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ABSTRACT

Rotational energy levels of axially asymmetric nuclei have been calculated in the manner of Davydov and Filippov, and a comparison with experiment shows good quantitative agreement. It is suggested that agreement may be improved by introducing the Bohr–Mottelson vibration–rotation interaction and a centrifugal stretching correction analogous to the type used in molecular spectra. The D–F method seems to be particularly useful for nuclei in the transition regions between "rotational" and "near-harmonic" modes of collective excitation.

Bohr and Mottelson (1953) have shown that even–even nuclei with spheroidal shapes may be expected to exhibit rotational states with energy levels given by

$$E_{\text{rot}} = \frac{\hbar^2}{2\mathfrak{I}} I(I+1), \qquad I = 0, 2, 4, 6, \ldots$$

where $\mathfrak{I}$ is the effective moment of inertia and I the total nuclear angular momentum. The ratios of the energies represented in this simple level scheme are, $E(4+)/E(2+) = 3.33$, $E(6+)/E(2+) = 7.00$, $E(8+)/E(2+) = 12.00$.

Davydov and Filippov (1958) have extended the treatment to include nuclei with ellipsoidal shapes, i.e. nuclei not possessing axial symmetry. Deviations from axial symmetry may be characterized by the nuclear shape parameter γ, a quantity which varies between 0 and $\pi/3$. Values of γ of 0 and $\pi/3$ characterize prolate and oblate spheroids respectively. The Hamiltonian operator for rotational motion is given by

$$\mathbf{H} = \frac{\hbar^2}{4B\beta^2} \sum_{i=1}^{3} \frac{\mathbf{I}_i^2}{2\sin^2\{\gamma - (2\pi/3)i\}}$$

where B is the mass transport parameter, β is the nuclear quadrupole deformation parameter, and the $\mathbf{I}_i$ are operators of the projections of the total angular momentum along the body-fixed axes of the nucleus. The Hamiltonian is symmetric about $\gamma = \pi/6$ and consequently a prolate ellipsoid of deformation γ ($<\pi/6$) will exhibit the same rotational spectrum as an oblate ellipsoid of deformation $\pi/3-\gamma$.

The energy equations of Dennison (1931) for an asymmetric top may be applied. The symmetry conditions of Bohr (1952) exclude certain energy levels. Resultant curves of relative energy vs. γ are given in Figs. 1 and 2. The $2+$, $4+$, $6+$, $8+$, . . . spin sequence appears and, as long as γ is small, the relative spacing of these levels does not differ appreciably from that of a spheroidal rotor. New levels, of energy $E(2'+)$, $E(3+)$, $E(4'+)$, . . . , appear,

