

# Mantle convection behavior with segregation in the core-mantle boundary

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**Abstract.** Dynamical effects at the core-mantle boundary have so far been modeled in the frame of thermal as well as double-diffusive convection, yet accumulating evidence supports the idea that the mantle material loses part of its heavy component at the CMB due to combined chemical and physical processes. The segregation of the heavy FeO component from the lowermost mantle to the top of the core occurs inside the lower thermal boundary layer and leads to a new kind of boundary layer instability. A mathematical description of mantle convection including segregation of FeO into the core is developed and the relevant model parameters are estimated. The numerical investigation of the flow structure shows the forming and up-welling of thermo-compositionally driven plumes, as well as down-welling of pure thermally driven plumes even for conditions where the simple thermal convection case has a steady behavior. Periods of quasi-stable plume activity interchange with periods of chaotic reorganization of the flow structure connected with strongly time dependent Nusselt.

## Introduction

Theoretical investigation on the solubility of FeO in molten Fe [Ringwood, 1984] as well as laboratory experiments on mantle silicates in contact with liquid iron [Knittle and Jeanloz, 1989], suggest that the core-fluid interacts chemically with the mantle silicates over geologic time [Artyushkov, 1972]. The mantle silicates are depleted in FeO by this process and a light residue consisting of iron-depleted enstatite and stishovite is left, which possibly rises in the form of plumes due to its enhanced buoyancy.

Theoretical investigation of McCammon *et al.* [1983] on the system Fe-FeO-MgO suggest that FeO may gradually ex-solve from (Mg, Fe)O to become a separate phase in CMB. The melting point measurement of FeO from Boehler [1992] suggests that free FeO should be liquid and mobile inside the lower thermal boundary layer (CMB). Because FeO is the dense mantle component and because it is miscible with Fe [Ringwood and Hibberson, 1991], FeO moves down to the core mantle interface and enters the core.

The theory of terrestrial heat flow shows that most of the Earth's heat loss comes from non-radiogenic sources

[Ozburgh and O'Nions, 1987]. To explain this, secular cooling and the release of gravitational potential energy due to growths of the core has been adopted [Vacquier, 1991].

The proposed loss of a heavy mantle component at the core/mantle interface is a density unstable situation leading to a new kind of boundary layer instability, whose influence on convective mantle dynamics is studied numerically.

## Equations

We use a two component, incompressible, infinite Prandtl number model-fluid in a 2D Cartesian geometry for investigating mantle convection behavior with segregation in CMB. The heavy component (described by a volume concentration  $C$ ) represents FeO and other siderophile elements that take part in the segregation process. The bulk fluid represents the residual silicate mantle. The density of the mixture linearly depends upon temperature and composition according to  $\rho_m = \rho_s (1 - \alpha(T - T_0) + \beta C)$ ,  $\alpha$  being the thermal expansivity and  $\beta = \frac{\rho_f - \rho_s}{\rho_s}$  describing density changes due to composition ( $\rho_f$  is density of pure FeO,  $\rho_s$  is the density of pure silicate).

In an effective medium approach, valid on geologic time and length scales, the segregation of FeO and other siderophiles is modeled by a phenomenological transport coefficient, the segregation velocity  $v_s$ , parameterizing a complex physical process. The segregation process can possibly occur within different hydrodynamical modes during the Earth's history [Stevenson, 1991], i.e. the segregation velocity naturally depends upon the temperature, pressure and chemical environment of the core/mantle reflecting for example kinetic limitations of the FeO-MgO ex-solution. For ease of discussion,  $v_s$  is only depth dependent here. The conservation law for iron and the other siderophiles including segregation is thus given by:

$$\frac{\partial \rho_f C}{\partial t} + \left\{ \begin{array}{l} \nabla \cdot (\rho_f C \vec{v}) \quad : \text{above } D'' \\ \nabla \cdot (\rho_f C (\vec{v} + v_s \frac{\vec{z}}{|\vec{z}|})) \quad : \text{inside } D'' \end{array} \right\} = 0 \quad (1)$$

With  $\vec{v}$  convective velocity and  $v_s$  segregation velocity. Because the diffusive transport of FeO is much smaller than the advective one, we have a priori neglected the diffusive transport of the FeO component.

