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A WATER QUALITY EVALUATION OF REGIONAL WASTEWATER MANAGEMENT

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ABSTRACT

A water quality evaluation of regional wastewater system centralization was undertaken to test the hypothesis that water quality improvement may result from the spatial and temporal variations of wasteloads attributed to decentralized regional systems. The evaluation employed water quality models developed for both deterministic and stochastic analyses. Each analysis considered a set of experiments which involved a determination of the water quality resulting from alternative degrees of regional wastewater centralization. Water quality was measured in terms of the minimum dissolved oxygen concentrations experienced by alternative systems. It was concluded that decentralized regional wastewater systems may result in significantly higher stream water qualities than that achieved by highly centralized systems.

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A WATER QUALITY EVALUATION OF REGIONAL WASTEWATER MANAGEMENT

The management of urban wastewaters should not only be exercised on a metropolitan or regional basis but it should also consider all components of the wastewater system: wastewater generation, collection, treatment, and disposal. The consideration of all wastewater system components in a regional framework offers more alternatives in wastewater management and affords a systematic evaluation of alternatives. However, the appreciation of the regional nature of wastewater management and the identification and evaluation of regional wastewater management alternatives is only evident in the literature of the last decade. In reviewing this literature, it is seen that the regional wastewater management alternatives identified include streamflow regulation by on-stream storage, degrees and types of wastewater treatment, effluent storage, instream aeration, wastewater reuse, and the size, number, and location of treatment plants.

SIZE, NUMBER, AND LOCATION OF REGIONAL PLANTS

The regional wastewater alternatives given by the size, number and location of wastewater treatment plants are addressed by this paper. The authors view this area as being particularly important because of the current management planning practice of highly centralized regional treatment systems. There exists some doubt to the wisdom of this practice on the basis of both the economic and the water quality implications of system centralization (Adams, et al, 1972). This paper confines itself to the water quality implications but the reader is referred to a study by Dajani and Gemmell (1973) for a consideration of the economic implications of wastewater system centralization.

There are two reasons to suggest the consideration of decentralized regional wastewater systems from the point of view of water quality. The first is that the facilities of a decentralized regional system will be spatiably distributed along the receiving waters, resulting in a more uniformly distributed contributory wasteload in the receiving waters, in comparison to the concentrated wasteloadings of the highly centralized system. Secondly, it is known that wastewater treatment plant performance is variable (Thomann, 1970; Adams and Gemmell, 1973). Thus, during times of poor effluent quality, the receiving waters will be subject to greater wasteloads than in times of good effluent quality. This is particularly evident in the case of large centralized facilities where the variations in effluent quality are amplified by the large flows of the effluent stream. Additionally, the facilities of a decentralized system do not produce the same effluent qualities simultaneously; that is, some plants may be producing high quality effluents while others are producing lower quality effluents. Since the fluctuating performances of various facilities are not in phase, or that the facilities operate independently, the receiving waters would not experience the sharp wasteloading peaks that might result from highly centralized systems (Adams and Gemmell, 1973).

These observations on the water quality implications of regional wastewater system centralization demand that an examination be made of the alternatives provided by the size, number and location of treatment facilities. In fact, these alternatives have received only a limited amount of attention. Most of the relatively few studies conducted on this aspect of regional wastewater treatment have been concerned with only the economics of system components of alternative plans involving various degrees of wastewater centralization. A series of waste sources is identified and the problem is to determine the economically optimal state of aggregation of these sources. The cost functions considered are those of interceptor sewers and treatment plants but not of networks. The problem has been solved by Deininger and Su (1971), Converse (1972), and Wanielista and Bauer (1972) with a variety of optimization techniques. The water quality implications of regional wastewater treatment have been studied for specific cases by Yao (1972) and Mendiratta and Davidson (1972) using only deterministic models.

The need for information on the generalized deterministic response and the stochastic response of receiving waters to regionalized wastewater treatment systems is clear. The remainder of this paper treats this area specifically. The water quality evaluation employs water quality models developed for both deterministic and stochastic analyses. Each analysis considers a set of experiments which involves a determination of the water qualities resulting from alternative degrees of wastewater aggregation. The experiments treat not only the degree of aggregation or equivalently the number of plants in the system but also the stream system length as an indicator of regional morphology and the dilution ratio as an indicator of relative stream size. The water quality assessment is made in terms of the minimum dissolved oxygen (DO) level experienced by the system.

THE WATER QUALITY MODEL

The structural elements of the receiving water quality model are subject to the water quality criteria of concern to the wastewater system planner. The criteria employed in any water quality system study are a function of the nature of the study and the characteristics of the study region's water uses. However, dissolved oxygen has long been recognized as a prime indicator of water quality and usually forms the basis of a water quality study. Since no other parameter depicts instream water quality as well as dissolved oxygen, its concentration governs stream ecology and many water uses, and since the standards for other pollutants are often met if the dissolved oxygen standard is met, the water quality model will be concerned with this parameter.

For the purpose of the study, a two reaction model as given by the following equation was adopted:

$$D = \frac{k_1 Lo}{k_2 - k_1} \left(e^{-k_1 t} - e^{-k_2 t} \right) + D_o e^{-k_2 t} \tag{1}$$

in which D is the oxygen deficit at time t, D = Do at time t = 0, L is the biochemical oxygen demand (BOD) at time t, L = Lo at time t = 0, k_1 is the deoxygenation rate constant, k_2 is the reaeration rate constant, and e is the base of natural logarithms (Streeter and Phelps, 1925). To apply this model, it is necessary to specify temporal conditions. These conditions are usually established by assumptions of the stream's hydraulic regime. This model assumes that the flow is uniform and steady and that there is no longitudinal dispersion. The values of Do, Lo and To (stream water temperature) are derived from mass/energy balances on the system as follows:

$$Do = Ds_T - \frac{QrDr + QwDw}{Qr + Qw} , \quad (2)$$

$$Lo = \frac{QrLr + QwLw}{Qr + Qw} , \quad (3)$$

and

$$To = \frac{QrTr + QwTw}{Qr + Qw} , \quad (4)$$

where Ds_T is the dissolved oxygen (DO) saturation concentration at temperature T ; Qr, Dr, Lr, and Tr are the flow, DO concentration, and BOD concentration, and temperature of the stream prior to waste input, respectively; Qw, Dw, Lw, Tw are the flow, DO concentration, and BOD concentration, and temperature of the waste stream, respectively.

The model as given by equation (1) is capable of predicting the DO deficit at any time of flow downstream due to a single point discharge. This formulation must be modified to accommodate multiple discharges. The stream system may be decomposed by lateral sections into reaches on the basis of either similarity of channel characteristics within a reach or location of points of outfall or both. The variables in the sag equation may then be viewed as subscripted variables with subscripts corresponding to reaches (see Figure 1). Since it is assumed that flow and heat transfers do not occur within the reach, the model for multiple outfalls may be written as:

$$D_i = \left(\frac{k1_i}{k2_i - k1_i}\right) Lo_i \left(e^{-k1_i} - e^{-k2_i t_i}\right) + Do_i e^{-k2_i t_i} \quad (5)$$

where

$$Lo_i = \frac{Qr_{i-1} Lr^*_{i-1} + Qw_i Lw_i}{Qr_{i-1} + Qw_i} , \quad (6)$$

$$Lr_i = Lo_i e^{-k1_i t_i} , \quad (7)$$

$$Do_i = Ds_{T_i} - \frac{Qr_{i-1} Dr^*_{i-1} + Qw_i Dw_i}{Qr_{i-1} + Qw_i} , \quad (8)$$

$$Dr_i = Ds_{T_i} - \left[\frac{k_{1_i} \, Lo_i}{k_{2_i} - k_{1_i}} \left(e^{-k_{1_i} t_i} - e^{-k_{1_i} t_i}\right) + Do_i e^{-k_{2_i} t_i} \right] , \qquad (9)$$

$$Qr_i = Qr_{i-1} + Qw_i , \qquad (10)$$

$$T_i = \frac{Qr_{i-1} Tr_{i-1} + Qw_i Tw_i}{Qr_{i-1} + Qw_i} , \qquad (11)$$

$$Qo_i = Qr_i = Qr^*_i , \qquad (12)$$

and $$To_i = T_i = Tr^*_i \qquad (13)$$

The asterisk superscript (*) denote parameter values at the end of each reach and t_i is the time of travel in reach i.

The value of the reaeration rate constant, k_2 , is a function of the hydraulic properties of the stream which are in turn a function of the streamflow. Thus, it is necessary to derive these functional relationships.

Analytic Input Functions

The shape of the channel section is of importance only in its effect on the stage discharge relationship. For the sake of providing an analytic function for stage-discharge, a rectangular channel section was assumed The steady-state analysis allows the use of a uniform flow, open channel formulation. Such a formulation is that of Manning and is given by

$$Q = \frac{1.486}{Mn} AR^{2/3} S^{\frac{1}{2}} \qquad (14)$$

where Q is the flow in cfs, Mn is the Manning number (a measure of channel roughness), A is the cross-sectional area of the channel in sq.ft., R is the hydraulic radius in feet, and S is the channel slope. Because the channel section is irregular, the following is evident;

$$A = XH^2 \tag{15}$$

and $$R = XH/(2+X) \tag{16}$$

where $$X = B/H \tag{17}$$

and H and B are the depth of flow in feet and width of the channel in feet, respectively Substituting equations (15) and (16) in equation (14) and solving for H yields

$$H = \left(\frac{(2+X)^{2/3} M_n Q}{1.486 X^{5/3} S^{\frac{1}{2}}} \right)^{3/8} \tag{18}$$

By the continuity equation

$$V = Q/A \tag{19}$$

where V is the mean stream velocity in fps. Substituting equation (15) into equation (18) gives

$$V = Q/XH^2 \qquad \text{in fps}$$

or $$V = 16.4Q/XH^2 \qquad \text{in mpd} . \tag{20}$$

The formulation exployed for the stream reaeration rate constant, k_2 , is that of O'Connor and Dobbins (1956) for isotropic turbulence given by

$$k_2 = \frac{(C_D V)^{\frac{1}{2}}}{H^{3/2}} \tag{21}$$

where C_D is the coefficient of molecular diffusivity (= 0.8×10^{-4} sq.ft/hr) for oxygen transfer through an aqueous film.

The DO saturation value is a function of temperature as given in the relationship

$$Ds_T = 14.652 - 0.41022T + 7.9910 \times 10^{-3}T^2 - 7.7774 \times 10^{-5}T^3 \tag{22}$$

in which Ds_T is the DO saturation level in mg/l at temperature T^oC (Committee on Sanitary Engineering Research, 1960).

