivity of particle and fluid

This subject has received considerable attention in the past and Soo (6) presents an extensive discussion of it in Chapter 2 of his text, where an equation for ϵ_p/ϵ_f in terms of Lagrangrian and Eulerian microscales of turbulence and of particle Reynolds number is given.

In trying to obtain a quantitative evaluation of ϵ_p/ϵ_f we found it more convenient to use the concept of frequency response developed by Hjelmfelt and Mocros (7).

The Basset, Boussinesq, Oseen equation for motion of a particle in a turbulent fluid as extended by Tchen (8) to the case of a moving fluid is

$$\frac{\pi}{6} d_p^3 \rho_p \frac{du_p}{dt} = 3\pi\mu d_p (u_f - u_p) - \frac{\pi}{6} d_p^3 \rho_f \frac{du_f}{dt} + \frac{1}{2}\frac{\pi}{6} d_p^3 \rho_f \left(\frac{du_f}{dt} - \frac{du_p}{dt}\right) + \frac{3}{2} d_p^2 \sqrt{\pi \rho_f \mu} \int_{t_o}^{t} \frac{\left(du_f/dt' - du_p/dt'\right)}{\sqrt{t - t'}} dt' + F_e \quad \ldots(3)$$

This equation is not exact but is valid when the following relations hold (9)

$$\frac{d_p^2}{\nu} \frac{\partial u}{\partial x} \ll 1 \quad , \quad \frac{u_r}{d_p^2 \left(\frac{\partial^2 u}{\partial x^2}\right)} \ggg 1 \quad \ldots(3a)$$

Following Hinze's approach (10) we write the equation in a simplified form, neglecting the external force Fe

$$\frac{du_p}{dt} + a' u_p = a' u_f + b' \frac{du_f}{dt} + c' \int_{o}^{t} \frac{\left(du_f/dt' - du_p/dt'\right)}{\sqrt{t - t'}} dt' \quad \ldots(4)$$

Expressing u_p and u_f by their Fourier integrals, we have :-

$$u_f = \int_0^\infty (\xi \cos \omega t + \lambda \sin \omega t)\, d\omega$$
$$u_p = \int_0^\infty (\sigma \cos \omega t + \varphi \sin \omega t)\, d\omega \qquad \ldots(5)$$

Substituting Eqn's (5) into (4) yields :-

$$\sigma = [1 - f_1]\,\xi + f_2 \lambda$$
$$\varphi = -f_2 \xi + [1 + f_1]\,\lambda \qquad \ldots(6)$$

where,
$$f_1 = \frac{\omega\,(\omega + c'\sqrt{\pi\omega/2})(b' - 1)}{(a' + c'\sqrt{\pi\omega/2})^2 + (\omega + c'\sqrt{\pi\omega/2})^2}$$
$$f_2 = \frac{\omega\,(a' + c'\sqrt{\pi\omega/2})(b' - 1)}{(a' + c'\sqrt{\pi\omega/2})^2 + (\omega + c'\sqrt{\pi\omega/2})^2}$$

$$a' = \frac{36\mu}{(2\rho_p + \rho_f)\,d^2} \qquad b' = \frac{3\rho_f}{2\rho_p + \rho_f}$$
$$c' = \frac{18}{(2\rho_p + \rho_f)\,d}\sqrt{\frac{\rho_f \mu}{\pi}} \qquad \ldots(7)$$

At this point, following Hjelmfelt and Mocros (7) we introduce the concept of amplitude ratio and phase angle between particle and fluid motion and express the velocity as follows :-

$$u_p = \int_0^\infty \left\{ \eta \left[\xi \cos(\omega t + \beta) + \lambda \sin(\omega t + \beta) \right] \right\} d\omega \qquad \ldots(8)$$

where the amplitude ratio $\eta = \sqrt{(1+f_1)^2 + f_2}$

and the phase angle $\beta = \tan^{-1}\left[\frac{f_2}{1+f_1}\right]$ ...(9)

The amplitude ratio between particle and fluid fluctuations is thus seen to be primarily a function of the angular frequency ω of fluid fluctuations, as well as being a function of fluid and particle properties. Fig. (1) shows a plot of the amplitude ratio η against ω for the three types of particles investigated in this work.* It is evident that particle frequency response decreases from unity to zero within about two decades of ω.

To determine actual particle eddy diffusivity as a function of Reynolds number we must obtain relations between eddy diffusivity and amplitude ratio and between angular frequency and Reynolds number.

In terms of the Lagrangrian energy spectra, ($E_f(n)$ for fluid and $E_p(n)$ for particles) the ratio of eddy diffusivities is (Ref. 10, p. 359)

$$\frac{\epsilon_p}{\epsilon_f} = \frac{\int_0^\infty E_p(n)\,dn}{\int_0^\infty E_f(n)\,dn} \qquad ...(10)$$

where n is the wave number, or frequency in cps.

Using the result obtained by Tchen (8)

$$\frac{E_p(n)}{E_f(n)} = (1+f_1)^2 + f_2 \qquad ...(11)$$

it follows from Eq. (9) that

$$E_p(n) = \eta^2 E_f(n) \qquad ...(12)$$

from which we obtain

* These particular types were chosen because they were readily available for experimental work, and were in the right size range. Information on them is given in Appendix A.

$$\frac{\epsilon_p}{\epsilon_f} = \frac{\int_0^\infty \eta^2 E_f(n)\, dn}{\int_0^\infty E_f(n)\, dn} \qquad \ldots(13)$$

To calculate ϵ_p/ϵ_f as a function of Reynolds number we need information on the spectral energy distribution function $E_f(n)$ at various Reynolds numbers. Unfortunately, this information is extremely sparse. The most reliable measurements are probably those of Comte-Bellot (11) who, in Fig. IV-49, -51, -52 gives plots of energy spectra of the v' component in 2-dimensional flow at three different Reynolds numbers and at a distance $y/h = 0.45$ from the wall (where 2h is the height of the 2-dimensional channel.)

Using these figures and our Fig. (1) the product $\eta^2 E_f(n)$ can be obtained as a function of n and hence, by a numerical integration, Eq. (13) can be solved to yield ϵ_p/ϵ_f for the particles under consideration.

The results are plotted in Fig. (2) for 30μ lycopodium spores. The curve is admittedly not very accurate, being based on only three points measured at locations which differred somewhat in each of the three tests, but it does indicate that a fourfold increase in Reynolds number, which causes a more than fourfold increase in ϵ_f, results in less than a twofold increase in particle diffusivity, ϵ_p.

These results are based on measurements taken at a large distance from the wall ($1000 < y^+ < 4000$). Near the wall, higher frequency oscillations contribute more to the value of $\int_0^\infty E_f(n)dn$ (Fig. IV-33 of Ref. 11) and hence the amplitude ratio η and consequently ϵ_p/ϵ_f will be even smaller than calculated here.

With 1.7μ iron particles, η is near unity for frequencies as high as 10^5 rads/sec. Reference (11) (Fig. IV-33) shows that the whole energy spectrum of v' oscillations lies within these frequencies (even near the wall) and in this case the assumption of $\epsilon_p/\epsilon_f = 1.0$ will not cause serious errors.

Particle Motion within the Laminar Sublayer

Friedlander and Johnstone (3) (4) as well as Beal (2) had to resort to the use of unreasonably high radial velocities within the boundary layer when they calculated the stopping distance, in order to get their model to agree with experimental data. This suggests that pure inertial coasting may not be the only means by which particles are transported across the laminar sublayer.

In viscous motion through a stagnant fluid in the absence of external forces, the Stokes drag is indeed the only force acting on the particle and this forms the basis for the calculation of Stokes stopping distance.

The laminar sublayer, however, is anything but stagnant. On the contrary, it is characterised by very steep velocity gradients. Now while it is a well known fact that a spinning sphere (or cylinder) in a moving fluid experiences a lateral lift force due to its rotation (Magnus effect) it is perhaps not so generally appreciated that a similar effect is present when a sphere moves through a viscous fluid in shear flow. Such effects were clearly demonstrated, however, by a number of investigators. Segre and Silberberg (12) observed that small, neutrally bouyant spheres in Poiseuille flow through a tube slowly migrated laterally away from the wall to a position 0.6 tube radii from the axis.

