LOW LYING NEGATIVE PARITY STATES IN A=17 NUCLEI ;

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Abstract.

The negative parity states of the nuclei ^{17}N, ^{17}O, ^{17}F, and ^{17}Ne were calculated using the shell model. All configurations whose unperturbed energies are one major shell spacing above the ^{17}O ground state were included in the calculation, and the spurious states were properly removed. The residual interactions were chosen from recent work on ^{16}O, ^{18}O, and ^{18}F. The calculated spacing of the levels agrees well with the available experimental data; however the binding is off by around 1.5 MeV. The lifetimes for the electromagnetic decay of the negative parity states to the ground and first excited states in ^{17}F and ^{17}O, and the log ft values for the allowed β^+ decays from ^{17}Ne to ^{17}F were calculated and where possible compared with experiment.

Preface .

I would like to express my gratitude to Prof. B. Margolis for the suggestion that started these calculations and for his constant guidance and sound advice. I wish to thank Dr. R. D. Lawson and Dr. S. Kahana for stimulating conversations, and Dr. T. D. Newton for checking some of the calculations with his generalized shell model program. I am grateful to Dr. T. Tamura for the subroutines used to calculate the 3-j , 6-j , and 9-j coefficients and to the staff of the McGill Computing Center for help in programming. I would finally like to thank the National Research Council for a studentship.

Table of contents :

Table of contents (cont'd) :

§1. Introduction

The calculations presented in this thesis were initially undertaken in an effort to understand the very slow β^+ decay from the ground state of ^{17}Ne to the 3.10 MeV $1/2^-$ level of ^{17}F which was observed in delayed proton experiments [1], and the similarly retarded "mirror" decay from ^{17}N to the 3.06 MeV state in ^{17}O [2]. A simplified shell model calculation based largely on experimental information showed [3] that an accidental cancellation occurs in the Gamow-Teller matrix element describing these decays, the transition probability being thereby decreased by two orders of magnitude.

It was then decided to extend this project into a general shell model study of the negative parity states of ^{17}N, ^{17}O, ^{17}F, and ^{17}Ne, which will be collectively designated as the "A=17 nuclei". Two circumstances prompted this course of action: first, the extensive calculations carried out on the nuclei ^{16}O, ^{18}O, and ^{18}F over the last few years indicated that a study of the A=17 nuclei would be very profitable, and secondly, the availability of a large computer at the McGill Computing Center assured the feasibility of the calculations.

Because of the close connection, which will become apparent in subsequent sections, between the negative parity states

of the A=17 nuclei on the one hand, and the positive parity states of ^{18}O, and ^{18}F and the negative parity states of ^{16}O on the other, it was found convenient to include these latter in the present calculation, although no new results are to be expected for them.

§2. Formalism

For the purposes of this calculation, we adopt the j-j coupling shell model [4] and use the usual second quantized notation [5]. Thus $a^{+}_{n\ell jm;m_t}$ is the creation operator for a nucleon in the state defined by the single particle quantum numbers $n\ell jm;m_t$, where n is the radial quantum number, ℓ is the orbital angular momentum, j and m are the total angular momentum and its z-component respectively, and m_t is the 3-component of isospin. This operator transforms as a proper spherical tensor in both configuration space and iso-space. The corresponding annihilation operator is denoted by $a_{n\ell jm;m_t}$ which behaves as the hermitian adjoint of a tensor operator [6]. For the sake of convenience, the single particle quantum numbers will be summarized by a single Greek index. Depending on the context, this index may or may not include the z-components of isospin and angular momentum. The above operators obey the usual fermion commutation rules:

$$\left[a_\alpha,\ a_\beta\right]_+ = \left[a^{+}_\alpha,\ a^{+}_\beta\right]_+ = 0$$

$$\left[a^{+}_\alpha,\ a_\beta\right]_+ = \delta_{\alpha\beta} \qquad \ldots(1)$$

The Hamiltonian can be written as :

$$H = H_o + V^{res}$$

where H_o includes the kinetic energy of all the particles and their average interaction with all the nucleons in closed shells and V^{res} is the residual interaction which acts only between particles in partially filled shells. The representation of the single particle states is taken to be the one that diagonalizes H_o. We can then write:

$$H_o = \sum_\alpha \varepsilon_\alpha \, a_\alpha^+ a_\alpha \qquad \ldots(2)$$

$$V^{res} = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \, a_\alpha^+ a_\beta^+ a_\gamma \, a_\delta \qquad \ldots(3)$$

The matrix elements $V_{\alpha\beta\gamma\delta}$ include both the direct and the exchange terms:

$$\begin{aligned} V_{\alpha\beta\gamma\delta} &\equiv \langle\psi_\alpha(1)|\langle\psi_\beta(2)| \; V(1,2) \; |\psi_\gamma(2)\rangle|\psi_\delta(1)\rangle_{AS} \\ &\equiv \langle\psi_\alpha(1)|\langle\psi_\beta(2)| \; V(1,2) \; |\psi_\gamma(2)\rangle|\psi_\delta(1)\rangle \\ &- \langle\psi_\alpha(1)|\langle\psi_\beta(2)| \; V(1,2) \; |\psi_\gamma(1)\rangle|\psi_\delta(2)\rangle \end{aligned} \qquad \ldots(4)$$

Clearly, then, this matrix element has the following symmetries;

$$V_{\alpha\beta\gamma\delta} = V_{\beta\alpha\delta\gamma} = -V_{\beta\alpha\gamma\delta} = -V_{\alpha\beta\delta\gamma}$$

The single particle energies ε_α and wavefunctions ψ_α should really be calculated in a self-consistent scheme such as

Hartree-Fock [5], however, it is usually assumed that the eigenfunctions of a reasonable average shell model potential are a close approximation to the single particle wavefunctions, and that the experimental single particle energies can be used for the $\mathcal{E}_\alpha$. For the sake of simplicity, spherical harmonic oscillator eigenfunctions are normally used for the radial part of ψ_α; we shall follow this practice.

All Coulomb effects are ignored in this calculation, hence isospin is a good quantum number. The vector $|0\rangle$ is defined as the ground state of ^{16}O which is assumed to consist of the completely filled 0s and 0p shells. All energies will be measured from the ground state of ^{16}O unless otherwise specified.

We must now construct the basis states for the negative parity states of ^{16}O, the positive parity states of ^{18}O, and ^{18}F, and the negative parity states of the A=17 nuclei. As a matter of notation, we shall usually refer to energy levels by their spin, parity and isospin, denoted as $J^\pi;T$. For instance, the lowest negative parity state in ^{16}O is $3^-;\ 0$. The simplest possible configurations are chosen for these states and these are illustrated in fig. 1. The justification for ignoring more complicated configurations is two-fold: first, the unperturbed energies of such configurations (i.e. their energies

with $V^{res}=0$) are at least two shell spacings greater than the energies of the configurations taken into account, and secondly, it is practically impossible to carry out these calculations if the higher excitations are included without some further simplifying assumptions. We now formally construct these basis states.

The negative parity states of ^{16}O are therefore taken to be simply particle-hole states :

$$| \eta^{-1}\nu ; J\ T\rangle = \underbrace{a_\eta\ a^+_\nu}_{J,T} | 0\rangle \qquad \ldots.(5).$$

where η^{-1} denotes a hole in the 0p shell and ν denotes a particle in the (0d,1s) shell . The bar under the operators indicates the angular momentum and isospin couplings :

$$\underbrace{a_\eta\ a^+_\nu}_{J,T} | 0\rangle = \sum_{\substack{m_\eta, m_\nu, \\ m_{t\eta}, m_{t\nu}.}} (-)^{j_\eta - m_\eta + 1/2 - m_{t\eta}} (j_\eta, -m_\eta, j_\nu, m_\nu \mid J,M) \times (1/2, -m_{t\eta}, 1/2, m_{t\nu} \mid T, M_t)\ a_\eta\ a^+_\nu\ | 0\rangle$$

$\ldots..(6).$

where the conventional notation for the vector coupling coefficients has been used [6]. The corresponding

bra, in the same notation, would be written:

$$\langle \eta^{-1}\nu ;JT| = \langle 0| \underbrace{a_\nu a^+_\eta}_{<JT}$$

...(7)

This definition of the particle-hole states differs from the conventional one by a phase factor. The matrix elements of the Hamiltonian, in this basis, are given by:

$$\langle \eta'^{-1}\ \nu';J\ T|\ H_o\ |\eta^{-1}\nu\ ;J\ T\rangle = \delta_{\eta\eta'}\delta_{\nu\nu'}(-\varepsilon_\eta+\varepsilon_\nu)$$

...(8)

$$\langle \eta'^{-1}\ \nu';JT|V^{res}|\eta^{-1}\nu\ ;JT\rangle = -\langle \psi_{\nu'}(1)|\overbrace{\langle \psi_\eta(2)|\ V(1,2)|\psi_{\eta'}(2)\rangle}^{<J;T}|\psi_\nu(1)\rangle_{AS}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{J;T}$$

...(9)

The explicit form of the particle-hole matrix elements is given in appendix A. With reasonable forces those matrix elements are attractive in T = 0 states and repulsive in T = 1 states [5].

The positive parity states of ^{18}O and ^{18}F are taken to be the two particle states:

$$|\nu\mu ;\tilde{J}\ \tilde{T}\rangle = N_{\nu\mu}\ \underbrace{a^+_\nu\ a^+_\mu}_{\tilde{J}\ \tilde{T}}\ |0\rangle$$

... (10)

where ν and μ both denote particle states in the 0d,1s shell. The normalization constant is defined by:

$N_{\nu\mu}$ = 1 if $\nu \neq \mu$

= $\sqrt{1/2}$ if $\nu = \mu$ and $\tilde{J}+\tilde{T}$ is odd,

= 0 otherwise.

The matrix elements of the Hamiltonian are given by :

$$\langle \nu'\mu';\tilde{J}\ \tilde{T} \mid H_o \mid \nu\mu;\tilde{J}\ \tilde{T}\rangle = N_{\nu'\mu'} N_{\nu\mu} (\varepsilon_\nu + \varepsilon_\mu)\left(\delta_{\nu\nu'}\delta_{\mu\mu'} + (-)^{j_\nu+j_\mu-\tilde{J}-\tilde{T}}\delta_{\nu\mu'}\delta_{\mu\nu'}\right) \quad .....(11).$$

$$\langle \nu'\mu';\tilde{J}\ \tilde{T}\mid V^{res}\mid \nu\mu;\tilde{J}\ \tilde{T}\rangle = N_{\nu'\mu'}N_{\nu\mu}\langle \underbrace{\psi_{\nu'}(1)\mid\langle\psi_{\mu'}(2)\mid}_{\rightarrow \tilde{J}\ \tilde{T}}\ V^{res}(1,2)\mid \underbrace{\psi_\mu(2)\rangle\mid\psi_\nu(1)\rangle}_{\tilde{J}\ \tilde{T}\ \leftarrow}\rangle_{AS} \quad .....(12).$$

The explicit form of the particle-particle matrix elements is again left to the appendix.

The T=1/2 negative parity states in the A=17 nuclei can be represented as linear combinations of one particle and two particle -one hole states whose unperturbed energies lie one major shell spacing above the ^{17}O ground state. The T=3/2 states consist only of two particle-one hole configurations. These can be written as :

$$|\rho\rangle = a^+_\rho\ |0\rangle$$

with ρ in the (0f,1p) shell , and

$$|\eta^{-1}(\nu\mu;\tilde{J}\ \tilde{T})J\ T\rangle = N_{\nu\mu}\ \underbrace{a_\eta\ \underbrace{a^+_\nu\ a^+_\mu}_{\rightarrow \tilde{J}\tilde{T}}}_{\rightarrow JT}\ |0\rangle$$

with ν and μ in the (0d,1s) shell, and η in the 0p shell. The normalization factor $N_{\nu\mu}$ is defined as before. We shall refer to the above states as "basis" states.

The matrix elements of H connecting the one particle states either to one particle states or to two-particle,one-hole states are:

$$\langle \rho' | H | \rho \rangle = \langle \rho' | H_o | \rho \rangle = \varepsilon_\rho \, \delta_{\rho\rho'} \qquad .....(15).$$

and,

$$\langle \rho' | H | \eta^{-1}(\nu\mu;\tilde{J}\,\tilde{T})J\,T \rangle = \langle \rho' | V^{res} | \eta^{-1}(\nu\mu;\tilde{J}\,\tilde{T})J\,T \rangle$$

$$= N_{\nu\mu} \, \delta_{J,j'_\rho} \, \delta_{T,1/2} \, \langle \psi_{\rho'}(1) | \langle \psi_\eta(2) | V^{res}(1,2) | \psi_\mu(2) \rangle | \psi_\nu(1) \rangle_{AS}$$

($\psi_\mu(2)\psi_\nu(1)$: $\tilde{J},\tilde{T}$; $\psi_\eta(2)$ with $\psi_\mu(2)\psi_\nu(1)$: J,T)

$$= N_{\nu\mu} \, \delta_{J,j'_\rho} \, \delta_{T,1/2} \left[\frac{(2\tilde{J}+1)(2\tilde{T}+1)}{2(2j'_\rho+1)} \right]^{1/2}$$

$$\times \langle \psi_{\rho'}(1) | \langle \psi_\eta(2) | V^{res}(1,2) | \psi_\mu(2) \rangle | \psi_\nu(1) \rangle_{AS}$$

($\psi_{\rho'}(1)\psi_\eta(2)$: $\tilde{J},\tilde{T}$; $\psi_\mu(2)\psi_\nu(1)$: $\tilde{J},\tilde{T}$)

These last off-diagonal matrix elements can be called "polarization" terms since they are similar to the matrix elements which, for instance , give rise to the effective charge of a single neutron outside the closed ^{16}O core . The matrix elements of H between

two particle-one hole states are considerably more complicated. For the purpose of calculating these matrix elements only, it is convenient to separate the residual interaction into two parts : V_{pp} , the particle-particle force, which acts only between particles in the same shell, and V_{ph} , the particle-hole force, that acts only across shells. We have therefore :

$$\langle \eta'^{-1}(\nu'\mathcal{M}';\tilde{J}'\tilde{T}')J\ T \mid H \mid \eta^{-1}(\nu\mathcal{M};\tilde{J}\ \tilde{T})J\ T\rangle =$$

$$= \langle \eta'^{-1}(\nu'\mathcal{M}';\tilde{J}'\tilde{T}')J\ T \mid H_o+V_{pp}+V_{ph} \mid \eta^{-1}(\nu\mathcal{M};\tilde{J}\ \tilde{T})J\ T\rangle$$

.....(17)

and ,

$$\langle \eta'^{-1}(\nu'\mathcal{M}';\tilde{J}'\tilde{T}')J\ T \mid H_o \mid \eta^{-1}(\nu\mathcal{M};\tilde{J}\ \tilde{T})J\ T\rangle = -\delta_{\tilde{J}',\tilde{J}}\ \delta_{\tilde{T}',\tilde{T}}\ \delta_{\eta\eta'}$$

$$\times N_{\nu\mathcal{M}}\ N_{\nu'\mathcal{M}'}\left[(-)^{j_\nu+j_\mu-\tilde{J}+1-\tilde{T}}\delta_{\nu\mathcal{M}'}\delta_{\mathcal{M}\nu'} - \delta_{\nu\nu'}\delta_{\mathcal{M}\mathcal{M}'}\right](\varepsilon_{\mathcal{M}}+\varepsilon_\nu-\varepsilon_\eta)$$

.....(18)

The particle-particle term is :

$$\langle \eta'^{-1}(\nu'\mathcal{M}';\tilde{J}'\tilde{T}')J\ T \mid V_{pp} \mid \eta^{-1}(\nu\mathcal{M};\tilde{J}\ \tilde{T})J\ T\rangle = \delta_{\tilde{J}',\tilde{J}}\ \delta_{\tilde{T}',\tilde{T}}\ \delta_{\eta',\eta}$$

$$\times\ N_{\nu'\mathcal{M}'}\ N_{\nu\mathcal{M}}\ \underbrace{\langle\psi_{\nu'}(1)|\langle\psi_{\mathcal{M}'}(2)|}_{\rightarrow \tilde{J}\tilde{T}}\ V^{res}(1,2)\ \underbrace{|\psi_{\mathcal{M}}(2)\rangle|\psi_\nu(1)\rangle}_{\leftarrow \tilde{J}\tilde{T}}{}_{AS}$$

.....(19)

The particle-hole term is given on the next page :

$$\langle \eta'^{-1}(\nu'\mu';\tilde{J}'\tilde{T}')J\ T \,|\, V_{ph} \,|\, \eta^{-1}(\nu\mu;\tilde{J}\ \tilde{T})J\ T\rangle = \sum_{\tilde{\tilde{J}},\tilde{\tilde{T}}} (-)^{j_\mu + j'_{\mu'}} (2\tilde{\tilde{J}}+1)(2\tilde{\tilde{T}}+1)\ N_{\nu'\mu'} N_{\nu\mu} \quad x$$

$$x \quad \left[(2\tilde{J}+1)(2\tilde{J}'+1)(2\tilde{T}+1)(2\tilde{T}'+1)\right]^{1/2} \quad W(1/2,1/2,T,1/2;\tilde{\tilde{T}},\tilde{T})\ W(1/2,1/2,T,1/2;\tilde{\tilde{T}},\tilde{T}')$$

$$x\Big\{ \delta_{\mu\mu'} W(j_\eta, j_\nu, J, j_\mu; \tilde{\tilde{J}}, \tilde{J})\ W(j'_\eta, j'_\nu, J, j'_{\mu'}; \tilde{\tilde{J}}, \tilde{J}')\ \langle \psi_{\nu'}(1) | \langle \psi_\eta(2) | \overleftarrow{\quad V^{res}(1,2) \quad} | \psi_{\eta'}(2)\rangle \overrightarrow{\quad} | \psi_\nu(1)\rangle_{AS}$$
$$\overleftarrow{\tilde{J},\tilde{T}} \qquad\qquad \overrightarrow{\tilde{J},\tilde{T}}$$

$$+(-)^{\tilde{J}+\tilde{J}'+\tilde{T}+\tilde{T}'} \delta_{\nu\nu'}\ W(j_\eta, j_\mu, J, j_\nu; \tilde{\tilde{J}}, \tilde{J})\ W(j'_\eta, j'_{\mu'}, J, j'_\nu; \tilde{\tilde{J}}, \tilde{J}')\ \langle \psi_{\mu'}(1) | \langle \psi_\eta(2) | \overleftarrow{\quad V^{res}(1,2) \quad} | \psi_{\eta'}(2)\rangle \overrightarrow{\quad} | \psi_\mu(1)\rangle_{AS}$$
$$\overleftarrow{\tilde{J},\tilde{T}} \qquad\qquad \overrightarrow{\tilde{J},\tilde{T}}$$

$$-(-)^{\tilde{J}'+\tilde{T}'} \delta_{\mu\nu'}\ W(j_\eta, j_\nu, J, j_\mu; \tilde{\tilde{J}}, \tilde{J})\ W(j'_\eta, j'_{\mu'}, J, j'_\nu; \tilde{\tilde{J}}, \tilde{J}')\ \langle \psi_{\mu'}(1) | \langle \psi_\eta(2) | \overleftarrow{\quad V^{res}(1,2) \quad} | \psi_{\eta'}(2)\rangle \overrightarrow{\quad} | \psi_\nu(1)\rangle_{AS}$$
$$\overleftarrow{\tilde{J},\tilde{T}} \qquad\qquad \overrightarrow{\tilde{J},\tilde{T}}$$

$$-(-)^{\tilde{J}+\tilde{T}} \delta_{\nu\mu'}\ W(j_\eta, j_\mu, J, j_\nu; \tilde{\tilde{J}}, \tilde{J})\ W(j'_\eta, j'_\nu, J, j'_{\mu'}; \tilde{\tilde{J}}, \tilde{J}')\ \langle \psi_{\nu'}(1) | \langle \psi_\eta(2) | \overleftarrow{\quad V^{res}(1,2) \quad} | \psi_{\eta'}(2)\rangle \overrightarrow{\quad} | \psi_\mu(1)\rangle_{AS} \Big\}$$
$$\overleftarrow{\tilde{J},\tilde{T}} \qquad\qquad \overrightarrow{\tilde{J},\tilde{T}}$$

.....(20).

The matrix elements that have been left uncalculated in the above expressions are standard particle-particle and particle-hole matrix elements which are given in the appendix along with the derivations of the preceding equations.

It is often convenient to have a diagrammatic representation of the various matrix elements. Such diagrams are shown in fig. 2 with all the angular momentum couplings of course supressed.

§ 3. Qualitative effects of the residual interaction

A qualitative discussion of the features expected in the A=17 nuclei now becomes appropriate. Let us initially restrict ourselves to the subspace of two-particle, one-hole states. It is evident from the way the states were constructed (see eq. 14 and fig. 1) that in some sense they can be considered to consist of a hole coupled to the even parity states of the A=18 nuclei, which we can call the "core" states. Indeed, if the particle-hole interaction were ignored, this description would be exact. One result of this would be, for instance, that the states having the ^{18}F ground state as "core" would lie somewhat lower ($\sim$ 1 MeV) than the states having the ^{18}O ground state as core.