The above mentioned approximations and the model approach for the transport of FeO leads to the following system of non-dimensional equations [Trubitsyn and Kharybin, 1987; Fradkov *et al.*, 1995]

$$\Delta \Delta \psi = R_T \left( \frac{\partial \tilde{T}}{\partial x} - R_p \frac{\partial \tilde{C}}{\partial x} \right) \quad (2)$$

$$\frac{\partial \tilde{T}}{\partial t} + \nabla \cdot \tilde{T} \vec{v} = \Delta \tilde{T}$$

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Paper number 96GL02062  
0094-8534/96/96GL-02062\$05.00

$$\frac{\partial \tilde{C}}{\partial t} + \nabla \cdot \tilde{C}\vec{v} = \begin{cases} 0 & \text{: above D''} \\ -\frac{\partial a \tilde{C}}{\partial z} & \text{: inside D''} \end{cases}$$

$$R_T = \frac{\alpha \Delta T g L^3}{\nu \kappa}, \quad R_p = \frac{\beta C_o}{\alpha \Delta T}, \quad a = \frac{v_s L}{\kappa}$$

where  $\psi$  is the stream function;  $\tilde{T}$  is the dimensionless temperature;  $\tilde{C}$  is the dimensionless volume concentration of FeO;  $R_T$  is the Rayleigh number;  $R_p$  is the density number;  $a$  is the segregation parameter;  $\Delta T$  is the temperature drop;  $g$  is the acceleration of gravity;  $L$  is the mantle depth;  $\kappa$  is the thermal diffusivity;  $\nu$  is the viscosity;  $C_o$  is the initial volume content of FeO.

Because initial conditions are very difficult to define on physical grounds, we used a homogeneous FeO-distribution and a steady thermal convection state from a time dependent run with one vortex of aspect ratio 1:1.5. The time evolution of the system will be studied for much more than 1 tectonic cycle (overtun), so initial conditions do not play any role in the interpretation of results. The numerical procedure was described in detail in [Fradkov et al., 1995].

### Parameter Estimation

**Density number.** For a first rough estimation of the density number  $R_p = \frac{\beta C_o}{\alpha \Delta T}$ , we use:  $\alpha = 1.4 \cdot 10^{-5} \text{ } ^\circ\text{K}^{-1}$  [Jarvis and Peltier, 1985];  $\Delta T \approx 2000 \text{ } ^\circ\text{K}$ ;  $\rho_f = 7.2 \frac{\text{g}}{\text{cm}^3}$  [Ringwood, 1984];  $\rho_s = 4.1 \frac{\text{g}}{\text{cm}^3}$ ;  $C_o$  is given by  $C_o \geq 0.045$ . This gives us the chemical 'expansivity'  $\beta = \frac{\rho_f - \rho_s}{\rho_s} \approx 0.756$  and the density number is given by  $R_p \geq 1.2$ . This estimate shows that the late stage of core formation can be described within the Boussinesq approximation.

**Segregation velocity.** The integral formulation of the conservation law (1) leads to a relation between the segregation velocity  $v_s$ , the averaged concentration of heavy component  $C(t)$  in the mantle, and the velocity of growing core radius  $\dot{R}_c$ . This is the case if the integral formulation is applied to the shrinking mantle when considering the whole Earth as a closed (mass) system (e.g. Earth's radius  $R_E$  is constant).

$$\frac{dM_c}{dt} = 4\pi R_c^2(t) \rho_f C(t) (v_s + \dot{R}_c)$$

$$\leftrightarrow C(t) = \frac{\dot{R}_c}{v_s + \dot{R}_c} \leftrightarrow \frac{dR_c(t)}{dt} = \frac{C(t)}{1 - C(t)} v_s \quad (3)$$

The differential equation governing  $R_c(t)$  is obtained by differentiating the equation of the Earth's mass, which is composed of core mass and mantle mass,

$$M_E = \frac{4}{3}\pi \left( \rho_f R_c^3(t) + (C(t)(\rho_f - \rho_s) + \rho_s)(R_E^3 - R_c^3(t)) \right)$$

and using the relation (3) for  $C(t)$ . The resulting ODE for  $R_c(t)$  is

$$0 = \frac{dR_c^3(t)}{dt} + \frac{d}{dt} \left( \frac{\dot{R}_c}{v_s + \dot{R}_c} (R_E^3 - R_c^3(t)) \right) \quad (4)$$

