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A SOLUTION PROCEDURE FOR THE DESIGN PROBLEM OF A MULTIPLE TREATMENT PLANT SYSTEM ALONG A STREAM

by
Demetrios Panagiotakopoulos

Department of Civil Engineering
and Applied Mechanics

McGill University
Montreal

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Working Paper

A SOLUTION PROCEDURE FOR THE DESIGN PROBLEM OF A MULTIPLE TREATMENT PLANT SYSTEM ALONG A STREAM

by

D.Panagiotakopoulos, Ph.D.
Department of Civil Engineering
McGill University
Montreal,Canada

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ABSTRACT

The design problem of liquid waste treatment plants in a multi-plant system along a stream entails the selection of a sequence of unit operations for each plant, as well as the determination of each operation's waste removing efficiency, so that the dissolved oxygen standards along the stream are met at minimum cost. The suggested solutions have so far been partial and suboptimal since they consider only the efficiencies on a fixed sequence of unit operations, neglecting the question of choice among them. This paper outlines an efficient two-phase procedure for completely solving this multi-plant system problem by determining for each plant both the best set of operations and their efficiencies. First, a network algorithm is developed for generating concave cost-efficiency curves for each plant such that each plant efficiency level corresponds to the optimal rather than to a pre-specified fixed sequence of operations. Second, a linear programming model with concave and separable objective function is employed for allocating treatment requirements among the plants. The procedure is compared with existing ones through an application to a real case.

A SOLUTION PROCEDURE FOR THE DESIGN PROBLEM
OF A MULTIPLE TREATMENT PLANT SYSTEM ALONG A STREAM

by D. Panagiotakopoulos, Ph.D.[1]

1. Introduction

Wastes can in general be looked upon as undesirable but unavoidable loads or burdens on various environmental media while waste treatment essentially entails reduction of these loads. With regard to liquid wastes, for which the underground or surface bodies of water receiving them constitute normally the burdened environmental media, one or several different forms of loads can be recognized and dealt with through various, often load-specific, treatment processes. Depending upon the level of the system one is dealing with, a waste treatment process might refer to a particular unit operation within a waste treatment plant (e.g., secondary clarifier), the whole treatment plant consisting of a number of unit operations in series, or even a system of such plants along a stream. The level of load reduction attained by a treatment process is a measure of the efficiency of the process.

Figure 1(a) shows a treatment process k which reduces an incoming waste load of I units by $e_k I$ units ($o \leq e_k < 1$) while $x_k I$ waste units are retained. An arc representation of this process is shown in Figure 1(b) where a waste inflow of I units at node i is reduced to $x_k I$ units along arc k; x_k is called the transformation coefficient of process k. Either e_k or x_k provides a measure of the process' efficiency. Typical cost-efficiency curves for a treatment process, whether it be a single unit operation or a system of plants, is shown in Figure 2; the process cost $C_k(e_k)$ is, in general, a nondecreasing function of e_k .

The design and management of a waste treatment plant along a stream involves two questions: first, the choice of the unit operations in series which will constitute the plant; second, the choice of efficiency level for each of these unit operations. The objective is to attain some desirable overall plant efficiency, with regard to a particular type of waste load, at minimum cost. A treatment process consisting of a number of operations in series is shown in Figure 3; if t_q is the transformation coefficient of operation q, the load

[1] The author wishes to thank Professor J. A. Buzacott and an anonymous referee for their helpful comments on revising an earlier draft of this paper.

fraction removed is $1-t_1t_2t_3 = 1-x_k$ while the process cost is $C_k(x_k) = C_1(t_1) + C_2(t_2) + C_3(t_3)$. Figure 4 shows a network model of a plan for a liquid waste treatment plant; the waste load is in terms of biochemical oxygen demand (BOD). As described in Table 1, this plan is identical to that studied by ECKER[1], ECKER AND MCNAMARA[2], and SHIH and KRISHNAN[7], and is also employed as an example in this work. Each path from node 1 to node 6 corresponds to a specific set of unit operations or, equivalently, to a specific plant design; the alternatives of "no plant" and "only primary treatment" correspond to paths along arcs 13 and 1-12, respectively. For a particular path (i.e., design) p, plant coefficient V_p is defined as,

$$V_p = \prod_{r \epsilon p} t_r \qquad (1)$$

whereupon the plant efficiency is $1-V_p$

The derivations of the cost functions for the unit operations listed in Table 1 are based on the work by SMITH[8] while they are further discussed and refined by Shih and Krishnan[7] and Ecker and McNamara[2]. The development of the cost-efficiency curve for a treatment plant requires the identification of the unit operations and their efficiencies which would attain various plant efficiencies at minimum cost. (In case of an existing plant which consists of a fixed set of operations, only the operations' efficiencies are to be selected.) A mathematical model for this problem is as follows:

