MICROWAVE ACOUSTIC PROPERTIES OF
FLUIDS BY BRAGG SCATTERING

Electrical

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M.Eng.

ABSTRACT

A thin film resonance technique is described for measuring the dispersion and absorption per wavelength of acoustic waves in lossy liquids from 0.5 to 2.0 GHz. The method uses Bragg scattering of laser light to measure the incident and reflected acoustic power from a 4.2μ liquid film formed between two sapphire rods. It is shown how the velocity and absorption are calculated from the measured acoustic reflection coefficient. The liquids studied are Dow Corning resins 276-V2 and 276-V9 both of which are found dispersionless within experimental error. The absorption per wavelength initially decreases approximately logarithmically with frequency for both resins. For resin V9, however, the absorption is about 25% smaller and also shows an increase at higher frequencies. The angular acoustic energy distribution in sapphire is also measured and found to be in good agreement with our theoretical calculation of the radiation field of a circular transducer.

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CHAPTER I
INTRODUCTION

Ultrasonic waves have been used extensively to study the mechanical properties of liquids.* Both temperature and frequency dependences of the velocity and attenuation are closely related to the energy exchange mechanisms and molecular structure of the liquids. Litovitz and Davis (1965), Lamb (1965), and Philippof (1965) have described several models which attempt to explain the dynamic properties of liquids in the long wavelength approximation. Piercy (1965) has reviewed these models in a more qualitative way and classified many liquids according to them.

As for the experimental aspect of the problem, the techniques most currently used for measuring dispersion and absorption in liquids may be classified in three categories. The travelling wave techniques described by McSkimmin (1964) involve the propagation of coherent ultrasonic waves through a pathlength of liquid which is normally many wavelengths long. However, in the gigahertz range of frequencies, the attenuation per wavelength ($\alpha\lambda$) approaches one neper and such measurements become unpractical. The standing wave methods are more amenable to use at these frequencies since they make

*For an extensive bibliography the reader is referred to McSkimmin (1964).

use of a few wavelengths of travel only. One of the experimental arrangements proposed by McSkimmin and Chambers (1967) utilizes this principle; the liquid specimen to be studied is shaped into a thin film between two solid delay lines called buffer rods. By means of the standard pulse echo technique, in which a transducer acts as a generator and a detector of acoustic energy, the amplitude of the various echoes reflected from the thin film may be measured; the information about the absorption and dispersion of the waves is extracted from these measurements. The latter type of experiment or modified version of it has been successfully used by McSkimmin (1962) and McSkimmin and Chambers (1964) in the 0.1 to 0.8 GHz frequency range and by Plass (1967, 1968) and Plass and Kehl (1968) in the 0.5 to 2.8 GHz range for liquids exhibiting low attenuation. Benedek, Lastokva, Fritsch and Greytak (1964) have described another experimental method of measurement which involves Brillouin scattering. Brillouin scattering is named after Leon Brillouin (1922) who first predicted that light would be scattered by thermal phonons. By studying the frequency spectrum and direction of the scattered light, the dispersion and absorption of the illuminated liquid can be obtained. This is a method which is specially well suited for frequencies in excess of a few gigahertz.

One of the shortcomings in McSkimmin's method lies in the means of detecting the acoustic echoes in the delay line. An alternative way is considered here. Recently, McMahon (1967) has made an experimental comparison between different methods of probing acoustic energy in solids and pointed out that Bragg scattering was one of the most reliable methods. Bragg scattering is a particular case of Brillouin scattering in which light is scattered from a coherent acoustic wave. Under an acoustic pressure the dielectric constant of the medium is perturbed locally. The perturbed region, as seen by an electromagnetic wave, then acts as a phase or volume grating which diffracts light into its many characteristic orders. When the acoustic and optical wavevector magnitudes are such that there is only one diffracted beam, the process is then called Bragg scattering. Quate, Wilkinson, and Winslow (1965), Cohen and Gordon (1965), and Gordon (1966) have extensively studied the acoustic beam probing in isotropic materials. They pointed out one of the important properties of the Bragg scattering technique which is the linear dependence between the diffracted light intensity and the acoustic power. Dixon (1967) and Reintjes and Schulz (1968) have measured the proportionality or efficiency factor in many solids.

In this thesis, it is intended (1) to design and test a Bragg scattering acoustic probe and (2) to show that such a probe, in combination with a modified version of McSkimmin's

buffer rod arrangement, is a useful method for obtaining the dispersive and absorptive characteristics of highly attenuating liquids in the frequency range 0.5 to 2.5 GHz; this method would then cover the frequency gap between the results obtained with the standard methods and the Brillouin scattering method.*

The bulk of the theory necessary for the application of the present method and for the interpretation of the results is given in the second chapter of this work. The first part consists of a description of the mathematical models used to predict the dispersion and absorption of acoustic waves in the liquids and solids considered here. A second section of this chapter is devoted to a summary of McSkimmin's buffer rod method and to the analytical and experimental description of the method used here which is called from now on the Bragg scattering method. Finally the last part consists of an analysis of the Bragg scattering acoustic probe behaviour.

The third chapter is concerned with the implementation of the Bragg scattering method. It describes the thin film

*To our knowledge the only other measurements covering this range of frequency are those published by Plass and Kehl (1968) for weakly attenuating liquids.

construction and the practical realization and testing of the acoustic probe. Finally, the actual experimental results are reported in the fourth chapter. The first section concerns the measurements of the buffer rod and transducer characteristics. The remainder of the chapter is devoted to the measurement of the acoustic propagation characteristics of two viscolastic liquids (Dow Corning resins 276-V2 and 276-V9). The chapter concludes with a discussion of the measurement accuracy and an interpretation of the results.

CHAPTER II

THEORY OF THE MEASURING TECHNIQUES

2.1 Introduction

The experimental methods generally used to study the absorption and dispersion of acoustic waves in liquids have been discussed briefly in Chapter I. The present chapter is concerned with the theoretical aspect of this problem; simple mathematical models describing the absorption and dispersion of acoustic waves in liquids and solids are presented and their applicability to our problem is discussed. Then, after a brief discussion of McSkimmin's buffer rod method, the theoretical fundamentals necessary to elaborate in detail the Bragg scattering method are developed. Finally, the respective merits of each method are compared.

2.2 Wave Propagation Characteristics

The absorption and dispersion of acoustic waves in liquids and solids have been studied extensively and many models have been proposed to describe their behaviour for different ranges of frequency and temperature. The models applicable in the conditions of the present experiment are given in the following sections.

2.2.1 Dispersion and Absorption of Ultrasound in Visco-elastic Liquids

Experimental evidence suggests that the dominant mechanisms causing absorption and dispersion in liquids are generally of a relaxational nature. Herzfeld and Litovitz (1959) and Litovitz and Davis (1965) define relaxation times and complex moduli for sinusoidally applied fluid pressures and shear stresses. For purely compressional waves the modulus has the following form:

$$K = K_0 + \frac{j\omega\tau_c K_r}{1+j\omega\tau_c}$$

and for pure shear waves,

$$G = \frac{j\omega\tau_s}{1+j\omega\tau_s} G_\infty$$

where K_0 is the static compressional modulus

K_∞ and G_∞ are respectively the compressional and shear moduli in the high frequency limit

$$K_r = K_\infty - K_0$$

τ_c and τ_s are respectively the relaxation times for compressional and shear mechanisms.

If heat conduction effects are neglected, the wave equation in the absence of volume forces may be written

$$\rho \frac{\partial^2 U}{\partial t^2} = M \frac{\partial^2 U}{\partial x^2},$$

where M is a combination of the previously defined moduli and, as derived by Thurston (1964), is given by

$$M = K + \frac{4}{3} G.$$

Assuming plane wave solutions of the form $\exp[j\omega t - (j\omega/V + \alpha)X]$, substitution in the wave equation yields for the case of $(\alpha\lambda/2\pi) < 1$

$$\alpha\lambda = \pi \left[\frac{\frac{K_r \omega \tau_c}{1 + \omega^2 \tau_c^2} + \frac{4}{3} \frac{G_\infty \omega \tau_s}{1 + \omega^2 \tau_s^2}}{K_o + \frac{K_r \omega^2 \tau_c^2}{1 + \omega^2 \tau_c^2} + \frac{4}{3} \frac{G_\infty \omega^2 \tau_s^2}{1 + \omega^2 \tau_s^2}} \right] \quad 2.1$$

and

$$V^2 = \frac{1}{\rho} \left[K_o + \frac{K_r \omega^2 \tau_c^2}{1 + \omega^2 \tau_c^2} + \frac{4}{3} \frac{G_\infty \omega^2 \tau_s^2}{1 + \omega^2 \tau_s^2} \right] \quad 2.2$$

Figure 2.1 shows a typical absorption and velocity frequency dependence.

In real liquids, the results are not so simple, particularly in liquids exhibiting structural relaxation in which a single relaxation time model does not account satisfactorily for the experimental results. In this case, a distribution of relaxation times can provide a reasonable description of the propagation characteristics. The distribution most frequently used and some applications to specific liquids are discussed by Litovitz and Davis (1965).

2.2.2 Wave Propagation Characteristics in Sapphire Near the Threefold Axis

Bommel and Dransfeld (1959, 1960) showed experimentally that for a few samples of synthetic or natural crystalline

Figure 2.1 Frequency Dependence of Absorption and Normalized Velocity According to Equations 2.1 and 2.2.

quartz, the attenuation at room temperature did not vary by more than a few percent. They also annealed and stressed the quartz and obtained the same results; they thus concluded that the attenuation in these crystals was an intrinsic property of the material and hence discarded possible contributions from dislocations and other defects. More recently, McBride et al. (1969) qualitatively accounted for the attenuation in sapphire over a wide range of temperature and frequency by assuming that the attenuation was due entirely to the interaction of the acoustic wave with thermal phonons. Such a mechanism predicts that at room temperature the attenuation in sapphire is of a relaxational nature; Mason and Bateman (1964) derive the following expression:

$$\alpha = \frac{\Delta c \omega^2 \tau}{2\rho V^3 (1 + \omega^2 \tau^2)} \quad 2.3$$

This model also yields for the velocity

$$V = V_0 \left[1 + \frac{\Delta c}{2C_0} \frac{\omega^2 \tau^2}{1 + \omega^2 \tau^2} \right], \quad 2.4$$

where Δc = change in elastic stiffness due to thermal phonon coupling

C_0 = unperturbed stiffness constant

V_0 = unperturbed phase velocity

τ = relaxational time for the thermal phonon coupling process.

