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MULTI-COMMODITY MULTI-TRANSFORMED NETWORK FLOWS WITH AN APPLICATION TO RESIDUALS MANAGEMENT

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ABSTRACT

This paper presents a network model for the analysis of waste management systems. In the "waste management network" that we develop waste-generating activities correspond to sources, each node corresponds to a specific flow (waste) form, and each arc to a flow transformation; a chain from a source to a destination represents in turn waste generation, treatment, and disposal, while a feasible flow pattern constitutes a possible waste management system. The model resembles the maximal multi-commodity flow model with positive gains (single transformation) along the arcs, but in addition it admits multi-transformations, interrelations among arc flows, and flow-dependent budget restrictions. The problem is formulated as a linear program where each column in the constraint matrix corresponds to a chain in the network. The solution procedure uses a column-generation scheme based on a shortest route algorithm adapted for multi-transformed flows. Recycling, discharge and effluent standards, charges, damage costs and various budgeting schemes can be handled. An illustrative hypothetical example is given.

1. Introduction

The capacity of a site to environmentally sustain a number of waste generating activities (industrial, commercial, consumptive) is a function of its environmental characteristics, the desired environmental quality, and the management of the generated wastes. A solution to the waste management problem attempts to strike a balance among pollution levels, waste management budget, and benefits from (or levels of) the waste generating activities. In other words, it aims at an efficient allocation of the limited waste absorbing capacity of the site to the various activities on the basis of some merit, which could be either maximum level of generated benefits for a fixed budget or minimum waste management spending for fixed levels of activities operations. This allocation takes place via a waste management system consisting of rules and processes for generation control, treatment, and discharge of wastes. The many available rules and processes yield a large number of alternative systems. Most of the existing methodologies for choosing among such systems deal with specific types of wastes or particular forms of pollution, often leading to unbalanced environmental burdens (e.g. high water pollution along with very clear air). This paper suggests a total-system waste management model along with a model optimization procedure as a potential method for identifying the most efficient balanced waste management system and for developing trade-offs among benefits, pollution, and spending.

A total-system analysis requires assessment of the impact of each waste generating activity on (a) the total environment, (b) each phase of the waste management effort, and (c) the other activities; this in turn requires identification of waste transformation sequences from each waste source to ultimate discharge. Through controls, regulations, and charges along the way the impact can be effectively controlled. The formulation of the model here is based on the above outlook of the waste management problem.

2. A Waste Management Network Model

The model is presented through the simple, hypothetical case of Figure 1 and Table 1, along with an outline of the model for the general case. There is one source node for each activity, with a single outgoing arc representing the activity operation and yielding wastes at levels proportional to the operation's level; in Figure 1, population (node 1) is the only waste generating activity. From generation to discharge wastes go through sequences of transformation processes. A node corresponds to a specific waste form, an arc to a transformation process; the levels of all processes are expressed in a uniform time rate. Differentiation of forms of wastes is based on costs, types, and products of further processing. A single sink node corresponds to the wastes absorbed by the environment; the arcs terminating at the sink are dummy processes conveniently inserted to allow for arc-capacity-type restrictions on the waste level accumulated at each arc's originating node. Here, S_A and S_R are the limits on discharges of particulates and Biochemical Oxygen Demand (BOD), respectively; S_L is a limit on the landfilling rate.

Each arc (i,j) is assigned three parameters: (1) cost F_{ij} as a (normally nonlinear) function of process level; (2) arc capacity $b_{ij} \geq 0$ arising from physical, technological, social, or legislated limitations on the process; and (3) the transformation coefficient $t_{ij} > 0$ defined as the amount of form j generated along (i,j) per unit flow of form i directed towards j. The flow transformation occurs at node i and the flow throughout (i,j) is of form j; along a chain i, (i,j), j, (j,k), k, the flow in (j,k) equals the flow in (i,j) times t_{jk}.

