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CONTRAFLOW CONTRAROTATING TURBOCOMPRESSORS[1]

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Abstract

The performance of a turbocompressor comprised of a large number of mechanically independent compressor and turbine stages is considered. Examination of the equilibrium conditions of any stage shows its performance, for a given design, to be dependent on four variables and hence the performance of the complete unit is also dependent on the same four variables. Exact arithmetical calculation would be possible, but very tedious. In order to render computation easier, the conception of reflected conditions is introduced. Physically this implies independence of each stage, in some respects, from its neighbors. The assumption is likely to be in error least for intermediate stages, and most for the first and last stages. Methods of expressing the aerodynamic performance of the blades of the compressor and turbine stages are introduced to render calculation easier. Some calculated performance figures are compared with experimental values and show by their reasonable agreement that the assumptions used are valid. The methods can be applied to other types of complex turbocompressor units by suitable modification of procedure and details.

Introduction

In 1929, A. A. Griffith put forward proposals for the use of the internal combustion turbine for aircraft propulsion (1). He showed that with the then existing knowledge such an engine was a practical proposition, and he also investigated mathematically the best arrangement of engine. A gas turbine engine consists essentially of an air compressor, combustion chamber, turbine to drive the compressor, and turbine to supply the useful output. Note that the two turbines may in fact form a single unit, and that in the common jet propulsion engine the second turbine is replaced by a propulsive jet. Griffith proved that in order to obtain a wide range of efficient operation it was desirable to arrange the compressor, and the turbine driving it, in a series of mechanically independent stages, each small compressor stage being driven by its own small turbine stage. The use of 'small' here denotes a stage over which the pressure ratio is small. The mechanical arrangement of such a unit is shown diagramatically in Fig. 1.

In this paper we discuss some aspects of the performance of such a unit, known as a contraflow compressor. The problem may be put as follows. Suppose we have such a unit of given geometrical design. We can express its

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over-all performance in various terms, for example, the mass flow, and initial density of the air entering the compressor, and its compression ratio and efficiency, and the four corresponding factors for the turbine. We shall show that in fact there are but four independent variables, and they may be conveniently

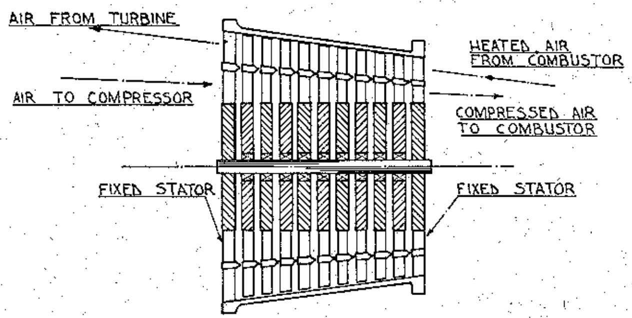

FIG. 1. *Diagram of a contraflow turbocompressor.*

taken as the mass flow and initial density of the air at the compressor inlet and at the turbine inlet. The problem then is to calculate from these four independent variables the values of the other four variables and also, the working conditions of any of the, possibly many, compressor or turbine stages.

Nomenclature

M — mass flow, pounds per second,
A — annulus area, square feet,
P — total pressure, poundals per square foot,
p — static pressure, poundals " " ",
T — total temperature, °K.,
t — static temperature, °K.,
ρ — density, pounds per cubic foot,
u — peripheral velocity of blade, feet per second,
v — gas velocity,
v_a — gas axial velocity,
v_w — gas whirl velocity,
ϕ — u/v_a,
r — mean blade radius,
α — angle between vector representing a gas velocity relative to a blade and the axial direction,
β — angle between vector representing a gas velocity relative to the frame of the machine and the axial direction,
Suffix 1 indicates condition at entry to a stage,
2 indicates condition at exit from a stage,
Prefix c indicates a compressor stage,
t indicates a turbine stage,
n denotes conditions at stage number n.

The stages are numbered from the H.P. end, i.e., the entry to the turbine and the exit from the compressor.

E.g., ${}_{nt}X_2$ indicates the value of quality X at exit from the n^{th} turbine stage. A bar over a quality denotes the value of the ratio.

E.g., $\overline{X} = \dfrac{{}_cX}{{}_tX}$.