The deoxygenation and reaeration rate constants are also functions of temperature given by the following relationships:

$$k_{1_T} = k_{1_{20^oC}} \theta^{T-20} \tag{23}$$

$$K_{2_T} = k_{2_{20^oC}} \phi^{T-20} \tag{24}$$

for any temperature T in degrees centigrade. The values for θ and ϕ are 1.047 (Gotaas, 1948) and 1.024 (Committee on Sanitary Engineering Research, 1961), respectively.

With the preceding set of analytic functions, the minimum dissolved oxygen level (Dr_{min}) in each reach of an n reach system may be computed.

The regional wastewater management problem of assessing the impact of treatment plant centralization on water quality is approached in both a deterministic and a stochastic manner. The deterministic treatment involves a water quality model with constant parameter values. These values are viewed as the nominal values of the system and are presented in Table 1. A nominal stream length of 64 miles was selected as being representative of a metropolitan region with a population equivalent of one million people. Although the nominal number of plants is 1, subsequent analyses consider up to 32 plants. The resulting minimum DO of the nominal system is 5.6 mg/l occurring 44.14 miles downstream of the wastewater discharge and requiring 1.08 days to be reached.

DETERMINISTIC ANALYSIS

The water quality model described previously was employed to examine the effects of the size, number, and location of wastewater treatment plant discharges on receiving waters of different sizes; that is, streams providing different dilution ratios. Six different stream system lengths were explored: 64, 128, 192,256,320, and 384 miles.

For each stream system length, six different plant systems were examined (1, 2, 4, 8, 16 and 32 plants) in a deterministic analysis. The configurations of these plant systems are illustrated in Figure 2. It was assumed that the treatment plant discharges of each system were equally spaced. The length of stream between discharge points defined a reach; thus, the stream system length, s, and the number of plants, n, specify the distance between plants (or reach length) as s/n and the number of reaches as n. Additionally, an extra reach following that of the last load point was considered to be of variable length. This physical arrangement is fundamental to all experiments with the water quality model.

For each combination of stream system length and the number of plants in the system, the programmed water quality model was run for a series of seven dilution ratios (1/1, 2/1, 4/1, 10/1, 20/1, 40/1 and 80/1). The dilution ratio is defined as the ratio of the initial stream flow and the total wastewater flow generated by the region. In the above experiments, it was assumed that tributary streamflow was due solely to wastewater discharges. An additional experiment was performed in which the streamflow was increased with length in accordance with a specified area-runoff function, the results of which are presented elsewhere. (Adams, 1973).

Size, Number and Location of Plants

The control or base system for this water quality evaluation involves a stream system 64 miles in length with one centralized regional wastewater treatment plant and a dilution ratio of 2/1. The water quality effects of disaggregating the region's wastewater through a multiple point discharge scheme is now examined. For each case in which the model was run, the numerical model parameter values were equal to those in the base system with the exception of wastewater flow which was equally divided among the plants serving the region. Thus, the flow from each plant in an n plant system was of the same strength and was equal to Qw/n where Qw is the wastewater flow in the single plant system of the control condition.

For each computer run, each reach in the system was searched for its minimum dissolved oxygen concentration, and the minimum DO (DO_{min}) of all reaches was determined. These results are summarized in Table 2 from which it is evident that disaggregating the region's wastewater from a single discharge point to 32 uniformly spaced discharge points results in a 0.29 mg/l improvement in the minimum dissolved oxygen concentration. Furthermore, plant systems with greater than about 8 plants result in a negligible water quality improvement over that number of plants.

Centralization and Stream Length

As the distance between discharge points increases or as the stream system length increases, there is a greater travel time between wastewater discharge points and a greater opportunity for self-purification of the water course. In order to quantify the degree of water quality improvement attributable to increased distance between discharges, the water quality model was run for a series of stream system lengths, each for a series of numbers of plants in the system. The shortest system length (64 miles) is viewed as being representative of a metropolitan region while the longer system lengths are viewed as being representative of more rural regions. For each combination of number of plants in the system and stream system length, the DO_{min} was calculated for each reach and the absolute DO_{min} for the entire system was determined. A summary of these results is presented in Table 3.

An examination of these results indicates a dramatic water quality improvement as the distance between plants is increased. This may be seen from Figure 3 in terms of the number of plants in the system or equivalently from Figure 4 in terms of the distance between plants. Figure 3 indicates that for a given stream system length, water quality improvement is experienced by additional numbers of plants in the system. However, the water quality improvements achieved by systems with greater than about 8 plants are only marginally greater than that achieved by an 8 plant system. Furthermore, as the stream system length is increased, substantial water quality improvements are experienced. Similarly, Figure 4 indicates that plant systems with large numbers of plants experience greater water quality improvement from small increments in distance between plants than do systems with small numbers of plants.

Centralization and Stream Size

It is noted that although a nominal dilution ratio of 2/1 was stipulated, the dilution ratio is a function of the region's physical environment: the low flow - drainage area function, length of stream under consideration, location of this length of stream, and the basin geometry. With this in mind, it would be useful to develop information concerning the effect of the dilution ratio on the water quality response to regional wastewater system centralization. An experiment was performed which employed the previously described water quality model to assess these effects. The model runs of the previous section for each combination of stream system length and number of plants in the system were repeated for a series of dilution ratios: 1/1, 2/1, 4/1, 10/1, 20/1, 40/1 and 80/1.

Figure 5 presents plots of the water quality improvement due to an n plant system, ΔDO_{min} (defined as the difference between DO_{min} of an n plant system and DO_{min} of a 1 plant system), for the number of plants in the system. A family of curves is presented for a variety of dilution ratios for the nominal 64 mile stream system It is evident from Figure 5 that the water quality improvement due to the disaggregation of plants is greatest at small dilution ratios and becomes negligible at higher dilution ratios. Again, a breakoff in water quality improvement is evident at an aggregation state of about 8 plants, beyond which the improvement over an 8 plant system is only marginal. The water quality improvements for a 32 plant system, ΔDO_{min} (32), at a 1/1 dilution ratio are 0.5 and 2.9 mg/l for the 64 and 384 mile systems, respectively.

Figure 6 presents a plot of DO_{min} and ΔDO_{min} versus dilution ratio for the 32 plant system. This figure presents a family of curves depicting various system lengths or, equivalently, distance between plants in the system. These plots serve to reinforce previous observations:

(a) as n is increased, ΔDO_{min} is increased at a decreasing rate such that at a disaggregation state of 8 plants the increase in ΔDO_{min} due to further disaggregation is negligible, (b) as the dilution ratio is increased, DO_{min} reaches a maximum for a given stream length and steadily decreases with increased dilution ratios, (c) for a given disaggreagation state, ΔDO_{min} increases with an increase in distances between plants at a decreasing rate, and (d) the dilution ratio at which DO_{min} is maximized decreases with an increase in distance between plants.

STOCHASTIC INPUT MODELS

The deterministic analysis examined the water quality impact due to the degree of wastewater treatment centralization assuming the wastewater and stream water quality and quantity parameter to be constant. Attention is now directed to the water quality impact due to system centralization when these parameters act in a stochastic manner. Before assessing the effects of variable wasteloads and stream conditions, it is necessary to develop stochastic models to generate values for these variables.

Models for Wastewater Treatment Plant Parameters

Models for wastewater flow and BOD and DO concentrations were based on a study of wastewater treatment plant performance (Adams and Gemmell, 1973). This study was concerned with the performance of both individual plants and regional groups of plants. It was concluded that the variance of plant performance was large, that individual plant performance was random on a short term basis except in the case of effluent DO, and that the variability of effluent BOD was weakly correlated from plant to plant in a regional group while discharge was strongly correlated.

Plots of the coefficients of variation of effluent BOD against plant sizes indicated a general decrease in the coefficient of variation (C_v) with an increase in plant size. This suggested the use of a regression equation to predict the coefficient of variation from the plant size. The form of the equation is given by the following:

$$C_v = a(\bar{Q}w)^b \tag{25}$$

in which $\bar{Q}w$ is the mean plant size in MGD and a and b are constants. The equation used was the following:

$$C_v = 0.806\ \bar{Q}w^{(-0.0898)} \tag{26}$$

It is then possible to draw values for effluent BOD by knowing the plant size and mean effluent BOD concentration ($\bar{L}w$) from a population distributed $N(\bar{L}w,\ \bar{L}wC_v)$

A similar relationship was established for effluent discharge as given by the following:

$$C_v = 0.265\ \bar{Q}w^{(-0.0778)} \tag{27}$$

This expression may not be used directly since the performance analysis indicated a dependence on discharge among regional plants. This cross correlation is considered by the following autoregressive model for discharge

$$Qw_i = \bar{Q}w_i + r(Qw_{i+1} - \bar{Q}w_i) + \varepsilon_i s(1-r^2)^{\frac{1}{2}} \tag{28}$$

in which Qw_i is the discharge of plant i, $\bar{Q}w_i$ is the mean flow of plant i, r is the correlation coefficient between flows Qw_i and Qw_{i-1} (= 0.82 from performance data), s is the standard deviation of flow (= $\bar{Q}wC_v$) , and the ε_i are random error terms drawn from a population distributed $N(0,1)$.

A study of DO concentrations of wastewater treatment plant effluents indicated that DO was decidedly nonrandom and varied only within a small range. For this reason, a constant effluent DO was assumed for all plants in the regions studied.

A model for wastewater temperature was established following the procedure of parametric time series analysis. The resulting model is autoregressive of order 1 and moving average of order 1 and is given by

$$Tw_t = 11.8 + 0.80\ Tw_{t-1} + 11.9 \cos (0.01721t - 3.9670)$$

$$-9.5 \cos (0.01721t - 3.9670) + a_t + 0.12\ a_{t-1} \qquad (29)$$

in which Tw_t is the wastewater temperature at time t (in ^oF) and the a_t are random error terms drawn from a population distributed N(0, 1.069) (Adams, 1973).

Models for Stream Parameters

A model similar to that for wastewater temperature was employed for stream water temperature as developed by McMichael and Hunter (1972) for Ohio River data. The model is autoregressive of order 1 and is given by

$$Tr_t = 5.066 + 0.915\ Tr_{t-1} + 22.4 \cos (0.01721t - 0.5426)$$

$$-20.2 \cos (0.01721t - 0.5426) + a_t \qquad (30)$$

in which Tr_t is the stream water temperature at time t (in ^oF) and the a_t are random error terms drawn from a population distributed N(0, 1.4).

The objective of the simulation experiment is to determine DO_{min} variability as a result of input and system variability. The initial stream DO (Dr_o) is not directly varied in this experiment because it is essentially the same parameter that is being measured. A different experimental time scale would warrant a function for Dr_o variability on a phenomenological basis. Inasmuch as Dr_o is a percentage of the saturation DO, it will vary indirectly with Tr_o.