The relaxation time, at room temperature, is less than 10^{-12} sec. and at 1.0 GHz, $\omega\tau \ll 1$. Hence equation 2.3 reduces to a frequency squared dependence, while equation 2.4 shows that dispersion may be neglected.

Although the phase velocity may be regarded as being independent of frequency, it cannot be considered a constant for a given anisotropic material since it varies with the direction of propagation. The problem of acoustic bulk waves in anisotropic media has been studied by Musgrave (1954), Buchwald (1959), and Farnell (1961). Our particular problem, the propagation of a compressional wave along a threefold axis, is a simple case since the wavevector is colinear with the Poynting vector. For such a pure mode direction, the magnitude of the wavevector is given by

$$K_3 = \omega \quad \rho / C_{33} \quad ,$$

where K_3 is the wavevector amplitude along the threefold axis, ρ is the density, and C_{33} is the appropriate stiffness matrix element as defined in Nye (1960). Waterman (1959) analyses the variations of the wavevector magnitude for directions near the pure mode axes where the wavevector surface can be approximated by a parabola. In the small angle approximation he shows that

$$|\underline{K}| = (K_3) (1 + b\phi^2), \quad 2.5$$

where ϕ is the angular deviation from the pure mode axis and b is an anisotropy factor which has a value of 0.16 for the threefold axis of sapphire. For such a case, the angle between the wavevector and the Poynting vector is given by

$$\theta = -2b\phi. \quad 2.6$$

2.3 Experimental Techniques

As noted earlier, the experimental study of the ultrasonic properties of highly attenuating liquids at gigahertz frequencies can only be carried out by using pathlengths of the order of a few wavelengths. This experimental condition is common to both McSkimmin's method and the one used here.

2.3.1 McSkimmin's Buffer Rod Technique

Figure 2.2 is a schematic description of the experimental arrangement proposed by McSkimmin and Chambers (1964) for performing the measurements. The liquid specimen to be studied is shaped into a thin film between two acoustic delay lines of the same material called buffer rods. A transducer, previously deposited on the end of one of the buffer rods, is used to launch a short pulse of rf acoustic energy which is partially reflected at the interface with the thin film and partially transmitted through the thin film into the second rod. If the transducer is used as a detector, the first echo to be seen (see Figure 2.2) is proportional to the displacement amplitude (E_p) of the reflected wave. The second echo (E_r^+) is a measure of the transmitted wave amplitude which is detected only after it has reflected at the far end of the second buffer rod and passed through the thin film a second time. Additional echoes arising from multiple reflections inside the second buffer rod are also present and it is important to choose the pulse duration short enough to permit time resolution of each echo, yet long

Figure 2.2 Schematic Presentation of the Buffer Rod
Experimental Arrangement Showing the Ray Tracing Pattern
After McSkimmin and Chambers (1964).

enough to achieve steady state conditions in the thin film, i.e. the pulse duration must be much longer than the transit time through the film.

McSkimmin's procedure for obtaining the velocity and absorption of acoustic waves in the liquid film from experimental values of $|E_b/E_r'|$ may be summarized in the following way: By analogy with electrical transmission line theory, it is shown that the ratio $|E_b/E_r'|$ exhibits minimum values at frequencies given by

$$F = \frac{nV_2}{2L}, \quad 2.7$$

in which L is the thickness of the film and V_2 is the phase velocity in the liquid. For the case of an attenuation per wavelength $(\alpha_2 \lambda_2) < 0.3$ these minima are given by

$$|E_b/E_r'|_{\min} \approx \frac{|K|}{2t_q} \{a[(\cosh 2\alpha L - 1) + \sinh 2\alpha_2 L]\} \quad 2.8$$

in which

$$K = 1/2 (Z_2/Z_1 - Z_1/Z_2)$$

$$a = \operatorname{Re} 1/2 (Z_2/Z_1 + Z_1/Z_2)$$

α_2 = attenuation in the liquid

Z = mechanical impedance. The subscripts 1 and 2 refer to buffer rods and liquid respectively.

t_q = loss in the second buffer rod due to thermal phonon coupling and diffraction

and $Z = Ap/\theta$

where A = area of acoustic beam

ρ = density of the medium

$\theta = -j\alpha/\omega + 1/V$.

McSkimmin now uses equation 2.8 to draw a family of curves for $2\alpha_2 L$ vs $|t_q^{E_b/E_r}|$ which has the ratio Z_1/Z_2 as a variable parameter. From equation 2.7, the velocity at a particular minimum can be calculated. Neglecting the imaginary contribution of the attenuation to the mechanical impedance, he obtains

$$Z_2 \approx \rho_2 A_2 V_2$$

The experimentally measured values $|t_q^{E_b/E_r}|$ and V_2 may thus be used to estimate $2\alpha_2 L$ from the computed curves.

We point out that this procedure is valid only for $\alpha_2 \lambda_2 \leq 0.3$. The liquids that we use may have larger attenuation than that. Hence, this method makes use of approximations that are difficult to justify a priori in our experiment. A slightly different approach is then utilized which is described below.

2.3.2 The Bragg Scattering Method

The experimental method that we use here is similar to that used by McSkimmin and described above. The difference lies in the method of detecting the acoustic wave energy; whereas McSkimmin uses a piezoelectric transducer, here we use Bragg scattering.

A laser beam is directed through the first buffer rod and interacts with the acoustic wave to produce a diffracted light beam whose energy is proportional to the acoustic energy flux. Referring to Figure 2.3, we see that this detection scheme allows us to monitor the incident energy which impinges on the thin film in addition to that of the echoes which are detectable using the transducer. Thus, if Bragg scattering is used, we can measure the quantity $|E_b/E_i|^2 e^{-2\alpha_1 l}$ directly. This is the reflection coefficient of the thin film $|\Gamma_F|^2$ times the attenuation factor $e^{-2\alpha_1 l}$ due to transit in sapphire. The problem remains to extract the dispersion and absorption of the acoustic wave in the liquid from such a measurement. For this purpose it will be shown in the remainder of this section that $|\Gamma_F|$ is a function of $\alpha_2 \lambda_2$, V_2 , ω and $|\Gamma|$ the reflection coefficient at an interface between the liquid and solid at normal incidence; this last quantity is experimentally measurable. The attenuation per wavelength ($\alpha_2 \lambda_2$) will then be shown to be explicitly expressible in terms of $|\Gamma|$ and $|\Gamma_F|_{\min}$ the thin film reflection coefficient measured at minima of reflection. The phase velocity V_2 will also be obtained from the location of these minima.

We first proceed to find $|\Gamma|$ as a function of the mechanical properties of the liquid. The general problem of

Figure 2.3 Experimental Arrangement Showing the Sequence of the First Three Pulses Which Are Detected Using the Bragg Scattering Probe.

the reflection and transmission of acoustic waves at the interface between anisotropic media has been analysed by Musgrave (1960). The present problem is a simple case of this, since the acoustic wavevector in the solid medium is along a pure mode direction normal to the interface and the second medium is isotropic. From Musgrave's general statement about the equality of all the wavevector projections on the interface, it is deduced for this case that all the wavevectors are colinear. From continuity of displacement and stress at the boundary, it is seen that the reflected and transmitted beams have the same polarization as the incident one and hence no other modes are produced at the interface. For this case, the relationship used by McSkimmin (1964) and analogous to electrical transmission line theory is valid and one obtains

$$\Gamma_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \quad 2.9$$

where Z has already been defined in equation 2.8. Substitution of the expression for Z into 2.9 yields

$$\Gamma_{12} = |\Gamma| e^{j\chi}, \quad 2.10(a)$$

where

$$|\Gamma| = \left[\frac{(\rho_1 V_1 - \rho_2 V_2)^2 + (\rho_1 V_1 \alpha_2 \lambda_2 / 2\pi - \rho_2 V_2 \alpha_1 \lambda_1 / 2\pi)^2}{(\rho_1 V_1 + \rho_2 V_2)^2 + (\rho_1 V_1 \alpha_2 \lambda_2 / 2\pi - \rho_2 V_2 \alpha_1 \lambda_1 / 2\pi)^2} \right]^{1/2},$$

and

$$\tan \chi = \frac{2 (\alpha_1 \lambda_1 / 2\pi - \alpha_2 \lambda_2 / 2\pi)}{\frac{\rho_1 V_1}{\rho_2 V_2} \left[1 + \left(\frac{\alpha_2 \lambda_2}{2\pi} \right)^2 \right] - \frac{\rho_2 V_2}{\rho_1 V_1} \left[1 + \left(\frac{\alpha_1 \lambda_1}{2\pi} \right)^2 \right]}$$

For most combinations of organic liquids and hard solids, one can safely assume that

$$\alpha_1 \lambda_1 \ll \alpha_2 \lambda_2$$

and

$$\rho_2 V_2 \leq \rho_1 V_1$$

For these conditions equation 2.10 becomes

$$|\Gamma| \approx \left[\frac{(\rho_1 V_1 - \rho_2 V_2)^2 + (\rho_1 V_1 \alpha_2 \lambda_2 / 2\pi)^2}{(\rho_1 V_1 + \rho_2 V_2)^2 + (\rho_1 V_1 \alpha_2 \lambda_2 / 2\pi)^2} \right]^{1/2} \quad 2.10 (b)$$

and

$$\chi \approx \frac{\rho_2 V_2}{\rho_1 V_1} \frac{\alpha_2 \lambda_2}{\pi} \quad 2.10 (c)$$

The next step is to develop a relationship for the reflection coefficient of the liquid thin film illustrated in figure 2.3. Again by analogy with a short length of transmission line, we have by definition

$$Z_F = \frac{Z_2 [1 + \Gamma_{23} \exp(-2j\omega\theta_2 L)]}{[1 - \Gamma_{23} \exp(-2j\omega\theta_2 L)]} \quad 2.11$$

where

Z_F = thin film impedance

Z_2 = liquid impedance

Γ_{23} = reflection coefficient [at interface between medium 2 and 3].