Karnis, Goldsmith and Mason (13), Denson, Christiansen and Salt (14) and Jeffrey and Pearson (15) all report a radial migration of particles, the latter stating that particles denser than the fluid migrate towards the wall in downward flow, and towards the center of the tube in upward flow.

Most researchers who considered the possibility of the shear-flow lift causing radial migration were reluctant to accept it on the grounds that the magnitude of the lift force was too small to produce such an effect.

Let us examine in more detail the magnitude of the forces involved. We shall use for this purpose the expression derived by Saffman (16) for the lift force experienced by a small sphere moving in an unbounded viscous shear flow :

$$F_L = \frac{K \mu V a^2}{\nu^{1/2}} \left[\frac{du}{dy}\right]^{1/2} \qquad \ldots(14)$$

where U is the difference between the velocity of the particle and the fluid, du/dy is the velocity gradient in the infinite shear flow, and K = 81.2 is a constant obtained from a numerical evaluation of an integral.

As a sufficient condition for validity of Eq. 14, Saffman stipulates

$$Re_U \ll Re_k^{1/2} \quad , \quad Re_k \ll 1 \quad , \quad Re_\Omega \ll 1$$

where the three Reynolds numbers are defined as follows :

$$Re_U = \frac{Ua}{\nu} \quad , \quad Re_k = \frac{a^2}{\nu}\frac{du}{dy} \quad , \quad Re_\Omega = \frac{\Omega a^2}{\nu}$$

(Ω is the rotational speed of the particle which, for free rotation, is equal to $\frac{1}{2}$ du/dy).*

If we compare the magnitude of the shear flow lift force (Eq. 14) to Stoke's drag force ($F_s = 6\pi\mu a V$) we obtain

$$\frac{|F_L|}{|F_s|} = \frac{K}{6\pi}\left[\frac{a^2}{\nu}\frac{du}{dy}\right]^{1/2} \frac{U}{V} \qquad \ldots(15)$$

* Validity of application of Eq. 14 to the present case is discussed in Appendix B.

Both U and V represent particle velocities relative to mean stream velocity and if their ratio is taken as unity, Eq. 15 reduces to

$$\frac{|F_L|}{|F_s|} = \frac{K}{6\pi} Re_k = 4.3\, Re_k$$

For $Re_K \ll 1$ which is one of the conditions of validity of Eq. 14, the lift force is indeed negligible compared to frictional drag, which is the conclusion reached by Soo, (Ref. 6, p. 28.)

If we apply the argument to the passage of the particle through the laminar sublayer, however, V will represent the transverse (radial) velocity which, very near the wall, must necessarily be very small and thus, even though the relative axial velocity U is not large in that region, the ratio U/V may be significant and thus the lift force may have a measurable effect on particle motion.

To verify this contention, we write the equations of motion for a dense particle moving in the laminar sublayer when the main flow is directed vertically upwards ;

$$-6\pi\mu a(u_p - u_f) - \frac{4}{3}\pi a^3(\rho_p - \rho_f)g = \frac{4}{3}\pi\rho_p a^3 \frac{du_p}{dt} \qquad \ldots(16a)$$

$$-6\pi\mu a v_p - K\mu(u_p - u_f)\left(\frac{du_f}{dy}\right)^{1/2}\frac{a^2}{\nu^{1/2}} = \frac{4}{3}\pi\rho_p a^3 \frac{dv_p}{dt} \qquad \ldots(16b)$$

Eq. 16a is a force-balance on the particle in the direction of flow (x-direction), x being taken as positive upwards. Eq. 16b is the force balance in the y-direction, y being taken as zero at the wall, increasing positively towards the centerline of the channel.

Subscripts p and f refer to particle and fluid respectively. The first terms in both equations are Stokes friction force terms. The second term in Eq. 16a is the gravitational term, negative for upward flow which was assumed here.

The second term in Eq. 16b is Saffman's shear-flow lift term, which is negative (towards the wall) when the particle velocity in the x-direction (u_p) is greater than the local stream velocity u_f.

In the sublayer the fluid velocity gradient can be taken as constant and hence

$$u_f = y\frac{du_f}{dy} = yk$$

where the magnitude k of the velocity gradient can be obtained from

$$k = \frac{du_f}{dy} = \frac{u^{*2}}{\nu}\frac{du^+}{dy^+} = \frac{u^{*2}}{\nu} = \frac{U_m^2}{\nu}\frac{f}{2}$$

Equations (16) were solved numerically using a predictor corrector method described in Ref. (17). Details of the program are given in Ref. (18).

It was assumed that the value of u_p at the edge of the sublayer ($y^+ = 5$) was equal to the local mean fluid velocity minus the velocity of gravitational settling, and different negative values of v_{p1} were tried at $y^+ = 5$.

Since the flow is upwards, u_p is lower than u_f and the lift force is directed towards the centerline. Thus the particle, whose initial transverse velocity v_{p1} causes it to start moving towards the wall, will decelerate because of the additive effect of lift and Stoke's drag.

If the initial velocity v_{p1} is low, the particle will soon reverse directions and start moving away from the wall.

If v_{p1} is high enough, however, the particle will be carried sufficiently close to the wall, so that it enters the region in which its velocity in the direction of flow (u_p) becomes greater than the local fluid velocity u_f.

Under these conditions the lift force is directed towards the wall and the particle will start accelerating in that direction. This is vividly demonstrated in Fig. 3 where the transverse velocity v is plotted against the distance from the wall for the 32μ lycopodium particle.

For this particular flow condition, an initial velocity $v_{p_1} = -0.0227$ ft/sec was sufficient to cause the particle to travel to the wall. This is a threshold value; a very small decrease in it would cause the particle to reverse directions and to return to the main stream.

By contrast, an initial velocity $v_{p_1} = -0.344$ ft/sec (or about 15 times the previous value) would be required for the particle to reach the surface if Stoke's drag was the only force acting on it in the sublayer.

With the initial velocity of $v_{p_1} = -.0.0227$ fps, the Stokes stopping distance is 0.435×10^{-4} ft, so that the particle would be able to penetrate only about 7% of the laminar sublayer thickness if Stokes drag was the only force acting on it.

Thus it appears that in the laminar sublayer the assumption of negligible shear flow lift is not valid and therefore the whole concept of the Stokes stopping distance becomes questionable.

The effect of the lift force was examined in some detail to determine its dependence on such factors as particle size and density and the flow Reynolds number. Results are presented for upflow of dense particles in terms of the following parameters :-

a) threshold velocity, v_{p_t} : the minimum velocity towards the wall which the particle must possess at the edge of the sublayer so that it will be deposited on the wall when the effect of the lift force is included.

b) Stokes-drag velocity, v_{p_s} : the minimum velocity towards the wall which the particle must possess at the edge of the sublayer so that it will be deposited on the wall when the effect of Stokes-drag only is considered.

Effect of Particle Size

Fig. 4 shows a plot of both v_{p_t} and v_{p_s} for various particle sizes as a function of Reynolds numbers.

At <u>low Reynolds numbers</u> (Re < 4000) the threshold velocity is the same as the Stokes-drag velocity for small particles, indicating that the effect of the lift force on the particles is indeed negligible. This is as expected since both the particle radius "a" and the velocity gradient du/dy are small so that F_L is very small (Eq. 14). As the size of the particle increases, the lift force which varies as the radius squared, increases more rapidly than the drag which varies linearly with radius. The inertia of the particle increases even more rapidly, however (varying with the cube of the radius), so that the particle is able to penetrate sufficiently far into the boundary layer for the lift force to reverse its direction and to start accelerating the particle towards the wall. With the lift force now operating to assist deposition a lower velocity is needed than if Stokes-drag was the only force acting on the particle and this is evident in Fig. 4 for particles greater than 8μ in diameter.

<u>As the Reynolds number increases</u>, du/dy becomes steeper and the particle reaches much more quickly the region where the direction of the lift force is reversed. This helps deposition and thus the threshold velocity of small particles drops several orders of magnitude within a relatively narrow Reynolds number range. This effect is much less pronounced with larger particles and the curve for the 32μ particles suggests that it may disappear completely with larger sizes. This again is reasonable because with large particles their inertia becomes the dominant factor and although the lift force is operating its effect is relatively small so that there is not much difference between v_{p_t} and v_{p_s}.