The same difficulty is experienced in assigning a function for Lr_o variability. Since the assumption of an unpolluted water upstream of the discharge point(s) is made, Lr_o will vary only within a small range. Further, it has been demonstrated that variations of 100 percent from the nominal value caused a deviation of only 10 per cent from the DO_{min} response. Since this study is not specifically concerned with the upstream BOD distribution, the assumption of its form is arbitrary. The distribution of Lr_o is assumed to be NID (μ,σ) with μ = 3.0 mg/l, the nominal value, and σ = 1.5 mg/l (C_v = 0.5) with the distribution truncated at 0 and 6 mg/l, providing a maximum deviation of 100 per cent from the nominal value.

The variation of k_1 has been studied by Kothandaraman (1968) for its randomness and distribution of random variation on the basis of 83 observations of k_1 from the Ohio River. Randomness was tested by the Runs Up and Down test and the hypothesis that the observations form a random sample was not rejected. The hypothesis that the sample was drawn from a normal distribution was tested by the Kolmogorov-Smirnov one-sample test and was not rejected at the 95 per cent confidence level. The values of the distribution parameters are μ = 0.173 and σ = 0.066 with C_v = 0.38. This coefficient of variation is used with the nominal value of k_1 as the mean, and values are generated independently from a normal distribution with these parameter values.

The variations in the geometry and the stream channel characteristics are of interest inasmuch as they vary the reaeration rate constant. Rather than arbitrarily assigning stochastic models to these parameters, a stochastic model for k_2 based on mean values of these parameters is proposed. Kothandaraman (1968) devised a technique for generating stochastic values of k_2 based on the mean values of parameters affecting k_2. From a series of regression analyses on reaeration measurements on TVA streams a relationship for predicting mean values of k_2 was developed. Recognizing that the residual variance was high (36.8 per cent of the estimated k_2), a study of the distribution of residual error was undertaken. The hypothesis that the residual error was normally distributed ($N(0,.368k_2)$) was not rejected by the Kolmogorov-Smirnov one-sample test. Values for k_2 are generated by estimating k_2 from V and H as described by equation (21) and adding this normally distributed random error.

STOCHASTIC ANALYSIS

It has been demonstrated that the distribution of nonvariable wastewater sources over the receiving waters has the effect of significantly improving the receiving water quality when the distance between outfalls is large and the dilution ratio is small. The objective at this point is to examine the water quality impact of variable wasteloads under variable stream conditions. In simulating the behaviour of the system, it must be given a temporal framework. The system was simulated in the framework of the lowest average 7 consecutive-day flow occurring in ten years, a common flow condition employed by water quality standards. The input models described previously were included in the water quality simulation model and the minimum dissolved oxygen frequency response of the receiving waters was determined after each set of simulation runs. Experiments were then conducted with stream systems of different lengths and dilution rations. These simulations are concerned with the day-to-day variabiliyt of the system, and each simulation is that of a steady-state system.

Hypothesis for Water Quality Improvement

Before discussing the results of these simulations, an explanation is presented for the hypothesis of improved low-frequency water quality response due to disaggregated regional wastewater systems. Consider the following theorem:

Let X be a random variable with expectation $E(X) = \mu$ and variance $V(X) = \sigma^2$ and let $\bar{X}$ be the sample mean of a random sample of size n.

Then (a) $E(\bar{X}) = \mu$, (31a)

(b) $V(\bar{X}) = \sigma^2/n$, (31b)

and (c) for large n, $(\bar{X} - \mu)/(\sigma/\sqrt{n}) \sim N(0, 1)$. (31c)

For the purposes of this discussion, consider a simplistic case where only the effluent BOD concentration is variable. To establish the variance of the system of plants,

let $(\sigma)^2$ = the variance of the plant, and

let $(\sigma^*)^2$ = the variance of the system.

From the regression relationships described previously

$$\sigma_i = \mu_i \, a(\frac{Q}{i})^{-b} \tag{32}$$

where i is the number of plants in the system and Q is the total wastewater discharged by the system For i = 1 and i = n, equation (32) becomes

$$\sigma_1 = \mu_1 \, a(\frac{Q}{1})^{-b} \quad , \quad i = 1 \tag{33}$$

and

$$\sigma_n = \mu_n \, a(\frac{Q}{n})^{-b} \quad , \quad i = n \quad . \tag{34}$$

If $\mu_1 = \mu_n$, then the division of equations (35) and (34) results in the solution

$$\sigma_n = \mu_1 \, n^b \tag{35}$$

and

$$\sigma_n^* = \sigma_n / \sqrt{n} = \sigma_1 n^b / \sqrt{n} \tag{36}$$

by the above theorem.

From equation (26) b = 0.09 and if n = 32

$$\sigma_{32}^* = \sigma_1 / (32^{(0.50 - 0.09)}) = 0.24 \, \sigma_1 = 0.24 \, \sigma_1^* \tag{37}$$

This simple example demonstrates that the variance of waste quantities of the 32 plant system may be considerably less than that of a single plant system.

Effect of Parameter Variability

The base physical system was employed to assess the effects of variability of the wastewater inputs and stream conditions. This system involves a 64 mile stream length with a 2/1 dilution ratio. The stochastic generator for each variable was incorporated in the water quality simulation model, and 100 simulation runs were executed for systems with 1,2,4,8,16 and 32 plants. The results of these simulations are summarized in terms of the water quality frequency response functions relating the per cent of time any particular water quality level is violated

The water quality frequency response functions determined by these simulations are displayed in Figure 7 as cumulative frequency distribution functions (cdf's) for the 1 and 32 plant systems. Low-frequency plots for the systems of various plant numbers are presented in Figure 8. From Figures 7 and 8 the general observation may be made: the lower the frequency the greater the difference between the minimum dissolved oxygen levels of the single and multiple plant systems.

At a 1 per cent frequency of occurrence, the single plant system is anaerobic at or around the point of critical deficit while the 32 plant system produces a minimum DO of 4.6 mg/l. At the 5, 10, and 20 per cent of frequencies, the differences between the minimum DO levels produced by the 1 and 32 plant systems are 2.5, 1.7, and 1.1 mg/l, respectively. The 2 plant system still results in anaerobic conditions at a finite probability level while the 4 plant system is essentially operating at a DO level above 2.5 mg/l A breakoff is seen in the neighborhood of an 8 plant system where an increase in plant number brings only marginally improved low-frequency response.

The Kolmogorov-Smirnov two-sample test was employed to determine the significance of the difference between the cdf's of system response to the 1 and the n plant systems. The differences between the distribution functions were calculated and the maximum difference was selected. These statistics were based on 500 simulations in each case. From statistical tables (Siegel, 1956) the critical value of d is given by

$$d_{\alpha} = 1.36 \sqrt{\frac{2}{n}} = 0.086 = 8.6\% \tag{38}$$

at $\alpha = 0.05$. A comparison of d_{max} and d_{α} indicates that $d_{max} > d_{\alpha}$ for the 4, 8, 16 and 32 plant systems (Adams, 1973). That is, in all but the 2 plant system, there is a significant difference between the 1 and n plant cdf's at the 5 per cent level.

Stream Length and the Water Quality Frequency Response

The question of how increasing the distance between plants affects the water quality frequency response is now addressed. The answer involves a determination of the effect of variability on the system response in conjunction with the effect of stream length. A series of simulations similar to those described previously were conducted on stream systems of varying length. One hundred simulations were executed for each plant number system (1,2,4,8,16 and 32 plants) of each stream system length (64,128,192,256,320 and 384 miles). The cumulative distribution functions from these simulations are presented in Figure 9 for the 1 and 32 plant systems. The length of the system does not affect the frequency response of the single plant system; however, with more than one plant in the system, a given DO level is violated increasing less frequently with an increase in distance between plants. The mean responses also improve in accordance with the results of the deterministic analysis.

Stream Dilution and the Water Quality Frequency Response

The previous experiments on the 64 mile stream system with a 2/1 dilution ratio were repeated for a series of dilution ratios: 1/1, 2/1, 4/1, 10/1,20/1,40/1 and 80/1. The dilution ratio is defined as the ratio of the streamflow before wastewater discharge to the wastewater flow. In each case, 100 simulations were executed employing the same models for the stochastic variables previously described. The results of these simulations are reported in terms of the water quality frequency response functions (Adams, 1973).

From an inspection of these functions, it is evident that the variance of the system response for any number of plants decreases as the dilution ratio increases and for any dilution ratio decreases as the number of plants increases. Correspondingly, there is an increase in the mean response with an increase in either dilution ratio or plant number or both, in accordance with the results of the deterministic analysis. Because the mean response increases and the variance of the response decreases with an increase in dilution ratio, the coefficient of variation decreases at an even greater rate. The difference in minimum stream DO response between the 1 and 32 plant systems at a given low frequency level decreases significantly with an increase in dilution ratio. For example, ΔDO_{min} is 3.9 mg/1 for a 1/1 dilution ratio and 0.4 at an 80/1 dilution ratio at the 5 per cent frequency. This conclusion is evident from an inspection of Figure 10 which presents plots of ΔDO_{min} for dilution ratios at the 1, 5, 10 and 20 per cent frequency levels.

SUMMARY AND CONCLUSIONS

The deterministic analysis revealed that for a given stream size, or dilution ratio, and a given stream length, an increase in the disaggregation state of wastewater treatment plants results in an improved water quality. This water quality improvement is negligible for short stream systems lengths typical of metropolitan regions while it is considerable for longer stream system lengths indicative of more rural regions. The magnitude of the improvement may be greater than a 2 mg/1 increase in the minimum stream dissolved oxygen level. It is noted that the greater part of the water quality improvement due to decentralization is achieved by a disaggregation state of approximately 8 plants. Further increases in disaggregation state result in only marginal improvements over an 8 plant system.

The water quality improvement resulting from a decentralized regional wastewater treatment system is also a function of the dilution ratio. For a given stream system length, the water quality improvement due to a fixed disaggregation state increases with a decrease in the dilution ratio. Thus, it is concluded that decentralization is more beneficial in cases with relatively small streamflows. Furthermore, for a given stream system length and a fixed number of plants in the system, there exists a dilution ratio at which DO_{min} is maximized. Beyond this dilution ratio, DO_{min} steadily decreases as previously explained.

Although results were not presented, the sensitivity of these experiments to the natural stream condition of augmented flow was assessed. It is concluded that although the values of DO_{min} are generally larger for short stream systems and smaller for longer stream systems, there exists the same general increase in water quality with an increase in disaggregation state.

The water quality impact of regional wastewater centralization due to stochastic variability was examined with stochastic simulation models. Experiments were conducted in the temporal framework of daily simulations of the critical low-flow season. Systems with various degrees of centralization of wastewater treatment facilities were examined for different stream lengths and dilution ratios.

Increasing the state of aggregation or equivalently increasing the number of wastewater discharge points resulted in minimum DO frequency responses with smaller variances. Thus, at lower frequency levels, the multi-plant system resulted in a minimum stream DO significantly larger than that of the single plant system. The mean responses for short system lengths increased only marginally with an increase in plant number, in accordance with the results of the deterministic analysis.