When a process yields jointly two or more waste forms, like "living" yielding solid wastes and sewage in the sample case, dummy nodes called splitting or secondary sources are inserted as shown in Figure 2: A transformation of form r_0 yields jointly forms r_1, r_2 and r_3 in amounts proportional to t_{01}, t_{02}, and t_{03}, respectively. This is reduced to the single transformation case as follows: r_2 is viewed as a byproduct of the transformation of r_0 into r_1, and r_3 as a byproduct of the generation of r_2. For each byproduct one secondary source is inserted (triangular node) having two arcs leaving it, the primary and the secondary; the secondary arc corresponds to the generation process of the byproduct. In Figure 2, if r_2 were the only byproduct, the secondary arc of s_1 would be (s_1, r_2); byproduct r_3, however, requires a second secondary source s_2. Form r_2 is now generated along (s_1, s_2) and "pushed" onto the primary arc (s_2, r_2) while r_3 is produced along secondary arc (s_2, r_3). For flow consistency, it is required that,

(1) (level of production of r_i) = g_i (level of production of r_{i+1})

where g_i is the splitting coefficient of secondary source s_i; accordingly, $g_1 = t_{01}/t_{02}$ and $g_2 = t_{02}/t_{03}$. For the optimization of the model it is more convenient to relate flow in the primary arc with flow in the secondary arc of a secondary source s_i; the assignment of t_{ij}'s in Figure 2 is such that

(2) (flow in primary arc of s_i) = g_i (flow in secondary arc of s_i) .

Thus, for a unit flow in (r_0, s_1), flow in (s_1, r_1) is t_{01}, in (s_1, s_2) it is $t_{01}/g_1 = t_{02}$, in (s_2, r_2) it is $t_{02} \cdot t_{s_2 r_2} = t_{02}$, and in (s_2, r_3) it is $t_{02}/g_2 = t_{03}$, in consistence with the requirement that t_{01}, t_{02}, and t_{03} units be generated of forms r_1, r_2 and r_3, respectively.

The detail and size of the network depends on the scope of the analysis. The model can be used for a particular waste management subsystem (e.g. solid wastes) or for an integrated total system, whereby interfaces among subsystems are identified and assessed. (In Figure 1, the lower part of the network corresponds to a liquid waste subsystem, the upper part to a solid waste one). Recycling is handled by actual cycling in the network. Pollution damages can be assigned as costs on the final arcs. When ambient standards are used, appropriate transformation coefficients obtained through dispersion/diffusion models are assigned to the final arcs. A larger, more complicated model is presented in [9].

For the mathematical programming formulation, a directed network is considered with nodes $i,j,\ldots,T$; each arc (i,j) is assigned parameters F_{ij}, t_{ij}, and b_{ij}. Let $P = \{p\}$ be the set of the source nodes corresponding to waste generating activities; call them <u>primary</u> sources. Let $S = \{s\}$ be the set of secondary sources; $Q = P + S = \{q\}$. Let C_k^q be the k^{th} chain of arcs joining source q with the network sink T. A chain <u>through</u> a splitting node follows the primary arc; the first arc on C_k^s is the secondary arc of source s. If x_k^q is the flow in the first arc of C_k^q and v^p is the value per unit flow leaving source p the value of the flow leaving P is $\sum_{p,k} v^p x_k^p$. The problem considered in this paper is to find the flow pattern which maximizes the flow value while: a. no arc flow exceeds the arc capacity, b. the flows leaving a secondary source satisfy (2), c. any other restrictions on arc flows are satisfied, and d. the total flow cost is within an available budget. (For the sample case, this would mean maximal population level sustainable by environmental and budget restrictions). A computationally equivalent problem is to find the minimum cost flow pattern which satisfies (a), (b), and (c) above while the flow leaving P or arriving at T is within a specified range. Clearly, a feasible flow pattern constitutes a possible waste management system.