(I)

From a network plan outlining all feasible plant designs, and for a minimum desirable plant efficiency 1-R, choose a path p and determine each t_r on it in order to

Minimize $C_p(V_p) = \sum_{r \epsilon p} C_r(t_r)$

such that

$$(V_p =) \prod_{r \epsilon p} t_r \leq R \qquad (2)$$

and

$$o < t_r^- \leq t_r \leq t_r^+ \leq 1 \quad \text{for each } t_r \qquad (3)$$

where $[t_r^-, t_r^+]$ is the feasible range of t_r values.

The above problem has attracted the interest of many analysts over the last decade and it will continue to do so in the future as the need for more and better-designed waste treatment plants keeps growing. In 1964, LYNN[5] simplified (I) into a linear programming network flow problem by assuming piece-wise linear cost functions; although highly unrealistic, that work provided a base for future applications of mathematical programming techniques to the problem. A few years later, Shih and Krishnan[7] employed dynamic programming for solving (I) in an attempt to deal with realistic cost functions. The major drawback of their approach is that the plant must be broken into sequential stages — a potentially complex exercise when more than one operation must be grouped into a single stage; for instance, arcs 2 to 6 in Figure 4 comprise stage 2 while arcs 7 to 11 comprise stage 3. Clearly, this is not a straightforward dynamic programming case and there can be no generalized solution scheme for any network plan. Finally, Ecker and McNamara[2] used geometric programming to solve (I); the disadvantages of this technique are, first, that the $C_r(t_r)$ curves must be smooth and continuous (as given in Table 1) when in reality they are discrete, and second, that a separate geometric program needs to be formed and solved for each and every path p of the network plan.

A simpler, more efficient, and generalized dynamic programming scheme for solving (I) without any of the drawbacks of the above mentioned methods is presented in this paper as a step towards the solution of the broader multi-plant problem that follows.

2. The Multi-Plant System Along a Stream

When several liquid-waste-discharging activities are operating along a stream, a need arises to allocate treatment requirements in order to meet stream dissolved oxygen (DO) standards along the stream at minimum cost; that is, an authority responsible for maintaining DO standards along the stream needs to specify the required level of waste load removal from each polluter's effluent. (Although DO is not the only measure of stream quality, it is the only one considered in this work whose scope is the improvement of existing solutions for a classical DO-related problem). Waste discharges are measured in terms of BOD;

accordingly, the waste treatment plant plan of Figure 4 can be considered as constituting all feasible choices of treatment operation combinations available to each polluter (with "no treatment" a possible alternative).

In order to develop a plan for BOD removal efficiencies by each treatment plant, it is necessary to relate BOD effluents with DO levels along the stream. The dissolved oxygen sag equation established by the pioneering work of STREETER AND PHELPS[10] provides such a relationship between BOD and DO. Essentially, this equation expresses the fact that a BOD discharge reduces the DO in the stream and that there is a natural replenishment of stream DO over time through reaeration or absorption from the air; obviously, a reduction of BOD discharge, through treatment, would result in an improvement of DO levels downstream. Several recent studies, such as those by REVELLE et al[6] and Ecker[1] have developed more refined and exact relationships between BOD discharges and DO levels. By dividing the stream into sections, called reaches, on the basis of stream characteristics, stream quality standard variations, and locations of effluent discharges, and by applying the dissolved oxygen sag equation to each reach, the minimum DO level at each reach can be expressed as a function of the BOD discharges and stream characteristics of every reach upstream. Thus, as outlined in detail by Ecker[1], the requirement that DO at reach i should not fall below a specified level can be reduced to the form

$$\alpha_{1i}(1-e_1) + \alpha_{2i}(1-e_2) + \cdots + \alpha_{ii}(1-e_i) \leq 1 \qquad (4)$$

where

α_{ji} is a positive constant depending on the specific parameters for reach j (namely, stream characteristics, DO and BOD levels at the top of the reach, and DO standard),

and e_j is the waste load reduction efficiency at reach j; that is, if I units of BOD would be discharged into the j^{th} reach in case of no treatment, only $(1-e_j)I$ units are discharged when treatment takes place. Clearly, waste treatment may not be necessary for every reach.