Water quality simulations were undertaken for systems of various lengths. Increasing system length had the effect of not only increasing the mean minimum DO response but also decreasing the variance of the response of multi-plant systems. The response of the single plant system is not length dependent, and the cdf's of minimum DO did not change from the above. Thus, system decentralization benefits from both the deterministic effect of increased mean response with length and the stochastic effect of decreased variance of the response.

Repeating this simulation experiment for stream systems with various dilution ratios indicated that the variance of the minimum DO response decreased with increased dilution ratios. Since the mean response increased with increased dilution ratios in accordance with the results of the deterministic analysis, the coefficient of variation of the DO response decreased at a rate faster than that at which mean response increased or variance of the response decreased. It is concluded that the decentralized wastewater system significantly outperforms the centralized system when the dilution ratio is small. As the dilution ratio increases, this difference in performance decreases; conversely, this difference increases with an increase in system length.

The results of these simulations indicate that there is variation not only in the value of the critical oxygen deficit but also in the location of this deficit. Thus, associated with the probability distribution of the minimum dissolved oxygen level is a probability distribution of the location of the critical DO deficit. A study of this distribution of the location of the critical deficit reveals that there is a tendency for the variance of the distribution to decrease with an increase in the number of plants in the system. This observation was noted for all stream system lengths and dilution ratios that were explored. Furthermore, systems with a fixed number of plants exhibited distributions of critical DO deficit location whose variances decreased with increased dilution ratios (Adams, 1973).

OTHER FACTORS

In addition to the factors addressed in this study, a number of other factors not specifically considered would have a bearing on the results of this study. It is felt that some comment on these factors is necessary.

The stochastic water quality model developed for the purposes of this study was a steady state model. Although input wasteloads and stream conditions were assigned day-to-day variability, they were assumed to be constant long enough for the system to achieve a steady state condition. In reality, there is short term, hour-to-hour, variability. The effect of short term variability on the system is of importance when longitudinal dispersion is considered. Li (1972) has shown that the effect of longitudinal dispersion is negligible in a steady state system while it may be considerable for the case of short term variability of input wasteloads. This was demonstrated analytically using a periodic BOD input without random variability. Although the knowledge of short term effluent BOD variability is scant at best and a simulation of this case is obstructed by this lack of knowledge, it should be noted that, on the basis of Li's study, longitudinal dispersion may significantly reduce the amplitudes of DO fluctuations along the stream (up to 50 per cent of the steady state value). It should also be noted that there are larger fluctuations due to short term variability which would cause a greater variance of the system response than that determined by the steady state analysis of day-to-day variability. Thus, the consideration of short term variability would combine the opposing forces of decreased variability of DO response due to dispersion and increased variability of DO response due to the increased variability of inputs on a short term basis.

Depending on the nature of the watercourse, the action of photosynthetic organisms may have an insignificant effect on the DO response or it may dominate the DO response. Dissolved oxygen is added and depleted by photosynthesis in a diurnal cycle with the oxygen added usually more than that depleted. A daily simulation would consider this net positive oxygen addition for applications in which photosynthesis is significant. An hourly simulation of dissolved oxygen would consider the diurnal fluctuation of dissolved oxygen. In the former case, the conclusions of this study would be essentially unaltered, and in the latter case, the conclusions regarding water quality improvement by decentralized systems would be amplified.

The effect of the second or nitrogenous stage of the biochemical oxygen demand may be significant. Typically, the nitrogenous BOD is not actively exerted until the 4th to 8th day after discharge from a treatment plant. This time lag is usually attributed to the development of an acclimatized nitrifying bacterial population. The exertion of nitrogenous BOD has been described as a first order reaction incorporating a lag by Thomas (1940) and as a second order reaction which inherently incorporates a lag by ReVelle, et al (1965). From studies on the Grand River by Courchaine (1963), the lag period of nitrogenous BOD exertion was found to be in the order of 4 days. In the 32 plant 64 mile stream system, the discharge from the first plant has mixed with the discharge from the last plant within 2 days. Thus, the effect of nitrogenous BOD on the n plant system would be experienced well downstream of the last plant in the system. This effect would be analogous to that of the carbonaceous demand but displaced in space and time. The assignment of values to parameters in the nitrogenous BOD formulation is made with less experience and confidence than with carbonaceous BOD formulations. Data on treatment plant effluent nitrogenous BOD variability is likewise more scant than that for carbonaceous BOD. These factors make a model for nitrogenous BOD effects less reliable than the model employed in this study. Since there is similitude in the impact of carbonaceous and nitrogenous BOD exertion on the stream and since the nitrogenous effects are less well defined, nitrogenous BOD exertion was not incorporated into the water quality model employed in this study although its potential importance is noted.

The deoxygenation and reaeration coefficients were assumed to be independent of the BOD concentration. It has been demonstrated that these coefficients are in fact not independent of pollution concentration. The coefficient of deoxygenation usually increases with increased concentration while the coefficient of reaeration decreases with concentration; however, little is known about the functional relationships involved (Tsivoglou and Wallace, 1972). Generally, it may be stated that the net effect of these changes in rate coefficients results in a more deteriorated water quality than that determined by holding these coefficients constant. This observation would magnify the water quality improvement attributed to decentralized wastewater systems by this study.

The characteristics of the physical system simulated were assumed to be as simple as possible to enable conclusions to be stated as generally as possible. Stream characteristics will vary from application to application and will become more and more complex. Such a complexity is that of a branched stream system. This case is cited as an example of how the general results reported above might be used to shed light on a more complex system.

A branched stream system would provide a greater opportunity for system alternatives. Two branches meeting at a confluence would allow the system designer to take advantage of the assimilative capacities of both branches. In effect, the branches could be viewed as parallel systems up to the point of confluence. Downstream from this point, the outputs of the branches are simply added to form an input to the mainstream. The effect of a multi-plant loading on each of the branches would be proportional to the relative sizes of the branches and discharge loads while the effect of the mainstream would be dependent on these factors plus the distance from the discharge points to the point of confluence. This reasoning could be extended to higher order streams.

Finally, it should be stated that this study was not directed specifically to a general purpose model applicable to an evaluation of wastewater centralization for all regional applications although the models developed in this study may be modified to accomplish such an end. Rather, the intention was to study the behaviour of water quality systems relative to the practice of regional wastewater centralization in order to contribute information for an evaluation of that practice. It is through such an evaluation that an understanding of man's environment may be developed to allow a rational approach to its management.

ACKNOWLEDGEMENTS

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NOTATION

a, b, θ, ϕ	-	constants
a_t	-	random error term
α	-	significance level
A	-	cross sectional area of channel
8	-	average width of channel
BOD	-	biochemical oxygen demand (mg/1)
cdf	-	cumulative distribution function
C_D	-	coefficient of molecular diffusivity (= 0.8×10^{-4} sq.ft/hr)
C_v	-	coefficient of variation
d	-	Kolmogorov-Smirnov one-sample statistic
D	-	oxygen deficit (mg/1)
DO	-	dissolved oxygen (mg/1)
Do	-	initial DO in stream
Dr	-	DO instream
Dw	-	DO in wastewater
D_{s_T}	-	DO saturation at temperature T
Do_{min}	-	minimum stream DO
ΔDO_{min}	-	DO_{min} (n plant system) - DO_{min} (1 plant system)
e	-	base of natural logarithms
E(X)	-	expectation of random variable X
ε	-	standard normal deviate
H	-	*average stream depth (feet)*
i	-	subscript denoting reach (or plant) number
k_1	-	deoxygenation rate constant (base e)
k_2	-	reaeration rate constant (base e)
K_1	-	deoxygenation rate constant (base 10)
K_2	-	reaeration rate constant (base 10)
L	-	BOD
L_o	-	initial BOD in stream
Lr	-	BOD in stream
Lw	-	BOD in wastewater

$\bar{L}w$ - mean of Lw

Mn - channel roughness coefficient

μ - mean

n - number of plants in system

NID - normal independently distributed

Q - flow (cfs)

Q_o - initial streamflow

Q_r - streamflow

Qw - treatment plant flow

$\bar{Q}w$ - mean of Qw

r - correlation coefficient

R - hydraulic radius of channel (feet)

s - standard deviation

S - channel slope, stream length (miles)

σ - standard deviation

T - temperature

To - initial stream temperature

Tr - stream temperature

Tw - wastewater temperature

V - average stream velocity

V(X) - variance of random variable X

X - stream width to depth ratio (= B/H), random variable

cfs - cubic feet per second

fps - feet per second

hr - hour

mg/l - milligrams per litre

mpd - miles per day

MGD - million gallons per day

sq.ft.- square feet

REFERENCES

Adams, B. J., "Regional wastewater management: a water quality evaluation of system centralization", Ph.D. thesis, Northwestern University, Evanston, Ill., 1973.

Adams, B. J., and Gemmell, R. S., "Performance of regionally related wastewater treatment plants", J. Water Poll. Contr. Fed., 45(10), 2088-2103, 1973.

Adams, B. J., Dajani, J. S., and Gemmell, R. S., "On the centralization of wastewater treatment plants", Water Resour. Bull., 8(4), 669-678,1972.

Committee on Sanitary Engineering Research, "Effect of temperature on stream reaeration", J. Sanit. Eng. Div., Amer. Soc. Civil Eng., 92(SA6), 59-71, 1961.

Committee on Sanitary Engineering Research, "Solubility of atmospheric oxygen in water", J. Sanit. Eng. Div., Amer. Soc. Civil Eng., 86(SA4), 1960.

Converse, A. O., "Optimum number and location of treatment plants", J. Water. Poll. Contr. Fed., 44(8), 1629-1636, 1972.

Courchaine, R. J., "The significance of nitrification in stream analysis - effects on the oxygen balance", Proc. 18th Industrial Waste Conf., Purdue Univ., Lafayette, Ind., 1963.

Dajani, J. S., and Gemmell, R. S., "Economic guidelines for public utilities planning", J. Urban Plann. and Devel. Div., Amer. Soc. Civil Eng., 99(UP3), 1973.

Deininger, R. A., and Su, S. Y, "Regional wastewater treatment systems", paper presented at the 18th Internat.Meeting. Inst. of Management Sciences, Washington, D.C., March, 1971.

Gotaas, H. B., "Effects of temperature on biochemical oxidation of sewage", Sewage Works J., 40(3), 441, 1948.

Kothandaraman, Veerasamy, "Probabilistic analysis of wastewater treatment and disposal systems", Ph.D. thesis, Univ. of Ill., Urbana, Ill., 1968.

Li, W.-H., "Effects of dispersion on DO-sag in a stream", J.Sanit.Eng.Div., Amer. Soc. Civil Eng., 98(SA1), 169-182, 1972.