General Theory

We start by considering the most general case. The assumptions we make are, that at any stage, we may typify air conditions over the annulus by referring to conditions at the mean radius, and that in any stage there is negligible

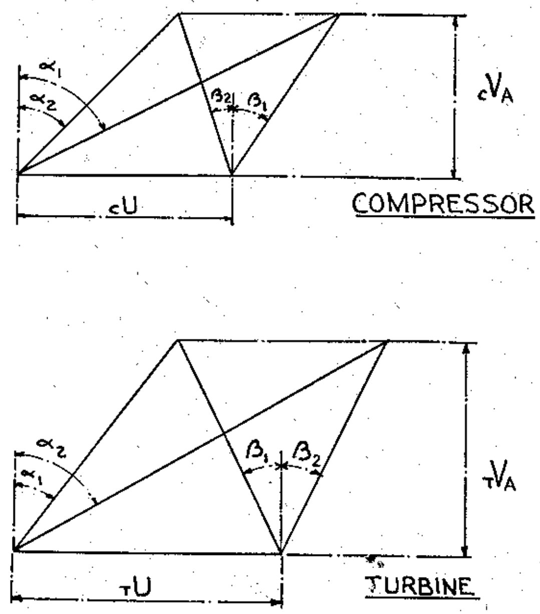

FIG. 2. *Contraflow compressor—typical velocity vector triangles.*

change in the axial velocity. This does not preclude a change over the whole compressor or turbine, but merely that such changes may be represented by small discontinuities between stages.

In Fig. 2 we show diagrams representing the velocity vectors for any compressor stage and for any turbine stage. Then in any compressor stage n, the work done per pound per second of air flowing is, by Euler's momentum theory

$$ {}_{nc}u\cdot{}_{nc}\Delta v_\omega = {}_{nc}u\cdot{}_{nc}v_a\,[\tan_{nc}\alpha_1 - \tan_{nc}\alpha_2], $$

where Δv_ω is the change of whirl velocity.

But $$\tan_{nc}\alpha_1 = \frac{{}_{nc}u}{{}_{nc}v_a} + \tan_{nc}\beta_1 = {}_{nc}\phi + \tan_{nc}\beta_1,$$

and $$\tan_{nc}\beta_1 = \tan_{(n+1)c}\beta_2 = {}_{(n+1)c}\phi - \tan_{(n+1)c}\alpha_2.$$

So the work done per pound per second =

$$ {}_{nc}v^2_a \cdot {}_{nc}\phi\,[{}_{nc}\phi + {}_{(n+1)c}\phi - \tan_{nc}\alpha_2 - \tan_{(n+1)c}\alpha_2]. \qquad (1)$$

Similarly in any turbine stage n, the work done per pound of gas per second is

$$ {}_{nt}v_a^2 \cdot {}_{nt}\phi[\tan_{nt}\alpha_2 - \tan_{nt}\alpha_1]\,.$$

But

$$\tan_{nt}\alpha_1 = {}_{nt}\phi - \tan_{nt}\beta_1$$

and

$$\tan_{nt}\beta_1 = \tan_{(n-1)t}\beta_2 = \tan_{(n-1)t}\alpha_2 - {}_{(n-1)t}\phi.$$

So the work done per pound-second =

$$ {}_{nt}v^2_a \cdot {}_{nt}\phi[\tan_{(n-1)t}\alpha_2 + \tan_{nt}\alpha_2 - {}_{(n-1)t}\phi - {}_{nt}\phi]\,. \qquad (2)$$

For the rotor to be in equilibrium, the total work done by the turbine stage must equal that absorbed by the compressor, plus any frictional or windage loss. We may write the equilibrium condition as

$$ {}_cM \cdot {}_{nc}v^2_a \cdot {}_{nc}\phi[{}_{nc}\phi + {}_{(n+1)c}\phi - \tan_{nc}\alpha_2 - \tan_{(n+1)c}\alpha_2] = $$
$$ k \cdot {}_tM \cdot {}_{nt}v^2_a \cdot {}_{nt}\phi[\tan_{(n-1)t}\alpha_2 + \tan_{nt}\alpha_2 - {}_{(n-1)t}\phi - {}_{nt}\phi]. \qquad (3)$$

Where ${}_cM$ and ${}_tM$ are the compressor and turbine mass flows and k is a factor nearly equal to 1, representing friction and windage losses.

Rearranging Equation (3) we have

$$\frac{\overline{M} \cdot {}_n\bar{v}_a^2 \cdot {}_n\bar{\phi}}{k} \cdot [{}_{nc}\phi + {}_{(n+1)c}\phi - \tan_{nc}\alpha_2 - \tan_{(n+1)c}\alpha_2] = $$
$$[\tan_{(n-1)t}\alpha_2 + \tan_{nt}\alpha_2 - {}_{(n-1)t}\phi - {}_{nt}\phi]$$

or

$$ {}_{(n-1)c}\phi\left[\frac{1}{{}_{(n-1)}\bar{\phi}}\right] + {}_{nc}\phi\left[\frac{\overline{M} \cdot {}_n\bar{v}_a^2 \cdot {}_n\bar{\phi}}{k} + \frac{1}{{}_n\bar{\phi}}\right] + {}_{(n+1)c}\phi\left[\frac{\overline{M} \cdot {}_n\bar{v}_a^2 \cdot {}_n\bar{\phi}}{k}\right] = $$
$$[\tan_{nc}\alpha_2 + \tan_{(n+1)c}\alpha_2] \cdot \frac{\overline{M} \cdot {}_n\bar{v}_a^2 \cdot {}_n\bar{\phi}}{k} + [\tan_{(n-1)t}\alpha_2 + \tan_{nt}\alpha_2]\,. \qquad (4)$$