**Table 1**

Initial Core Radius	Segregation velocity
$3.25 \times 10^6 \text{ m}$	$0.30 \text{ cm/y}$
$3.30 \times 10^6 \text{ m}$	$0.19 \text{ cm/y}$
$3.35 \times 10^6 \text{ m}$	$0.12 \text{ cm/y}$

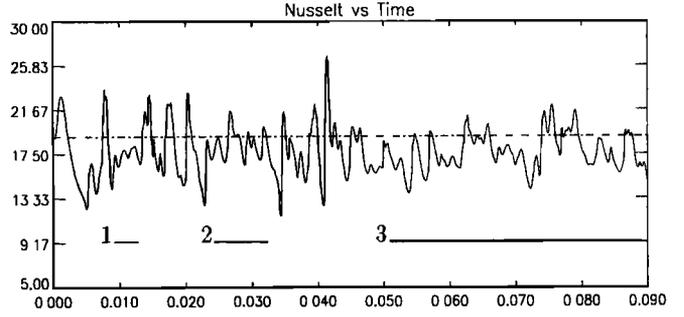


Figure 1. Nusselt vs time for the parameters  $R_T = 10^6$ ,  $R_p = 1$ ,  $a = 20$ . The dashed line shows the pure thermal convection case. Branch 1 & 2 indicate quasi-stable plume activity. Branch 3 shows a quasi-stable flow structure with one thermal plume from the top.

This equation can be solved analytically to give

$$t(R_c) = \frac{R_E^3 - a^3}{v_s} \left\{ \frac{1}{a^2 \sqrt{3}} \arctan \left( \frac{2R_c + a}{a\sqrt{3}} \right) - \frac{1}{6a^2} \ln \left( \frac{(a - R_c)^2}{a^2 + aR_c + R_c^2} \right) \right\} + b \quad (5)$$

where  $a$  and  $b$  are constants of integration. These are determined by the boundary conditions (core radius at the beginning and core radius at a later time). The additional constraint determining the separation velocity is given by the initial or final volume concentration  $C$  of the 'iron' composition.

To get a conservative estimate, we stretch the time interval in which differentiation occurs to 3.0 Gy. The final core radius is  $3.48 \times 10^6 \text{ m}$  and the heavy material able to segregate into the core present in the initial state is  $C(t_i) = 0.045$ . The estimates of  $v_s$  for the initial core radius  $R_i$  are listed in table 1.

### Numerical Results

Figure 1 demonstrates the change in the time behavior of the Nusselt going from pure thermal convection (dashed line) to thermal convection with segregation (solid line), showing that the time behavior becomes irregular in the case of segregation. The time averaged Nusselt in figure 1 is smaller than for the pure thermal convection case, which is contrary to what was expected by Davies [1980] considering the thermal regime of the Earth during segregation. In figure 2 we present the corresponding temperature, concentration and stream function fields for selected time points. The first three panels of figure 2 show the development of a density-unstable temperature and concentration distribution near the lower boundary after the beginning of segregation. At the upper boundary, the FeO composition — carried under the top of the cell — has a density-stable distribution (positive buoyancy) and hinders the convection process (fig. 2.1; first minimum in  $Nu(t)$ ). The material thus stays longer at the horizontal boundaries, leading to over-cooling at the top (temperature unstable; concentration stable) and over-heating plus differentiation (temperature and concentration unstable) at the bottom of the model domain, i.e. the segregation stabilizes the uppermost part of the cell and destabilizes the lower thermal boundary