McMichael, F. C., and Hunter, J. S., "Stochastic modeling of temperature and flow in rivers", Water Resour. Res., 8(1), 87-98, 1972.

Mendiratta, A. K., and Davidson, B., "Best combination of waste treatment and spatially distributed discharges of effluent", Water Resour.Res., 8(3), 565-585, 1972.

O'Connor, D. J., and Dobbins, W. E., "The mechanics of reaeration in natural streams", J. Sanit. Eng. Div., Amer Soc.Civil Eng., 82(SA6), 1956.

ReVelle, C. S., Lynn, W. R., and Rivera, M. A., "Bio-oxidation kinetics and a second-order equation describing the BOD reaction", J.Water Poll. Contr. Fed., 37(12), 1679-1692, 1965.

Siegel, Sidney, Nonparametric Statistics, McGraw-Hill, New York, 1956.

Streeter, H. W., and Phelps, E. B., "A study of the pollution and natural purification of the Ohio River", U. S. Public Health Service Bulletin 14-6, USPHS, Washington, D.C., 1925.

Thomann, R. V., "Variability of waste treatment plant performance", J. Sanit. Eng. Div., Amer. Soc. Civil Eng., 96(SA3), 819-837, 1970.

Thomas, H. A., "Analysis of the biochemical oxygen demand curve", Sewage Works J., 12(3), 504-512, 1940.

Tsivoglou, E. C., and Wallace, J. R., "Characterization of stream reaeration capacity", U.S.Government Printing Office, Washington,D.C., 1972.

Wanielista, M. P., and Bauer, C. S., "Centralization of waste treatment facilities", J. Wat. Poll. Contr. Fed., 44(12), 2229-2238, 1972.

Yao, K. M., "Regionalization and Water Quality Management", J.Water Poll.Contr. Fed., 45(3), 407-411, 1973.

TABLE 1

Nominal Values of Parameters

Parameter	Nominal Value
Manning number (Mn)	0.060
Channel slope (S)	0.001 ft/ft
Width/depth ratio (X)	5
Streamflow (Qr_o)	527 cfs
Stream BOD (Lr_o)	3 mg/1
Stream DO (Dr_o)	90% of saturation
Stream temperature (Tr_o)	16.8°C
Deoxygenation coefficient (K_1)	0.20 (base 10, 20°C)
Wastewater flow (Qw)	263 cfs
Wastewater BOD (Lw)	30 mg/1
Wastewater DO (Dw)	4 mg/1
Wastewater temperature (Tw)	21.5°C
Reaeration coefficient (K_2)	0.40 (base 10, 20°C)
Minimum stream DO (DO_{min})	5.57 mg/1
DO saturation level (Ds_T)	9.24 mg/1

TABLE 2

Effect of Size, Number, and Location of Plants on Water Quality

Number of Plants	Reach Length (miles)	Critical Reach	DO_{min} (mg/1)	ΔDO_{min}[1] (mg/1)
1	64	1	5.56	0
2	32	2	5.75	0.19
4	16	5	5.81	0.25
8	8	9	5.84	0.28
16	4	17	5.85	0.29
32	2	33	5.85	0.29

[1] $\Delta DO_{min} = DO_{min}$ (n Plant) $- DO_{min}$ (1 Plant)

TABLE 3

Effect of System Length and Number of Plants on Water Quality

System Length (miles)	64		128		192		256		320		384	
Number of Plants	DO^{1}_{min}	DIST[2]	DO_{min}	DIST	DO_{min}	DIST	DO_{min}	DIST	DO_{min}	DIST	DO_{min}	DIST
1	5.56		5.56		5.56		5.56		5.56		5.56	
2	5.75	32	6.10	64	6.49	96	6.73	128	6.73	160	6.73	192
4	5.81	16	6.27	32	6.75	48	7.13	64	7.31	80	7.44	96
8	5.84	8	6.32	16	6.83	24	7.24	32	7.50	40	7.65	48
16	5.85	4	6.35	8	6.85	12	7.28	16	7.57	20	7.74	24
32	5.85	2	6.36	4	6.87	6	7.30	8	7.60	10	7.78	12

[1] DO_{min} = minimum stream dissolved oxygen concentration (mg/1)

[2] DIST = distance between plants (miles)