Letting $V_j = 1 - e_j$, a mathematical programming model for the design problem facing the authority above is as follows:

(II)

For each reach j choose a plant design p_j (i.e., a path p_j on the network plan) and determine each t_r on it in order to,

$$\text{Minimize} \quad \sum_{j=1}^{n} C_j(V_j) \tag{5}$$

such that

$$\left.\begin{aligned} &\alpha_{11}V_1 \leq 1 \\ &\alpha_{12}V_1 + \alpha_{22}V_2 \leq 1 \\ &\vdots \\ &\alpha_{1n}V_1 + \alpha_{2n}V_2 + \dots + \alpha_{nn}V_n \leq 1 \end{aligned}\right\} \tag{6}$$

$$V_j = \prod_{r \varepsilon p_j} t_r \,, \quad j = 1,2,\dots,n \tag{7}$$

$$o < t^-_{r,j} \leq t_r \leq t^+_{r,j} \leq 1, \text{ for each } r \text{ in every plant } j \tag{8}$$

$$Q_j \leq V_j \leq S_j \,, \quad j = 1,2,\dots,n \tag{9}$$

where:

n is the total number of reaches; $[t^-_{r,j}, t^+_{r,j}]$ is the feasible range of t_r for plant j;
and $[Q_j,S_j]$ is the feasible range for the plant coefficient in reach j .

A typical constraint of type (8) could be a technical limitation on the efficiency attainable by a primary clarifier; of type (9), a requirement that the plant at reach j must attain a specified minimum efficiency imposed by local authorities.

To date, there exists no successful analytical scheme for solving (II), even though the problem has arisen and solution plans have been developed and implemented. In a number of studies, solutions have been obtained for a simplified version of (II) where each plant j is treated as a "black-box" characterized by a $C_j(V_j)$ function corresponding to a fixed plant design p_j . This simplification amounts to solving a "reduced" problem (II), without (7) and (8), for establishing the least-cost set of plant efficiencies; the manager of each plant j is then faced with problem (I) where the design path is fixed and R is the established level of V_j. The obvious drawback of this approach is that, in order to determine the best plant design p_j for each plant j, one "reduced" problem (II) and n problems (I) must be solved for each and every combination of p_j's for the n-plant system.

The "reduced" problem of determining a least-cost set of plant efficiencies for maintaining DO standards along a stream was solved by SOBEL[9] and REVELLE et al[6] using linear programming; LIEBMAN[4] and FUTAGAMI [3] solved the problem using dynamic programming and they applied their method to Willameter River in Oregon and Todo River System in Japan, respectively. Recently, Ecker[1] considered this problem for several reaches of the Upper Hudson River; he employed geometric programming and proceeded a step further by determining not only plant efficiencies but also, simultaneously, the efficiencies of all unit operations on the plant design paths (in other words, he reintroduced (7) and (8) into problem (II). However, Ecker[1] still does not solve (II) since his paths p_j are fixed and pre-specified, not decision variables; as he clearly states, a combinatorial scheme is needed for choosing the best p_j for each of the n plants.

In this paper: 1. An efficient solution procedure for solving (II) is outlined which is based on a dynamic programming scheme for solving (I) and generating the $C_j(V_j)$ cost functions; and 2. This procedure is applied to the real case considered by Ecker[1].

3. The Solution Procedure

The major drawback of all existing procedures for solving (II) is the need for a combinatorial scheme to choose the best p_j for each of the n plants. In this work this drawback is completely eliminated through the following two-phase strategy:

Phase I: For each plant j, solve (I) and obtain $C_j(V_j)$ values for all plant efficiency levels 1-R such that $Q_j \leq R \leq S_j$. A cost-efficiency curve can thus be generated giving the minimum cost $C_j(e_j)$ for attaining a plant efficiency not less than e_j . The crucial characteristic of this $C_j(e_j)$ curve is that each point corresponds to the least-cost plant design p_j for attaining that particular efficiency.