Hence, by definition also, the reflection coefficient at the thin film first rod interface is given by

$$\Gamma_F = \frac{Z_F - Z_1}{Z_F + Z_1} .$$

Substituting for Z_F from equation 2.11 and using equation 2.9 one obtains

$$\Gamma_F = \frac{\Gamma_{12} + \Gamma_{23} \exp(-2j\omega\theta_2 L)}{1 + \Gamma_{12} \Gamma_{23} \exp(-2j\omega\theta_2 L)} .$$

Since the media 1 and 3 are identical, we have

$$\Gamma_{12} = -\Gamma_{23}$$

and the expression further simplifies to

$$\Gamma_F = \frac{\Gamma_{12} [1 - \exp(-2j\omega\theta_2 L)]}{(1 - \Gamma_{12}^2 \exp(-2j\omega\theta_2 L))} . \quad 2.12$$

Finally, substituting for Γ_{12} from equation 2.10 (a), the magnitude of the reflection coefficient is obtained as

$$|\Gamma_F|^2 = |\Gamma|^2 \left[\frac{1 - 2\cos\left(\frac{2\omega L}{V_2}\right) \exp\left(\frac{-2\omega L \alpha_2 \lambda_2}{V_2 2\pi}\right) + \exp\left(\frac{-4\omega L \alpha_2 \lambda_2}{V_2 2\pi}\right)}{1 - 2|\Gamma| \cos\left(\frac{2\omega L}{V_2} - 2\chi\right) \exp\left(\frac{-2\omega L \alpha_2 \lambda_2}{V_2 2\pi}\right) + |\Gamma|^2 \exp\left(\frac{-4\omega L \alpha_2 \lambda_2}{V_2 2\pi}\right)} \right] \quad 2.13$$

In the limit, if $\alpha_2 \lambda_2$ tends to zero, χ tends to zero and the exponential terms of equation 2.13 tend to 1. Under these conditions equation 2.13 exhibits minima at frequencies

$$F_{\min} = nV_2/2L , \quad 2.14$$

where n = all positive integers.

For non-zero value of $\alpha_2 \lambda_2$, differentiating equation 2.13 with respect to frequency and equating the result to zero yields the location of the minima. If χ is small and $|\Gamma|$ is assumed to be slowly varying function of frequency, the minima of equation 2.13 are found at

$$F_{\min} = \frac{V_2}{2L} \left\{ n - \frac{\beta_2}{(2\pi)^2} \left[\frac{1 - (|\Gamma| + 1) \exp(-n\alpha_2 \lambda_2) + |\Gamma| \exp(-2n\alpha_2 \lambda_2)}{1 - |\Gamma| \exp(-2n\alpha_2 \lambda_2)} \right] \right\} \quad 2.15$$

where

$$\beta_2 = \left(F \frac{d(\alpha_2 \lambda_2)}{dF} + \alpha_2 \lambda_2 \right) n$$

The difference between equation 2.14 and 2.15, as expected, tends to zero if $\alpha_2 \lambda_2$ tends to zero. Let us study the case of a large attenuation, i.e. $\alpha_2 \lambda_2 \approx 1$. Successive substitution of equation 2.14 and equation 2.15 into equation 2.13 reveals that the values thus obtained for $|\Gamma_F|^2$ differ by less than 0.2% from each other. This discrepancy is negligible compared to the expected experimental error and, for the purpose of extraction of $\alpha_2 \lambda_2$ from experimental measurements, equation 2.14 is sufficiently precise. Equation 2.13 then yields

$$\alpha_2 \lambda_2 = \frac{1}{n} \ln \left(\frac{1 - |\Gamma|}{1 - |\Gamma_F|_{\min}} \frac{|\Gamma_F|_{\min}}{|\Gamma|} \right) \quad 2.16$$

The velocity measurements normally require more precision than the absorption. For this reason, the attenuation obtained from equation 2.16 is substituted into equation 2.15 which gives for the velocity

$$V_2 = \frac{2L F_{\min}}{n - \frac{\beta_2}{(2\pi)^2} \left[\frac{1 - (|\Gamma|+1)\exp(-n\alpha_2\lambda_2) + |\Gamma|\exp(-2n\alpha_2\lambda_2)}{1 - |\Gamma|\exp(-2n\alpha_2\lambda_2)} \right]} \quad 2.17$$

Thus an experimental curve of the thin film reflection coefficient versus frequency can yield both the attenuation per wavelength and the velocity even in highly viscous liquids. The extent to which the precision of the results is dependent on the experimental measurement accuracy is shown by the following expression for the proportional error developed in appendix A

$$\frac{\Delta\alpha_2\lambda_2}{\alpha_2\lambda_2} \approx \pm \frac{(\Delta f + \Delta g)}{2(1-|\Gamma|^2)} \frac{[1 - \exp(n\alpha_2\lambda_2)]}{n\alpha_2\lambda_2}, \quad 2.18$$

where $\Delta f = \Delta|\Gamma_F|_{\min}^2/|\Gamma_F|_{\min}^2$ and $\Delta g = \Delta|\Gamma|^2/|\Gamma|^2$ are respectively the relative experimental error of the interface and thin film reflection coefficient as measured using the Bragg scattering probe. The minimum error occurs when $\alpha_2\lambda_2$ is small. For this case, the proportional error simplifies to

$$\frac{\Delta\alpha_2\lambda_2}{\alpha_2\lambda_2} \approx \pm \frac{(\Delta f + \Delta g)}{2(1-|\Gamma|^2)}$$

and is greater than the average experimental error

$\frac{1}{2}(\Delta g + \Delta f)$ by the factor $1/(1-|\Gamma|^2)$ which may in some cases be quite large since $|\Gamma|^2$ is of the order of 0.6-0.9 for typical liquids and hard solids.

2.4 Theory of Bragg Scattering

The main characteristics of the Bragg scattering probing technique have been assumed in the previous section. This section now provides an analytical description of this technique and of its main features.

Several authors have discussed Bragg scattering and its use as an acoustic probe; Cohen and Gordon (1965), Quate, Wilkinson and Winslow (1965), Gordon (1966) and Lean (1967) are amongst the earliest. Here, the work of Gordon is adapted to the case of Bragg scattering interaction with an acoustic beam traveling along the threefold axis of sapphire.

An acoustic wave propagating in a material produces a periodic change in refractive index which in turn scatters light coherently in a manner similar to a volume diffraction grating. At low acoustic frequencies light is diffracted into its many characteristic orders; this is the Raman Nath regime. At higher acoustic frequencies, a point is reached where only one diffracted beam is present. This is the case of interest here and it is called the Bragg scattering regime. Another way of thinking about this phenomenon is to assume that in passing through such a perturbed region the incident light wave produces an induced light polarization which becomes the source of the scattered light. The amplitude of the induced polarization depends on the strain amplitude

and on the acoustooptic characteristics of the medium. Reintjes and Schulz (1968) have developed an expression for the induced polarization which is valid in any material

$$P_i = \epsilon_{ij} D_k^i p_{jklm} S_{lm} , \quad 2.19$$

where P_i is the induced polarization vector
 ϵ_{ij} dielectric tensor element
 D_k^i incident optical displacement vector
 p_{jklm} photoelastic tensor element
 S_{lm} strain tensor element.

Summation over repeated indices is assumed. Now, for a parametric interaction, if plane waves are considered, the energy and momentum are conserved. Thus, we obtain for the interaction of a light wave with an acoustic wave the frequency and wavevector relations:

$$\omega^i = \omega^d \pm \Omega \quad 2.20$$

$$\underline{k}^d = \underline{k}^i \pm \underline{K} \quad 2.21$$

where uppercase symbols refer to the acoustic wave and lower case symbols refer to optical waves. The superscripts d and i indicate the diffracted and incident waves respectively. When acoustic and light beams of finite cross sections are considered, the scattered light intensity distribution is no more so well defined and there is some angular spread. However, equation 2.21 still determines the geometry for optimum scattering. This is the case for the present work as shown in Figure 2.4. Here, a longitudinal wave is generated by a circular transducer and propagated along the threefold

Figure 2.4 Schematic Presentation of the Acoustooptic Interaction Geometry.

axis of a sapphire rod, which is itself coplanar with the incident light wavevector and optical displacement vector. Uniformity of the light intensity is assumed over the whole interaction region. It is well known that a threefold axis is a pure mode axis for longitudinal waves and hence the acoustic displacement is colinear with the wavevector (Farnell, 1961). Hence, the strain element involved here is S_{33} . Furthermore, the incident optical displacement vector is in the X_1 - X_2 plane and for a trigonal crystal such as sapphire the photoelastic tensor elements satisfy the relation

$$P_{ij33} = 0 \text{ unless } i = j,$$

$$\text{and } P_{1133} \ll P_{3333}.$$

Hence, the induced polarization, as given by equation 2.19, becomes for the present case

$$P_3 = \epsilon_{33} D_3^i P_{3333} S_{33}. \quad 2.22$$

The momentum conservation relation equation 2.21 holds here too, but due to the optical anisotropy of sapphire, care must be exercised in applying this relation. In general, for any given direction, only two independent light waves, having different polarization and wavevector magnitude, can be propagated. The magnitude of the two wavevectors are obtained from the following quadratic relation derived by Born and Wolf (1959)

$$\frac{\mu \epsilon_{kk} k^2}{|k|^2 - \mu \epsilon_{kk}} = 0, \quad 2.23$$

where ℓ_k are the direction cosines of the given direction and ϵ_{kk} are the principal values of the dielectric constant tensor. This equation describes in ℓ space a surface of two sheets and each sheet corresponds to the wavevector locus of one particular mode. The momentum conservation relation assures that the three wavevectors involved are coplanar. Since, here, $\underline{k}^i$ and $\underline{K}$ are both in the ℓ_1 - ℓ_3 plane, so will be $\underline{k}^d$ the diffracted wavevector. Hence, the portion of the wavevector surface which governs the behaviour of both the incident and diffracted optical wavevector is the intersection of the wavevector surface and the ℓ_1 - ℓ_3 plane. Figure 2.5 shows the resulting two curves for the case of sapphire. The circle is the wavevector surface for the ordinary polarization where the optical displacement is normal to the ℓ_1 - ℓ_3 plane. The other curve applies to the so-called extraordinary polarized wave where the optical displacement vector is in the ℓ_1 - ℓ_3 plane. Here, the induced polarization (and hence the diffracted optical displacement) is in this plane. Since the incident optical displacement is also in the same plane, the curve corresponding to the extraordinarily polarized waves describes the behaviour of both the incident and diffracted wavevectors if these oscillate at the same frequency. This is the case for the acoustic frequency range used here where $\Omega \ll \omega^i$. We then obtain from equation 2.20 the condition

$$\omega^d \approx \omega^i .$$

Figure 2.5 Optical Wavevector Surfaces in a Positive Uniaxial Crystal Such as Sapphire. The Momentum Conservation Relation Is Also Illustrated.