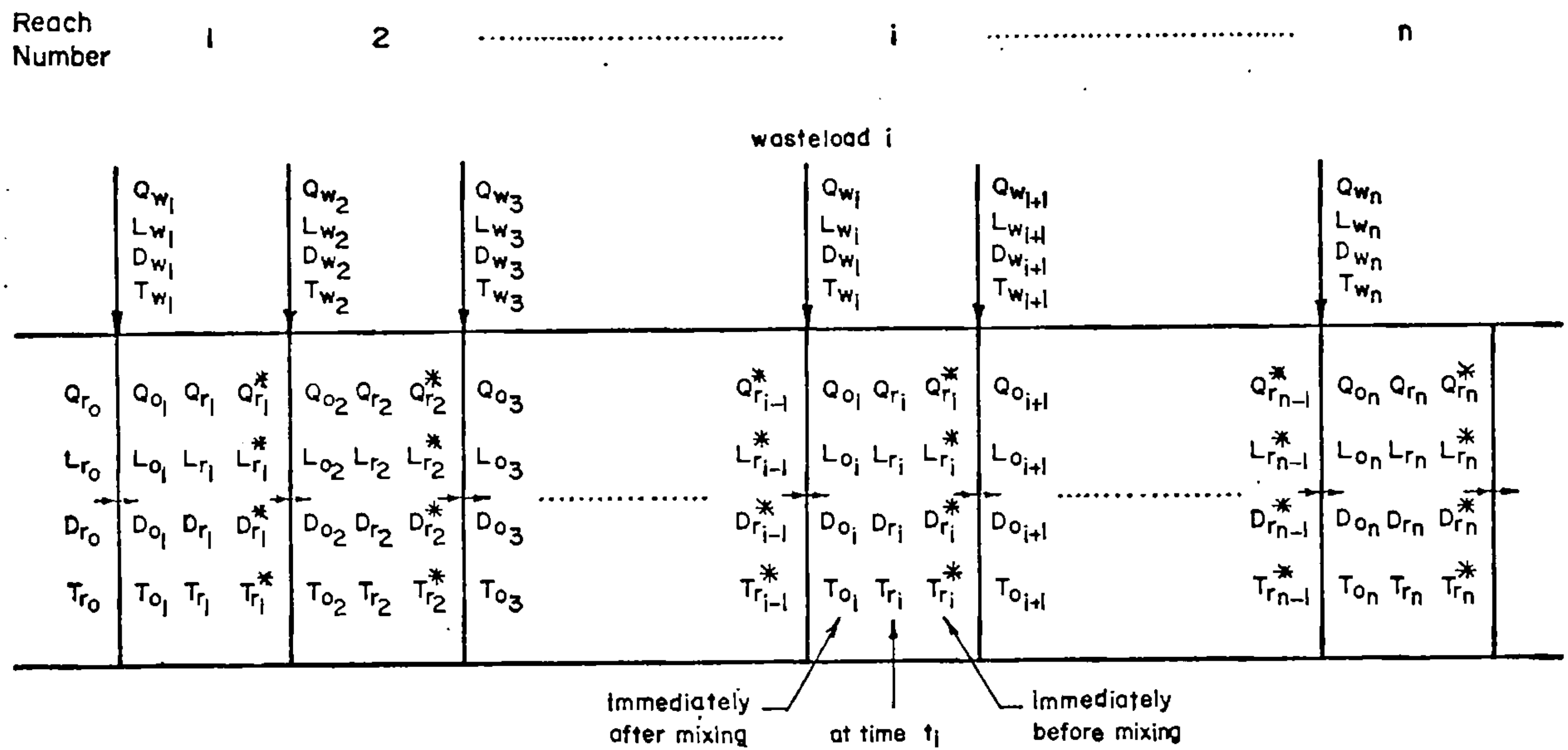

Figure 1. Schematic Diagram for Model Notation

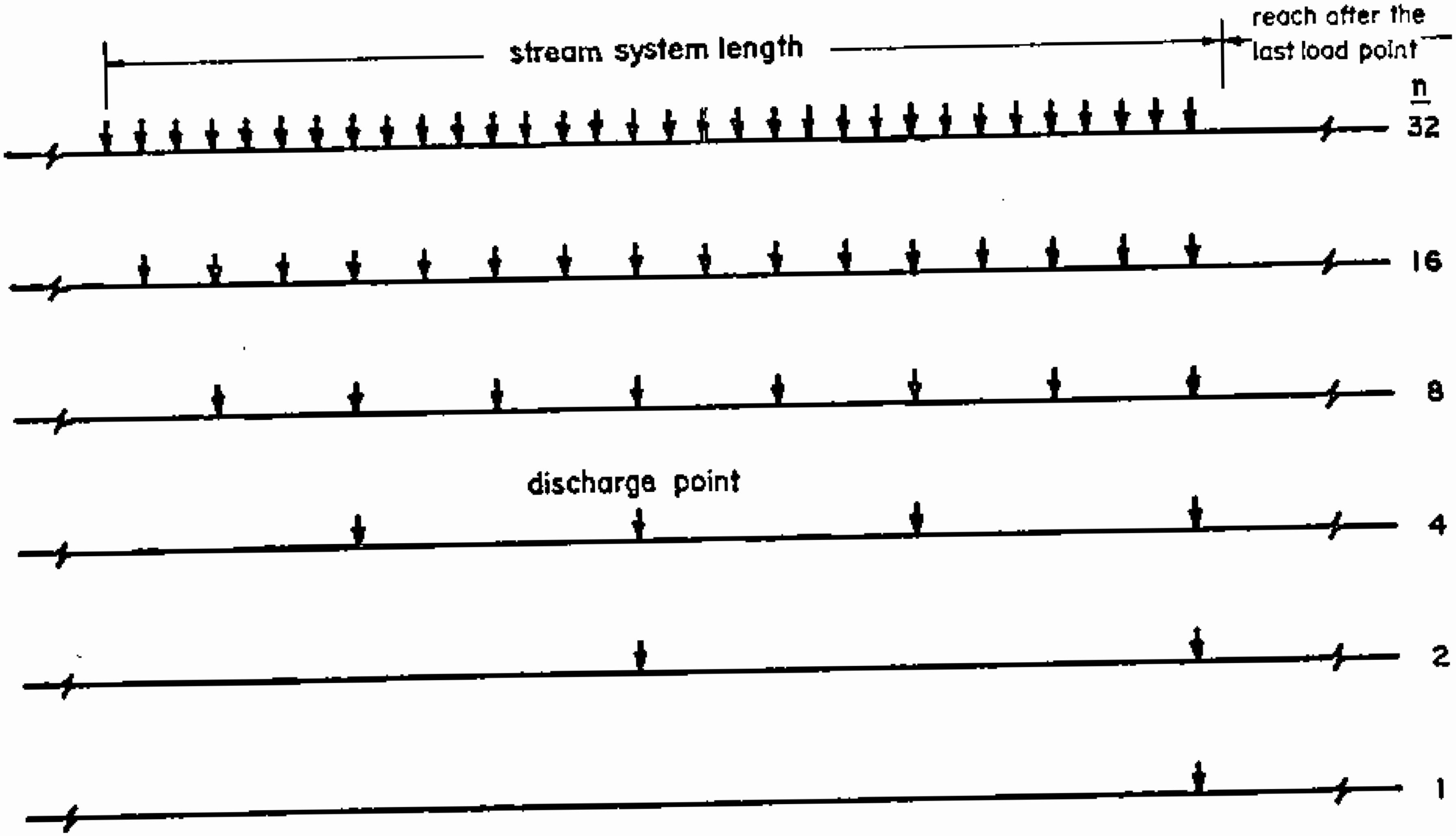

Figure 2. Regional Wastewater System Configurations

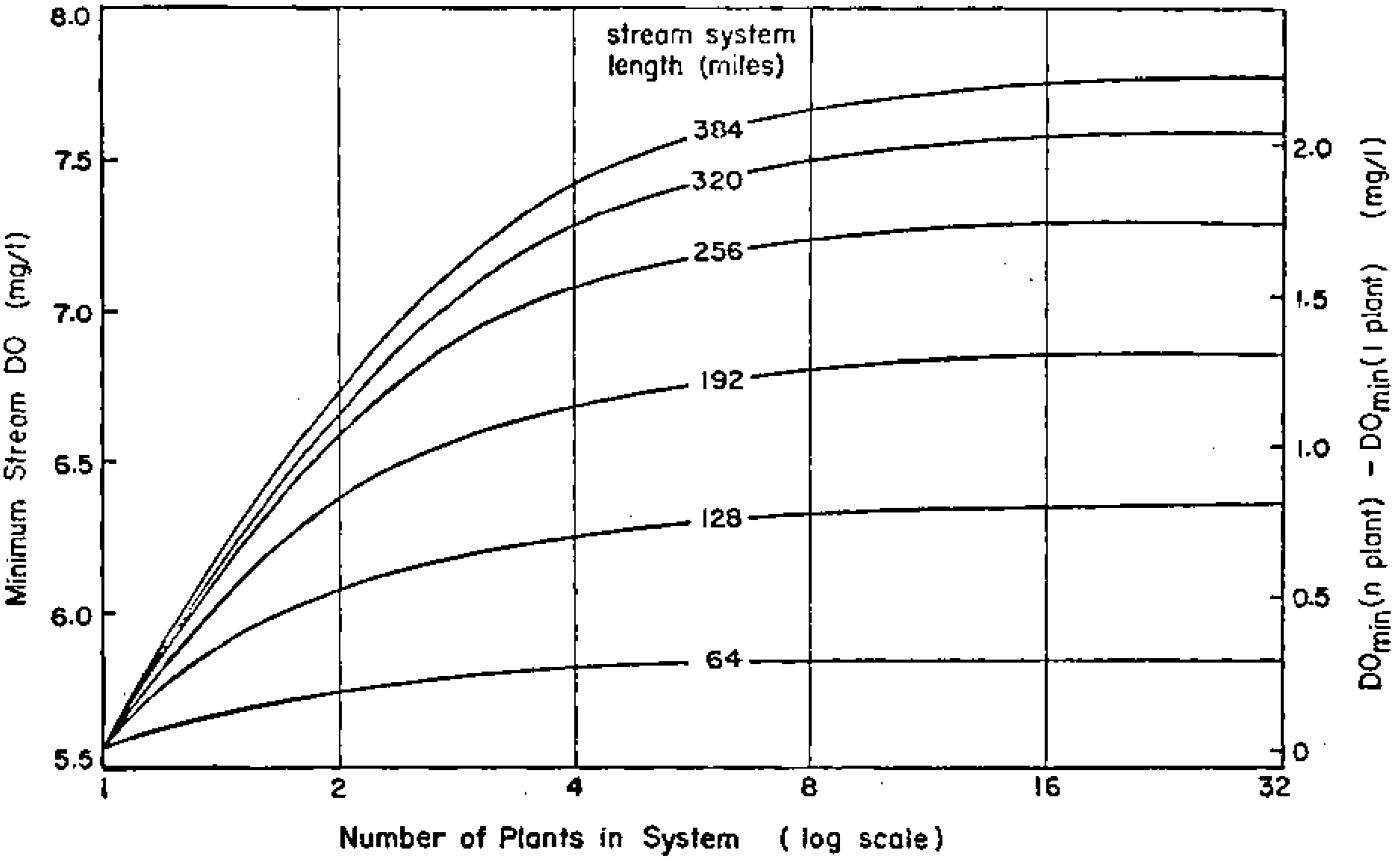

Figure 3. Effect of Plant Numbers on Water Quality for Various System Lengths

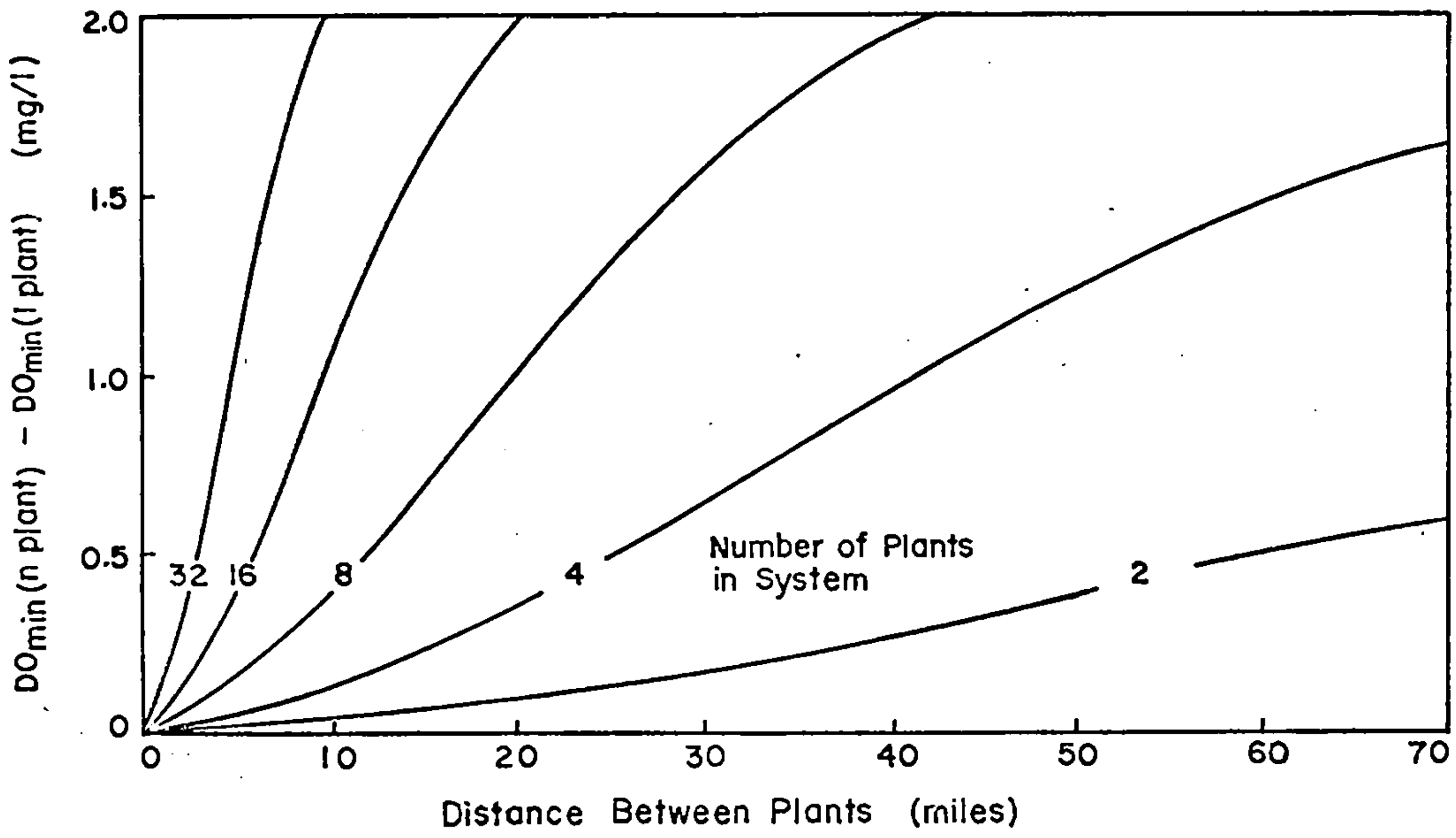

Figure 4. Effect of Distance between Plants on Water Quality for Various Plant Numbers