Now we know that the particle-hole interaction is in fact very strong, and that it is repulsive in relative T=1 states and attractive in relative T=0 states. In order to estimate the effect of this particle-hole force on the two particle-one hole states, let us ignore all factors other than isospin and recouple these states to display relative isospin of any one of the particles and the hole.

$$\underbrace{|\chi_h\rangle \underbrace{|\chi_{p_1}\rangle |\chi_{p_2}\rangle}_{\tilde{T}}}_{T} = \sum_{\tilde{\tilde{T}}} \left[(2\tilde{T}+1)(2\tilde{\tilde{T}}+1)\right]^{1/2} W(\tfrac{1}{2}\ \tfrac{1}{2}\ T\ \tfrac{1}{2};\tilde{T}\ \tilde{\tilde{T}})\ \underbrace{|\chi_h\rangle|\chi_{p_1}\rangle}_{\tilde{\tilde{T}}}|\chi_{p_2}\rangle_{T}$$

Here, χ_h is the hole isospinor and χ_{p_1} and χ_{p_2} are the isospinors of the two particles respectively. Clearly if T=3/2 , then :

$$|\chi_h\rangle\,|\chi_{p_1}\rangle|\chi_{p_2}\rangle \;=\; |\chi_h\rangle\,|\chi_{p_1}\rangle|\chi_{p_2}\rangle$$

(1 → 3/2) (1 → 3/2)

Such states are going to be strongly pushed up by the repulsive T=1 particle-hole force. The T=1/2 states can be separated into two classes : those with T=1 "cores" and those with T=0 "cores". For the former we can write :

$$|\chi_h\rangle|\chi_{p_1}\rangle|\chi_{p_2}\rangle \;=\; \frac{1}{2}|\chi_h\rangle|\chi_{p_1}\rangle|\chi_{p_2}\rangle \;+\; \sqrt{\frac{3}{4}}|\chi_h\rangle|\chi_{p_1}\rangle|\chi_{p_2}\rangle$$

(1 → 1/2) (1 → 1/2) (0 → 1/2)

The diagonal particle-hole matrix element will therefore be $0.25\ V_{ph}(T=1) + 0.75\ V_{ph}(T=0)$. Assuming roughly equal strengths, these states should be strongly depressed. On the other hand, the states with T=0 "cores" can be written as :

$$|\chi_h\rangle|\chi_{p_1}\rangle|\chi_{p_2}\rangle \;=\; \sqrt{\frac{3}{4}}\,|\chi_h\rangle|\chi_{p_1}\rangle|\chi_{p_2}\rangle \;-\; \frac{1}{2}|\chi_h\rangle|\chi_{p_1}\rangle|\chi_{p_2}\rangle$$

(0 → 1/2) (1 → 1/2) (0 → 1/2)

Hence the diagonal matrix element of the particle-hole force becomes $0.75\ V_{ph}(T=1) + 0.25\ V_{ph}(T=0)$ with the result that

the repulsive T=1 particle hole force should be dominant and these levels should be pushed up.

It is therefore to be expected that due to the strong particle-hole forces, the T=1/2 states with T=1 "cores" will lie much lower than the T=1/2 states with T=0 "cores", which in turn will lie well below the T=3/2 states. The introduction of the different possible single particle states, and the angular momentum factors of course complicate the picture but the above discussed effects are nevertheless expected to show up clearly.

The T=1/2 basis also contains one-particle states from the (0f,1p) shell, which will be mixed with the two-particle-one hole states both by means of the polarization matrix elements and by the orthogonalization process involved in the elimination of spurious states (this is discussed in the following section.) We shall give reasons for omitting the polarization terms from the calculation and it will turn out that, in general, the mixing caused by the elimination of the spurious states doesn't affect the low lying levels of the A=17 nuclei. Thus the above discussion remains essentially valid.

§ 4. Spurious states

The principal characteristic of the shell model is that all the particles are treated individually, each moving in a fixed central potential whose origin doesn't necessarily coincide with the center of mass of the system. Consequently, the many particle states that can be constructed using the shell model may contain configurations which correspond to an overall motion of the center of mass with respect to the fixed potential. Such configurations cannot in any way approximate true physical states, hence they are called spurious and must be eliminated from the shell model basis states. Now in the general case of an arbitrary average potential it is very difficult to deal with such center of mass motions. However, if the potential is that of a spherically symmetric harmonic oscillator then the spurious states can be eliminated by a very convenient technique due to E. Baranger and C. W. Lee [7].

The zero-order shell model Hamiltonian with a harmonic oscillator potential is:

$$H_{sm} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \frac{1}{2} mw^2 \sum_{i=1}^{A} r_i^2$$

whose solutions are denoted by Ψ_{sm}

$$\Psi_{sm} = \psi_{n_1 \ell_1}(r_1) \ldots\ldots\ldots\ldots \psi_{n_A \ell_A}(r_A)$$

where the $\psi_{n\ell}(r)$ are the well known oscillator functions[4].

The corresponding eigenvalue is:

$$E = (2n_1 + \ell_1 + 2n_2 \ldots\ldots\ldots\ldots + 2n_A + \ell_A)\hbar w + \frac{3}{2}\hbar w$$

Now we can rewrite H_{sm} as:

$$H_{sm} = H_{int} + H_{CM}$$

with

$$H_{CM} = \frac{1}{2Am} P^2 + \frac{1}{2} Amw^2 R^2$$

$$H_{int} = \frac{1}{2m} \sum_{i=1}^{A} p_i'^2 + \frac{1}{2}mw^2 \sum_{i=1}^{A} r_i'^2$$

where the following definitions have been used:

$$\vec{R} = \frac{1}{A} \sum_{i=1}^{A} \vec{r}_i \qquad \vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

$$\vec{r}_i' = \vec{r}_i - \vec{R} \qquad \vec{p}_i' = \vec{p}_i - \frac{\vec{P}}{A}$$

In order to avoid introducing another three degrees of freedom, one of the variables in H_{int} must be eliminated by using $\sum_i \vec{r}_i' = 0$. We do not want to find explicitly the solutions

of H_{int}, but the solutions of H_{CM} are again the oscillator functions $\psi_{NL}(R)$, and so we can write the eigenstates of H_{sm} in the alternate form,

$$\Psi'_{SM} = \psi_{NL}(R) \quad \psi_{int}$$

with the eigenvalue

$$E = (2N + L + \frac{3}{2}) \hbar w + E_{int}$$

Clearly, Ψ_{sm} with eigenvalue E can be written as a linear combination of the $\psi_{NL}(R)\psi_{int}$ belonging to the same eigenvalue and viceversa.

We normally want to restrict the shell model states used in any calculation to those that correspond to the center of mass being in its ground state : N = 0 = L.

Now all the shell model wavefunctions of A particles that correspond to the lowest possible oscillator eigenvalue say E_o compatible with the Pauli principle necessarily have their center of mass in its ground state. All the states of this A particle system that have their center of mass in its first excited state therefore correspond to the eigenvalue $E_o + \hbar w$ and can be constructed explicitly by operating on all the states with center of mass in the ground state with the operator $\vec{C}^+$ (c.m.) which adds an oscillator quantum to the

center of mass function ψ_{CM} (R).

$$\vec{C}^{+} \text{(c.m.)}\ \psi_{NL}(R) = \psi_{N\pm1,\ L\mp1}(R)$$

The explicit form of $\vec{C}^{+}$(c.m.) is:

$$\vec{C}^{+} \text{(c.m.)} = \frac{1}{A} \sum_{i=1}^{A} \frac{-i}{2m\hbar w} (-i\hbar\vec{\nabla}_i + imw\ \vec{r}_i) = \frac{1}{A} \sum_{i=1}^{A} \vec{c}_i^{+}$$

We can repeat this procedure to generate all the excited states with the center of mass in its second excited state $E_o + 2\hbar w$, and so on. All such states are called spurious states. One of the most useful aspects of this generating procedure is that the spurious states so constructed turn out to be orthonormal to each other.[7]

We can therefore formulate the prescription for dealing with spurious states in the harmonic oscillator shell model. If we are dealing with the ground state of a nucleus and its excited states corresponding to the lowest oscillator eigenvalue only (i.e. no across shell excitations) then there are no spurious states involved. If we are dealing with states involving the excitation of one or more neutrons or protons across an oscillator shell, then we must construct all relevent spurious states and orthogonalize the basis states to these. This of course implies that all the states corresponding to a given oscillator eigenvalue must be included in the calculation.

In ^{16}O, all the excited shell model configurations may involve spurious components, however, if we restrict ourselves to particle-hole excitations then there is only one spurious state with quantum numbers $J^{\pi};T = 1^{-};0$. It turns out that this state remains essentially isolated after diagonalization of the Hamiltonian, consequently it is not necessary to subtract it initially from the basis.

It is clear that the two particle configurations used to describe the nuclei ^{18}O and ^{18}F do not contain any spurious components, so no problems are encountered in this case as well.

The basis states used to describe the negative parity T=3/2 states of the A=17 nuclei also do not contain spurious states since they correspond to the lowest oscillator eigenvalue allowed by the Pauli principle. On the other hand, the T=1/2 negative parity basis states contain a large number of spurious states which must be eliminated. The T=1/2 states that correspond to the lowest allowed oscillator eigenvalue are the positive parity states of one particle outside the ^{16}O core, hence the spurious states can be generated by operating on these states with $\vec{C}^{+}$ (c.m.). These spurious states will be labelled by the quantum number σ which defines the single particle state from which they were generated, and by the total angular momentum J' of the resulting state. The isospin T' is

of course 1/2 . We can then write :

$$\left|sp.(\sigma)J',1/2\right\rangle = \frac{\sqrt{2}}{\sqrt{3A}} \sum_{\alpha\beta} (-)^{j_\alpha + j_\beta} \left\langle \psi_\alpha \left\| \vec{c}^{+} \right\| \psi_\beta \right\rangle \underbrace{a_\alpha^+ a_\beta}_{1^-;0} \underbrace{a_\sigma^+}_{J';1/2} |0\rangle$$

where Schwinger's definition of the reduced matrix element (35) was used. We can rewrite this state in the coupling scheme used in the previous section :

$$\left|sp.(\sigma)J',1/2\right\rangle = \frac{1}{\sqrt{17}} \sum_{\rho} \frac{\left\langle \psi_\rho \left\| \vec{c}^{+} \right\| \psi_\sigma \right\rangle}{\sqrt{2j_\rho + 1}} \delta_{j_\rho, J'} \quad a_\rho^+ |0\rangle$$

$$- \sum_{\alpha\beta\tilde{J}\tilde{T}} \sqrt{\frac{2(2J'+1)(2T'+1)}{17}} \left\langle \psi_\alpha \left\| \vec{c}^{+} \right\| \psi_\beta \right\rangle W(\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ ;\ 0\ \tilde{T})$$

$$\times\ W(j_\beta, j_\alpha, J', j_\sigma; 1, \tilde{J}) \quad \underbrace{a_\beta \underbrace{a_\alpha^+ a_\sigma^+}_{\tilde{J}\ \tilde{T}}}_{J;1/2} |0\rangle$$

The reduced matrix elements are :

$$\left\langle \psi_{n'\ell'j'} \left\| \vec{c}^{+} \right\| \psi_{n\ell j} \right\rangle = (-)^{3/2-j-\ell'} \sqrt{(2j+1)(2j'+1)}\ W(\ell, j, \ell', j'; \tfrac{1}{2}, 1)$$

$$\times \left\langle \psi_{n'\ell'} \left\| \vec{c}^{+} \right\| \psi_{n\ell} \right\rangle$$

where the $\vec{r}$-space matrix element is :

$$\langle \psi_{n'\ell'} \| \vec{c}^{+} \| \psi_{n\ell} \rangle = \sqrt{(\ell+1)(2n+2\ell+3)} \qquad \text{if } n'=n \text{ and } \ell'=\ell+1$$

$$= \sqrt{2\,(n+1)} \qquad \text{if } n' = n+1 \text{ and } \ell' = \ell - 1$$

$$= 0 \text{ otherwise}$$

We now have to construct a set of "parent states" which are equivalent to the basis states but which have the spurious states isolated. This is done by using the Schmidt orthonormalization process as follows. All the spurious states of given J are numbered sequentially from 1 to K_{sp}, the k^{th} state being $|\text{sp. } (\sigma_k)\ J;1/2\rangle$. Next, all the two particle-one hole states are numbered sequentially from K_{sp} up, the k^{th} state being $|\eta_k^{-1}(\nu_k \mu_k;\ \tilde{J}_k\ \tilde{T}_k)\ J;1/2\rangle$. This set of states spans the same space as the set of basis states. It is now merely necessary to orthonormalize this set to obtain the required "parent states" which will be denoted by $|k,J;1/2\rangle$ where k is simply a label running from 1 to k_{max}, this being the number of "parent states" involved. For convenience we omit the labels J and T (=1/2). We can write the "parent states" in terms of the "basis states":

$$|k\rangle = \sum_{\rho} B(k|\rho)\,|\rho\rangle + \sum_{\eta \nu_{\mathcal{H}} \tilde{J}\ \tilde{T}} D(k|\ \eta^{-1}(\nu_{\mathcal{H}};\ \tilde{J}\ \tilde{T}))\ \ |\eta^{-1}(\nu_{\mathcal{H}};\ \tilde{J}\ \tilde{T})\rangle$$

For $1 \leq k \leq K_{sp}$, we have:

$$B(k \mid \rho) = \frac{1}{\sqrt{17}} \frac{\langle \psi_\rho \| \vec{c}^{+} \| \psi_{\sigma_k} \rangle}{\sqrt{2j_\rho + 1}} \delta_{j_\rho , J}$$

$$D(k \mid \eta^{-1}(\nu_\mu ; \tilde{J}\ \tilde{T})) = -\sqrt{\frac{2}{17}(2\tilde{J}+1)(2\tilde{T}+1)}\ W(\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ ;\ 0\ \tilde{T})$$

$$\times \Big[\delta_{\mu\sigma_k} W(j_\eta j_\nu J\ j_\mu\ ;\ 1\ \tilde{J}) \langle \psi_\nu \| \vec{c}^{+} \| \psi_\eta \rangle + $$

$$+ (-)^{1-\tilde{J}-\tilde{T}} \delta_{\nu\sigma_k} W(j_\eta j_\mu J\ j_\nu\ ;\ 1\ \tilde{J}) \langle \psi_\mu \| \vec{c}^{+} \| \psi_\eta \rangle \Big]$$

For $k > K_{sp}$, all the coefficients can be defined in terms of a recurrence relation:

$$B(k \mid \rho) = -\ N(k) \sum_{k'=1}^{k-1} B(k' \mid \rho)\ D(k' \mid \eta_k^{-1}(\nu_k \mu_k ; \tilde{J}_k\ \tilde{T}_k))$$

$$D(k \mid \eta^{-1}(\nu_\mu ; \tilde{J}\ \tilde{T})) = N(k)\ \delta_{\eta,\eta_k}\ \delta_{\nu,\nu_k} \delta_{\mu,\mu_k}\ \delta_{\tilde{J},\tilde{J}_k}\ \delta_{\tilde{T},\tilde{T}_k}$$

$$- \sum_{k'=1}^{k-1} D(k' \mid \eta^{-1}(\nu_\mu ; \tilde{J}\ \tilde{T}))\ D(k' \mid \eta_k^{-1}(\nu_k \mu_k ; \tilde{J}_k\ \tilde{T}_k))$$

$$N(k) = \left\{ 1 - \sum_{k'=1}^{k-1} \left| D(k' \mid \eta_k^{-1}(\nu_k \mu_k ; \tilde{J}_k\ \tilde{T}_k)) \right|^2 \right\}^{-1/2}$$

If $\{N(k)\}^{-1} = 0$, then the corresponding parent state is completely dependant, and hence must be omitted from the set.

§ 5. Choice of the Hamiltonian

The zero order shell model Hamiltonian H_o is completely defined by the choice of the single particle energies ε_α. These energies, which were taken from experiment as far as possible, are listed in table 1 . Now the positions of the levels in the 0p and {0d ,1s} shells are quite well known experimentally, however this is not the case for the levels in the 0f,1p shell. The work of Salisbury and Richards [8] on elastic proton scattering off ^{16}O indicates that there is very little single particle f or p state strength below 8.3 MeV. proton energy. Beyond this, there is no conclusive experimental evidence available. The $0f_{7/2}$ and $1p_{3/2}$ energies listed in table 1 were therefore chosen quite arbitrarily from the ones used by Gillet and Vinh-Mau [9], while the $0f_{5/2}$ and $1p_{1/2}$ energies were obtained by assuming the same spin-orbit force as in the 0d,1s shell. It is probable that the $0f_{7/2}$ energy so chosen is a few MeV too low.

In view of the fact that the single particle energies in the 0f,1p shell are so badly known, it was decided to omit from the calculation the off-diagonal matrix elements of the residual interaction which connect the single particle states to the two-particle,one-hole states. This is probably not too

drastic an approximation for the low lying states, since although both the two particle-one hole configurations and the single-particle configuration start off with the same unperturbed energy, the two-particle, one-hole states are depressed by the diagonal parts of the residual force. In any case, all eigenstates obtained by diagonalization of H that contain a large admixture of the single particle 0f or 1p states and other states of the same spin that lie nearby must be viewed with some suspicion.

Within our approximations, the only matrix elements of the residual interaction that are involved in the A=17 problem are the particle-particle matrix elements that also determine the spectra of ^{18}O and ^{18}F, and the particle-hole matrix elements which are also involved in the spectrum of ^{16}O. We shall therefore choose V_{pp} and V_{ph} from the interactions currently used in these three nuclei. This will generally involve using different potentials for V_{pp} and V_{ph}.

The particle-hole force will be taken from the work of Gillet and Vinh-Mau [9] who derived it using the R. P. A.[5, 10] by a χ^2 fit to those negative parity states of ^{16}O that are strongly suspected of being particle-hole states. This potential is purely central and can be written:

$$V(r) = V_o f(r) \left\{ W + BP + HP_\sigma P_x + MP_x \right\}$$

where f(r) defines the radial dependence of the force, and V_o fixes its strength. The spin exchange operator is denoted by P_σ and the space exchange operator by P_x . The exchange mixture is specified by the constants W , B , H , and M which obey the relation :

$$W + B + H + M = 1$$

Forces of this general type will be called forces of "conventional form" . The parameters derived by Gillet and Vinh-Mau are listed in table 2 , and this force will be called the Gillet force.

A great variety of particle-particle forces have been used in recent years for the study of ^{18}O and ^{18}F . We have adopted four of these for the purposes of our calculations. The first of these is a force used by Inoue et al.[11] to calculate the even parity states of nuclei between ^{18}O and ^{20}Ne. This is a force of conventional form whose parameters are listed in table 2 . This force will be called the Inoue force.

The second particle-particle force was taken from the

work of Kallio and Engeland [12] on the nuclei ^{18}O and ^{20}O. This effective interaction was derived by means of the Scott-Moszkowski separation technique [13] which simulates the effect of the hard core that appears in the free nucleon-nucleon potential but neglects the tensor forces. This effective interaction is assumed to act in relative s-states only, and is written as:

$$V(r) = V_o \exp\left[- \alpha(r-a)\right] \qquad \text{if } r \geqslant d$$

$$= 0 \qquad \text{if } r \leqslant d$$

with the parameters listed in table 3 . This force will be called the "Kallio" force.

The third force adopted for these calculations was a Rosenfeld mixture with Gaussian radial dependence [15]. This is also a force of "conventional form" which was used in this case with the parameters listed in table. 2 . This type of force with a Yukawa shape has been used by Elliott and Flowers [14] in calculations on ^{18}O and ^{16}O as well as in many other calculations. The particular parameters shown in table 2 were suggested by T. D. Newton [15]. This force can be used for both V_{pp} and V_{ph}.

Finally Cohen, Lawson, Macfarlane and Soga [16] derived

the T=1 matrix elements of the particle-particle force by a direct χ^2 fit to the known levels of the oxygen isotopes using only the $0d_{5/2}$ and $1s_{1/2}$ single particle states. This set of matrix elements are listed in table 4; they will be called the "Lawson" force. It is clear that this force can only be used for the T=3/2 states of the A=17 nuclei.

The forces used in the calculation of the spectra of the A=17 nuclei will be labelled by two names, the first referring to the particle-particle force and the second to the particle-hole force: e.g. the "Kallio-Gillet force".

§6. Negative parity states of ^{16}O and positive parity states of ^{18}O and ^{18}F

As mentioned previously, there should exist a close relationship between the spectra of the A=17 nuclei and those of the nuclei ^{18}O, ^{18}F, and ^{16}O, which have been extensively studied over the last few years. We expect to be able to understand both the similarities and the differences between the calculated and the experimental spectra of the A=17 nuclei in terms of the comparative characteristics of the calculated and experimental spectra of ^{18}O, ^{18}F, and ^{16}O, which are shown in figs. 3, 4, & 5.

We have calculated the energy level schemes of the nuclei ^{18}O, ^{18}F, and ^{16}O as predicted by the shell model partially because not all the results were already available in the literature, and partially in order to test various subroutines used in the A=17 calculations. The results presented in this section are therefore not new, but are intended to outline the state of our knowledge of ^{18}O, ^{18}F, and ^{16}O insofar as this will help us to discuss the A=17 nuclei.