This equation is the general equation relating the speed of any rotor to the speeds of rotors on either side, and can be used for any rotor. It is a second order difference equation and in its general solution has two arbitrary constants. It is evident that these two constants are related to the direction of the air before entering the compressor and the turbine, i.e., the effect of any stationary guide vanes. To solve Equation (4) as it stands, is very tedious, and for the purposes of this analysis, we can make certain simplifying assumptions, the validity of which will be discussed later.

Let us suppose

${}_n\bar{v}_a = \bar{v}_a$	for all n,
${}_n\bar{\phi} = \bar{\phi}$	for all n,
${}_{nc}\alpha_2 = c$	for $1 \leq n \leq N$,
${}_{nt}\alpha_2 = t$	for $1 \leq n \leq N$.

Note that since $\bar{\phi} = \bar{u}/\bar{v}_a = \bar{r}/\bar{v}_a$ then $\bar{v}_a^2\bar{\phi} = \bar{r}\,\bar{v}_a$.

Equation (4) then becomes

$$ {}_{(n-1)c}\phi \cdot \frac{\bar{v}_a}{\bar{r}} + {}_{nc}\phi\left[\frac{\overline{M}\,\bar{v}_a\bar{r}}{k} + \frac{\bar{v}_a}{\bar{r}}\right] + {}_{(n+1)c}\phi\left[\frac{\overline{M}\,\bar{v}_a\bar{r}}{k}\right] = \frac{\overline{M}\,\bar{v}_a\bar{r}}{k} \times 2c + 2t.$$

For further convenience put $\dfrac{\overline{M}\,\bar{v}_a\bar{r}}{k} = k_1, \quad \dfrac{\bar{v}_a}{\bar{r}} = k_2$

then

$$ {}_{(n-1)c}\phi \cdot k_2 + {}_{nc}\phi[k_1 + k_2] + {}_{(n+1)c}\phi \cdot k_1 = k_1 \cdot 2c + 2t. \qquad (5)$$

Put

$$ {}_{nc}\phi = Ax^n + \frac{k_1 \cdot 2c + 2t}{2[k_1 + k_2]}\,.$$

Then substituting in Equation (5) we have

$$Ax^{n-1}[k_2 + (k_1 + k_2)x + k_1x^2] = 0.$$

So either $Ax^{n-1} = 0$ which means that ${}_{nc}\phi$ is constant for all n, irrespective of any boundary conditions, which is obviously absurd, or else

$$k_1x^2 + (k_1 + k_2)x + k_2 = 0$$
$$x^2 + (1 + k_2/k_1)x + k_2/k_1 = 0.$$

The roots of this equation are $x_1 = -1,\ x_2 = -\dfrac{k_2}{k_1}$.

So the general solution of Equation (5) becomes

$$ {}_{nc}\phi = A[-k_2/k_1]^n + B[-1]^n + \frac{k_1 \cdot 2c + 2t}{2(k_1 + k_2)}\,. \qquad (6)$$

It remains to determine A and B. We do this by considering rotor $n = 1$, and rotor $n = N$. At $n = 1$, i.e., the first turbine stage, the direction of the air, relative to the frame of the machine, is given by $\tan^{-1}\beta_t$, say.

Then Equation (5) becomes

$$ {}_{1c}\phi[k_1 + k_2] + {}_{2c}\phi \cdot k_1 = k_1 \cdot 2c + t + \beta_t. \qquad (7)$$

At $n = N$, if the direction of the air, entering the compressor, relative to the frame of the machine is given by $\tan^{-1}\beta_c$, then we have

$$ {}_{(n-1)c}\phi \cdot k_2 + {}_{nc}\phi(k_1 + k_2) = (c - \beta_c)k_1 + 2t. \qquad (8)$$

Substitution of Equation (6) in Equations (7) and (8) yields

$$(k_1 + k_2)\left[-\frac{k_2}{k_1} \cdot A - B + \frac{k_1 \cdot 2c + 2t}{2(k_1 + k_2)}\right] + k_1\left[\left(\frac{k_2}{k_1}\right)^2 A + B + \frac{k_1 \cdot 2c + 2t}{2(k_1 + k_2)}\right]$$
$$= k_1 2c + t + \beta_t$$

$$k_2\left[A\left(-\frac{k_2}{k_1}\right)^{N-1} + B(-1)^{N-1} + \frac{k_1 \cdot 2c + 2t}{2(k_1 + k_2)}\right]$$
$$+ (k_1 + k_2)\left[A\left(-\frac{k_2}{k_1}\right)^N + \beta(-1)^N + \frac{k_1 \cdot 2c + 2t}{2(k_1 + k_2)}\right] = (c - \beta_c)k_1 + 2t.$$