The flow in an arc along C_k^q, and due to C_k^q, is x_k^q times every t_{ij} from q to this arc. (Notation: $t_{ij}\, t_{jk}\, t_{kf} = t_{ijkf}$) Letting

$$(3)\quad a_{ij}^{qk} = \begin{cases} t_{q\ldots ij} & \text{, if } (i,j) \in C_k^q \\ 0 & \text{, otherwise} \end{cases}$$

the total flow in (i,j) is $\sum_{q,k} a_{ij}^{qk}\, x_k^q$. For the arc capacity constraint this flow is less than or equal to b_{ij}. For a secondary source s, the flow in the primary arc (s,w) is related to the flow in the secondary arc (s,u) according to

$$(4) \qquad \sum_{q,k} a_{sw}^{qk} \; x_k^q - g_s \sum_k \; x_k^s = 0$$

A more general constraint interrelating the flows in any two arcs (i,j) and (m,n) is,

$$(5) \sum_{q,k} a_{ij}^{qk} \; x_k^q + R_{mn}^{ij} \sum_{q,k} a_{mn}^{qk} \; x_k^q \leqq d_{mn}^{ij}$$

where R_{mn}^{ij} and d_{mn}^{ij} are constants corresponding to the pair [(i,j), (m,n)]. The set of all such arc pairs is called H, with H1 the set of the first arcs and H2 the set of the second arcs; $|H| = |H1| = |H2|$. Such a constraint arises quite frequently from requirements such as: the combined level of two similar processes can not exceed a set limit, a process cannot begin until another has reached a certain level, the level of an activity is proportional to the level of another, fixed arc flows, or lower bounds on flows.

In general, a budget consists of a fixed portion B plus a variable portion G per unit flow value. For the linear cost case with $F_{ij} = \left\{ \sum_{q,k} a_{ij}^{qk} \; x_k^q \right\} c_{ij}$, where c_{ij} is the cost per unit flow through (i,j), the budget constraint for the problem becomes

$$(6) \sum_{q,k} \left\{ \sum_{(i,j)} a_{ij}^{qk} \; c_{ij} \right\} x_k^q \leq B + G \sum_{p,k} v^p x_k^p$$

The more frequent case of nonlinearity could be reduced to the linear case as discussed in the next section.

The problem has thus been cast into the linear programming model,

$$\text{Maximize} \sum_{p,k} v^p x_k^p$$

such that,

$$\sum_{q,k} a_{ij}^{qk}\, x_k^q \leq b_{ij}, \text{ all } (i,j)$$

(7)
(4) for each $s \in S$
(5) for each pair in H
(6)

and $x_k^q \geqq 0$ for all C_k^q.

If A_k^q is the column in (7) corresponding to C_k^q, it has $m + |S| + |H| + 1$ components, where m is the number of capacitated arcs. The A_k^q component in the capacity constraint of arc (i,j) and the component in (5) for $(i,j) \in H1$ is a_{ij}^{qk}; for a primary arc (s,w) the A_k^q component in (4) is a_{sw}^{qk} ; if C_k^q originates at $q \in S$ the component in (4) is $-g_q$; for an arc $(i,j) \in H2$ the component in (5) is $R_{ij}^{mn}\, a_{ij}^{qk}$. The last component of A_k^q is the cost of having a flow $x_k^q = 1$ go through C_k^q, minus the flow value $G\, v^q$ for $q \in P$.

The matrix of coefficients for the sample problem with $v^1 = 1$, B = \$2000/day, G=0, and secondary arcs (2,4), (6,14), (9,10), and (11,13) is shown in Table 2.

3. The Solution Algorithm

The problem identified in the previous sections is a multi-commodity multi-transformed network flow problem. The large size of the problem for a real case calls for a model with a solution scheme more efficient than the normal simplex. The related problems of multi-commodity network flows and flows-with-gains (or transformed flows) have been analyzed by several investigators and solution algorithms superior to the simplex have been developed; yet, none of these is applicable to the enlarged problem presented here. The flows-with-gains problem has been examined by, among others, Jewell [6], Jarris and Jezior [5], Johnson [7], Maurras [8], and Grinold [3]. The problems they consider, however, do not include multi-transformation of flows or relations among arc flows; moreover, when a maximal flow version is considered it is without a budget constraint. Ford and Fulkerson [2] and Hu [4] suggest an arc-chain formultion of the multi-commodity flow problem and solve it via an efficient column-generation technique; Swoveland [10] takes the same approach to solve the multi-commodity distribution problem. This approach is also taken here since it is especially efficient for network problems with a large number of chains and comparatively few constraints, as is the case in a waste management network.