The experimental spectrum of ^{18}O was taken from refs. 17 and 18. The first few levels exhibit a "vibrational" spacing:

a "one phonon" 2^+ level at 2 MeV followed by a "two phonon" triplet between 3.5 and 4.0 MeV. These states are followed by many levels only a few of which have been well identified; among these are a 2^+, 0^+, 3^+ triplet at 5.3 MeV and 4^+ level at 7.1 MeV. The 0^+ and 2^+ states near 5.3 MeV seem to belong to higher configurations and can be described as deformed states [18, 19, 20]. There is still some uncertainty about the effects of such deformations on the "vibrational" states, but the evidence, both experimental [21] and theoretical, indicates that there is, for instance, less than 15% admixture of deformed configurations in the ground state.

In our shell model calculations we have ignored the possibility of deformation, thus the calculated spectra do not show any levels corresponding to the third 0^+ and 2^+ states. It can be seen that while the first 2^+ state is given quite well by both the Kallio and Inoue forces, the closely spaced "two phonon" states are reproduced with only moderate success. The Kallio force pulls the second 0^+ state about 0.7 MeV too low. The other "two phonon" states are given to better than 0.3 MeV. The first 3^+ and second 4^+ levels are given quite badly by both forces, but the deviations are in opposite directions, hence, it is likely that the use of a moderately different exchange mixture would

give these states correctly.

The spacing of all the states as predicted by the Rosenfeld force is consistently too small.

The calculated binding energies of the ground state are listed in table 5 along with the corresponding experimental values. All these energies are measured from the ^{16}O ground state. The Kallio force underestimates the binding by 0.5 MeV, the Rosenfeld force by nearly one MeV , while the Inoue force gives it perfectly.

The spectra obtained using the Lawson force are not illustrated in the figures. This force predicts the spacing between the ground state, the one and two phonon states and the first 3^+ state to better than 0.1 MeV in each case (16). It cannot of course predict the second 4^+ state because it only takes into account the $0d_{5/2}$ and $1s_{1/2}$ single particle states. The binding energy of the ground state is predicted to be 0.3 MeV too small.

The ^{18}O spectrum was also calculated using the Gillet force. The binding energy turned out to be 2.7 MeV too weak, and the spacing between the "vibrational" states

was too small by a factor of two. The Gillet force is therefore totally inadequate for the evaluation of the particle-particle matrix elements.

The calculated and experimental [17, 22] spectra of ^{18}F are shown in fig. 4 . It is clear that the levels at 1.08, 1.70, 2.10, and 2.53 MeV cannot be predicted by any of the forces used, so that they probably arise from more complicated configurations. The Inoue force predicts very well all the other identified levels. The Kallio force pulls the 1^+, 0 ground state down too low with respect to the rest of the levels, whose spacing is otherwise quite good. The Rosenfeld force gives a fairly poor agreement with the experimental spectrum.

The binding energies with respect to the ^{16}O ground state and with the Coulomb forces subtracted out are shown in table 5 . The Inoue force gives the correct binding energy, the Kallio force gives 0.3 MeV too much binding, while the Rosenfeld force gives 0.3 MeV too little.

The experimental negative parity spectrum of ^{16}O below 16 MeV [17] is characterized by a closely spaced quartet of T=1 levels near 13 MeV, imbedded in a more or less evenly distributed set of T=0 levels starting from 6 MeV. By analogy with the spectrum

of ^{16}N, all other T=1 states should lie above 16 MeV, hence the three observed 2^- states with no experimental isospin assignments are probably T=0. Very few spin and isospin assignments are known above 16 MeV; Isabelle and Bishop [23] have tentatively identified two $1^-;1$ states at 22.5 and 25.5 MeV respectively and de Forest et al. [24] have some evidence for a $2^-;1$ level at 20.2 MeV. Finally, there is a 0^- level listed in ref 17 at 16.3 MeV.

The Gillet force gives all the known T=1 levels reasonably well. The quartet at 13 MeV lies about 1/4 MeV too high and the $0^-;1$ and $3^-;1$ levels are inverted. The Rosenfeld force gives the quartet about 1/4 MeV too low, but the level sequence is a little better since the $0^-;1$ level is now the lowest T=1 state; however the $3^-;1$ state is still too low compared with the center of gravity of the quartet.

The lowest calculated $1^-;0$ state should be entirely spurious and should come in at zero energy if ground state correlations were properly included [5]. It can in fact be verified that the lowest $1^-;0$ state predicted using the Gillet force, the Rosenfeld force, or the Inoue force contains at least 80% of the spurious state constructed according to the prescription of Baranger and Lee [7].

A glance at fig. 5 shows that for every T=0 state predicted

by the shell model using one particle-one hole configurations only there are about two experimentally observed levels of the same spin and parity. It is therefore clear that the presence of higher configurations is not negligible. Although attempts have been made to sort out the effects of higher configurations[25], the situation is not yet completely clear; it appears however that one can say the following. The $3^-;0$ and the $0^-;0$ states observed at 6.13 MeV and 10.9 MeV respectively are probably good particle-hole states, however the $3^-;0$ level is strongly affected by ground state correlations [9]. The $2^-;0$ and $1^-;0$ states appear to involve considerable configuration mixing.

The Gillet force gives the correct position of the first $0^-;0$ level, but it gives the lowest $3^-;0$ state about 1.3 MeV too high; if the ground state correlations were taken into account by using the RPA*, then this state would come down to its correct position also [9]. In spite of the likelihood of strong configuration mixing, it appears that the lowest non-spurious $1^-;0$ particle-hole state is predicted too high, probably by around one MeV. Generally then, the Gillet force doesn't pull the T=0 states down low enough; the clear exception to this statement is the $0^-;0$ level which consists of the pure $(0p_{1/2}^{-1} ; 1s_{1/2})$ configuration, and which is quite correctly predicted.

* "Random Phase Approximation" : see refs. 5, 9, and 10.

The Rosenfeld force pulls the first $3^-;0$ level about 0.7 MeV too low; since ground state correlations would lower it further still, this indicates that the force is somewhat too strong. The first $1^-;0$ state is now nearer to its expected position than with the Gillet force, but the $0^-;0$ state is raised nearly 3 MeV too high.

The Inoue force yields no improvement on the Gillet force as far as the T=1 spectrum is concerned, while the first $3^-;0$ state is pulled nearly 2.5 MeV too low. Since the $0^-;0$ state ends up around one MeV too high, it appears that the fit is generally quite poor. The Inoue force therefore doesn't seem to be useful for the calculation of the particle-hole matrix elements.

§7. Negative parity states of ^{17}F and ^{17}O

The calculated and experimental energy levels of ^{17}F are shown in fig. 6 . The experimental level scheme was taken from the work of Salisbury and Richards [8] who studied these states by elastic proton scattering on ^{16}O. The level at 3.86 MeV which they labelled as $7/2^-$ from the previous work of Laubenstein and Laubenstein [26] has since been identified as $5/2^-$ by Segel et al.[27]. The experimental spectrum of ^{17}O is identical to that of ^{17}F below 6.1 MeV; above this energy, the level density increases, fewer unequivocal spin and parity assignments have been made, and the similarity between the two spectra ceases to be quite as startling.

The calculated spectra have been normalized so that the lowest $1/2^-;1/2$ state lies at 3.10 MeV. The level spacings obtained using either the Kallio-Gillet or the Inoue-Gillet forces agree with the experimental spacings as well as can be expected considering the quality of the fits obtained in ^{16}O, ^{18}O, and ^{18}F. The use of the Rosenfeld force in both the particle-particle and the particle-hole matrix elements yields much worse results, which is of course a reflection of the poor fits obtained in the A=18 nuclei. We shall return to the

spectrum obtained using the Rosenfeld force a little later, but for the moment the discussion will be restricted to the other two spectra.

Because of the large dimension of the basis used to describe these states, it is not practical to tabulate the complete wavefunctions , although we could of course list the main components. For the purposes of the following discussion , however , it is more convenient to describe the negative parity states of the A=17 nuclei in terms of a $0p_{1/2}$ or $0p_{3/2}$ hole coupled to a "core" consisting of those eigenstates of the A=18 nuclei that can be described in terms of two-particle configurations . The wavefunctions of the first few T=1/2 states are given in tables 8,9,10,and 11 using this type of decomposition . Tables 6 and 7 show the wavefunctions of the relevant ^{18}O and ^{18}F states. One can note that the configurations which account for less than a few percent of any state have generally been omitted from the tables .The figure in brackets at the foot of each column indicates the fraction of the normalization that is accounted for by the listed components.

Let us now discuss the $1/2^-;1/2$ levels in some detail. There are three such states predicted below 9 MeV. ; the lowest of which of course appears at the correct energy because of the arbitrary normalization of the calculated spectra. Because of the

β decay information to be seen later, the second $1/2^-$; 1/2 level, experimentally seen at 6.04 MeV, must be identified with the level predicted at 5.94 MeV (Kallio-Gillet forces) or at 6.39 MeV (Inoue-Gillet forces) whose structure is essentially $0p_{1/2}^{-1}$ (first 1^+,0) , that is to say, a $0p_{1/2}$ hole coupled to the ground state of ^{18}F as a "core". This leaves an " extra " $1/2^-$;1/2 state predicted in the region just above 6.0 MeV whose structure is dominantly $0p_{1/2}^{-1}$ (second 0^+;1). The inversion of the order of the two $1/2^-$;1/2 states near 6 MeV can be directly attributed to the characteristics of the Kallio and the Inoue forces as observed in ^{18}O and ^{18}F . We may recall that the Kallio force pulls the 1^+,0 ground state of ^{18}F down too low as compared to the other states, whereas the Inoue force gives the second 0^+,1 state in ^{18}O too low.

These $1/2^-$;1/2 states also serve to illustrate the arguments presented in section §3 . The state based on the configuration $0p_{1/2}^{-1}$ (first 1^+,0) which would lie one MeV below the state based on the configuration $0p_{1/2}^{-1}$ (first 0^+,1) in the absence of particle-hole forces turns out in fact to lie 3 MeV above this state. The first $1/2^-$;3/2 state lies far above both of them.

The predicted and observed energies of the first $3/2^-$;1/2

level agree to $\sim$ 1/2 MeV, which is satisfactory. However, the second predicted $3/2^-$ state lies halfway between the two experimental levels at 5.5 and 6.7 MeV respectively. There is, furthermore, another experimental $3/2^-$ level at 7.0 MeV while the nearest calculated levels are at least one MeV above this. We could of course identify sequentially the second, third and fourth experimental and calculated levels, noting that the predicted energies are then too high by 0.6, 1.3, and 1.4 MeV respectively. However, a more satisfactory explanation can be given if we note that the experimental proton reduced (8) widths γ_p^2 of the two levels at 6.70 and 7.03 MeV are two orders of magnitude smaller than the reduced widths of the two levels seen at 5.52 and 4.69 MeV. This suggests that the upper two levels may be due to higher configurations (presumably four-particle,three-hole), since γ_p^2 for a given level is proportional to the admixture of single particle configurations in the level,and, whereas the polarization diagrams provide an immediate mechanism for mixing these with the two-particle,one-hole configurations, there are no matrix elements connecting the single particle configurations to the four-particle,three-hole configurations. These two upper $3/2^-$ levels can probably be understood as arising from two of the states between 1.0 and 3.0 MeV in ^{18}F which also cannot be described by two-particle

configuration. The energies of these states turn out to be roughly right if we assume that the effects of the particle-hole forces are the same for all states involving T=0 "cores" as their dominant configuration. The presence of these two extra states would tend to depress the second calculated $3/2^-;1/2$ state closer to its observed position at the expense of some configuration mixing. The calculated $3/2^-;1/2$ states above 7.5 MeV can probably be identified with some $3/2^-$ states seen in ^{17}O at 7.72, 8.20, and 8.68 MeV [8,28] which have neutron reduced widths of the right order of magnitude to be two-particle, one-hole states.

The calculations give the first $5/2^-$ level about 0.4 MeV above its experimental position, which is within the accuracy expected of the model. However, two additional $5/2^-$ states are predicted near 6MeV and 7 MeV respectively. There is no experimental evidence for such levels in ^{17}F; however, two 5/2 levels (parity not known) have been seen in ^{17}O at 7.16 and 7.37 MeV respectively[8], one of which may correspond to the calculated state near 7 MeV. The state predicted near 6 MeV seems too far away to be identified with any of these levels; however, if its width is as narrow as that of the known state at 3.86 MeV (i.e. 3KeV), it is not impossible

that it would have been missed in the experiments.

The interpretation of the $7/2^-;1/2$ states is a little more complicated, first because these are the only low lying states that involve the badly predicted $3^+,1$ and $4^+,1$ levels of ^{18}O as "cores", and secondly because large admixtures of the single particle configuration are involved.

Using the Inoue-Gillet force, the model predicts a $7/2^-$ state at 5.67 MeV. in excellent agreement with experiment. The dominant configuration in this state is $0p_{1/2}^{-1}$ (first $3^+,0$). The next three states, which lie between 7 and 9 MeV, are mixtures of four configurations; in particular, they contain 65% of the single particle $0f_{7/2}$ strength. Now experimentally, there should not be much single particle strength below 9 MeV [8, 28], consequently it seems clear that the unperturbed energy of the $0f_{7/2}$ state was chosen too low. Raising the $0f_{7/2}$ energy by two or three MeV would then leave only two $7/2^-$ states with predicted energies between 7 and 8 MeV. One of these should in fact lie near 6.5 MeV, since one of the dominant configurations in these states is $0p_{1/2}^{-1}$ (first $3^+,1$) and it was noted in section 6 that the Inoue force gave the first $3^+,1$ state of ^{18}O approximately one MeV too high. The other predicted $7/2^-$ state presumably

stays between 7 and 8 MeV and can perhaps be identified with the $7/2^-$ level observed in ^{17}F at 7.55 MeV.

Now from the calculated ^{18}O spectrum we know that the Kallio force pulls the first $3^+,1$ and the first $4^+,1$ "core" states about one MeV too low. Because of this, the two $7/2^-$ states arising from the $0p_{1/2}^{-1}$ (first $3^+,0$) and the $0p_{1/2}^{-1}$(first $3^+,1$) configurations are found to be nearly degenerate at 5.7 MeV when the ^{17}F spectrum is calculated with the Kallio-Gillet force. Clearly one of these states should in fact lie one MeV higher. The next two $7/2^-$ states lie between 7 and 9 MeV, but a little lower than with the Inoue-Gillet force, and they again contain most of the single particle $0f_{7/2}$ strength.

It seems therefore clear that the shell model predicts a $7/2^-$ state at 5.6 MeV whose principal component is the $0p_{1/2}^{-1}$ (first $3^+,0$) configuration. This state can be identified with the $7/2^-$ level observed in ^{17}F at the same energy. Furthermore the calculations show that with a reasonable particle-particle force and a realistic choice of the $0f_{7/2}$ energy, two more $7/2^-$ states should appear, one around $6.5 \rightarrow 7$ MeV and the other around $7 \rightarrow 8$ MeV. The latter may perhaps be identified with

the state observed at 7.55 MeV.

We now discuss the absolute energy of the lowest $1/2^-$; 1/2 state which in fig. 6 was normalized to 3.10 MeV. Table 12 shows the experimental and calculated energies of this level measured from the ^{16}O ground state with the Coulomb energies subtracted out (see appendix D). It is clear that the calculated level is too weakly bound. The reason for this result can be found in the discussion of the ^{16}O negative parity states where it was observed that the Gillet potential was not strong enough to pull many of the T=0 states down to their observed position. The magnitude of this defect in the particle-hole force is quite adequate to account for the discrepancy in the binding of the lowest $1/2^-$;1/2 state of ^{17}F.

We can also use the above arguments to suggest the probable reason for the appearance of an extra $1/2^-$;1/2 state in the calculated spectrum of ^{17}F. As discussed in section §6, it is principally the $0p_{1/2}^{-1}, 0d_{5/2}$ T=0 matrix elements that are too small. The only low lying negative parity state in ^{17}F that does not depend strongly on these particle-hole matrix elements is precisely the extra $1/2^-$;1/2 state which is dominantly $0p_{1/2}^{-1}$ $(1s_{1/2}^2;0^+,1)$. If the $0p_{1/2}^{-1}, 1s_{1/2}$ particle-hole matrix

elements are more or less correctly given by the Gillet force, then this extra state should lie one MeV or so above its position shown in fig. 6 , which puts it just high enough to be near the region where there are many experimental levels with no spin assignments.

It must also be noted that the above arguments apply only to T=1/2 states. The T=3/2 states are not affected by the T=0 particle-hole matrix elements, hence they should not be lowered with the T=1/2 states. This then puts the $1/2^-;3/2$ levels shown in fig. 6 about 1.5 MeV above their indicated position.

The ^{17}F spectrum obtained using the Rosenfeld force for both the particle-particle and the particle-hole matrix elements has broadly speaking the correct characteristics, however the levels are shifted from their correct positions often by more than one MeV and in a fairly disorganized manner, which reflects the poor spectra obtained for the A=18 nuclei with this force. The following seems to be true: a central potential that predicts the particle-hole states of ^{16}O reasonably well is too weak to give the proper spectrum for the A=18 nuclei (this is the case for the Gillet potential); while

apotential which yields good spectra for the A=18 nuclei is too strong to reproduce the particle-hole states of ^{16}O (this is the case for the Inoue potential). In a sense the Rosenfeld force that we have used is a compromise: it is a little too strong in ^{16}O (i.e. it pulls the first $3^-,0$ state too low) and it is also too weak to give the correct spacing between the levels of ^{18}O. The results for the spectrum of ^{17}F are consequently not very good when the same force is used in both the particle-particle and the particle-hole matrix elements.

It might appear that the Kallio force would be useful both as a particle-particle force and as a particle-hole force since it has already been used to describe both ^{16}O [29] and ^{18}O [12]. However, although the parameters of the force were left the same in both these calculations, the harmonic oscillator constant was changed from $\hbar\omega$=17 MeV in the former to $\hbar\omega$=13.3 MeV in the latter. Now the radial spread of the wavefunction depends on $(\frac{m\omega}{\hbar})^{-1/2}$, consequently the oscillator functions used in ^{18}O are more spread out than those used in ^{16}O, which results in an effectively stronger interaction in ^{18}O for this particular type of force. The Kallio force also has the drawback that it actually raises the first $2^-,0$ state in ^{16}O from its unperturbed position instead of lowering it. This reflects very strongly on the energies of the low lying states of ^{17}F. The Kallio force was therefore not used as a particle-hole force in our calculations of the spectra of the A=17 nuclei.

§ 8. Negative parity states of ^{17}N and ^{17}Ne

The calculated and experimental[30] spectra of ^{17}N below 7.0 MeV are shown in fig. 7 . It is evident from the figure that very little experimental information is available on the levels of ^{17}N, in particular no spin or parity assignments have yet been made except for the ground state. The spectrum of the mirror nucleus ^{17}Ne is even less well known.

The spectrum calculated using the Lawson-Rosenfeld force is thought to be the most reliable since the Lawson force predicts the ^{18}O spectrum very well, while the Rosenfeld force gives the low lying T=1 states of ^{16}O somewhat better than the Gillet potential. We shall therefore compare this spectrum to the available experimental data and then discuss the variations found in the other calculated spectra. The first excited state found at 1.37 MeV is apparently not given by the calculation, and hence it probably arises from configurations not included in the calculation. The two states of the doublet at 1.9 MeV are well predicted, their principal component being the $0p^{-1}_{1/2}$ (first 2^+,1) configuration. The next five calculated levels arise from the "two phonon" triplet of ^{18}O; these states should therefore be characterized

by a weak electromagnetic cross-over transition to the ground state, but strong transitions to the doublet at 1.9 MeV. These levels are followed by two states based on the first 3^+,1 of ^{18}O as "core". It is possible to tentatively identify the $1/2^-$ state,calculated to be at 2.83 MeV,with the experimentally observed state at 2.54 MeV; however, the identification of the other states will have to await the availability of some experimental spin assignments.

Since the 3.21 MeV state is observed to have a 50% branching ratio to the ground state [30], it is possible that this state does not correspond to any of the calculated states. This would then leave five experimental and five calculated levels between 3.0 and 5.0 MeV. The great number of experimental levels seen above 5 MeV probably correspond to the many levels observed in ^{18}O at about the same distance above the ground state, consequently most of these must be assigned to higher configurations.

The deviations of the other calculated spectra from the one obtained using the Lawson-Rosenfeld force can be traced directly to the differences between the calculated and observed spectra of ^{18}O. Thus, for example, the $5/2^-$ level calculated to lie at 5.37 MeV using the Lawson-Rosenfeld force is given

one MeV below this position by the Kallio-Gillet force and one MeV above this position by the Inoue-Gillet force. This is a precise reflection of the shifts of the 3^+,1 level of ^{18}O as given by these forces.