Solving for A and B, and noting that, for an aircraft engine, N would desirably be even, so as to balance gyroscopic loadings, we have

$$A = \frac{\dfrac{2k_1c + k_1\beta_c + t}{k_1} - \dfrac{k_1 2c + t + \beta_t}{k_2}}{1 + (k_2/k_1)^N(2 + k_2/k_1)}$$

$$B = \frac{t - k_2c - \beta_t(1 + k_2/k_1)}{k_2(1 + k_2/k_1)} - A.$$

These then are the two arbitrary constants to go in Equation (6). We note that Equation (6) gives us ${}_{nc}\phi$ as a function of

n,	the stage position and N the total number of stages,
c, t,	the gas leaving angles from the moving blades,
β_c, β_t,	the gas entry angle to the compressor and turbine,
k_1, k_2, i.e.,	$\dfrac{\bar{M}\bar{v}_a\bar{r}}{k}$ and $\dfrac{\bar{v}_a}{\bar{r}}$.

If we consider a given machine, c, t, β_c, β_t, $\bar{r}$, and k are all fixed. If now we specify $\bar{M}_a$ and $\bar{v}_a$, or what is the same, $\bar{M}$ and $\bar{\rho}$ since $\bar{v}_a = \bar{M}/\bar{\rho}\bar{A}$ Equation (6) is completely defined, and the value of ${}_c\phi$, and thence ${}_t\phi$ can be calculated for each stage of the compressor and turbine. Now for any given value of ${}_cM$ and the density at entry to the compressor, we can calculate ${}_{Nc}v_a$, and since we know ${}_{Nc}\phi$, ${}_{Nc}u$ is known. Once the blade velocity and axial flow velocity are known, the performance of the stage is completely fixed. Thus the conditions leaving the stage can be calculated and the next stage considered, and so on. In short, if we know the four independent variables $\bar{M}$, $\bar{\rho}$, and ${}_cM$, ${}_c\rho$ the performance of the whole unit is completely specified. There is no loss of generality in the simplifying assumptions made and this conclusion applies to any machine of this type.

Reflected Conditions

It is evident that detailed calculations are tedious, and, in any event, the blade angles in any real compressor will not be equal for all compressor stages, so that the mathematical expression for the parameter ${}_{cn}\phi$ becomes most complicated. Fortunately a simpler method can be used for most cases, without introducing errors greater than those implicit in necessarily assumed data concerning blade performance, to which we will refer later.

When ${}_{nc}\beta_1 = {}_{nc}\beta_2$, and ${}_{nt}\beta_1 = {}_{nt}\beta_2$, we have a symmetrical or reflected variation of the absolute velocity of the gas in its passage through the machine, which can be regarded as the condition to be aimed at in design. If we assume that this condition is satisfied, the mathematics simplifies considerably. Physically, we are in effect considering each rotor on its own without influence being exerted on it by its neighbors. The justification for this rather drastic assumption is that it makes the sum practicable, and experience shows that the results given are sufficiently near the truth to be of great use.

Referring to the vector triangles of Fig. 2, it is evident that if we suppose $\beta_1 = \beta_2$ in both cases, then the change of tangential velocity component can be written

$${}_c\Delta v_\omega = 2({}_cu - {}_cv_a \tan {}_c\alpha_2)$$
$${}_t\Delta v_\omega = 2({}_cv_a \tan {}_t\alpha_2 - {}_tu)\,.$$

Expressing the equilibrium of the rotor, we have

$$\begin{aligned} {}_cM.{}_cu.2({}_cu - {}_cv_a \tan {}_c\alpha_2) &= k.{}_tM{}_tu.2({}_tv_a \tan {}_t\alpha_2 - {}_tu) \\ {}_cM.2.{}_c\phi{}_cv_a^2({}_c\phi - \tan {}_c\alpha_2) &= k.{}_tM.2{}_t\phi{}_tv_a^2(\tan {}_t\alpha_2 - t\phi) \\ \frac{\bar{M}\bar{\phi}\bar{v}_a^2}{k}({}_c\phi - \tan {}_c\alpha_2) &= \tan {}_t\alpha_2 - \frac{{}_c\phi}{\bar{\phi}}, \end{aligned}$$

or

$${}_c\phi = \frac{\dfrac{\bar{M}\bar{\phi}\bar{v}_a^2}{k}.\tan {}_c\alpha_2 + \tan {}_t\alpha_2}{\dfrac{\bar{M}\bar{\phi}\bar{v}_a^2}{k} + \dfrac{1}{\bar{\phi}}}.$$