<u>At high Reynolds numbers</u> the small particles benefit most from the effect of the lift force. Since the boundary layer is very thin, the distance that the particle must travel into the sublayer for the lift force to reverse directions is very small and although that force is not very large (a^2 being very small though $du/dy^{1/2}$ is large), the particle begins to accelerate towards the wall at a very early stage of its travel through the sublayer. Thus a very low threshold velocity is required for

deposition, while a relatively high Stokes-drag velocity would be needed because of the small inertia of the particle.

This is most vividly demonstrated in Fig. 5 where the ratio of the threshold velocity to Stokes-drag velocity is plotted. At high Reynolds numbers small particles need less than 1% of Stokes-drag velocity in order to traverse the sublayer. Large particles, however, whose large inertia enables them to reach the wall without difficulty even without the assistance of the lift force, benefit but little from its presence. As a matter of fact, the ratio v_{p_t}/v_{p_s} exceeds unity for the 32μ particle at large Reynolds numbers. This again is logical, for both "a" and du/dy are now very large and the lift force acts so effectively that it tends to return the particle to the main stream, so that the threshold velocity must be increased considerably to prevent this from happening.

An interesting result is obtained when data of Fig. 5 are cross-plotted to show the dependence of v_{p_t}/v_{p_s} on particle diameter at constant Reynolds number. There appears to exist a definite particle size which is most affected by the action of the lift force at any given Reynolds number. (See Fig. 6).

At low Reynolds numbers, this optimum size is fairly large (~16μ) because the large particle diameter produces a large lift force and the large inertia helps to overcome the initial deleterious effect of the lift.

At high Reynolds numbers, on the other hand, small particles benefit most from the action of the lift force because with their low inertia they would be stopped very quickly if drag force alone acted on them.

Effect of Particle Density

Fig. 7 shows the effect of particle density on the ratio v_{p_t}/v_{p_s}. It can be seen that, except at low Reynolds numbers, the light particles benefit most from the lift force.

This is understandable since, without the help of the lift force, the low inertia of these light particles prevents them from penetrating the sublayer to any great depth. This effect is not so pronounced at low Reynolds numbers, because here again, the magnitude of the lift force becomes small (because of small du/dy) and viscous drag predominates so that v_{p_t}/v_{p_s} will tend to approach unity.

Wall Impact

There is another conclusion which can be drawn from this analysis which is highly pertinent to the study of particle deposition from turbulent streams.

According to the Stokes stopping distance concept the particles approach the wall with an exponentially decaying transverse velocity, so that many of them would just barely reach the surface and might tend to skip or roll along it without being firmly deposited on it even at low stream velocities.

Thus the efficiency of collection would become a problematic quantity. Most researchers, however, have not found this to be the case except at high Reynolds numbers, and not even then if the surface was coated with an adhesive substance. To verify this point, we observed under a microscope the surface on which particles were being deposited. The particles seemedto land with a definite impact and to embed themselves to a considerable depth in the adhesive with which the surface was coated. This behaviour would not be likely with an exponentially decaying approach velocity.

A somewhat similar phenomenon was observed by Cousins and Hewitt (19) who report photographic studies in two-phase upflow which indicate that droplets "reach the surface with high velocities" without being "appreciably slowed as they enter the region of low gas velocity adjacent to the surface." They ascribe this to the fact that the stopping distance for the large droplet sizes they investigated is very large and therefore, "the boundary sublayer is irrelevant in the type of mass transfer considered" by them.

In many of their tests, however, the mean droplet diameter was no greater than 50μ, with 70% to 80% of the total number of droplets being less than 25μ in diameter. Since their photographic tests were made with water droplets suspended in air flowing at room temperature and pressure, the latter being also the conditions assumed for calculation of Fig. 7, the 1.0 gm/cc density curve in Fig. 7 (calculated for 20μ particles) can be used to obtain an estimate of the effect of lift on the particles photographed by Cousins and Hewitt. This effect is seen to be substantial, the threshold velocity being about an order of magnitude lower than the Stokes-drag velocity for all Reynolds numbers, thus indicating that the lift force effect may well be responsible for the observed large velocities of at least the smaller droplets.

It is, of course, realized that the presence of a wavy liquid surface in this case renders the whole concept of a laminar sublayer even more questionable than it already is, it is remarkable nevertheless that in a study of deposition on solid wavy surfaces (to be published shortly) the authors have found that many of the observed deposition phenomena can be explained in terms of the effects of the Saffman lift force.

Conclusions

The foregoing discussion indicates clearly that the two fundamental assumptions (that of equality of particle and fluid diffusivity and that of purely inertial coasting within the viscous sublayer) on which existing deposition models are based can be completely invalid under most actual conditions.

A simple method of calculating the dependence of the ratio of particle to fluid diffusivity on Reynolds number is outlined, based on the integration of turbulent energy spectra.

The concept of Stokes stopping distance is shown to be invalid under most flow conditions, since the effect of shear-flow induced transverse lift force, which heretofore has been disregarded, is not negligible when considering the passage of a dense particle through the viscous sublayer. Due to the action of this force much lower radial velocities are required at the edge of the sublayer to insure particle deposition on the wall than would be the case if Stokes-drag were the only force present. This explains why deposition models based on Stokes stopping distance concept must resort to the use of unrealistically high radial velocities within the sublayer to insure agreement with experimental data.

Acknowledgements

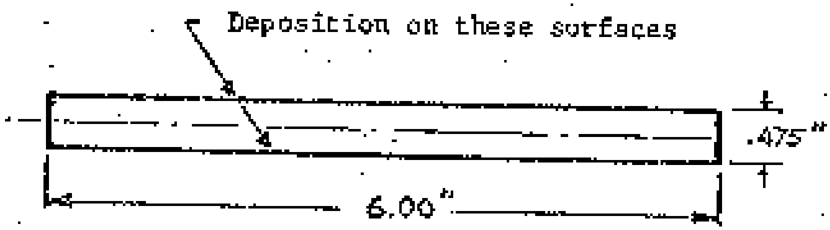
The work reported in this paper was performed under an Atomic Energy of Canada Limited research contract.

Appendix A

Properties of Particle used in Computation

	Lycopodium Spores	Glass Beads	Iron Particles
Shape	Spherical	Spherical	Spherical
Size, microns	1.69	32	32
Specific gravity	0.621	2.5	7.8

Configuration of Channel used in Computation



Equivalent diameter = 0.0734 ft.

APPENDIX B

a) Proximity of Wall

Saffman derived this equation for a particle in an unbounded flow. Therefore the particle must be far enough from the wall if the equation is to hold. Fig. 3 shows the comparative thickness of the sublayer and of a 32μ particle. It also shows that the most crucial effects of the lift force occur near the edge of the laminar sublayer, or sufficiently far away from the wall so that its effects may be neglected.

b) Magnitude of Reynolds Number

As mentioned in the main text, the first condition is $Re_u \ll Re_k^{1/2}$ or in another form

$$\frac{u_p - u_f}{(\nu k)^{1/2}} \ll 1$$

This quantity was calculated for the case of lycopodium spores and for a mean air velocity of 20 ft/sec at every step of the integration of Eq. 16.

It was found to have a maximum value of 0.05 at the edge of the sublayer, when v_{p_1} is near its threshold value. As the Reynolds number increases k increases also, reducing the value $(u_p - u_f)/(\nu k)^{1/2}$ even further. Thus the requirement of $Re_u \ll Re_k^{1/2}$ is well satisfied.

The second requirement was that $Re_k \ll 1$ and $Re_\Omega \ll 1$. Since for a freely rotating particle $Re_\Omega = 1/2\ Re_k$ we shall examine the behaviour of Re_k only.

Re_k was calculated for all types of particles used in this study. In Fig. 8 Re_k for 32μ and 1.69μ particles is plotted as a function of the duct Reynolds number. For the smaller particles, the Re_k requirement is well satisfied - for the larger ones, it holds reasonably well at lower Reynolds numbers.

c) Constancy of Velocity Gradient

This assumption holds in the sublayer as far as mean flow velocity is concerned. Any velocity fluctuations which may be present there would tend to make the quantitative results inaccurate, but qualitative conclusions would remain the same.