The disturbing feature of these calculations of the ^{17}N spectrum is of course the fact that the 1.37 MeV level apparently cannot be explained as a two particle-one hole state. The presence of higher configurations so near the ground-state implies the possibility of a great deal of configuration mixing if the parity of this first excited state were to be negative. It might be pointed out in this connection that a fairly accurate prediction of the energies of the various states does not necessarily imply that all the more important configurations have been properly taken into account. For example, the first four excited states of ^{18}O are quite well predicted by the shell model using only two particle configurations; on the other hand it now appears that these states may contain a fair amount (perhaps 15%) of "deformed" configurations [18,19,20,21] which is enough to greatly affect some properties such as transition probabilities for instance.

The experimental and calculated binding energies of the ^{17}N ground state are given in table 12; these energies

are as usual measured from the ^{16}O ground state with the Coulomb energies subtracted out. It can be seen that the calculated values are too low by roughly one MeV. We have no explanation for this discrepancy.

§ 9. Electromagnetic transition probabilities in ^{17}O and ^{17}F

Fig. 8 shows the calculated transition probabilities for the electromagnetic decay of the low-lying negative parity states of ^{17}F to the ground state and first excited state. The wavefunctions of the initial states were those obtained using the Inoue-Gillet force, while the ground state and the $1/2^+$ level at 0.5 MeV were assumed to be the single particle $0d_{5/2}$ and $1s_{1/2}$ states. The wavefunctions obtained using all three sets of forces were used to calculate those transition probabilities in ^{17}F and ^{17}O that have been experimentally studied [27, 32]; these results are shown in table 13. The limit given in the table for the transition from the first $5/2^-$ level of ^{17}O to the level at 0.88 MeV is derived from the upper limit set by Broude et al. [32] on the branching ratio to this $1/2^+$ state, and the transition probability measured by Segel et al. [27] for the decay of the analogue $5/2^-$ level in ^{17}F; since this is an indirect estimate it is marked in table 13 by a question mark.

The measured E1 transitions are two orders of magnitude

slower than the corresponding single particle transitions. Such very long E1 lifetimes can be understood as follows. If we ignore the single particle configurations which appear as very small components in the low lying negative parity states of ^{17}O and ^{17}F , then these states can be described in terms of a particle coupled to an excited ^{16}O core . Now the giant dipole resonance in ^{16}O lies above 20 MeV. , so it is unlikely that the extra particle in ^{17}O or ^{17}F will pull much E1 strength down into the low lying states. Hence , there is only a very weak E1 transition between the two particle-one hole part of the first few excited states and the single particle even parity states.

The calculations generally bear out these arguments. In detail, the transition amplitudes turn out to be small for two reasons . First, of the five particle-hole combinations that contribute to E1 transitions , only one appears in the first few negative parity states with a moderately large amplitude. Secondly, a considerable amount of cancellation occurs between the matrix elements to the many configurations that enter with small amplitude. Thus lifetime calculations based only on the dominant configurations can turn out to be very inaccurate : for example, the transition

probability from the first $1/2^-$ level of ^{17}O to the 0.88 MeV. level, calculated using the wavefunctions obtained with the Inoue-Gillet forces, is equal to 0.25×10^{12} sec.$^{-1}$; using the same wavefunctions, but omitting those configurations that are based on the $0p_{3/2}$ hole, we obtain $\Lambda_{E1} = 0.4 \times 10^{14}$ sec.$^{-1}$, in spite of the fact that the configurations based on the $0p_{3/2}$ hole account for only 6% of the initial state.

How reliable , then , are the calculated transition probabilities ? In general , the amplitudes of the small components vary by less than a factor of two with changes in the forces used; consequently , if too much cancellation does not occur , the calculated lifetimes should be accurate to a factor of four or five . On the other hand , when cancellations become excessive , then the calculated lifetimes are no better than order of magnitude estimates. It can be seen from table 13 that the calculated and experimental electromagnetic transition probabilities agree to within the limitations of the calculation.

§10. The β^+ decay of ^{17}Ne

We can now calculate the log ft values for the β-decays from ^{17}Ne to ^{17}F using the previously calculated wavefunctions. Since this is a $\Delta T=1$ transition, it is purely Gamow-Teller and we can write (31)

$$\log ft = 3.64 - \log \langle \vec{\sigma} \rangle^2$$

where $\langle \vec{\sigma} \rangle$ is the conventional symbol for the Gamow-Teller matrix element. It is given by:

$$\langle \vec{\sigma} \rangle^2 = \frac{1}{2(2J+1)(2T'+1)} \left| (1, M_{op}, T, M_t \mid T', M_t') \right|^2 \left| \langle J'T' \| \sum_{i=1}^{A} \vec{\sigma}(i) \; \vec{\tau}(i) \| JT \rangle \right|^2$$

with $M_{op} = +1$ for β^- decay and $M_{op} = -1$ for β^+ decay. The reduced matrix element in the "basis" states is shown on the next page, equ. 21 . It can be seen that this matrix element separates into two contributions, one which corresponds to a hole transition and the other which corresponds to a "core" transition. The derivation of equ.21 and the details of the evaluation of the various factors are given in appendix $\mathcal{B}$.

$$\langle \eta'^{-1}(\nu'\mathcal{H}';\tilde{J}'\tilde{T}')J'T' \| \vec{\sigma}_{\vec{\tau}} \| \eta^{-1}(\nu\mathcal{H};\tilde{J}\tilde{T})J\,T\rangle = \left[(2J+1)(2J'+1)(2T+1)(2T'+1)\right]^{1/2}$$

$$\times \Big\{ \delta_{\eta\eta'} (-)^{-J'-\tilde{J}+j_\eta} (-)^{-T'-\tilde{T}+1/2} W(\tilde{T},T,\tilde{T}',T';1/2,1)\, W(\tilde{J},J,\tilde{J}',J';j_\eta,1)$$

$$\times \langle (\nu'\mathcal{H}')\tilde{J}'\tilde{T}' \| \vec{\sigma}_{\vec{\tau}} \| (\nu\mathcal{H})\tilde{J}\,\tilde{T}\rangle$$

$$- \delta_{\tilde{J},\tilde{J}'}\, \delta_{\tilde{T},\tilde{T}'}\, N_{\nu'\mathcal{H}'} N_{\nu\mu} \left(\delta_{\nu\nu'}\delta_{\mathcal{H}\mathcal{H}'} - (-)^{j_\nu+j_\mu-\tilde{J}+1-\tilde{T}} \delta_{\nu\mathcal{H}'}\delta_{\mu\nu'} \right)$$

$$\times (-)^{-J+\tilde{J}+j_\eta-T+\tilde{T}+1/2} W(1/2,T,1/2,T';\tilde{T},1)\, W(j\ ,J,j',J';\tilde{J},1)$$

$$\times \langle \psi_\eta \| \vec{\sigma}_{\vec{\tau}} \| \psi_{\eta'} \rangle \Big\}$$

.....(21).

The $\vec{\sigma}\vec{\tau}$ operator does not connect the single particle basis states to the two particle-one hole states so the former do not enter the expression for the log ft.

The measured [1] and calculated log ft values for the decay $^{17}Ne \xrightarrow{\beta^+} {}^{17}F$ are shown in fig. 9 . The experimental log ft values for the decay $^{17}N \xrightarrow{\beta^-} {}^{17}O$ are essentially similar [2].

Experimentally the decay to the lowest $1/2^-$; 1/2 state of ^{17}F is strongly inhibited with a log ft value greater than 5.8. The mirror decay to the lowest $1/2^-$ state of ^{17}O has a log ft = 6.8. The calculations also predict a weak decay to this state which can be understood in terms of a cancellation as follows. The $0p_{1/2}^{-1}$ (first 0^+,1) configuration accounts for 96% of the ground state of ^{17}Ne, while the lowest $1/2^-$;1/2 state of ^{17}F is made up primarily of the configuration $0p_{1/2}^{-1}$ (first 0^+,1) and a small admixture of $0p_{1/2}^{-1}$ (first 1^+,0) which together account for 90% of the state. Now, let A be the amplitude of the configuration based on the 0^+,1 "core" and B the amplitude of the configuration based on the 1^+, 0 "core". Then the Gamow-Teller reduced matrix element can be written according to eq. 21 . as the sum of a hole part and a "core" part

$$A \langle 0p_{1/2} \| \vec{\sigma}\vec{\tau} \| 0p_{1/2} \rangle + B \langle \text{first } 1^+,0 \| \vec{\sigma}\vec{\tau} \| \text{first } 0^+,1 \rangle$$

where, for the sake of clarity, we have omitted all the angular momentum and isospin coefficients which in this case turn out to be almost equal for both terms. Now $B << A$ according to table 8 ; on the other hand, the "core" matrix element turns out to be nearly three times larger than the hole matrix element. The relative signs of A and B are determined by the relevant off-diagonal matrix elements of the particle-hole force; it turns out that two terms interfere destructively and thus give rise to the very large observed log ft value.

The $1/2^-;1/2$ state predicted at 6.39 MeV using the Inoue-Gillet force or at 5.94 MeV using the Kallio-Gillet force is essentially the state built on the same $0^+,1$ and $1^+,0$ "cores" as the lowest $1/2^-;1/2$ and orthogonal to it. It therefore has the values of A and B interchanged with the opposite relative sign. The interference is therefore constructive and gives rise to a very fast calculated decay rate with a log ft = 3.4. This is much faster than the observed log ft which is $\sim$ 4.3.

These interference effects illustrate the great

sensitivity of the β-decay matrix elements to the small admixtures in the wavefunctions. In this sense the log ft values are an excellent test of the finer details of the calculation; on the other hand they are often too sensitive to verify the broad features of the model. This can again be illustrated by the third low lying $1/2^-;1/2$ (this is the "extra" state which has not been identified with any experimental level). This state contains as dominant component the $0p_{1/2}^{-1}$ (second 0^+,1) configuration which is not connected by the $\vec{\sigma}\vec{\tau}$ operator to the main components of the ^{17}Ne ground state. The transition therefore proceeds entirely via the small components of these wavefunctions. The amplitude of the $0p_{1/2}^{-1}$ (first 1^+,0) configuration in this state is found to be essentially zero using the Kallio-Gillet force and 0.155 using the Inoue-Gillet force: correspondingly the log ft values change from $\sim$ 6.7 in the former case to $\sim$ 4.9 in the latter.

The decays to the $3/2^-;1/2$ levels of ^{17}F are quite similar to the ones discussed above. The dominant component of the first $3/2^-$ state is the $0p_{1/2}^{-1}$ (first 1^+,0) configuration; the decay is therefore calculated to be very fast with log ft = 3.3, which is much smaller than the experimental value of 4.3. On the other hand, the log ft of the decay to the second $3/2^-$ level has a calculated value of 4.85 (Kallio-Gillet forces) or 6.7 (Inoue-Gillet forces) which is much

larger than the experimental value of 3.8, This occurrs because the $\vec{\sigma}\vec{\tau}$ operator doesn't connect the main components of this state to the ground state of ^{17}Ne, the transition being therefore forced to proceed via the small components only. The difference between the two calculated log ft values can be largely accounted for by the different small admixtures of the $0p^{-1}_{1/2}$ (first 1^+,0) configuration predicted by the two forces. The log ft value of the decay to the third $3/2^-$ state predicted near 8 MeV is calculated to be 4.2; it is therefore tempting to try to identify this state with the level observed in the delayed proton experiments near 8 MeV with a log ft value ~ 4.0. At any rate this fast β-decay certainly rules out any identification of the third calculated $3/2^-$ level with the two levels seen in the proton scattering work [8] at 6.7 and 7.0 MeV since these are not observed to have any β decay strength.

It appears therefore that, although the decay to the lowest $1/2^-;1/2$ state is well predicted, these calculations cannot explain all the other observed β^+-decays from ^{17}Ne to ^{17}F . In particular, some mechanism must be provided for a better sharing of the decay strength between the two lowest $3/2^-$ levels of ^{17}F. Presumably the same mechanism

should also weaken the very fast calculated decay to the $1/2^-$ state at 6 MeV, since all these discrepancies seem to arise from the concentration of the $0p^{-1}_{1/2}$(first 1^+,0) configuration in one $1/2^-$ state and one $3/2^-$ state. The presence of higher configurations as indicated by the two $3/2^-$ states near 7 MeV could perhaps provide the required mechanism. On the other hand, it must be noted that higher configurations very probably also affect the ground state of ^{17}Ne (see section § 8), and this could also seriously alter the log ft values.

§ 11 . Conclusions .

These calculations show that the negative parity states of ^{17}O and ^{17}F up to 6 MeV. can be adequately described in terms of two particle-one hole configurations . There are however indications that levels arising mainly from higher configurations appear above 6 MeV. ; some mixing of these into the lower states is therefore to be expected . Whether such mixing can explain the discrepancies in the log ft values for the decay $^{17}Ne \xrightarrow{\beta^+} {}^{17}F$ can only be decided by further calculations.

In the absence of experimental spin assignments, it is difficult to say much about the spectrum of ^{17}N . It seems that the first few observed energy levels (with the notable exception of the 1.37 MeV. level) can be identified as negative parity two particle-one hole states. The 1.37 MeV. level either has positive parity, or it must arise from higher configurations.

This study of the negative parity states in the A=17 nuclei can be extended in two ways : first , the effect of four particle-three hole configurations should be investigated in detail, and secondly, the possibility of ground state correlations should be included in the calculations. This latter point is particularly important if good absolute energies are required.

References

(1) R. Barton, Ph. D. thesis, McGill University (1963)
R. McPherson, J. C. Hardy and R. E. Bell, Phys. Lett. 11, 65 (1964)
J. C. Hardy and R. E. Bell, Can. J. Phys. 9, 1671 (1965)
A. M. Poskanzer, Invited paper at the New York Meeting of the A. P. S. (1965)

(2) M. G. Silbert and J. C. Hopkins, Phys. Rev. 134, B 16 (1964)

(3) B. Margolis and N. de Takacsy, Phys. Lett. 15, 329 (1965)

(4) I. Talmi and A. de-Shalit, "Nuclear Shell Theory" Academic Press (1963)
M. A. Preston, "Physics of the Nucleus" Addison-Wesley (1962)

(5) G. E. Brown, "Unified Theory of Nuclear Models" North-Holland 1964

(6) A. R. Edmonds, "Angular Momentum in Quantum Mechanics" (Princeton 1957)

(7) E. V. Baranger and C. W. Lee, Nucl. Phys. 22, 157 (1961)

(8) S. R. Salisbury and H. T. Richards, Phys. Rev. 126, 2147 (1962)

(9) V. Gillet and N. Vinh-Mau, Nucl. Phys. 54, 321 (1964)

(10) A. M. Lane, "Nuclear Theory" W. A. Benjamin (1964)

(11) T. Inoue, T. Sebe, H. Hagiwara, and A. Arima, Nucl. Phys. 59, 1 (1964)

(13) B. L. Scott and S. A. Moszkowski, Nucl. Phys. 29, 665 (1962)
Ann. of Phys. (New York) 11, 65 (1960)

(14) J. P. Elliott and B. H. Flowers , Proc. Roy. Soc. A-229, 536 (1955)

(15) T. D. Newton , private communication (1965)

(16) S. Cohen, R. D. Lawson, M. H. Macfarlane and M. Soga, Phys. Lett. 9, 180 (1964)

(17) F. Ajzenberg-Selove and T. Lauritsen,Nucl. Phys. 11,1 (1959)

(18) G. E. Brown,Rapporteur's talk Paris Conference (1964)

(19) T. Engeland,Nucl. Phys. 72, 68 (1965)

(20) P. Federman and I. Talmi,Phys. Lett. 15, 165 (1965)

(21) J. R. Erskine, R. E. Holland, R. D. Lawson, M. H. Macfarlane, J. P. Schiffer, Phys. Rev. Lett. 14, 915 (1965)

(22) A. R. Poletti and E. K. Warburton,Phys. Rev. 137 B 595 (1965)

(23) D. B. Isabelle and G. R. Bishop, Report LAL-1017, Laboratoire de l'accélérateur linéaire, Orsay

(24) T. de Forest Jr., J. D. Walecka, G. Vanpraet, and W. C. Barber , Phys. Lett. to be published and preprint (1965)

(25) G. E. Brown and A. M. Green , Phys. Lett. 15, 168 (1965)

(26) R. A. Laubenstein and M. J. W. Laubenstein , Phys. Rev. 84, 18 (1951)

(27) R. E. Segel, P. P. Singh, R. G. Allas and S. S. Hanna, Phys. Rev. Lett. 10, 345 (1963)

(28) B. K. Barnes, T. A. Belote and J. R. Risser , Phys. Rev. Lett. 10, 345 (1963)

(29) A. Kallio and K. Kolltveit,Nucl. Phys. 53, 87 (1964)

(30) V. P. Hart, E. Norbeck and R. R. Carlson , Phys. Rev. 137, B 17 (1965)

(31) J. Bahcall , Phys. Rev. 135, B 137 (1964)

(32) T. K. Alexander, C. Broude and A. E. Litherland , Nucl. Phys. 53, 593 (1964)

(33) J. M.Kennedy and M. J. Cliff , Chalk River Report CRT-609 (1955)

(34) M. Moshinsky , Nucl. Phys. 13, 104 (1959)

(35) J. M. Kennedy and W. T. Sharp , Chalk River Report CRT-580 (1954)

(36) J. Jänecke , Report KFK 185 Karlsruhe (1963)

Level	$0p_{3/2}$	$0p_{1/2}$	$0d_{5/2}$	$1s_{1/2}$	$0d_{3/2}$	$0f_{7/2}$	$1p_{3/2}$	$0f_{5/2}$	$1p_{1/2}$
Energy (MeV.)	-21.8	-15.65	-4.15	-3.27	0.93	6.70	11.7	13.7	14.7

TABLE 1 . The single particle energies used in the study of nuclei near ^{16}O .

Name	f(r)	V_o (MeV.)	α ($f.^{-1}$)	W	B	H	M
Gillet	$\exp(-\alpha^2 r^2)$	-40.0	0.5682	0.35	-0.10	0.40	0.35
Rosenfeld	$\exp(-\alpha^2 r^2)$	-50.0	0.5774	-0.13	0.46	-0.26	0.93
Inoue	$\frac{\exp(-\alpha r)}{r}$	-35.0	0.6026	0.25	-0.136	0.25	0.636

TABLE 2 . Parameters for the residual forces of conventional form.

Spin state	V_o (MeV.)	α (f.$^{-1}$)	a (f.)	d (f.)
Singlet	-330.8	2.402	0.40	1.045
Triplet	-475.0	2.521	0.40	0.925

TABLE 3 . Parameters for the Kallio residual force .

Two particle matrix elements	(MeV.)
$\langle (0d_{5/2})^2 J=0 \mid V^{res} \mid (0d_{5/2})^2 J=0 \rangle$	-3.33
$\langle (1s_{1/2})^2 J=0 \mid V^{res} \mid (1s_{1/2})^2 J=0 \rangle$	-2.07
$\langle (0d_{5/2})^2 J=0 \mid V^{res} \mid (1s_{1/2})^2 J=0 \rangle$	-0.93
$\langle (od_{5/2})^2 J=2 \mid V^{res} \mid (0d_{5/2})^2 J=2 \rangle$	-1.38
$\langle (0d_{5/2}, 1s_{1/2}) J=2 \mid V^{res} \mid (0d_{5/2}, 1s_{1/2}) J=2 \rangle$	-0.87
$\langle (0d_{5/2})^2 J=2 \mid V^{res} \mid (0d_{5/2}, 1s_{1/2}) J=2 \rangle$	-0.69
$\langle (0d_{5/2}, 1s_{1/2}) J=3 \mid V^{res} \mid (0d_{5/2}, 1s_{1/2}) J=3 \rangle$	1.00
$\langle (0d_{5/2})^2 J=4 \mid V^{res} \mid (0d_{5/2})^2 J=4 \rangle$	0.02

TABLE 4 . The T=1 particle-particle matrix elements of the Lawson force .