Remembering that $\bar{\phi} = \bar{u}/\bar{v}_a = \bar{r}/\bar{v}_a$

and $\bar{v}_a = \bar{M}/\bar{\rho}\bar{A}$,

we have

$${}_c\phi = \frac{\dfrac{\bar{M}^2\bar{r}}{k\bar{A}\bar{\rho}}.\tan {}_c\alpha_2 + \tan {}_t\alpha_2}{\dfrac{\bar{M}^2\bar{r}}{k\bar{A}\bar{\rho}} + \dfrac{\bar{M}}{\bar{r}\bar{A}\bar{\rho}}}. \tag{9}$$

Hence for any given stage ${}_c\phi$ is a function of $\bar{M}$ and $\bar{\rho}$, just as we found in the more general case, Equation (6). Also since ${}_t\phi = \dfrac{\bar{M}}{\bar{\rho}\bar{A}\bar{r}}\cdot{}_c\phi$

$${}_t\phi = \frac{\tan {}_t\alpha_2 + \dfrac{\bar{M}^2\bar{r}}{\bar{\rho}\bar{A}k}\tan {}_c\alpha_2}{\left(1 + \dfrac{\bar{M}\bar{r}^2}{k}\right)}. \tag{10}$$

Note further that from the vector triangles and the assumed symmetrical conditions

$$\tan {}_c\alpha_1 = 2{}_c\phi - \tan {}_c\alpha_2, \tag{11}$$
$$\tan {}_t\alpha_1 = 2{}_t\phi - \tan {}_t\alpha_2. \tag{12}$$

Finally we define

$${}_t\psi = 2{}_t\phi(\tan {}_t\alpha_2 - {}_t\phi)$$

and

$${}_c\psi = 2{}_c\phi({}_c\phi - \tan {}_c\alpha_2). \tag{13}$$

The stage total temperature rise is ${}_c\Delta T = \dfrac{{}_c\psi\cdot{}_cv_a^2}{{}_ck_p}$, (14)

and the turbine temperature drop is ${}_t\Delta T = \dfrac{{}_t\psi\cdot{}_tv_a^2}{{}_tk_p}$. (15)

Blade Performance

It remains now to consider, in more detail, the aerodynamic performance of the blades. For this we rely on cascade tests (2) in which a pack of blades is mounted in a wind tunnel and the angle α_2, and the losses of pressure are measured for a range of values of α_1 and M_{n1}. Such tests are two dimensional and when the results are applied to predict the flow in an annulus some care is necessary. It is found by experience (3), that the values of the leaving gas angle measured in two dimensional tests may safely be applied to the annular flow, whilst to the two dimensional loss measured must be added due allowance for annulus and wall friction loss, and also an allowance for the fact that the

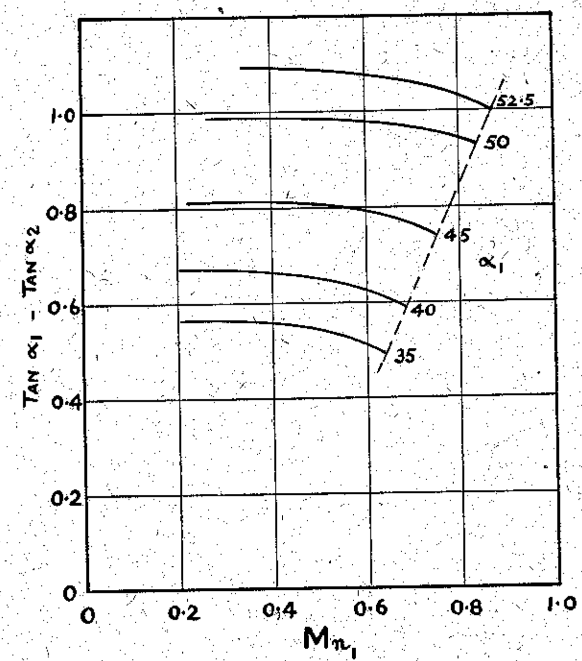

FIG. 3. *Compressor blade performance curves—section Gottingen 436, 50 deg. Camber,—20.5 deg. stagger.*

flow is in fact three dimensional and that there are slight variations in incidence up and down the blade, leading to secondary losses. These may be allowed for and in Fig. 3 we show a typical set of curves, relating the tangent of α_2 to tan α_1, and the Mach number. Fig. 4 shows the stage efficiency as a function of α_1 and Mach number.

A third factor to be considered is the value of the axial velocity. This may be calculated from the mass flow and area and density to give a mean value, but in practice the growth of the boundary layer in an adverse pressure gradient

increases the axial velocity above this value. This may be allowed for if necessary by putting in an empirical factor multiplying the right hand side of Equations (14) and (15), known as a work factor, or it may be taken care of by putting in a factor multiplying the nominal flow areas by a so-called contraction factor. Either, since the factors must be determined from analysis of test results, will yield reasonable results afterwards. The contraction factor is probably sounder physically.