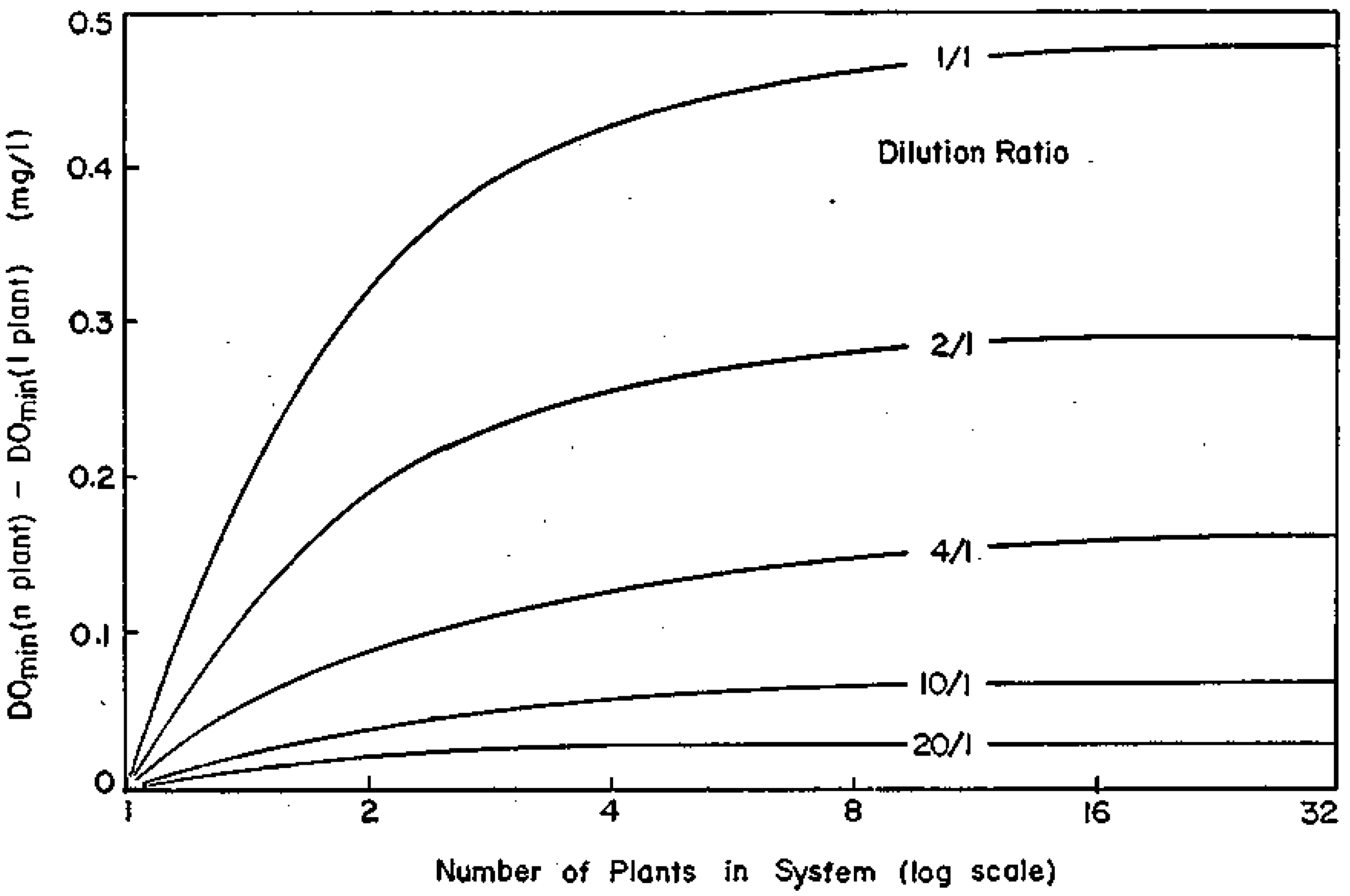

Figure 5. Effect of Plant Numbers on Water Quality for Various Dilution Ratios

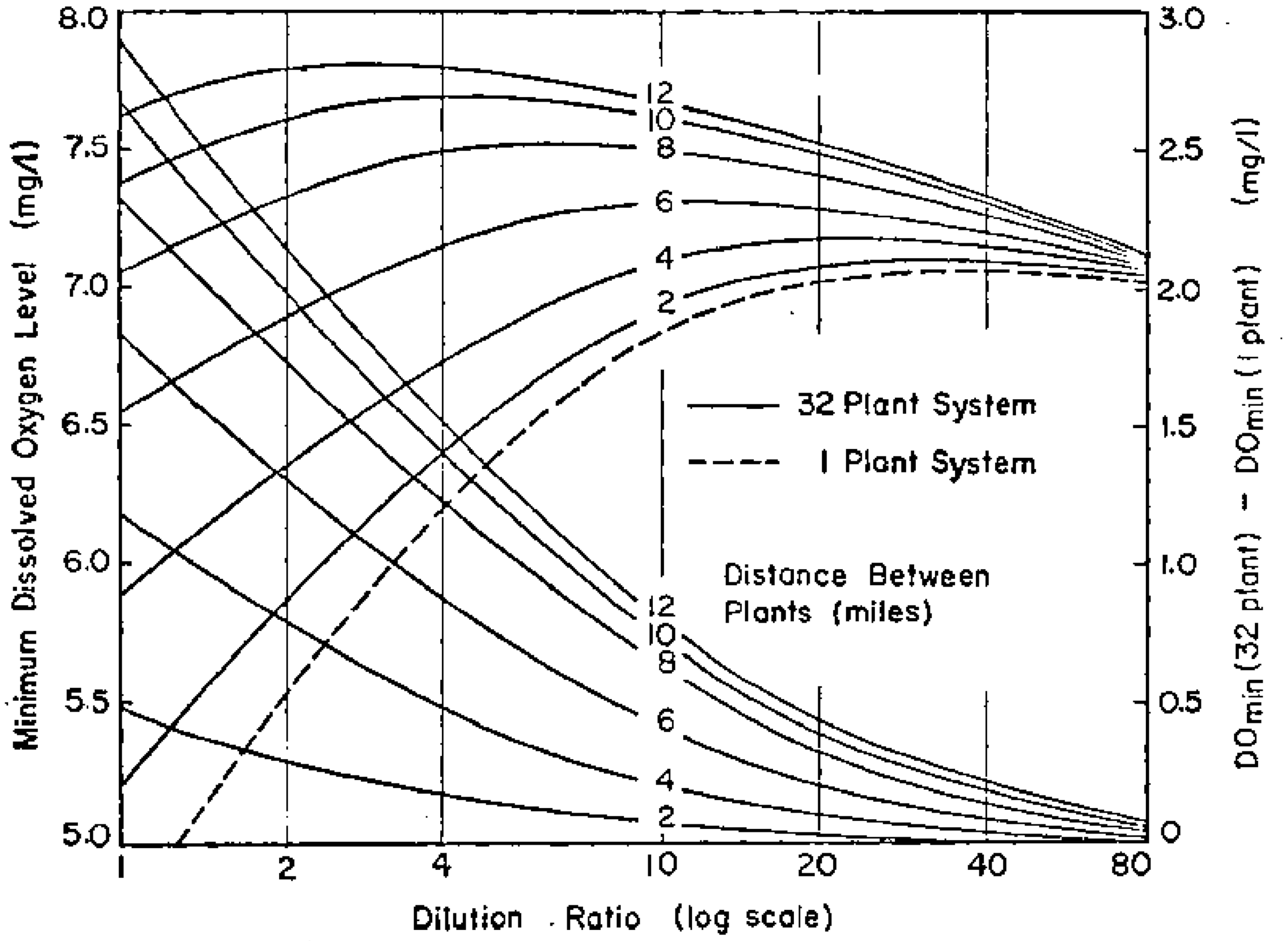

Figure 6. Effect of Dilution Ratio on Water Quality for Various Distances Between Plants

