

Possible Consistency Requirement for Kinetic Exponents

During a prototypical first-order phase transition, one quenches a system from a high temperature, far above a critical temperature T_c , to a low temperature below T_c .¹ Domains of ordered phase form and grow to macroscopic size, as time goes on. For late times, an average domain of size R grows in time t via

$$R \sim t^n,$$

where n is the growth exponent. The driving force for the growth is the thermodynamic curvature of the interfaces separating domains from the surface free energy. In this Comment, I give an argument which relates n to the dynamical critical exponent z .

If one quenches close to T_c , time scales are affected by critical slowing down. Milchev, Binder, and Heermann² have shown that this can be incorporated above, using a simple scaling argument: near T_c , time scales must be measured in units of the correlation time τ and length scales in units of the correlation length ξ . This gives $R/\xi \sim (t/\tau)^n$. The dynamical scaling assumption is³ $\tau \sim \xi^z$. Thus²

$$R/t^n \sim (T_c - T)^{\nu n(z - 1/n)},$$

where ν is the correlation-length exponent. It is clear, however, that since the driving force for a first-order phase transition is due to thermodynamic forces, and not fluctuations, thermal fluctuations can only slow down domain growth, or leave it unaffected. Certainly, fluctuations will not speed up growth. Thus I obtain the inequality,

$$z \geq 1/n.$$

For model A , the nonconserved Ising model,³ the growth exponent has been found to be $n=1/2$, in dimension $d=2$ and 3, by many methods.¹ This gives $z \geq 2$. This is a good bound on z since it is found that $z \approx 2$ in two and three dimensions.⁴ For model C , the nonconserved Ising model with coupling to a conserved field,³ it is also found that $n=1/2$, so $z \geq 2$. This is consistent with the exact result,³ $z=2+\alpha/\nu$, where α is the specific-heat exponent, which is true for positive α . For negative α , models C and A have the same value for z .

For model B , the Ising model with a conserved order parameter, there is an exact result from critical dynamics³: $z=4-\eta$, where η is the correlation-function exponent. This gives

$$n \geq 1/(4-\eta).$$

This provides a good bound on n , whose value is controversial. In a recent Letter, Mazenko and Valls⁵ obtained $n=1/4$ in an analytic and numerical study of the Langevin model B , in two dimensions. While their numerical results are consistent with the bound, their analytic

results are not, since $\eta=1/4$ in two dimensions. It should be noted that the classical result for Ostwald ripening due to Lifshitz and Slyozov⁶ gives $n=1/3$, which is consistent with this bound.

The result should hold for kinetic Ising-type models, but not for systems such as models³ F and G which have a propagating critical mode near T_c .⁷ I expect the equality to hold as one approaches the lower critical dimension, which is $d=1$ for the Ising model, for then ordering dynamics and fluctuational dynamics should have the same strength. Note that $z \rightarrow 2$ for models A and C as $d \rightarrow 1$, which is consistent with $n=1/2$; while for model B , $\eta \rightarrow 1$, at the lower critical dimension,⁸ which is consistent with $n=1/3$.

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¹Reviews are given by, J. D. Gunton, M. San Miguel, and P. S. Sahni, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, London, 1983), Vol. 8; and K. Binder, *Physica (Amsterdam)* **140A**, 35 (1986).

²A. Milchev, K. Binder, and D. W. Heermann, *Z. Phys. B* **63**, 521 (1986).

³P. C. Hohenberg and B. I. Halperin, *Rev. Mod. Phys.* **49**, 435 (1977).

⁴For $d=2$, it is found numerically that $z \approx 2.125$ [see J. K. Williams, *J. Phys. A* **18**, 49 (1985)]. The $d-1$ expansion of the drumhead model, which is closely related to model A , by R. Bausch, V. Dohm, H. K. Janssen, and R. K. P. Zia, *Phys. Rev. Lett.* **47**, 1837 (1981), gives $z=2+(d-1)-\frac{1}{2}(d-1)^2+O(d-1)^3$. The ϵ expansion gives (Ref. 3)

$$z=2+0.01345(4-d)^2-0.02268(4-d)^3+O[(4-d)^4].$$

⁵G. F. Mazenko and O. T. Valls, *Phys. Rev. Lett.* **59**, 680 (1987); G. F. Mazenko, O. T. Valls and M. Zannetti, *Phys. Rev. B* **38**, 520 (1988).

⁶I. M. Lifshitz and V. V. Slyozov, *J. Phys. Chem. Solids* **19**, 35 (1961). In this regard, note that recent numerical studies of the Langevin model B are found to be more consistent with $n=1/3$ than $n=1/4$: T. M. Rogers, K. R. Elder, and R. C. Desai, *Phys. Rev. B* **37**, 9638 (1988); E. T. Gawłinski, J. Viñals, and J. D. Gunton (to be published); R. Toral, A. Chakrabarti, and J. D. Gunton, *Phys. Rev. Lett.* **60**, 2311 (1988).

⁷I thank D. A. Huse (private communication) for pointing out that the result probably does not apply to models F and G .

⁸For the values of α and η , see, for example, M. E. Fisher, *Rev. Mod. Phys.* **46**, 597 (1974).