REFERENCES

1. LIN, C.S., MOULTON, R.W., PUTNAM, G.L., "Mass Transfer between Solid Wall and Fluid Streams," Ind. Eng. Chem. 45, p. 667, (1953).

2. BEAL, S.K., "Transport of Particles in Turbulent Flow to Channel or Pipe Walls," Bettis Atomic Power Laboratory, Pittsburgh, Pa., Westinghouse Electric Corporation Report, WAPD-TM-765, (1968).

3. FRIEDLANDER, S.K., "Deposition of Aerosol Particles from Turbulent Gases," Ph.D. Thesis, University of Illinois, Urbana, Illinois, (1954).

4. FRIEDLANDER, S.K., JOHNSTONE, H.F., "Deposition of Suspended Particles from Turbulent Gas Streams," Ind. Eng. Chem. 49, p. 1151, (1951).

5. DAVIES, C.N., "Deposition of Aerosols from Turbulent Flows Through Pipes," Proc. Roy. Soc. (London), 289 A, p. 235, (1966).

6. SOO, S.L., "Fluid Dynamics of Multiphase Systems," Blaisdell Publishing Company, Waltham, Mass., (1967).

7. HJELMFELT, A.T., Jr., MOCROS, L.F., "Particles in a Turbulent Fluid," Appl. Sci. Res. Vol. 16, p. 149.

8. TCHEN, C.M., "Mean Value and Correlation Problems Connected with the Motion of Small Particles Suspended in a Turbulent Field," Ph.D. Thesis, Delft, (1947).

9. CORRSIN, S., LUMLEY, J.L., "On the Equation of the Motion of a Particle in Turbulent Fluid," Appl. Sci. Res. Section A, Volume 6.

10. HINZE, J.O., "Turbulence," McGraw-Hill, New York, p. 352, (1959).

11. COMTE-BELLOT, G., "Ecoulement Turbulent Entre Deux Parois Paralleles," Publications Scientifiques et Techniques du Ministère de l'Air, No. 418, (1965).

12. SEGRE, G., SILBERBERG, A., "Behaviour of Macroscopic Rigid Spheres in Poisseuille Flow," J. Fluid Mech. 14, Part 1, p. 115; Part 2, p. 136, (1962).

13. KARNIS, A., GOLDSMITH, H.L., MASON, S.G., "The Flow of Suspensions through Tubes. V. Inertia Effects," Canadian J. of Chem. Eng., August (1966).

14. DENSON, C.D., CHRISTIANSEN, E.B., SALT, D.L., "Particle Migration in Shear Field," A.I.Ch.E. Journal, Vol. 12, No. 3, p. 589, (1966).

15. JEFFREY, R.C., PEARSON, J.R.A., "Particle Motion in Laminar Vertical Tube Flow," J. Fluid Mech., Vol. 22, Part 4, p. 721, (1965).

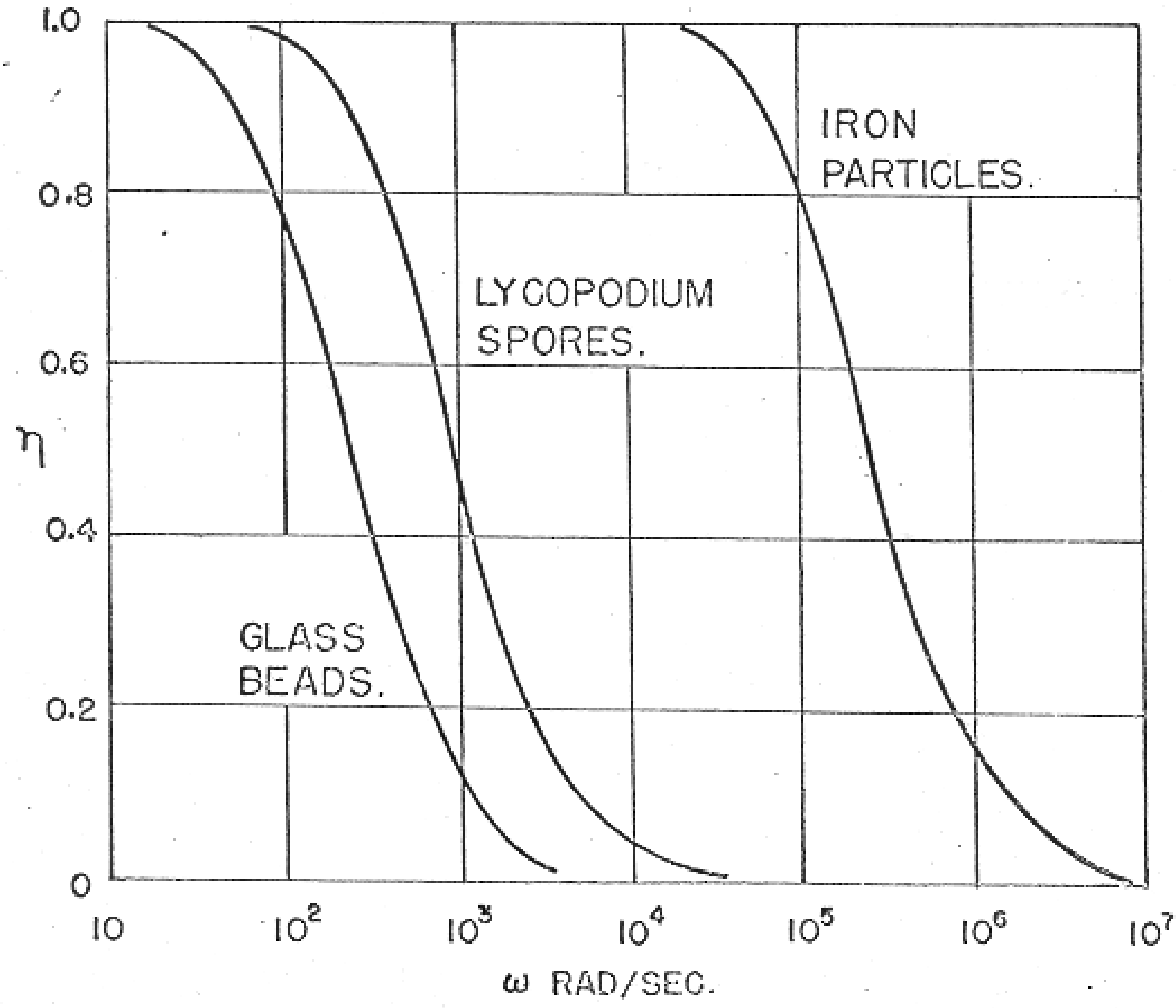
16. SAFFMAN, P.Q., "The Lift on a Small Sphere in a Slow Shear Flow," J. Fluid Mech. 22, p. 385, (1965).