This is accomplished via a Least-cost Plant Coefficient (LPC) algorithm.

Phase II: Using the $C_j(e_j)$ curves just derived, and noting that

$V_j = 1 - e_j$, solve (II) by linear programming techniques considering only constraints (6) and (9), since (7) and (8) have already been considered in Phase I. Each chosen V_j corresponds to an already determined best design for plant j.

A. Development of the LPC Algorithm

An acyclic network corresponding to a plan for a treatment plant is given with nodes 1,2,...,N and each arc (i,k) corresponding to a specific unit operation. A unit operation (i,k) is characterized: 1. by a feasible range $[t_{ik}^-, t_{ik}^+]$ for the transformation coefficient t_{ik} , and 2. by a function $C_{ik}(t)$ giving the minimum cost for attaining a removal efficiency not less than 1-t. A realistic property of $C_{ik}(t)$ is that, for $t^1 < t^2$, $C_{ik}(t^1) \geq C_{ik}(t^2)$.

The following notation needs to be introduced:

- A_i : the set of all nodes s connected with i through (i,s)
- W_i : a sequential product of feasible values of transformation coefficients, one for each arc, on a path from i to N.
- W_i^-/W_i^+ : The minimum/maximum possible value of W_i
- $Z_i(w)$: minimum cost for attaining a W_i not greater than w. Thus $Z_1(R)$ is the optimal value of the objective function in (I), while Phase I calls for a calculation of $Z_1(w)$ over values $Q_j \leq w \leq S_j$, for each plant j .

According to the definition of W_i^- and W_i^+ , if W_k^- and W_k^+ are known for every $k \epsilon A_i$, then

$$W_i^- = \underset{k \epsilon A_i}{\text{minimum}} \left\{ t_{ik}^- W_k^- \right\} \qquad (10)$$

and

$$W_i^+ = \underset{k \epsilon A_i}{\text{maximum}} \left\{ t_{ik}^+ W_k^+ \right\} \qquad (11)$$

In view of (9), (10) and (11) the values of w for which $Z_1(w)$ should be computed are those such that $\max\{Q_j; W_1^-\} \leq w \leq \min\{S_j; W_1^+\}$. Moreover, since $t_{ik} \leq 1$ for any (i,k), it follows that on a path through arc (i,k) at optimality,

$$\max(Q_j, W_1^-) \leq W_1 \leq W_i \leq W_k \quad \text{or} \quad Q_j \leq W_k \tag{12}$$

and the range of w values for $Z_k(w)$ should be

$$Y_k^- \leq w \leq W_k^+ \qquad \text{for any } k>1 \tag{13}$$

where

$$Y_k^- = \max\{Q_j ; W_k^-\} \tag{14}$$

Assuming now that all $Z_k(w)$ values have been obtained for some node k which is the only node in A_i, then

$$Z_i(w) = \underset{t}{\text{minimum}} \{Z_k(w/t) + C_{ik}(t)\} \tag{15}$$

where $t_{ik}^- \leq t \leq t_{ik}^+$ and $Y_k^- \leq w/t \leq W_k^+$, or equivalently,

$$\max\{t_{ik}^-; w/W_k^+\} \leq t \leq \min\{t_{ik}^+; w/Y_k^-\} . \tag{16}$$

When several arcs leave node i, then

$$Z_i(w) = \underset{k \in A_i}{\text{minimum}} \{\underset{t}{\text{minimum}} [Z_k(w/t) + C_{ik}(t)]\} \tag{17}$$

where, for each (i,k),

$$\min\{t_{ik}^+ ; \max[t_{ik}^-; w/W_k^+]\} \leq t \leq \min\{t_{ik}^+ ; w/Y_k^-\} \tag{18}$$

The modification of the lower bound of (16) into that of (18) is necessary for cases where, for some k in A_i, $W_i^+ > W_k^+ t_{ik}^+$; then, for values of $w > W_k^+ t_{ik}^+$, the lower bound on t_{ik} would have been w/W_k^+ which is greater than t_{ik}^+ rendering (16) meaningless.