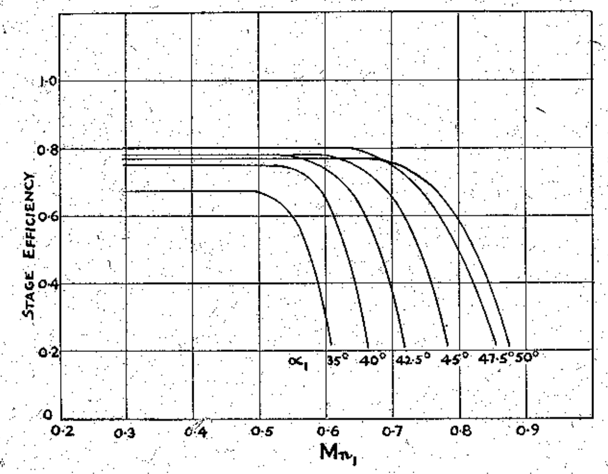

FIG. 4. *Estimated compressor stage efficiencies, assuming annulus and secondary losses to be 150% of the minimum profile drag.*

Performance Integration

We are now in a position to combine Equations (9) to (15) with the data of Figs. 3 and 4 and the corresponding data for the turbine blades to produce a complete performance analysis.

The combination of Equation (11), with Fig. 3, yields a relation between $_c\phi$, tan $_c\alpha_2$ and $_cMn_1$ which is shown on Fig. 5. There is a corresponding relation between $_t\phi$, tan $_t\alpha_2$, and $_tMn_1$, but in this case we do not show on Fig. 6 any variation due to $_tMn_1$, as in turbines of this kind, the turbine Mach numbers are always a long way from the critical values. From Figs. 5 and 6 we can solve Equation (9). For typical values of $\bar{A}$ and $\bar{r}$, and $\bar{M} = 1.0$ this is plotted on Fig. 7 as a curve of $_c\phi$ against $\bar{\rho}$.

This is the key to the solution and there is no need to proceed in any detail further.

For any contraflow compressor of specified geometry we can prepare curves such as Figs. 7 and 8 for each rotor. Then our four independent variables are

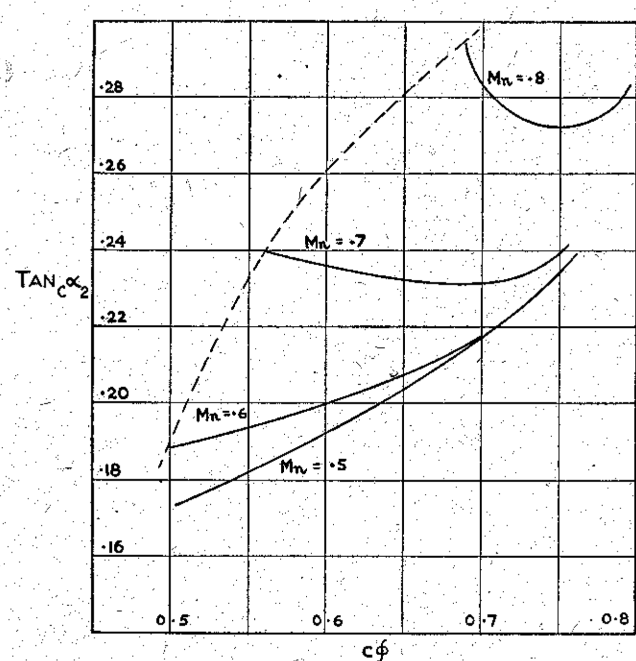

FIG. 5. *Compressor leaving angles as a function of* $_c\phi$ *and* $_cMn_1$

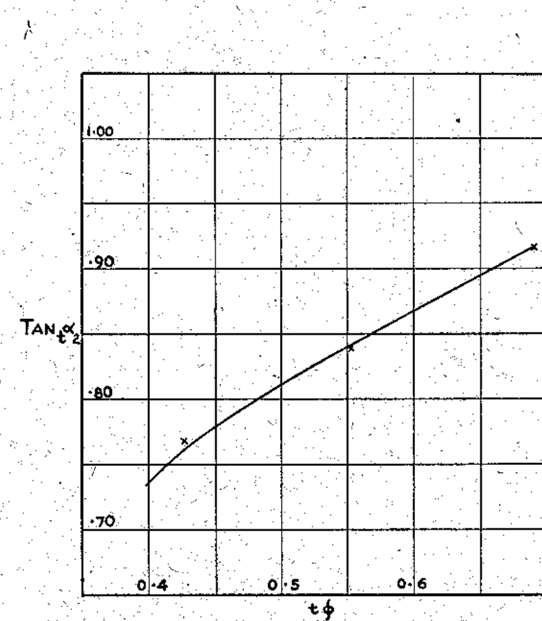

FIG. 6. *Turbine leaving angles as a function of* $_t\phi$.