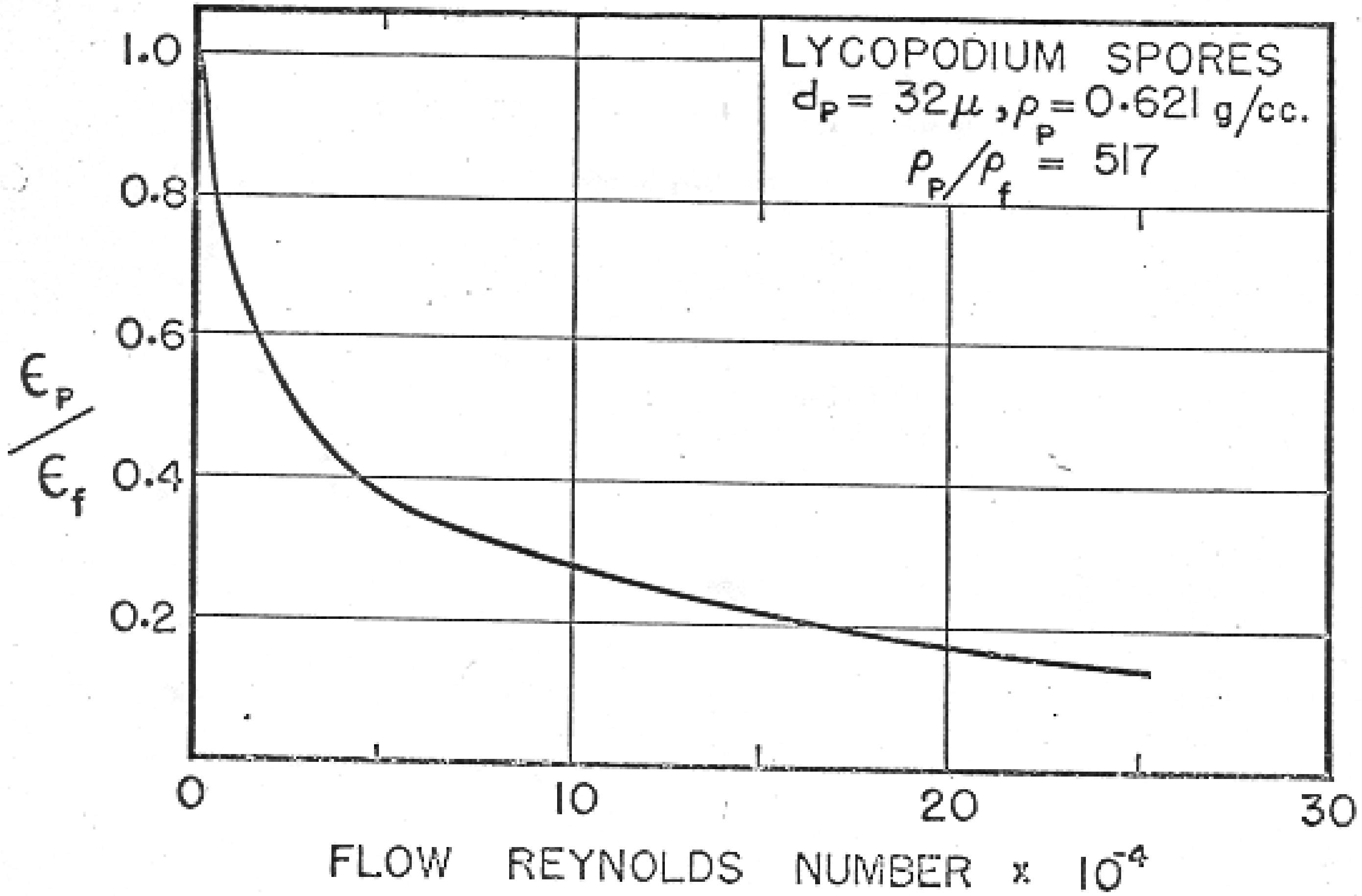
17. RALSTON, A., WILF, H.S., "Mathematical Methods for Digital Computers," John Wiley and Sons, Inc., New York.

18. ROUHIAINEN, P.O., "Deposition of Airborne Particles on Walls of Two-Dimensional Channels," M. Eng. Thesis, McGill University, (1969).

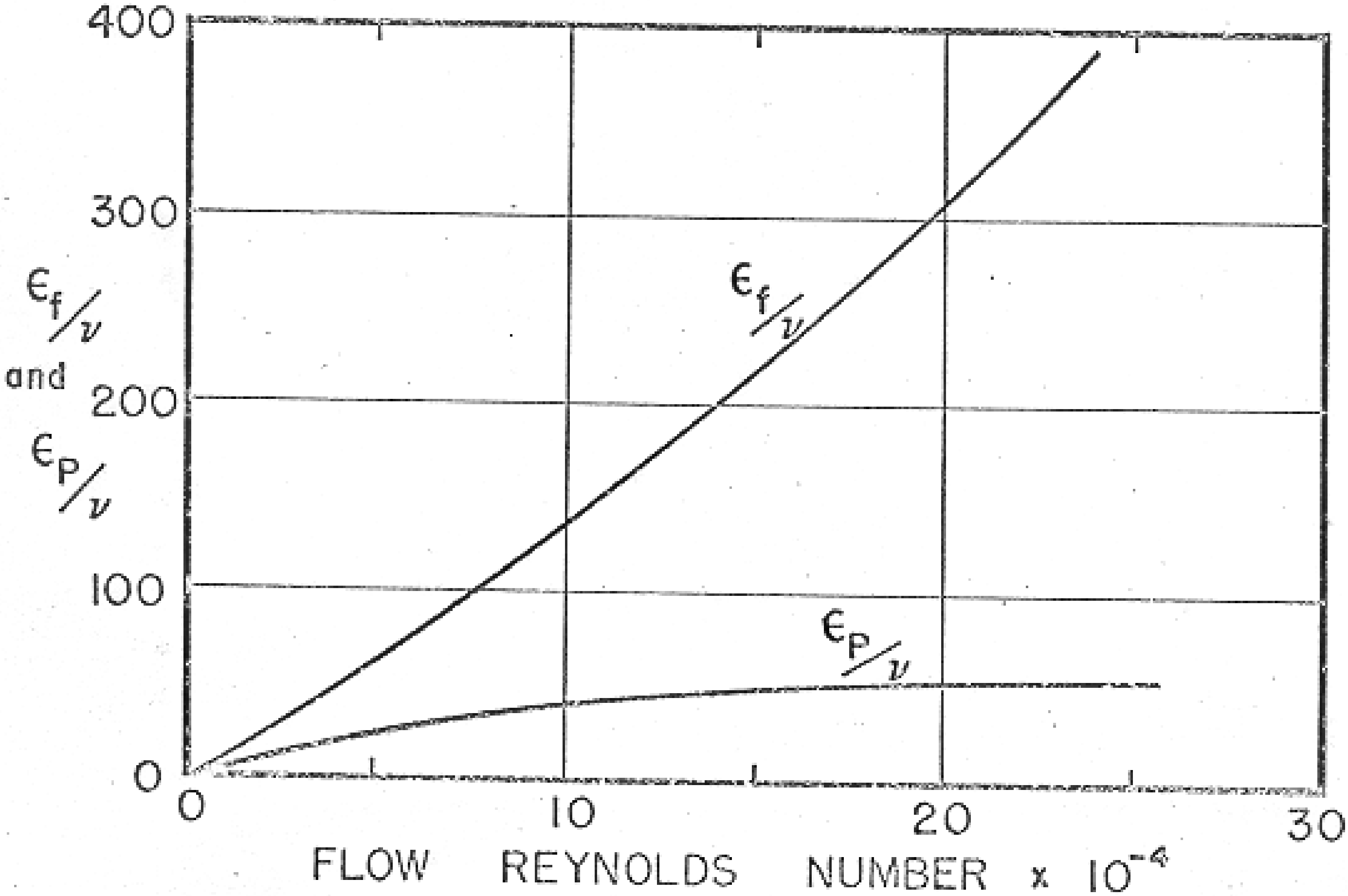
19. COUSINS, L.B., HEWITT, G.F., "Liquid Phase Mass Transfer in Annular Two-Phase Flow : Droplet Deposition and Liquid Entrainment," AERE-R 5657, U.K.A.E.A., Harwell, (1968).