Figure 2.6 Wavevector Diagram for Acoustic Waves Propagating Near the Threefold Axis of Sapphire.

The extraordinary wavevector sheet is symmetrical with respect to the ℓ_1 axis; hence, as shown in Figure 2.5, the maximum signal will occur when

$$\theta^i = \theta^d = \theta,$$

where θ is the Bragg angle which is then given by

$$\sin \theta = \frac{|\underline{K}|}{2|\underline{k}^i|} . \quad 2.24$$

Thus, in the bragg regime, the available information about the acoustic energy in the solid may be obtained by investigating the diffracted light intensity in the neighbourhood of the Bragg angle; the actual angular energy distribution of the diffracted light might be expected to depend on the acoustic strain distribution in the crystal. Hence, the effects of diffraction of the acoustic wave must be investigated. For longitudinal waves propagating near the threefold axis of sapphire, the wavevector surface is a parabola as given by equation 2.5 and shown in Figure 2.6. Hence, for small angles, the wavevector surface approximates very closely the isotropic sphere and, keeping in mind that even for small angles the Poynting vector direction does not coincide with the wavevector direction, we use the isotropic approximation. In the weak scattering approximation, Gordon (1966) obtains in the isotropic case, the ratio of the diffracted light energy density to the incident light energy density to be

$$|V^d(\theta^i)/V^i|^2 = \frac{1}{16} \frac{|\underline{k}_0|^2 p^2 n^6}{\cos^2(\theta^i)}$$

$$\cdot \left| \int_{-\infty}^{+\infty} dx'_1 S(x'_1, x'_2) \exp[-j|\underline{k}| \left(\frac{\sin\theta^i - \sin\theta}{\cos\theta^i} \right) x'_1] \right|^2 \quad 2.25$$

where θ^i is the angle that the incident light wavevector makes with the normal to the acoustic wavevector

x'_1 and x'_2 refer to the transducer coordinates

$S(x'_1, x'_2)$ is the strain distribution at the transducer

n is the refractive index of the medium.

$|\underline{k}_0|$ is the magnitude of the light wavevector in vacuum.

As noted by Gordon, the integral term is proportional to the angular distribution of the acoustic energy and, when attenuation is neglected, it does not change with distance from the transducer. Thus, diffraction of the acoustic wave controls the angular distribution of the acoustic energy and consequently of the diffracted light intensity. This effect is, however, independent of the distance of the interaction region position from the transducer along the acoustic beam direction since the latter quantity does not appear in equation 2.25. If in the magnitude term of equation 2.25 the appropriate quantities applicable to sapphire and obtained from equation 2.22 are substituted, the former equation becomes in the small angle approximation:

$$\left| \frac{v^d(\theta^i)}{v^i} \right|^2 = \frac{1}{16} |\underline{k}^d|^2 p_{3333}^2 \epsilon_{33}^2$$

$$\cdot \left| \int_{-\infty}^{+\infty} s(x_1', x_2') \exp(-j|\underline{K}|\Delta\theta x_1') dx_1' \right|^2$$

where $\Delta\theta = \theta^i - \theta$.

The quantity of interest in this experiment is the scattering efficiency defined as the ratio of the total diffracted energy to the portion of the incident light which is intercepted by the acoustic beam. For a circular transducer of radius a and a light beam of width b , we obtain for these quantities,

$$I^i = |v^i|^2 2 ab,$$

$$I^d = |v^i|^2 b \int_{-\infty}^{+\infty} dx_2' \left| \frac{v^d(\theta^i)}{v^i} \right|^2.$$

This integral is evaluated in appendix B for a circular transducer generating a uniform beam of acoustic energy into an isotropic medium. The expression obtained is

$$\frac{I^d}{I^i} = \frac{|\underline{k}^d|^2}{2} p_{3333}^2 \epsilon_{33}^2 a^2 |s|^2 \frac{G(\gamma)}{\gamma^2} \quad 2.26$$

where $\gamma = 2|\underline{K}|\Delta\theta a$.

$$G(\gamma) = 1 - J_0(\gamma) - 2 \sum_{n=1}^{\infty} \frac{J_{2n}(\gamma)}{1-4n^2}$$

and $J_m(\gamma)$ is the Bessel function of the m^{th} order. The magnitude term of this expression has been modified to account for the sapphire anisotropy but the distribution function

$G(\gamma)$ is only valid for the isotropic case. Cohen (1967) has predicted and verified experimentally that the angular acoustic energy distribution for a wave propagating near a threefold axis is the same as for the isotropic case except for a width factor $1/1-2b$. Hence, the distribution function $G(\gamma)$ is unchanged but γ now becomes

$$\gamma = \frac{2|\underline{k}|\Delta\theta a}{1-2b} . \quad 2.27$$

The total acoustic power available at the transducer is related to the strain magnitude at a distance ℓ from the transducer according to

$$|S|^2 = \frac{2 P_o \exp(-2\alpha\ell)}{\rho V^3 \pi a^2} .$$

Substituting this expression in equation 2.26 one obtains

$$\frac{I^d}{I^i} = \frac{|\underline{k}^d|^2 p_{3333}^2 \epsilon_{33}^2 P_o \exp(-2\alpha\ell)}{\rho V^3 \pi} \frac{G(\gamma)}{\gamma^2} , \quad 2.28$$

where P_o is the total acoustic power available from the transducer,

α is the attenuation of the acoustic wave,

ℓ is the distance of the interaction region from the transducer.

Equations 2.27 and 2.28 can be further simplified by noting that in the small angle approximation

$$\epsilon_{33} \approx n_e^2 , \text{ the extraordinary refractive index}$$

and $|\underline{k}^d| \approx n_e |k_o| .$

Equation 2.28 then becomes

$$\frac{I^d}{I^i} = |\underline{k}_0|^2 M_2 P_0 \exp(-2\alpha l) \left[\frac{G(\gamma)}{\pi \gamma^2} \right], \quad 2.29$$

where

$$M_2 = \frac{n_e^6 p_{3333}^2}{\rho V^3},$$

and equation 2.24 becomes

$$\sin \theta = \frac{|K|}{2n_e |k_0|}. \quad 2.30$$

At Bragg angle, $\gamma = 0$ and $G(\gamma)$ reaches a maximum value of $1/12$. For this case, equation 2.29 has the same form as that obtained by Dixon (1966) for a square transducer. It differs only by a factor of $8/3\pi$. It features the same important property that the total diffracted energy is proportional to the acoustic energy in the interaction region through the factor $P_0 \exp(-2\alpha l)$ and is not affected by the spatial dependence of diffraction of the acoustic wave.

Equations 2.29 and 2.30 govern the diffracted light angle and energy inside the crystal while the actual measured quantities are obtained externally. The effects of the crystal side reflections must then be taken into account. We note from Figure 2.7 that the diffracted and transmitted wavevector inside the crystal impinge on the outward surface at the same angle and thus have the same reflection losses so that

$$\frac{I^d}{I^t} = \frac{I_o^d}{I_o^t},$$

Figure 2.7 Wavevector Diagram for the Present Experimental Conditions.

where the subscript o refers to outside measurements and I^t is the transmitted energy. Since here very little energy goes into the diffracted beam (less than 1%) we make the approximation that $I^t \approx I^i$ and hence at Bragg angle

$$\frac{I_o^d}{I_o^t} \approx \frac{I^d}{I^t} \approx \frac{M_2 k_o^2 P_o \exp(-2\alpha l)}{3\pi} \quad 2.31$$

Furthermore, optical refraction must also be taken into account. Using Snell's law, equation 2.30 becomes

$$\sin \theta_o = \frac{|K|}{2|k_o|} \quad 2.32$$

These are the relations used to describe the behaviour of the Bragg scattering acoustic probe in our experiment.

Within experimental error, the linear dependence of the diffracted light energy to the acoustic energy predicted by equation 2.31 is expected to be observed within certain acoustic power and frequency ranges which are determined by considering the approximations made by Gordon in deriving equation 2.25. They are:

- 1) The diffracted portion of the incident optical energy is small.
- 2) Higher order diffracted beam energy is neglected.

Quate, Wilkinson, and Winslow (1965) have shown experimentally that the former approximation is justified for acoustic powers that diffract up to 10% of the incident light which corresponds to an acoustic power of the order of 10 Watts in our case.

The latter approximation is the condition for the existence of Bragg scattering as discussed by the same authors. This condition is expressed as:

$$|\underline{K}|^2 a / \pi |\underline{k}^i| > 1$$

which sets a lower limit on the acoustic wave frequency to 0.5 GHz for our experimental conditions.

It is useful at this point to obtain an approximation of the angular resolution which is required to perform the Bragg scattering experiment. A numerical computation shows that for $\gamma \approx \pi$, the distribution function $G(\gamma)$ has decreased to half the maximum value. Hence the angular width at mid-power is given by

$$2\Delta\theta|_{1/2} = \frac{n_e (1-2b) V_3}{2aF} . \quad 2.33$$

For a transducer 1 mm in diameter, an order of magnitude for the angular width is 10 mrad for the range frequencies used here. Thus the resolution of the equipment must be within 1 mrad.

2.5 Comparison Between the Two Methods

The principal difference between the method used here and McSkimmin's is in the technique for detecting the acoustic energy. When Bragg scattering is used, as in our method, we have the advantage that the acoustic wave energy can be measured at any point along the direction of propagation. This allows the acoustic energy to be measured before and right after

reflection with the thin film, thus determining the reflection coefficient directly. It also permits us to acquire the necessary information after a minimum pathlength of travel through the buffer rods, and hence to minimize the effects of the intrinsic attenuation and inhomogeneities of the crystal.

