First Order Phase Transition in Intermediate-Energy Heavy Ion Collisions

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(Received 5 May 1997)

We model the disassembly of an excited nuclear system formed as a result of a heavy ion collision. We find that, as the beam energy in central collisions in varied, the dissociating system crosses a liquid-gas coexistence curve, resulting in a first-order phase transition. Accessible experimental signatures are identified: a peak in the specific heat, a power-law yield for composites, and a maximum in the second moment of the yield distribution. [S0031-9007(97)05236-8]

PACS numbers: 25.70.Pq, 21.65.+f, 24.10.Pa, 64.60.My

Nuclear matter is a fictitious arbitrarily large $N = Z$ system in which the Coulomb interaction is switched off. Mean-field theory for nuclear matter has been applied many times, and it is well known that such a system shows a van-der-Waals-type liquid-gas phase transition. It was suggested in the early 1980s [1,2] that in heavy ion collisions at intermediate energies one might be able to probe this liquid-gas phase transition region. In heavy ion collisions, matter would be heated as well as compressed. This compressed blob would then expand passing through the liquid-gas coexistence phase. One might be able to extract information about this region from selected experimental data. Earlier, the Purdue group had conjectured that the breakup of large nuclei by energetic protons would show signatures of critical phenomena [2].

There are several complications which make the study of phase transitions in nuclei difficult. The systems are small, thus singularities get replaced by broad peaks. The collision is over quickly. The existence of thermal equilibrium has sometimes been questioned and replaced by quite complicated transport equation approaches. Often one has been content to do calculations where the objective is to fit the experimental data. Once this is achieved the question of a possible liquid-gas phase transition is not addressed. There are models where such questions are irrelevant, or at least very indirect, such as various models based on sequential decays. The literature on calculations for fragment yields in intermediate energy heavy ion collisions is huge. We will not attempt to mention all approaches.

In this paper we will focus on the liquid-gas phase transition using a lattice gas model [3]. Previously we have used the model to fit data on central collisions [3], on peripheral collisions [4], and for central Au on Au collisions [5]. The model gives a fair description of data in those instances—does it say anything definite about the experimental signature of liquid-gas phase transitions?

In the lattice gas model we place n nucleons in N cubes, where n is the number of nucleons in the disassembling system, and $N/n = \rho_0/\rho_f$, where ρ_0 is the normal nuclear density, and the disassembly is to be calculated at ρ_f, the "freeze-out" density beyond which nucleons are too far apart to interact. An attractive nearest-neighbor interaction is assumed. We place the n nucleons in N cubes by Monte Carlo sampling using the Metropolis algorithm. Once the nucleons have been placed we also ascribe to each of them a momentum. The momenta are generated by a Monte Carlo sampling of a Maxwell-Boltzmann distribution. Various observables can be calculated in a straightforward fashion. Experiments usually measure the distributions of cluster sizes, i.e., the yield $Y(Z)$, which is a function of the number of protons in the composite Z. In our model, two neighboring nucleons are considered to be part of the same cluster if their relative kinetic energy is insufficient to overcome the attractive bond: $p_r^2/2\mu + \epsilon < 0$. This can be immediately turned into a temperature and ϵ dependent bonding probability [3,5] much like Coniglio-Klein's method [6]. The prescription allows us to calculate the distribution of clusters.

The equation of state in the lattice gas model has been well studied in condensed matter physics. The grand canonical ensemble for the lattice gas corresponds to a three-dimensional Ising model in the presence of a magnetic field. It is thereby possible to translate many well known results to the present situation. Figure 1 depicts the phase diagram of the lattice-gas model in the thermodynamic limit, which is adapted from Ref. [7]. In drawing the coexistence curve, *D-C-E*, the series expansion given in Ref. [8] was used. The point "*C*" is the thermal critical point which occurs at $T \approx 1.1275|\epsilon|$ and density $\rho_f/\rho_0 \equiv 1/2$. The coexistence curve is *D-C-E*, and *C-B* is the line along which percolation sets in. The line *C-B* is only slightly different in our and Coniglio-Klein's prescription.

In an experimental situation involving the collision of heavy ions, one gates on central collisions and varies the beam energy. At low beam energy, the temperature at freeze-out will be below the coexistence curve. As the beam energy increases, this point will cross the coexistence curve (indicated by an arrow in Fig. 1). Note that there is no reason for the system, under heating, to "tune itself" to the second-order point at density $\rho = 1/2$, since there is no symmetry between the clustered and unclustered phases involved in the heavy-ion collision.