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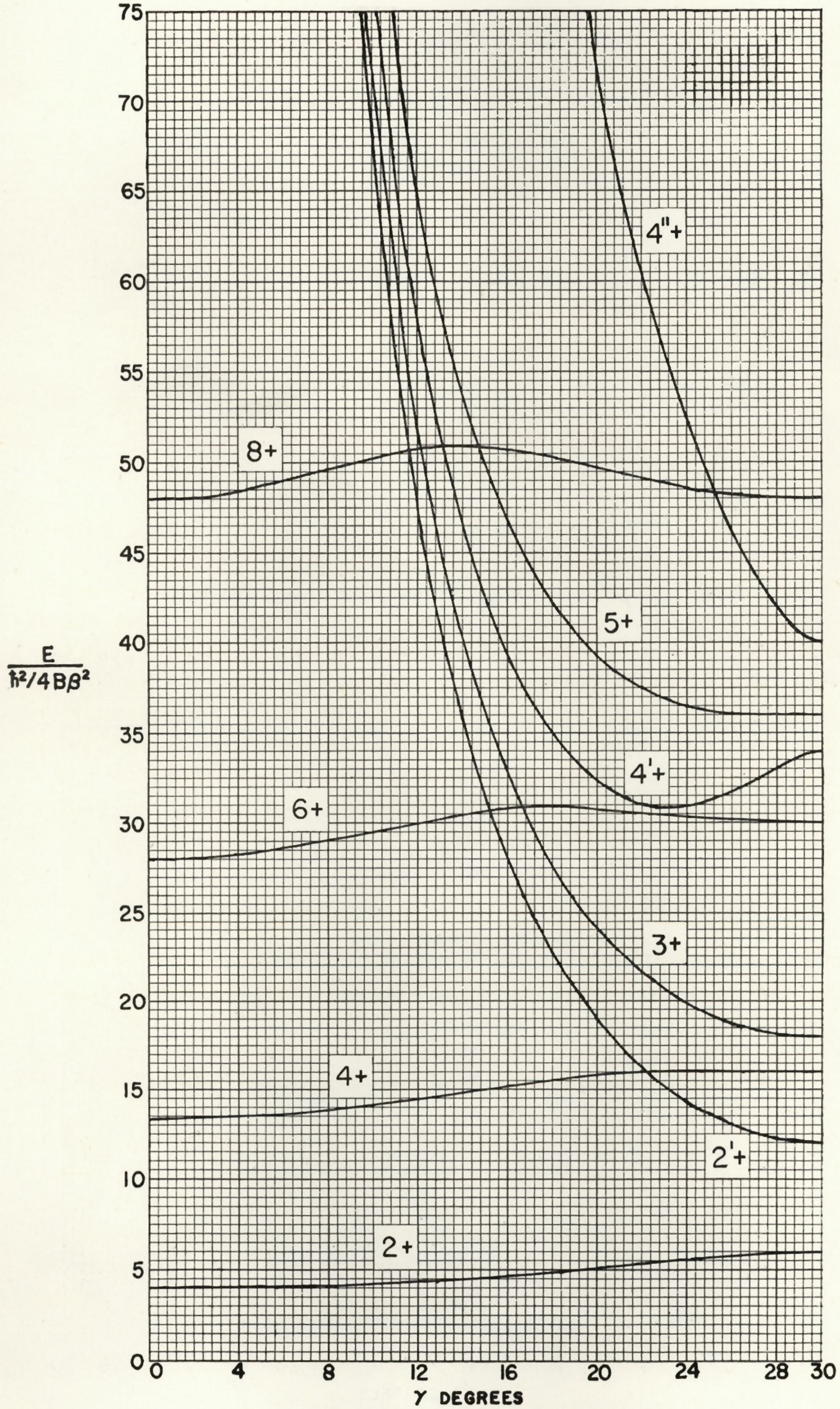

FIG. 1. Rotational energy levels of axially asymmetric even–even nuclei.