Thus, it appears that an algorithm can be developed whereby one proceeds from node N to node 1, in decreasing order, at each node k computing $Z_k(w)$ according to (13), (17), and (18). This resembles a shortest-route algorithm, $Z_k(w)$ being the label assigned to each node k; it is also a dynamic programming procedure where at each stage (node) a minimum cost is computed as a function of the portion of W_1 "remaining to be allocated" on the arcs from k to N. At the end of such an algorithm for plant j, curves $C_j(V_j)$ and $C_j(e_j)$ will be obtained from the $Z_1(w)$ values; repeating the algorithm for each plant completes Phase I of the solution procedure whereupon Phase II can be performed.

B. The Algorithm

Step 1: Let $W_N^- = W_N^+ = 1$ and $Z_N(1) = 0$

Step 2: Proceeding from node N-1 to 1, in decreasing order, at each node i:

(a) Let $W_i^- = \underset{k \epsilon A_i}{\text{minimum}} \left\{ t_{ik}^- W_k^- \right\}$

$$W_i^+ = \underset{k \epsilon A_i}{\text{maximum}} \left\{ t_{ik}^+ W_k^+ \right\}$$

$$Y_i^- = \max \left\{ Q_j ; W_i^- \right\}$$

(b) For w values: $Y_i^- \leq w \leq W_i^+$ for $i \neq 1$ and $Y_1^- \leq w \leq \min \left\{ S_j ; W_1^+ \right\}$

- for i = 1, compute

$Z_i(w) = \underset{k \epsilon A_i}{\text{minimum}} \{\underset{t}{\min} [Z_k(w/t) + C_{ik}(t)]\}$, where for each (i,k)

$\min\{t_{ik}^+ ; \max[t_{ik}^- ; w/W_k^+]\} \leq t \leq \min\{t_{ik}^+ ; w/Y_k^-\}$

(c) Check that, for $w_1 < w_2$, $Z_i(w_1) \geq Z_i(w_2)$; if not, let $Z_i(w_2) = Z_i(w_1)$.

(d) For each w considered, record the arc (i,k) and the value of t_{ik} corresponding to $Z_i(w)$.

C. Accuracy vs. Efficiency

Like in other dynamic programming procedures with continuous state variables, a grid method is used here for the computation of $Z_i(w)$'s . Each $C_{ik}(t)$ curve is replaced by M_{ik} cost-cöefficient pairs $(c_m, t_m)_{ik}$, $m = 1,2,\ldots,M_{ik}$; if arc (i,k) is the r^{th} arc of the network, an equivalent notation is $(c_m,t_m)_r$, m = 1,2,...,M_r. For a real problem, this replacement is not a 'simplifying assumption' since it was from data sets in the form of $(c_m,t_m)_{ik}$ pairs that $C_{ik}(t)$ functions were derived in the first place. The cost functions of Table 1 for the example case considered here are replaced by the $(c_m,t_m)_r$ pairs of Table 2.

Regarding the range of w values, $Z_i(w)$ will be computed for a number of w values from Y_i^- to W_i^+ . According to the definition of $Z_i(w)$, if only $Z_i(a)$ and $Z_i(b)$ are computed in the interval $a \leq w \leq b$, then for any $a \leq w < b$, $Z_i(w)$ is assumed to equal $Z_i(a)$; this,however, could result in cost overestimation. Clearly, the larger the number of w values for which $Z_i(w)$ is computed, the greater the accuracy. Moreover, since marginal costs increase as transformation coefficients decrease, the size of intervals in which the range $[Y_i^-, W_i^+]$ is broken should be smaller near Y_i^- than near W_i^+ . A possible consequence of large intervals in the w-range is to obtain values $Z_i(w_1) < Z_i(w_2)$ for $w_1<w_2$; by the definition of $Z_i(w)$, this is unrealistic and Step 2c of the algorithm corrects it.

4. An Application

The case study on the Upper Hudson River reported by Ecker[1] is now considered and solved. The problem is of the form in (II) involving the design of a treatment plant, if there is a need for one, in each of six reaches along the river so as to meet the physical stream quality constraints implied by the α_{ji} values of Table 3. The design alternatives for each plant are as in Figure 4 and Table 2. As an added feature of the geometric programming approach, Ecker[1] imposes and handles constraints requiring the product of t_r's over only part of a design path to be within specified bounds (e.g., $t_1t_2t_6 \geq 0.15$). Although such a requirement can be easily considered in the LPC algorithm, it was left out of this work for simplicity. Thus, the example solved here is slightly different than that treated by Ecker[1] .