The dual of (7), with y_s, y_{mn}^{ij}, y_B, and y_{ij} the simplex multipliers for (4), (5), (6), and arc capacity constraint, respectively, is,

(8) Minimize $\sum_{(i,j)} b_{ij}\, y_{ij} + \sum_{H} d_{mn}^{ij}\, y_{mn}^{ij} + By_B$

such that,

(9) $\sum_{(i,j)} a_{ij}^{qk} \left\{ y_{ij} + c_{ij} y_B + \sum_{(m,n)\in H2} y_{mn}^{ij} + \sum_{(m,n)\in H1} R_{ij}^{mn}\, y_{ij}^{mn} \right\} +$

$+ \sum_{(s,w)} a_{sw}^{qk} y_s - g_q y_q \geq v^q$, all C_k^q

$y_B \geq 0$; $y_{ij} \geq 0$, all (i,j); $y_{mn}^{ij} \geq 0$, all pairs in H,

where,

1. $g_q y_q = Gv^q y_B$, for $q \in P$
2. $v^q = 0$ for $q \in S$,

and 3. (s,w) refers to the primary arcs along C_k^q.

Given a feasible basis for (7) the dual vector y is obtained following the revised simplex method; if some y_{ij}, y_{mn}^{ij}, or y_B is negative, the corresponding slack column is brought into the basis until the nonnegativity restrictions are satisfied. From (9), if $\underset{k}{\text{minimum}} \left\{ yA_k^q \right\} \geq v^q$ for all q, optimality has been reached; otherwise, a column A_j^q such that $yA_j^q < v^q$ enters the basis. The structures of A_k^q and of (9) suggest that by assigning prescribed combinations of duals as "costs" on the arcs the product yA_k^q, called transformation cost of C_k^q, could be found by refering only to the network. Moreover, a least transformation cost algorithm could be used to obtain the $\underset{k}{\text{minimum}}\left\{ yA_k^q \right\}$ for the simplex optimality test; as outlined in [2], this would generate columns only as they are needed for the simplex process maintaining throughout only the columns in the basis.

Transformation Cost: Each arc (i,j) is assigned a "cost" λ_{ij} equal to: $(c_{ij}\, y_B + y_{ij})$ plus y_{mn}^{ij} for each pair [(i,j),(m,n)] in H s.t. $(m,n) \in H2$ plus $R_{ij}^{mn} y_{ij}^{mn}$ for each pair [(m,n), (i,j)] in H s.t. $(m,n) \in H1$ plus y_i if (i,j) is a primary arc; for a noncapacitated arc, $y_{ij} = 0$. The transformation cost of $C_k^q = y\, A_k^q = \sum_{(i,j) \in C_k^q} a_{ij}^{qk}\, \lambda_{ij} - g_q y_q$. For example, if at some iteration of the problem in Table 2, $(y_{13,16}, y_{14,16}, y_{15,16}, y_2, y_6, y_9, y_{11}, y_B) = (5,0,0.1,0.18,0,4,2.5,\ 0.03)$, then

$$yA_1^6 = (a_{6,14}^{6,1})\ (0) + (a_{14,16}^{6,1})\ (0) - (g_6)\ (0) = 0, \text{and}$$

$$yA_1^1 = (a_{1,2}^{1,1})\ (0) + (a_{2,3}^{1,1})\ (y_2) + (a_{3,15}^{1,1})\ (c_{3,15} y_B) + (a_{15,16}^{1,1})\ (c_{15,16} y_B + y_{15,16}) - Gv^1 y_B =$$

$$= 0 + (2.5)\ (0.18) + (2.5)\ (0.09) + (2.5)\ (0.03 + 0.10) - (0)\ (1)\ (0.03) = 1 .$$

Least Transformation Cost Algorithm

1. Assign $Z_T = 0$, $Z_i = \infty$ for $i \neq T$, and $W_s = \infty$ for $s \in S$; assign each arc (i,j) a "cost" λ_{ij} as outlined above.
2. a. For a node $i \notin S$, if, for some j, $Z_i > t_{ij}\ (\lambda_{ij} + Z_j)$, set $Z_i = t_{ij}\ (\lambda_{ij} + Z_j)$.

   b. For a node $s \in S$, from the primary arc (s,w) set $Z_s = t_{sw}\ (\lambda_{sw} + Z_w)$; from the secondary arc (s,u) set $W_s = -g_s y_s + Z_u$.