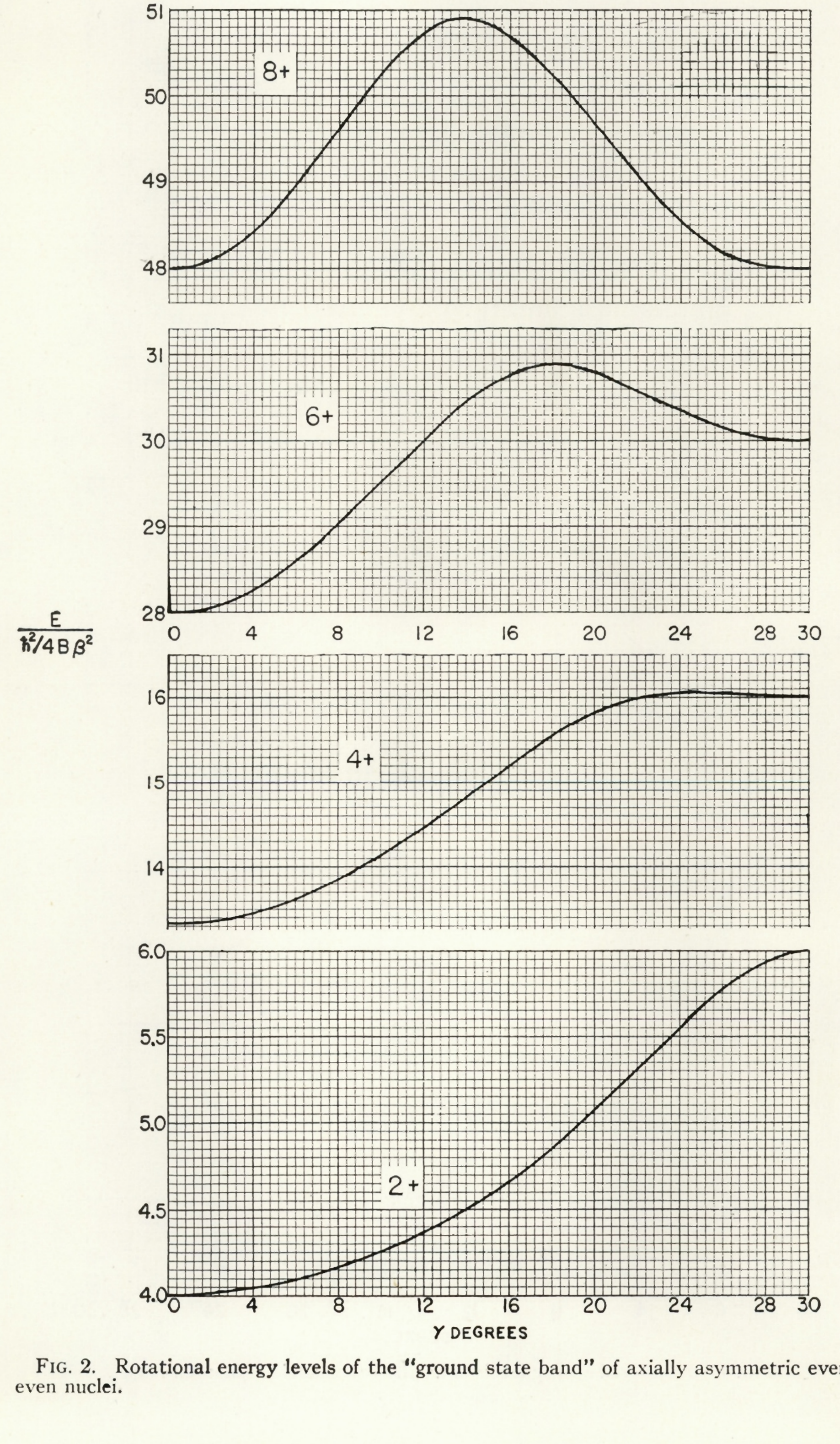

FIG. 2. Rotational energy levels of the "ground state band" of axially asymmetric even-even nuclei.

TABLE I

Rotational energy levels of some even–even nuclei. Theoretical values are for γ determined from $E(2'+)/E(2+)$

				E (3+)		E (4+)		E (6+)		E (8+)		E (4′+)		E (5+)		E (6′+)		
Nucleus	E (2+)	E (2′+)	γ	Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	References
$_{62}Sm^{152}$	121.8	1092	13.2	1240	1214	367	402											
$_{64}Gd^{154}$	123.1	998	13.9	1130	1121	371	406											
$_{66}Dy^{160}$	86.5	966	11.9	1049	1052	283	287					1156	1165					Grigor'ev (1958)
$_{74}W^{182}$	100.1	1222	11.4	1332	1322	329	332	677	690			1501	1439	1620	1612	1810	1770	Gallagher (1958)
$_{74}W^{184}$	111.2	904	13.8	1006	1015	364	367											
$_{76}Os^{186}$	137.2	768	16.6			434	445	869	896									Diamond (1958)
$_{76}Os^{188}$	155.0	633	19.1			478	490											Diamond (1958)
$_{76}Os^{190}$	186.7	558	22.2	754	745	548	560	1048	1070	1670	1716	956	1084					Diamond (1958) Nielsen (1959)
$_{76}Os^{192}$	205.8	489	25.0	691	695													
$_{78}Pt^{192}$	316.5	613	~30	921	929	785	844											Kawamura (1958)
$_{90}Th^{228}$	57.5	965	9.7	1017	1022	186	191											
$_{94}Pu^{238}$	44.1	1030	~8	1071	1074	146	147	304	307	514	525							
$_{94}Pu^{240}$	42.9	1017	~8	1060	1060	142	143	292	299									Davydov (1958)

NOTE: Experimental energy values have been taken for the most part from Strominger, Hollander, and Seaborg (1958). Additional references are given in the last column. Energies are given in kev and γ in degrees.