In Phase I, the LPC algorithm is applied to the network plan for each of the six possible plants. For the network of Figure 4 (which is the plan for all six plants), and for a plant efficiency range from 0 to 0.970, the algorithm yields the $Z_1(w)$ values listed in Table 4 (this table is condensed from one of 60 w-intervals). The cost-efficiency curve for each plant is shown in Figure 5.

Turning to Phase II of the solution procedure, the IBM's MPSX option for linear programs with separable, concave objective functions is employed to solve the model which consists of (5), (6), and (9) with V_j replaced by $1-e_j$. (For the case here, there are no type (9) constraints). The convexity of the plant's $C_j(e_j)$ curve for $e_j<0.05$ may require a translation of the origin to point (18,0) and an adjustment of any solution e_j such that $e_j<0.5$; in reality, however, this possibility does not arise since, if e_j is not zero, it will be required to be much greater than 0.05. Extensive investigation with similar problems has shown that the optimal e_j values are rather insensitive to minor varia tions in the slopes of the segments of the approximated piecewise linear cost functions, especially when these functions are identical for several plants; this clearly reduces the desirability of a very fine w-grid in the LPC algorithm.

The optimal design is shown in Table 5; with the MPSX-generated e_j value for plant j, the optimal w-interval in $Z_1(w)$ is identified along with the already recorded best t_r values. The insignificant difference between e_j from MPSX and $1-W_1$ becomes even smaller as the w-grid in the LPC algorithm gets finer. A requirement that a plant, if built, should operate at $e_j \geq .95$ yields $e_5=0$ and $e_j= .95$ for $j\neq 5$ corresponding to $t_1 = .60$, $t_2 = .80$, $t_6 = .10$, and $t_{11} = 1.0$ for a cost of 182.3 per plant. As a final comment, the single-plant design obtained by the LPC algorithm for $e_j \geq 0.971$ is almost identical to that obtained by Ecker and McNamara(2) and Shih and Krishnan(7) for the same problem.

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TABLE 1: The Waste Treatment Plant

Arc in Fig. 4 (r)	Unit Operation	Cost Function $c_r(t_r)$
1	Primary Clarifier (PC)	$19.4\ t_1^{-1.47}$
2	Trickling Filter (TF)	$16.8\ t_2^{-1.66}$
3	Activated Sludge after TF (AS/T)	$91.5\ t_3^{-0.30}$
4	Activated Sludge after PC (AS/P)	$86.0\ t_4^{-0.38}$
5	Aerated Lagoon after PC (AL/P)	$45.9\ t_5^{-0.45}$
6	Aerated Lagoon after TF (AL/T)	$27.4\ t_6^{-0.63}$
7	Coagulation /Sedimentation /Filtration after AS (CSF)	$152.0\ t_7^{-0.27}$
8	Carbon Adsorption after AS (CA)	$120.0\ t_8^{-0.33}$
10	Coagulation /Sedimentation /Filtration after AL	$179.0\ t_{10}^{-0.37}$

Arcs 9, 11, 12, 13 correspond to No Treatment (NT) with

$t_r = 1$ and $c_r = 0$

TABLE 2: Cost-Coefficient Pairs $(c,t)_r$

r = 1		r = 2		r = 3		r = 4		r = 5	
c	t	c	t	c	t	c	t	c	t
53.74	0.50	39.02	0.60	182.57	0.10	206.30	0.10	129.36	0.10
41.11	0.60	34.20	0.65	161.66	0.15	176.84	0.15	107.79	0.15
32.77	0.70	30.26	0.70	148.29	0.20	158.53	0.20	94.70	0.20
26.93	0.80	27.01	0.75	131.35	0.30	135.89	0.30	85.65	0.25
22.65	0.90	24.28	0.80	120.44	0.40	121.82	0.40	78.91	0.30
20.92	0.95								

r = 6		r = 7		r = 8		r = 10	
c	t	c	t	c	t	c	t
116.88	0.10	174.48	0.60	178.54	0.30	216.24	0.60
90.53	0.15	170.75	0.65	169.68	0.35	209.93	0.65
75.53	0.20	167.37	0.70	162.37	0.40	204.25	0.70
58.50	0.30	164.28	0.75	156.18	0.45	199.10	0.75
48.80	0.40	161.44	0.80	150.84	0.50	194.41	0.80
42.40	0.50						
37.80	0.60						