3. Continue until there is no arc (i,j) s.t. $Z_i > t_{ij} \; (\lambda_{ij} + Z_j)$ and until Z_s and W_s are as required by 2b for all $s \in S$.
4. For each $p \in P$, reduce Z_p by $Gv^P y_B$.

At the end of the algorithm $Z_p = \min_k \left\{ y \, A_k^p \right\}$ and $W_s = \min_k \left\{ y \, A_k^s \right\}$. The best chains are along arcs (i,j) s.t. $Z_i = t_{ij} \; (\lambda_{ij} + Z_j)$. The simplex optimality test is now performed; columns entering the basis are constructed as already outlined.

In acyclic networks convergence is guaranteed since there is a finite number of arcs and a lower bound on Z_p and W_s (because the duals and all parameters are finite). In networks with cycles, the algorithm will not converge when, for a node n on a cycle, Z_n decreases with no bound or it approaches a bound assymptotically. The way Z_n is computed suggests that the case of no bound occurs only on a cycle with no capacitated arcs for which: either the cost for a unit flow entering it and going around it is negative or this cost is zero but the cycle is absorbing (product of t_{ij}'s around it less than unity); existence of such a cycle indicates the unbounded solution of the original problem and the solution process is terminated. On the other hand, an assymptotically approached bound for Z_n can be approximated in terms of the t_{ij}'s and λ_{ij}'s of the cycle's arcs.

For a starting basis, a Phase I/Phase II procedure is used only when artificial or surplus variables are needed in (5); an artificial variable introduced in (4) remains at zero level while in the basis, leaves the basis when a chain through or from the corresponding secondary source enters it, and never enters the basis again. In large networks the optimal flow may bypass a number of secondary sources and capacitated arcs; to avoid unnecessary constraints of types (4) and arc capacity, a <u>row-generation</u> scheme is employed whereby, as columns are generated by the least transformation cost algorithm, a new row (constraint) is added in updated form to the current simplex tableau for each secondary source or capacitated arc encountered for the first time.

The more realistic case of nonlinear costs can be reduced to the linear case by piece-wise linear approximation and insertion in the network of one arc per linear segment. For a two-segment approximation of F_{ij}, (i,j) is replaced by arcs $(i,j)^1$ and $(i,j)^2$ with unit costs c^1_{ij} and c^2_{ij}, respectively, flow ranges 0 to R_{ij} for the first and 0 to $b_{ij} - R_{ij}$ for the second, and $t^1_{ij} = t^2_{ij} = t_{ij}$. In the optimization procedure, a chain through $(i,j)^1$ will be chosen over an identical chain but for $(i,j)^2$ in place of $(i,j)^1$ only if $y^1_{ij} + c^1_{ij}\, y_B < y^2_{ij} + c^2_{ij} y_B$. In the convex cost case, when arc flow limits are not reached this will always hold, yielding a consistent flow. For the concave cost case, the following procedure has been tested with variable success:

1. Solve problem as for the linear case. A consistent solution is optimal; an inconsistent one provides an upper bound on the flow value.

2. Re-solve problem with the stipulation in step 2.a. of the least cost algorithm that: for node i of concave-cost arc (i,j), arc $(i,j)^2$ is ignored if the slack s^1_{ij} of the capacity constraint of arc $(i,j)^1$ is basic. If no chain through some $(i,j)^2$ violates (9), the solution is optimal; otherwise, it provides a lower bound on the value of consistent flow.

3. Choose a chain through a $(i,j)^2$ violating (9), restrict flow in $(i,j)^1$ to equal R_{ij}, and get a consistent solution whose flow value replaces the current lower bound if the new one is higher; if not, a violating chain through another $(i,j)^2$ is chosen.