The use of light scattering has several advantages other than those enumerated above. In particular, it is shown here that it is practically insensitive to diffraction losses, an advantage which is not shared by the transducer detector. This also implies that the method is insensitive to the anomalies in the radiating pattern of the generating transducer due to pinholes or other imperfections which would normally render a transducer unusable as a detector.

As for the experimental procedure itself, the only advantage resides in the fact that it allows extraction of data with a minimum of assumptions on the size of the quantities to be measured. On the other hand, it requires an additional experimental measurement, the reflection coefficient at an interface.

Most of the advantages enumerated above become more and more important at high frequencies; this is why it is believed that the Bragg scattering probe will permit the frequency range of study to be extended by a factor of three to four over McSkimmin's method.

CHAPTER III
EXPERIMENTAL APPARATUS

3.1 Introduction

In this chapter, the design and testing of the equipment is described. The various components studied are:

- 1) Bragg scattering probe and auxiliary equipment
- 2) Acoustic delay line or buffer rods
- 3) Liquid film
- 4) Transducer
- 5) rf driving and power measuring equipment.

The description of the experimental procedure and results for evaluating the above components is included and the overall performance of the designed optical probe is compared with theory.

3.2 Optical and Mechanical Apparatus

3.2.1 The Bragg Scattering Probe

The important features of the Bragg scattering acoustic probe are its translational and angular mobility as seen from the discussion in the preceding chapter. Hence, the design of the probe consists in fabricating a mechanical system that will allow a variable and precise alignment of the acoustic and optical beams. Figure 3.1 is a schematic presentation of such an apparatus. The mechanical and optical system consists of three distinct sections mounted independently on an optical bench: the source section comprises

Figure 3.1 Schematic Presentation of the Bragg Scattering Probe Arrangement

an helium neon laser radiating at $6328 \text{ \AA}$ (Spectra Physics, model 124), a lens and a spatial filter which is used to "clean" the laser beam. The lens in combination with the spatial filter provides for the necessary divergence and diameter enlargement of the laser light beam. This whole assembly is kept fixed except for the position of the lens which can be varied over a distance equal to one fifth of its focal length.

The specimen under study is mounted on two translation stages, one horizontal and one vertical. These allow a variable spatial alignment of the acoustic beam with respect to the light beam. Accurate angular rotation of the specimen about the vertical axis is achieved through the use of a solid bar 8 cm long fixed to the mount at one end and driven by a micrometer screw at the other end. The third section consists of the optical detection apparatus. The optical detector itself is mounted on a base along with a variable iris and a second lens of short focal length which is used to enlarge the field of view of the detector. In order to obtain alignment of the detector with the diffracted light beam, the base is mounted on a horizontal translation stage. A rigid bar, free to rotate about the specimen vertical axis and rigidly attached to the base, keeps the detector axis in line with the interaction region.

Each of the sections described above is attached to the optical bench by means of a baseplate which can be translated along the optical bench axis, thus adding another

degree of freedom and improving the mechanical stability. All the angular and translational motions of the specimen and the lateral motion of the detector are monitored using a ten turn potentiometer attached to the micrometer screws.

The diffracted light intensity is detected using a photomultiplier (S-20 R.C.A., type 7265) while the video signal is displayed on an oscilloscope equipped with a differential plug in amplifier that has a variable gain vertical output (Tektronix 1A7A). This arrangement provides a visual display of the energy of the rf acoustic pulses as they pass in front of the light beam but it cannot be used for accurate magnitude measurements due to the photomultiplier noise present on the signal. To overcome this inconvenience, a boxcar integrator is included as shown in the circuit of Figure 3.2.

The amplified signal available at the vertical output of the plug in unit is fed into a boxcar integrator (Princeton Applied Research, model CW-1) whose minimum gate width is 1 μ sec. If the gate is centered on a particular impulse and the gated signal integrated with a suitable time constant (30 msec.), the photomultiplier noise is greatly reduced and the output voltage of the integrator is proportional to the pulse amplitude. The integrator also features an X-Y recorder output which is very useful in obtaining permanent records of the experimental results.

Figure 3.2 Electronic Circuit Used for Measuring the Diffracted Light Intensity.

One of the principal assumptions used in developing the Bragg scattering interaction theory is that the light beam has a constant intensity distribution and is much larger than the acoustic beam in a direction transverse to both the acoustic and light wavevectors. This condition is approximated experimentally by making the laser beam 5 mm in diameter (about 4 times the acoustic beam diameter) and reduced laterally to about 1.5 mm in passing through a vertical slit before reaching the crystal. The divergence of the beam is kept to a minimum which should be near the diffraction limit. The gate of the integrator is now centered on a particular pulse by making the gate signal coincide with its expected position on the oscilloscope display. Using the normal to the crystal side as a reference, the crystal and the detector angular positions are set to the calculated Bragg angle. At this point, a small signal is visible on the integrator meter. Final optimization of angles, relative beam position, and gate timing is obtained by maximizing the signal output using the different fine adjustments described in the preceding section. The signal thus obtained is fed to the Y axis of the recorder which should then read a voltage proportional to the energy content of the monitored acoustic pulse. By connecting the different potentiometers to the X axis, the acoustic energy can now be recorded as a function of various parameters like position, angle, delay time, etc.

3.2.2 The Buffer Rods

Since the acoustic probing is carried out in the buffer rods and since these serve as physical boundaries for the thin film, the choice of the buffer rod material is important; it must offer a combination of suitable elastooptic, optical transmission, and mechanical properties. A suitable buffer rod material should have low attenuation, small acoustic wave velocity, and a large coupling between the light and acoustic displacement. The latter condition is described quantitatively in equation 2.29 as the elastooptic coupling constant M_2 . The former conditions are arrived at in the following way: Since the minimum time between two pulses is determined by the minimum gate width of the integrator and the pulse length, the attenuation between two pulses is proportional to $\alpha_1 V_1$ the attenuation per unit time which is to be chosen small. In addition, consideration of equations 2.18 and 2.10 shows that the acoustic impedance ($\tilde{\rho}_1 V_1$) must be kept to a minimum if the experimental errors are to be minimized. The material chosen for this work was C-oriented α - Al_2O_3 . Its characteristics are compared with two similarly oriented crystals in Table 3.1.

Table 3.1

Material	$\alpha \text{VdB}/\mu\text{sec. at 1 GHz}$	$Z \times 10^{-5}$	$M_2 \times 10^{12}$
LiNbO_3	0.25	33.0	7.00
$\alpha\text{-Al}_2\text{O}_3$	0.20	44.6	0.66
$\alpha\text{-SiO}_2$	~3.00	16.7	1.30

Although a LiNbO_3 crystal would have been preferable, the $\alpha\text{-Al}_2\text{O}_3$ crystal was chosen because it is much cheaper and was readily available in good optical quality and suitable sizes.

The sizes of the rods are governed by the following considerations:

- 1) Their cross sections must be large enough so that, due to diffraction, a negligible amount of acoustic energy strikes the walls.
- 2) The length of the first rod must allow time resolution of 1 μsec . pulses.
- 3) The length of the second rod is chosen so as to eliminate the interference effects due to overlapping pulses.

The two sizes of sapphire rods available, that is 1" X 1/4" X 1/4" and 1/2" X 1/4" X 1/4", met these specifications. Their optical quality and polish were found suitable for the purpose of the experiment.

3.2.3 The Liquid Thin Film

The liquid thin film must now be formed between these two rods. For this purpose two strips of chromium to serve as spacers are sputtered on each side of one end of the shortest buffer rod. Chromium is chosen because of its hardness and good adherence to sapphire. The thickness of the liquid film is chosen in such a way as to yield ten or

more minima of reflection in the frequency range available (0.5 to 2.5 GHz). According to equation 2.14, the thickness of the liquid film should be approximately

$$L = \frac{Vn}{2\Delta f} \approx 5\mu.$$

The strips effectively produced were 4.2 μ thick as measured using the Fizeau plate optical method.

The two ends of the rods are then carefully cleaned under a microscope to eliminate all the dust particles. A small drop of liquid is placed between the strips; the amount of liquid must be just sufficient to form a film that will not touch the strips once formed. The crystal is also heated to about 60°C to increase the fluidity of the liquid and hence aid in the formation of the film. The two buffer rods are then brought in close contact and held together by a spring loaded teflon sphere which serves the purpose of applying only axial forces on the buffer rods. This arrangement is shown in Figure 3.3. To avoid any further changes in the geometry of the liquid film thus formed, the temperature of the film is cycled a few times.

By looking through the side of the crystal at approximately Brewster angle, the film is clearly seen. If one uses an extended monochromatic source (Cesium lamp radiating at 0.54 μ), localized fringes appear in the film. The thickness uniformity of the liquid film is considered acceptable if it exhibits half a fringe or less over its whole area; this

Observation Direction

Figure 3.3 Apparatus for the Construction and Testing of the Liquid Thin Film

indicates a uniformity better than $\lambda/10$ at the maximum acoustic frequency available (2.5 GHz). A careful examination of the liquid film under the microscope is also necessary to detect the presence of gas bubbles that would scatter the acoustic energy.

Many attempts were made before obtaining an acceptable film and the realization of such a film turned out to take an important period of time.

3.2.4 The Transducer

The piezoelectric transducer used in this work is a thin film ZnO layer.* This film is sputtered on the end of the first buffer rod on an intermediate gold electrode. The ZnO film used here is 1.5μ thick and C-axis oriented. Such a transducer has a fundamental frequency for longitudinal waves of 2 GHz. The effective diameter of the transducer is defined by a gold disc electrode 1.3 mm in diameter sputtered on top of the ZnO layer.

The circuit that is used to drive the transducer and measure the power is described in Figure 3.4. It consists of a standard combination of high power microwave generator (Airborne Instrument Laboratory, type 125) and pulse generator (Hewlett-Packard, model 214A). The impedance matching is

*Supplied by E. Lefebvre-Ganne and fabricated according to the procedures outlined in Lefebvre-Ganne (1969).