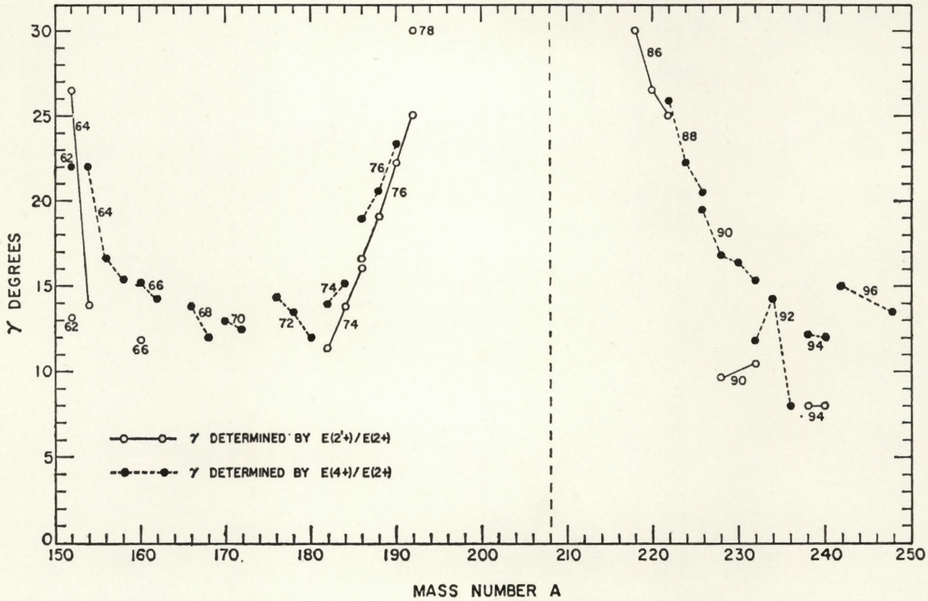

FIG. 3. Variation of the nuclear shape parameter γ with mass number A. Figures indicate proton number.

however, as a direct consequence of axial asymmetry. If, as proposed by Davydov, γ is determined from the ratio $E(2'+)/E(2+)$ and the experimental value of $E(2+)$ assumed, the energies of the remaining levels may be predicted for even–even nuclei with large quadrupole deformations. A comparison of these predictions with experimental data in the range $150 < A < 250$ is shown in Table I. All spin assignments are those given by the authors of the reference quoted. In general, agreement is good—especially in the tungsten–osmium transition region and in the region of heavy nuclei. In these regions discrepancies are typically of the order 1 to 3%, with the exception of the $4'+$ level of Os^{190} where the discrepancy is 13%.

Figure 3 shows the variation of γ with mass number A for the range $150 < A < 250$. Since the $2'+$ level is not always known, γ has been calculated from the ratio $E(4+)/E(2+)$ in some cases. γ calculated in this manner is always somewhat larger than that given by the $E(2'+)/E(2+)$ ratio. The three transition regions from "rotational" to "near-harmonic" level schemes (Scharff-Goldhaber *et al.* 1958) are clearly shown for $A \sim 150$, 190, and 220. The samarium–gadolinium group ($A \sim 150$) exhibits a very fast transition, whereas the tungsten–osmium group ($A \sim 190$) and the radon–radium–thorium group show a more gradual transition.

Qualitative arguments indicate that the minimum finite value of γ for a stable deformation cannot be small, since the frequencies of vibration and rotation would then be comparable, and separation into rotational and vibrational levels meaningless. The data in Fig. 3 would seem to support this. However, most of the low values of γ were obtained from the $E(4+)/E(2+)$ ratio. It is well to keep in mind that low values of γ obtained from

TABLE II

Energy levels of some even–even nuclei. Theoretical values are for γ determined from levels within the "ground state rotational band"

Nucleus	E (2+)	γ	E (4+)		E (6+)		E (8+)		E (2′+)		References
			Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	Expt.	Theor.	
$_{72}Hf^{176}$	88.4	14.4	290.5*	—	596	596					
$_{72}Hf^{178}$	93.1	13.5	306.8*	—	633	633	1060	1060			
$_{72}Hf^{180}$	93.3	11.9	308.8	308.8	642	642	1085*	—			Scharff-Goldhaber (1957)
$_{74}W^{182}$	100.1	14.1	329.3	329.6	677.1*	—			1222	794	Gallagher (1958)
$_{76}Os^{186}$	137.2	18.3	434.1	438	869*	—			768	618	Diamond (1958)
$_{76}Os^{190}$	186.7	23.2	547.9	549.5	1048	1042	1670*	—	558	510	Diamond (1958) Nielsen (1959)
$_{92}U^{232}$	47.2	13.5	156.3	156	321*	—					
$_{92}U^{234}$	43.5	13.1	143	144	296	297	499*	—			
$_{92}U^{236}$	45.3	10.9	151	150	313*	—					
$_{94}Pu^{238}$	44.1	11.6	145.9	145.7	303.5*	—	514	515	1030	517	
$_{94}Pu^{240}$	42.9	12.0	142*	—	292	295			1017	466	Davydov (1958)
$_{96}Cm^{242}$	42.1	14.4	138	139	284*	—					

NOTE: γ has been determined from the ratio of the energy of the level marked with an asterisk to that of the first 2+ level. The choice of level takes into account the relative sensitivities of the various ratios to changes in γ, as well as the experimental accuracy of the energy measurements.