4. Continue until an acceptable difference between upper and lower bounds is reached.

The above procedure works well in networks with physical interpretation since an analyst familiar with the physical requirements of the solution may: (a) after steps 1 and 2, drop arcs considered unlikely to be in an optimal solution, and eliminate some nonlinearities by imposing lower flow bounds on arcs likely to be in an optimal solution; (b) in step (3), choose simultaneously more than one chain; and (c) reduce the upper bound through reasonable cost approximations based on already obtained (non-optimal) solutions. The number of concave-cost arcs may also be significantly reduced by combining processes (and their costs) into superprocesses in a way that does not violate the logic of the original network. Fixed costs are treated as concave costs by assuming for the first linear segment of the approximation a high c_{ij}^1 and a small R_{ij}. For a real problem, the magnitude of the effort required to deal with concave costs by this procedure is expected to be acceptable, especially when one considers the great effort required to model the problem and the considerable expense involved with decisions to be made.

Extensive computational experience has been obtained with the McGill IBM 360/75 computer for both the max-flow and min-cost versions. For the sample problem, the optimum solution is obtained in 1.02 sec CPU time (1.14 sec without row generation). The allowable population is 86,300; 28 tons of solid refuse are incinerated, the rest taken to a transfer station; 0.24 tons of untreated BOD goes in the river, the rest going through up to secondary treatment; the limits are reached for budget, S_R, and S_L, suggesting a shift, if possible, of monies from particulates control to sewage treatment or landfilling. In general, solution times depend on the number and type of constraints and, much less, on network size. With linear costs, for 170 arcs and 26 constraints (including 10 splitting nodes) CPU time is 3.40 sec (3.50 without row generation); for 110 arcs and 22 constraints (12 splitting nodes) it is 3.10 sec (4.39); and for 23 arcs and 23 constraints (no splitting nodes) it is 1.90 (2.57).

4. Conclusion

A model and its optimization algorithm have been presented for the multi-commodity multi-transformed network flow problem, the arc flows of which may be interrelated and their total cost is subject to a flow-dependent budget. The maximal multi-commodity flow problem and the maximal flow-with-gains problem (for positive gains) can be solved as special cases of this model. A minimum-cost version of the model can also be optimized in an almost identical manner.

The model has been developed for the waste management problem which it treats in an integrated total-system approach, taking into account the interrelationships among the different forms of the environment, balancing the burdens on them, and evaluating a large number of waste management systems. It can be used for modifying existing systems over the short and long runs or for planning new ones as in the case of a new town. The model is simple in concept yet flexible enough to be responsive to the special needs of different controllers at different levels of the system. The large size of the network for a real problem is dealt with through column and row generation schemes, while the simplex-based optimization procedure allows for simple sensitivity analyses. It is quite possible that there may be more than one optimal solution to the problem or a range of solutions that differ very little in flow value or cost; this can be detected by the proximity of the final transformation cost of nonbasic chains to their flow unit value. Different optimal or near-optimal systems can then be evaluated by criteria that could not have been incorporated directly in the model.

Although several complications that arise in a real situation can be handled by the model, mainly because of the generality of constraint (5) and the possibility of inserting extra arcs in the network, the model is not without major drawbacks. When environmental standards are ill-defined or when wastes interreact chemically the practicality of the model is limited. Data availability is as serious a problem as for currently used methodologies. The existence of concave cost functions prevents attainment of optimality, but good solutions are expected. Another drawback may be the implicit assumption throughout this work that t_{ij} is constant and independent of the process level when, in fact, the efficiency of a process may change as its level increases; this resembles the nonlinear cost problem, convexity being the more frequent case. A variable t_{ij} is often accompanied by a variable unit cost, presenting a more complex case currently under study.

Finally, partial independence of agencies in charge of specific subsystems of the waste management system can be retained, similar to that described by Dantzig [1], with the duals of (7) used by a cental authority as points in a merit system providing incentives for good performance. If $v^q + g_q y_q$ is offered to the controller of source q as reward per unit flow leaving q, and λ_{ij} is charged per unit flow in (i,j) due to that unit from q, source controllers would look for processes such that: reward $\geqq$ disposal charge. Over a long horizon when charges can be adjusted, the central authority can start off with a rough network and, as source controllers discover new "profitable" chains which violate (9), the charges are increased to levels suggested by the improved network model, until no "profitable" chains exist.