LIST OF FIGURE CAPTIONS

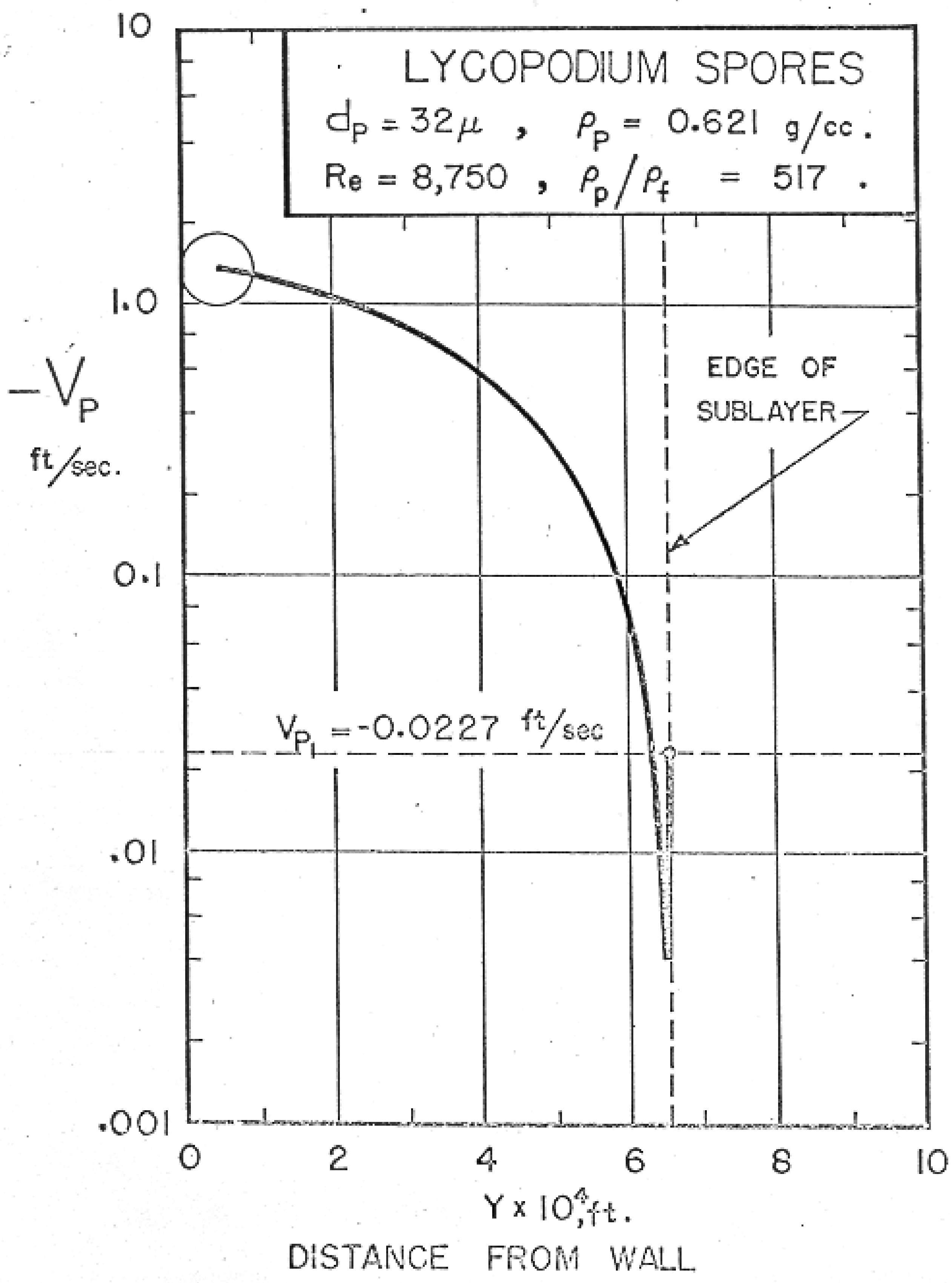

1.0
0.8
0.6
0.4
0.2
0
η
IRON
PARTICLES.
LYCOPODIUM
SPORES.
GLASS
BEADS.
10
10^2
10^3
10^4
10^5
10^6
10^7
ω RAD/SEC.



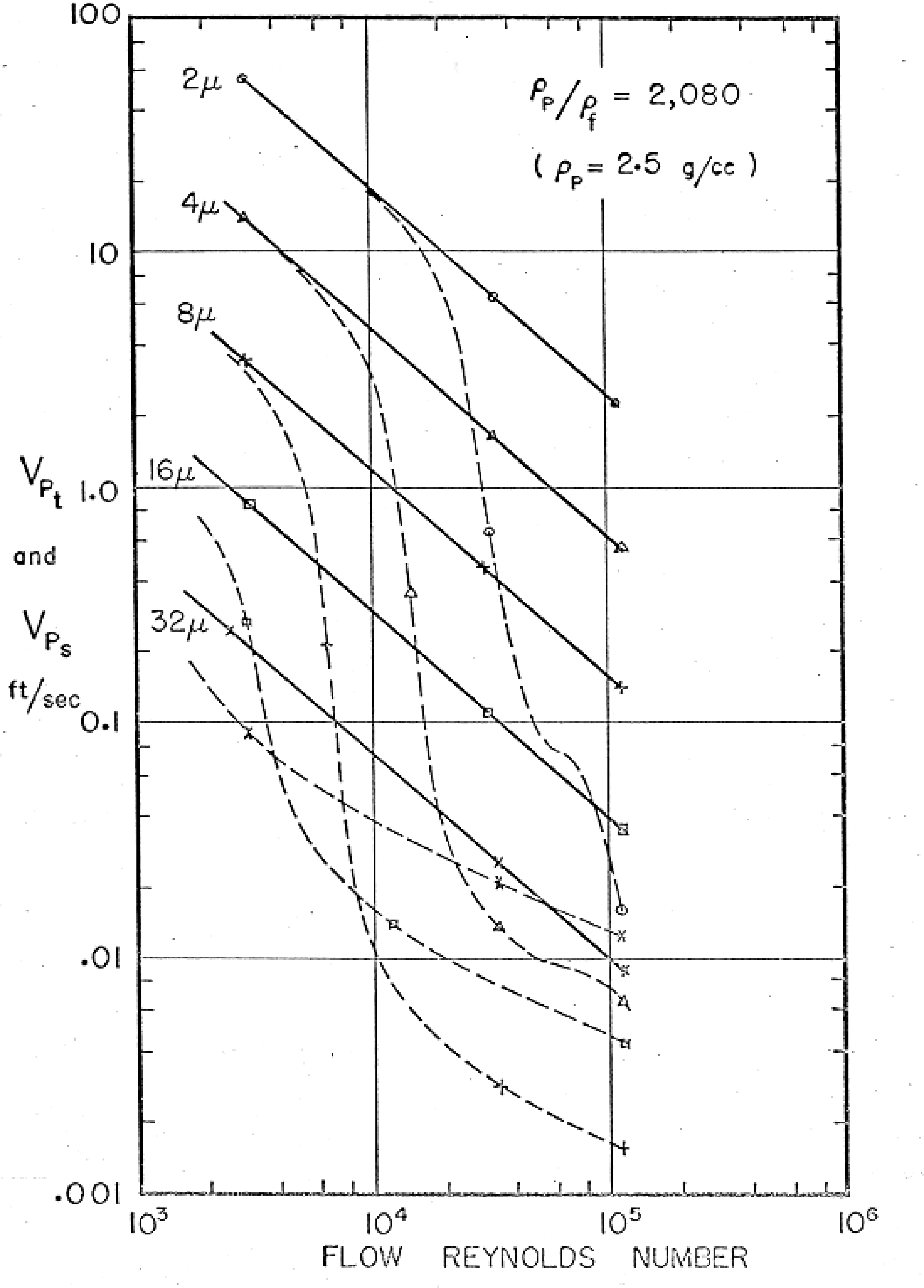
LYCOPODIUM SPORES
$d_P = 32\mu$, $\rho_P = 0.621$ g/cc.
$\rho_P/\rho_f = 517$
ϵ_P/ϵ_f
1.0
0.8
0.6
0.4
0.2
0
10
20
30
FLOW REYNOLDS NUMBER x 10^{-4}