	Exp't.	Kallio force	Inoue force	Rosenfeld force
^{18}O ground state	-12.21	-11.71	-12.20	-11.26
^{18}F ground state	-13.26	-13.54	-13.28	-13.00

TABLE 5 . The energies of the gound states of ^{18}O and ^{18}F measured from the ground state of ^{16}O. The Coulomb energies have been subtracted out, and all the energies are given in MeV.

configuration	$J=0^+, T=1$		$J=2^+, T=1$		$J=4^+, T=1$	$J=1^+, T=0$		$J=2^+, T=0$	$J=3^+, T=0$
	first	second	first	second	first	first	second	first	first
$0d_{5/2}^2$	0.924	-0.337	0.773	-0.622	0.974	0.716	-0.524		0.650
$0d_{5/2}{}^{1}s_{1/2}$			0.598	0.779				0.872	0.734
$0d_{5/2}{}^{0}d_{3/2}$			0.121	0.017	0,226	-0,512	-0.015	0,375	-0.192
$1s_{1/2}^2$	0.331	0.942				0.452	0.841		
$1s_{1/2}{}^{0}d_{3/2}$			0.153	0.076		0.027	-0.117	0.314	
$0d_{3/2}^2$	0.190	-0.002	0.074	0.009		-0.142	0.068		-0.041

TABLE 6 . The wavefunctions of the two particle eigenstates of ^{18}O and ^{18}F as calculated using the Kallio force.

configuration	$J=0^+, T=1$		$J=2^+, T=1$		$J=4^+, T=1$	$J=1^+, T=0$		$J=2^+, T=0$	$J=3^+, T=0$
	first	second	first	second	first	first	second	first	first
$0d_{5/2}^2$	0.862	-0.422	0.644	0.700	0.885	0.707	-0.534		0.619
$0d_{5/2}{}^{1s}{}_{1/2}$			0.677	-0.665				0.869	0.759
$0d_{5/2}{}^{0d}{}_{3/2}$			0.209	0.163	0.466	-0.474	0.062	0.340	-0.198
$1s_{1/2}^2$	0.429	0.902				0.504	0.835		
$1s_{1/2}{}^{0d}{}_{3/2}$			0.256	-0.183		0.027	-0.086	0.358	
$0d_{3/2}^2$	0.268	-0.086	0.132	0.085		-0.142	0.084		-0.042

TABLE 7 . The wavefunctions of the two particle eigenstates of ^{18}O and ^{18}F as calculated using the Inoue force .

configuration	Kallio-Gillet forces			Inoue-Gillet forces		
	3.10 MeV.	5.94 MeV.	6.65 MeV.	3.10 MeV.	6.09 MeV.	6.39 MeV.
$0p_{1/2}^{-1}$ (first 0^+,1)	0.919	0.276	0.000	0.934	0.042	0.230
$0p_{1/2}^{-1}$ (second 0^+,1)	-0.021	0.009	0.930	-0.025	0.925	-0.136
$0p_{1/2}^{-1}$ (first 1^+,0)	-0.281	0.947	0.007	-0.225	0.160	0.949
$0p_{1/2}^{-1}$ (second 1^+,0)	0.036	0.047	-0.182	0.028	-0.155	0.087
	(92.5%)	(97.5%)	(89.8%)	(92.4%)	(90.7%)	(98.0%)

TABLE 8 . The wavefunctions of the $J=1/2^-$,$T=1/2$ states of ^{17}O and ^{17}F . The wavefunctions are expressed in terms of a hole coupled to the "A=18 cores" and only the main components are shown.

configuration	Kallio-Gillet forces				Inoue-Gillet forces			
	4.55 MeV.	6.13 MeV.	7.90 MeV.	8.81 MeV.	5.05 MeV.	6.15 MeV.	8.01 MeV.	8.45 MeV.
$0p_{1/2}^{-1}$ (first 2^+, 1)	-0.212	0.893	-0.025	-0.237	-0.114	0.897	-0.066	0.224
$0p_{1/2}^{-1}$ (second 2^+,1)	0.146	0.055	-0.854	0.276	-0.172	-0.029	0.830	0.374
$0p_{1/2}^{-1}$ (first 1^+,0)	0.892	0.227	0.188	-0.051	0.900	0.117	0.192	0.086
$0p_{1/2}^{-1}$ (second 1^+,0)	-0.033	0.193	0.271	0.825	-0.018	0.222	0.388	-0.795
	(86.3%)	(88.9%)	(83.9%)	(81.6%)	(85.3%)	(86.8%)	(88.1%)	(83.2%)

TABLE 9 . The wavefunctions of the $J=3/2^-, T=1/2$ states of ^{17}O and ^{17}F . The wavefunctions are expressed in terms of a hole coupled to the "A=18 cores" and only the main components are shown.

configurations	Kallio-Gillet forces			Inoue-Gillet forces		
	4.29 MeV.	5.75 MeV.	7.03 MeV.	4.38 MeV.	6.11 MeV.	6.95 MeV.
$0p_{1/2}^{-1}$(first 2^+,1)	0.925	0.053	0.138	0.927	0.062	0.183
$0p_{1/2}^{-1}$ (second 2^+,1)	-0.055	0.919	-0.061	0.038	-0.900	0.189
$0p_{1/2}^{-1}$ (first 3^+,1)	-0.017	-0.036	0.403	-0.018	0.022	0.152
$0p_{1/2}^{-1}$ (first 2^+,0)	-0.052	-0.140	-0.444	-0.065	-0.199	-0.368
$0p_{1/2}^{-1}$ (first 3^+,0)	-0.221	0.018	0.608	-0.236	0.130	0.776
	(91.0%)	(86.9%)	(75.2%)	(92.1%)	(87.1%)	(83.0%)

TABLE 10 . The wavefunctions of the $J=5/2^-,T=1/2$ states in ^{17}O and ^{17}F . The wavefunctions are expanded in terms of a hole coupled to the "A=18 cores" and only the main components are shown.

configuration	Kallio-Gillet forces				Inoue-Gillet forces			
	5.56 MeV.	5.75 MeV.	7.28 MeV.	8.38 MeV.	5.67 MeV.	7.17 MeV.	7.70 MeV.	8.53 MeV.
$0p_{1/2}^{-1}$(first 3^+,1)	-0.712	0.483	0.162	0.123	-0.102	-0.546	0.617	0.181
$0p_{1/2}^{-1}$(second 3^+,1)	-0.043	0.047	0.040	0.211	0.015	-0.009	0.044	0.109
$0p_{1/2}^{-1}$(first 4^+,1)	-0.121	-0.365	0.701	0.318	-0.174	0.453	0.458	0.519
$0p_{1/2}^{-1}$(second 4^+,1)	-0.052	0.110	0.040	0.299	0.080	-0.086	0.022	0.212
$0p_{1/2}^{-1}$(first 3^+,0)	0.559	0.616	0.426	0.042	0.905	0.116	0.198	0.063
$0p_{1/2}^{-1}$(second 3^+,0)	-.050	0.181	-0.333	0.106	0.063	-0.523	-0.300	0.322
$0f_{7/2}$	-0.177	0.107	0.281	-0.809	0.018	-0.134	0.408	-0.686
	(87.2%)	(80.4%)	(89.2%)	(91.8%)	(87.1%)	(81.6%)	(88.8%)	(93.7%)

TABLE 11 . The wavefunctions the $J=7/2^-, T=1/2$ states of ^{17}O and ^{17}F . The wavefunctions are written in terms of a hole coupled to the "A=18 cores" and only the main components are shown.

Level:	Experimental energies (MeV.)	Calculated energies (MeV.)			
		Kallio-Gillet forces	Inoue-Gillet forces	Rosenfeld force	Lawson-Rosenfeld forces
$J=1/2^-, T=1/2$	-1.09	0.88	0.62	0.55	——
$J=1/2^-, T=3/2$	7.0	6.55	6.05	6.68	6.28

TABLE 12 . The energies of the lowest T=1/2 and T=3/2 states in the A=17 nuclei measured from the ground state of ^{16}O. The Coulomb energies have been subtracted out.

	Initial state	Final state	Λ (exp.) ($sec.^{-1}$)	Λ (calc.) ($sec.^{-1}$)		
				Inoue-Gillet	Kallio-Gillet	Rosenfeld
^{17}O	$1/2^-$ 3.06 MeV.	$5/2^+$ g.s.	$< 0.3 \times 10^{12}$	0.44×10^{11}	0.53×10^{11}	0.46×10^{11}
	$1/2^-$ 3.06 MeV.	$1/2^+$ 0.88 MeV.	0.8×10^{13}	0.25×10^{12}	0.33×10^{12}	0.12×10^{14}
	$5/2^-$ 3.85 MeV.	$5/2^+$ g.s.	$> 0.4 \times 10^{14}$	0.41×10^{14}	0.94×10^{14}	0.82×10^{14}
	$5/2^-$ 3.85 MeV.	$1/2^+$ 0.88 MeV.	$< 0.8 \times 10^{13}$ (?)	0.37×10^{9}	0.31×10^{9}	0.51×10^{8}
^{17}F	$5/2^-$ 3.86 MeV.	$5/2^+$ g.s.	0.17×10^{15}	0.39×10^{14}	0.97×10^{14}	

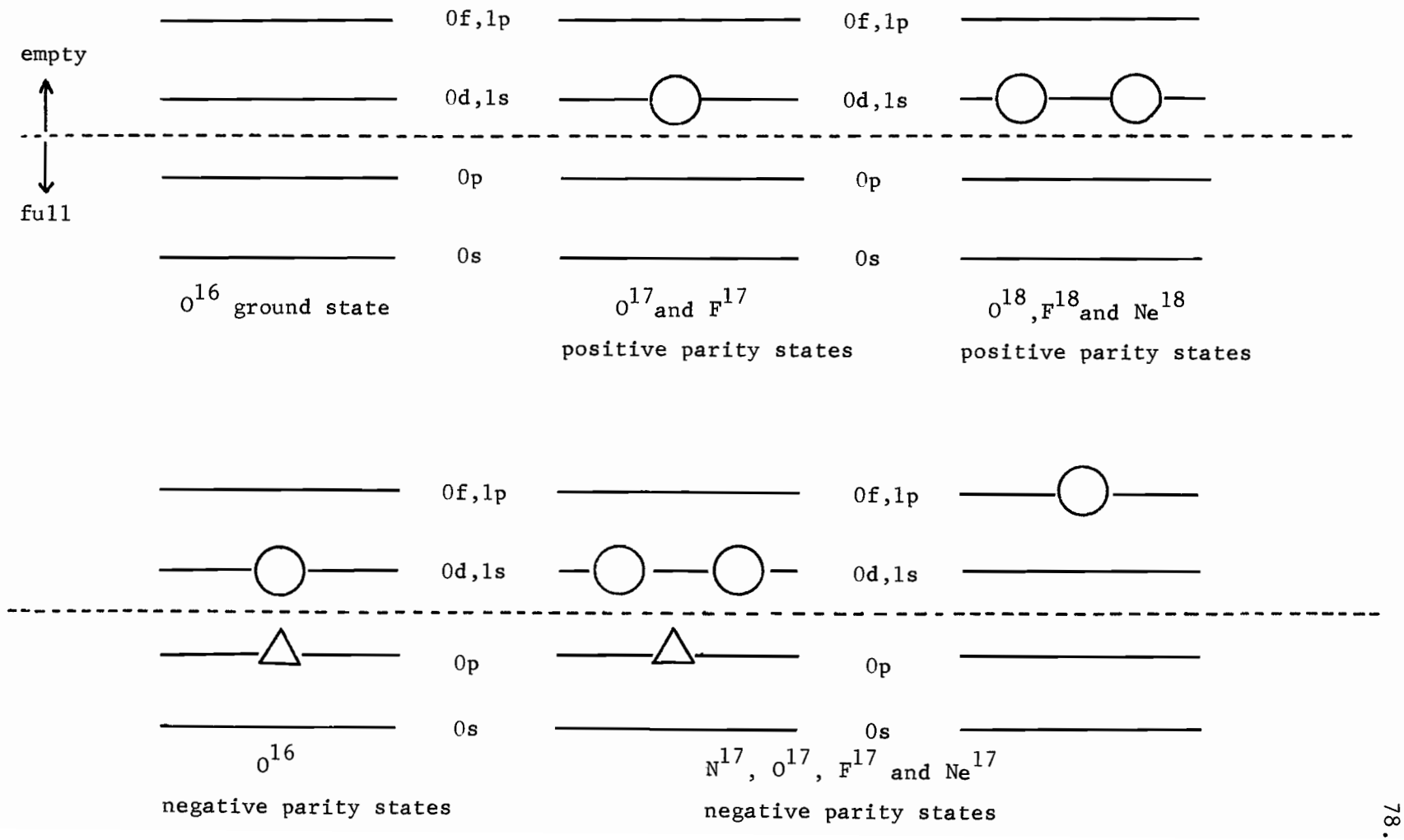
TABLE 13. Some calculated and experimental electromagnetic transition probabilities in ^{17}O and ^{17}F .

configurations	Lawson-Rosenfeld forces	Kallio-Gillet forces	Inoue-Gillet forces
$0p_{3/2}^{-1}\ (0d_{5/2}^{2};2^{+},1)$	0.249	0.159	0.180
$0p_{1/2}^{-1}\ (0d_{5/2}^{2};0^{+},1)$	0.899	0.900	0.811
$0p_{1/2}^{-1}\ (1s_{1/2}^{2};0^{+},1)$	0.353	0.353	0.480
$0p_{1/2}^{-1}\ (0d_{3/2}^{2};0^{+},1)$	0.000	0.182	0.253
	(99.5%)	(99.3%)	(98.4%)

TABLE 14 . The wavefunction of the ground state of ^{17}N or ^{17}Ne ($J=1/2^{-},T=1/2$) as calculated using various forces. The wavefunctions are expressed in the "basis" states and only the main components are shown.

Fig. 1 : Shell model configurations for the low lying states of nuclei near O^{16}

(△ is a hole in a filled shell; ◯ is a particle in an empty shell)



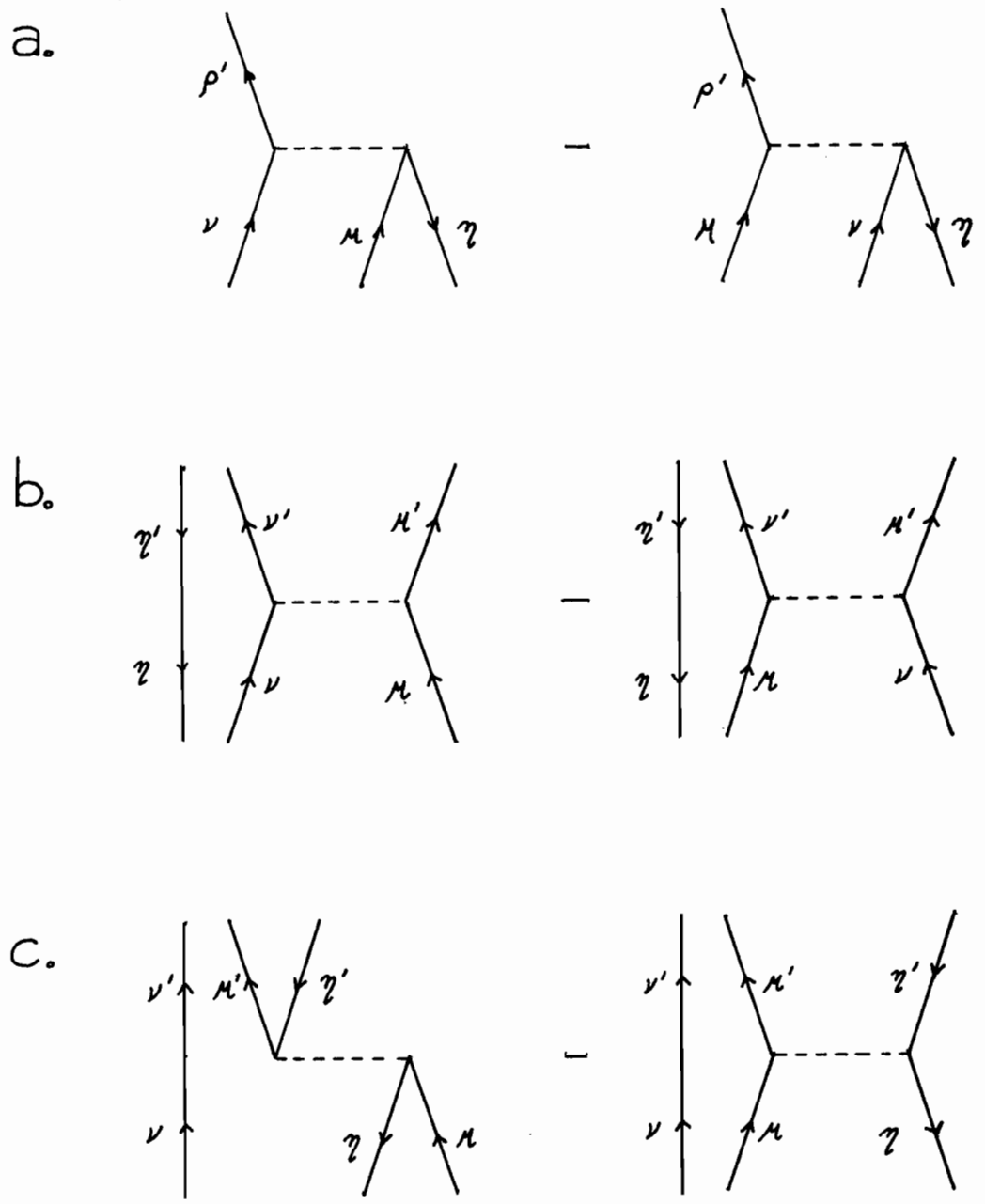


Fig. 2 : Examples of the three types of diagrams representing the matrix elements of the residual interaction : (a) the "polarization" diagrams, (b) the "particle-particle" diagrams, (c) the "particle-hole" diagrams.

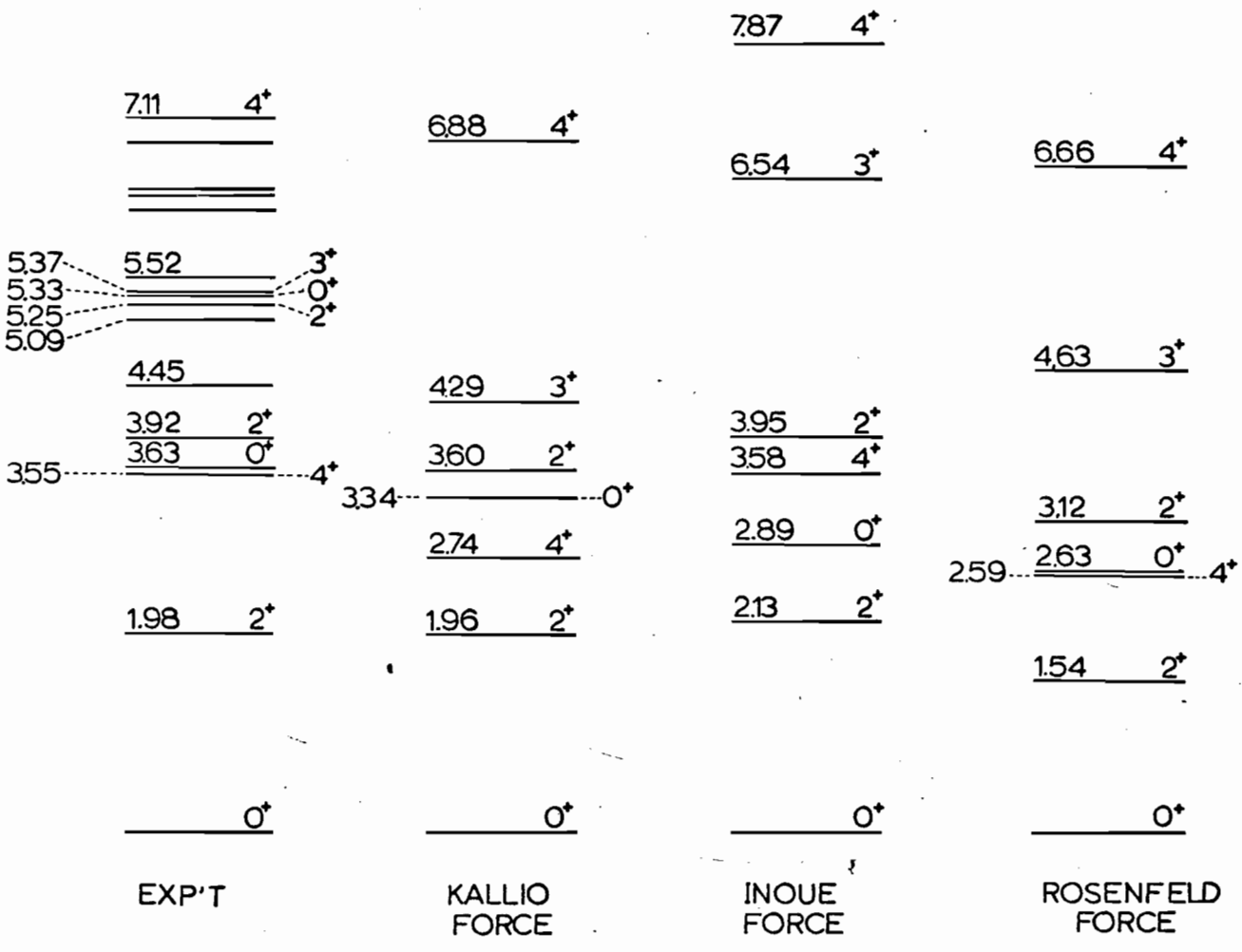


FIG. 3 : Energy levels in ^{18}O . The ground state has been set at 0 MeV.