Figure 3.4 rf Power Transducer Drive and Measuring Circuit.

achieved through the use of an rf bridge (Alford, type 1318 and 1134) and a double stub tuner (Weinschell Engineering, model DS-109-2). When the impedance of the transducer is properly matched, the input power splits equally between ports 2 and 3 of the bridge, thus permitting measurement of the power input to the transducer by means of a peak power detector connected at port 2 (Boonton Radio, type 8900A) and a precision step attenuator (Hewlett-Packard, model 355-C). In addition, a standard detecting circuit (local oscillator + mixer + i.f. amplifier) is connected at port 4 which allows a visual display on the oscilloscope of the echoes detected by the transducer. Attenuators A1 and A4 are included for convenience.

3.3 Preliminary Experiments

Before proceeding with the main experiment the performance of the equipment is evaluated. In particular, the linearity of the probe, the various experimental errors, and the dependence of the total diffracted light power on the angle of incidence of the laser light beam are investigated.

The first experiment consists in testing the linear dependence of the detected optical signal on the rf input power to the transducer. For this experiment only the first buffer rod is used. The rf power source is tuned to 1.0 GHz and set for maximum output while the input power to the transducer is maximized using the double stub tuner. After aligning the probe to monitor the first detected acoustic

pulse, the power to the transducer is varied by means of a step attenuator and measured using the peak power detector arrangement described earlier, while the output of the integrator is recorded. The results are shown in Figure 3.5.

Within the precision of the peak power detector and attenuator (± 0.1 dB), the relation is found to be linear over a 20 dB range. The origin of Figure 3.5 corresponds to the maximum rf power available from the generator and measured to be 9 Watts* while the -20 dB point corresponds to a detected signal to noise ratio of approximately one as seen on the oscilloscope; this determines the lowest limit of acoustic power that can be detected to within ± 0.1 dB since at that point the ratio of the fluctuations of the integrator output to the signal begins to increase. Consequently, through the increase in the buffer rod attenuation and the transducer insertion loss, this determines also the highest frequency usable experimentally (here $F_{\max} \approx 2.3$ GHz). This procedure gives the linearity of the entire system which includes the Bragg scattering probe and auxiliary equipment.

The observed fluctuations of the integrator output are present at any acoustic level or frequency. In order to

*For the transducer used, this corresponds approximately to an acoustic power of 0.1 Watts. This is well within the linearity limits of equation 2.29.

Figure 3.5 Relative Intensity of Bragg Scattered Signal (Integrator Signal) vs Electrical R.F. Input Power to the Transducer ($F = 1.0$ GHz).

measure the extent to which they influence the results, the signal is recorded during a period of 1 minute for different power levels and frequencies. The ratio of the amplitude of the fluctuations to the signal level was observed to be about 2% and independent of frequency for the frequency range of interest here. This behaviour and the magnitude of the fluctuations may be attributed to variations of the laser output. These are specified by the manufacturer to be 2%.

In the main experiment two pulses whose magnitudes differ by less than 3 dB are compared. For such a case, the non-linearity error may be neglected and the main source of error is the observed fluctuations. The corresponding uncertainty on the ratio of the magnitude of the pulses is ± 0.09 dB.

Another meaningful experiment is the measurement of the diffracted light intensity as a function of the angular position of the acoustic beam with respect to the incident light beam. This experiment is performed using the same alignment procedure as for the preceding experiment except that here the potentiometer related to the angular position of the crystal is connected to the X input of the recorder. The integrator signal is recorded on the chart as a function of the angular position for several frequencies in the range 0.5 to 2.3 GHz for the first three pulses. Typical results thus obtained are shown in Figure 3.6 for a frequency of 1.0 GHz. The Bragg angles obtained from these records are plotted as a function of frequency in Figure 3.7. This is linear and it agrees

Figure 3.6 Relative Intensity of Bragg Scattering Signal vs Angle of Incidence (θ_0^i) for the Experimental Conditions Shown in Figure 3.8 and a Frequency of 1 GHz.

Figure 3.7 Angle of Incidence Corresponding to the Maximum Diffracted Light Intensity (Bragg Angle) vs Acoustic Frequency for the Experimental Conditions Shown in Figure 3.8

Figure 3.8 Actual Transducer Delay Line Arrangement and Suggested Acoustic Wavevector Deflection.

with equation 2.30 in the small angle approximation. However, an extrapolation of the data to zero frequency yields a Bragg angle of 13.6 mrad instead of zero. This deviation from zero is a measure of the angle between the acoustic wavevector and crystal face whose normal was used as a reference in obtaining the data in Figure 3.7. We also note that the Bragg angle of the second pulse differs from that of the first by 0.8 ± 0.15 mrad while the Bragg angle of the third pulse coincides with the first within experimental errors. This discrepancy cannot be accounted for by non-parallelism of the end faces which were supplied parallel within ± 0.02 mrad. Lean (1967) suggests the following explanation: The transducer initially launches an rf pulse whose wavevector is normal to the interface. Due to some residual stress distribution in the delay rod, the wavevector is deflected for both forward and backward paths and then impinges on the transducer as shown in Figure 3.8. This accounts for the discrepancy between the first and second pulse. The rest of the explanation is due to Cohen and Gordon (1965). Since the impedances of ZnO and $\alpha\text{-Al}_2\text{O}_3$ are well matched, little energy is reflected from the transducer-delay rod interface and some portion of it reaches the back surface where it is completely reflected. If, as shown in Figure 3.8, the transducer has a wedge shape this adds up an additional deflection to the wavevector of the third pulse. In the present experiment these two effects just happen to cancel out and the wavevector of the third echo is reflected back in the same direction as of the first

one. An interferometric optical thickness measurement effectively revealed that the ZnO layer was slightly concave and the small gold electrode had been deposited off center, thus yielding an effective transducer which was wedge-shaped.

Additional important data are still obtained from such an experiment. For this purpose Figure 3.9 shows the experimental curves after normalization and corrections are made for the angular shift to permit direct comparison between pulses. These are compared with the predicted curve given by equation 2.29 for the following parameters value:

$$\begin{aligned} F &= 1.0 \text{ GHz} \\ V &= 11.15 \times 10^3 \text{ m/s} \\ b &= 0.16 \\ n_e &= 1.75 \\ a &= 0.57 \times 10^{-3} \text{ m.} \end{aligned}$$

The effective transducer radius "a" is fixed empirically in such a way that for the first pulse, theory and experiment coincide at midheight. With this choice of parameter, the agreement between predicted and measured results, for the first two echoes, is seen to hold within experimental errors for all angles and hence two conclusions may be drawn:

- 1) The theoretical model used to simulate the experimental conditions is valid especially for the assumptions on the uniformity of the mechanical displacement over the transducer area and of the electric field in the interaction region.

Figure 3.9 Normalized Integrator Signal vs Angular Deviation
From Bragg Angle for the First Three Pulses at 1.0 GHz.

In fact, as discussed by Cohen (1967) any non-uniformity would be brought to evidence by asymetries in the diffracted energy angular dependence. 2) The close match obtained for the first two echoes permits us to state that the Bragg scattering probing method is insensitive to diffraction losses, otherwise the distribution would differ in shape. This experiment is also in agreement with Waterman's theory. In equation 2.33 it was demonstrated that the half power deviation angle was given by

$$2\Delta\theta|_{1/2} = \frac{n_e(1-2b)V}{2aF} .$$

Hence, from Figure 3.10 we obtain

$$\frac{n_e(1-2b)V}{2a} = 11.7 \times 10^6 \text{ rad/sec.}$$

Substituting for the actual values of $a = 1.3 \text{ mm}$, we find

$$b \approx 0.18 ,$$

which is near the theoretical value (0.16) predicted by Waterman.

The results for the third pulse deviate from the others. A certain part of the energy (5% to 10%) is seen to be scattered away from the normal beam. This effect was barely detectable at 0.6 GHz, increased up to a maximum at 1.1 GHz, and decreased slowly with increasing frequency. It is noted that similar but not identical irregularities in the angular energy distribution of the third echo was observed for a few different transducers deposited earlier

Figure 3.10 Angular Width at Midheight of the Integrator Signal vs Frequency for the Experimental Conditions Given in Figure 3.8.

on the delay rod, thus suggesting a transducer effect. Since no experiments involved the use of the third pulse, no attempt was made to investigate this effect further.

The purpose of the probe is to measure the ratio of two successive pulses by moving the integrator gate from one to the other. On the other hand, we have seen that the Bragg angle differs by 0.8 mrad for the first two pulses thus requiring readjustment of the angle between the two measurements at high frequencies. In order to avoid this, the laser beam is given a slight divergence which has for effect to broaden the diffracted energy distribution in such a way that their maximum overlap. By setting the angles at the mean value of the two previously measured Bragg angles, a reliable amplitude reading is obtained for both pulses without angular readjustment.

In summary, in this chapter it has been verified experimentally that the designed Bragg scattering probe exhibits the following characteristics:

- 1) It is linear for the range of parameters of our experiment.
- 2) It is insensitive to diffraction losses.
- 3) It allows the measurement of the ratio of the energy of two pulses with an accuracy of ± 0.09 dB.

The design of the Bragg scattering probe has also been finalized and found suitable for the realization of the main experiment.

CHAPTER IV

EXPERIMENTAL RESULTS

4.1 Introduction

In this chapter the experimental results obtained using the Bragg scattering acoustic probe described in Chapter III are presented. The first part covers the measurements of the transducer efficiency and the wave propagation characteristics of the buffer rod material. The overall performance of the acoustic probe is evaluated by comparing these results with values obtained from other sources. In the second part, the results obtained using this method applied to two Dow Corning resins (276-V2 and 276-V9) are given. From the frequency dependence of the thin film reflection coefficient, described by equation 2.13, the absorption and dispersion of sound in the resins are calculated. Finally the results are discussed in the light of the theory given in Chapter II.