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TABLE 1

Description of the Problem in Figure 1

Arc	Process Description
1,2	"living", 1000 people per day
2,3	generation of solid wastes; 5 lbs/cap/day = 2.5 tons/unit (1,2)
2,4	generation of sewage; 0.167 lbs BOD/cap/day = 0.0835 tons BOC/unit (1,2)
3,6	refuse collection and incineration; \$6 per ton is used based on \$0.10 per ton-mile for collection, a 5-mile average travel distance, and \$5.50 for incinerator capital and operating cost.
3,8	refuse collection and transport to a transfer station
4,5	sewage collection, including building the sewarage system; available cost figures for per capita annual expenditures are reduced to daily costs per ton of BOD.
4,13	raw discharge of sewage into the river
5,9	primary treatment (37.5% BOD removal) yielding digested sludge and BOD; \$90/ton of BOD treated is derived from \$2.75 annual cost per capita
6,7	generation and handling of residue; cost has been included in (3,6)
6,14	generation and handling of particulates: 10 lbs per ton incinerated
7,15	residue transfered to a landfill site
8,15	usage of a transfer station plus refuse transport to the landfill; there is a fixed daily cost of \$100 plus \$1.5 per ton handled.
9,10	BOD load reduction; 0.625 tons remain per ton treated
9,12	sludge generation; 0.5 tons of sludge per ton of BOD treated.
10,11	secondary treatment (89% BOD removal); cost figures have been reduced to per ton of BOD reaching node 11.
10,13	discharge of effluent from primary treatment
12,15	sludge from primary and secondary treatments taken to landfill
13,16	total BOD discharge into river; must not exceed S_R = 1 ton per day
14,16	total particulates discharge; S_A = 0.50 lbs per day
15,16	use of the landfill site; S_L = 200 tons per day