TABLE 3: Coefficients α_{ji} for Problem (II)

j \ i	1	2	3	4	5	6
1	4.266					
2	3.975	4.741				
3	4.356	10.57	.5055			
4	1.710	8.812	.6592	.7926		
5	1.186	6.434	.4870	.6009	.0168	
6	0.6272	3.792	.2932	.3792	.0289	1.254

TABLE 4: $Z_i(w)$ Values From the LPC Algorithm

w	i = 1	i = 2	i = 3	i = 4	i = 5
0.030	389.96	371.95	361.11		
0.034	381.22	365.75	361.11		
0.038	197.01	360.42	352.25		
0.042	189.65	355.29	344.94		
0.046	183.85	351.04	388.75		
0.052	178.01	344.85	333.41		
0.060	165.84	155.90	324.03		
0.068	157.50	151.08	317.84		
0.080	147.38	141.16	310.66		
0.100	138.54	124.73	116.88		
0.150	115.77	102.54	90.53		
0.200	109.71	92.70	75.53		
0.300	93.61	75.81	58.50	178.54	
0.400	87.46	66.68	48.80	162.37	
0.500	53.74	66.68	42.40	150.84	
0.600	41.11	66.68	37.80	150.84	216.24
0.700	32.77	66.68		150.84	204.25
0.800	26.93	66.68		150.84	194.41
0.900	22.65	66.68		150.84	194.41
1.000	0	0		0	0

TABLE 5: The Optimal Design for the Example Case

Plant j	MPSX e_j	Unit Operation Design t_1	t_2	t_6	t_{11}	t_{12}	t_{13}	Actual Efficiency $1 - W_1$	Plant Cost
1	.930	.70	.65	.15	1			.932	157.5
2	.960	.60	.65	.10	1			.961	192.2
3	.454	.50				1		.500	53.7
4	.789	.70	.75	.40	1			.790	108.6
5	0						1	0	0.0
6	.573	.95	.75	.60	1			.573	85.7
									597.7

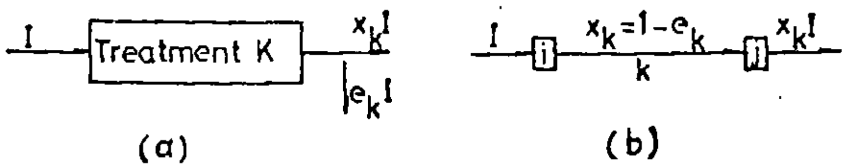

FIGURE 1: A Treatment Process Representation

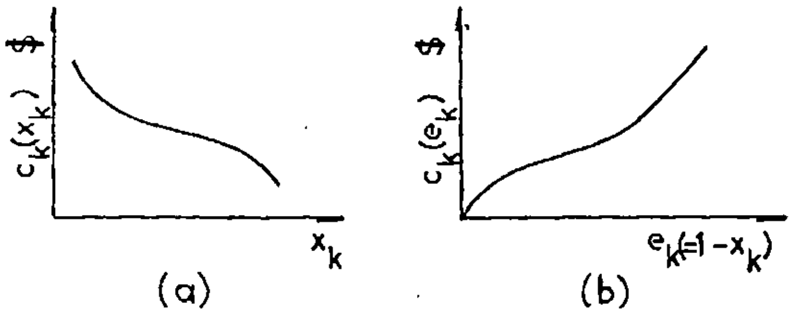

FIGURE 2: Cost-Efficiency Curves: Minimum Cost For Attaining Corresponding Efficiency