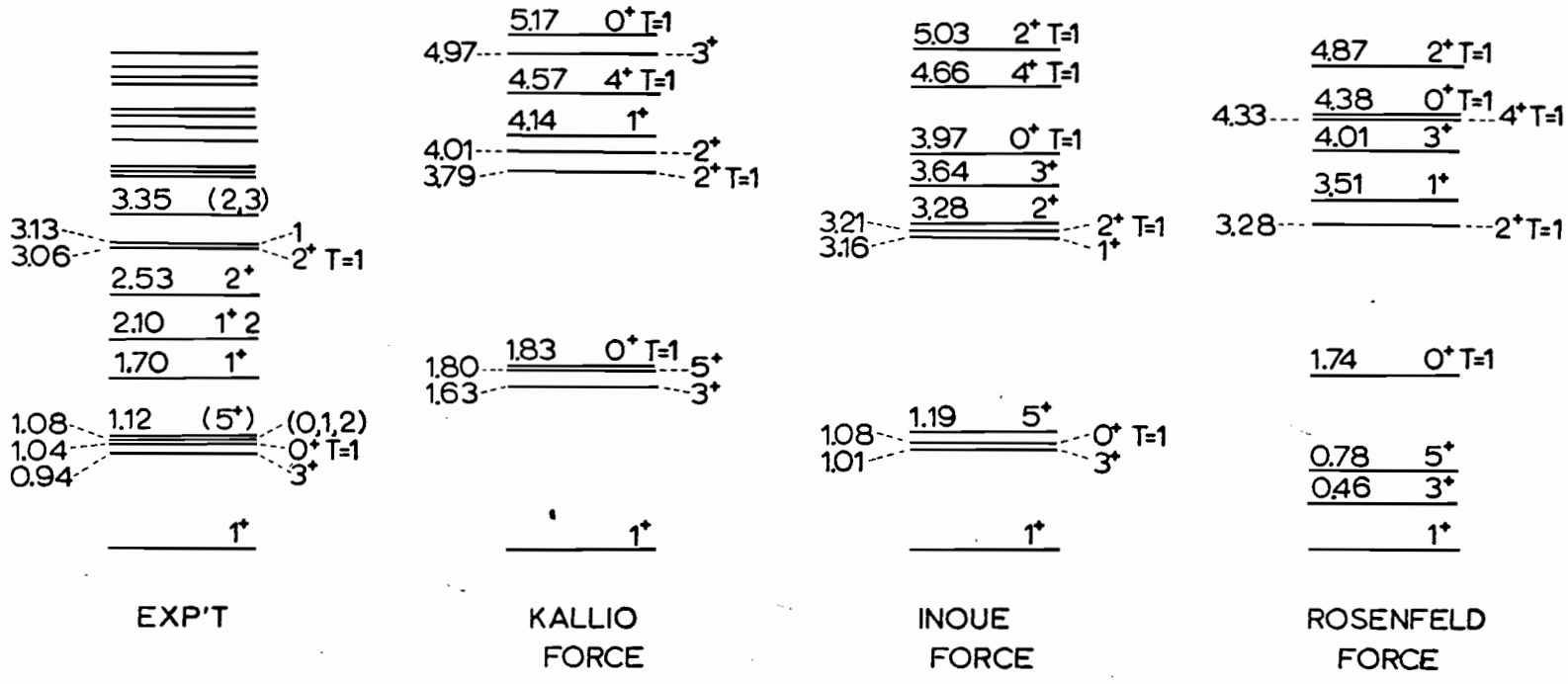


FIG. 4 : Energy levels in ^{18}F . The ground state has been set at 0 MeV.

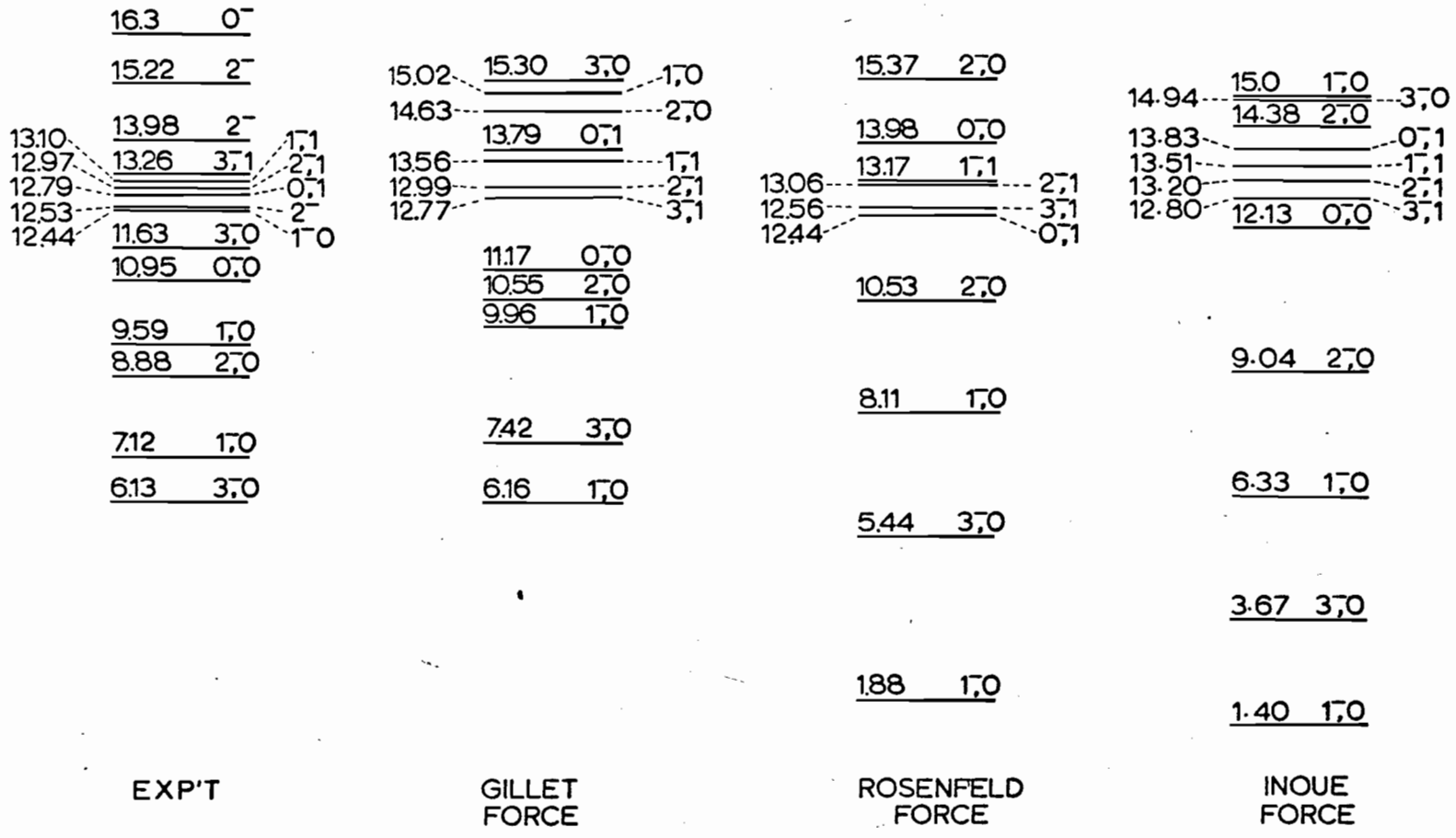


FIG. 5 : ENERGY LEVELS IN ^{16}O LABELED BY SPIN AND ISOSPIN.

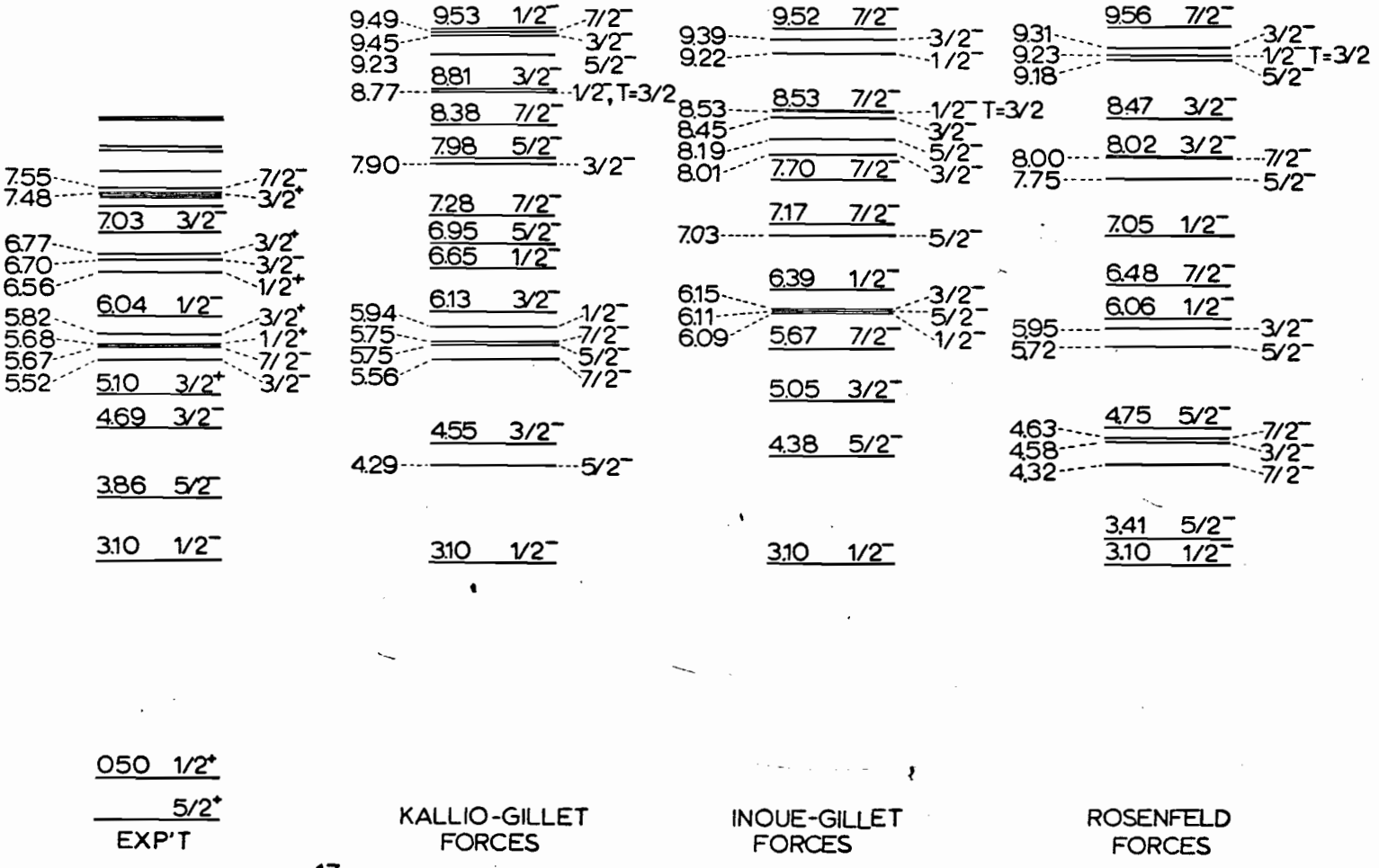


FIG. 6 : Energy levels in ^{17}F. The first $1/2^-$ level has been set at 3.10 MeV.

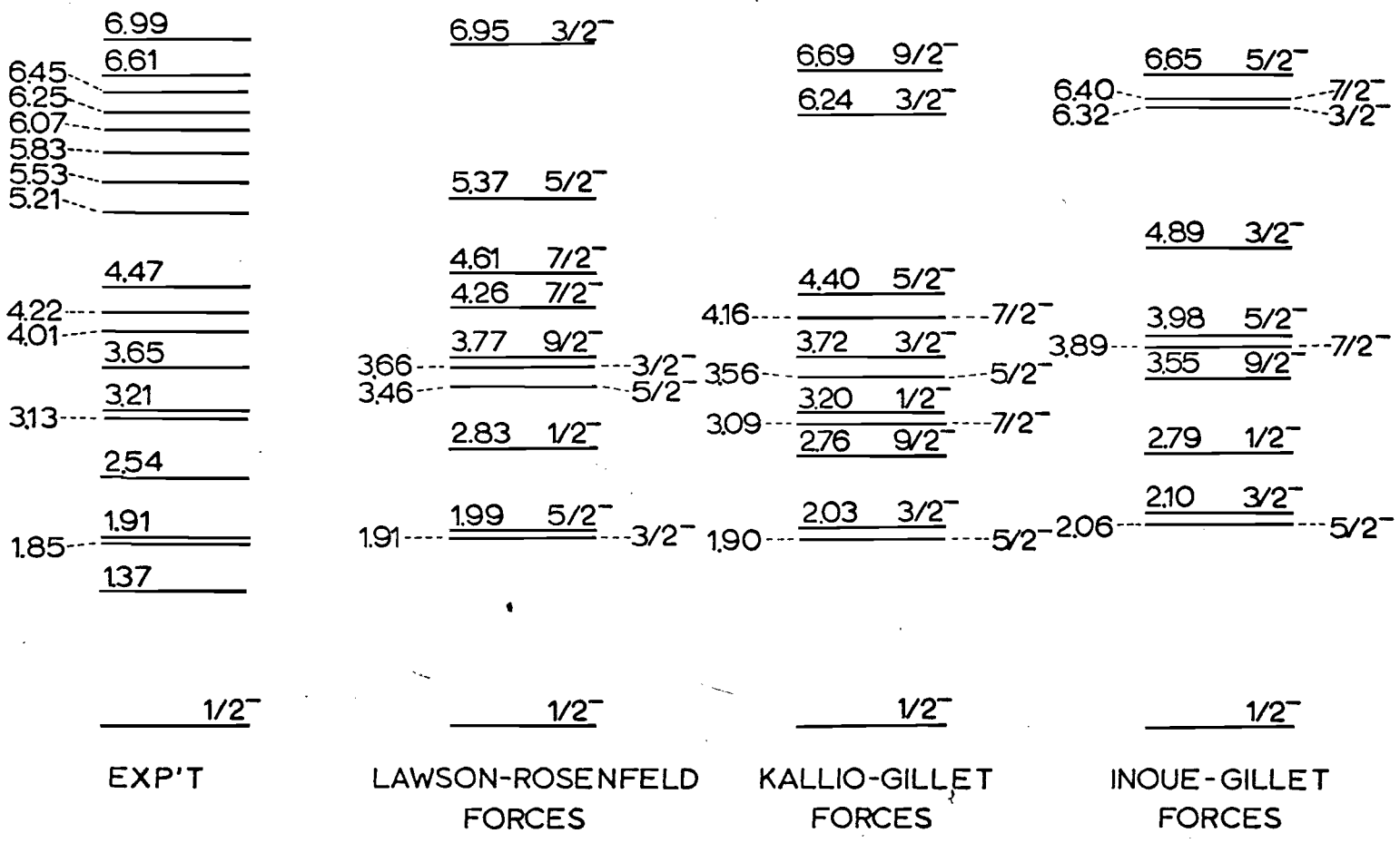
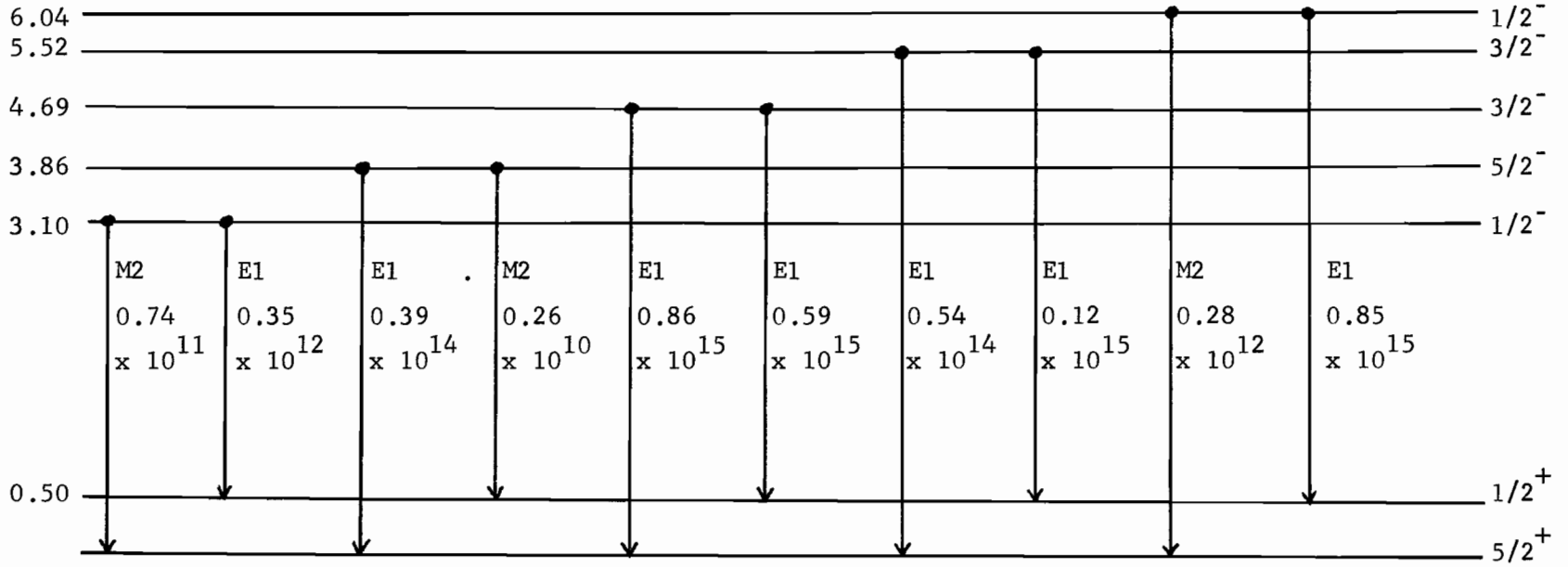


FIG. 7 : Energy levels in ^{17}N . The ground state has been set at 0 MeV.

Fig. 8 Electromagnetic transition probabilities in F^{17} calculated with the wavefunctions obtained using the Gillet particle-hole force and the Inoue particle-particle force.



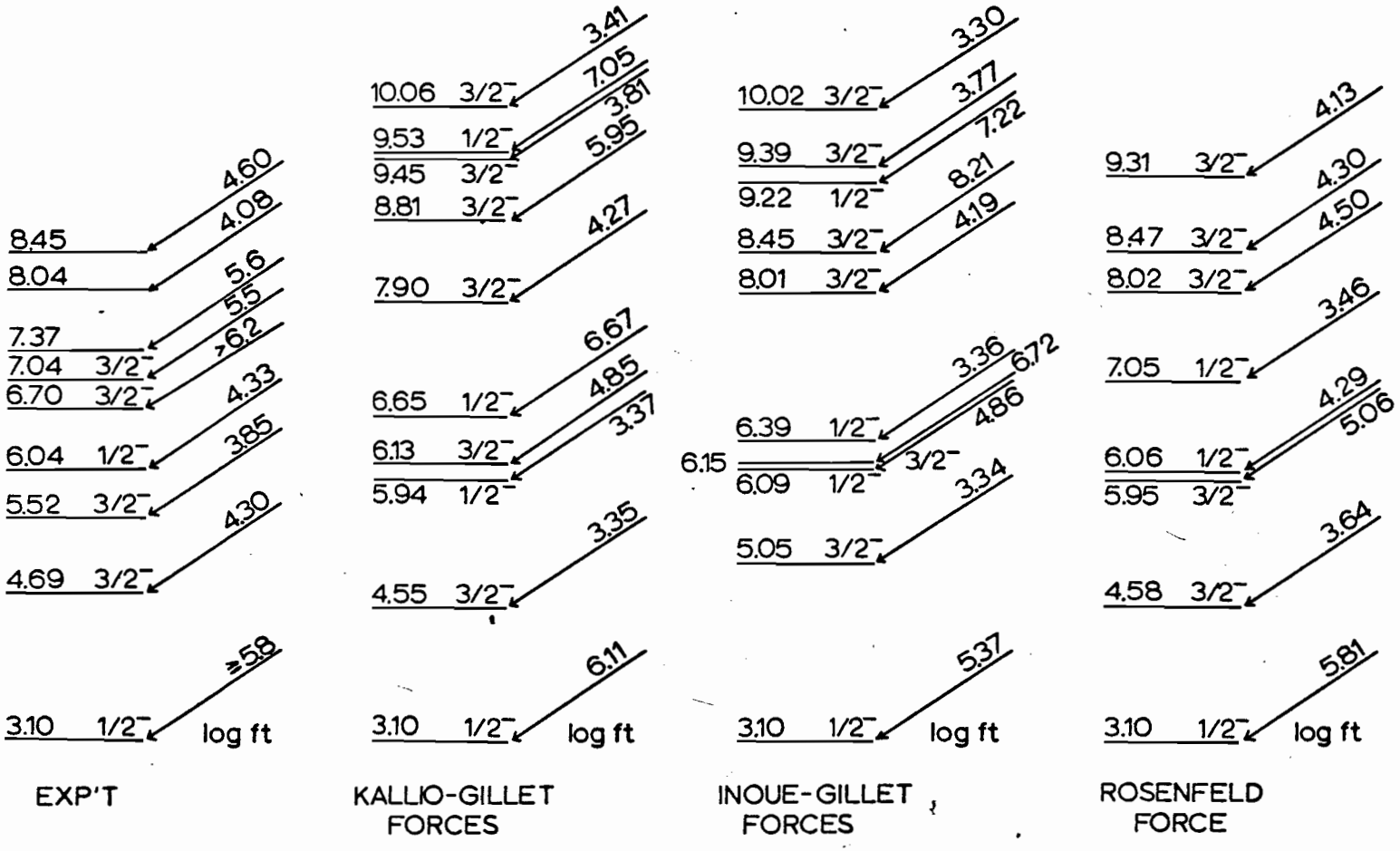


FIG. 9 : Log ft values for the beta decay of ^{17}Ne to ^{17}F.

Appendix A : The matrix elements of V^{res}.