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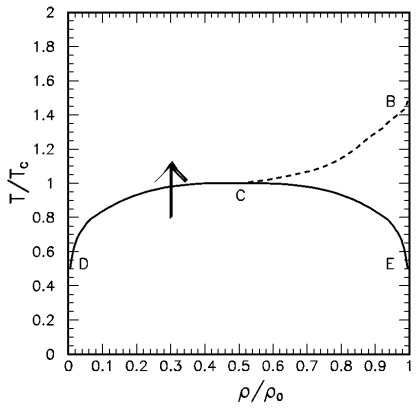

FIG. 1. Phase diagram of three-dimensional lattice gas model. The line *D-C-E* is the coexistence curve. *C-B* is the percolation line. The arrow demonstrates the crossing of the coexistence line as the beam energy for central collisions increases. In the Ising model *D-C-E* is the line of spontaneous magnetization. With the usual convention the point 0 on the abscissa corresponds to magnetization $-M$ and the point 1 to M.

Indeed, in statistical models of disassembly of which we know [9,10], the freeze-out density is less than $0.5\rho_0$. In our model, we find that the data are best fitted by a ρ_f between $0.3\rho_0$ and $0.4\rho_0$. There is good support from experiment that the value of ρ_f is significantly below $0.5\rho_0$ [11]. Therefore the phase transition is *first order*.

As one crosses the coexistence curve, experiments would see various signatures of the first-order transition. In the thermodynamic limit these are well defined. For example, since first derivatives of the free energy are discontinuous in the thermodynamic limit, there are delta-function peaks in second derivatives [12,13], e.g., in the heat capacity at constant pressure C_p, and in the isothermal compressibility κ_T. There is also a clear signature of the transition in the density-density fluctuation correlation function $\Gamma(r)$ [whose integral gives the compressibility by a thermodynamic sum rule [14] $\int d\vec{r}\ \Gamma(r) \propto \kappa_T$]. This correlation function decays exponentially above and below the transition point. At the transition point itself, however, $\Gamma(r)$ is flat [14,15], as is evident from, for example, the sum rule.

This singular behavior is smeared out in finite-size systems [15]. Instead of delta-function singularities, κ_T and C_p have broad Gaussian peaks at the first-order transition, whose height is proportional to the system's volume $\sim L^3$. Indeed, these features are somewhat analogous to those at a continuous second-order transition: In the thermodynamic limit at a continuous transition, C_p and κ_T have power-law singularities, but in a finite-size system, these singularities are replaced by bumps of height $\sim L^{\gamma/\nu}$, where γ and ν are critical exponents. This similarity implies why it is often difficult to distinguish the two types of transitions in a finite-size system [14,15].

While C_p and κ_T are both infinite at a first-order liquid-gas transition, the heat capacity at constant volume C_v has only a step discontinuity in the thermodynamic limit, giving a bump in a finite-size system. The extent of the discontinuity is given in terms of thermodynamic relations in Ref. [16]. This is in contrast to the more pronounced behavior at a second-order transition where $C_v \sim L^{\alpha/\nu}$, and α is a critical exponent.

In Fig. 2 we show results of our numerical calculation. We fix N at 7^3 and vary n to obtain a variable freeze-out density. Half of the nucleons are labeled as protons. Although this system is small, there is a well-defined bump in C_v signifying the transition. An arrow marks the nearby point at which the transition occurs in the thermodynamic limit.

In nuclear experiments locating the peak in C_v as the beam energy increases is difficult, although in recent years pioneering progress has been made in the measurement of temperature [17,18]. However, other characteristics of the transition appear which are more readily measurable. In particular, the yield $Y(Z)$ is readily measurable, which gives the distribution of clusters of charge Z, $n(Z)$. The probability of clusters of a given size is related (through its second moment), to the density correlation function [7,19,20]: Roughly, if the clusters are not fractals, $Z^2 n(Z) \sim \Gamma(r)$, in the disordered phase, where r is the diameter of a cluster of charge Z. At the transition, a droplet of thermodynamic size spans the system. Hence the yield includes an infinite cluster, or