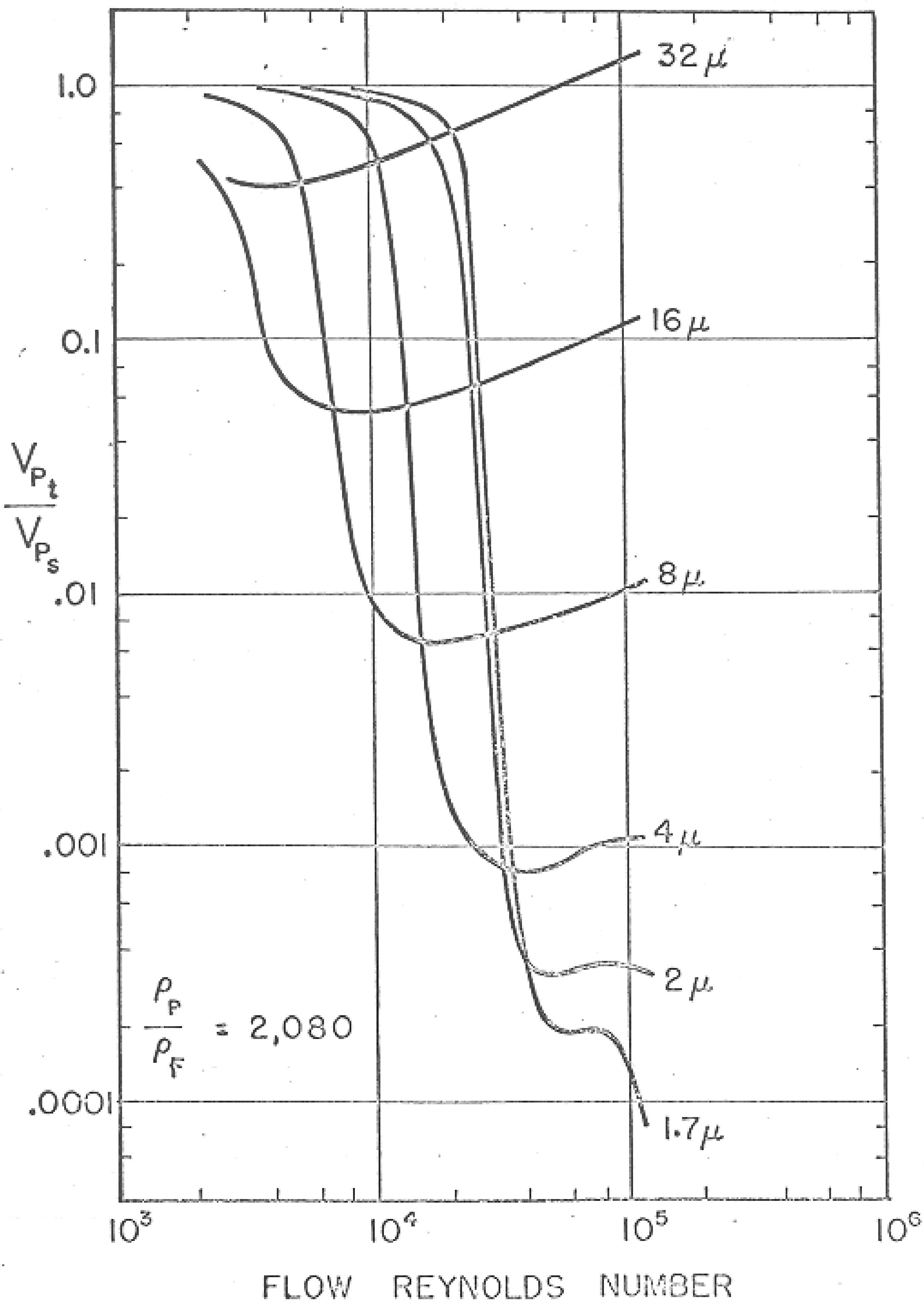
ϵ_f/ν and ϵ_P/ν
400
300
200
100
0
ϵ_f/ν
ϵ_P/ν
0
10
20
30
FLOW REYNOLDS NUMBER x 10^{-4}



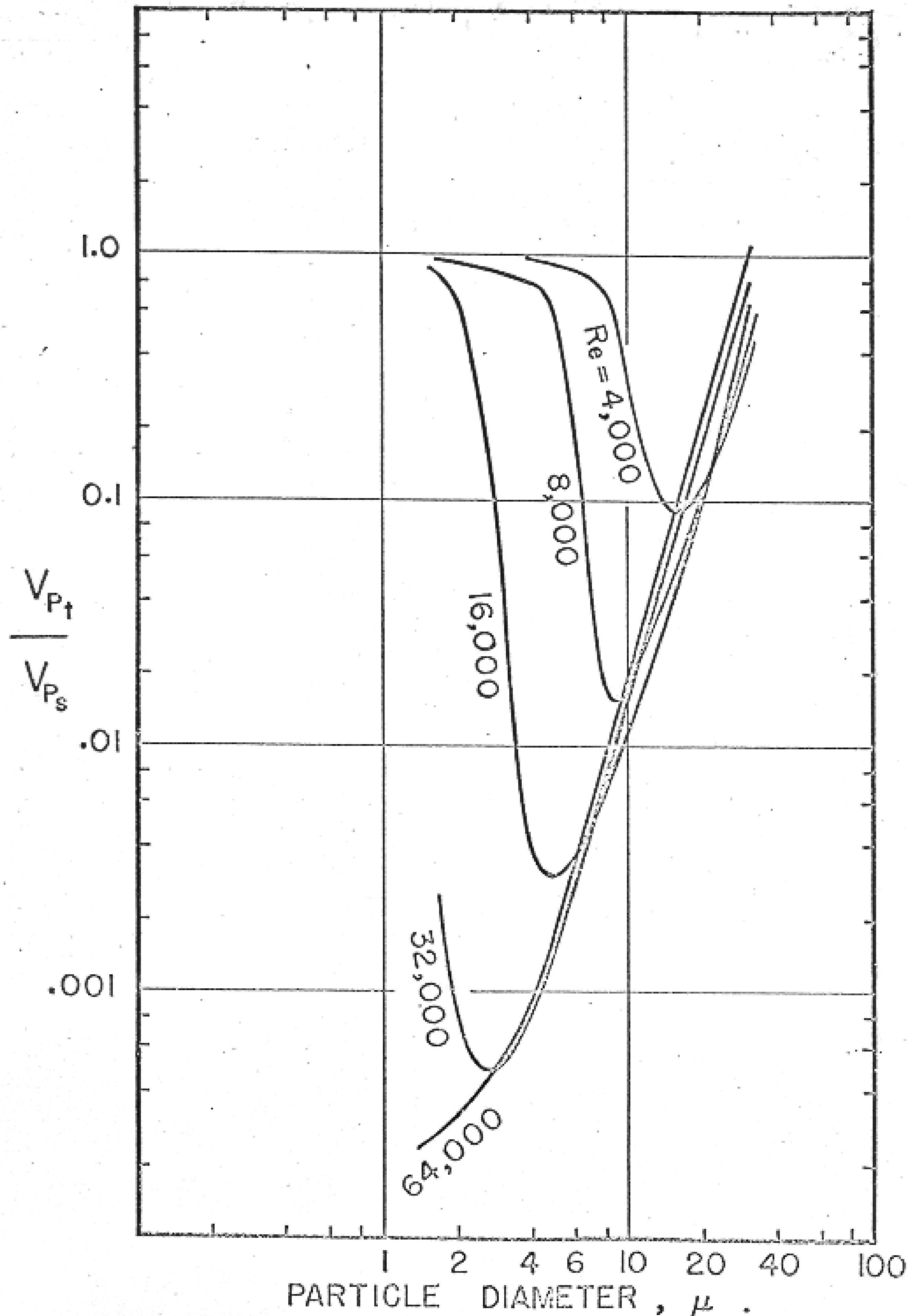
LYCOPODIUM SPORES
$d_P = 32\mu$, $\rho_P = 0.621$ g/cc.
Re = 8,750 , ρ_p/ρ_f = 517 .
EDGE OF SUBLAYER
$V_{P_1} = -0.0227$ ft/sec
$-V_P$ ft/sec.
10
1.0
0.1
.01
.001
0
2
4
6
8
10
Y x 10^4, ft.
DISTANCE FROM WALL