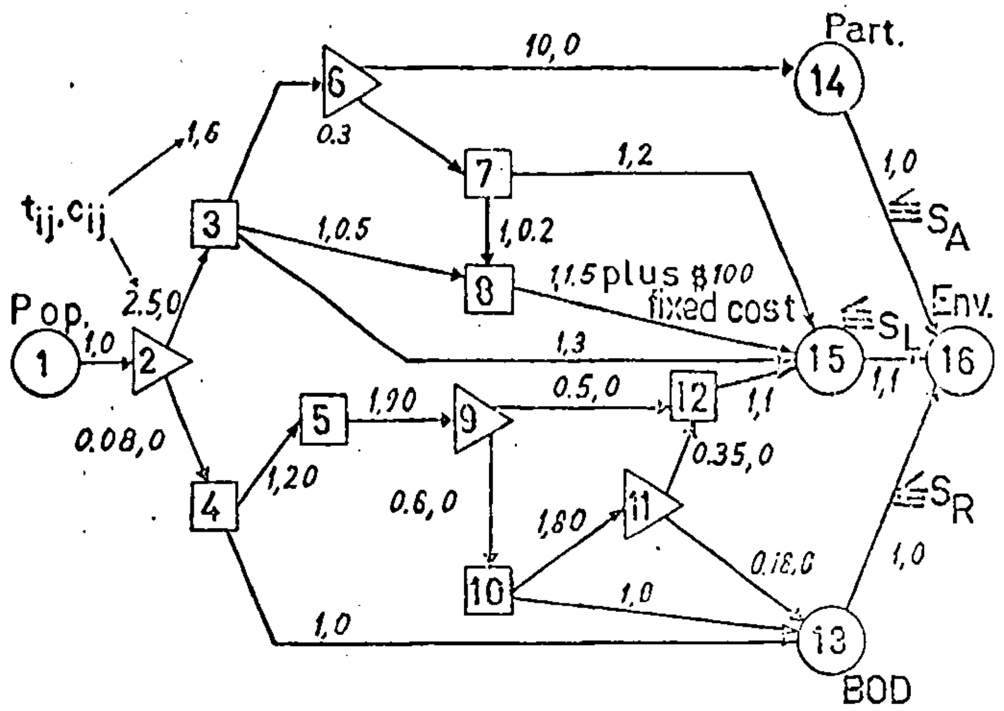

FIGURE 1: A Waste Management Network

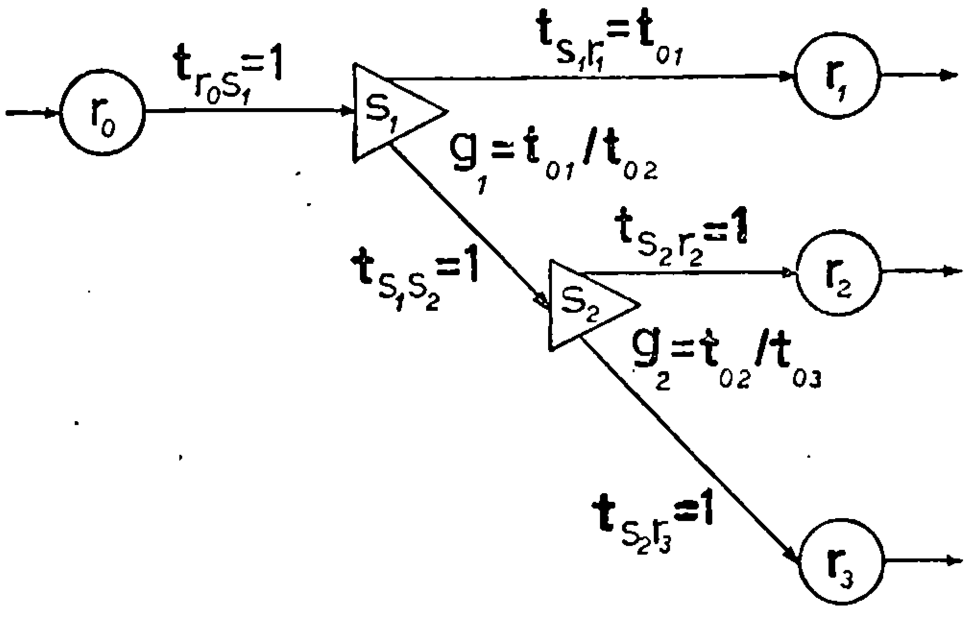

FIGURE 2: Multi-Transformation of r_0 into r_1, r_2, and r_3.

TABLE 2

Matrix of Coefficients for the Problem in Figure 1

A_k^q	A_1^1	A_2^1	A_3^1	A_4^1	A_1^2	A_2^2	A_1^6	A_1^9	A_2^9	A_1^{11}	
v^q	1	1	1	1	0	0	0	0	0	0	
S_R	0	0	0	0	1[b]	0	0	1	0	1	1.0
S_A	0	0	0	0	0	0	1	0	0	0	0.5
S_L	2.5	2.5	0.75	0.75	0	0.5	0	0	0.35	0	200
s=2	2.5	2.5	2.5	2.5	$-\frac{2.5}{0.08}$	$-\frac{2.5}{0.08}$	0	0	0	0	0
s=6	0	0	0.75	0.75	0	0	$-\frac{0.3}{0.005}$	0	0	0	0
s=9	0	0	0	0	0	0.5	0	$-\frac{0.5}{0.6}$	$-\frac{0.5}{0.6}$	0	0
s=11	0	0	0	0	0	0	0	0	0.35	$-\frac{0.35}{0.18}$	0
B	10	7.5	17.25	17.03[a]	0	111	0	0	80.7	0	2000

C_1^1: 1,2,3,15,16; C_3^1: 1,2,3,6,7,15,16; C_1^2: 2,4,13,16; C_1^9: 9,10,13,16;

C_2^1: 1,2,3,8,15,16; C_4^1: 1,2,3,6,7,8,15,16; C_2^2: 2,4,5,9,12,15,16; C_2^9: 9,10,11,12,15,16

[a] plus the fixed cost of (8,15)

[b] t_{ij} of a secondary arc is set at 1 (See Figure 2)