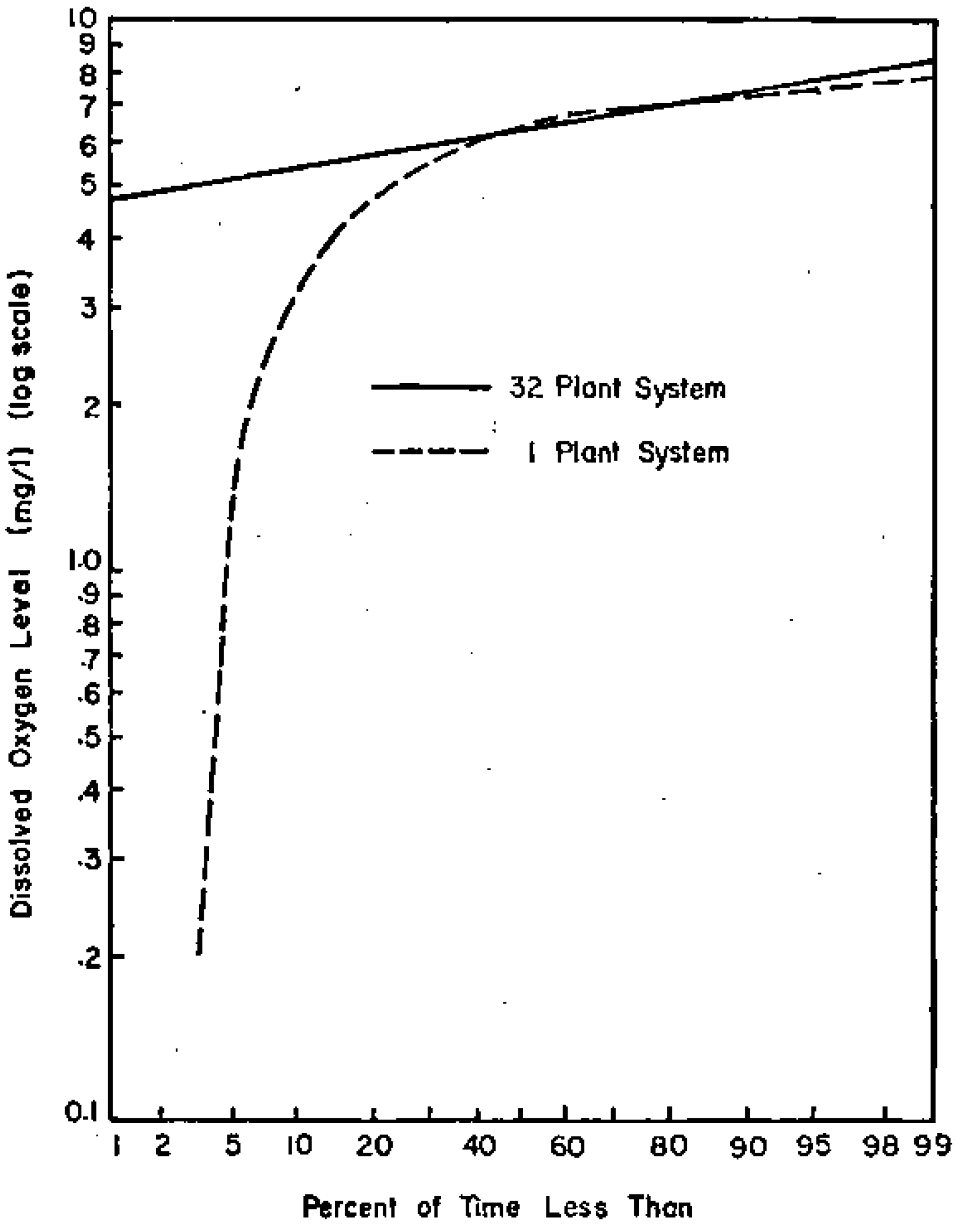

Figure 7. Probability Plots of Water Quality Response

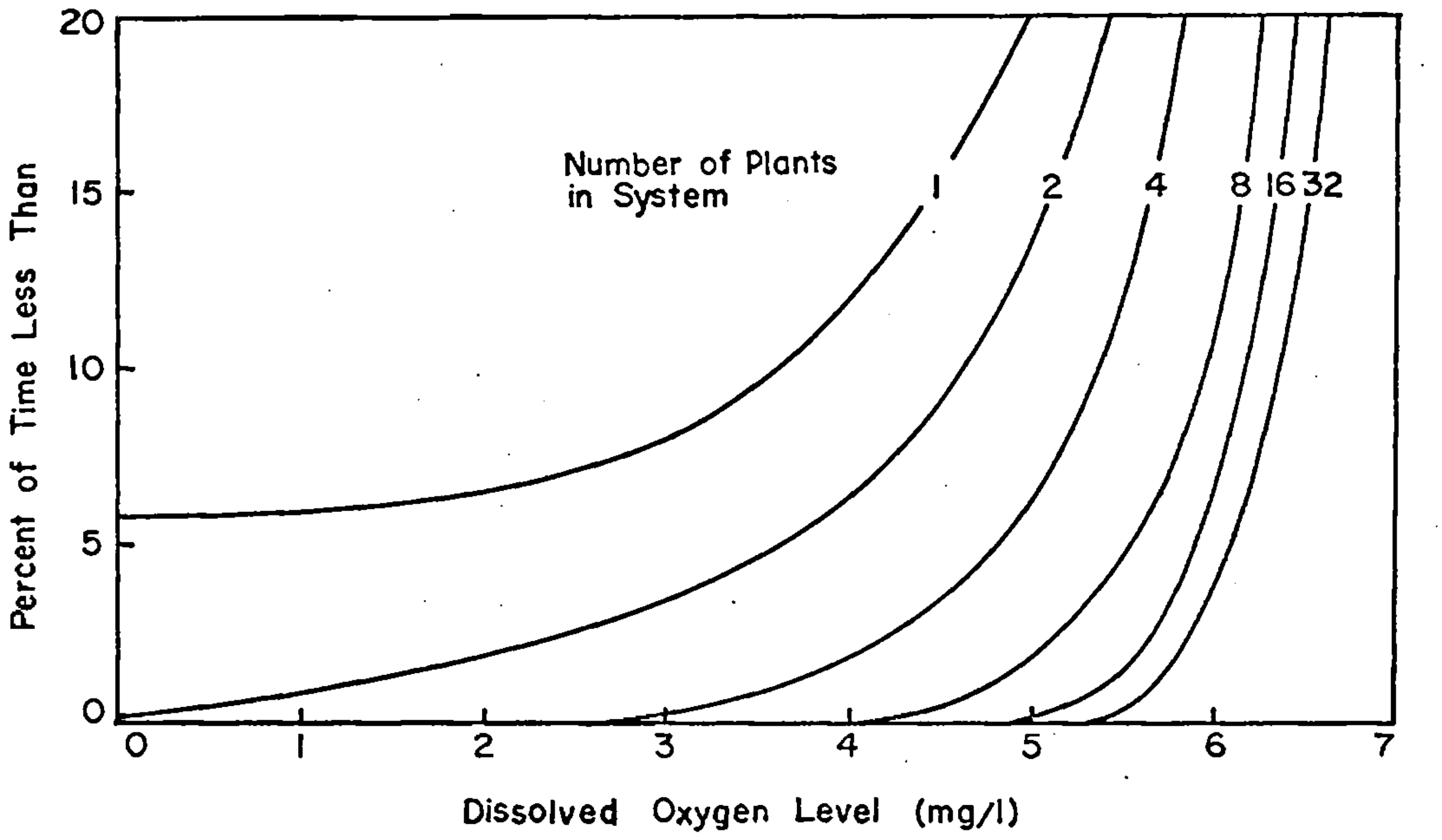

Figure 8. Water Quality Low Frequency Plots for Various Plant Number Systems

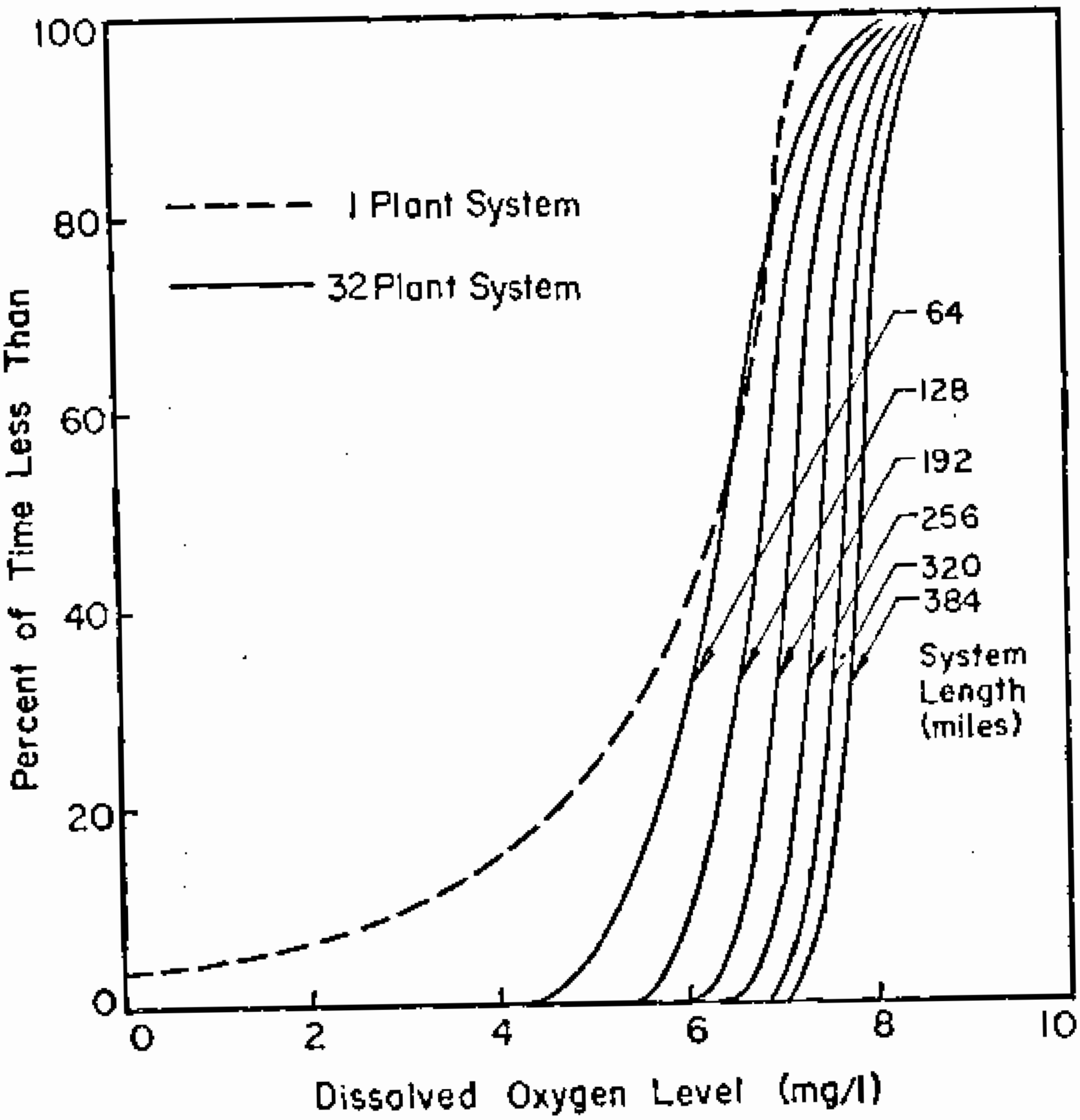

Figure 9. Water Quality Frequency Plots for Various System Lengths

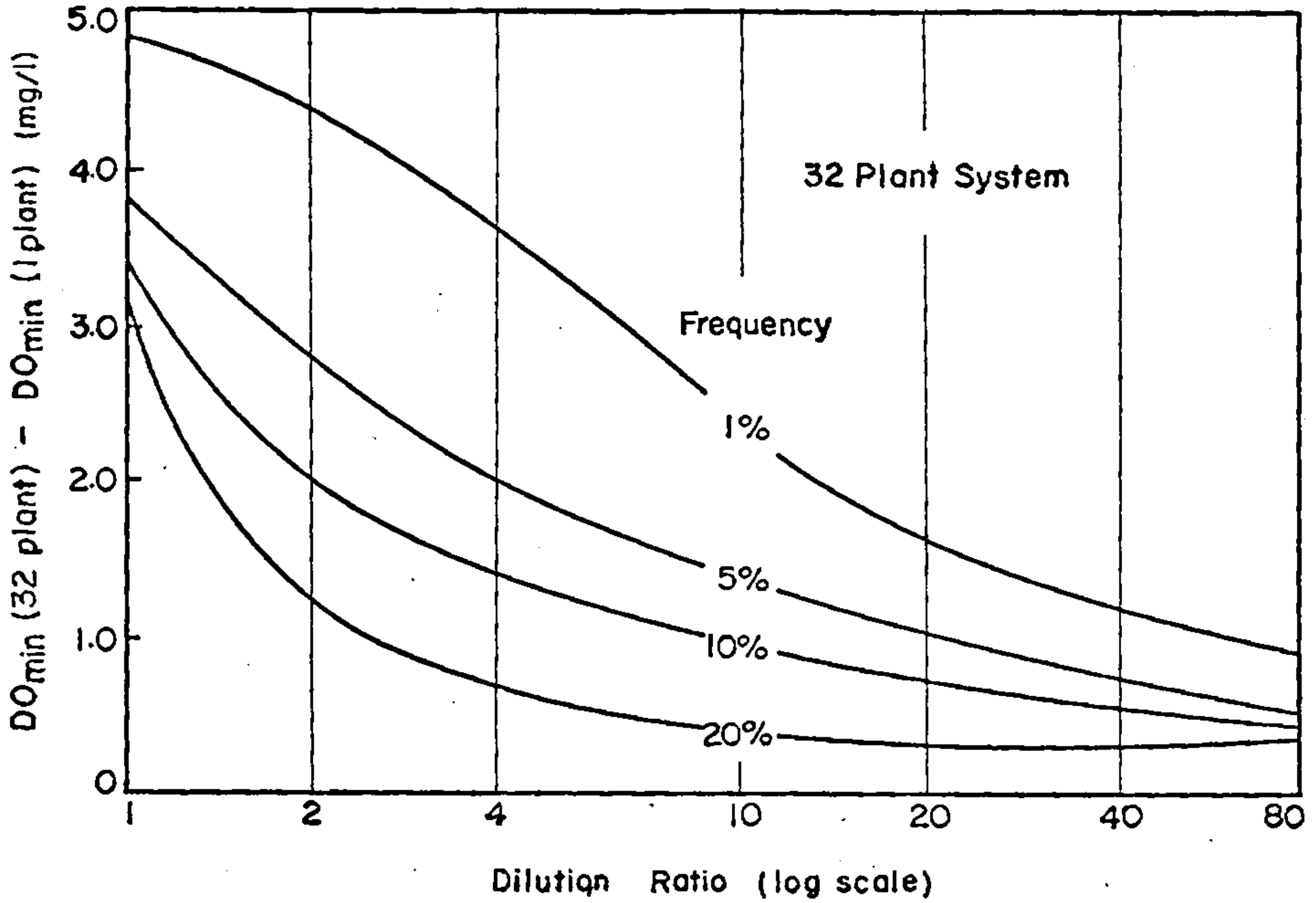

Figure 10. Low Frequency Water Quality Improvement