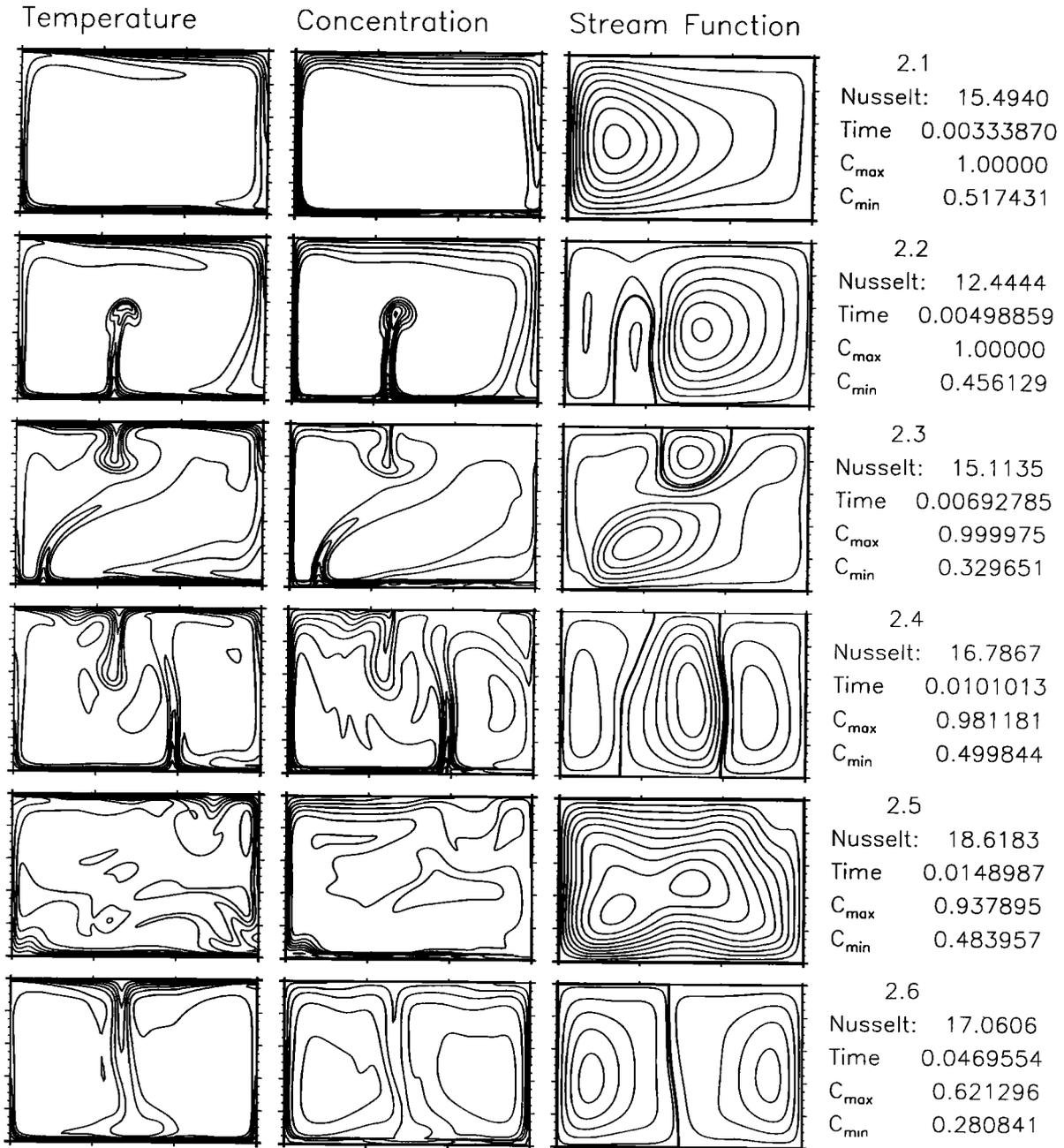


Figure 2. Contour plots of temperature, concentration and stream function in the different flow stages occurring during time evolution. Same parameters as in figure 1. Isocontours of concentration are equally spaced between  $C_{min}$  and  $C_{max}$ . Nullisoline of stream-function is drawn thick. Panel 2.4 shows typical flow structure of quasi-steady plume activity (Branch 1 & 2 fig.1). Panel 2.5 shows the transition state (between Branch 1 & 2 fig.1). Panel 2.6 show final flow stage which evolve to pure thermal convection case due to growths of one whirl.

layer. This density-instabilities are eased by the ascending (fig. 2.2, second maximum in  $Nu(t)$ ) and descending (fig. 2.3) of the plumes.