4.2 Experimental Results

4.2.1 Acoustic Generation Efficiency

The usual insertion loss measurements utilize the tuned system shown in Figure 4.1. The power source delivers rf electrical power through a matching network to the transducer which converts this power into mechanical power in the form of an acoustic wave propagating in the delay line. The insertion loss is a measure of the efficiency of the conversion process and, as expressed by Liebman (1968),

Figure 4.1 Diagram Showing the Acoustic Wave Generator Arrangement and the Various Loss Contributions to the Insertion Loss Measurement. P_o and P_i Are the Pulses Detected by the Standard Echo Method while P_1 , P_2 , and P_3 are Detected by the Bragg Scattering Method.

it is given by the relation

$$T.I.L. = 10 \log (\eta_{EM} \times \eta_{ME}) + 10 \log (T_1 \times T_2),$$

where η_{EM} is electrical to mechanical conversion efficiency defined as the ratio of mechanical power supplied by the transducer to the mechanical load, to electrical power delivered to the transducer. Conversely, η_{ME} is the mechanical to electrical efficiency and is defined as the ratio of electrical power supplied by the transducer to the electrical load, to mechanical power delivered to the transducer. T_1 is the portion of electrical power transmitted from the line to the transducer and T_2 is the portion of acoustic power transmitted from the acoustic delay line to the transducer. Figure 4.1 illustrates the effect of these factors on the power as it passes through the different parts of the system. Comparison of the amplitude of the input power to the power of the first echo by the standard pulse echo technique gives

$$10 \log P_o/P_i = T.I.L. + 10 \log L_1 L_2 e^{-4\alpha l}$$

where L_1 is the matching network efficiency for the electrical to mechanical conversion process and L_2 is the matching network efficiency for the mechanical to electrical conversion process. Note that the usual procedure of subtracting the delay line loss factor from the measured value of $10 \log P_o/P_i$ only correctly gives the transducer insertion loss if the

matching network is lossless, i.e. $L_1 = L_2 = 1$. Hence, plots of "tuned insertion loss" are not usually a true measure of the transducer quality.

In doing this calculation, two important practical factors have been omitted. First, the acoustic diffraction losses and, second, the apparent losses contributed by angular deviations of the acoustic wavevector in the acoustic delay line due to non-parallelism of the end faces or slight inhomogeneities in the delay line material. These effects cannot be minimized simultaneously since curing one problem has adverse effect on the other.

The Bragg scattering probing technique is preferable since it is insensitive to diffraction losses and the angular position of the laser beam is adjustable. As shown by equation 2.30, the absolute value of the total acoustic power propagating in the delay line may be obtained by comparing the transmitted light power with the light power diffracted from the first acoustic pulse which is proportional to P_1 in Figure 4.1.

The electrical power is measured using the arrangement described in section 3.2.4. By considering Figure 4.1, we see that such an experiment yields

$$10 \log P_1/P_i = 10 \log \eta_{EM} L_1 T_1 e^{-\alpha l}.$$

Except for the delay line loss factor this quantity describes completely the efficiency of the matching network and transducer arrangement used as a generator of acoustic energy. However, since an absolute measurement of the acoustic energy (P_1) was not made, only relative results obtained by taking the ratio of the diffracted light power to the electrical input power are shown in Figure 4.2. The curve shows a sharp decrease in efficiency at 1.35 GHz which was also present in the insertion loss measurements using the standard echo method. This effect is attributed to an electrical resonance in the crystal holder.

4.2.2 Dispersion and Absorption of Longitudinal Waves

Propagating Near the Threefold Axis of Sapphire

The acoustic wave velocity can be obtained from the plot of the Bragg angle vs frequency given in Figure 3.7. More precisely, by writing equation 2.31 as an explicit function of frequency, and by differentiating it with respect to frequency yields in the small angle approximation

$$V = (\lambda_0/2) \frac{\partial \theta}{\partial F} .$$

From the slope of Figure 3.7, we obtain

$$V = (11.08 \pm .08) \text{ km/s} .$$

Since the rods were oriented to within ± 10 mrad, and as mentioned in Chapter III, the wavevector was within ± 15 mrad

Figure 4.2 Relative Acoustic Energy Generator Efficiency vs Frequency as Measured Using the Bragg Scattering Probe.

of the crystal sides, the total deviation from the threefold axis is negligible and the velocity can be directly compared to

$$V_3 = 11.15 \text{ km/s},$$

as quoted by Dixon (1967) and the measured value is within experimental error.

The arrangement for performing all the subsequent experiments is basically the same and is shown in Figure 3.8. The Bragg scattering probe is set to monitor the acoustic energy in the middle of the first buffer rod and the ratio of the second to the first pulse amplitude (P_2/P_1) is measured using the experimental arrangement described in Chapter III. This ratio (R) is in all cases

$$R = \rho e^{-2\alpha l}, \quad 4.1$$

where ρ is a reflection coefficient which depends on the acoustic termination of the buffer rod. If the end of the rod is carefully cleaned, $\rho = 1$, and this procedure will simply give the attenuation losses in the buffer rod material. Figure 4.3 shows a plot of the measured attenuation vs frequency. The full line is the frequency squared dependence given by equation 2.3 and which has been fitted to the value of 0.46 db/in at 1.0 GHz quoted by Oliver and Slack (1966). The experimental error is shown by error bars and is a direct consequence of the uncertainty of the power measurements

Figure 4.3 Attenuation of Longitudinal Waves Propagating Along the Threefold Axis of Sapphire ($T \approx 23^{\circ}\text{C}$).

($\pm .09$ dB). It is noted that all but one point at 1.3 GHz agree with the predicted curve within the experimental error. This discrepancy can be explained by the fact that the little power available from the transducer in this frequency (see Figure 4.2) sets the probe in the power range where the signal to noise ratio is smaller than one and the experimental error increases as explained in section 3.3.

In summary, the Bragg scattering acoustic probe yields results which compare favourably with those obtained by other experimenters. These results, together with those of Chapter III, establish the linearity and reliability of the probe designed for the range of acoustic power and frequencies used in this work.

4.2.3 Reflection Coefficient of the Liquid Film

In order to measure the reflection coefficient of a film the experimental arrangement described in the preceding section is simply modified by acoustically terminating the first buffer rod with the thin film structure. For this case equation 4.1 becomes

$$R = |\Gamma_F|^2 e^{-2\alpha L};$$

the data thus obtained are plotted in Figure 4.4 and 4.5 respectively for thin films made of the Dow Corning resins 276-V2 and 276-V9 according to the procedure outlined in

Figure 4.4 Dow Corning Resin 276-V2 Film Reflection Coefficient vs Frequency as Measured Using the Bragg Scattering Probe. ($L = 4.2\mu$).

Figure 4.5 Dow Corning Resin 276-V9 Film Reflection Coefficient vs Frequency as Measured Using the Bragg Scattering Probe. ($L = 4.2\mu$).

section 3.2.3. The attenuation loss in sapphire is also shown; the reflection coefficient of the liquid film ($|\Gamma_F|^2$) is obtained by simply subtracting the attenuation losses from the experimental data. The points of particular interest, i.e. the reflection coefficients at minima and maxima are given in Figures 4.6 and 4.7 for resins V2 and V9 respectively. Again the experimental errors are shown by error bars. In the two cases, a smooth curve is drawn through all the points of minimum and maximum reflection.

4.2.4 Reflection Coefficient at the Liquid Solid Interface

As discussed in section 2.3.2 we also require a knowledge of the reflection coefficient at a single solid liquid interface. This quantity is obtained experimentally by placing a drop of liquid on the end of the sapphire rod and measuring the incident and reflected acoustic power as described previously. The interface reflection coefficient ($|\Gamma|^2$) is obtained directly by subtracting the attenuation losses from the measured data. The results thus obtained are shown in Figures 4.6 and 4.7 for resins V2 and V9 respectively. As predicted by equation 2.13 the reflection coefficient of the liquid film ($|\Gamma_F|^2$) approaches the interface reflection coefficient ($|\Gamma|^2$) for high frequencies where attenuation losses in the liquid are high. Also for both liquids the value obtained for ($|\Gamma|^2$) is independent of frequency within experimental error for the frequency range

Figure 4.6 Reflection Coefficient at an Interface Between Sapphire and Resin V2. The Maxima and Minima of Reflection of the Corresponding Thin Film Are Also Shown.

Figure 4.7 Reflection Coefficient at an Interface Between Sapphire and Resin V9. The Maxima and Minima of Reflection of the Corresponding Thin Film Are Also Shown.

studied. This suggests from equation 2.10(b) that the liquids are likely to be dispersionless although a knowledge of the attenuation of the liquid is required to justify this conclusion.

4.3 Dispersion and Absorption of Acoustic Waves in the Liquid Resins

4.3.1 Calculation and Accuracy of the Results

The experimental results given in section 4.2.3 and 4.2.4 are now used to calculate the velocity and attenuation of the liquid resins V2 and V9 by means of equations 2.16 and 2.17. The results thus obtained are shown in Figures 4.8 and 4.9. In Figure 4.9, the low frequency measurements of McSkimmin and Chambers (1964) are also shown.

Although care was taken to measure the reflection coefficient accurately ($\pm 2\%$), it is noted that the corresponding absorption per wavelength deduced from these data can have an accuracy of only ($\pm 12\%$). This large uncertainty is a consequence of the functional form of equation 2.18. This error which increases exponentially with film thickness is the reason for using very thin liquid films. To obtain the frequency dependence of velocity and absorption per wavelength using a single liquid film, the film must be several wavelengths thick at the highest frequency studied and in this work the film thickness used was approximately four wavelengths at 2.0 GHz.

Figure 4.8 Absorption Per Wavelength and Velocity of Longitudinal Waves in Dow Corning Resin 276-V2.

Figure 4.9 Absorption Per Wavelength and Velocity of Longitudinal Waves in Dow Corning Resin 276-V9.

The accuracy of the calculated velocity of the liquid is limited by

- 1) Errors in the measured film thickness ($\sim 1\%$)
- 2) Errors in estimating the location of the minima due to attenuation broadening of the resonance peaks ($\sim 1\%$),

thus yielding an error of $\pm 2\%$ on the calculated value of the velocity.