This appendix contains a brief derivation of the various matrix elements that were required for the calculations discussed above. The fundamental matrix element is of course the two-particle matrix element, and this is evaluated in section A-1 below. The remaining sections are then devoted to reducing all the other types of matrix elements to sums over two particle matrix elements.

Throughout this appendix the residual interaction is assumed to be purely central. It can therefore be written as :

$$V^{res}(r) = V_o\ f(r) \left[W + BP_\sigma + HP\ P_x + MP_x \right]$$

where f(r) defines the radial dependence of the force, and V_o fixes its strength.The spin and space exchange operators are denoted by P_σ and P_x respectively. For the purposes of calculation, it is easier to rewrite the expression for V^{res} in terms of the singlet spin and triplet spin projection operators P^s_σ and P^t_σ :

$$V^{res}(r) = V_o f(r) \left\{ \left[(W-B) + (M-H)P_x \right] P^s_\sigma + \left[(W+B) + (M+H)P_x \right] P^t_\sigma \right\} \quad \ldots\ldots(A\text{-}1)$$

A - 1 . The matrix elements of V^{res} between two-particle states

$$\langle \nu' \mu' ; \tilde{J}\ \tilde{T} \,|\, V^{res} \,|\, \nu\mu ; \tilde{J}\ \tilde{T} \rangle = N_{\nu\mu}\ N_{\nu'\mu'} \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}$$

$$\langle 0 \,|\, \underbrace{a_{\mu'}\, a_{\nu'}}_{\leftarrow}\ a^{+}_{\alpha} a^{+}_{\beta} a_{\gamma} a_{\delta}\ \underbrace{a^{+}_{\nu}\, a^{+}_{\mu}}_{\rightarrow} \,|\, 0 \rangle \qquad \text{.....(A-2)}$$

$$\langle \tilde{J},\tilde{T} \qquad\qquad \tilde{J},\tilde{T}$$

The matrix element involving the creation and annihilation operators can be calculated in a straightforward manner by using the commutation rules given in equ. 1 . The symmetries of $V_{\alpha\beta\gamma\delta}$ allow the resulting expression to be written as :

$$\langle \nu' \mu' ; \tilde{J}\ \tilde{T} \,|\, V^{res} \,|\, \nu\mu ; \tilde{J}\ \tilde{T} \rangle =$$

$$= N_{\nu'\mu'} N_{\nu\mu} \langle \underbrace{\psi_{\nu'}(1) \,|\, \langle \psi_{\mu'}(2)}_{\rightarrow} \,|\, V^{res}(1,2) \,|\, \underbrace{\psi_{\mu}(2) \rangle \,|\, \psi_{\nu}(1)}_{\leftarrow} \rangle_{AS}$$

$$\langle \tilde{J}\ \tilde{T} \qquad\qquad\qquad\qquad \tilde{J}\ \tilde{T}$$

$$\text{.....(A-3)}$$

The single particle wavefunctions may be written:

$$|\psi_{n\ell jm;m_t}(1)\rangle = \sum_{m_\ell m_s} (\ell, m_\ell, 1/2, m_s \,|\, j, m)\ R_{n\ell}(r_1)\ Y_{\ell m_\ell}(\Omega_1)$$

$$\times\ \chi^{(s)}_{m_s}(1)\ \ \chi^{(T)}_{m_t}(1)$$

where $Y_{\ell m_\ell}$ is a spherical harmonic as defined by Edmonds [6], $R_{n\ell}$ is a harmonic oscillator radial wavefunction [4], and $\chi^{(s)}$ and $\chi^{(T)}$ are the spin and isospin wavefunctions respectively. In order to lighten the notation somewhat, we shall write :

$$|n\ell\rangle_1 = R_{n\ell}(r_1)\ Y_{\ell m_\ell}(\Omega_1)$$

$$|s\rangle_1 = \chi^{(s)}_{m_s}(1)$$

$$|t\rangle_1 = \chi^{(T)}_{m_t}(1)$$

We shall also suppress the z components of angular momentum and isospin.

The residual interaction defined in equ. A-1 can be written as a sum of terms $g_x g_\sigma$ where g_x acts only in coordinate space, and g_σ acts only in spin space, and both g_x and g_σ are scalars. We can therefore easily evaluate the matrix elements on the right hand side of equ.A-3 by going over to L-S coupling ; the direct term will then be :

$$\underbrace{\langle\psi_\nu(1)|\langle\psi_{\nu'}(2)|}_{\langle \tilde{J}\ \tilde{T}}\ g_x g_\sigma\ \underbrace{|\psi_\mu(2)\rangle|\psi_\nu(1)\rangle}_{\tilde{J}\ \tilde{T}} =$$

$$= \sum_{L\ S} A \begin{Bmatrix} \ell'_\nu & 1/2 & j'_\nu \\ \ell'_\mu & 1/2 & j'_\mu \\ L & S & \tilde{J} \end{Bmatrix} A \begin{Bmatrix} \ell_\nu & 1/2 & j_\nu \\ \ell_\mu & 1/2 & j_\mu \\ L & S & \tilde{J} \end{Bmatrix}$$

$$\times\ {}_1\underbrace{\langle n'_\nu \ell'_\nu|}_{\langle L}\,{}_2\underbrace{\langle n'_\mu \ell'_\mu|}_{\langle L}\, g_x\, |n_\mu \ell_\mu\rangle_2 \underbrace{|n_\nu \ell_\nu\rangle_1}_{L} \quad {}_1\underbrace{\langle s'_\nu|}_{\langle S}\,{}_2\underbrace{\langle s'_\mu|}_{\langle S}\, g_\sigma\, |s_\mu\rangle_2 \underbrace{|s_\nu\rangle_1}_{S}$$

The A coefficient is the one defined by Kennedy and Cliff [33].

Since we are using harmonic oscillator wavefunctions, the coordinate space matrix element can be evaluated by transforming to the relative and center of mass coordinates of the two particles. If g_x doesn't contain the space exchange operator, i.e. if $g_x = f(r_{12})$, then :

$$\underbrace{{}_1\langle n'_\nu \ell'_\nu |}_{\to L} \underbrace{\langle n'_\mu \ell'_\mu |}_{2} \; f(r_{12}) \; \underbrace{|n_\mu \ell_\mu\rangle_2}_{\leftarrow L} |n_\nu \ell_\nu\rangle_1 = \sum_{n\, n' \ell \mathcal{N} \mathcal{L}} \langle n_\nu \ell_\nu \, n_\mu \ell_\mu ; L | \, n \ell \mathcal{N} \mathcal{L} ; L \rangle$$

$$\times \langle n'_\nu \ell'_\nu \; n'_\mu \ell'_\mu ; L | \, n' \ell \mathcal{N} \mathcal{L} ; L \rangle \int_0^\infty R_{n'\ell}(r) \; f(r_{12}) \; R_{n\ell}(r) \; r^2 \, dr$$

where $r_{12} = |\vec{r}_1 - \vec{r}_2|$ and $r = \frac{1}{\sqrt{2}} r_{12}$. The transformation brackets are defined in the paper of Moshinsky [34]. The matrix element of $f(r_{12}) P_x$ is identical except for a factor of $(-)^\ell$ inside the summation sign.

Since we can take g_σ to be either the singlet or the triplet projection operators, the spin space matrix elements are trivial.

The exchange term differs from the direct term only by a phase factor $(-)^{\ell + S - \tilde{T}}$ inside all the summation signs. Using all the above results, we can write the total matrix element for the $\tilde{T} = 0$ and the $\tilde{T} = 1$ cases as follows.

If $\tilde{T} = 0$:

$$\langle \nu'\mu';\tilde{J}\ \tilde{T}=0 |\ V^{res}\ | \nu\mu;\tilde{J}\ \tilde{T}=0\rangle = 2\ A\begin{Bmatrix} \ell_\nu' & 1/2 & j_\nu' \\ \ell_\mu' & 1/2 & j_\mu' \\ \tilde{J} & 0 & \tilde{J}\end{Bmatrix} A\begin{Bmatrix} \ell_\nu & 1/2 & j_\nu \\ \ell_\mu & 1/2 & j_\mu \\ \tilde{J} & 0 & \tilde{J}\end{Bmatrix} \times$$

$$\times\Bigg[N_{\nu'\mu'}\,N_{\nu\mu}\sum_{\substack{n\ n'\kappa\lambda \\ \ell=\text{odd}}}\langle n_\nu'\ell_\nu'\ n_\mu'\ell_\mu';\tilde{J}\ |\ n'\ell\kappa\lambda;\tilde{J}\rangle\langle n_\nu\ell_\nu\ n_\mu\ell_\mu;\tilde{J}\ |\ n\ell\kappa\lambda;\tilde{J}\rangle \quad \times$$

$$\times\ (W-B+H-M)\int_o^\infty R_{n'\ell}(r)\ V(r_{12})\ R_{n\ell}(r)\ r^2\,dr\Bigg]$$

$$+\sum_L 2\ A\begin{Bmatrix} \ell_\nu' & 1/2 & j_\nu' \\ \ell_\mu' & 1/2 & j_\mu' \\ L & 1 & \tilde{J}\end{Bmatrix} A\begin{Bmatrix} \ell_\nu & 1/2 & j_\nu \\ \ell_\mu & 1/2 & j_\mu \\ L & 1 & \tilde{J}\end{Bmatrix} \times$$

$$\times\Bigg[N_{\nu'\mu'}\,N_{\nu\mu}\sum_{\substack{n\ n'\kappa\lambda \\ \ell=\text{even}}}\langle n_\nu'\ell_\nu' n_\mu'\ell_\mu';L\ |\ n'\ell\kappa\lambda;L\rangle\langle n_\nu\ell_\nu\ n_\mu\ell_\mu;L\ |\ n\ell\kappa\lambda;L\rangle$$

$$\times\ (W+B+M+H)\int_o^\infty R_{n'\ell}(r)\ V(r_{12})\ R_{n\ell}(r)\ r^2\,dr\Bigg]$$

.....(A-4)

where $V(r_{12})$ is equal to $V_o f(r_{12})$.

The $\tilde{T} = 1$ matrix element is the same as above with the replacements :

$(W - B + H - M)$ to be replaced by $(W - B + M - H)$

$(W + B + M + H)$ to be replaced by $(W + B - M - H)$.

A-2. Matrix elements of V^{res} between particle-hole states.

$$\langle \eta'^{-1} \nu' ; J\ T | V^{res} | \eta^{-1} \nu ; J\ T \rangle = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V^{res}_{\alpha\beta\gamma\delta}$$

$$\times \langle 0 | \underbrace{a_{\nu'} a^{+}_{\eta'}}_{<J\ T} a^{+}_{\alpha} a_{\beta} a_{\gamma} a_{\delta} \underbrace{a_{\eta} a^{+}_{\nu}}_{J\ T} | 0 \rangle$$

The use of the commutation rules again leads to a straightforward evaluation of the occupation number space matrix element. Using the symmetries of $V^{res}_{\alpha\beta\gamma\delta}$ we get :

$$\langle \eta'^{-1} \nu' ; J\ T | V^{res} | \eta^{-1} \nu ; J\ T \rangle =$$

$$= -\langle \psi_{\nu'}(1) | \overbrace{\langle \psi_{\eta}(2) | V^{res}(1,2) | \psi_{\eta'}(2) \rangle}^{<J\ T} | \psi_{\nu}(1) \rangle_{AS}$$

$$\underbrace{\qquad\qquad}_{J\ T}$$

.....(A-5)

Since V^{res} is a scalar, then as far as the angular momentum couplings are concerned the matrix element on the right hand side of equ. A-5 can be thought of as a product of four spherical tensors coupled together in a given order to zero total angular momentum. We can therefore change the order of coupling by the use of an A-coefficient [33] :

$$A \begin{Bmatrix} j_\eta & j_\nu & J \\ j'_\nu & j'_\eta & J \\ \tilde{J} & \tilde{J} & 0 \end{Bmatrix}$$

which, in this special case, is equal to a Racah coefficient to within a factor (6). The argument can of course be repeated for the isospin couplings. The final result becomes :

$$\overbrace{\langle \psi_{\nu'}(1) | \langle \psi_\eta(2) |}^{<\ J\ T} V^{res}(1,2) \underbrace{| \psi_{\eta'}(2) \rangle | \psi_\nu(1) \rangle}_{J\ T}{}_{AS} =$$

$$= \sum_{\tilde{J}\ \tilde{T}} (2\tilde{J}+1)(2\tilde{T}+1)\ W(j_\eta, j_\nu, j'_\eta, j'_\nu; J, \tilde{J}) \quad x$$

$$x \quad W(1/2,1/2,1/2,1/2; T, \tilde{T}) \quad x$$

$$x \underbrace{\langle \psi_{\nu'}(1) | \langle \psi_\eta(2) |}_{<\tilde{J}\ \tilde{T}} V^{res}(1,2) \underbrace{| \psi_{\eta'}(2) \rangle | \psi_\nu(1) \rangle}_{\tilde{J}\ \tilde{T}}{}_{AS}$$

.....(A-6)

The two particle matrix element can now be evaluated using equ. A-4 without any difficulty.

A-3. The matrix elements of V^{res} between two-particle, one-hole states.

$$\langle \eta'^{-1}(\nu'\mu';\tilde{J}'\tilde{T}')J\ T|\ V^{res}|\ \eta^{-1}(\nu\mu;\tilde{J}\ \tilde{T})J\ T\rangle \quad =$$

$$= N_{\nu'\mu'} N_{\nu\mu}\ \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V^{res}_{\alpha\beta\gamma\delta} \langle 0|\ a_{\mu'}\, a_{\nu'}\, a^{+}_{\eta'}\, a^{+}_{\alpha} a^{+}_{\beta} a_{\gamma} a_{\delta}\ a_{\eta}\ a^{+}_{\nu}\ a^{+}_{\mu}\ |0\rangle$$

($a_{\mu'}\, a_{\nu'}$: $\tilde{J}\tilde{T}'$; $a^{+}_{\eta'}$: $< JT$; a_{η}: JT; $a^{+}_{\nu}\ a^{+}_{\mu}$: $\tilde{J}\tilde{T}$)

Let us for the moment ignore all the angular momentum and isospin couplings. We then have :

$$\langle \eta'^{-1}\ \nu'\mu'|\ V^{res}|\ \eta^{-1}\nu\mu\rangle \quad = \quad \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V^{res}_{\alpha\beta\gamma\delta} \qquad x$$

$$x\ \langle 0|\ a_{\mu'}\ a_{\nu'}\, a^{+}_{\eta'}\, a^{+}_{\alpha}\, a^{+}_{\beta}\, a_{\gamma}\ a_{\delta}\ a_{\eta}\ a^{+}_{\nu}\ a^{+}_{\mu}\ |0\rangle$$

$$= \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V^{res}_{\alpha\beta\gamma\delta} \Big[\delta_{\eta\eta'} \langle 0|\ a_{\mu'}\, a_{\nu'}\, a^{+}_{\alpha} a^{+}_{\beta} a_{\gamma} a_{\delta}\ a^{+}_{\nu}\ a^{+}_{\mu}\ |0\rangle$$

$$- 4\, \delta_{\beta\eta}\, \delta_{\gamma\eta'} \langle 0|\ a_{\mu'}\, a_{\nu'}\, a^{+}_{\alpha} a_{\delta}\ a^{+}_{\nu}\ a^{+}_{\mu}\ |0\rangle \Big]$$

where we have used the symmetries of $V^{res}_{\alpha\beta\gamma\delta}$. Evaluating the remaining second quantized matrix elements, we obtain :

$$\langle \eta'^{-1}\ \nu'\mu' \mid V^{res} \mid \eta^{-1}\ \nu\mu \rangle \quad =$$

$$= \delta_{\eta\eta'} \langle \psi_{\nu'}(1) \mid \langle \psi_{\mu'}(2) \mid V^{res}(1,2) \mid \psi_{\mu}(2) \rangle \mid \psi_{\nu}(1) \rangle_{AS}$$

$$- \Big[\delta_{\mu\mu'} \langle \psi_{\nu'}(1) \mid \langle \psi_{\eta}(2) \mid V^{res}(1,2) \mid \psi_{\eta'}(2) \rangle \mid \psi_{\nu}(1) \rangle_{AS}$$

$$+ \delta_{\nu\nu'} \langle \psi_{\mu'}(1) \mid \langle \psi_{\eta}(2) \mid V^{res}(1,2) \mid \psi_{\eta'}(2) \rangle \mid \psi_{\mu}(1) \rangle_{AS}$$

$$- \delta_{\mu\nu'} \langle \psi_{\mu'}(1) \mid \langle \psi_{\eta}(2) \mid V^{res}(1,2) \mid \psi_{\eta'}(2) \rangle \mid \psi_{\nu}(1) \rangle_{AS}$$

$$- \delta_{\nu\mu'} \langle \psi_{\nu'}(1) \mid \langle \psi_{\eta}(2) \mid V^{res}(1,2) \mid \psi_{\eta'}(2) \rangle \mid \psi_{\mu}(1) \rangle_{AS} \Big]$$

.....(A-7)

It is not very difficult to put back the angular momentum and isospin couplings if we use the following relations :

$$\underset{JT}{\Big|}\ a_\eta \underbrace{a_\nu^+ a_\mu^+}_{\tilde{J}\tilde{T}} = \sum_{\tilde{\tilde{J}}\,\tilde{\tilde{T}}} \left[(2\tilde{J}+1)(2\tilde{\tilde{J}}+1)(2\tilde{T}+1)(2\tilde{\tilde{T}}+1) \right]^{1/2}$$

$$\times\ W(j_\eta, j_\nu, J, j_\mu; \tilde{\tilde{J}}, \tilde{J})\ W(1/2,1/2,T,1/2;\tilde{\tilde{T}},\tilde{T})\ \underbrace{a_\eta\ a_\nu^+}_{\tilde{\tilde{J}}\tilde{\tilde{T}}}\ a_\mu^+ \Big|$$

$$JT$$

or, when we want to isolate a_ν^+ rather than a_μ^+ :

$$\left[a_\eta \; \underbrace{a^+_\nu \; a^+_\mu}_{\tilde{J}\tilde{T}} \right]_{JT} = \sum_{\tilde{\tilde{J}}\,\tilde{\tilde{T}}} \left[(2\tilde{\tilde{J}}+1)(2\tilde{J}+1)(2\tilde{\tilde{T}}+1)(2\tilde{T}+1) \right]^{1/2} (-)^{j_\mu + j_\nu + 1 - \tilde{J} - \tilde{T}}$$

$$\times \; W(j_\eta, j_\mu, J, j_\nu; \tilde{\tilde{J}}, \tilde{J}) \; W(1/2, 1/2, T, 1/2; \tilde{\tilde{T}}, \tilde{T}) \left[\underbrace{a_\eta \; a^+_\nu}_{\tilde{\tilde{J}}\tilde{\tilde{T}}} \; a^+_\mu \right]_{JT}$$

When we put the couplings back into equ. A-7 , the first term gives rise to the "particle-particle" matrix element given in equ. 19 of the text, while the term within the square brackets gives rise to the "particle-hole" force whose matrix element is shown in equ. 20 of the text.

A-4. Matrix element of V^{res} connecting the two-particle, one-hole states to the single particle states.

$$\langle \rho' | V^{res} | \eta^{-1}(\nu\mu ; \tilde{J}\,\tilde{T}) J\,T \rangle = N_{\nu\mu}\ \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V^{res}_{\alpha\beta\gamma\delta}$$

$$\times \langle 0 | a_{\rho'} a^{+}_{\alpha} a^{+}_{\beta} a_{\gamma} a_{\delta} a_{\eta} a^{+}_{\nu} a^{+}_{\mu} | 0 \rangle$$

($a^{+}_{\nu} a^{+}_{\mu}$: $\tilde{J}\tilde{T}$; a_{η} : JT)

The evaluation of the second quantized matrix element is straightforward, and gives :

$$\langle \rho' | V^{res} | \eta^{-1}(\nu\mu ; \tilde{J}\,\tilde{T}) J\,T \rangle = N_{\nu\mu}\ \delta_{J,j_\rho}\ \delta_{M,m_\rho}\ \delta_{T,1/2}\ \delta_{M_t,m_{t\rho}}$$

$$\times \langle \psi_{\rho'}(1) | \langle \psi_{\eta}(2) | V^{res}(1,2) | \psi_{\mu}(2) \rangle | \psi_{\nu}(1) \rangle_{AS}$$

($\psi_{\mu}(2)$, $\psi_{\nu}(1)$: $\tilde{J}\,\tilde{T}$; $\psi_{\eta}(2)$, $\psi_{\rho'}(1)$: $J\,T$)

where M , m_ρ , M_t , and $m_{t\rho}$ are the z-components of J , j_ρ , T , and of the isospin of ρ respectively. Making use of the fact that the above matrix element is independant of these z-components, we can rewrite it in the form :

$$\langle \rho' |\ V^{res} |\ \eta^{-1}(\nu_{\mathcal{H}}\ ;\tilde{J}\ \tilde{T})J\ T\rangle \;=\; N_{\nu_{\mathcal{H}}} \left[\frac{(2\tilde{J}+1)(2\tilde{T}+1)}{2\ (2j_{\rho}+1)}\right]^{1/2}$$

$$\times\ \langle \underbrace{\psi_{\rho'}(1)|\langle\psi_{\eta}(2)|}_{\langle \tilde{J}\ \tilde{T}}\ V^{res}(1,2)\ \ |\underbrace{\psi_{\mathcal{H}}(2)\rangle|\psi_{\nu}(1)\rangle}_{\tilde{J}\ \tilde{T}}{}_{AS}$$

..... (A-8)

The two particle matrix element that is left on the right hand side of the above equation can be calculated as shown in section A-1 of this appendix.