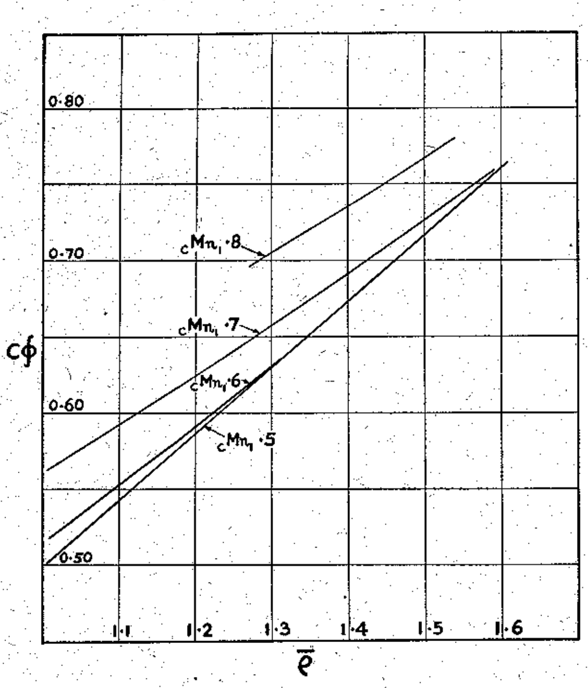

FIG. 7. *Relation between* $_c\phi$, $\bar{\rho}$ *and* $_cMn_1$ *at equilibrium running conditions.*

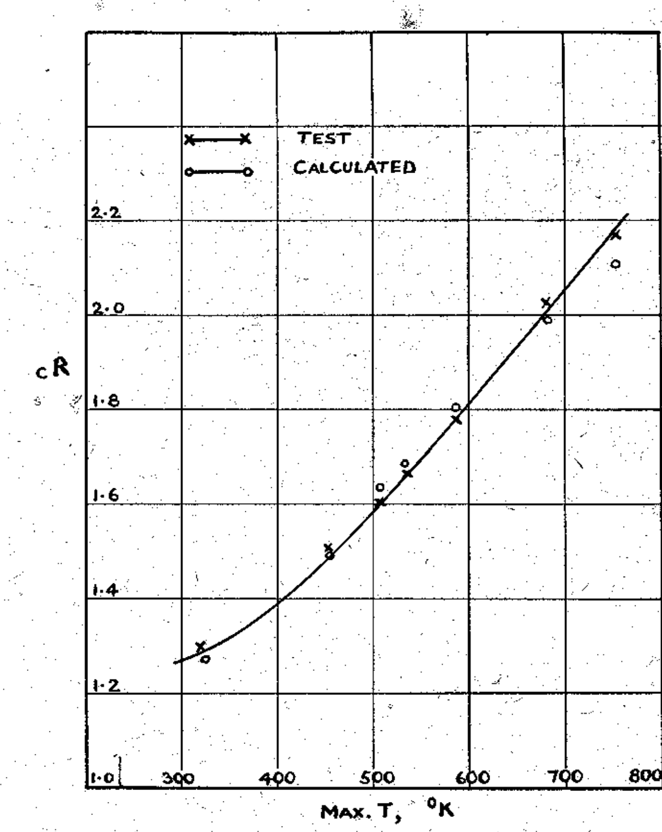

FIG. 8. *Comparison of predicted and measured variation of compression ratio with flame temperature.*

${}_cM$, ${}_c\rho$, $\bar{\rho}$, and $\bar{M}$. The $\bar{\rho}$ is most conveniently expressed at the low pressure end of the unit. Then since $\bar{\rho}$ fixes ${}_c\phi$ and ${}_cM$ and ${}_c\rho$ fix ${}_cv_a$, all the conditions at the L.P. rotor are known, in particular all gas velocities and directions.

The value of ${}_c\psi$ gives us the temperature rise in the compressor stage, and Fig. 4 yields the stage efficiency, permitting the pressure ratio to be calculated. In a similar manner we can work backwards through the turbine stages. Thus we can calculate all the conditions on the far side of the L.P. rotor, which are then the entry conditions to the next stage, through which we can work in turn, and so on, integrating through all the compressor and turbine stages until the H.P. end is reached.