Experimental energy values have been taken for the most part from Strominger, Hollander, and Seaborg (1958). Additional references are given in the last column. Energies are given in kev and γ in degrees.

ratios of the energies $E(2+)$, $E(4+)$, $E(6+)$, $E(8+)$ do not necessarily prove the existence of a stable deformation of this magnitude. For values of γ up to approximately 15°, the effect of assuming a γ is equivalent to introducing the Bohr–Mottelson vibration–rotation interaction correction of the form $-bI^2(I+1)^2$. Values of γ obtained from the $E(2'+)/E(2+)$ ratio are not subject to this objection.

Table II gives the results of calculations of energies of what has been termed the "ground state rotational band", 2+, 4+, 6+, 8+, ... If γ is determined not from the ratio $E(2'+)/E(2+)$ but rather from any two members of the "ground state band", then the energies of the remaining two levels of this band may be predicted to an accuracy usually well within 1%. However, energies of the 2′+, 3+, 4′+, . . . levels cannot then be predicted with any precision. The data is consistent with assuming a higher value of γ for the "ground state band" than for the 2′+, 3+, 4′+, . . . levels.

A qualitative argument may help to clarify this point. Two corrections which may be of importance in the Davydov–Filippov treatment are the vibration–rotation interaction correction and a centrifugal stretching correction. As in molecular spectra these corrections have the form $-bI^2(I+1)^2$ and become more important as the equilibrium deformation decreases. For the "ground state band" it is the β deformation that is important and for the 2′+, 3+, 4′+, . . . levels it is the γ deformation that is important. The effect of this correction to the energies of the levels of the "ground state band" is equivalent to an upward shift of γ of several degrees. Since γ is relatively insensitive to the ratio $E(2'+)/E(2+)$, the correction which must be applied to the 2′+ level does not appreciably affect γ as determined from this level. Thus the difference between γ calculated from the "ground state band" energies and γ calculated from the $E(2'+)/E(2+)$ ratio is quite naturally accounted for by the type of correction arrived at by analogy with molecular spectra.

This paper has shown that the Davydov and Filippov treatment can successfully account for a large number of nuclear energy levels. It cannot, however, predict all the low-lying excited levels, for it is known that other 0+ and 2+ levels exist which cannot arise as excited states of a rigid asymmetric rotor. This point has been clearly shown in a recent paper by King and Johns (1959) concerning the energy levels of Os^{188}.

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REFERENCES

Bohr, A. 1952. Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 14.
Bohr, A. and Mottelson, B. R. 1953. Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16.
Davydov, A. S. and Filippov, G. F. 1958. Nuclear Phys. **8**, 237.
——— 1958. Acta Phys. Acad. Sci. Hung. **9**, 169.
Dennison, D. 1931. Revs. Modern Phys. **3**, 280.

DIAMOND, R. M. and HOLLANDER, J. M. 1958. Nuclear Phys. **8**, 143.
GALLAGHER, C. J., JR., and RASMUSSEN, J. O. 1958. Phys. Rev. **112**, 1730.
GRIGOR'EV, E. P., ZOLOTAVIN, A. V., and KRATSIK, B. 1958. Izvest. Akad. Nauk S.S.S.R. Ser. Fiz. **22**, 785.
KAWAMURA, M., AOKI, A., and HAYASHI, T. 1958. J. Phys. Soc. Japan, **13**, 1071.
KING, W. J. and JOHNS, M. W. 1959. Can. J. Phys. **37**, 755.
NIELSEN, O. B., POULSEN, N. O. R., SHELINE, R. K., and JENSEN, B. S. 1959. Nuclear Phys. **10**, 475.
SCHARFF-GOLDHABER, G. 1957. Proceedings of the University of Pittsburgh Conference on Nuclear Structure.
SCHARFF-GOLDHABER, G., ALBURGER, D. E., HARBOTTLE, G., and MCKEOWN, M. 1958. Phys. Rev. **111**, 913.
STROMINGER, D., HOLLANDER, J. M., and SEABORG, G. T. 1958. Revs. Modern Phys. **30**, 585.

NOTE ADDED IN PROOF.—The authors are pleased to note that Davydov and Rostovsky (Nuclear Phys. **12**, 58, 1959) in a paper on transition probabilities give energy levels nearly identical with the above values.