APPENDIX B ; Matrix elements of some one body operators .

We now want to calculate those matrix elements of an arbitrary one-body operator that connect the two-particle,one-hole basis states to each other. Let the operator $\mathcal{O}^{\lambda,\lambda_t}$ be a spherical tensor operator of rank λ in coordinate space and rank λ_t in isospin space. Then with Schwinger's definition of the reduced matrix element (35), and using equation 6 of ref. 35, we can write almost immediately that :

$$\langle \eta'^{-1}(\nu'\mathcal{M}';\tilde{J}'\tilde{T}')J'T' \| \mathcal{O}^{\lambda\lambda_t} \| \eta^{-1}(\nu\mathcal{M};\tilde{J}\,\tilde{T})J\,T\rangle =$$

$$= \left[(2J+1)(2J'+1)(2T+1)(2T'+1)\right]^{1/2}$$

$$\times \Big[\delta_{\eta\eta'}(-)^{-J'-\tilde{J}+\lambda+j_\eta}\,(-)^{-T'-\tilde{T}+\lambda_t+1/2}\,W(\tilde{T},T,\tilde{T}',T';1/2,\lambda_t) \times$$

$$\times W(\tilde{J},J,\tilde{J}',J';j_\eta,\lambda)\,\langle \nu'\mathcal{M}';\tilde{J}'\tilde{T}' \| \mathcal{O}^{\lambda,\lambda_t} \| \nu\mathcal{M};\tilde{J}\,\tilde{T}\rangle$$

$$- \langle \nu'\mathcal{M}';\tilde{J}'\tilde{T}' \,|\, \nu\mathcal{M};\tilde{J}\,\tilde{T}\rangle\,(-)^{-J+\tilde{J}+j_\eta}\,(-)^{-T+\tilde{T}+1/2} \times$$

$$\times W(1/2,T,1/2,T';\tilde{T},\lambda_t)\;W(j_\eta,J,j'_\eta,J';\tilde{J},\lambda) \times$$

$$\times \langle j_\eta \| \mathcal{O}^{\lambda,\lambda_t} \| j'_\eta\rangle \Big]$$

.....(B-1)

where we have used the fact that the hole-hole matrix element is related to the particle-particle matrix element by :

$$\langle \eta'^{-1} \| \mathcal{O}^{\lambda\lambda_t} \| \eta^{-1} \rangle = (-)^{j_\eta + j'_\eta + \lambda} (-)^{1+\lambda_t} \langle \eta \| \mathcal{O}^{\lambda\lambda_t} \| \eta' \rangle$$

If we substitute $\mathcal{O}^{\lambda\lambda_t} = \vec{\sigma}\vec{\tau}$ then equ. 21 is obtained immediately.

The two-particle matrix element can also be rewritten in terms of single particle matrix elements by using equ. 6 of ref. 35 again. This yields :

$$\begin{aligned}
\langle \nu'\mu'; \tilde{J}'\tilde{T}' \| \mathcal{O}^{\lambda\lambda_t} \| \nu\mu; \tilde{J}\,\tilde{T} \rangle &= \left[(2\tilde{J}+1)(2\tilde{J}'+1)(2\tilde{T}+1)(2\tilde{T}'+1)\right]^{1/2} \times \\
&\times N_{\nu'\mu'} N_{\nu\mu} (-)^{\lambda - \tilde{J} + j_\mu - j'_\nu} (-)^{\lambda_t + \tilde{T}} W(1/2,\tilde{T},1/2,\tilde{T}';\, 1/2,\lambda_t) \times \\
&\times \Big[\delta_{\mu\mu'} W(j_\nu,\tilde{J},j'_\nu,\tilde{J}';j_\mu,\lambda) \langle \nu' \| \mathcal{O}^{\lambda\lambda_t} \| \nu \rangle \\
&+ \delta_{\nu\nu'} (-)^{\tilde{J}+\tilde{J}'+\tilde{T}+\tilde{T}'} W(j_\mu,\tilde{J},j'_\mu,\tilde{J}';j_\nu,\lambda) \langle \mu' \| \mathcal{O}^{\lambda\lambda_t} \| \mu \rangle \\
&- \delta_{\mu\nu'} (-)^{\tilde{J}'+\tilde{T}'} W(j_\nu,\tilde{J},j'_\mu,\tilde{J}';j_\mu,\lambda) \langle \mu' \| \mathcal{O}^{\lambda\lambda_t} \| \nu \rangle \\
&- \delta_{\nu\mu'} (-)^{\tilde{J}+\tilde{T}} W(j_\mu,\tilde{J},j'_\nu,\tilde{J}';j_\nu,\lambda) \langle \nu' \| \mathcal{O}^{\lambda\lambda_t} \| \mu \rangle \Big]
\end{aligned}$$

.....(B-2).

The single particle matrix elements are of course determined by the choice of the one body operator $\mathcal{O}^{\lambda\lambda_t}$. If we take this to be $\vec{\sigma}\vec{\tau}$,

and if we assume unit overlap for the radial wavefunctions of given ℓ but different j, then we can write :

$$\langle \psi_{n'\ell'j'}(1) \| \vec{\sigma}\vec{\xi} \| \psi_{n\ell j}(1) \rangle = 6\,(-)^{\ell'+1/2-j'} \left[(2j+1)(2j'+1)\right]^{1/2} \times W(1/2,j,1/2,j';\ell,1) \qquad \text{.....(B-3)}$$

these are all the formulae necessary for the calculation of the log ft values for the β decay of the ground states of ^{17}Ne and ^{17}N.

The matrix elements of a one body operator $\mathcal{O}^{JJ_t}$ that connect the two particle-one hole states to the single particle states will now be calculated. In this case, it is better not to skip over the explicit calculation of the second quantized matrix elements .

$$\mathcal{O}^{JJ_t} = \sum_{\alpha\beta} \langle \psi_\alpha(1) | \mathcal{O}^{JJ_t}(1) | \psi_\beta(1) \rangle \; a^+_\alpha a_\beta$$

$$= \sum_{\alpha\,\beta} \mathcal{O}^{JJ_t}_{\alpha\beta} \; a^+_\alpha a_\beta$$

Omitting the angular momentum and isospin coupling coefficients, we have :

$$\langle \rho | \mathcal{O}^{JJ_t} | \eta^{-1}\nu\mu \rangle = \sum_{\alpha\beta} \mathcal{O}^{JJ_t}_{\alpha\beta} \langle 0 | a_\rho \, a^+_\alpha a_\beta \, a_\eta \, a^+_\nu a^+_\mu | 0 \rangle$$

$$= \delta_{\rho\nu} \mathcal{O}^{JJ_t}_{\eta\mu} - \delta_{\rho\mu} \mathcal{O}^{JJ_t}_{\eta\nu}$$

Hence :

$$\langle \rho \,|\, \mathcal{O}^{\mathcal{J}\mathcal{T}_t} \,|\, \eta^{-1}(\nu\mu\,;\tilde{J}\,\tilde{T})J\;T\rangle = \sum_{\substack{m_\nu m_\mu \\ m_\eta \tilde{M}}} \sum_{\substack{m_{t\nu} m_{t\mu} \\ m_{t\eta} \tilde{M}_t}} (-)^{j_\eta - m_\eta + 1/2 - m_{t\eta}}$$

$$\times\ (1/2, -m_{t\eta}, \tilde{T}, \tilde{M}_t \,|\, T, M_t)\quad (1/2, m_{t\nu}, 1/2, m_{t\mu} \,|\, \tilde{T}, \tilde{M}_t)$$

$$\times\ (j_\eta, -m_\eta, \tilde{J}, \tilde{M} \,|\, J, M)\quad (j_\nu, m_\nu, j_\mu, m_\mu \,|\, \tilde{J}, \tilde{M})$$

$$\times\ N_{\nu\mu} \left[\delta_{\rho\nu}\, \mathcal{O}^{\mathcal{J}\mathcal{T}_t}_{\eta\mu} - \delta_{\rho\mu}\, \mathcal{O}^{\mathcal{J}\mathcal{T}_t}_{\eta\nu} \right]$$

After doing some angular momentum algebra, we can write :

$$\langle \rho \,|\, \mathcal{O}^{\mathcal{J}\mathcal{T}_t} \,|\, \eta^{-1}(\nu\mu\,;\tilde{J}\,\tilde{T})J\;T\rangle = \left[\frac{(2J+1)(2\tilde{J}+1)(2T+1)(2\tilde{T}+1)}{2(2j_\rho+1)} \right]^{1/2}$$

$$\times\ (\mathcal{J}_t, \mathcal{M}_t, T, M_t \,|\, 1/2, m_{t\rho})\quad (\mathcal{J}, \mathcal{M}, J, M \,|\, j_\rho, m_\rho)$$

$$\times\ W(1/2, 1/2, T, 1/2; \tilde{T}, \mathcal{T}_t)\quad W(j_\rho, j_\mu, J, j_\eta; \tilde{J}, \mathcal{J})$$

$$\times\ N_{\nu\mu}\, \delta_{\rho\nu}\quad (-)^{J-\tilde{J}+j_\eta+T-\tilde{T}+1/2} \langle \psi_\eta(1) \| \mathcal{O}^{\mathcal{J}\mathcal{T}_t}(1) \| \psi_\mu(1) \rangle$$

$$-\ (-)^{j_\nu + j_\mu - \tilde{J} + 1 - \tilde{T}}\quad \text{(same as above with } \nu \rightleftharpoons \mu \text{)}$$

.....(B-4).

This equation will be used in Appendix C to calculate some electromagnetic transition probabilities.

APPENDIX C . Some electromagnetic transitions .

The theory of low energy electromagnetic transitions is given in all textbooks on nuclear theory (see for example M.A. Preston , ref.4) . In this appendix we shall use the formulae given by Kennedy and Sharp [35] , but we shall rewrite them using the isospin formalism. The charge and magnetic moment operators can then be written :

$$\text{charge} = e\left[\left(\frac{\mathcal{E}_p + \mathcal{E}_n}{2} \right) + \tau_z \left(\frac{\mathcal{E}_p - \mathcal{E}_n}{2} \right) \right]$$

$$= e\left[\mathcal{E}^0 + \mathcal{E}^1 \tau_z \right]$$

$$\text{magnetic moment} = \frac{e\hbar}{mc}\left[\left(\frac{\mathcal{M}_p + \mathcal{M}_n}{2} \right) + \quad _z \left(\frac{\mathcal{M}_p - \mathcal{M}_n}{2} \right) \right]$$

$$= \frac{e\hbar}{mc}\left[\mathcal{M}^0 + \mathcal{M}^1 \tau_z \right]$$

where e is the magnitude of the charge of an electron, τ_z is the z-component of the isospin operator, $\mathcal{E}_p$ and $\mathcal{E}_n$ are the effective charges of the proton and the neutron respectively, and $\mathcal{M}_p$ and $\mathcal{M}_n$ are their magnetic moments.

The electric and magnetic multipole operators of order L can then be written :

$$e\, Q_{L,M_L}(1) = e(\varepsilon^0 + \varepsilon^1 \tau_z)\; r^L\, Y^*_{L,M_L}$$

$$e\, Q'_{L,M_L}(1) = \frac{i\,k}{(L+1)} \left(\frac{e\hbar}{2mc}\right) (\mu^0 + \mu^1 \tau_z) \left\{ (\vec{\nabla} r^L\, Y^*_{L,M_L}) \cdot (\vec{r} \times \vec{\sigma}) \right\}$$

$$e\, M_{L,M_L}(1) = \frac{1}{(L+1)} \left(\frac{e\hbar}{mc}\right) (\varepsilon^0 + \varepsilon^1 \tau_z) \left\{ (\vec{\nabla} r^L\, Y^*_{L,M_L}) \cdot \vec{L} \right\}$$

$$e\, M'_{L,M_L}(1) = \left(\frac{e\hbar}{2mc}\right) (\mu^0 + \mu^1 \tau_z) \left\{ (\vec{\nabla} r^L\, Y^*_{L,M_L}) \cdot \vec{\sigma} \right\}$$

where the label "1" stands for all the single particle coordinates and is explicitly shown to distinguish the above operators from the corresponding second quantized operators which are :

$$\mathbf{Q}_{L,M_L} = \sum_{\alpha\beta} \langle \psi_\alpha(1) \,|\, Q_{L,M_L}(1) \,|\, \psi_\beta(1) \rangle \; a^+_\alpha\, a_\beta$$

and similar expressions for the other three operators. These operators transform under rotations as the hermitian adjoints of spherical tensors[6], and they contain both an isoscalar and an isovector part.

We shall from now on restrict the discussion to electric multipole transitions , since all the formulae for magnetic multipole transitions are identical except for the replacement of all the Q and Q' by M and M' .

The transition probabilities Λ_{EL} for an electric transition of multipole order L , going from an initial state $|J_i,M_i,M_{ti}\rangle$ to a final state $|J_f,M_f,M_{tf}\rangle$ is given by :

$$\Lambda_{EL} = \frac{8\pi\ (L+1)}{L\ [(2L+1)!!]^2}\ \frac{e^2\ c\ k^{2L+1}}{\hbar c}\ \frac{1}{(2J_i+1)}\ x$$

$$x \sum_{M_i,M_f,M_L} \left| \langle J_f,M_f,M_{tf} |\ Q_{L,M_L} + Q'_{L,M_L}\ |\ J_i,M_i,M_{ti}\rangle \right|^2$$

Now we can write :

$$\langle J_f,M_f,M_{tf}|\ Q_{L,M_L} + Q'_{L,M_L}\ |\ J_i,M_i,M_{ti}\rangle =$$

$$= \frac{(-)^{M_L}\ (\ L,-M_L,J_i,M_i\ |\ J_f,M_f\ )}{\sqrt{2\ (2J_f+1)}} \langle J_f,M_{tf} \| Q_L + Q'_L \| J_i,M_{ti}\rangle$$

which defines the quantity $\langle \| \| \rangle$; it is essentially a reduced matrix element in spin and coordinate space only . Using the completeness of the Clebsch-Gordan coefficients, we can perform the sums over the magnetic quantum numbers :

$$\Lambda_{EL} = \frac{8\pi\ (L+1)}{L\ [(2L+1)!!]^2}\ c\ k^{2L+1}\ \left(\frac{e^2}{\hbar c}\right)\ \frac{1}{2(2J_i+1)}\ x$$

$$x\ \left| \langle J_f,M_{tf} \| Q_L + Q'_L \| J_i,M_{ti}\rangle \right|^2$$

Now , we are interested in electromagnetic transitions in ^{17}F and ^{17}O going from some of the low lying negative parity states to the ground state and first excited state. The latter are assumed to be pure single particle states whereas the former involve both single particle and two particle-one hole configurations. We therefore need the matrix elements that connect single particle configurations either to single particle configurations (eq. C-1) or to two particle-one hole configurations (eq. C-2).

$$\langle \rho' \| Q_L + Q'_L \| \rho \rangle = \sqrt{2}\, \langle \psi_{\rho'}(1) \| \varepsilon^0 \tilde{Q}_L(1) + \mu^0 \tilde{Q}'_L(1) \| \psi_\rho(1) \rangle$$

$$- \sqrt{6}\ (1,0,1/2,m_{t\rho} \mid 1/2,m_{t\rho'})\ \langle \psi_{\rho'}(1) \| \varepsilon^1 \tilde{Q}_L(1) + \mu^1 \tilde{Q}'_L(1) \| \psi_\rho(1) \rangle$$

.....(C-1)

The operators $\tilde{Q}_L$ and $\tilde{Q}'_L$ are the same as Q_L and Q'_L except that the factors $\varepsilon^0 + \varepsilon^1 \tau_z$ and $\mu^0 + \mu^1 \tau_z$ are omitted. The double bar matrix elements that are left in eq. C-1 and C-2 are reduced matrix elements in configuration space only; they are given explicitly by Kennedy and Sharp [35].

$$\langle \rho' \| Q_L + Q'_L \| \eta^{-1}(\nu\mu;\tilde{J}\,\tilde{T})J\,T\rangle \quad =$$

$$= \delta_{\nu\rho'} \Big\{ N_{\nu\mu} \; (-)^{T-\tilde{T}+1/2+J-\tilde{J}+j_\eta} \sqrt{(2J+1)(2\tilde{J}+1)(2T+1)(2\tilde{T}+1)} \quad W(j_{\rho'}, j_\mu, J, j_\eta; \tilde{J}, L)$$

$$\Big[\delta_{m_{t\rho'}, M_t} \; W(1/2,1/2,T,1/2;\tilde{T},0) \; \sqrt{2} \; \langle \psi_\eta(1) \| \varepsilon^0 Q_L(1) + \mu^0 Q'_L(1) \| \psi_\mu(1)\rangle$$

$$- \sqrt{6} \; (1,0,T,M_t \,|\, 1/2, m_{t\rho}) \; W(1/2,1/2,T,1/2;\tilde{T},1) \; \langle \psi_\eta(1) \| \varepsilon^1 Q_L(1) + \mu^1 Q'_L(1) \| \psi_\mu(1)\rangle \Big] \Big\}$$

$$- \delta_{\mu\rho'} (-)^{1-\tilde{T}+j_\nu+j_\mu-\tilde{J}} \Big\{ \text{same as above with } \mu \rightleftharpoons \nu \Big\}$$

.....(C-2).

APPENDIX D ; Coulomb energies near ^{16}O.

The rest mass or total energy of a nucleus in a given state can be divided into three contributions : first, the energy due to the strong interactions of the constituent nucleons (E_n), secondly, the energy due to the Coulomb repulsion between the protons (E_c) , and finally the rest energy of the nucleons. Since it is conventional to tabulate the atomic masses rather than the nuclear masses, we must also include in the total energy the rest masses of the orbital electrons. Since we are only interested in energy differences between neighbouring light nuclei, we can neglect the binding energy of the electrons. We can then write for the total energy E_t :

$$E_t = E_n + E_c + (A-Z)\, m_n c^2 + Z\, m_p c^2 + Z\, m_e c^2$$

where as usual, A is the number of nucleons , Z is the number of protons, and m_n, m_p , and m_e are the masses of the neutron , proton, and electron respectively.

Now our shell model calculations yield the nuclear energies E_n measured with respect to the ground state of ^{16}O , whereas the experimental information relates the mass differences. It turns out that an explicit estimate of the Coulomb energies E_c is not necessary

if we only want the relative nuclear energies of the T=0 and T=1 states in the A=16 and A=18 nuclei since the experimental positions of both the lowest T=0 and T=1 states are known in ^{16}O and also in ^{18}F . However,the T=3/2 states in ^{17}F or ^{17}O have not yet been experimentally observed; we must therefore use the known $^{17}N \leftrightarrow ^{17}O$ mass difference and an estimate of the Coulomb energies to determine the nuclear energy difference between the T=1/2 and the T=3/2 states in the A=17 system.

If we make the crude assumption that each proton is uniformly distributed inside a sphere of radius proportional to $A^{1/3}$ then (see M.A. Preston , ref.4) :

$$E_c = \frac{1}{2} C \left(\frac{16}{A}\right)^{1/3} Z(Z-1)$$

where Z is a constant that we shall determine from experiment, and the radius of ^{16}O is used as a convenient reference radius. The energy difference between two nuclei of the same A but different Z can be written :

$$E_t(A,Z+1) - E_t(A,Z) = E_n + C \left(\frac{16}{A}\right)^{1/3} Z - 0.783$$

where all the energies are in MeV.

We can now use the known energy differences [17,22] between nuclei near ^{16}O that are members of isospin multiplets to estimate C .

C = 0.454 ... from the $^{18}O \longleftrightarrow {}^{18}F$ mass difference and the known position of the T=1 states in ^{18}F.

= 0.462 ... from the $^{18}F \longleftrightarrow {}^{18}Ne$ mass difference.

= 0.453 ... from the $^{17}O \longleftrightarrow {}^{17}F$ mass difference.

= 0.456 ... from the $^{16}O \longleftrightarrow {}^{16}N$ mass difference and the known position of the T=1 states in ^{16}O.

= 0.495 ... from the $^{15}N \longleftrightarrow {}^{15}O$ mass difference.

Hence we have for the mean value of C,

$C_{mean} = 0.464$

We can therefore write for the A=17 system,

E_c (A=17, Z+1) - E_c (A=17, Z) = 0.455

and thus, using the ^{17}N - ^{17}O mass difference:

$E_n(^{17}N$ g.s.) - $E_n(^{17}O$ g.s.) = $E_n(^{17}O$ first T=3/2 state) - $E_n(^{17}O$ g.s.)

= 11.1 MeV.

The mass difference between ^{17}Ne and ^{17}F can also be estimated from the above to be:

$E_t(^{17}Ne$ g.s.) - $E_t(^{17}F$ g.s.) = 14.4 MeV.

which is the same as Janecke's estimate [36].