4.3.2 Discussion of Results

The dispersion and absorption of acoustic waves in V2 and V9 resins will now be discussed from the point of view of relaxation processes as first mentioned in section 2.2.1. As shown in Figure 2.1, the liquid is dispersionless except in the neighbourhood of a relaxation frequency whose position relative to a particular frequency range may be inferred from the increasing or decreasing character of the absorption per wavelength with frequency. For resin V2 the absorption per wavelength in the frequency range studied (0.5 to 2.0 GHz) was observed to decrease with increasing frequency. The velocity, on the other hand, is observed to be constant within experimental error in the same range of frequency. This behaviour suggests the presence of a relaxation process resonant at frequencies lower than the frequency range studied here. For resin V9, below 1.3 GHz, the behaviour of the absorption and

velocity is similar to that described for resin V2. This behaviour is in qualitative agreement with the lower frequency measurements (0.2 to 0.8 GHz) of McSkimmin and Chambers (1964). Although no errors are quoted in their work, these are expected to be similar to those found here since Bommel and Dransfeld (1960) in the same laboratory as McSkimmin reported an accuracy of only (10%) on the absolute measurements of attenuation in solids and the error is expected to be worse in liquids. Thus, for reasonable errors on their part and noting the difference in sources of the material, the data obtained are seen to be consistent with that of McSkimmin and Chambers. At frequencies larger than 1.3 GHz, the absorption per wavelength for resin V9 increases with increasing frequency. This suggests the existence of a second relaxational process which would account for the anomalously large attenuation reported by McSkimmin and Chambers and tentatively attributed by them to hysteresis losses. The frequency range studied here does not include the relaxation frequencies of any process. Hence, characterizing quantitatively the mathematical model applicable here must await further experimentation either by the present method using thinner films or by the Brillouin scattering method.

CHAPTER V

SUMMARY AND CONCLUSIONS

The theory, design, and construction of a Bragg scattering technique for measuring the acoustic properties of viscoelastic liquids has been presented. The method uses Bragg scattering of laser light to measure the reflection coefficient of acoustic waves due to a liquid film, formed between two c-axis oriented sapphire rods, as the frequency is varied.

The theory of Bragg scattering for the geometry used is derived in Chapter II together with an analysis of the measuring technique. Relations have also been derived, which relate the sound velocity and attenuation per wavelength in the liquid to the reflection coefficient of the liquid film.

In Chapter III, the actual construction and evaluation of the probe is described; it was found to be linear and accurate to within ± 0.1 dB over a 20 dB range of acoustic power. Using this probe, the velocity and attenuation of longitudinal sound waves in sapphire were measured to be (0.43 ± 0.1) dB/in at 1.0 GHz and $(11.08 \pm .08)$ km/s respectively, as compared to the value 0.46 dB/in quoted by Oliver and Slack (1966) and 11.15 km/s quoted by Dixon (1967). The angular distribution of acoustic energy was also determined and found to agree with the calculations

carried out in section 2.4 for a circular transducer deposited on the end of the rod. The probe was then used to measure the reflection coefficient of two 4.2 μ Dow Corning liquid resin films. From these measurements the liquid resins 276-V2 and 276-V9 were found to be nondispersive within experimental error with values given by $(2.00 \pm .04)$ and $(2.11 \pm .04)$ km/s respectively. The frequency dependence of the attenuation for resin V2 can be described by the relation $\alpha_2 \lambda_2 = - (0.05 \pm 0.02) \ln (F) + (0.15 \pm .01)$ neper, where F is the acoustic frequency in gigahertz. At low frequencies, the frequency dependence of the absorption per wavelength for resin V9 is similar although smaller by about 25%. At the higher frequencies studied, however, the attenuation per wavelength increases with frequency.

The accuracy of these experiments was limited by the available equipment. Further improvements should include the use of a pulse comparator system (Anderson and Chick, 1969). Such a system compares two pulses of the same train and consequently would eliminate the major source of error viz. the fluctuations of the laser light intensity.

Although the results described here are for a liquid film, the method may be used to characterize the acoustic properties of any thin film, liquid or solid, placed on the end of an optically transparent acoustic delay line. A particularly useful application could be the characterization

of the acoustic properties of thin film piezoelectric transducers. This method would then provide additional information about the homogeneity or quality of the transducer.

Appendix A

Uncertainty in Attenuation Per Wavelength Calculation

The value of $\alpha_2 \lambda_2$ is obtained from equation 2.16 as

$$\alpha_2 \lambda_2 = \frac{1}{n} \ln \left(\frac{1 - |\Gamma_F| |\Gamma|}{1 - |\Gamma_F| / |\Gamma|} \right)$$

The error in $\alpha_2 \lambda_2$ can be approximated by the following first order expansion:

$$\Delta \alpha_2 \lambda_2 \approx |\Gamma|^2 \frac{\partial \alpha_2 \lambda_2}{\partial |\Gamma|^2} (\pm \Delta g) + |\Gamma_F|^2 \frac{\partial \alpha_2 \lambda_2}{\partial |\Gamma_F|^2} (\pm \Delta f) \quad A.1$$

where $\Delta g = \Delta |\Gamma|^2 / |\Gamma|^2$

$$\Delta f = \Delta |\Gamma_F|^2 / |\Gamma_F|^2$$

and the min subscript has been omitted. Performing the required partial differentiation yields

$$\Delta \alpha_2 \lambda_2 = - \frac{1}{2n} [(A+B)(\pm \Delta g) + (A-B)(\pm \Delta f)] \quad A.2$$

where $A = \frac{|\Gamma_F| |\Gamma|}{1 - |\Gamma_F| |\Gamma|}$

and $B = \frac{|\Gamma_F| / |\Gamma|}{1 - |\Gamma_F| / |\Gamma|}$

If the value of $|\Gamma_F|$ at a minimum in terms of $\alpha_2 \lambda_2$ and $|\Gamma|$ is substituted, equation A.2 becomes

$$\Delta \alpha_2 \lambda_2 = \left[\frac{\exp(n \alpha_2 \lambda_2) - 1}{2n(1 - |\Gamma|^2)} \right] \{ [|\Gamma|^2 + \exp(n \alpha_2 \lambda_2)] (\pm \Delta g) + [|\Gamma|^2 - \exp(n \alpha_2 \lambda_2)] (\pm \Delta f) \} \quad A.3$$

Since for any value of $\alpha_2 \lambda_2$ we have

$$\exp(\alpha_2 \lambda_2) > 1$$

$$\text{and } 0 < |\Gamma|^2 < 1$$

$$\text{then, } |\Gamma|^2 + \exp(n\alpha_2 \lambda_2) > 0$$

$$\text{and } |\Gamma|^2 - \exp(n\alpha_2 \lambda_2) < 0.$$

Hence, the maximum error on $\alpha_2 \lambda_2$ will occur when the errors on $|\Gamma|^2$ and $|\Gamma_F|^2$ differ in sign and have maximum magnitude.

If $\Delta g = \Delta f$, equation A.3 becomes

$$\frac{\Delta \alpha_2 \lambda_2}{\alpha_2 \lambda_2} = \pm \frac{[1 - \exp(n\alpha_2 \lambda_2)]}{n\alpha_2 \lambda_2 (1 - |\Gamma|^2)} \frac{\Delta g + \Delta f}{2} \quad \text{A.4}$$

If $n\alpha_2 \lambda_2$ is small

$$\frac{1 - \exp(n\alpha_2 \lambda_2)}{n\alpha_2 \lambda_2} \approx 1,$$

and the error is minimized when $\alpha_2 \lambda_2$ is small; it is then given by

$$\frac{\Delta \alpha_2 \lambda_2}{\alpha_2 \lambda_2} = \frac{(\Delta g + \Delta f)}{2(1 - |\Gamma|^2)}$$

Equation A.4 is the relation which is used in Chapter IV to calculate the uncertainty of the absorption per wavelength from the experimental errors in the reflection coefficients.

Appendix B

Bragg Scattering Efficiency for a Circular Transducer

The scattering efficiency has been defined as

$$I^d/I^i = \frac{1}{2a} \int_{-\infty}^{+\infty} dx'_2 |v^d(\theta^i)/v^i|^2 \quad \text{B.1}$$

where

$$|v^d(\theta^i)/v^i|^2 = \frac{|k^d|^2 p_{3333}^2 \epsilon_{33}^2}{16} \left| \int_{-\infty}^{+\infty} S^*(x'_1, x'_2) \exp(-j|\underline{K}|\Delta\theta x'_1) dx'_1 \right|^2. \quad \text{B.2}$$

For a uniform circular transducer the integral in equation B.2 becomes

$$S^* \int_{-\sqrt{a^2-x_2'^2}}^{+\sqrt{a^2-x_2'^2}} \exp(-j|\underline{K}|\Delta\theta x'_1) dx'_1 = 2S^* \frac{\sin|\underline{K}|\Delta\theta\sqrt{a^2-x_2'^2}}{|\underline{K}|\Delta\theta}.$$

Equation B.1 after substitution yields

$$I^d/I^i = \frac{|k^d|^2 p_{3333}^2 \epsilon_{33}^2 |S|^2}{8(|\underline{K}|\Delta\theta)^2 a} \int_{-a}^{+a} \sin^2(|\underline{K}|\Delta\theta\sqrt{a^2-x_2'^2}) dx'_2. \quad \text{B.3}$$

This integration can be evaluated by substituting

$$x'_2 = a \cos\phi.$$

The integral of equation B.3 then becomes

$$\frac{a}{2} \int_0^\pi [1 - \cos(\gamma \sin\phi)] \sin\phi \, d\phi, \quad \text{B.4}$$

where $\gamma = 2|\underline{K}|\Delta\theta a$.

The cosine term may be expanded in its Bessel series

$$\cos(\gamma \sin \phi) = J_0(\gamma) + 2 \sum_{n=1}^{\infty} J_{2n}(\gamma) \cos 2n\phi.$$

Substituting in equation B.4 and performing the integral yields for the scattering efficiency

$$I^d/I^i = |k^d|^2 p_{3333}^2 \epsilon_{33}^2 |s|^2 a^2 \frac{G(\gamma)}{2\gamma^2}, \quad \text{B.5}$$

where

$$G(\gamma) = 1 - J_0^2(\gamma) - \sum_{n=1}^{\infty} \frac{2J_{2n}^2(\gamma)}{1-4n^2}.$$

This is the expression used in equation 2.25 as a starting point to obtain the Bragg scattering efficiency of a circular acoustic beam along the threefold axis of sapphire.

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