Example

To show the results obtained by the method, Figs. 8 and 9 compare the measured and calculated performance, for an experimental turbocompressor

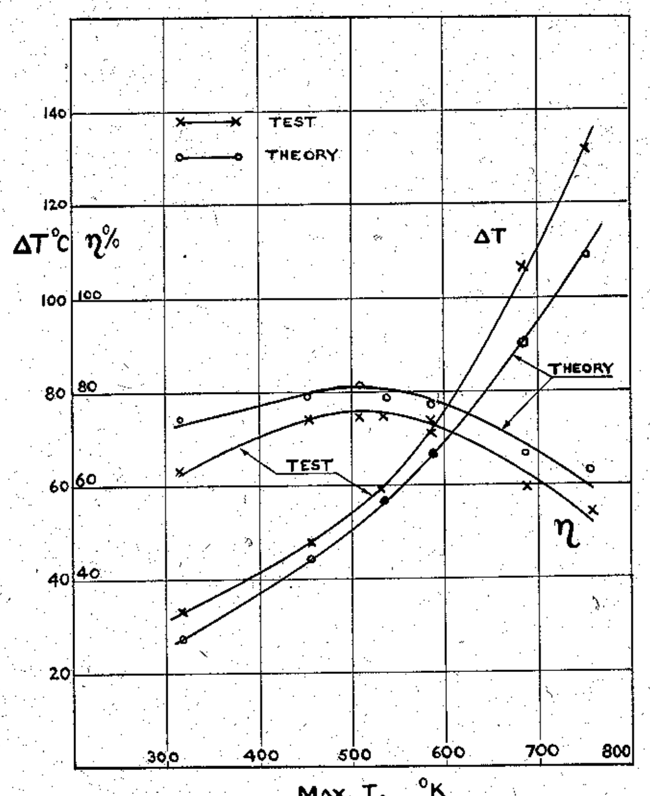

FIG. 9. *Comparison of predicted and measured variation of compression temperature rise and efficiency with flame temperature.*

that was run at a constant value of ${}_cM$, ${}_c\rho$, and $\bar{M} = 1.0$, while $\bar{\rho}$ was varied. $\bar{\rho}$ was varied by varying the turbine entry temperature, and the results are plotted against this variable. It is evident that the compression ratio is in

close agreement. The observed temperature rise was greater than that predicted, which is the reason why the efficiency is low. The explanation of this can be seen by reference to the illustration of Fig. 1. Any leakage between stages from turbine to compressor will raise the apparent temperature rise. Considering this effect, the agreement is regarded as satisfactory. Fig. 10

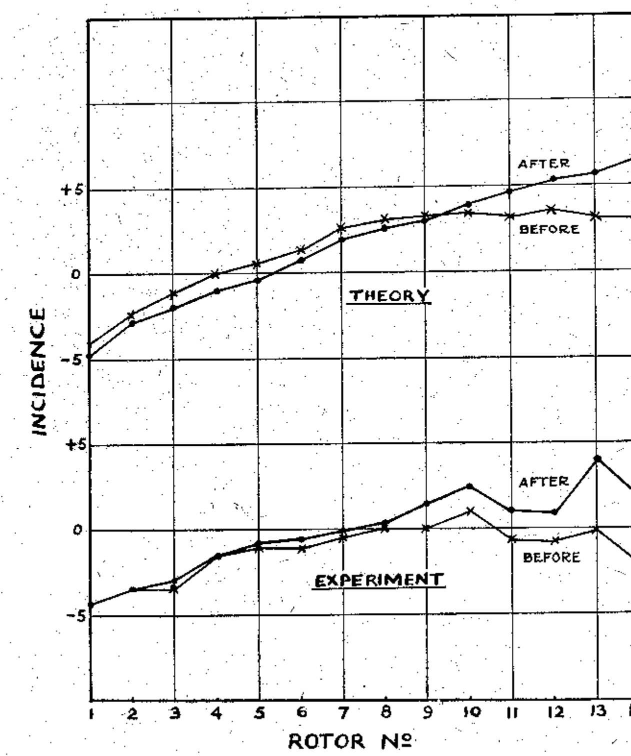

FIG. 10. *Comparison of predicted and measured value of compressor blade incidence.*

shows the calculated and observed values of the incidence on the compressor blades for all the rotors. In this figure two sets of predicted and observed results are shown, as predictions were made of the estimated results following a modification in the turbine blades to alter their angles. Again the agreement is fairly satisfactory. It is interesting to note the peculiarities of the observed results at rotors Nos. 10 to 14. It will be remembered that the theory used neglects any effect of stator angle. In other words it assumes that the stator angle can be always adjusted to suit operating conditions. In fact, of course, a compromise is necessary, and the lower curves show the effect of the error introduced.

Application to Other Types of Unit

The method described applies particularly to a certain type of turbocompressor unit. With suitable modifications it is applicable to any type of compound engine, having one or more compressor stages each of which is independently driven by its own turbine stage. In such cases, it often happens, if there are relatively few stages, that the turbine stages may be operating under "choked" conditions which makes calculation easier, by controlling the mass flow ${}_tM$.

Acknowledgments

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