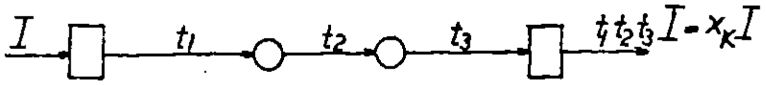

FIGURE 3: Treatment Process k as a Combination of Processes

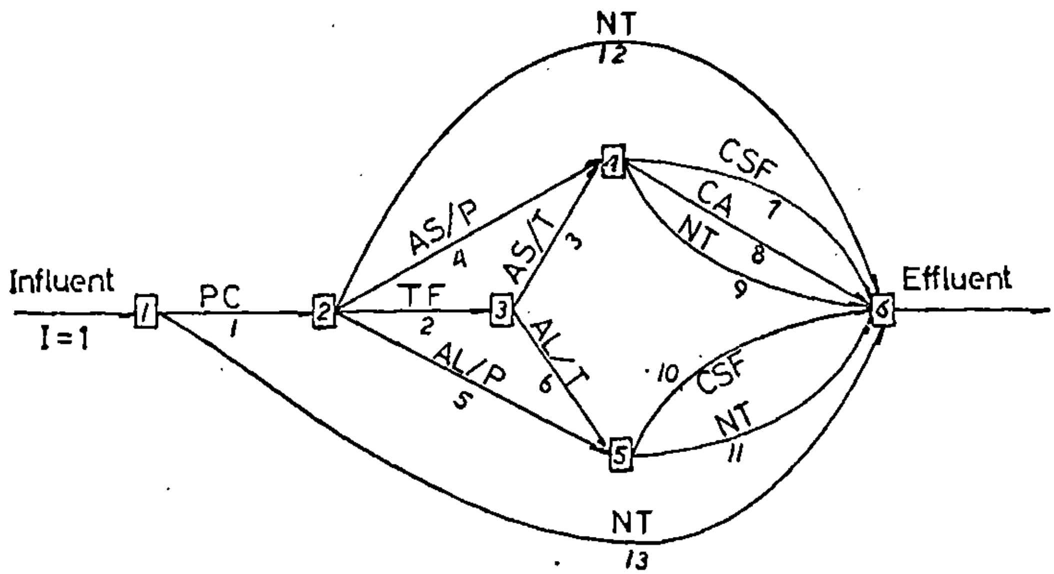

FIGURE 4: A Plan For a Waste Treatment Plant

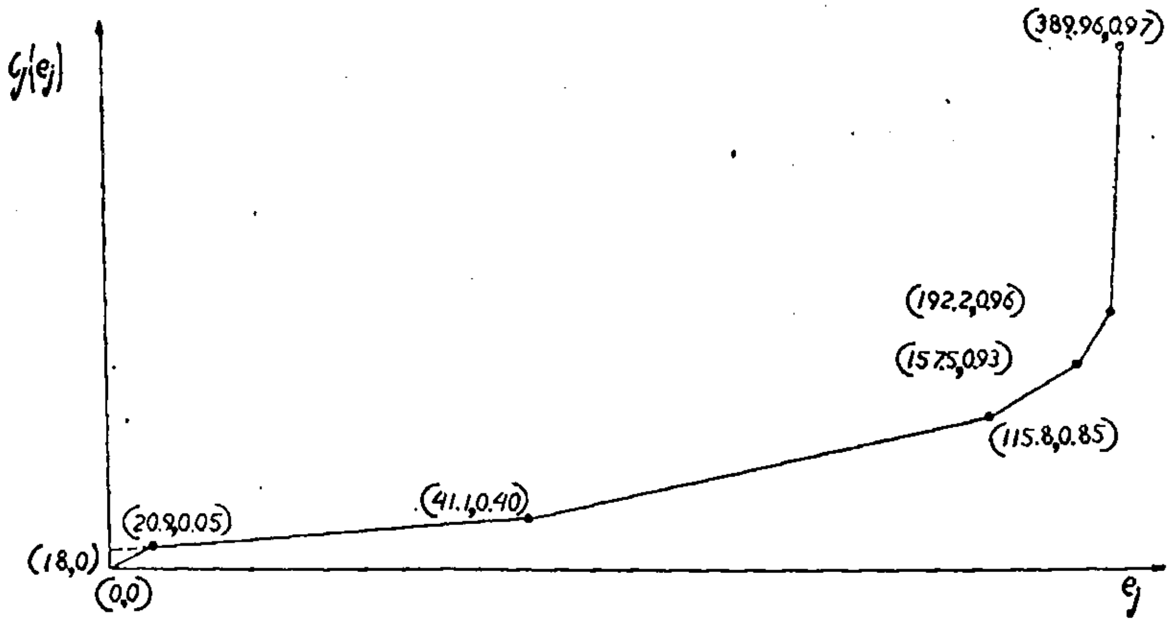

FIGURE 5: Cost-Efficiency Curve For the Example Case.