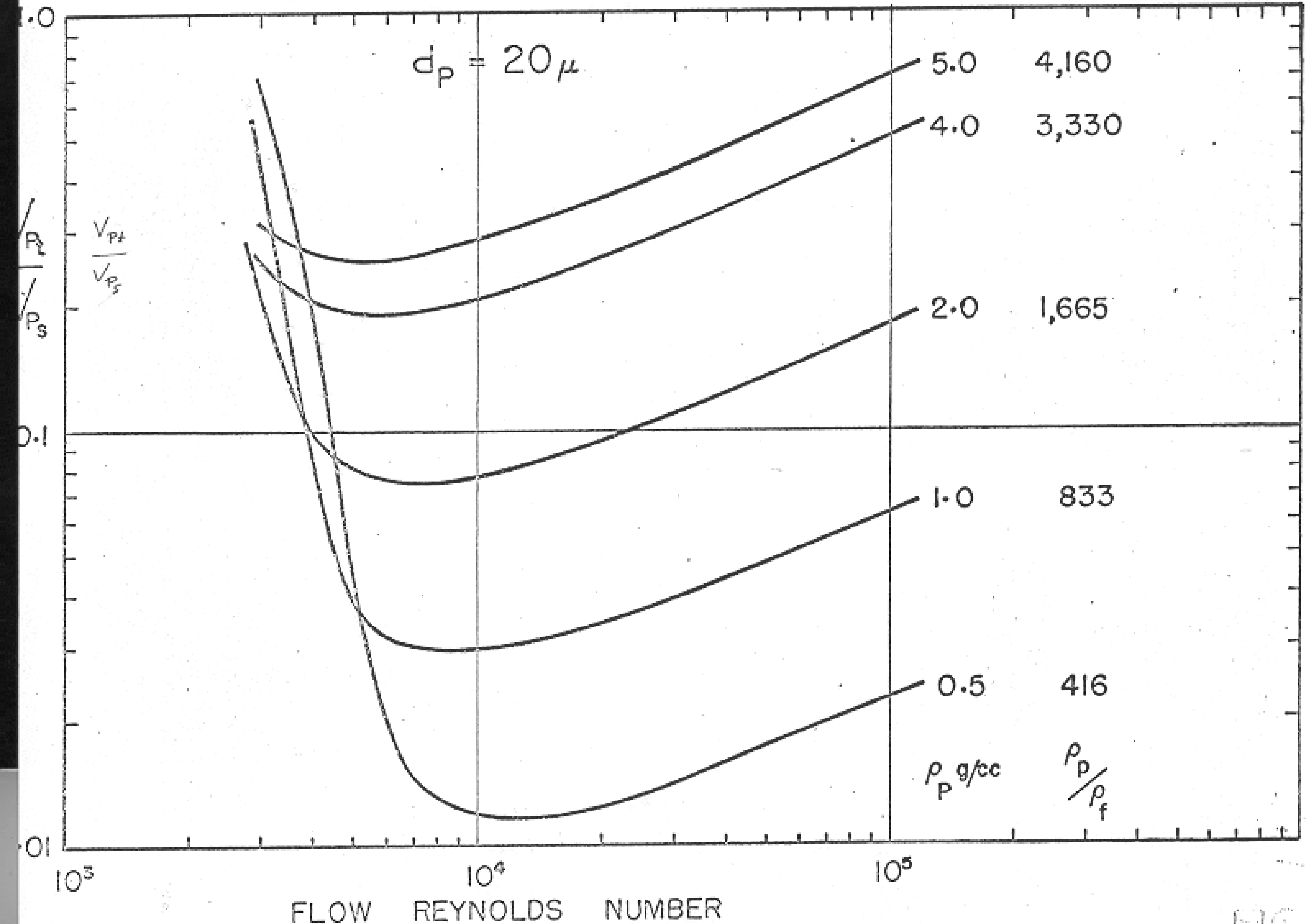


ρP/ρf = 2,080
(ρP = 2·5 g/cc)
2μ
4μ
8μ
16μ
32μ
VPt and VPs ft/sec
100
10
1.0
0.1
.01
.001
10³
10⁴
10⁵
10⁶
FLOW REYNOLDS NUMBER

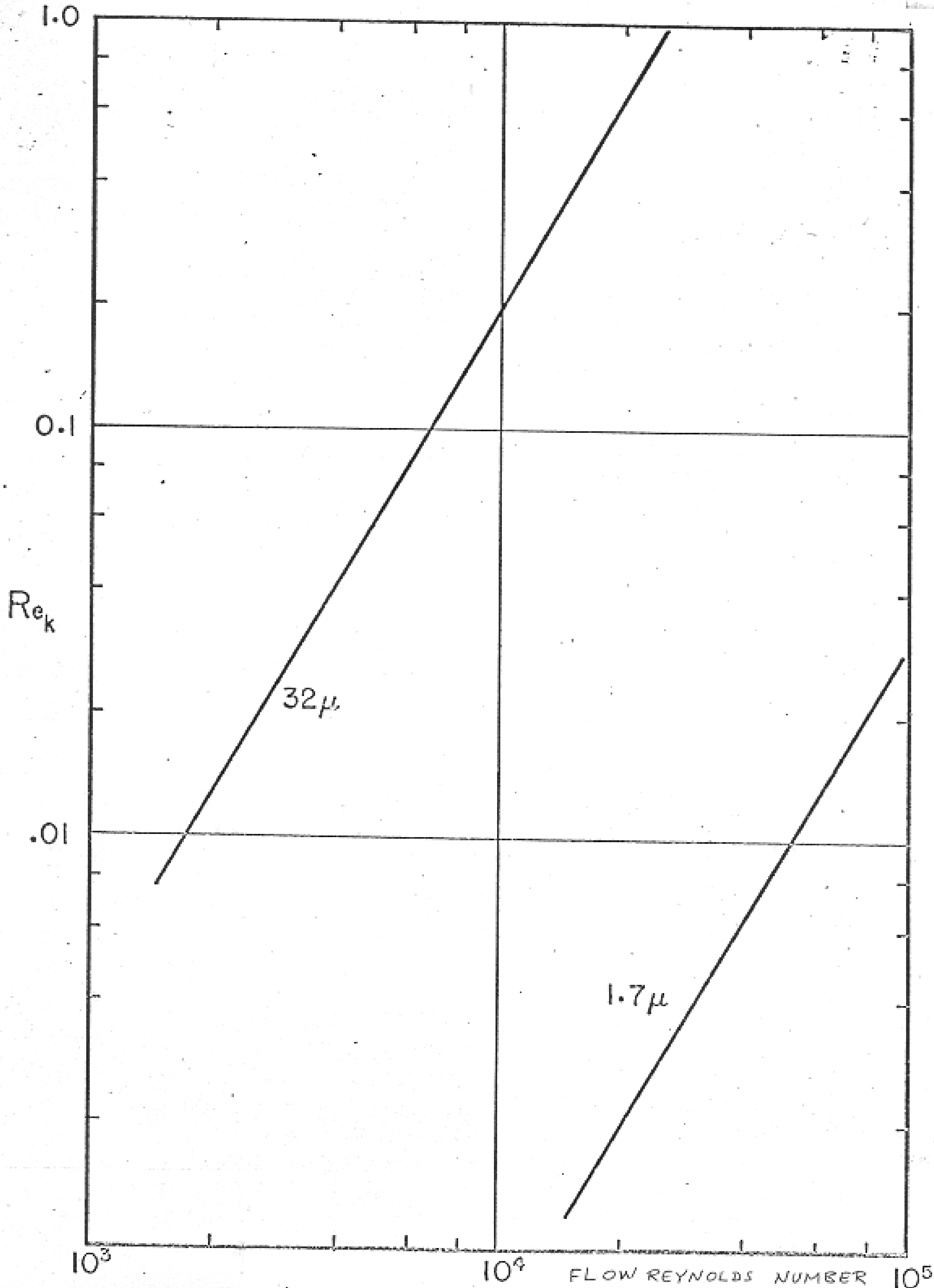


32 μ
16 μ
8 μ
4 μ
2 μ
1.7 μ
1.0
0.1
.01
.001
.0001
$\frac{V_{P_t}}{V_{P_s}}$
$\frac{\rho_P}{\rho_F}$ = 2,080
10^3
10^4
10^5
10^6
FLOW REYNOLDS NUMBER


1.0
0.1
.01
.001
V_{P_t} / V_{P_s}
Re = 4,000
8,000
16,000
32,000
64,000
1
2
4
6
10
20
40
100
PARTICLE DIAMETER, μ



$d_P = 20\mu$
5.0
4,160
4.0
3,330
2.0
1,665
1.0
833
0.5
416
ρ_P g/cc
ρ_P/ρ_f
1.0
0.1
.01
V_{Pt}/V_{Ps}
10^3
10^4
10^5
FLOW REYNOLDS NUMBER


1.0
0.1
.01
Re_k
32μ
1.7μ
10^3
10^4
10^5
FLOW REYNOLDS NUMBER