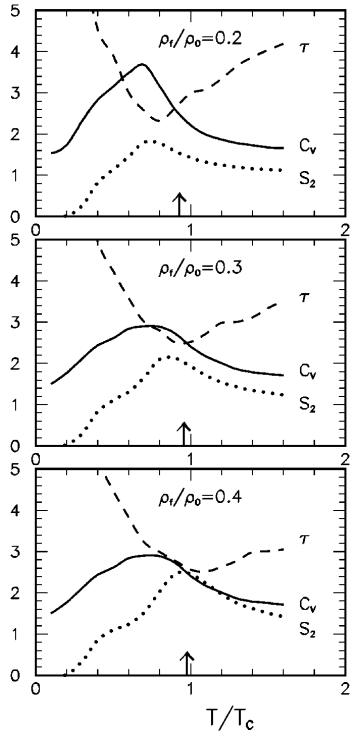

FIG. 2. Curves for τ, C_v, and S_2 at $\rho_f/\rho_0 = 0.2$, 0.3, and 0.4. At each point 1000 events were taken. C_v is in units of k_B; the kinetic energy contributes 1.5 to it at all temperatures; $T_c \approx 1.1275|\epsilon|$.

in percolation language, a cluster which spans the system. The remainder of the distribution describes fluctuations in that phase, giving the density correlation function, whose integral is the compressibility. The delta-function peak in the compressibility implies that the correlation function, and hence the yield, is broad. Following standard practice [21], the yield is fit to a power law form $Y(Z) \propto 1/Z^{\tau}$, giving an effective exponent τ, even when the distribution has deviated from a power law. In fact, at a *continuous* phase transition, $\tau = 2 + \beta/(\beta + \gamma)$, where β is a critical exponent. Here, however, where the transition is first order, the correlation function decays exponentially in either bulk phase (i.e., the effective $\tau \rightarrow \infty$ above or below the transition), while, since the correlation function is flat at the transition itself, and is related to the second moment of the cluster distribution, $\tau = 2$ there, for an infinite system. Of course, in a finite-size system, there will be an effective τ, which is neither two nor infinity, so that the transition will look somewhat analogous to a continuous transition.

It is also useful to consider the second moment of the cluster distribution function $S_2 = \sum_A' A^2 n(A)/n$, where $n(A)$ is the number of nucleons with mass number A, and n is the total number of nucleons. The primed sum excludes the largest cluster, and so we expect it [20] to be proportional to the compressibility κ_T. The usefulness of the second moment was emphasized by Campi [22].

For experimental determination of τ in one case, see [23]. Campi has, for example, used experimental data for the extraction of the second moment [24,25].

Figure 2 shows that these quantities do indeed provide clear experimental signatures of the first-order transition in a finite-size system. The crossing of the coexistence line is evident in the minimum value of the effective τ, which is close to 2, and the prominent maximum in S_2 and C_v. All of these occur at approximately the same point, close to the transition in the thermodynamic limit, indicated by an arrow.

For our analysis, it is important that the freeze-out density is on the low density side of the coexistence curve. Beyond the critical point, "*C*", the bump in C_v continues on the line *C-E* whereas the minimum of τ will follow line *C-B*, which corresponds to a change in short-range order. We should also note that very large Coulomb forces will alter this picture. We now describe briefly those effects, following Ref. [5]. In our lattice gas model one generates an event in which the nucleons have been placed with appropriate momenta. Using our prescription we can immediately obtain the cluster distribution. Starting from this configuration we can propagate the system by molecular dynamics with a suitably chosen short range nucleon-nucleon interaction (to correspond to the nearest-neighbor interaction) with or without the Coulomb interaction. By propagating molecular dynamics for a considerable time, one can unambiguously obtain the cluster distribution and determine the effect of the Coulomb interaction. Reference [5] concluded that without the Coulomb interaction there is little difference between lattice gas results and molecular dynamics results, but that Coulomb interactions can be important for large mass numbers. For mass number $A = 85$, the Coulomb interaction is a small perturbation, while at $A = 394$ it is so strong as to totally alter the picture. In the latter case there is no minimum in τ; at a temperature of 1 MeV, the value of τ is slightly higher than 1 and it continues to rise monotonically with temperature. We have now studied this in much greater detail and find that a minimum in the value of τ continues to be obtained for mass number as large as $A = 200$. Details of this as well as further applications of the lattice gas model will be published in a longer paper.

To conclude, we have modeled the disassembly of nuclear matter following a heavy-ion collision. We find that the transition is first order, with the standard signatures of such a phase transition.

We thank Charles Gale, Martin Zuckermann, and Hong Guo for useful discussions. This work is supported in part by the Natural Sciences and Engineering Research Council of Canada and by *le Fonds pour la Formation de Chercheurs et l'Aide à la Recherche du Québec.*

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