The above analysis of the temperature and concentration fields shows, that we can distinguish two kinds of plumes by their different causes: i) Up-welling plumes are driven by temperature and concentration (called thermo-compositional plumes (tcP)), ii) Down-welling plumes are driven only by temperature (called thermal plumes (tP)).

The analysis of the flow structure reveals that the system's evolution can be grouped into different kinds of stages, showed in the  $Nu(t)$  diagram.

The first stage (fig. 2.4; fig. 1 branches 1 & 2) has a relatively steady flow structure on whole cell scale with a stably localized tcP plus tP and small amplitude oscillations of the  $Nu(t)$ . The time duration is  $\Delta t \approx 4.9 \cdot 10^{-3}$  for branch 1 and  $\Delta t \approx 11.2 \cdot 10^{-3}$  for branch 2.

The second stage, the so called transition state, (fig. 2.5; fig. 1, between branches 1 & 2 and between

branches 2 & 3) has greater amplitude oscillations in the Nusselt as compared to branch 1 or 2. The flow structure, extending over the whole cell, leads to the destruction of the plume structure.

Finally, we make some general remarks on the role of the density number and segregation parameter.

The density number has an important influence on system behavior in two ways: i) The density number triggers the birth of tcP's and stabilizes the localization of the tcP's, because  $R_p$  together with the segregation parameter  $a$  defines the magnitude of the 'chemical-buoyancy production' at the lower boundary. ii) The ascend of tcP's is facilitated by the additional chemical buoyancy of the depleted material. This gives the tcP's a greater power to punch through the whole cell. The ability of a tcP to penetrate into the upper mantle depends on the density change in 670 km [Kellogg, 1991] and is not estimated quantitatively here. Contrary to the findings of Liu and Chase [1991], the chemical buoyancy persists as an important parameter for the ascend of the up-welling plume even for the 'infinite Lewis number' presented here because the plume source has a finite thickness in our case so that the thickness of the chemical plume stays finite in the limit  $Le \rightarrow \infty$ .

An increase of the segregation parameter shortens the time duration where the segregation process influences convective behavior of the mantle. This duration is not a simple linear function of the segregation parameter due to the nonlinearity of the problem. The effectiveness of mass removal of 'FeO' depends also on the 'refreshing rate' of the lower layer where segregation occurs. This is a function of the overlying flow structure.

## Summary and Conclusions

The main features of convection with segregation in the lower boundary are:

- Segregation of a heavy component in D" leads to a new kind of boundary layer instability. For that reason, the segregation process can strongly influence the convective behavior in the whole cell even if the additional mass transport due to segregation is small.
- Segregation destabilizes the lower thermal boundary layer and stabilizes the uppermost part of the cell.
- The flow structure shows two different kinds of plumes, distinguished by their various causes:
  - plumes from the top are due to over-cooling (i.e. pure thermal origin)
  - plumes from the bottom resulting from both over-heating and segregation (i.e. thermal & compositional origin).
- The temporary behavior of the flow structure and the Nusselt can pass through different stages during the time evolution of the system:
  - relatively steady small scale convection with small amplitude oscillation of the Nusselt; this flow structure has stably localized thermo-compositional plumes
  - strong time-dependent convection with one whirl and great amplitude oscillation in the Nusselt.

The dynamics of mantle convection with segregation shows new traits for the Rayleigh number and geometry

under consideration. We can conclude that this process can have structure dominating influence on mantle dynamics during Earth's history.

**Acknowledgments.** G.N. and A.S.F. would like to thank J. Arkani-Hamed (McGill-University Canada) for helpful discussions about geophysical applications of results. We would like to thank two anonymous reviewers and Mian Liu (University of Missouri) for their constructive reviews. We are grateful to D.J. Stevenson (Caltech) whose criticism removed an error in the early version of the paper. A.S.F. thanks the DFG (Germany) and NCERC (Canada) for financial support during the progression of this work.

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(Received October 12, 1995; revised March 4, 1996; accepted May 16, 1996.)